Chapter 10: Finding Distances to Saturn's Moons

Student Worksheet

Objective: Use Kepler's 3rd Law and observation records to calculate distances to three of Saturn's moons

Engage: Look at the Cassini Spacecraft images below. The three moon pictures in Figures 1, 2, and 3 are the moons you will use in your investigation. Engage by letting your mind wander on the topic of moons and Saturn. What questions or thoughts come to mind? Use the space on the side of the images to record your thoughts.

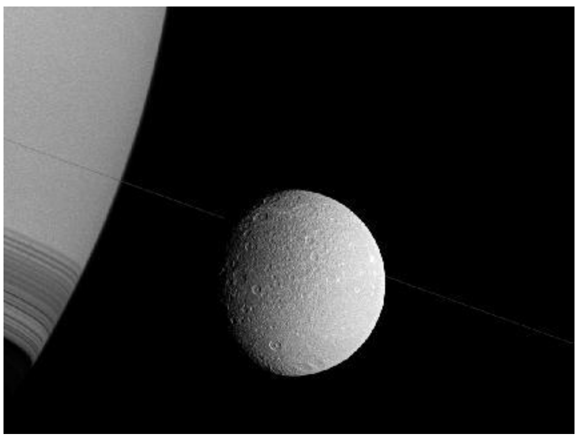

Figure 1 Saturn and Dione; NASA/JPL-Caltech/Space Science Institute

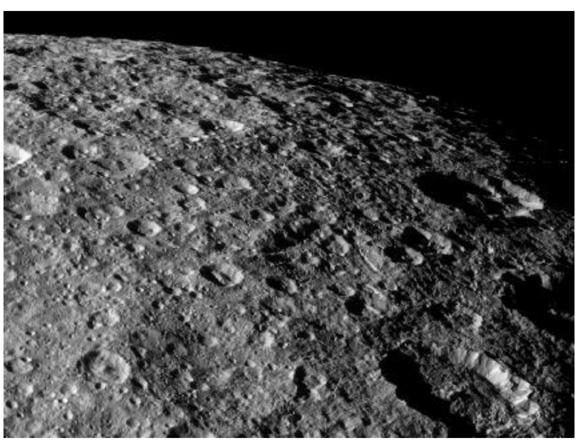

Figure 2 Rhea's surface; NASA/JPL-Caltech/Space Science Institute

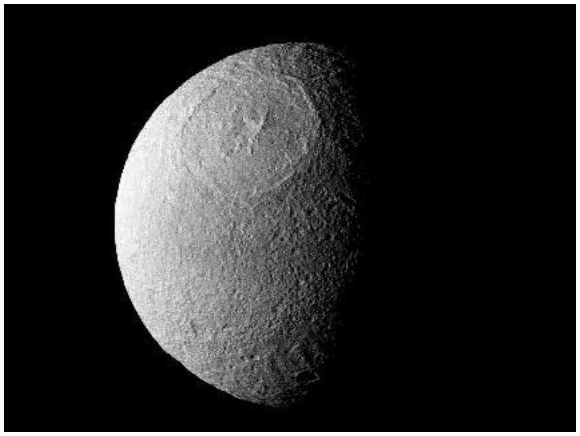

Figure 3 The Odysseus Crater on Tethys; NASA/JPL/Space Science Institute

Introduction:

In the late seventeenth century, Giovanni Cassini studied Saturn. Looking through his telescope, he found moons around the ringed planet, much like Galileo had found moons around Jupiter a few decades earlier. Three of the first moons Cassini found are Tethys, Rhea, and Dione. Today there are 62 known moons of Saturn, 53 have names.

Because Giovanni Cassini discovered so much about Saturn through his meticulous observations, the first spacecraft sent to orbit and study Saturn was named for him. The Cassini Spacecraft was launched from Earth in 1997 and arrived at Saturn in 2004. Since then it has been capturing scientifically rich and visually stunning images of the planet and its many moons in the UV, IR, and visible wavelengths.

Giovanni Cassini observed the moons of Saturn and charted their positions as they changed with time. It is not too hard to measure the period of revolution of a moon through observation – the process is straightforward, though with the telescopes available in the late 1600s it cannot be called an easy task. To measure the period of a moon through observation you time how long it takes for the moon to make an orbit.

Just a few decades before Cassini discovered moons around Saturn, Johannes Kepler published his 3 laws of planetary motion. The third law relates the period of a planet traveling in an elliptical orbit to the semi major axis—i.e, P^2 is proportional to a^3. Kepler knew that the period of an object and the distance from the body were related. For planets orbiting stars, P^2 very nearly equaled a^3. For a planet orbiting a star, the mass of the planet is negligible compared to that of the star, but for moons orbiting planets the case is not the same. Isaac Newton's work with forces and gravity unlocked the constant that held Kepler's 3rd law to be true:

$$P^2 \frac{GM}{4\pi^2} = a^3$$

In this activity, you will use Newton's version of Kepler's 3rd law.

The chart you will use to analyze the periods and semi-major axes of the moons is much like the one Galileo made to study the moons of Jupiter, and much like the one Cassini made to study the moons of Saturn. While it does not reflect actual observation, the periods and distances are true to the moons represented.

In your chart you will see the positions of each moon as observed every 12 hours. The moons are marked with the following symbols:

Dione : ■
Rhea: ✜
Tethys: ⊙

Your Task:

Use a chart of the locations of Dione, Rhea, and Tethys to determine the period of revolution of each moon, and subsequently, the distance from Saturn.

Procedure:

1. On Figure 4 below, start with Dione and draw a smooth line connecting the positions of Dione every 12 hours. You should see a wave-like pattern emerge down the page.

2. Repeat (1) above for both Rhea and then Tethys. Once these are completed, you will have 3 separate, wave-like lines on the chart.

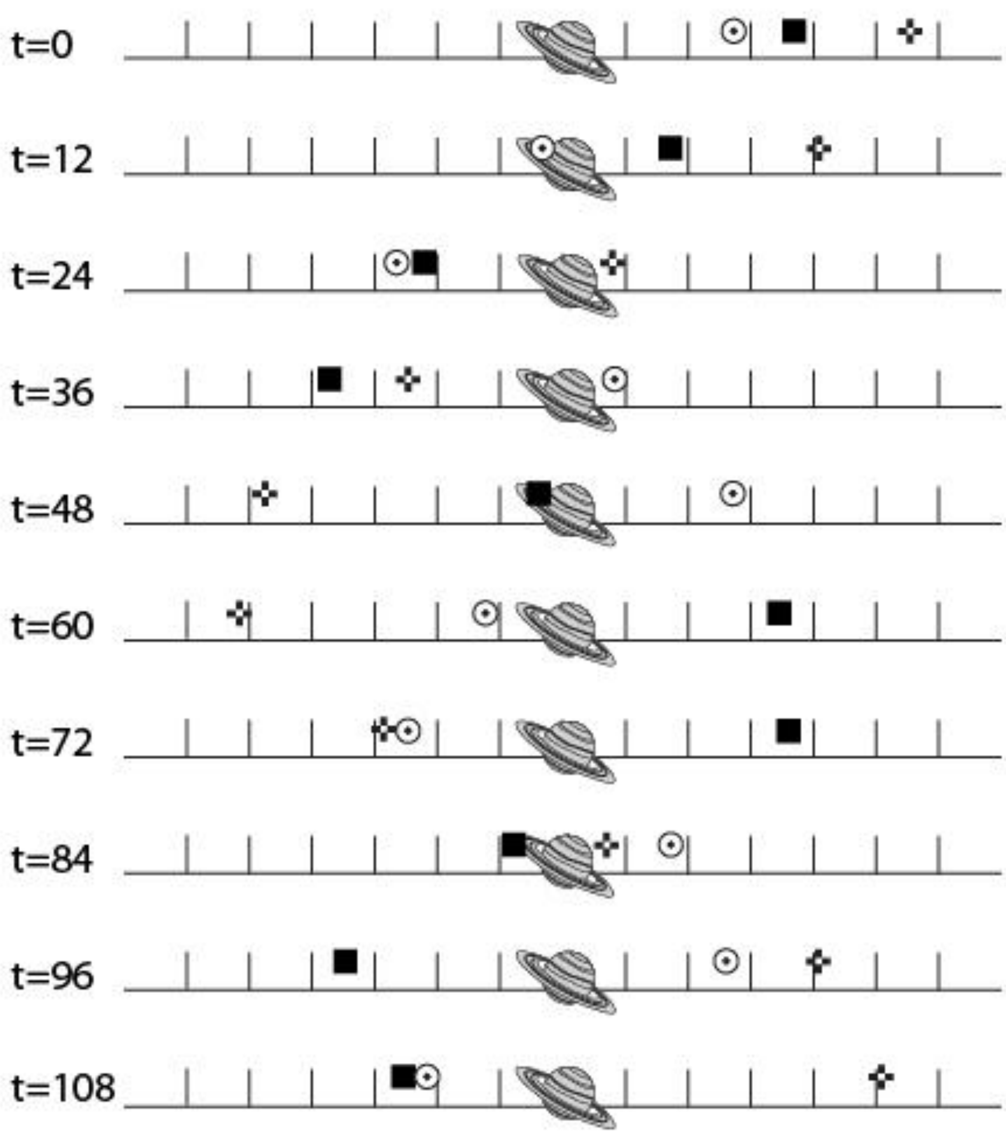

Figure 4 Moon positions charted every 12 hours

3. Use the pattern of each line to calculate the period of each moon. Record your findings in Table 1 below.

Table 1

Moon	Period, P (convert hours to seconds)	Semi-major axis, a (answer will be in meters)
Dione ■		
Rhea ✚		
Tethys ⊙		

4. Now, recall Newton's version of Kepler's 3rd Law:

$$P^2 \frac{GM}{4\pi^2} = a^3 \qquad G = 6.67 \times 10^{-11} \qquad M_{Saturn} = 5.68 \times 10^{26}\ kg$$

Use the period you calculated for each moon in step 3 and Newton's version of Kepler's 3rd Law to determine the approximate distance between Saturn and each moon. Record your findings in Table 1. Remember, your units must be in years and AU's .

Conclusion:

1. In figure 4, each tic mark represents 100,000 km from Saturn. Does this match up with your semi-major axis calculations?

2. How could the accuracy of this exercise be enhanced?

3. If a new moon were discovered with a period of 6.5 days, what would you expect the distance of its semi-major axis to be?

4. If a moon were discovered at a maximum distance from Saturn of 1,278,000 km from Saturn, what might its period be?

5. Earlier in your astronomy course you learned that parallax and the small angle formula are alternate methods for finding the distance from one object to another. If you timed the period of one of Saturn's moons via observation and measured its distance from Saturn via parallax or the small angle formula, you would be able to solve Newton's version of Kepler's 3rd Law for something else….Yes!— for the mass of Saturn! This is how the mass of Saturn was found; Newton's version of Kepler's 3rd law was rearranged to solve for mass. Try it for a different moon of Saturn—one of the strangest, darkest, most cratered, backwards moons, Phoebe. Phoebe orbits with a semi-major axis of 1.29 x 1010m and an orbital period of 550.6 days. Solve Newton's version of Kepler's 3rd Law to see if the mass of Saturn can be calculated from Phoebe. Make sure to convert the period of Phoebe to seconds.

$$M = \frac{4\pi^2 a^3}{GP^2}$$

Extend:

- Why do gas giants have so many moons?
- Learn more about the life and many astronomical contributions of Giovanni Cassini.
- What does the term "Roche limit" mean? How might it relate to Saturn's moons and rings? Calculate the distance of the Roche limit for each of the gas giants.