The profit-maximizing rule

A competitive firm needs to know how much labor and how much capital to employ in order to obtain the maximum profit. In math note 12.2, “The least-cost rule,” we saw that producing a given output at minimum cost requires that the ratio of labor’s marginal product to its price equals the ratio of capital’s marginal product to its price. But is this particular output level the one that maximizes the firm’s profit? We begin by obtaining an expression for the firm’s profit in terms of its input levels.

The firm produces output, Q, by combining labor, L, and capital, K, according to the production function $Q = g(L, K)$. The firm’s cost is the sum of its expenditures on labor, wL, and capital, rK, where w is the price of labor and r is the price of capital. Profit (π) is revenue minus cost, or in symbols: $\pi(L, K) = Pg(L, K) - (wL + rK)$.

Differentiating $\pi(L, K)$ with respect to both L and K, we obtain the first-order conditions for a maximum (we assume the second-order conditions are fulfilled.)

$$\frac{\partial \pi}{\partial L} = P\frac{\partial g}{\partial L} - w = 0, \text{ or } \frac{P\frac{\partial g}{\partial L}}{w} = 1$$

$$\frac{\partial \pi}{\partial K} = P\frac{\partial g}{\partial K} - r = 0, \text{ or } \frac{P\frac{\partial g}{\partial K}}{r} = 1$$

In each expression, we observe that the price of the product times the input’s marginal product, divided by the input’s price, must equal 1. Product price times marginal product is the firm’s marginal revenue product, so this is the condition as stated in the text: maximum profit requires that each input’s marginal revenue product divided by its price must equal one. An equivalent condition states that each input’s marginal revenue product must equal its price.