## Core-Plus Mathematics, Course 1

MA.912.A.2.In.c Identify the mathematical relationship (function) and the type of information represented in a function table or simple graph.

Plant Growth Display the "Plant Growth" table shown below.

| Plant Growth |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Week | 1 | 2 | 3 | 4 |
| Height (cm) | 25 | 30 | 35 | 40 |

Have students describe the relationship shown in the table.
Sample answers: The relationship is positive; the plant grows 5 centimeters each week; $y=5 x+20$, where $x$ represents the number of weeks and $y$ represents the plant height.

MA.912.A.2.In.d Use function tables and simple graphs to determine the mathematical relationship between two numbers representing real-world situations.

Growth of a Bean Sprout Have students plant a bean seed in a disposable cup. Every day, students can collect data about the height of the sprout in centimeters. At the end of two weeks, have students graph their data. Discuss what the graph shows about the growth of the sprout.

## MA.912.A.2.Su.c Identify number patterns and relationships using physical and visual models representing real-world situations.

Pattern Blocks and Addition Patterns Provide pairs of students with sets of pattern blocks, including triangles, squares, and hexagons. Have students find the perimeter of 1 square. Next, students place two squares together, as shown below, and find the perimeter of the newly formed rectangular-shaped figure. Students continue to add squares one at a time and find the perimeter of each new rectangular-shaped figure. Repeat the activity using triangles and hexagons.


Paper Folding and a Multiplication Patterns Provide students with a plain 8.5 " by 11 " sheet of paper. Have students fold the paper in half horizontally, open the paper, and count the number of equal-sized sections. Students repeat the process by refolding the paper horizontally, and then folding the paper vertically. After recording the number of equal-sized sections, the folding activity continues, alternating between horizontal and vertical folds.


Patterns After the completion of each activity, discuss each resulting pattern. Have students describe patterns to their partners and write the pattern rules in their own words. Then show examples of numerical patterns, such as $3,8,13,18$, and ask students to describe each pattern rule.

# MA.912.A.2.Pa.b Compare sets to 10 of objects, pictures, or symbols using one-to-one correspondence and identify which has more or less. 

Connecting Cubes Connect cubes to form two sets of 2 cubes, two sets of 4 cubes, two sets of 6 cubes, two sets of 8 cubes, and two sets of 10 cubes. You may wish to form one of each set using red cubes and one of each set using blue cubes. Place one set of each quantity in an opaque bag, such as a brown lunch bag. Place the remaining sets in a second bag. Have volunteer students randomly select one set from each bag. Students determine which set has less cubes and which set has more cubes. Repeat until all students have had at least one opportunity to select the sets and make comparison statements.

MA.912.A.3.In.a Solve equations with one unknown (variable) involving addition,
multiplication, subtraction, and division of whole numbers representing problems in
real-world situations.
Whole Numbers When equations for solving real-world problems involve fractions or decimals, change the numbers in the problem so that corresponding equations have whole number constants, coefficients, and solutions. This will allow students to focus on the process and not become frustrated with more challenging computations.

## MA.912.A.3.In.b Use the commutative, associative, and equality properties of addition as strategies to solve equations involving real-world situations.

Properties Once students have written an equation that represents a real-world problem, encourage them to use commutative and associative properties whenever possible to simplify left- and right-sides, as well as when combining like terms. For example, the equation $12 x+16 x+8 x+14 x-20=130$ can be rewritten as shown below.

$$
\begin{array}{ll}
12 x+16 x+8 x+14 x-20=130 & \\
12 x+8 x+16 x+14 x-20=130 & \text { commutative property } \\
(12 x+8 x)+(16 x+14 x)-20=130 & \text { associative property }
\end{array}
$$

Students can combine like terms to obtain the equation $50 x-20=130$. Then they can use the equality properties to determine that $x$ equals 3 .

## MA.912.A.3.In.c Use the commutative and associative properties of multiplication and the properties of one and zero for multiplication as strategies to solve equations involving real-world situations.

Properties Once students have written an equation that represents a real-world problem, encourage them to use commutative and associative properties, as well as other multiplication properties, whenever possible to simplify left- and right-sides and combine like terms. For example, the equation $(2 x)(8)(5)=240$ can be rewritten as shown below.

$$
\begin{array}{ll}
(2 x)(8)(5)=240 & \\
(2 x)(5)(8)=240 & \text { commutative property } \\
(2 x \cdot 5)(8)=240 & \text { associative property }
\end{array}
$$

Students can multiply 2 by 5 and then 10 by 8 to obtain the equation $80 x=240$. Then they can divide by 80 to determine that $x$ equals 3 .

## MA.912.A.3.In.d Solve equations involving common literal formulas related to real-world situations.

Formulas Before asking students to solve literal formulas, have them solve a number of one-variable equations based on the applicable formula. For example, substitute values for $r$, the rate, and $d$, the distance, as a means of leading students to solve the formula $r \cdot t=d$ for $t$, the time. Have students describe what step is being done to solve for $t$ in each equation.

| $25 t=50$ | $30 t=60$ | $45 t=90$ | $50 t=100$ | $60 t=120$ |
| :--- | :--- | :--- | :--- | :--- |
| $25 t=75$ | $30 t=90$ | $45 t=135$ | $50 t=150$ | $60 t=180$ |
| $25 t=100$ | $30 t=120$ | $45 t=180$ | $50 t=200$ | $60 t=240$ |

Repeat until students see the pattern and are able to work with the more abstract literal formula.

## MA.912.A.3.In.e Solve real-world equations and inequalities with one unknown (variable) using visual models to represent the procedure.

Algebra Tiles Have students use algebra tiles to model and solve equations, such as $x-5=-4$.

Step 1: Model $x-5=-4$ using tiles.

Step 2: To isolate the $x$-tile, add 5 positive tiles to the left mat to make 5 zero pairs. Add 5 positive tiles to the right mat.

Step 3: There are 5 zero pairs on the left mat and 4 zero pairs on the right mat. Remove the zero pairs.


Step 4: Count the tile(s) on the right mat. So, $x=1$.

## MA.912.A.3.In.f Create function tables and simple graphs that show the mathematical relationship between number pairs.

Multiple Representations Place students in pairs. Give each pair of students two equations. One student creates a function table for the first equation while the other student creates the graph. Students compare representations and check each other's work. Then students switch roles to create a table and graph for the second equation. Have students present their representations.

## MA.912.A.3.In.g Use function tables and simple graphs representing equations to make predictions for real-world situations.

Make Predictions Students perform a simple activity, such as making X's with their dominant hand, for $5-, 10-$ - $15-$, 20-, and $25-$ second intervals, record their results as ordered pairs, and graph their data. Assuming that they would continue the activity at the same rate, students predict the outcome if they performed the activity for 30 seconds.

As an extension, informally introduce the concepts of interpolation and extrapolation by asking students to predict the outcomes at 18 seconds and 60 seconds. Then have students perform the activity for each of these intervals to test the reliability of their predictions. Finally, lead students in a discussion of factors that may have resulted in inaccurate predictions, especially when extrapolating to 60 seconds. In the case of making X's, students may not have been able to sustain their initial rate of X-making for the entire 60 seconds.

## MA.912.A.3.Su.a Solve number sentences (equations) involving addition and subtraction of one-digit and two-digit whole numbers based on real-world situations using visual models.

Connecting Cubes Allow students to use connecting cubes to solve real-world problems involving addition and subtraction, such as the one shown below.

MONEY Teresa saved money to spend at her school's May Day festival. She saved $\$ 6$ in January, $\$ 4$ in February, $\$ 7$ in March, and $\$ 5$ in April. She spent $\$ 20$ at the festival. How much money did she have left? \$2

To solve the problem, students can use one connecting cube to represent each dollar saved and add 6 cubes, 4 cubes, 7 cubes, and 5 cubes. You may wish for students to connect cubes in groups of 10. Then students will remove 20 cubes to determine that Teresa had $\$ 2$ left.

Have students present their solutions to the class. Then guide students through the process of writing an equation that represents the real-world situation. Repeat with other real-world problems.

MA.912.A.3.Su.b Use the commutative property and the additive identity property of addition as a strategy to solve number sentences (equations).

Commutative Property Have students use the commutative property to obtain friendly numbers when solving equations, such as $9+13+11=x$.
$19+13+1=x$
$19+1+13=x$
$20+13=x$
So, $x=33$.

MA.912.A.3.Su.C Use the concepts of equality and inequality as strategies to solve problems involving real-world situations.

Problem-Solving Work Mats, Counters, and Equality Create and make photocopies of a problem-solving work mat from an 8.5 " by 11 " sheet of paper.

Provide two-color counters and a work mat to each student. Students place counters, with the red-side facing up, to model real-world situations, such as the following School Supply problem. Then they add counters, with the yellow-side facing up, to either the left or right side of the mat in order to create an equal number of counters on both left and right sides. Students share their solutions with a partner and/or present their solutions to the class.

SCHOOL SUPPLIES Mr. Lopez is buying school supplies for his class. He wants each of his 16 students to have a ruler. He currently has 11 rulers. How many rulers does he need to buy? 5 rulers

| Left Side: Students | Right Side: Rulers |
| :---: | :---: |
| 00000 | 00000 |
| 00000 | 00000 |
| 00000 | 0 |
| 0 |  |

## MA.912.A.3.Su.d Solve equations involving addition and subtraction using visual models, such as a number line, in real-world situations.

Use Number Lines and Manipulatives Have students use a number line and counters to solve real-world problems, such as the following "Charity Food Drive" problem.

CHARITY FOOD DRIVE Rafael wants to donate 10 cans of food to the canned good drive. He currently has 6 cans of food. How many more cans of food does Rafael need?
$x+6=10$ models the situation. Students can place one counter on 6 , place a second counter on 10 , and count the number of spaces between the counters.

## MA.912.A.3.Su.e Identify the mathematical relationship between number pairs in

 function tables, such as +2 or -3 .Function Tables Present students with two sequences of numbers, such as 1, 3, 5, 7, 9 and 30, 27, 24, 21, 18. Have students identify the change between numbers in each sequence. Then present the same values in tables, such as the ones shown below, and have students identify the change between numbers.

| 1 | 3 | 5 | 7 | 9 |
| :--- | :--- | :--- | :--- | :--- |


| 30 | 27 | 24 | 21 | 18 |
| :--- | :--- | :--- | :--- | :--- |

Next, present students with the same values in a function table, such as the one shown below. Have students identify the changes in both number sequences.

| $x$ | 1 | 3 | 5 | 7 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 30 | 27 | 24 | 21 | 18 |

Repeat the process with other pairs of sequences until students can work directly with a function table.

## MA.912.A.3.Su.f Use function tables and simple graphs representing equations to make predictions for real-world situations.

Students can make a function table to solve real-world problems, such as the following "Earnings" problem.

Earnings Teresa earns $\$ 7$ an hour as a customer service representative. The equation $y=7 x$ represents her earnings, where $x$ is the amount of hours she works. How much money does Teresa earn for a 10-hour workday? $\$ 70$

Have students predict the amount earned for 10 hours of work based on calculations for $6,7,8$, and 9 hours of work before substituting 10 for $x$. You may wish to have students confirm their answer by plotting and connecting the first four ordered pairs and extending the line to determine the $y$-value when $x$ equals 10 .

| $\boldsymbol{x}$ | $\boldsymbol{y}=\mathbf{7 x}$ | $(\boldsymbol{x}, \boldsymbol{y})$ |
| :--- | :--- | :--- |
| 6 | $7(6)=42$ | $(6,42)$ |
| 7 | $7(7)=49$ | $(7,49)$ |
| 8 | $7(8)=56$ | $(8,56)$ |
| 9 | $7(9)=63$ | $(9,63)$ |
| 10 | $7(10)=70$ | $(10,70)$ |

## MA.912.A.3.Pa.a Identify quantities to 9 or more and add 1 more in real-world situations.

Manipulatives Allow students to use manipulatives to solve real-world problems, such as the one shown below.

SCHOOL SUPPLIES Elena is buying notebooks for school. She selects 8 notebooks and then adds 1 more. How many notebooks does Elena have altogether? 9 notebooks

Have students present their solutions to the class.

## MA.912.A.3.Pa.b Identify quantities to 10 or more and take 1 away in real-world situations.

Manipulatives Allow students to use manipulatives to solve real-world problems, such as the one shown below.

PETS Mr. Numkena manages a pet store. On Monday, he had a litter of 7 puppies available for adoption. One puppy was adopted on Monday evening. How many puppies were available for adoption on Tuesday? 6 puppies

Have students present their solutions to the class.

## MA.912.A.3.Pa.c Identify quantities to 10 as equal or unequal.

Connecting Cubes Connect cubes to form two sets of 2 cubes, two sets of 4 cubes, two sets of 6 cubes, two sets of 8 cubes, and two sets of 10 cubes. You may wish to form one of each set using red cubes and one of each set using blue cubes. Place one set of each quantity in an opaque bag, such as a brown lunch bag. Place the remaining sets in a second bag. Have volunteer students randomly select one set from each bag. Students determine whether selected sets represent equal or unequal quantities. Repeat until all students have had at least one opportunity to select the sets and make comparison statements.

## MA.912.A.3.Pa.d Sort sets of objects to $\mathbf{1 0}$ into groups by quantity.

Manipulatives Give each student 10 connecting cubes. Prompt students to count out various numbers of cubes and connect them into groups of specified quantities. Possibilities include: 4 cubes in groups of 2 , 6 cubes in groups of 2,6 cubes in groups of 3, 8 cubes in groups of 2 , 8 cubes in groups of 4,9 cubes in groups of 3,10 cubes in groups of 2, and 10 cubes in groups of 5 .

## MA.912.A.3.Pa.e Count objects, pictures, or symbols used in a pictograph or chart and identify which category has the largest quantity.

Sticky Notes Give each student one small sticky note for each letter in his/her first name. For example, students named Ricky and Carla receive five sticky notes. Draw a number line on the board and have students stack their sticky notes vertically from the number line, forming a bar graph-like structure, as shown below.


Have students count the number of sticky notes used for each name and determine whose name has the greatest number of sticky notes.

The activity can be extended by having students draw pictographs or bar graphs of the class sticky note structure.

## MA.912.A.4.In.a Simplify expressions with one unknown (variable) by identifying like terms.

Color Coding Students use colored pencils or markers to underline like terms. For example, before simplifying the expression $10 x+3 y-6 x+8 y+5$, students can underline like terms as shown below.

$$
\underline{10 x}+\underline{3 y}-\underline{6 x}+\underline{8 y}+\underline{5}
$$

Students then simplify the expression by adding/subtracting the coefficients of the terms underlined in the same color. The expression $10 x+3 y-6 x+8 y+5$ simplifies to $4 x+11 y+5$.

## MA.912.A.4.In.b Solve equations with one unknown (variable) involving addition and subtraction, and multiplication.

One-Variable Equations Working with a partner, have students solve one-step equations, such as the ones shown below.

$$
x+10=17 \quad y-10=5 \quad 4 z=28
$$

Then have students write equations for their partners to solve. As students progress, equations with fraction/decimal solutions can be included.

Once one-step equations have been mastered, repeat the process for two-step equations, such as $5 x+1=21$.

## MA.912.A.4.In.c Combine like and unlike terms in number sentences representing real-world situations.

Combine Like Terms Equations can be simplified before solving by combining like terms on the left and right sides of equations. Have students use colored pencils or markers to underline like terms before solving equations representing real-world situations.

For example, before solving the equation $10 x+3-6 x=11$, students can underline the like terms, as shown below.

$$
\underline{10 x}+3-\underline{6 x}=11
$$

After students combine like terms, they can use the equality properties to solve for the unknown. In the previous example, students would combine like terms to obtain $4 x+3=11$. They would then subtract 3 from both sides, and divide both sides by 4 to find that $x$ has a value of 2 .

## MA.912.A.4.In.d Identify factors of expressions with whole numbers by dividing.

Binomials and Trinomials Have students use a table, like the one shown below, to identify greatest common factors when factoring binomial expressions. Structure the table so that students supply more information as they gain fluency with the process, as shown in the first and second rows, the third and fourth rows, and the fifth row. Allow students to use a calculator when finding the greatest common factor (GCF). Continue to add binomials to the table until students have mastered working with binomials.

| Binomial | GCF | Factored <br> Binomial |
| :---: | :---: | :---: |
| $2 x+8$ | 2 | $\underline{2}(x+4)$ |
| $3 y-6$ | 3 | $\underline{3}(y-2)$ |
| $4 z+20$ | 4 | $\underline{4(z+5)}$ |
| $7 d-49$ | 7 | $\underline{7(d-7}(d)$ |
| $10 m+80$ | 10 | $\underline{10}(\underline{m}+\underline{8})$ |

Once students have mastered factoring out the greatest common factor for binomials, use a similar table to factor out greatest common factors for trinomials.

## MA.912.A.4.Su.a Solve number sentences (equations) with one unknown involving addition and subtraction facts using physical and visual models.

Algebra Tiles Have students use algebra tiles to model and solve addition and subtraction one-step equations, such as $x+4=9$.


Step 1: Model $x+4=9$ using tiles.
Step 2: To isolate the $x$-tile, remove 4 tiles from both sides of the mat.

Step 3: Count the remaining tiles on the right side of the mat to determine that $x=5$.

## MA.912.A.4.Su.b Identify like and unlike terms in number sentences representing real-world situations.

Color Coding Have students use colored pencils or markers to underline like terms, using different colors for unlike terms. For example, before solving $7 x+10 x+8=42$, students can underline like terms as shown below.

$$
\underline{7 x}+\underline{10 x}+\underline{8}=\underline{42}
$$

Students then isolate the variable by combining like terms on the left and right sides of the equation and apply the equality properties. $17 x+8=42 ; 17 x=34 ; x=2$

## MA.912.A.4.Su.C Identify factors of whole numbers by using division facts.

Factor Challenge Game Divide an 8.5 " by 11 " sheet of paper into 16 equal-sized sections. Write one of the following numbers in each section.

$$
12,16,18,20,24,36,40,42,48,50,54,60,66,70,80,100
$$

Photocopy and cut the paper to create sets of cards. Arrange students in groups of 3 or 4 . Give a set of cards and a calculator to each group of students.

To start play, students "shuffle" the cards and place them face down in the middle of the group. In each round of the game, one student per group serves as the referee. The referee is responsible for selecting that round's card and, as needed, verifying student responses. The referee may use a calculator to verify answers.

Competing students write down as many whole number factors as they can within a designated period of time. Points are awarded to all competing students for all correct factor pairs. Consider the number 24 . A student who writes $2 \times 12$ and $3 \times 8$ receives two points and a student who writes $2 \times 12,4 \times 6$ and $3 \times 8$ receives 3 points.

Students take turns serving as the referee. The game continues until time expires or all numbers have been factored. The winner is the student with the greatest number of total points.

## MA.912.A.4.Pa.a Identify a missing item from two or more sets.

Pattern Blocks Provide students with identical sets of pattern blocks. For example, each student receives 3 triangles, 2 squares, 1 trapezoid, 2 parallelograms, and 1 hexagon. Have students work with a partner. One student in each pair removes an item from his/her set and displays the remaining pattern blocks. The other student identifies the missing item. Next, the students switch roles. The activity can be repeated with students working in groups of three or four, with one student identifying missing items from group members' sets.

## MA.912.A.4.Pa.b Recognize that joining sets of objects results in a larger quantity and separating sets of objects results in a smaller quantity.

Flashcards Model the process of joining (adding objects) and separating (subtracting or dividing objects) sets of objects, making the connection with larger and smaller quantities. Next, provide students "smaller" and "larger" cards. Show students flashcards with statements such as, "add 4 objects", "join two sets", and "divide into 3 groups". Have students indicate whether the resulting set will be "smaller" or "larger" by holding up the appropriate card.

MA.912.A.4.Pa.C Separate groups of objects to 10 into sets with the same quantity.

Manipulatives Provide each student with 10 counters or cubes and an 8.5 " by 11 " sheet of paper divided in fourths as shown below.

Have students separate 6 counters/cubes into the following equal groups and draw an illustration of each grouping in one of the sections.

- One group of six
- Two groups of three
- Three groups of two
- Six groups of one


Repeat with 8, 9, and 10 counters/cubes.

## MA.912.A.6.In.a Identify perfect squares and their factors, including 1, 4, 9, 16, $25,49,64,100$, and 144 using visual models.

Grid Paper Rectangles and Squares Provide students with rectangles and squares cut from grid paper.

Possible rectangle dimensions: 1 unit by 4 units, 1 unit by 16 units, 2 units by 8 units, 4 units by 16 units, 5 units by 20 units, and 9 units by 16 units.

Possible square dimensions: 1 unit by 1 unit, 2 units by 2 units, 3 units by 3 units, 4 units by 4 units, 5 units by 5 units, 7 units by 7 units, 8 units by 8 units, 10 units by 10 units, and 12 units by 12 units

Have students identify the squares, count and record the number of area units to create a list of perfect squares, and record the side lengths that correspond with each square.

## MA.912.A.6.In.b Use factors of perfect squares to solve problems in real-world situations.

Graph Paper Students can use graph paper to draw diagrams of real-world problems involving perfect squares, such as the following problem.

AREA Mr. Johnson wants to plant a square-shaped vegetable garden. He has enough seeds to cover a 25 -square foot area. What will be the dimensions of Mr. Johnson's garden? 5 feet by 5 feet

MA.912.A.6.Su.a Use physical models of perfect squares, including 1, 4, 9, 16, 25 , and 100 to solve problems.

Manipulatives and Grid Paper Allow students to use manipulatives, such as square tiles or cubes, and/or to make sketches when solving real-world problems involving perfect squares, such as the following "Area" problem

AREA Antonio made a bulletin board as a gift for his sister. The bulletin board was square-shaped with side lengths measuring 3 feet. What was the display area of the bulletin board? 9 square feet

## MA.912.A.6.Pa.a Use one-to-one correspondence to identify equal sets of objects to solve problems.

Match Manipulatives Allow students to use manipulatives to solve problems involving one-to-one correspondence, such as the one below.

BOOKS As part of a summer reading project, Ben, Yolanda, Gomda, and Rosalinda recorded the number of books they read during the months of June, July, and August. Which pairs of students read the same number of books? Ben and Rosalinda, Yolanda and Gomda

| Month | Books Read by Ben | Books Read by Yolanda | Books Read by Gomda | Books Read by Rosalinda |
| :---: | :---: | :---: | :---: | :---: |
| June | 2 | 2 | 3 | 2 |
| July | 4 | 3 | 2 | 3 |
| August | 1 | 1 | 1 | 2 |

For students to solve this problem using manipulatives, provide students with 4 sets of 7 or more manipulatives. Each set of manipulatives should be a different color. For example, students receive sets of blue cubes, yellow cubes, green cubes, and red cubes. Students select a color to represent each person's read books and count out the total number of cubes that represent each total number of books. Students use one-to-one correspondence, comparing sets of cubes, to determine which pairs of students read the same number of books.

MA.912.A.10.In.a Use a variety of problem-solving strategies, such as finding key information to determine the correct operation and using graphic representations for numbers, to solve real-world problems.

Problem-Solving Strategies Have students use the four-step problem solving plan when solving real-world problems.

Step 1: Students understand key words and identify unnecessary information.

Step 2: Students plan to solve the problem using a problem-solving strategy. Strategies include:

Act it out.
Solve a simpler problem.
Draw a diagram.
Make a model.
Make a table.
Use a graph.
Work backward.
Use an equation or formula.
Step 3: Students solve the problem using the selected strategy.
Step 4: Students check the solution using a different strategy.

## MA.912.A.10.In.b Use estimation strategies, such as rounding, grouping, and comparing, to determine if answers are reasonable.

Estimation Work with students to estimate using several different strategies. Teach them how to round fractions to $0, \frac{1}{2}$, or 1 , how to round whole numbers to the largest place value, and how to round decimals to the nearest whole number. Provide students with real-world problems, such as the one below. Encourage them to determine if their answer is reasonable by estimating the answer and comparing it to their exact answer.

GUITAR PRACTICE Maria practiced her guitar for $\frac{1}{4}$ hour on Monday, $\frac{5}{8}$ hour on Tuesday, and $\frac{8}{12}$ hour on
Wednesday. So far this week, how much time has Maria spent practicing?

Exact answer: $\frac{1}{4}+\frac{5}{8}+\frac{8}{12}=\frac{37}{24}=1 \frac{13}{24}$
Estimated answer: $0+\frac{1}{2}+1=1 \frac{1}{2}$
The exact answer is the very close to the estimate.
The exact answer is reasonable.

## MA.912.A.10.Su.a Use visual and physical models as strategies for solving real-world mathematical problems.

Bar Diagrams Have students use bar diagrams when solving real-world problems, as shown in the following example.

HOBBIES Joshua and Rosita collect trading cards. Joshua has three times as many trading cards as Rosita. They have 60 cards altogether. How many cards does Joshua have? 45 cards


## MA.912.A.10.Su.b Use a variety of problem-solving strategies, such as finding key information to determine the correct operation and using graphic representations for numbers, to solve real-world problems.

Problem-Solving Strategies Many times students want to rush through real-world problem-solving exercises. Encourage students to follow the four-step problem-solving strategy, and emphasize the importance of checking their answers.

Step 1: Students understand the information provided in the exercise, as well as the information needed to supply the correct response.

Step 2: Students plan to solve the problem using a problem-solving strategy. Strategies include:

Act it out.
Solve a simpler problem.
Draw a diagram.
Make a model.
Make a table.
Use a graph.
Work backward.
Use an equation or formula.
Step 3: Students solve the problem.
Step 4: Students check their answers.
Technology provides a valuable resource that students can use to check their answers. Many times students understand the problem and are able to plan their strategy. However, mathematical errors can lead students to provide incorrect answers. Allowing students to check their calculations, using technology such as calculators, can empower them to find their errors and build confidence in their ability to solve word problems.

## MA.912.A.10.Pa.a Solve real-world problems involving quantities to $\mathbf{1 0}$, matching the result to the correct answer to determine accuracy.

Classroom Routines Establish a procedure for solving real-world problems using play money. Provide opportunities for students to "earn" the play money for "a job well-done." Give them one dollar at a time for following established procedures, such as turning in their work on time, following classroom rules, or showing respect to other students. Once students have earned ten dollars, allow them to trade the money for a privilege, such as being the class leader for a week. Ask students to use a money chart to match their "dollars" to the number of dollars required to earn the classroom privilege.

Adapt this idea in accordance with classroom and school requirements in order to meet the needs of the students.

## MA.912.G.1.In.a Find the length and midpoint of line segments in real-world situations.

Materials: picture frames, rulers, maps
Real-Life Measures Students can work alone or in pairs. Ask students to find the length of one side of an $8 \times 10$ picture frame. Next, have them find the midpoint along the "top" of the picture frame in order to determine the place to set the hanger. Ask: How would your answer change if you turned the frame?

Repeat this activity using a variety of real-world examples. Another option is to ask students to plan a long-distance trip and decide to stop at a city at the midpoint. For example, the distance between Dayton, OH and Sarasota, FL is about 1,000 miles. How far should they travel before stopping at the midpoint? Provide students with maps and rulers and encourage them to calculate the distance using the map's key.

## MA.912.G.1.In.b Locate angles formed when a line intersects two parallel lines and classify the angles as obtuse, acute, or right angles.

Using Grid Paper Emphasize to students that grid paper is a good manipulative for identifying angles as acute, obtuse, or right. Show students how to align one corner of a grid box with the vertex of the angle, and how to align one ray of the angle with a line of the grid (which includes the vertex). Students can use the grid boxes to determine if the angle is less than $90^{\circ}$ (acute), equal to $90^{\circ}$ (right), or greater than $90^{\circ}$ (obtuse).

Next, have students draw parallel lines on the grid paper and then draw one line that intersects both of the parallel lines (called a transversal). Students should locate 8 separate angles. Students will then be able to determine if the created angles are acute, right, or obtuse. Ask students to compare the angles and note any similarities between the angles.

MA.912.G.1.In.c Locate and identify points on coordinate planes, such as line graphs or maps, using ordered pairs of numbers.

Maps Locate and identify locations on a city or state map using ordered pairs. Discuss the purpose of using the same order of letters and numbers to give directions and site locations. As an extension, have students create a map of the classroom or school, supplying the ordered pairs of specific objects.

## MA.912.G.1.Su.a Determine the midpoint of a line.

Midpoint Provide each pair of students with a six-inch paper ruler. Explain to students that a ruler can be seen as a line segment. Ask them to identify the number in the middle of the ruler and explain that the point in the middle of the line segment is called the midpoint.

Ask students to explain why 3 is a good guess for the midpoint. Students should recognize that $6 \div 2=3$. Students can check their answers by folding the ruler in half. (Be certain that the edges of the ruler are marked as zero and six inches, as some rulers start the zero mark about an eighth-inch away from the edge.)

This activity can be repeated using other common "linear" objects, such as plastic straws, pipe cleaners, and ribbon. First, ask students to measure the length of the object. Then, instruct them to determine the midpoint by using division before folding the object into two equal parts.

## MA.912.G.1.Su.b Differentiate between intersecting and parallel lines.

Real-World Examples Provide students with a variety of real-world examples that show parallel and intersecting lines. Street maps of downtown areas often show parallel and intersecting streets in a grid pattern. Other examples, such as railroad tracks, bookshelves, and architectural drawings can also be used.

## MA.912.G.1.Su.C Match types of angles, such as obtuse, acute, and right angles, using physical models and drawings.

Using Grid Paper Emphasize to students that grid paper is a good manipulative for identifying angles as acute, obtuse, or right. Show students how to align one corner of a grid box with the vertex of the angle, and how to align one ray of the angle with a line of the grid (which includes the vertex). Students can use the grid boxes to determine if the angle is less than $90^{\circ}$ (acute), equal to $90^{\circ}$ (right), or greater than $90^{\circ}$ (obtuse). Choose grid paper with a lighter weight so that the angles can be seen through the grid paper.

## MA.912.G.1.Su.d Locate specified points on a coordinate plane, such as a simple map represented on a grid.

Manipulatives Use grid paper to practice naming and plotting points on coordinate planes. Students can use counters to graph points. As an extension, have students create a geometric shape, such as a rectangle, and then supply the ordered pairs of the shape's vertices.

## MA.912.G.1.Pa.a Recognize the ends and middle of a line.

Midpoint and Ends Provide each pair of students with a six-inch paper ruler. Explain to students that a ruler can be seen as a line segment. Ask them to identify the numbers at each end and the number in the middle of the ruler. Explain that the point in the middle of the line segment is called the midpoint.

Ask students to explain why 3 is a good guess for the midpoint. Students should recognize that $6 \div 2=3$. Students can check their answers by folding the ruler in half. (Be certain that the edges of the ruler are marked as zero and 6 inches, as some rulers start and end about an eighth-inch away from the edge.)

This activity can be repeated using other common "linear" objects, such as plastic straws, pipe cleaners, and ribbon. First, ask students to measure the length of the object by identifying the ends of the "line." Then, instruct them to determine the midpoint by using division before folding the object into two equal parts.

## MA.912.G.1.Pa.b Recognize angles in two-dimensional shapes.

Angles Draw Diagram A on the board.

Diagram A


Diagram B


Ask students to identify the two-dimensional figure that is created when these lines intersect. A square is created.

Next, indicate the vertices of the square, or the points at which the lines intersect. Students may also find it helpful to label the vertices as $A, B, C$, and $D$. Erase the arrow heads to leave only the four line segments that created the square. Explain how to identify the four angles of the square, using the vertices, by placing a small star inside each vertex. (See Diagram B.)

Allow student pairs to create two-dimensional figures using grid paper. Ask them to place a small star inside each vertex. Finally, encourage students to identify the figure according to the number of angles. Allow them to use a reference chart or table that names the figures, if necessary.

## MA.912.G.1.Pa.c Solve real-world problems involving points, lines, angles, and areas (planes) using directional and positional language.

Classroom Map Display a large grid-like map showing the location of various classroom objects, such as teacher and student desks, chalkboard, door, and windows. Provide students with smaller versions of the same map. Have a volunteer show the horizontal and vertical moves needed to go from one object to a second object on the large map, and describe the moves using directional and positional language, while other students follow along at their seats. Repeat with other objects and volunteers.

## MA.912.G.2.In.a Determine if polygons have all sides and angles equal (regular) or have sides or angles that are not equal (irregular) using physical and visual models.

Materials: cut-out figures, rulers, sorting mats
Equal Lengths Provide students with various cut-out shapes. Provide a mixture of regular and irregular polygons. Begin using familiar figures such as squares and rectangles (that are not squares). Ask students to explain the difference between a square (with sides equal to 3 inches) and a rectangle (with a length equal to 3 inches and a width equal to 5 inches). Students should notice that the rectangle has "long" sides and "short" sides, but that all the sides of a square are equal in length.

Ask them to use rulers and the cut-out shapes to sort the figures into "regular" shapes like the square, and "irregular" shapes, like the rectangle. Provide a sorting mat with a
 square on the left and a rectangle on the right. Students can glue the cut-out shapes to the correct side of their sorting mat.

## MA.912.G.2.In.b Use tools to measure angles, including 45 degrees and 90 degrees.

Real-World Triangles Provide students with pictures of architecture from around the world. Have students find examples of triangles in the buildings. Include examples which have $45^{\circ}$ and $90^{\circ}$ angles. Students should trace the triangles onto a sheet of paper. Prompt students to classify the triangles by their sides and angles. Then have the students measure the angles with their protractors.

## MA.912.G.2.Su.a Identify polygons with all sides and angles equal (regular) in the environment.

Regular Polygons Define regular polygons. Show students examples of pattern blocks, attribute blocks, or cut-out shapes. Demonstrate measuring the lengths of the sides, and the size of the angles using a ruler and a protractor.

Provide students with magazines and newspapers, as well as real-world objects. Ask them to find examples of regular polygons. Street signs, CD cases, self-stick notes, and square picture frames are a few real-world examples of regular polygons. When appropriate, have students measure the lengths of the sides and the size of the angles using rulers and protractors.

## MA.912.G.2.Su.b Use a model of a right triangle to compare the size of angles, such as acute, obtuse, and right angles.

Useful Tool Draw a right triangle on a sheet of paper. Have student determine if the angles are larger than $90^{\circ}$, smaller than $90^{\circ}$, or equal to $90^{\circ}$, you can use the corner of an index card to show a $90^{\circ}$ angle. You can also use a protractor. Angle measures from $0^{\circ}$ to $180^{\circ}$ are labeled. A protractor will assist students in determining how to classify a triangle by its angle measures.

## MA.912.G.2.Pa.a Identify objects or pictures with polygons.

Polygon Collage Allow students to cut out pictures from magazines and newspapers that show examples of different polygons. They may choose to cut out photos of CD cases to show squares, or picture frames to show rectangles.

If necessary, provide a work mat with a sample of different polygons. Place examples of polygons, such as squares, rectangles, pentagons, and hexagons, in columns at the top of the mat. Ask students to glue pictures of real-world objects in each column that exemplify each of these shapes.

MA.912.G.4.In.a Discriminate between triangles that have equal sides and angles (equilateral), triangles that have two equal sides and two equal angles (isosceles), and triangles that have one right angle (right triangle) using visual and physical models.

Materials: protractors, rulers, triangle cut-outs, sorting mat
Equal Lengths Provide students with various cut-out triangles, including equilateral, isosceles, and right triangles, made from paper or cardstock.

Ask students to use rulers to measure the length of each side of the cut-out triangles. Students should use protractors to measure each angle of the triangles. Allow them to write their measurements on the triangles. Finally, have students sort the figures into three categories; equilateral, isosceles, and right. Provide a sorting mat with one column for each type of triangle. Students can glue the cut-out shapes to the correct side of their sorting mat.


Triangles Create a set of three index cards. On each card, draw an image of an isosceles triangle, an equilateral triangle, and a scalene triangle. Arrange students into pairs and give each group a geoboard. Have one student select an index card. The other student makes the figure on the geoboard. Point out that finding an angle measure on a geoboard is difficult, so try to classify the figures by side lengths and parallel sides.

## MA.912.G.4.In.b Identify the height (altitude) in equilateral and isosceles triangles using physical and visual models.

Number Cubes Have students measure the height of equilateral and isosceles triangles using number cubes. Provide construction paper cutouts of equilateral and isosceles triangles of various sizes. They should line the cubes up along the altitude and count the number of cubes to determine the measurement. The can use grid paper or the corner of a rectangular piece of paper to create a right angle from the vertex of the correct angle to the midpoint of the correct side. Students can compare the sizes of triangles based upon the number cube measurements.

MA.912.G.4.Su.a Discriminate between triangles that have equal sides and angles (equilateral) and triangles that have two equal sides and two equal angles (isosceles) using physical models.

Geoboards Draw or print pictures of isosceles and equilateral triangles on index cards, write the name of the triangle on the card. The student turns over a card with the name of a figure and says the name. The student models the figure on a geoboard. Students should then compare the figure on the card to the model on the geoboard to see if they match.

MA.912.G.4.Pa.a Identify objects, pictures, or signs with a triangle in real-world situations.

Materials: paper, scissors, glue, magazines
Shape Search

- Have students look through magazines to find pictures of objects containing triangles.
- When students find a triangle, have them cut it out and glue it on their paper.
- Encourage students to explain their choices and allow them to share their work with a partner.

MA.912.G.4.Pa.b Match two or more objects with a triangle based on a given feature, such as the length of the side or the size of the angle, to complete tasks in real-world situations.

Side Lengths and Angle Measures Students work in pairs to match triangles according to side lengths and angle measures.
Provide each pair of students with a set of triangles, such as those shown below. You may wish to include right triangles in the set. Some triangles have equal side lengths while other triangles have equal angle measures.

Instruct students to find triangles that have equal side lengths. Students can cut out figures and physically match up congruent sides or measure triangle side lengths with a ruler. Students using a ruler can be challenged to find sides with specified lengths. Then instruct students to find triangles with equal angle measures.


MA.912.G.5.In.a Compare the length of the straight sides in a right triangle with the length of the side opposite the right angle (hypotenuse).

Materials: grid paper, rulers, right triangle cut-outs
Right Triangles Distribute rulers, grid paper, and cut-outs of various sizes of right triangles. Use common right triangles that are easily measured, such as a triangle with sides that measure 3 inches, 4 inches, and 5 inches.

Emphasize to students that grid paper is a good manipulative for identifying angles as acute, obtuse, or right. Show students how to align one corner of a grid box with each vertex of the triangle, and how to align one side of the triangle with a line of the grid (which includes the vertex). Students can use the grid boxes to determine which angle is the right angle.

Once students have identified the right angle, have them find the hypotenuse. Provide a model of a right triangle which shows the right angle and the hypotenuse labeled. Ask students to identify the hypotenuse and measure its length using the ruler. Next, have students measure the length of the other two sides. Students should compare the length of the hypotenuse with the lengths of the other two sides of each triangle. Have them note any patterns they find.

## MA.912.G.5.Su.a Identify right triangles in the environment using physical models.

Materials: right triangle cut-outs, real-world right triangle examples

Right Triangles Emphasize to students that the paper cut-outs are good manipulatives for comparing angles. Point out that each right triangle has exactly one right angle. Encourage students to identify and mark each right angle using the appropriate symbol.

Once students are comfortable identifying right angles, have them evaluate different real-world examples. Provide various examples, such as sketches of roof trusses, children's toys, or decorating tiles. Ask students to identify a right triangle by finding the right angle.

## MA.912.G.5.Pa.a Identify objects, pictures, or signs with a right triangle.

Materials: right triangle cut-outs, real-world right triangle examples

Right Triangles Emphasize to students that the paper cut-outs are good manipulatives for comparing angles. Point out that each right triangle has exactly one right angle. Encourage students to identify and mark each right angle using the appropriate symbol.

Once students are comfortable identifying right angles, have them evaluate different real-world examples. Provide various examples, such as sketches of roof trusses, children's toys, or decorating tiles. Ask students to identify a right triangle by finding the right angle.

## MA.912.G.5.Pa.b Match objects, pictures, or signs with a right triangle by a given feature, such as length of sides.

Materials: rulers, a variety of cut-out triangles on paper or cardstock, photos of real-world objects

Match Triangles Distribute a ruler, a variety of cut-out triangles, and several photos of real-world objects to each pair of students. Use photos of real-world objects, such as a bridge's trusses. Make sure that one of the cut-out shapes is congruent to each photo.

Students should compare the length of each side on the paper figures with the length of each side of various photos of real-world objects using a ruler. Ask students to match the cut-outs to a photo based on lengths of the sides.

## MA.912.G.8.In.a Use problem-solving strategies including visual and physical models and tools, for solving real-world problems involving geometry concepts and skills.

Real-World Solutions Provide students with a variety of real-world situations which include geometric concepts and skills. They may be asked to determine the preferred shape of a vegetable garden, the area of a garden, or the amount (or volume) of top soil they might need for a small vegetable garden. If possible, allow students to use the measurement to plant a small garden outdoors or use a large plastic basin to create an indoor herb garden.

## MA.912.G.8.Su.b Use resources, such as calculators and conversion charts to verify accuracy of solutions to problems involving geometry concepts.

Metric and Customary Volume Practice comparing volume measured in metric and standard units of measurement. Provide a variety of three-dimensional solids with one open end, such as a cylindrical container without the lid. Allow students to measure the dimensions of the containers using rulers in metric and customary units of measure. Students can compare the measurements and place them in a table. Ask them to review their measurements by using conversion charts and completing the mathematical calculations using a calculator. Students should explain their results in a math journal.

## MA.912.G.8.Pa.a Solve real-world problems involving objects with two- and three-dimensional shapes and match the result to the correct answer to determine accuracy.

Real-World Solutions Provide students with real-world problem-solving opportunities, such as the exercise below.

CRAFT KITS Damion is building a bird house for the backyard. The craft kit comes with two square boards and 4 rectangular boards. What three-dimensional shape is his birdhouse?

Allow students to use square and rectangular paper cut-outs and tape to assemble a model of the birdhouse. After students have had a chance to complete their birdhouse model, provide a rectangular prism and ask them to compare their model to the rectangular prism.

