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1-1 Enrichment

California Redwoods

The tallest trees in the world, California's redwoods, are located along the state's aptly named 'Redwood Coast.' The redwoods are thousands of years old, and are preserved for posterity in state and national parks. The Rockefeller Tree, officially the world's tallest, measures 362 feet and has a diameter of 13 feet, 6 inches.

Find each sum, difference, product or quotient.

1. 24.3 − 13.6 = K
K = _____

2. 38(12.5) = S
S = _____

3. 84.2(2.6) = W
W = _______

4. 7.3 − 4.9 = P
P = _____

5. 13.2 + 2.5 = B
B = _____

6. 23,876 ÷ 520 = T
T = ______

7. \$12.75 − \$6.80 = E
E = ______

8. 3.6 + 4.9 = D
D = _____

9. 1790 ÷ 310 = O
O = _____

10. 7.06 − 6.84 = L
L = _____

11. 17.6 + 6.2 = A
A = _____

12. 3.7(12.3) = R
R = ______

13. \$7350 ÷ 150 = M
M = _____

14. 485 − 234 = U
U = _____

15. 8.62 − 6.93 = H
H = _____

Match the numeric answer to each equation above with the numeric answers above the spaces below. Then place each corresponding letter on the appropriate blank to form the answer to this trivia question.

Where is the world's tallest tree located?

____	____	____	____	____	____	____	____		
1.69	251	\$49	15.7	5.77	0.22	8.5	45.92		
____	____	____	____	____	____	____	____		
45.51	\$5.95	8.5	218.92	5.77	5.77	8.5	475		
____	____	____	____	____		____	____	____	____
475	45.92	23.8	45.92	\$5.95		2.4	23.8	45.51	10.7

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1-2 Enrichment

Operations Search

This is a fun activity that you can try on your own as well as with your family or classmates.

In each exercise below, you are given some numbers. Insert operations symbols (+, −, ×, ÷) and parentheses so that a true mathematical sentence is formed. Follow the specific instructions for each problem, remembering to observe the order of operations.

Do not change the order of the numbers.

1. 5 4 3 2 1 = 3

2. 5 4 3 2 1 = 0

3. 5 4 3 2 1 = 1

4. 5 4 3 2 1 = 50

Do not change the order of the numbers. You may put two numbers together to form a two-digit number.

5. 1 2 3 4 5 6 7 8 = 90

6. 8 7 6 5 4 3 2 1 = 25

Change the order of the numbers and put numbers together to form a two- or three-digit number, as needed.

Use these four digits: 4 3 2 1

Make these totals:

7. 1312

8. 2

9. 16

10. 1

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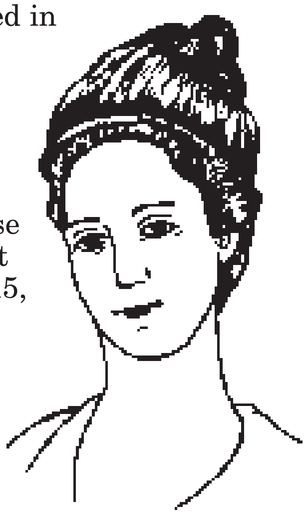
1-3 Enrichment

Hypatia

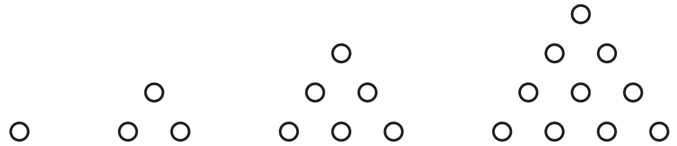


Hypatia, pronounced *hi PAY sha,* was the first woman to be mentioned in the history of mathematics. Born about A.D. 370, Hypatia lived in Alexandria and served as a professor at the famous Library of Alexandria. Hypatia wrote important commentaries on the works of the mathematician Appollonius and the scientist Ptolemy. She also excelled in the fields of astronomy, medicine, and philosophy.

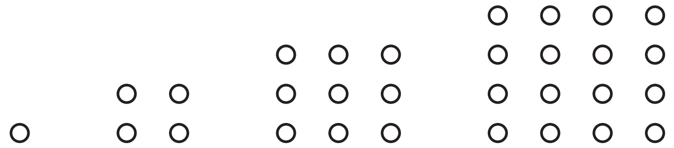
Egypt was in great political turmoil during Hypatia's lifetime. Because of her influence among scholars of the day, Hypatia became the target of criticism from those who equated science with paganism. In A.D. 415, she was murdered by an angry mob. Soon after her death, the library was destroyed and the Dark Ages began. The serious study of mathematics was limited for the next 500 years.

One of the things Hypatia studied was the relationship between number patterns and geometry. Investigate the geometric patterns below.

Triangular Numbers



Square Numbers



1. Draw the fifth and sixth figures in the pattern of triangular numbers. Then write the first six triangular numbers.

2. Draw the fifth and sixth figures in the patterns of square numbers. Then write the first six square numbers.

3. Draw the first four pentagonal and hexagonal numbers.

4. Use counters or drawings to determine if there is a number that is both square and triangular.

1-4 Enrichment

Algebraic Proof

Axioms are statements assumed to be true without being proven. They are used in the proofs of theorems. The following properties are examples of algebraic axioms. Abbreviations for these properties are used in the examples below.

Commutative Property of Addition (CPA)
Associative Property of Addition (APA)
Subtraction Property of Equality (SubPE)
Addition Property of Equality (APE)
Substitution Property of Equality (SPE)
Additive Identity Property (AIP)

Prove: $a + (b + c) = c + (a + b)$

Example 1

Statement	Reason
$a + (b + c) = (a + b) + c$	APA
$= c + (a + b)$	CPA

Prove: $5 + (x + 2) = 7 + x$

Example 2

Statement	Reason
$5 + (x + 2) = 5 + (2 + x)$	CPA
$= (5 + 2) + x$	APA
$= 7 + x$	SPE

Write the reason for each statement.

1. Prove: $(9 + 6) + (x + 3) = x + 18$

Statement	Reason
$(9 + 6) + (x + 3) = 15 + (x + 3)$	**a.** _______
$= (15 + x) + 3$	**b.** _______
$= (x + 15) + 3$	**c.** _______
$= x + (15 + 3)$	**d.** _______
$= x + 18$	**e.** _______

Prove each of the following. Identify a reason for each statement.

2. $9 + (x + 4) = 13 + x$

3. $5 + (x + 11) = x + (10 + 6)$

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1-5 Enrichment

Solution Sets

Consider the following open sentence.

It is a robot that starred in STAR WARS.

You know that a replacement for the word *It* must be found in order to determine if the sentence is true or false. If *It* is replaced by either *R2D2* or *C3PO*, the sentence is true.

The set {*R2D2, C3PO*} can be thought of as the *solution set* of the open sentence given above. This set includes all replacements for the word that make the sentence true.

Write the solution set of each open sentence.

1. It is the name of a state beginning with the letter *A*.

2. It is a primary color.

3. Its capital is Harrisburg.

4. It is a New England state.

5. He was one of the Seven Dwarfs.

6. It is the name of a month that contains the letter q.

7. During the 1990s, she was the wife of a U.S. President.

8. It is an even number between 1 and 13.

9. $x + 4 = 10$

10. $31 = 72 - k$

11. It is the square of 2, 3, or 4.

Write a description of each set.

12. {A, E, I, O, U}

13. {1, 3, 5, 7, 9}

14. {June, July, August}

15. {United States, Canada, Mexico}

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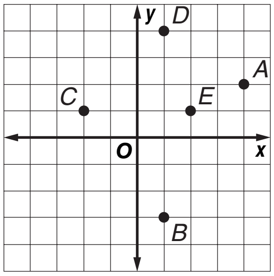
1-6 Enrichment

Archaeological Dig

Archaeology is a scientific study into the past that often involves unearthing artifacts or structures from an earlier time. Archaeological digs for these artifacts take place all over the world. One of the earliest examples of archaeology took place between 1550 and 1070 B.C. in Egypt when the pharaohs unearthed and reconstructed the Sphinx, which had been built about one thousand years earlier.

Archaeologists sometimes use a coordinate plane record where artifacts are unearthed at their dig site.

1. Complete the table to shown the coordinates for the locations of artifacts A–E.

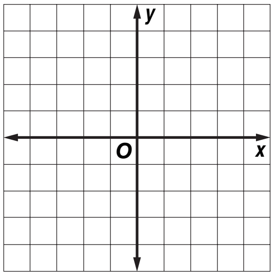


Artifact	Coordinates
A	
B	
C	
D	
E	

2. Several days after A–E were found, archaeologists unearthed two new artifacts: F, located at (1,−1), and G, located at (−3,−2). Graph the location of artifacts F and G on the grid from Exercise 1.

3. Suppose you were an archaeologist working on this dig. In what location would you most likely dig next? Why?

4. At a new dig site, archaeologists found seven artifacts. Their locations are listed in the table below. Use the coordinate plane at the right to graph the location of each artifact.



Artifact	Coordinates
A	(1, 1)
B	(0, 3)
C	(3, 0)
D	(3, 2)
E	(2, -1)
F	(2, 2)

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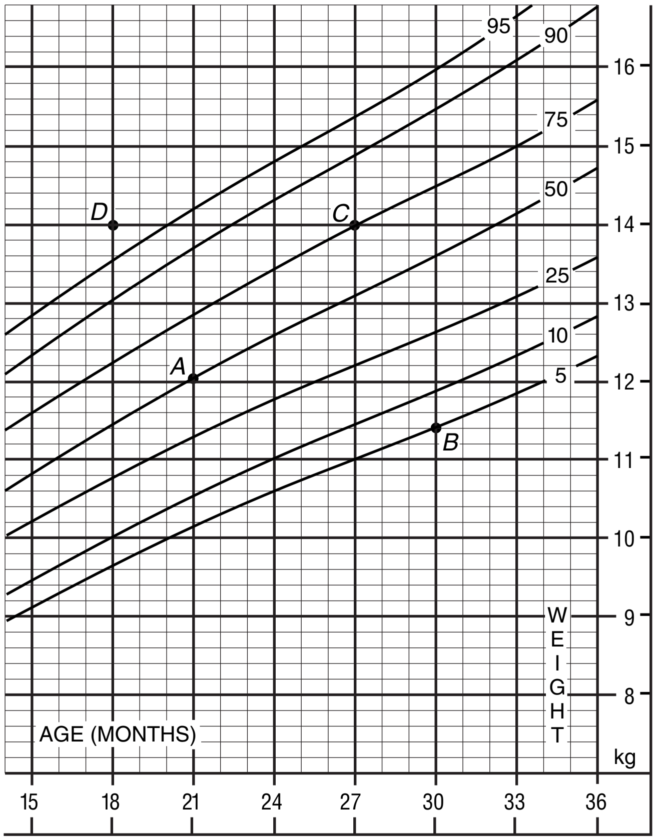
1-7 Enrichment

Growth Charts

Scatter plots are often used by doctors to show parents the growth rates of their children. The horizontal scale of the chart at the right shows the ages from 15 to 36 months. The vertical scale shows weight in kilograms. One kilogram is about 2.2 pounds. The slanted lines are used to show how a child's weight compares with others of his or her age.

Look at the point labeled *A*. It represents a 21-month-old who weighs 12 kilograms. It is located on the slanted line labeled 50. This means the child's weight is in the "50th percentile." In other words, 50% of all 21-month-olds weigh more than 12 kilograms and 50% weigh less than 12 kilograms.

The location of Point *B* indicates that a 30-month-old who weighs 11.4 kilograms is in the 5th percentile. Only 5% of 30-month-old children will weigh less than 11.4 kilograms.



Solve.

1. Look at the point labeled *C*. How much does the child weigh? How old is he? What percent of children his age will weigh more than he does?

2. Look at the point labeled *D*. What is the child's age and weight? What percent of children her age will weigh more than she does?

3. What is the 50th percentile weight for a child 27 months old?

4. How much weight would child *B* have to gain to be in the 50th percentile?

5. If child *D* did not gain any weight for four months, what percentile would he be in?

6. How much heavier is a $2\frac{1}{2}$-year-old in the 90th percentile than one in the 10th percentile?

2-1 Enrichment

Football Statistics

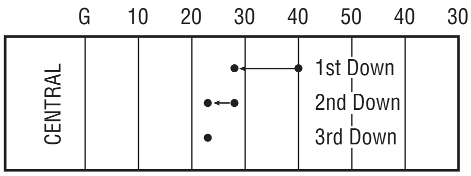
In football, one of the key offensive positions is running back. The job of the running back is to gain as many yards as possible with the ball. The line where the play begins is called the line of scrimmage. If the running back gets beyond the line of scrimmage when he is given the ball, he gains yards on the carry. However, if he is tackled behind the line of scrimmage, he loses yards. When he gains yards, the integer describing the carry is positive. However, when yards are lost, the integer describing the carry is negative.

Example 1 Sam is a running back for Central High School. He gains 3 yards on his first carry. The integer describing the carry is 3.

Example 2 Then Sam is then tackled 7 yards behind the line of scrimmage on his second carry. The integer describing the carry is -7.

Exercises

Sam carried the ball from the quarterback 15 times in their game against Southwest High School, and three consecutive times in one series of plays. Sam's three consecutive carries are illustrated at the right.

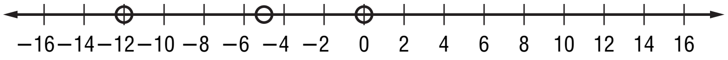


1. On 1st down, Sam was tackled 12 yards behind the line of scrimmage. What integer describes the carry?

2. On 2nd down, he was tackled 5 yards behind the line of scrimmage. What integer describes the number of yards lost on the play?

3. On 3rd down, Sam was tackled on the line of scrimmage. What integer represents the yardage gained on the play?

4. Graph the integers from each of the three consecutive carries on the number line below.



5. If you were the coach, would you play Sam in next week's game against North High School based on the five carries shown in the Examples and Exercises 1–3? Why or why not?

6. The table to the right shows the yards gained or lost each of the first five times that the Southwest High School running back carried the ball. Using this information, write the relationship between the yardage gained or lost on each carry by the Southwest High School running back compared to the Central High School's running back, Sam.

Carry	Yards Gained
1	2
2	5
3	−6
4	−2
5	26

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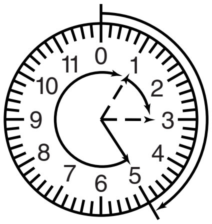
2-2 Enrichment

Modular Arithmetic

In modular arithmetic, there is a finite set of numbers. An example of a finite set would be the numbers on the face of a clock. No matter how many numbers are added together, the answer will still be one of the twelve numbers found on the face of the clock.

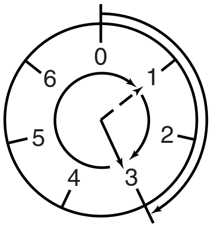
Example 1 In mod 12, 5 + 8 + 2 = 3.

To find the sum, start at 0. Count 5 units in a clockwise direction, then 8 units, and then 2 units. You should end at the number 3.



Example 2 In mod 7, 3 + 5 + 2 = 3.

To find the sum, start at 0. Count 3 units in a clockwise direction, then 5 units, and then 2 units. You should end at the number 3.



Exercises

Find each sum in mod 12.

1. 6 + 8 + 4

2. 7 + 9 + 11 + 5

3. 6 + 4 + 11 + 12 + 5

4. 1 + 4 + 9 + 11 + 6

5. 7 + 10 + 9 + 12 + 10

6. 6 + 12 + 5 + 12 + 8 + 12

7. 12 + 12 + 12 + 12 + 6

8. 4 + 9 + 3 + 7 + 8 + 11 + 5

9. 6 + 10 + 7 + 8 + 4 + 3 + 9

10. 9 + 2 + 4 + 3 + 10 + 12 + 4 + 11

Find each sum in mod 7.

11. 2 + 4 + 3

12. 5 + 1 + 6 + 3

13. 2 + 2 + 4 + 6 + 7

14. 5 + 3 + 4 + 6 + 1

15. 7 + 7 + 7 + 7 + 5

16. 6 + 5 + 1 + 2 + 4

17. 4 + 3 + 1 + 5 + 6 + 2

18. 4 + 2 + 1 + 6 + 7 + 5 + 4 + 3

19. 6 + 5 + 4 + 3 + 2 + 1 + 7 + 6

20. 5 + 7 + 4 + 4 + 2 + 6 + 1 + 2

21. Can you find a pattern to these exercises? Explain.

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2-3 Enrichment

Mental Math: Compensation

To add or subtract in your head, work with multiples of 10 (20, 30, 40, . . .) or 100 (200, 300, 400, . . .) and then adjust your answer.

To add 52, first add 50, then add 2 more.

To subtract 74, first subtract 70, then subtract 4 more.

To subtract 38, first subtract 40, then add 2.

To add 296, first add 300, then subtract 4.

Write the second step you would use to do each of the following.

1. Add 83.
1) Add 80.
2) ________________

2. Add 304.
1) Add 300.
2) ________________

3. Subtract 62.
1) Subtract 60.
2) ________________

4. Add 27.
1) Add 30.
2) ________________

5. Subtract 79.
1) Subtract 80.
2) ________________

6. Subtract 103.
1) Subtract 100.
2) ________________

7. Add 499.
1) Add 500.
2) ________________

8. Add 294.
1) Add 300.
2) ________________

9. Subtract 590.
1) Subtract 600.
2) ________________

Use the method above to add 59 to each of the following.

10. 40

11. 72

12. 53

13. 15

Use the method above to subtract 18 from each of the following.

14. 96

15. 45

16. 71

17. 67

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2-4 Enrichment

Cyclic Numbers

Look closely at the products below. Do you see a pattern?

1 × 142,857 = 142,857

2 × 142,857 = 285,714

3 × 142,857 = 428,571

4 × 142,857 = 571,428

5 × 142,857 = 714,285

6 × 142,857 = 857,142

The same six digits repeat in all of the products. Numbers like 142,857 are called *cyclic numbers*.

1. Cyclic numbers are related to prime numbers. A prime number is a number that has exactly two factors, 1 and itself. You can use a calculator and the decimal equivalents of fractions of the form $\frac{1}{p}$, where p is a prime number, to find cyclic numbers. Use a calculator to find the decimal equivalent of each fraction below.

a. $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{5}$ **d.** $\frac{1}{7}$

2. Study the decimal equivalents you found. Do you observe a pattern in any of the digits?

3. The cyclic number 142,857 has six digits. The next largest cyclic number has sixteen digits. What fraction do you think might help you find the next cyclic number? Explain.

4. Explain why the next largest cyclic number cannot be determined using a scientific calculator.

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2-5 Enrichment

Divisibility Rules for 7 and 11

Example 1 **Determine whether 4032 is divisible by 7.**

$403\cancel{2}$ Cross out the ones digit.

$-\ \ 4$ Subtract twice the value of the ones digit from the rest of the number.

$39\cancel{9}$ If the difference is a number that you know is divisible by 7, stop. If not, repeat.

-18

21

Since 21 is divisible by 7, 4032 is divisible by 7.

Example 2 **Determine whether 5159 is divisible by 11.**

Method 1

$515\cancel{9}$ Cross out the ones digit.

$-\ \ 9$ Subtract the value of the ones digit from the rest of the number.

$50\cancel{6}$ If the difference is a number that you know is divisible by 11, stop. If not, repeat.

$-\ 6$

44

Since 44 is divisible by 11, 5159 is divisible by 11.

Method 2

5159

$5 + 5 = 10$ Add the odd-numbered digits (first and third).

$1 + 9 = \underline{10}$ Add the even-numbered digits (second and fourth).

0 Subtract the sums. If the difference is divisible by 11, the number is divisible by 11.

Since 0 is divisible by 11, 5159 is divisible by 11.

Example 3 **Determine whether 62,382 is divisible by 11.**

$6 + 3 + 2 = 11$ Add the odd-numbered digits.

$2 + 8 = \underline{10}$ Add the even-numbered digits.

1 Subtract the sums.

Since 1 is not divisible by 11, 62,382 is not divisible by 11.

Exercises

Determine whether each number is divisible by 7 or 11.

1. 266

2. 4312

3. 8976

4. 936

5. 13,293

6. 7085

7. 2957

8. 3124

9. 6545

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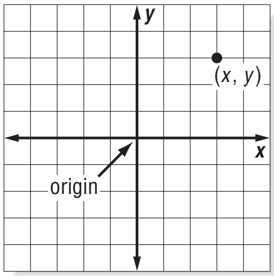
2-6 Enrichment

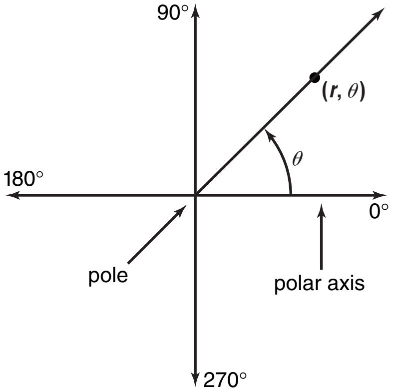
Polar Coordinates

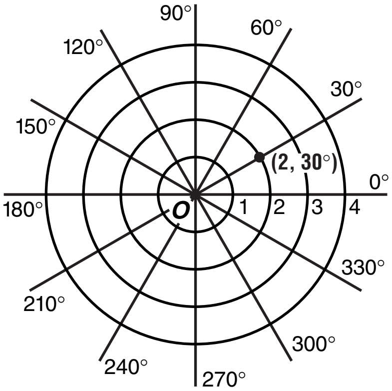


In a rectangular coordinate system, the ordered pair (x, y) describes the location of a point x units from the origin along the x-axis and y units from the origin along the y-axis.

In a polar coordinate system, the ordered pair (r, θ) describes the location of a point r units from the pole on the ray (vector) whose endpoint is the pole and which forms an angle of θ with the polar axis.

The graph of (2, 30°) is shown on the polar coordinate system at the right below. Note that the concentric circles indicate the number of units from the pole.





Locate each point on the polar coordinate system below.

1. (3, 45°)

2. (1, 135°)

3. $\left(2\frac{1}{2}, 60°\right)$

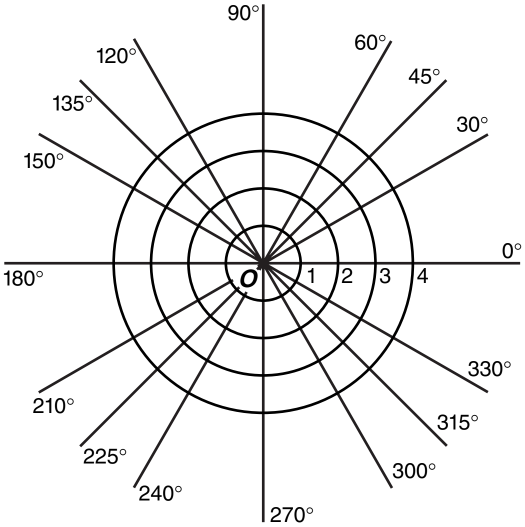
4. (4, 120°)

5. (2, 225°)

6. (3, −30°)

7. (1, −90°)

8. (−2, 30°)



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3-1 Enrichment

What Day Was It?

To find the day of the week on which a date occurred, follow these steps.

- Use the formula $s = d + 2m + \left[\frac{3(m+1)}{5}\right] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$

  where s = sum,
  d = day of the month, using whole numbers from 1 to 31,
  m = month, where March = 3, April = 4, and so on, up to December = 12; then January = 13 and February = 14, and
  y = year except for dates in January or February when the previous year is used.

- Evaluate expressions inside the special brackets [ ] by dividing, then discarding the remainder and using only the whole number part of the quotient.

- After finding the value of s, divide s by 7 and note the remainder.

- The remainder 0 represents Saturday, 1 represents Sunday, 2 represents Monday, and so on to 6 represents Friday.

Example **On December 7, 1941, Pearl Harbor was bombed. What day of the week was that?**

Let $d = 7$, $m = 12$, and $y = 1941$.

$$s = d + 2m + \left[\frac{3(m+1)}{5}\right] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$$

$$s = 7 + 2(12) + \left[\frac{3(12+1)}{5}\right] + 1941 + \left[\frac{1941}{4}\right] - \left[\frac{1941}{100}\right] + \left[\frac{1941}{400}\right] + 2$$

$$s = 7 + 24 + \left[\frac{39}{5}\right] + 1941 + \left[\frac{1941}{4}\right] - \left[\frac{1941}{100}\right] + \left[\frac{1941}{400}\right] + 2$$

$$s = 7 + 24 + 7 + 1941 + 485 - 19 + 4 + 2$$

$$s = 2451$$

Now divide s by 7. $2451 \div 7 = 305$ R1

Since the remainder is 1, December 7, 1941, was a Sunday.

Exercises **Use the formula to solve each problem.**

1. Verify today's date.
2. What will be the day of the week for April 13, 2012?
3. On what day of the week was the signing of the Declaration of Independence, July 4, 1776?
4. On what day of the week were you born?

3-2 Enrichment

Algebraic Proof

Recall that properties are statements that are true for any numbers. These properties are used to prove theorems. Use the properties you have learned to complete each proof.

Abbreviations for some properties you may need to use are listed below.

Commutative Property—Addition (CPA)
Commutative Property—Multiplication (CPM)
Associative Property—Addition (APA)
Associative Property—Multiplication (APM)
Additive Identity Property (AIP)
Multiplicative Identity Property (MIP)
Inverse Property of Addition (IPA)
Inverse Property of Multiplication (IPM)
Multiplicative Property of Zero (MPZ)
Distributive Property (DP)

Write the reason for each statement.

1. Prove: $-(y - x) = x - y$

Statement	**Reason**
$-(y - x) = -1(y - x)$	**MIP**
$= -1y - (-1x)$	**a.** ______________________
$= -y - (-x)$	**b.** ______________________
$= -y + x$	**c.** ______________________
$= x + (-y)$	**d.** ______________________
$= x - y$	**e.** ______________________

2. Prove: $3x - 4 - x = 2x - 4$

Statement	**Reason**
$3x - 4 - x = 3x + (-4) + (-x)$	**a.** ______________________
$= 3x + (-x) + (-4)$	**b.** ______________________
$= 3x + (-1x) + (-4)$	**c.** ______________________
$= [3 + (-1)]x + (-4)$	**d.** ______________________
$= 2x + (-4)$	**e.** ______________________
$= 2x - 4$	**f.** ______________________

3. Prove: $-2x + 6 + 2x = 6$

Statement	**Reason**
$-2x + 6 + 2x = -2x + 2x + 6$	**a.** ______________________
$= (-2 + 2)x + 6$	**b.** ______________________
$= 0x + 6$	**c.** ______________________
$= 0 + 6$	**d.** ______________________
$= 6$	**e.** ______________________

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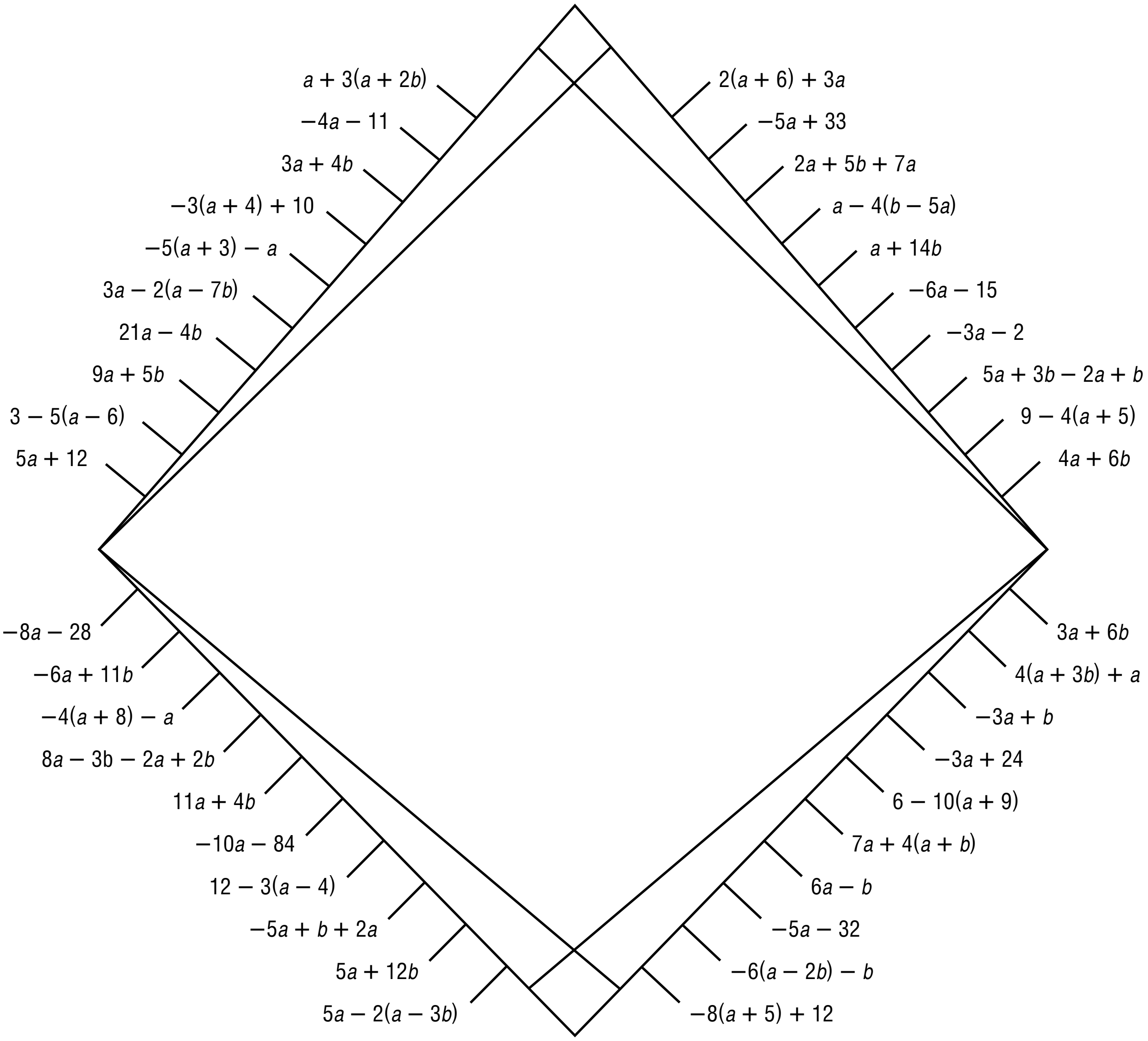
3-3 Enrichment

Creating a Line Design

Connect each pair of equivalent expressions with a straight line segment. Describe the finished design.



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3-4 Enrichment

Puzzling Equations

Solve each equation. Notice that the first equation is completed.

1. $\frac{m}{12} = 13$	**1.** 156 = I
2. $17v = -578$	**2.** ______ = A
3. $\frac{c}{75} = 18$	**3.** ______ = E
4. $-252d = -5796$	**4.** ______ = J
5. $64 \cdot w = 5568$	**5.** ______ = A
6. $g \div 29 = 61$	**6.** ______ = M
7. $p(85) = -7225$	**7.** ______ = R
8. $39x = 663$	**8.** ______ = S
9. $\frac{k}{18} = 30$	**9.** ______ = Y
10. $\frac{z}{-94} = -32$	**10.** ______ = R
11. $-112q = 1456$	**11.** ______ = O
12. $201y = -1608$	**12.** ______ = N
13. $\frac{a}{14} = -17$	**13.** ______ = R
14. $-8045 = -5k$	**14.** ______ = S
15. $m \div (-105) = 8$	**15.** ______ = H

Use the letter beside each of your answers to decode the answer to this question.

What woman led a 125-mile march from Pennsylvania to Long Island in 1903 to bring the practice of child labor to the attention of President Theodore Roosevelt?

1769	−34	3008	540

−840	87	−238	−85	156	1609

23	−13	−8	1350	17

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3-5 Enrichment

Al-Khowarizmi: The Father of Algebra

The title “Father of Algebra” should be awarded to the Arabian mathematician Al-Khowarizmi. In the ninth century, he wrote a work entitled *Hisab al-jabr-w’al muqubalah*, meaning “the science of restoring and canceling.” In this work, he gave a clear and complete explanation of how to solve an equation by performing the same operation on both sides of the equation.

In the thirteenth century, Al-Khowarizmi’s work was translated into Latin, the language of educated people in Europe, which launched algebra into the Western world. It is from the title of his work that we get the word *algebra*.

Here is an example of how Al-Khowarizmi solved an equation like $x - 5 = 10$. To solve this equation, 5 must be added, or restored, to each side. Thus, $x - 5 + 5 = 10 + 5$, or $x = 15$.

Another example of restoring can be used to solve $x + 5 = 10$. To solve this equation, -5 must be restored to each side. This is the same as subtracting 5 from each side. Thus, $x + 5 - 5 = 10 - 5$, or $x = 5$.

Here is an example of solving an equation by canceling (or dividing). To solve $3x = 9$, each side must be canceled by a factor of 3.

Thus, $\frac{3x}{3} = \frac{9}{3}$, or $x = 3$.

Solve each equation and label it with an *R* for restoring or with a *C* for canceling.

1. $x - 10 = 20$

2. $y - 3 = 2$

3. $4x = 12$

4. $5y = 15$

5. $y - 6 = 5$

6. $3x = 18$

7. $x + 2 = 3$

8. $2y = 10$

9. $3y = 21$

10. Make up your own equations to solve. Three should use restoring and three canceling.

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3-6 Enrichment

Systems of Equations

A **system of equations** is a set of equations with the same variables. The equations shown below are an example of one kind of system of equations.

$y = x + 2$
$3x - 5 = 16$

The solution of this system must be a pair of numbers, x and y, that make both equations true.

To solve this type of system, first solve the equation that contains only one variable. Then substitute that answer into the second equation and solve for the remaining variable.

Example **Solve each system of equations.**

a. $\mathbf{y = x + 2}$
$\mathbf{3x - 5 = 16}$

Solve $3x - 5 = 16$ first.

$$\begin{aligned} 3x - 5 &= 16 \\ 3x - 5 + 5 &= 16 + 5 \\ 3x &= 21 \\ \frac{3x}{3} &= \frac{21}{3} \\ x &= 7 \end{aligned}$$

Substitute 7 for x in the other equation.
$y = x + 2$
$y = 7 + 2$ or 9

The solution is $x = 7$ and $y = 9$.

b. $\mathbf{4d - 1 = 19}$
$\mathbf{c = d - 3}$

Solve $4d - 1 = 19$ first.

$$\begin{aligned} 4d - 1 &= 19 \\ 4d - 1 + 1 &= 19 + 1 \\ 4d &= 20 \\ \frac{4d}{4} &= \frac{20}{4} \\ d &= 5 \end{aligned}$$

Substitute 5 for d in the other equation.
$c = d - 3$
$c = 5 - 3$ or 2

The solution is $c = 2$ and $d = 5$.

Solve each system of equations.

1. $40 - 2t = 10$
$3t - s = 35$

2. $4a + 2b = 22$
$25 = 11a - 8$

3. $82.5 = 1.5s$
$d = 3s + 35$

4. $\frac{m}{5} + 1.5 = 2$
$7m + n = 17.5$

5. $6x + \frac{y}{2} = 43$
$22 + 3x = 43$

6. $\frac{c}{5} + p = 4$
$20 = 4p - 3$

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3-7 Enrichment

Applications of Geometric Sequences

Populations often grow according to a geometric sequence, which is the sequence in which the quotient of any two consecutive terms is the same. If the population of a city grows at the rate of 2% per year, then the common ratio, *r*, is 1.02. To find the population of a city of 100,000 after 5 years of 2% growth, use the formula ar^{n-1}, where *r* is the common ratio and *n* is the number of years.

$$\begin{aligned} ar^{n-1} &= 100{,}000\,(1.02)^{5-1} && a = 100{,}000,\ r = 1.02,\ n = 5 \\ &= 100{,}000\,(1.02)^4 \\ &\approx 108{,}243 && \text{So, the city has a population of 108,243 people.} \end{aligned}$$

After a few years, a small change in the annual growth rate can cause enormous differences in the population.

Assume that the nation of Grogro had a population of one million in 2000. Using a calculator, find the population of the country in the years 2020, 2070, and 2120 at growth rates of 1%, 3%, and 5% per year. Record your results in the table below.

Population of Grogro

	Growth Rate	Year		
		2020	2070	2120
1.	1%			
2.	3%			
3.	5%			

Suppose we want to find the total upward distance a bouncing ball has moved. It bounces up 36 inches on the first bounce and $\frac{3}{4}$ times its height on each of five more consecutive bounces.

1st bounce		2nd bounce		3rd bounce
36 in.	$\xrightarrow{\times\frac{3}{4}}$	27 in.	$\xrightarrow{\times\frac{3}{4}}$	$20\frac{1}{4}$ in.

The distances form the terms of a geometric sequence. We want to find the sum of the distances, or the sum of the six terms in the sequence.

The sum of the terms of a geometric sequence is called a geometric series. The formula for the sum S_n of the first *n* terms of a geometric series is $S_n = \frac{a - ar^n}{1 - r}$, where a = the first term and r = the common ratio ($r \neq 1$).

$$\begin{aligned} S_n &= \frac{a - ar^n}{1 - r} && \text{Write the formula.} \\ &= \frac{36 - 36\left(\frac{3}{4}\right)^6}{1 - \frac{3}{4}} && a = 36,\ r = \frac{3}{4},\ n = 6 \\ &\approx 118.4 && \text{So, the ball bounces upward approximately 118.4 inches.} \end{aligned}$$

Use the formula above to find each sum. Then check your answer by adding.

4. 5 + 10 + 20 + 40 + 80

5. 80 + 240 + 720 + 2160 + 6480

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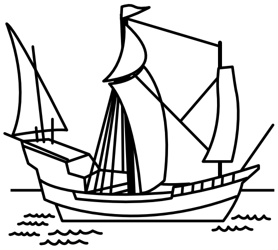
3-8 Enrichment

Mathematics and Social Studies



Since the discovery that Earth is round, people have been fascinated with the prospect of making ever faster trips around the world. Ferdinand Magellan's ship *Victoria* set sail on September 20, 1519, and completed the first voyage around the world about three years later on September 6, 1522. With more consistent modes of transportation came new records in circling Earth.

Solve each problem by finding the average speed it took to circle Earth. Use 24,900 miles to approximate the distance around Earth.

1. **DIRIGIBLE** In 1929, Graf Zeppelin made the first round-the-world dirigible flight in 21 days, 8 hours on the *LZ127*. What was the Zeppelin's average speed in miles per hour?

2. **AIRPLANE** Air Force bomber *Lucky Lady II* made the first nonstop flight around the world in 1949. The flight took 94 hours. What was the average speed of the *Lucky Lady II* on that trip?

3. **SPACESHIP** In 1961, Yuri Gagarin and Bherman Titov of Russia each circled Earth in a little over an hour and 45 minutes. At what speed must one travel to circle Earth at its surface in an hour and a half?

AIRCRAFT In Exercises 4–6, assume that each aircraft travels at a constant rate. Use 24,900 miles as the distance traveled. How long would it take each aircraft to circle Earth?

4. a commercial plane of the 1930s traveling at 168 mph

5. a Boeing 707 cruising at 640 mph

6. a Concorde flying at 1450 mph

7. **BALLOONING** Jules Verne wrote about circling Earth in a hot air balloon in his novel *Around the World in Eighty Days* (1873). Suppose it were possible for a hot air balloon to circle Earth in 80 days. What would be the average speed in miles per hour?

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4-1 Enrichment

Exponents

Numbers can be expressed in several ways. Some numbers are expressed as sums. Some numbers are expressed as products of factors, while other numbers are expressed as powers.

Two ways to express 27 are $3 \cdot 3 \cdot 3$ and 3^3.

The number 1 million can be expressed in the following ways.

1,000,000	$1000 \cdot 1000$	$100 \cdot 100 \cdot 100$	$10^2 \cdot 10^2 \cdot 10^2$
$1{,}000{,}000^1$	1000^2	100^3	10^6

Write names for each number below using the given exponents.

1. 16; exponents: 2 and 4

2. 81; exponents: 2 and 4

3. 64; exponents: 2 and 6

4. 256; exponents: 2 and 8

5. 625; exponents: 2 and 4

6. 729; exponents: 2 and 6

7. 2401; exponents: 2 and 4

8. 4096; exponents: 2 and 12

9. 6561; exponents: 2 and 8

10. 390,625; exponents: 2 and 8

Numbers that can be named as powers with like bases can be multiplied by adding the exponents.

$$
\begin{aligned}
8 \cdot 8 &= 2^3 \cdot 2^3 \\
&= 2^{3+3} \\
&= 2^6
\end{aligned}
$$

Write the product of each pair of factors in exponential form.

11. $9 \cdot 9$

12. $4 \cdot 4$

13. $16 \cdot 8$

14. $125 \cdot 25$

15. $27 \cdot 9$

16. $81 \cdot 27$

17. $49 \cdot 49$

18. $121 \cdot 121$

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4-2 Enrichment

Lesson 4–2

Prime Pyramid

A **prime number** is a whole number that has exactly two factors—itself and 1. The pyramid below is called a *prime pyramid*. Each row begins with 1 and ends with the number of that row. So, row 2 begins with 1 and ends with 2, row 3 begins with 1 and ends with 3, and so on. In each row, the numbers from 1 to the row number are arranged such that the sum of any two adjacent numbers is a prime number.

For example, look at row 4:

- It must contain the numbers 1, 2, 3, and 4.
- It must begin with 1 and end with 4.
- The sum of adjacent pairs must be a prime number:
  $1 + 2 = 3,\ 2 + 3 = 5,\ 3 + 4 = 7$

*
1 2
1 2 3
1 2 3 4
1 4 3 2 5
1 4 3 2 5 6
1 ___ ___ ___ 5 ___ 7
1 ___ ___ ___ ___ ___ ___ 8
1 2 3 4 7 6 5 8 9
1 ___ ___ ___ ___ ___ ___ ___ ___ 10
1 ___ ___ ___ ___ ___ ___ ___ ___ ___ 11
1 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ 12

1. Complete the pyramid by filling in the missing numbers.

2. Extend the pyramid to row 13.

3. Explain the patterns you see in the completed pyramid.

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4-3 Enrichment

GCFs by Successive Division

Another way to find the greatest common factor (GCF) of two numbers is to use **successive division**. This method works well for large numbers.

Find the GCF of 848 and 1325.

Step 1 Divide the smaller number into the greater number.

$$\begin{array}{r} 1\text{ R477} \\ 848\overline{)1325} \\ \underline{848} \\ 477 \end{array}$$

Step 2 Divide the remainder into the divisor. Repeat this step until you get a remainder of 0.

$$\begin{array}{rrrr} 1\text{ R371} & 1\text{ R106} & 3\text{ R53} & 2\text{ R0} \\ 477\overline{)848} & 371\overline{)477} & 106\overline{)371} & 53\overline{)106} \\ \underline{477} & \underline{371} & \underline{318} & \underline{106} \\ 371 & 106 & 53 & 0 \end{array}$$

The last divisor is the GCF of the two original numbers. So the GCF of 848 and 1325 is 53.

Use the method above to find the GCF of each pair of numbers.

1. 187; 578

2. 161; 943

3. 215; 1849

4. 453; 484

5. 432; 588

6. 279; 403

7. 1325; 3498

8. 9840; 1751

9. 3484; 5963

10. 1802; 106

11. 45,787; 69,875

12. 35,811; 102,070

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4-4 Enrichment

Simplifying Algebraic Fractions

Alaska became the 49th state when President Andrew Johnson purchased its territory from Russia in 1867. The highest mountain peak in North America, Mount McKinley, is in the state's Alaska Mountain Range, which measures 600 miles in length. Mount McKinley is 20,320 feet high. Solve the problems below to discover the name and location of the tallest mountain peak in the lower 48 states.

Write each fraction in simplest form.

1. M = $\frac{32}{46}$

M = ___

2. R = $\frac{18}{56}$

R = ___

3. Y = $\frac{7}{84}$

Y = ___

4. I = $\frac{12}{64}$

I = ___

5. E = $\frac{9}{24}$

E = ___

6. N = $\frac{26}{48}$

N = ___

7. W = $\frac{6x^2y}{18yx}$

W = ___

8. V = $\frac{38}{57}$

V = ___

9. A = $\frac{12ab^2}{16bc}$

A = ___

10. U = $\frac{62}{70}$

U = ___

11. S = $\frac{14xy}{26y^2}$

S = ___

12. T = $\frac{28bc}{32ab^2}$

T = ___

13. O = $\frac{64}{84}$

O = ___

14. D = $\frac{22}{88}$

D = ___

15. H = $\frac{36x^2y^2}{44yx^3}$

H = ___

Match the numeric answer to each equation above with the numeric answers above the spaces below. Then place each corresponding letter on the appropriate blank to form the answer to this trivia question.

What mountain has the highest peak in the lower 48 states?

___	___	___	___	___	___	___	___	___	___	___	___
$\frac{16}{23}$	$\frac{16}{21}$	$\frac{31}{35}$	$\frac{13}{24}$	$\frac{7c}{8ab}$	$\frac{x}{3}$	$\frac{9y}{11x}$	$\frac{3}{16}$	$\frac{7c}{8ab}$	$\frac{13}{24}$	$\frac{3}{8}$	$\frac{1}{12}$

In which mountain range is this peak located?

___	___	___	___	___	___	___	___	___	___	___	___
$\frac{7x}{13y}$	$\frac{3}{16}$	$\frac{3}{8}$	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{3ab}{4c}$	$\frac{13}{24}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{3ab}{4c}$	$\frac{1}{4}$	$\frac{3ab}{4c}$

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4-5 Enrichment

Dividing Powers with Different Bases

Some powers with different bases can be divided. First, you must be able to write both as powers of the same base. An example is shown below.

$$\frac{2^5}{8^2} = \frac{2^5}{(2^3)^2}$$ To find the power of a power, multiply the exponents.

$$= \frac{2^5}{2^6}$$

$$= 2^{-1} \text{ or } \frac{1}{2}$$

This method could not have been used to divide $\frac{2^5}{9^2}$, since 9 cannot be written as a power of 2 using integers.

Simplify each fraction using the method shown above. Express the solution without exponents.

1. $\frac{8^2}{2^2}$

2. $\frac{16^4}{8^3}$

3. $\frac{9^3}{3^3}$

4. $\frac{81^4}{3^4}$

5. $\frac{3^9}{81^2}$

6. $\frac{32^4}{16^4}$

7. $\frac{125^2}{25^3}$

8. $\frac{6^6}{216^2}$

9. $\frac{10^6}{1000^3}$

10. $\frac{64^3}{8^5}$

11. $\frac{27^5}{9^4}$

12. $\frac{343^3}{7^5}$

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4-6 Enrichment

Proving Definitions of Exponents

Recall the rules for multiplying and dividing powers with the same base. Use these rules, along with other properties you have learned, to justify each definition. Abbreviations for some properties you may wish to use are listed below.

Associative Property of Multiplication (APM)	Additive Identity Property (AIP)
Multiplicative Identity Property (MIP)	Inverse Property of Addition (IPA)
Inverse Property of Multiplication (IPM)	

Write the reason for each statement.

1. Prove: $a^0 = 1$

Statement	**Reason**
Let m be an integer, and let a be any nonzero number.	**a.** Given
$a^m \cdot a^0 = a^{m+0}$	**b.** ____________
$a^m \cdot a^0 = a^m$	**c.** ____________
$\frac{1}{a^m} \cdot (a^m \cdot a^0) = \frac{1}{a^m} \cdot a^m$	**d.** ____________
$\left(\frac{1}{a^m} \cdot a^m\right) \cdot a^0 = \frac{1}{a^m} \cdot a^m$	**e.** ____________
$1 \cdot a^0 = 1$	**f.** ____________
$a^0 = 1$	**g.** ____________

2. Prove: $a^{-n} = \frac{1}{a^n}$

Statement	**Reason**
Let n be an integer, and let a be any nonzero number.	**a.** Given
$a^{-n} \cdot a^n = a^{-n+n}$	**b.** ____________
$a^{-n} \cdot a^n = a^0$	**c.** ____________
$a^{-n} \cdot a^n = 1$	**d.** ____________
$(a^{-n} \cdot a^n) \cdot \frac{1}{a^n} = 1 \cdot \frac{1}{a^n}$	**e.** ____________
$a^{-n} \cdot \left(a^n \cdot \frac{1}{a^n}\right) = 1 \cdot \frac{1}{a^n}$	**f.** ____________
$a^{-n} \cdot 1 = 1 \cdot \frac{1}{a^n}$	**g.** ____________
$a^{-n} = \frac{1}{a^n}$	**h.** ____________

4-7 Enrichment

Scientific Notation

It is sometimes necessary to multiply and divide very large or very small numbers using scientific notation.

To multiply numbers in scientific notation, use the following rule.

For any numbers a and b, and any numbers c and d,
$(c \times 10^{a})(d \times 10^{b}) = (c \times d) \times 10^{a+b}$.

Example 1

$$\begin{aligned}(3.0 \times 10^{4})(-5.0 \times 10^{-2}) &= [3.0 \times (-5.0)] \times 10^{4+(-2)}\\ &= -15.0 \times 10^{2}\\ &= -1.5 \times 10^{3} \text{ or } -1500\end{aligned}$$

To divide numbers in scientific notation, use the following rule.

For any numbers a and b, and any numbers c and d, $(d \neq 0)$
$(c \times 10^{a}) \div (d \times 10^{b}) = (c \div d) \times 10^{a-b}$.

Example 2

$$\begin{aligned}(24 \times 10^{-4}) \div (1.5 \times 10^{2}) &= (24 \div 1.5) \times 10^{-4-2}\\ &= 16 \times 10^{-6}\\ &= 1.6 \times 10^{-5} \text{ or } 0.000016\end{aligned}$$

Exercises

Multiply or divide. Express each product or quotient in scientific notation.

1. $(2.7 \times 10^{9}) \times (3.1 \times 10^{2})$

2. $(6.1 \times 10^{-2}) \times (1.3 \times 10^{5})$

3. $(5.4 \times 10^{-3}) \div (1.8 \times 10^{2})$

4. $(6.9 \times 10^{-3}) \div (3.0 \times 10^{-8})$

5. $(1.1 \times 10^{-5}) \times (9.9 \times 10^{-1})$

6. $(4.0 \times 10^{0}) \div (1.0 \times 10^{-2})$

Solve. Write your answers in standard form.

7. The distance from Earth to the Moon is about 2.0×10^{5} miles. The distance from Earth to the Sun is about 9.3×10^{7} miles. How many times farther is it to the Sun than to the Moon?

8. If each of the 3.0×10^{4} people employed by Sunny Motors earned 4.0×10^{4} dollars last year, how much money did the company pay out to its employees?

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5-1 Enrichment

Matching Equivalent Fractions and Decimals

Cut out the pieces below and match the edges so that equivalent fractions and decimals meet. The pieces form a 4 × 6 rectangle. The outer edges of the rectangle formed will have no fractions or decimals.

0.9 $\frac{15}{35}$ $\frac{5}{9}$ $\frac{3}{24}$	$\frac{1}{7}$ $\frac{12}{42}$ $\frac{3}{8}$ 0.2	$\frac{11}{12}$ $\frac{16}{28}$ $\frac{5}{8}$ $\frac{45}{50}$	$\frac{12}{13}$ 0.4 $\frac{4}{7}$ $\frac{40}{45}$	$\frac{8}{9}$ $\frac{5}{15}$ $\frac{3}{7}$ $\frac{3}{21}$
$\frac{9}{11}$ $\frac{15}{21}$ $\frac{40}{44}$	0.3 $\frac{1}{2}$ $\frac{52}{56}$	$\frac{1}{9}$ $\frac{35}{77}$ $\frac{7}{13}$	$\frac{1}{8}$ $\frac{18}{48}$ $\frac{4}{9}$ $\frac{2}{18}$	0.8 $\frac{27}{33}$
$\frac{1}{12}$ $\frac{24}{54}$ $\frac{2}{20}$	0.1 $\frac{49}{91}$	$\frac{6}{8}$ $\frac{4}{5}$ $\frac{21}{33}$	0.875 $\frac{1}{3}$ $\frac{3}{18}$	$\frac{7}{11}$ $\frac{9}{15}$ $\frac{5}{7}$ $\frac{44}{48}$
$\frac{4}{6}$ 0.75 $\frac{9}{33}$	$\frac{1}{5}$ $\frac{14}{56}$ $\frac{5}{11}$	$\frac{1}{4}$ $\frac{2}{8}$	$\frac{1}{6}$ $\frac{2}{7}$ 0.25	$\frac{13}{14}$ $\frac{2}{5}$ $\frac{35}{40}$
$\frac{2}{3}$ $\frac{6}{20}$	$\frac{6}{7}$ $\frac{25}{45}$ $\frac{3}{36}$		$\frac{3}{11}$ $\frac{3}{6}$ 0.6 $\frac{48}{52}$	$\frac{10}{11}$ 0.625 $\frac{30}{35}$

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5-2 Enrichment

Irrational Numbers

Decimals in which a digit or group of digits repeats are called **repeating decimals**.

Example1

$0.333\ldots = 0.\overline{3}$
$1.454545\ldots = 1.\overline{45}$ The bar indicates the repeating digit or digits.

Decimals in which only a 0 repeats are called **terminating decimals**.

Example 2

$0.5000\ldots = 0.5$
$42.19500\ldots = 42.195$

Consider the decimal 0.757757775 . . . . It does not terminate in zero nor does it have a group of digits that repeat. Numbers that are represented by nonterminating, nonrepeating decimals are irrational numbers.

Example 3

$\pi = 3.14159\ldots$
$\sqrt{5} = 2.236068\ldots$

Exercises

Determine whether each decimal is repeating or nonrepeating.

1. 0.373373337 . . .

2. 24.15971597 . . .

3. 5.71571571 . . .

4. 0.5795579555 . . .

5. 8.3121121112 . . .

Name the next three digits in the following irrational numbers.

6. 0.13141516 . . .

7. 0.96796679666 . . .

Name an irrational number between each pair of numbers.

8. 6.7 and 6.8

9. 17.3 and 17.4

10. 0.1231233 . . . and 0.1231134111 . . .

11. 2.333 . . . and 2.444 . . .

Find each sum.

12. 0.232232223 . . . + 0.323323332 . . .

13. 0.131131113 . . . + 0.868868886 . . .

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5-3 Enrichment

Finding Sums of Arithmetic Series

The numbers 7, 11, 15, 19, 23, 27, 31, 35, . . . form an arithmetic sequence. It might take a long time to find the sum of the **series** 7 + 11 + 15 + . . . + 35 even if you had a computer or a calculator. Formulas have been developed to make solving these problems much easier.

An **arithmetic series** is the sum of the first n terms of an arithmetic sequence, where n is the number of terms in the arithmetic series. To add an arithmetic series using a formula, you need to know the first term, the common difference, and the number of terms.

Example **Find the sum of the arithmetic series $7 + 11 + 15 + 19 + 23 + 27 + 31 + 35$.**

The first term a is 7.
The common difference d is 4.
The number of terms n is 8.

Use the formula $S_n = \frac{n}{2}[2a + (n - 1)d]$ and substitute to find the sum.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_8 = \frac{8}{2}[2(7) + (8 - 1)4]$$

S_8 means the sum of the first eight terms of the series.

$$S_8 = 4[14 + (7)4]$$

$$S_8 = 4(42)$$

$$S_8 = 168$$

The sum of the series is 168.

Use the formula to find the sum of each arithmetic series.

1. $7 + 11 + 15 + 19 + 23$

2. $12 + 17 + 22 + 27 + 32 + 37$

3. $0 + 6 + 12 + 18 + \ldots + 36$

4. $50 + 57 + 64 + \ldots + 92$

5. $5 + 10 + 15 + 20 + \ldots + 65$

6. $9 + 18 + 27 + 36 + \ldots + 72$

7. $9 + 20 + 31 + 42 + \ldots + 86$

8. $21 + 29 + 37 + 45 + 53 + \ldots + 93$

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5-4 Enrichment

Number Theory Proof

Use your knowledge of properties to demonstrate why the reciprocal of the opposite of a number multiplied by the opposite of that number is equal to 1. In other words, show why $-\frac{1}{a} \cdot (-\ a) = 1$.

Write the reason for each statement.

1. Prove: $-\frac{1}{a} \cdot -a = 1$

Statement	**Reason**
Let a be any number.	**a.** Given
$-\frac{1}{a} \cdot (-a) = -a^{-1} \cdot (-a)$	**b.** ________________
$-\frac{1}{a} \cdot (-a) = -1a^{-1} \cdot (-1a)$	**c.** ________________
$-\frac{1}{a} \cdot (-a) = 1a^{-1+1}$	**d.** ________________
$-\frac{1}{a} \cdot (-a) = 1a^{0}$	**e.** ________________
$-\frac{1}{a} \cdot (-a) = 1 \cdot 1$	**f.** ________________
$-\frac{1}{a} \cdot (-a) = 1$	**g.** ________________

Use fractions to write an example showing each property of multiplication.

2. Commutative Property

____

3. Associative Property

____

4. Identity Property

____

5. Inverse Property

____

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5-5 Enrichment

Magic Squares

A *magic square* is a square array of numbers in which all rows, columns, and diagonals have the same sum, called the magic sum.

To make a magic square with fractions, follow these steps.

1. Draw a 3-by-3 square with 9 spaces. Then add a square on each side as shown by the dashed lines in Figure 1 below. Number the squares as shown.
2. Choose any fraction and put it in box 5. See Figure 2.
3. Choose any fraction to add to box 5 and put the answer in box 2. Then add that same fraction to the result in box 2 and put this answer in box 1. See Figure 2.
4. Choose any fraction to add to box 5 and put the answer in box 10. Then add that same fraction to the result in box 10 and put this answer in box 13. See Figure 2.
5. Complete the upward diagonals by adding $\frac{3}{24}$ each time.
6. Complete the downward diagonals by adding $\frac{5}{24}$ each time.
7. Now complete the magic square by putting the fraction in box 5 into box 8, the fraction in box 9 into box 6, the fraction in box 1 into box 11, and the fraction in box 13 into box 3. See Figure 3.

Figure 1

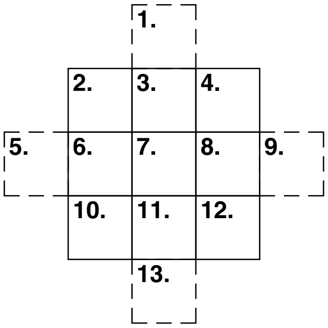


Figure 2

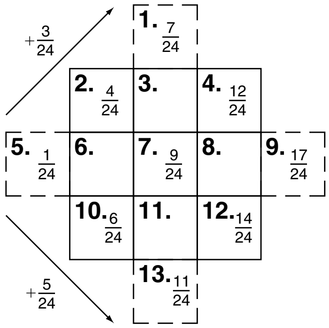


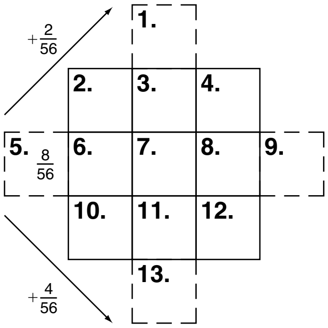
Figure 3

$\frac{4}{24}$	$\frac{11}{24}$	$\frac{12}{24}$
$\frac{17}{24}$	$\frac{9}{24}$	$\frac{1}{24}$
$\frac{6}{24}$	$\frac{7}{24}$	$\frac{14}{24}$

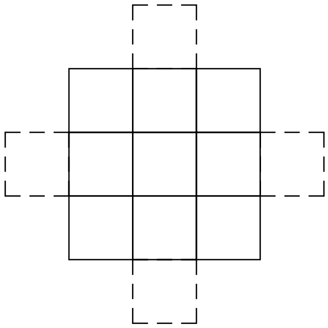
The magic sum is $\frac{27}{24}$ or $1\frac{1}{8}$.

Use the steps above to complete each exercise.

1. Complete the magic square below.



2. Make your own magic square. Check the rows and diagonals to be sure they have the same sum.



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5-6 Enrichment

GCF, LCM, and Ladders

When two or more numbers have common factors, the greatest of these is called the **greatest common factor (GCF)**. When two or more numbers have nonzero common multiples, the least of these is called the **least common multiple (LCM)** of the numbers. A method of determining the GCF and LCM for two or more numbers is the ladder.

Example **Find the GCF and LCM of 24 and 36.**

6	24	36
2	4	6
1	2	3
	2	3

Find any number that will divide into both numbers.

Both 24 and 36 can be divided evenly by 6.

Continue to choose divisors until the only remaining divisor is 1.

To find the GCF, multiply all the divisors.

$6 \cdot 2 \cdot 1 = 12$ The GCF is 12.

To find the LCM, multiply the GCF and the numbers remaining on the bottom of the ladder.

$12 \cdot 2 \cdot 3 = 72$ The LCM is 72.

Find the GCF and LCM for each pair of numbers by using a ladder.

1. 18, 27

2. 24, 42

3. 44, 66

4. 36, 81

5. 42, 48

6. 100, 125

7. 64, 72

8. 90, 150

9. 250, 300

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5-7 Enrichment

Cooking Class

Before beginning a recipe, it is a good idea to check your pantry to approximate how much of each ingredient you have on hand. Then, make a shopping list so you can get any groceries you need in one trip.

Raul and Elizabeth have been invited to brunch at a friend's house and have each been asked to bring muffins. They decided to bake blueberry muffins and banana nut muffins together at Elizabeth's house for the brunch.

Blueberry Muffins
INGREDIENTS:

$2\frac{1}{3}$ cups all-purpose flour
$\frac{1}{2}$ cup white sugar
3 teaspoons baking powder
$\frac{1}{2}$ teaspoon salt
$\frac{3}{4}$ cup milk
$\frac{1}{3}$ cup vegetable oil
1 egg
1 cup fresh blueberries

Banana Nut Muffins
INGREDIENTS:

$2\frac{1}{4}$ cups all-purpose flour
$1\frac{1}{3}$ cup white sugar
$\frac{1}{2}$ teaspoon salt
$\frac{1}{2}$ cup vegetable oil
3 ripe bananas, mashed
$\frac{1}{4}$ cup chopped walnuts
$1\frac{1}{3}$ teaspoon baking soda
1 egg

Elizabeth's Pantry

Ingredients	Amounts
Sugar	$\approx \frac{1}{2}$ of a 5 pound bag
Salt	unopened 26 ounce container
Vegetable Oil	$\approx \frac{1}{2}$ of a 24 ounce bottle
Baking Powder	$\approx \frac{1}{2}$ of a 10 ounce can
Milk	$\approx \frac{1}{2}$ gallon
Flour	5 pound bag and $\approx$ 1 cup

1. How much of each ingredient will Raul and Elizabeth need to make both recipes?

2. Which ingredients will they need to purchase?

3. For some of the ingredients, Raul and Elizabeth may have to buy more than they need depending on what size of packages they find at the store. Of the ingredients on their list, which one might they be able to buy only as much as they need?

4. Why would Raul and Elizabeth never use the combined amounts when preparing their ingredients?

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5-8 Enrichment

Unit Price

Large grocery store chains often display unit prices to help shoppers compare between different brands of products and different sizes. The unit price is the price of a product per unit of volume and can be written as an equation: unit price $= \frac{\text{price}}{\text{volume}}$. You can determine the unit price of a product using this equation to comparison shop.

Example **An 18-ounce can of chicken noodle soup costs $2.19, while the cost of a 22-ounce can is $2.39. Would you be getting more for your money if you purchased the 18 ounce can or the 22 ounce can?**

18-ounce can:	22 ounce-can:
unit price = $2.19/18 ounces	unit price = $2.39/22 ounces
unit price ≈ $0.12/ounce	unit price ≈ $0.10/ounce

The 22-ounce can has a lower unit price than the 18-ounce can, making it the better value.

Exercises

1. **DOG FOOD** A 7-pound bag of dog food costs $11.75 while a 20-pound bag is priced at $18.99. What is the unit price for each bag? Which size is the better value?

2. **CEREAL** Your favorite cereal comes in three sizes, 15 ounces, 17 ounces and 20 ounces. The smallest size costs $3.79, the medium size is priced at $3.99 and the large size is 20 cents more than the medium size. Which should you buy to get the most for your money?

3. **CUPCAKES** You can buy 12 cupcakes for $4.99. If the unit price for 16 is $0.38 cents, which package is the better deal?

4. **ICE CREAM** You prefer the name brand of a certain ice cream flavor over the less expensive store brand. The unit price of the name brand is $1.39 per quart, while the unit price of the store brand is $4.36 per gallon.
   - **a.** Which product is less expensive per gallon? (*Hint:* 1 gallon = 4 quarts)
   - **b.** How much would the name brand ice cream have to be discounted per quart to make it the same price as the store brand?

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5-9 Enrichment

Mean Variation

Mean variation is the average amount by which the data differ from the mean.

Example The mean for the set of data at the right is 17.

12, 16, 27, 16, 14

Find the mean variation as follows.

Step 1 Find the difference between the mean and each item in the set.

$$\begin{array}{rr} 17 - 12 = & 5 \\ 17 - 16 = & 1 \\ 27 - 17 = & 10 \\ 17 - 16 = & 1 \\ 17 - 14 = & \underline{+\ 3} \\ & 20 \end{array}$$

Step 2 Add the differences.

Step 3 Find the mean of the differences. This is the mean variation.

$$\frac{20}{5} = 4$$

The mean variation is 4.

Exercises

Find the mean variation for each set of numbers.

1. 100, 250, 200, 175, 300
The mean is 205.

205 − 100 = ________
250 − 205 = ________
205 − 200 = ________
205 − 175 = ________
300 − 205 = ________

Add the differences above.

Then, divide by 5.

The mean variation is ______.

2. 124, 128, 121, 123
The mean is 124.

124 − 124 = ________
128 − 124 = ________
124 − 121 = ________
124 − 123 = ________

Add the differences above.

Then, divide by 4.

The mean variation is ______.

For each set of data, find the mean and the mean variation.

3. 68, 43, 28, 25

4. 13, 18, 22, 28, 35, 46

5. 68, 25, 36, 42, 603, 16, 8, 18

6. 79, 81, 85, 80, 78, 86, 84, 83, 75, 88

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6-1 Enrichment

Comparison Shopping

Rates are useful and meaningful when expressed as a unit rate. For example, which is the better buy—one orange for $0.50 or 8 oranges for $3.49?

To find the unit rate for 8 oranges, divide $3.49 by 8. The result is $0.44 per orange. If a shopper needs to buy at least 8 oranges, then 8 oranges for $3.49 is the better buy.

For each exercise below, rates are given in Column A and Column B. In the blank next to each exercise number, write the letter of the column that contains the better buy.

		Column A	**Column B**
______	**1.**	1 apple for $0.19	3 apples for $0.59
______	**2.**	20 pounds of pet food for $14.99	50 pounds of pet food for $37.99
______	**3.**	A car that travels 308 miles on 11 gallons of gasoline	A car that travels 406 miles on 14 gallons of gasoline
______	**4.**	10 floppy discs for $8.99	25 floppy discs for $19.75
______	**5.**	1-gallon can of paint for $13.99	5-gallon can of paint for $67.45
______	**6.**	84 ounces of liquid detergent for $10.64	48 ounces of liquid detergent for $6.19
______	**7.**	5000 square feet of lawn food for $10.99	12,500 square feet of lawn food for $29.99
______	**8.**	2 compact discs for $26.50	3 compact discs for $40.00
______	**9.**	8 pencils for $0.99	12 pencils for $1.49
______	**10.**	1000 sheets of computer paper for $8.95	5000 sheets of computer paper for $41.99

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6-2 Enrichment

Direct Variation

If the relationship between two quantities is such that when one quantity increases, the other increases proportionally, or when one quantity decreases, the other decreases proportionally, the quantities are said to **vary directly**.

Example 1 **MONEY If 2 loaves of bread cost $1.60, how much will 3 loaves of the same type of bread cost?**

As the number of loaves increases, so does the cost.
If 2 loaves cost $1.60, then 1 loaf costs $0.80.
Use this information to complete a proportion table.

Loaves	1	2	3
Cost	$0.80	$1.60	$2.40

Use a proportion table to solve.

1. **MONEY** Three gallons of gasoline cost $3.36. How much do 5 gallons cost?

2. **DISTANCE** At a rate of 50 mph, a car travels a distance of 600 miles. How far will the car travel at a rate of 40 mph if it is driven the same amount of time?

3. **MONEY** If the rent for two weeks is $500, how much rent is paid for 5 weeks?

4. **MONEY** If 8 newspapers cost $3.20, how much will 6 newspapers cost?

5. **WEIGHT** If 9 fully loaded trucks carry a total of 140,400 pounds, how many pounds can 3 trucks carry?

6. **TECHNOLOGY** Twelve floppy disks can hold 16.8 million bytes of data. How many bytes will 20 floppy disks hold?

7. **TECHNOLOGY** If 12 floppy disks hold 16.8 million bytes of data, how many floppy disks are needed to hold 10 million bytes of data?

8. **HEALTH** There are a total of 2100 Calories in 5 candy bars (all the same kind). How many total Calories are there in 3 dozen of these candy bars?

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6-3 Enrichment

Cross Products Proof

Recall the Cross Products Property: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. Use the statements below to justify this property.

Write the reason for each statement.

1. Prove: $ad = bc$

Statement	Reason
$\frac{a}{b} = \frac{c}{d}$	**a.** Given
$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$	**b.** ____________________________________
$bd \cdot \left(a \cdot \frac{1}{b}\right) = bd \cdot \left(c \cdot \frac{1}{d}\right)$	**c.** Rewrite division as multiplication.
$\left(b \cdot \frac{1}{b}\right)ad = \left(d \cdot \frac{1}{d}\right)cb$	**d.** ____________________________________
$1ad = 1cb$	**e.** ____________________________________
$ad = bc$	**f.** ____________________________________

Solve each proportion problem.

2. AGE The ratio of Brandi's age to Jason's age is 4:5. In eight years the ratio will be 6:7. How old are Brandi and Jason now?

3. AGE The ratio of Drew's age to Stacey's age is 3:4. Four years ago the ratio was 2:3. How old were Drew and Stacey four years ago?

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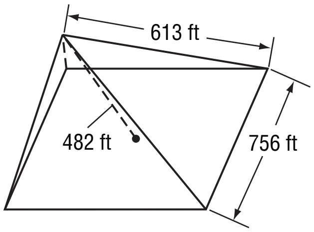
6-4 Enrichment

Great Pyramid

The Great Pyramid of Giza is the only surviving wonder of the Seven Wonders of the World. Construction began on this remarkable project in 2680 BC and lasted many years. The pyramid is a solid mass of limestone blocks with a base measuring 756 feet on every side and a height of 482 feet. The length of each edge is 613 feet.



1. If you wanted to make a model of the Great Pyramid that was 1/125 the size, how long would the sides of the base be?

2. What would the length of each edge be?

3. What would be the height?

4. If you made a model of the Great Pyramid that was 3 feet tall, what would be the scale of the model?

5. Using the information and answer from Exercise 4, determine the length of the base and sides of the model.

6. If you made a scale drawing of the Great Pyramid where 1 inch = 150 feet, what would be the measurements for the base, sides and height of the drawing?

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6-5 Enrichment

Political Polling

In politics, polling is used to gauge public opinion and can help shape the policies adopted by state legislatures and the United State Congress. Polling is a scientific process that involves very specific rules and techniques. The purpose of polling is to get an idea of the views of a large group or population without having to interview each person individually.

A state representative from Austin, Texas, decided to have polling done of his legislative district in order to determine the 'pulse' of his constituents on various state issues. For the poll, five hundred thirty people were surveyed by telephone over a three-day period. Here are the results:

Question	Yes	No	Undecided
1. Do you feel that Texas's economy is strong?	46%	39%	15%
2. Is our state headed in a positive direction?	38%	49%	13%
3. Should we raise taxes to support state funding of primary and secondary education?	17%	63%	20%
4. Should eligibility for HICAP be expanded?	29%	44%	27%
5. Is the state legislature in touch with the views of the people of Texas?	45%	46%	9%

1. Write the percentages of each answer from question 2 as fractions in the simplest form.

Yes

No

Undecided

2. Express the percent of people who feel that Texas is headed in a positive direction as a decimal.

3. Fill in the table by expressing each percent from the chart above as a fraction and a decimal. Three are completed for you.

Question	Yes	No	Undecided
1. Do you feel that Texas's economy is strong?			
2. Is our state headed in a positive direction?		$\frac{49}{100}$, 0.49	
3. Should we raise taxes to support state funding of primary and secondary education?			$\frac{1}{5}$, 0.20
4. Should eligibility for HICAP be expanded?	$\frac{29}{100}$, 0.29		
5. Is the state legislature in touch with the views of the people of Texas?			

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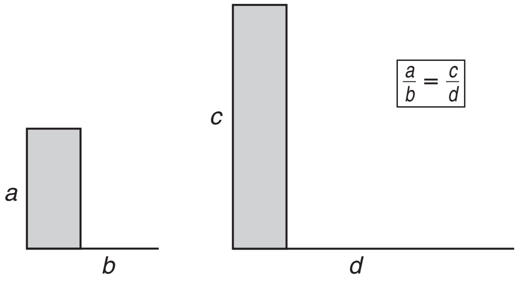
6-6 Enrichment

Using Proportions to Approximate Height

A proportion can be used to determine the height of tall structures if three variables of the proportion are known. The three known variables are usually the height, a, of the observer, the length, b, of the observer's shadow, and the length, d, of the structure's shadow. However, a proportion can be solved given any three of the four variables.



$\frac{a}{b} = \frac{c}{d}$

This chart contains information about various observers and tall buildings. Use proportions and your calculator to complete the chart of tall buildings of the world.

	Height of Observer	Length of Shadow	Building Location	Height of Building	Length of Shadow
1.	5 ft	8 in.	Natwest, London		80 ft
2.	4 ft 9 in.		Columbia Seafirst Center, Seattle	954 ft	318 ft
3.	5 ft 10 in.	14 in.	Wachovia building, Winston-Salem	410 ft	
4.		15 in.	Waterfront Towers, Honolulu	400 ft	83 ft 4 in.
5.	6 ft		CN Tower, Toronto	1821 ft	303 ft 6 in.
6.	5 ft 9 in.	1 ft 11 in.	Gateway Arch, St. Louis	630 ft	
7.	5 ft 3 in.	1 ft 9 in.	Eiffel Tower, Paris		328 ft
8.		8 in.	Texas Commerce Tower, Houston	1002 ft	167 ft
9.	5 ft 6 in.	11 in.	John Hancock Tower, Boston		131 ft 8 in.
10.	5 ft 4 in.	8 in.	Sears Tower, Chicago		181 ft 9 in.
11.	5 ft 6 in.		Barnett Tower, Jacksonville	631 ft	105 ft 2 in.

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6-7 Enrichment

Using Mental Math While Shopping

One way of estimating percents to find the sale price of an item while shopping is to think of the percent in terms of how much you save for each $10 or for each $1 you spend.

Example 1 **A sweater that you would like to buy is priced at $40, but is on sale for 30% off. What is the sale price?**

Think: At that rate, for every $10, I save $3.
$40 = 4 × $10, so the savings for $40 would be 4 × $3 or $12.
Subtract the savings from the original price: $40 − $12 = $28.
The sale price is $28.

Example 2 **A $36 pair of jeans is on sale for 40% off. How much are the jeans?**

Think: For every $10, I save $4, and for every $1, I save $0.40.
$36 = 3 ×$10 and 6 × $1, so the savings would be 3 × $4 + 6 × $0.40 or $14.40.
Round $14.40 to $14 and subtract from the original price: $36 − $14 = $22.
The jeans cost approximately $22.

Exercises

Sarah is going to the mall and has $75 to buy a new pair of jeans, a hooded sweatshirt, and shoes. Store 1 is having a sale where everything is 20% off, while all the items in Store 2 are 30% off.

Store 1	
Item	**Price Before Sale**
Jeans	$30
Shoes	$25
Hooded Sweatshirt	$20

Store 2	
Item	**Price Before Sale**
Jeans	$34
Shoes	$31
Hooded Sweatshirt	$25

1. How much would each item cost if purchased at Store 1?

2. Is Sarah within her budget if she buys all of her items there? If so, how much money will she have left once she makes her purchases?

3. Does she have enough money to purchase all her items at Store 2? How much will she have left over?

4. What is the approximate sale price of each item at Store 2?

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6-8 Enrichment

Compound Interest

Interest may be paid, or compounded, annually (each year), semiannually (twice per year), quarterly (four times per year), monthly (once per month), or daily.

Example FINANCE **George had $100 in an account for $1\frac{1}{2}$ years that paid 8% interest compounded semiannually. What was the total amount in his account at the end of $1\frac{1}{2}$ years?**

At the end of $\frac{1}{2}$ year: Interest: $\$100 \times 0.08 \times \frac{1}{2} = \$4.00$

New Principal: $100 + $4 = $104

At the end of 1 year: Interest: $\$104 \times 0.08 \times \frac{1}{2} = \$4.16$

New Principal: $104 + $4.16 = $108.16

At the end of $1\frac{1}{2}$ years: Interest: $\$108.16 \times 0.08 \times \frac{1}{2} = \$4.33$

New Principal: $108.16 + $4.33 = $112.49

Exercises

Find the total amount for each of the following.

	Principal	Rate	Time	Compounded	Total Amount
1.	$200	6%	$1\frac{1}{2}$ years	semiannually	
2.	$300	5%	2 years	semiannually	
3.	$100	6%	1 year	quarterly	
4.	$500	8%	$\frac{3}{4}$ year	quarterly	
5.	$500	10%	4 months	monthly	
6.	$800	8%	$\frac{1}{4}$ year	monthly	
7.	$1000	8%	4 years	annually	
8.	$700	6%	$1\frac{1}{2}$ years	semiannually	

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6-9 Enrichment

Finance Charges

Previous Balance	Total Purchases	Payments Returns & Other Credits	*FINANCE CHARGE	This Is Your New Balance	Payments Overdue	This Is Your Minimum Payment
245.67	*138.56*	*184.23*	*2.94*	*202.94*		*25.00*

*ANNUAL PERCENTAGE RATE OF 15% COMPUTED
ON THE AVERAGE DAILY BALANCE OF $235.19.

Eileen Farrell received her monthly statement for her department store charge account. She left part of her balance unpaid last month. Her statement says that her finance charge is $2.94. This is computed at the annual rate of 15% on an average daily balance of $235.19. Is the charge correct?

Example FINANCE **Compute the monthly finance charge for Eileen's unpaid balance.**

The annual percentage rate is 15%. To find the monthly rate, divide by 12. Let B = average daily balance, R = monthly rate, and P = annual percentage rate.

Monthly rate: $15\% \div 12 = 1.25\%$ Compute 1.25% of $235.19.

$B \cdot R = P \rightarrow 235.19 \cdot 0.0125 = P$ 1.25% = 0.0125

$2.939875 = P$

$2.94 = P$ Round to the nearest cent.

The finance charge is $2.94. The statement is correct.

Exercises

Find the monthly finance charge to the next cent on each average daily balance.

1. Balance: $165.00
Annual percentage rate: 18%

2. Balance: $231.00
Annual percentage rate: 15%

3. Balance: $713.00
Annual rate: 12%

4. Balance: $147.93
Annual rate: 15.5%

5. Balance: $419.60
Annual rate: 14%

6. Balance: $175.14
Annual rate: 16%

7. Balance: $450.00
Annual rate: 17%

8. Balance: $368.50
Annual rate: 18%

9. Balance: $250.50
Annual rate: $18\frac{1}{2}\%$

10. Balance: $654.90
Annual rate: 12.5%

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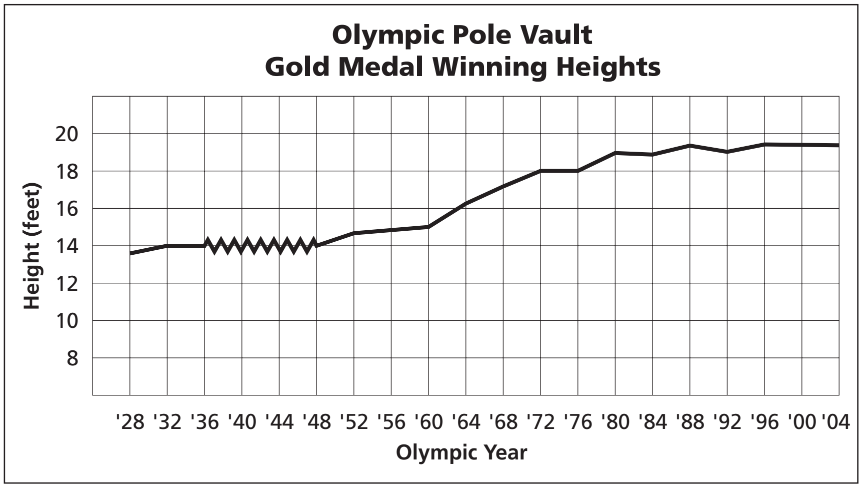
6-10 Enrichment

Using Graphs to Predict

SPORTS Refer to the graph at the right for Exercises 1–6.



Source: *The World Almanac*

1. Estimate the height to the nearest foot that pole vaulters probably cleared in 1964.

2. Estimate the year when 14 feet was cleared for the first time.

3. Estimate the height to the nearest foot that pole vaulters probably cleared in 1968.

4. Estimate the year when 18 feet was cleared for the first time.

5. If the Olympics had been held in 1940, predict what the winning height would have been (to the nearest foot).

6. Based on the trend from 1960 through 2004, would you predict the winning height in 2008 to be over or under 19 feet?

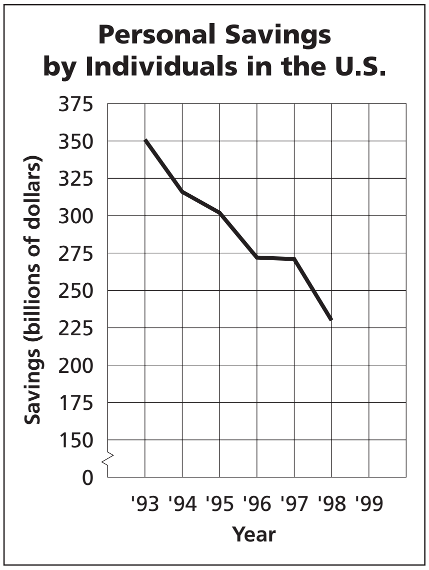
FINANCE Refer to the graph at the right for Exercises 7–10.

7. Based on the trend from 1993 through 1996, what level of savings would you predict for 1997?

8. How does your prediction for 1997 compare with the actual level of savings for 1997?

9. Based on the trend from 1993 through 1998, what level of savings would you predict for 1999?

10. The actual level of savings in 1999 was $156.3 billion. How does your prediction for 1999 compare with this actual level?



Source: *Statistical Abstract for the United States*

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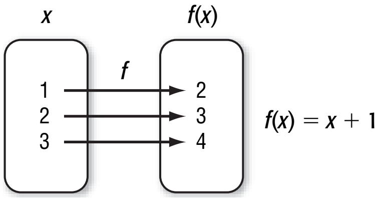
7-1 Enrichment

Emmy Noether

Emmy Noether (1882–1935) was a German mathematician and a leading figure in modern abstract algebra. Her contributions helped change the role of women in German universities and advanced the mathematical progress of the time. Noether fought and overcame rules that once prevented her from becoming a faculty member. Her most notable work pertains to linear transformations of non-commutative algebras and their structures.

A linear equation is an example of a **transformation**, unless it is the equation for a vertical line. In a transformation, a given rule transforms each number in one set, the **domain**, into one and only one number in another set, the **range**. In graphing linear equations, the domain is usually the set of real numbers and the range is either the set of real numbers or a subset of the real numbers. Each value of x is transformed into some value of y.

Transformation f is shown in the diagram below. The transformation takes each element of the domain {1, 2, 3} and adds 1 to produce the corresponding value in the range. Functional notation is used to show the rule $f(x) = x+1$.



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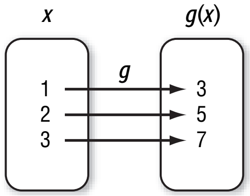

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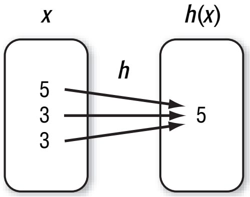
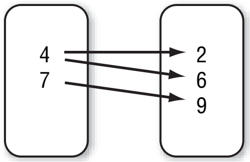


3. Explain whether the diagram below shows a transformation.



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7-2 Enrichment

Equations with Two Variables

Complete the table for each equation.

1. $y = 7 + x$

x	y
−4	
	5
	1

2. $y = 2x + 4$

x	y
6	
	12
	2

3. $y = x - 9$

x	y
3	
	−4
	−9

4. $y = 3x - 2$

x	y
2	
	−5
	7

5. $y = \frac{x}{4}$

x	y
8	
	4
	−6

6. $y = -6x + 1$

x	y
1	
	−11
	13

7. $y = 9 - 2x$

x	y
3	
	7
	1

8. $y = \frac{x + 5}{3}$

x	y
4	
	4
	−1

9. $y = \frac{x}{2} + 5$

x	y
8	
	8
	10

10. $y = x^2$

x	y
2	
	1
	16

11. $y = x^2 - 3$

x	y
3	
	22
	−3

12. $y = 1 - 2x$

x	y
−1	
	7
	11

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7-3 Enrichment

Fibonacci Sequence

The first fifteen terms of the **Fibonacci sequence** are shown below.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610

In addition to the fact that each term is the sum of the two previous terms, there are other interesting relationships.

Complete.

1. Find the sum of the first four terms.

2. Find the sum of the first five terms.

3. Find the sum of the first six terms.

4. What is the relationship between the sums and terms in the sequence?

5. Use your answer from Exercise 4 above to predict the sum of the first 10 terms, and of the first 12 terms. Check your answers by finding the sums.

6. Divide each of the first 15 terms by 4. Make a list of the remainders. What pattern do you see in the remainder sequence?

7. The sums of the squares of consecutive terms of the Fibonacci sequence are shown below. Complete the next three lines of this pattern.

$1^2 + 1^2 = 1 \times 2$

$1^2 + 1^2 + 2^2 = 2 \times 3$

$1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \times 8$

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7-4 Enrichment

Curves With Varying Rates of Change

If you know the graph is going to be a straight line, then you only need two points (a third one as a check). But if the graph is going to be a curved line, then you need enough points to be able to sketch a nice smooth curve. This may be 5 or more points.

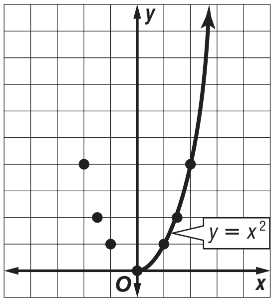
Step 1 Make a table of ordered pairs that satisfy the equation.

Step 2 Graph each point.

Step 3 Draw a smooth curve connecting the points.

Sketch a smooth curve through the points shown. The right side of the first curve has been done for you.

1.



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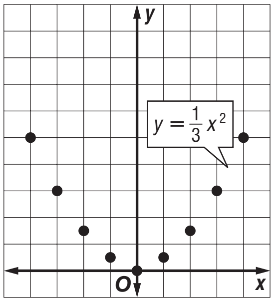

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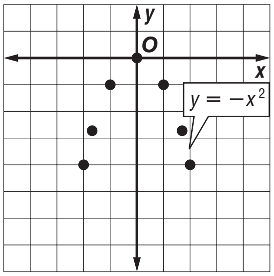
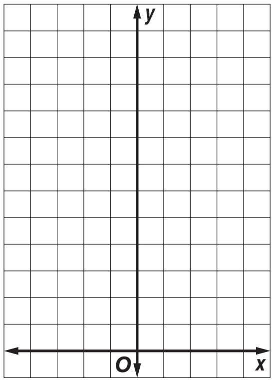


Graph each equation.

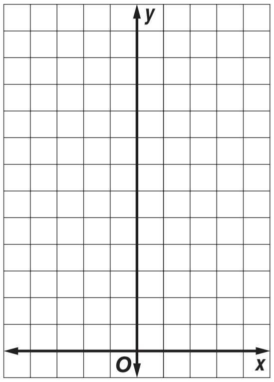
4. $y = 2x^2$

x	−2	−1.5	−1	0	1	−1.5	2
y	8	4.5	2	0			



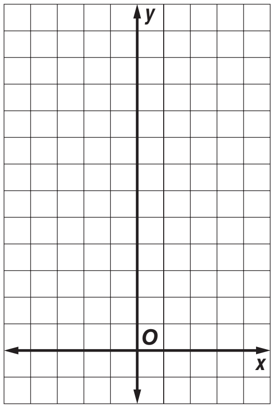
5. $y = x^2 + 2$

x	−3	−2	−1	0	1	2	3
y							



6. $y = x^2 - 1$

x	−3	−2	−1	0	1	2	3
y							



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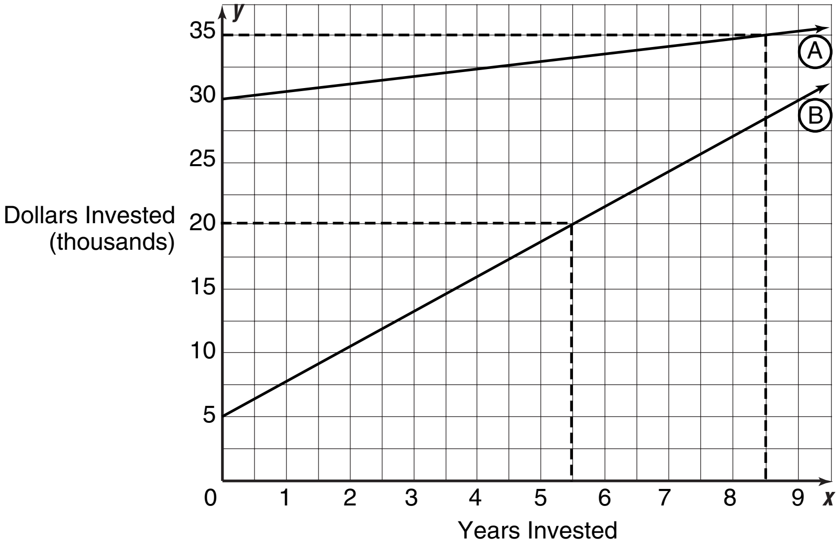
7-5 Enrichment

Investments

The graph below represents two different investments. Line A represents an initial investment of $30,000 at a bank paying passbook-savings interest. Line B represents an initial investment of $5000 in a profitable mutual fund with dividends reinvested and capital gains accepted in shares. By deriving the equation, $y = mx + b$, for A and B, a projection of the future can be made.



Solve.

1. The y-intercept, b, is the initial investment. Find b for each of the following.

a. line A **b.** line B

2. The slope of the line, m, is the rate of return. Find m for each of the following.

a. line A **b.** line B

3. What are the equations of each of the following lines?

a. line A **b.** line B

Answer each of the following, assuming that the growth of each investment continues in the same pattern.

4. What will be the value of the mutual fund after the 11th year?

5. What will be the value of the bank account after the 11th year?

6. When will the mutual fund and the bank account be of equal value?

7. In the long term, which investment has the greater payoff?

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7-6 Enrichment

Skiing

Ski slopes in the United States are often labeled according to a system that informs skiers of the level of skill that is required to safely ski the slope. The Black Diamond designation is used to indicate a difficult slope that is best attempted only by an advanced skier. A Double Diamond is even steeper, and is truly for experts only. These symbols represent Black Diamond and Double Diamond slopes at ski resorts throughout the country.

◆	Most Difficult
◆ ◆	Expert

The slope of the mountain is related to the slope of the line representing the height in comparison to time the skier is skiing.

Exercises

1. A skier takes a lift to the top of a Black Diamond mountain with a height of 1700 feet. Every minute the skier travels down the mountain, his altitude is 500 feet less.

a. What part of a linear equation does the 1700 feet represent?

b. Using the above information, what is the slope of the line that represents the change in the skier's altitude as he travels down the mountain?

c. Write a linear equation that calculates the skier's altitude.

2. On the skier's next run, he decides to try the Double Diamond run, which is at 1800 feet altitude. When skiing down the mountain, he is losing altitude at a rate of 645 feet per minute.

a. What is the slope representing the change in altitude?

b. How long will it take the skier to reach sea level (an altitude of 0)?

c. Write the linear equation that represents his altitude while skiing.

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7-7 Enrichment

Water Pressure

Water pressure is measured in atmospheres, which are abbreviated as *at*. One atmosphere is equal to the weight of Earth's atmosphere at sea level which is 14.6 pounds per square inch. Water pressure below sea level can be determined using a linear equation since pressure increases at a constant rate. For example, water pressure at 30 meters below sea level is twice what it would be at 15 meters. More specifically, for every 10 meters in depth, water pressure increases by 1 atmosphere.

Divers and researchers have to be very aware of water pressure in order to choose equipment that will withstand the water pressure at the depths where they want to explore. The equation used to determine water pressure is $P = 0.1d + 1$ where P = pressure in atmospheres and d = depth in meters.

Example **What is the water pressure when a diver is at a depth of 41 meters?**

$$\begin{aligned} P &= 0.1d + 1 \\ &= 0.1(41) + 1 \\ &= 5.1 \end{aligned}$$

The pressure at 41 meters below sea level is 5.1 atmospheres.

Exercises

1. Use the equation for water pressure to complete the table at the right.

2. If a diver is at a depth of 75 meters and his equipment can only withstand 10.4 atmospheres of pressure, how much deeper can he dive?

3. If a submarine is constructed to withstand submersion at a depth of 1100 meters, how many atmospheres of pressure can it support?

4. A diver is at a depth where the water pressure is 9.5 atmospheres, but he wants to dive down 15 additional meters to reach a sunken ship. If his equipment can withstand up to 9.9 atmospheres of pressure, can he reach the site? Explain.

Depth (in m)	Water Pressure (in atmospheres)
	16.1
475	
1567	
	23.5
4680	
	9.64
	13.75
	27.6
3169	
9237	
	772.1
14	

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7-8 Enrichment

Graphing Systems of Equations

Because checking accounts vary from one financial institution to another, educated consumers should carefully weigh the options offered by various institutions when choosing a checking account.

The four equations below describe the cost of four different checking accounts. In each equation, C represents the monthly cost in dollars, and n represents the number of checks written. Find the cost for the number of checks written in each account.

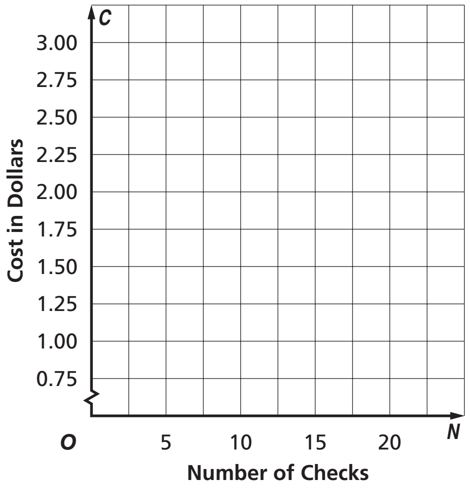
Account 1 $C = 0.1n + 1.50$	
n	C
0	
2	
5	
8	
12	
15	

Account 2 $C = 0.2n + 1.00$	
n	C
0	
2	
4	
5	
8	
10	

Account 3 $C = 0.25n + 0.75$	
n	C
0	
3	
5	
6	
8	
9	

Account 4 $C = 0.05n + 1.75$	
n	C
0	
2	
5	
9	
14	
20	

Graph and label each account on the grid.



1. The *break-even point* of the graph is the point at which the costs of the accounts are the same. What is the break-even point for these accounts?
2. Which account should be chosen by a consumer who writes fewer than five checks per month?
3. Which account should be chosen by a consumer who writes more than five checks per month?

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8-1 Enrichment

Fractional Equations

To solve equations containing fractions, multiply both sides by the least common denominator. Then solve as usual.

Example Solve $\frac{2x}{5} - \frac{x}{10} = 6$.

$$\frac{2x}{5} - \frac{x}{10} = 6$$

The least common denominator is 10.

$$10\left(\frac{2x}{5} - \frac{x}{10}\right) = 10(6)$$

$$10\left(\frac{2x}{5}\right) - 10\left(\frac{x}{10}\right) = 10(6)$$

$$4x - x = 10(6)$$

$$3x = 60$$

$$\frac{3x}{3} = \frac{60}{3}$$

$$x = 20$$

Exercises

Solve each equation.

1. $\frac{3x}{2} - x = 1$

2. $\frac{3x}{8} = \frac{x}{3} + \frac{4}{3}$

3. $\frac{y}{6} - \frac{y}{4} = 5$

4. $2a + \frac{a}{3} = \frac{a}{4} + 5$

5. $\frac{x-2}{3} = \frac{x+1}{4}$

6. $\frac{x-1}{2} + \frac{x-2}{3} = 1$

7. $\frac{x-3}{5} - \frac{x+2}{15} + \frac{2}{3} = 0$

8. $\frac{x+4}{3} - 4 = \frac{x-11}{4}$

9. $-\frac{d}{4} + d = \frac{1}{8}$

10. $\frac{x-7}{5} + 2 = \frac{x+8}{10}$

11. $z + \frac{z}{4} = 14 - \frac{z}{2}$

12. $\frac{y+3}{16} - \frac{y-4}{6} = \frac{1}{3}$

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8-2 Enrichment

Sonya Vasilievna Kovalesky

Sonya Vasilievna Kovalevsky (1850−1891), a Russian mathematician, achieved a good education and a successful career despite the disadvantage at that time of being female. Discover her accomplishments by working the problems at the bottom of the page. Then if you find your answer in the chart, circle the fact in that square. Each circled fact is a true statement about Kovalevsky's life.

SONYA VASILIEVNA KOVALEVSKY (1850–1891)

She was denied an education in Russia. $\frac{1}{4}$	She was married just so the European universities would allow her to attend higher education classes. 3	Upon receiving her doctorate, she was offered several professional appointments all over Europe. 156
Sonya was a Russian mathematician and physicist. 33	She wrote a play that was produced in Moscow. 10	Sonya was born in Poland and became a well-known mathematician and novelist. -5
She became a Lecturer and later Professor of Higher Mathematics in Stockholm. $-\frac{1}{2}$	She began her formal study at the Naval Academy in St. Petersburg. 6	She was taught privately by renowned mathematician Karl Weierstrass in Berlin. $\frac{1}{2}$

1. $2z - 8 = 12$

2. $28 = 11x - 5$

3. $n - 12 = 54 - n$

4. $3x + 8 = 5x - 6$

5. $6 + 2s = 7$

6. $\frac{x}{9} - 3 = 17$

7. $9 - (2a + 6) = -9$

8. $x + 6 = 5 - x$

9. $x + 3(2x - 1) = 4(2x + 1)$

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8-3 Enrichment

Make Up a Problem

You have seen that some problem situations can be solved using variables and open sentences. Usually in math, you are asked to solve problems. In this activity, you will be writing the problems.

Use the open sentence, story idea, and your imagination to write an interesting word problem.

1. $x + 4 = 83$

scores on a chapter test

2. $15 - x \leq 10$

money spent on clothes

3. $12 + v = 20$

a team's win/loss record

4. $9 > p - 10$

people at a party

5. $13 + y \geq 21$

ages

6. $36 + h = 47$

temperatures

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8-4 Enrichment

Solving Inequalities Using Addition and Subtraction

Inequalities can be written many different ways in English.

English Phrase	Mathematical Phrase
at most x	$\leq x$
at least x	$\geq x$
5 less than x	$x - 5$
5 more than x	$x + 5$
5 is less than x	$5 < x$
x is between 4 and 6	$4 < x < 6$ or $x > 4$ and $x < 6$

To solve compound inequalities, you must perform the same operation on each of the three parts of the inequality.

Example **Solve the compound inequality $3 < x - 2 < 5$.**

$3 < x - 2 < 5$

$3 + 2 < x - 2 + 2 < 5 + 2$ To get x by itself, add 2 to each part.

$5 < x < 7$

Thus, x is all numbers between 5 and 7.

Exercises

1. Write a mathematical phrase for each of the following English phrases.

a. 8 more than x.

b. x is at least 3.

c. x is between 6 and 12.

d. x cannot exceed -5.

2. For what values of x, if any, is the mathematical phrase $5 > x > 7$ true?

Solve each compound inequality.

3. $-2 < x + 1 < 4$

4. $-4 \leq x - 1 \leq 0$

5. $3 < 15 + x < 10$

6. $-12 < x - 3 \leq 1$

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8-5 Enrichment

Hidden Word

In each group of five inequalities, only two have the same solution set. For each group, write the solution of each inequality and then circle the letters of the two inequalities having the same solution set. After completing all four groups, use the circled letters to form a one-word answer to the question at the bottom of the page.

GROUP 1

E. $3x< 30$

B. $-3x < -30$

F. $-30 > 3x$

D. $-30 < -3x$

D. $-3x < 30$

GROUP 2

M. $\frac{x}{5} \leq -2$

N. $\frac{-x}{5} \leq 2$

W. $\frac{x}{-5} \leq -2$

R. $\frac{-x}{2} \leq -5$

R. $\frac{-x}{5} < -2$

GROUP 3

Q. $\frac{3x}{2} < 0$

K. $0 \leq -12x$

L. $-x \leq 0$

L. $0.73x \geq 0$

A. $0 \geq -912x$

GROUP 4

L. $200 > 0.05x$

A. $0.05x < 20$

D. $\frac{1}{5}x > 20$

E. $200 > 5x$

E. $\frac{x}{0.1} < 4000$

Unscramble the circled letters to find the name of the first of the original 13 colonies to ratify the U.S. Constitution.

Lesson 8-5

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8-6 Enrichment

Conditional and Unconditional Inequalities

When the replacement set is the set of real numbers, the inequality $2x < 16$ is called a **conditional inequality** because it is true for at least one but not all values of the replacement set. Other examples of conditional inequalities are $x + 5 > 8$ and $2y - 6 < 10$.

If the replacement set is the set of real numbers, $x + 5 > x$ is true for every element of the replacement set. Such an inequality is called an **unconditional inequality**. Other examples of unconditional inequalities are $2x + 9 > 2x$ and $x - 7 < x$.

Solve each inequality. Then determine whether each inequality is conditional or unconditional.

1. $x - 2 > 4$

2. $3x - 2 < 2x + 4$

3. $4x + 5 \geq 4x$

4. $2(3x + 5) > 6x + 5$

5. $7y - 4 > 6 + 2y$

6. $8y - 3y > 5y - 10$

7. $5x \leq 10 + 2(3x - 4)$

8. $x < 7 + x$

9. $2c + 5 < 8 + 2c$

10. $6x + 3x - 1 < 8x + x$

11. $2x + 3x \geq 4x + 1$

12. $7x + x + 10 \geq 8x$

13. $x + 8 < 8$

14. $8x < 5(2x + 4)$

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9-1 Enrichment

Roots

The symbol $\sqrt{}$ indicates a square root. By placing a number in the upper left, the symbol can be changed to indicate higher roots.

Examples

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[4]{81} = 3$ because $3^4 = 81$

$\sqrt[5]{100{,}000} = 10$ because $10^5 = 100{,}000$

Exercises

Find each of the following.

1. $\sqrt[3]{125}$

2. $\sqrt[4]{16}$

3. $\sqrt[8]{1}$

4. $\sqrt[3]{27}$

5. $\sqrt[5]{32}$

6. $\sqrt[3]{64}$

7. $\sqrt[3]{1000}$

8. $\sqrt[3]{216}$

9. $\sqrt[6]{1{,}000{,}000}$

10. $\sqrt[3]{1{,}000{,}000}$

11. $\sqrt[4]{256}$

12. $\sqrt[3]{729}$

13. $\sqrt[6]{64}$

14. $\sqrt[4]{625}$

15. $\sqrt[5]{243}$

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9-2 Enrichment

Hero's Formula

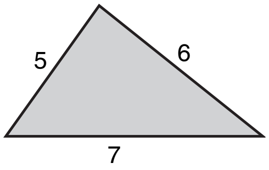
Hero of Alexandria (or Heron) was a mathematician and inventor who lived sometime around the first or second century A.D. In his work, there was a strong emphasis on the applications of mathematics to the real world. He made discoveries in geometry and physics, and he is also believed to have invented a steam engine.

Hero's formula, named after Hero, is a formula for finding the area of a triangle in terms of the lengths of the three sides.

Hero's Formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are lengths of sides, and s is half the perimeter of the triangle or $\frac{1}{2}(a+b+c)$.

Example **Find the area of the triangle.**

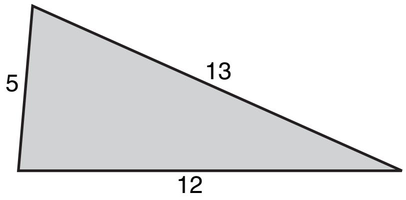
$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{9(9-5)(9-6)(9-7)} \\ &= \sqrt{9 \times 4 \times 3 \times 2} \\ &= 6\sqrt{6} \text{ square units} \end{aligned}$$



Exercises

Use Hero's formula to find the area of each triangle.

1.



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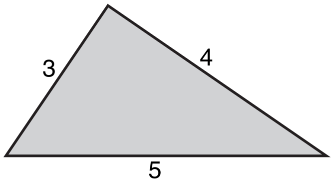

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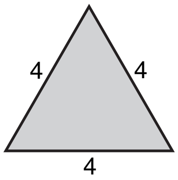

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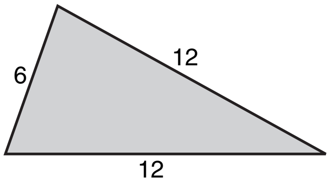

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9-3 Enrichment

Angle Relationships

Angles are measured in degrees (°). Each degree of an angle is divided into 60 minutes (′), and each minute of an angle is divided into 60 seconds (″).

$$60' = 1^\circ$$

$$60'' = 1'$$

$$67\frac{1}{2}^\circ = 67^\circ 30'$$

$$70.4^\circ = 70^\circ 24'$$

$$90^\circ = 89^\circ 60'$$

Two angles are complementary if the sum of their measures is 90°. Find the complement of each of the following angles.

1. $35^\circ 15'$

2. $27^\circ 16'$

3. $15^\circ 54'$

4. $29^\circ 18' 22''$

5. $34^\circ 29' 45''$

6. $87^\circ 2' 3''$

Two angles are supplementary if the sum of their measures is 180°. Find the supplement of each of the following angles.

7. $120^\circ 18'$

8. $84^\circ 12'$

9. $110^\circ 2'$

10. $45^\circ 16' 24''$

11. $39^\circ 21' 54''$

12. $129^\circ 18' 36''$

13. $98^\circ 52' 59''$

14. $9^\circ 2' 32''$

15. $1^\circ 2' 3''$

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9-4 Enrichment

Reduced Triangle Principle

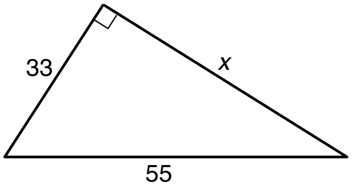
The following steps can be used to reduce the difficulty of a triangle problem by converting to easier side lengths.

Step 1 Multiply or divide the three lengths of the triangle by the same number.

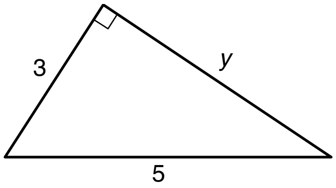
Step 2 Solve for the missing side of the easier problem.

Step 3 Convert back to the original problem.

Example 1



33 and 55 are both multiples of 11. Reduce the problem to an easier problem by dividing the side lengths by 11. Let y represent $\frac{x}{11}$.

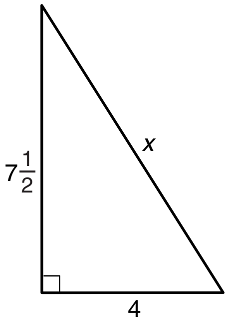


$$\begin{aligned} 3^2 + y^2 &= 5^2 \\ 9 + y^2 &= 25 \\ y^2 &= 16 \\ y &= 4 \end{aligned}$$

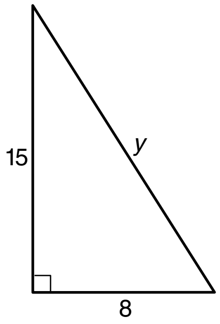
Now convert back.

$$\begin{aligned} \frac{x}{11} &= y \\ x &= 11y \\ x &= 11(4) \text{ or } 44 \end{aligned}$$

Example 2



Multiply each side by 2. Let y represent $2x$.



$$\begin{aligned} 8^2 + 15^2 &= y^2 \\ 64 + 225 &= y^2 \\ 289 &= y^2 \\ 17 &= y \end{aligned}$$

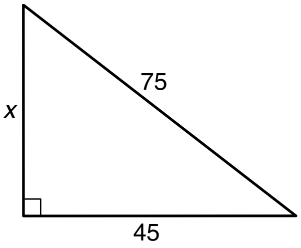
Now convert back.

$$\begin{aligned} 2x &= y \\ x &= \frac{y}{2} \\ x &= \frac{17}{2} \text{ or } 8\frac{1}{2} \end{aligned}$$

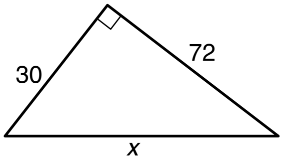
Exercises

Use the reduced triangle method to find the value of *x*.

1.



2.



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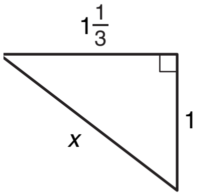

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9-5 Enrichment

The Midpoint Formula

On a line segment, the point that is halfway between the endpoints in called the **midpoint**. To find the midpoint of a segment on the coordinate plane, you can use the Midpoint Formula.

Midpoint Formula	The coordinates of the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example **Find the midpoint of $\overline{AB}$ for $A(3, 2)$ and B(2, 0).**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 2}{2}, \frac{2 + 0}{2}\right) \qquad x_1 = 3,\ y_1 = 2,\ x_2 = 2,\ y_2 = 0$$

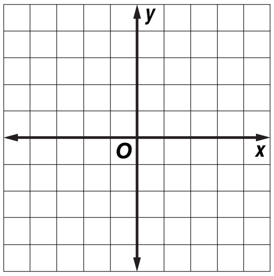
$$= (2.5, 1)$$

Exercises

Use the coordinate plane at the right for Exercises 1 and 2.

1. Graph the following points and connect them to form a triangle.

$A(4,0)$ $\quad B(0,-2)$ $\quad C\ (1, 2)$



2. Calculate the midpoint of each line segment using the Midpoint Formula. Graph each midpoint and connect the midpoints to form a triangle.

3. Using the Distance Formula, find the perimeter of both the larger and smaller triangles you graphed in Exercises 1 and 2.

4. Based on your finding in Exercise 3, make a conjecture about the relationship between the perimeter of a triangle and the perimeter of a triangle that could be created by connecting the midpoints of each side.

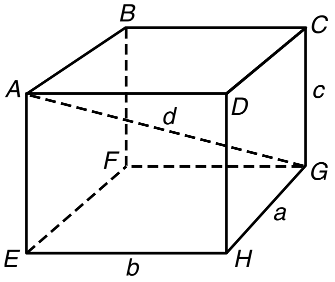
9-6 Enrichment

Diagonals of Prisms

To find the length of diagonals in cubes and rectangular solids, a formula can be applied. In the example below, the length of diagonal $\overline{AG}$, or d, can be found using the formula

$$d^2 = a^2 + b^2 + c^2 \text{ or } d = \sqrt{a^2 + b^2 + c^2}.$$

Example 1



The diagonal, d, is equal to the square root of the sum of the squares of the length, a, the width, b, and the height, c.

Example 2

Find the length of the diagonal of a rectangular prism with length of 8 meters, width of 6 meters, and height of 10 meters.

$d = \sqrt{8^2 + 6^2 + 10^2}$ Substitute the dimensions into the equation.

$= \sqrt{64 + 36 + 100}$ Square each value. Add.

$= \sqrt{200}$ Find the square root of the sum.

$= 14.1$ m Round the answer to the nearest tenth.

Exercises

Solve. Use $d = \sqrt{a^2 + b^2 + c^2}$. Round answers to the nearest tenth if necessary.

1. Find the diagonal of a cube with sides of 6 inches.

2. Find the diagonal of a cube with sides of 2.4 meters.

3. Find the diagonal of a rectangular solid with length of 18 meters, width of 16 meters, and height of 24 meters.

4. Find the diagonal of a rectangular solid with length of 15.1 meters, width of 8.4 meters, and height of 6.3 meters.

5. Find the diagonal of a cube with sides of 34 millimeters.

6. Find the diagonal of a rectangular solid with length of 8.9 millimeters, width of 6.7 millimeters, and height of 14 millimeters.

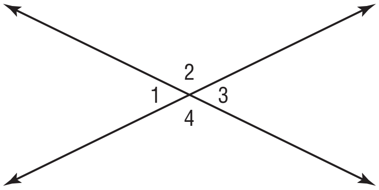
10-1 Enrichment

Geometric Proof

Use definitions and theorems for angle congruence to complete the proofs.

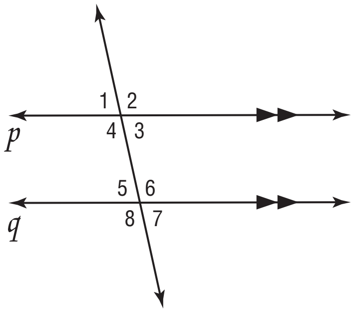
Write the reason for each statement.

1. Prove: $\angle 1 \cong \angle 3$



Statement	Reason
a. $\angle 1$ and $\angle 3$ are vertical angles.	**a.** Given
b. $m\angle 1 + m\angle 2 = 180°$; $m\angle 3 + m\angle 2 = 180°$	**b.** ____________________________
c. $m\angle 1 = 180° - m\angle 2$; $m\angle 3 = 180° - m\angle 2$	**c.** ____________________________
d. $m\angle 1 = m\angle 3$	**d.** ____________________________
e. $\angle 1 \cong \angle 3$	**e.** ____________________________

2. Prove: $m\angle 3 \cong m\angle 7$



Statement	Reason
a. Line *p* is parallel to line *q*.	**a.** Given
b. $m\angle 3 \cong m\angle 5$	**b.** ____________________________
c. $m\angle 5 \cong m\angle 7$	**c.** ____________________________
d. $m\angle 3 \cong m\angle 7$	**d.** ____________________________

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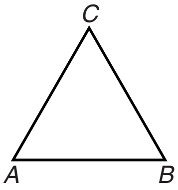
10-2 Enrichment

Constructions: Congruent Triangles

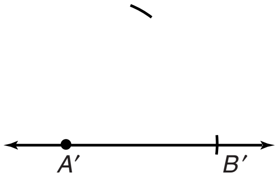
Construct a triangle congruent to the given triangle using the SSS rule.

(Three sides of one triangle are congruent to three sides of another triangle.)

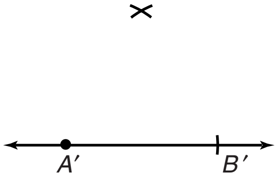
Given △ *ABC*



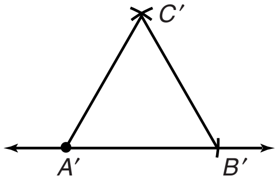
Step 1



Step 2



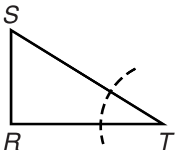
Step 3



Construct a triangle congruent to the given triangle using the SAS rule.

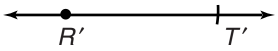
(Two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle.)

Given △ *RST*

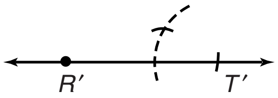


Use ($\overline{RT}$, ∠*T*, $\overline{ST}$).

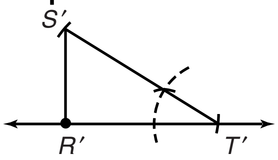
Step 1



Step 2



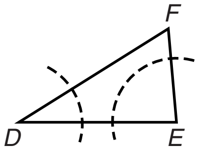
Step 3



Construct a triangle congruent to the given triangle using the ASA rule.

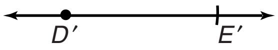
(Two angles and the included side of one triangle are congruent to two angles and the included side of another triangle.)

Given △ *DEF*

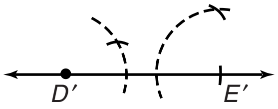


Use (∠*D*, $\overline{DE}$, ∠*E*).

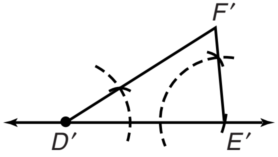
Step 1



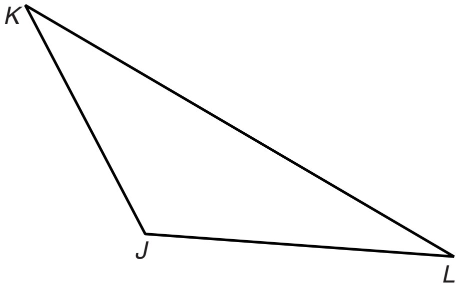
Step 2



Step 3



Construct a triangle congruent to triangle *KJL*.
Use the rule and parts specified.



1. SSS; Use: $\overline{JL}$, $\overline{JK}$, $\overline{KL}$

2. SAS; Use: $\overline{JL}$, $\angle L$, $\overline{KL}$

3. ASA; Use: $\angle K$, $\overline{KL}$, $\angle L$

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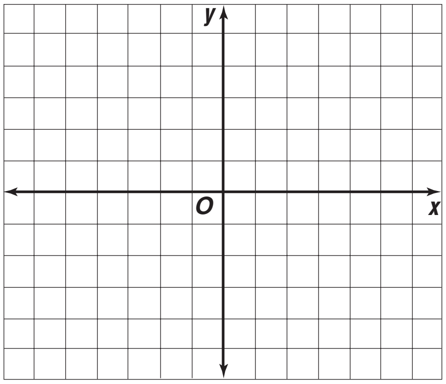
10-3 Enrichment

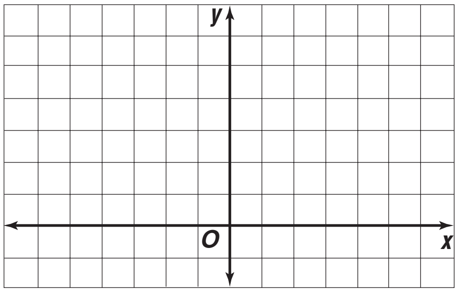
Translations and Reflections

The lines on graph paper can help you draw slide images of figures.

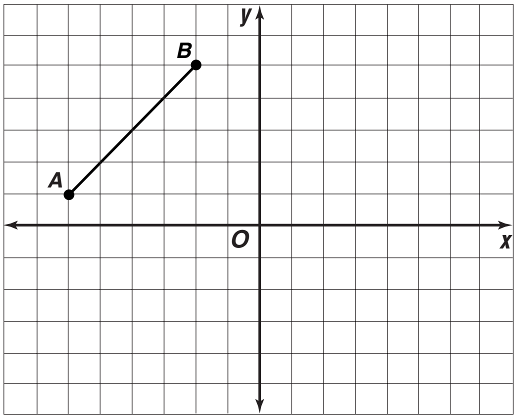
1. Graph $\triangle ABC$ with vertices $A(1, 1)$, $B(-3, 4)$, and $C(-3, -4)$. Draw $\triangle A'B'C'$, the translation image of $\triangle ABC$, where the slide is 3 units to the right. Name the coordinates of the image of each vertex.



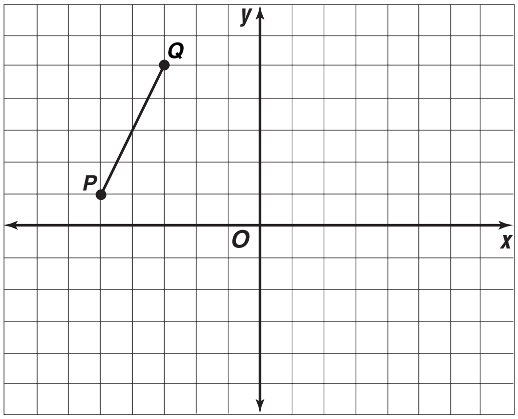
2. Draw $\triangle JKL$ with vertices $J(-4, 3)$, $K(0, 2)$, and $L(-2, 0)$. Let $\triangle J'K'L'$ be the image of $\triangle JKL$ under a slide of 4 units to the right and then a slide of 3 units up. Graph $\triangle J'K'L'$. Name the coordinates of the vertices of $\triangle J'K'L'$.



3. Draw $\overline{A'B'}$, the image formed by reflecting $\overline{AB}$ over the y-axis. Then draw $\overline{A''B''}$, the image formed by reflecting $\overline{A'B'}$ over the x-axis. What are the coordinates of A'' and B''? What is the relationship between the coordinates of the endpoints of $\overline{AB}$ and those of $\overline{A''B''}$?



4. Draw $\overline{P'Q'}$, the reflection image of $\overline{PQ}$ over the y-axis. Draw $\overline{P''Q''}$, the reflection image of $\overline{P'Q'}$ over the x-axis. Find the slopes of $\overline{PQ}$, $\overline{P'Q'}$, and $\overline{P''Q''}$. What is the relationship between the slopes of $\overline{PQ}$ and $\overline{P''Q''}$?



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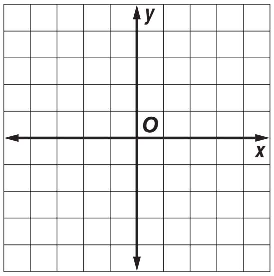
10-4 Enrichment

Using Coordinates

For Exercises 1–4, use the coordinate grid at the right.



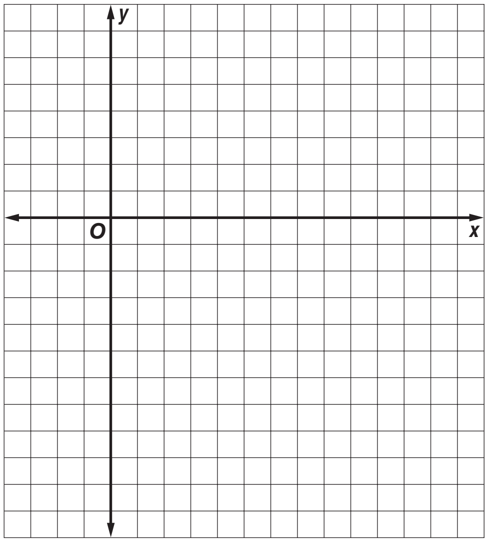
1. Graph the points (1, 1), (4, 4), and (2, 4). Connect the dots. Name the figure formed.

2. Multiply each y-coordinate in Exercise 1 by −1. Graph the points. How is this triangle related to the one in Exercise 1?

3. Multiply each coordinate in Exercise 1 by −1. Graph the points.

4. What would you have to do to get the coordinates of a triangle in Quadrant II congruent to the ones in Excercises 1–3?

For Exercises 5–10, use the coordinate grid at the right.



5. Graph the points (3, 1), (2, 3), (4, 6), and (5, 4). Connect the dots. The figure formed is called a parallelogram.

6. Add 2 to both coordinates of each point and graph the new coordinates. Name the figure formed.

7. Add −4 to the x-coordinate of each point in Exercise 5 and graph the new coordinates. Is this figure also a parallelogram?

8. Graph the points (5, −2), (6,−3), (5, −5), and (3, −3) on the coordinate plane at the right. Name the figure formed.

9. Multiply both coordinates of each point in Exercise 8 by 2 and graph the new coordinates. This is an enlargement.

10. Multiply both coordinates of each point in Exercise 8 by $\frac{1}{2}$ and graph the new coordinates. This is a reduction.

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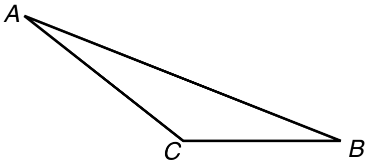
10-5 Enrichment

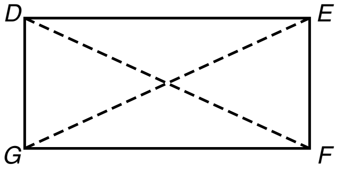
Polygons and Diagonals

A **diagonal** of a polygon is any segment that connects two nonconsecutive vertices of the polygon. In each of the following polygons, all possible diagonals are drawn.

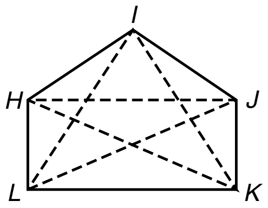
Example



In △*ABC*, no diagonals can be drawn. Why?



In quadrilateral *DEFG*, 2 diagonals can be drawn.



In pentagon *HIJKL*, 5 diagonals can be drawn.

Complete the chart below and try to find a pattern that will help you answer the questions that follow.

	Polygons	Number of Sides	Number of Diagonals from One Vertex	Total Number of Diagonals
	triangle	3	0	0
	quadrilateral	4	1	2
	pentagon	5	2	5
1.	hexagon	6		
2.	heptagon	7		
3.	octagon	8		
4.	nonagon	9		
5.	decagon	10		

Find the total number of diagonals that can be drawn in a polygon with the given number of sides.

6. 6

7. 7

8. 8

9. 9

10. 10

11. 11

12. 12

13. 15

14. 20

15. 50

16. 75

17. *n*

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10-6 Enrichment

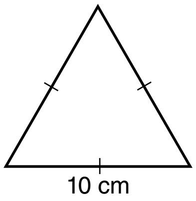
Area of an Equilateral Triangle

The area of an equilateral triangle is the product of one fourth of the square of a side times the square root of 3 (which is approximately 1.732).

$$A = \frac{1}{4}s^2(\sqrt{3})$$

$$\text{or } A \approx \frac{s^2}{4}(1.732)$$

Example



$$A \approx \frac{10^2}{4}(1.732)$$

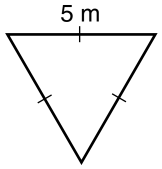
$$\approx \frac{100}{4}(1.732)$$

$$\approx 43.3$$

The area of the triangle is approximately 43.3 cm^2.

Find the area of each equilateral triangle. Round each answer to the nearest tenth.

1.



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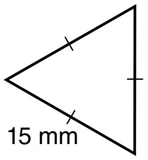

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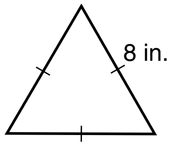

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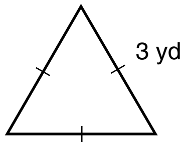

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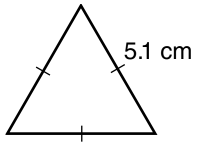

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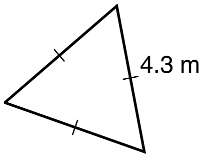

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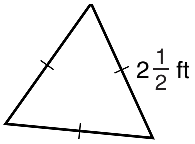

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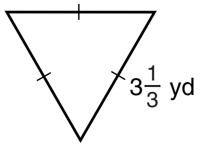

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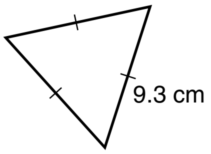

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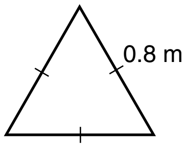
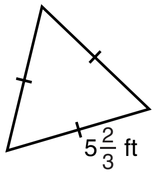


11.



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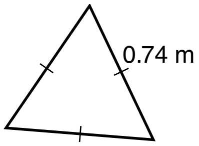

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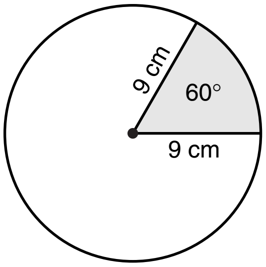
10-7 Enrichment

Sector of a Circle

The sector of a circle is the region bounded by two radii and the arc of the circle. The area of a sector is a fractional part of the area of the circle.

$A = \frac{n}{360} \times \pi \times r^2$ where n is the degree measure of the central angle.

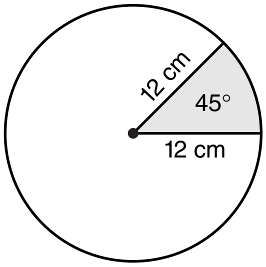
Example



$$\text{Area of a sector} \approx \frac{60}{360} \times 3.14 \times 81$$
$$\approx 42.39 \text{ cm}^2$$

Find the area of each sector. Use 3.14 for π.

1.



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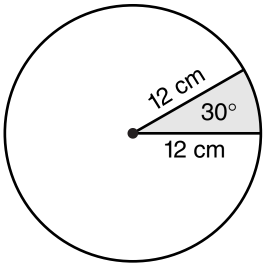

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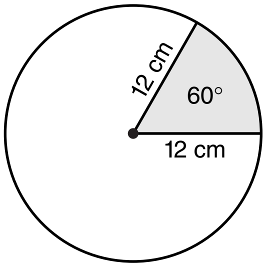

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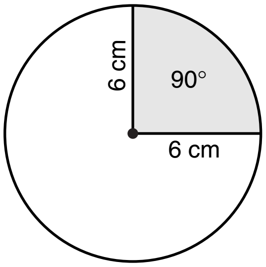

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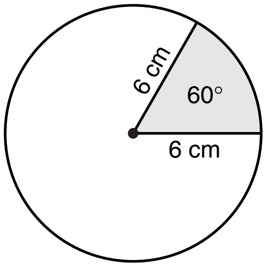

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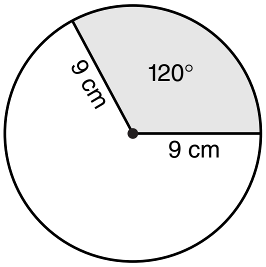

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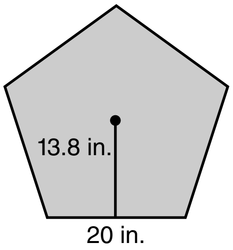
10-8 Enrichment

Area of a Regular Polygon

The area of a regular polygon is equal to one-half the product of the **apothem** and the **perimeter**. The apothem is the distance from the center of the polygon to a side. The perimeter is the sum of the lengths of all of the sides.

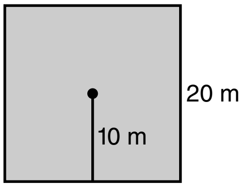
Example



$$\begin{aligned} A &= \frac{1}{2}ap \quad a = 13.8, p = 100 \\ &= \frac{1}{2}(13.8) \cdot (100) \\ &= 690 \text{ in}^2 \end{aligned}$$

Find the area of each regular polygon.

1.



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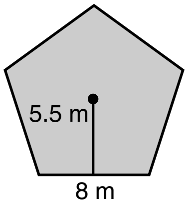

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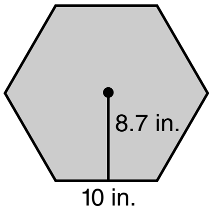

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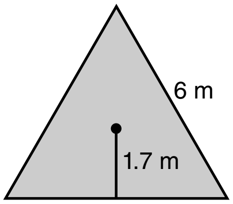

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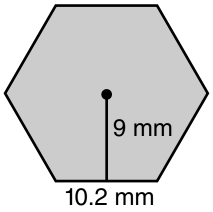

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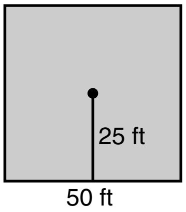

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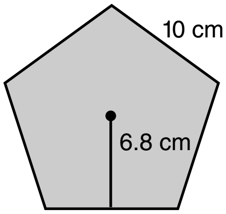

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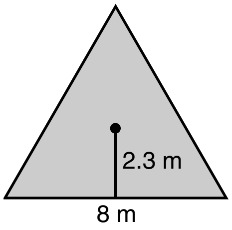

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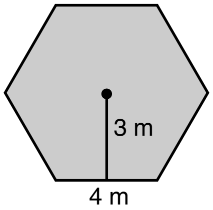

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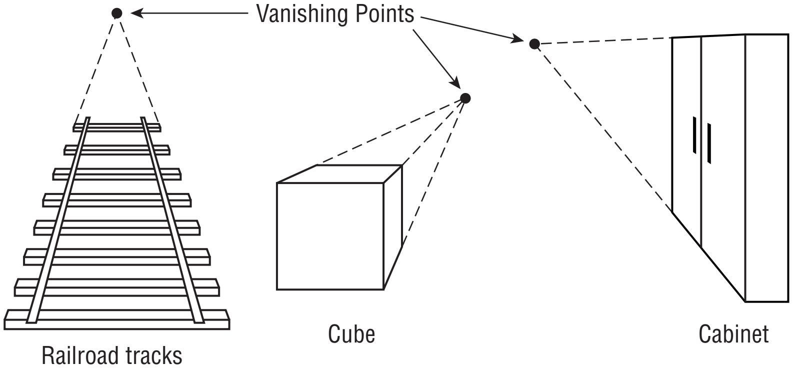
11-1 Enrichment

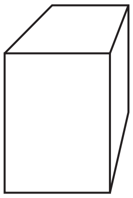
Perspective Drawings

To draw three-dimensional objects, artists make **perspective drawings** such as the ones shown below. To indicate depth in a perspective drawing, some parallel lines are drawn as converging lines. The dotted lines in the figures below each extend to a **vanishing point**, or spot where parallel lines appear to meet.

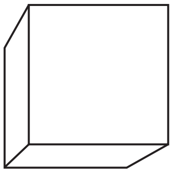


Draw lines to locate the vanishing point in each drawing of a box.

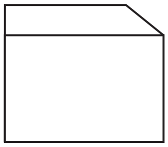
1.



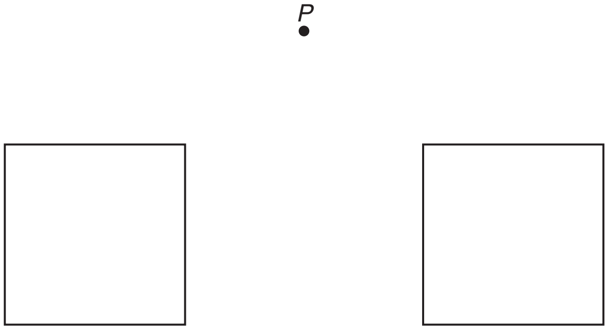
2.



3.



4. The fronts of two cubes are shown below. Using point *P* as the vanishing point for both cubes, complete the perspective drawings of the cubes.



5. Find an example of a perspective drawing in a newspaper or magazine. Trace the drawing and locate a vanishing point.

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11-2 Enrichment

Density

Density is the mass per unit of volume of a substance and is an important scientific concept. The formula for determining density is Density $= \frac{\text{Mass}}{\text{Volume}}$.

Example **The mass of an object is 27g and its volume is 130 cm^3. What is its density?**

Density $= \frac{\text{Mass}}{\text{Volume}}$

Density $= \frac{27\text{ g}}{130\text{ cm}^3}$

Density = 0.21 g/cm^3

Exercises

1. If the base of a prism is 4 centimeters by 2 centimeters and the prism has a height of 10 centimeters, what is the volume?

2. If you know the density of the prism from Exercise 1 is 8.75 g/cm^3, what is its mass?

3. Use the density formula to complete the table.

Mass (g)	Volume (cm^3)	Density (g/cm^3)
45		8.9
23	158	
	3462	6.97

For Exercises 4-6, use the table at the right that lists the density for some common substances.

Substance	Density (g/cm^3)
Lead	11.3
Gold	19.3
Aluminum	2.7
Ice	0.93

4. What is the volume of a cube of ice with a mass of 15 g?

5. A prism has a mass of 291.6 g. Its base measures 3 centimeters by 3 centimeters and its height is 12 centimeters. Assuming it is one of the substances in the table, what substance is it?

6. A cylinder has a radius of 6 centimeters and a height of 8 centimeters. If the cylinder is lead, what is its mass? Use 3.14 to approximate the value of π.

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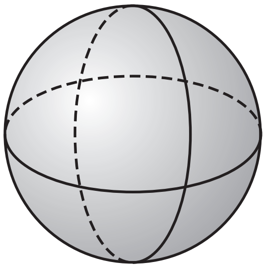
11-3 Enrichment

Great Circles

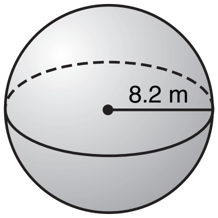
The *great circle* on a sphere is the intersection of the sphere with a plane through the center. Both circles shown to the right are great circles. They are congruent and have circumferences that measure $2\pi r$.



Volume of sphere: $V = \frac{4}{3}\pi r^3$

Surface area of a sphere: $S = 4\pi r^2$

Example



The *area of the great circle* is about 211.24 m^2.
The *circumference of the great circle* is about 51.52 m.
The *surface area of the sphere* is about 844.96 m^2.
The *volume of the sphere* is about 2309.56 m^3.

Exercises

Solve. Round answers to the nearest hundredth.

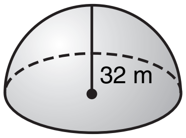
1. Find the volume of a sphere with a radius of 4 meters.

2. Find the surface area of a sphere with a radius of 8 centimeters.

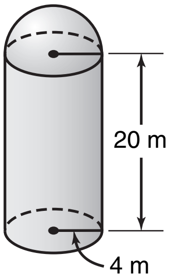
3. A hemispherical dome (half of a sphere) has a height of 32 meters.

a. What is the volume of the dome?

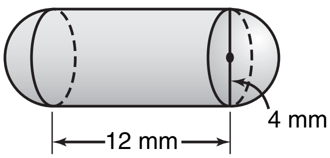
b. What is the surface area?



4. Find the volume of the grain silo shown below.



5. A cold medicine capsule is 12 millimeters long and 4 millimeters in diameter. Find the volume of medicine it can contain.



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11-4 Enrichment

Euler's Formula

Leonard Euler (oi'ler), 1707–1783, was a Swiss mathematician who is often called the father of topology. He studied perfect numbers and produced a proof to show that there is an infinite number of primes. He also developed the following formula, relating the numbers of vertices, faces, and edges of a *polyhedron*.

Euler's formula: $V + F = E + 2$

V = number of vertices

F = number of faces

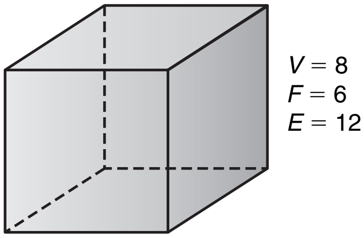
E = number of edges

For a cube, the following is true.

$V + F = E + 2$

$8 + 6 = 12 + 2$

$14 = 14$ ✓

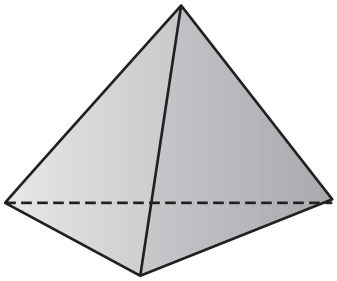


Cube

Another name for a cube is **hexahedron**.

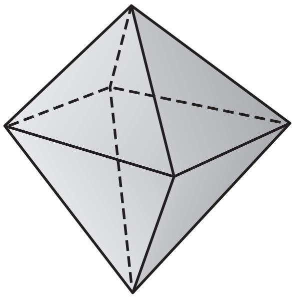
Use Euler's formula to find the number of faces of each polyhedron.

1. $V = 4, E = 6$



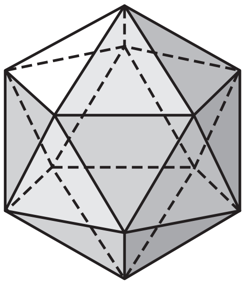
Tetrahedron

2. $V = 6, E = 12$



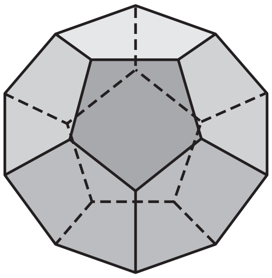
Octahedron

3. $V = 12, E = 30$



Icosahedron

4. $V = 20, E = 30$



Dodecahedron

The suffix *hedron* comes from the Greek language meaning "face." Find the meaning of each of the following prefixes.

5. hexa **6.** tetra **7.** octa **8.** icosa **9.** dodeca

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11-5 Enrichment

Using Area and Volume

Solve. If neccessary, round to the nearest hundredth.

1. Mrs. Bartlett wants to put carpeting in a room that is 15 feet long and 12 feet wide. How many square feet of carpeting does she need?

2. Tom is making a cover for a box 4.5 feet long and 23 inches wide. What will be the area of the cover, in square feet?

3. How many 2-inch by 3-inch tickets can be cut from a 2-foot by 3-foot sheet of paper?

4. What is the area of the top of a tree stump that is 42 centimeters in diameter?

5. Mrs. Stabile has a one-story house that is 32 feet wide and 48 feet long. She plans to build an addition 12 feet by 16 feet. What will the total floor area of the house be then?

6. How many cubic meters of dirt must be removed when digging the foundation of a building if the excavation is 32 meters long, 12 meters wide, and 8 meters deep?

7. Mr. Mendez has a whirling sprinkler for watering his lawn. The sprinkler waters a circle with a radius of 12 meters. Find the area of the lawn that he can water at one time.

8. A cylindrical storage tank has a diameter of 6 meters and a height of 5 meters. What is the volume of the storage tank?

9. In Mr. Curin's living room there is a circular mirror with a diameter of 18 inches. What is the area of the mirror?

10. Find the cost of cementing a driveway 9 meters by 16 meters at $48 per square meter.

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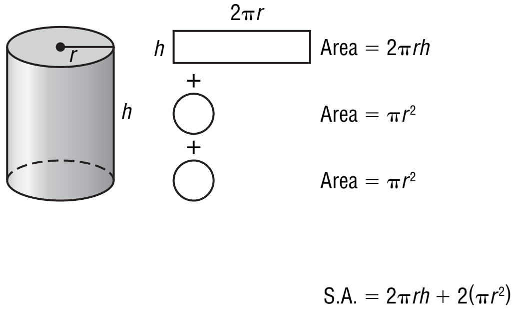
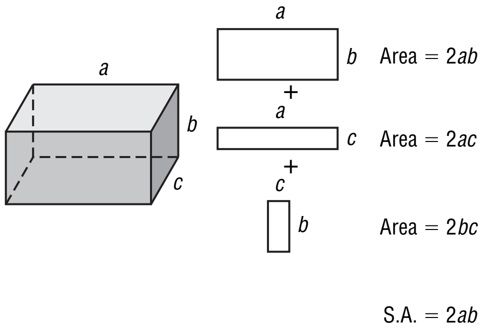
11-6 Enrichment

Surface Area of Solids

The surface area (S.A.) of a solid is equal to the sum of the areas the shapes that make up the surface of that solid. A circular cylinder has bases that are circles and its lateral surface is a rectangle. A prism is composed of six rectangles.





Example1 Find the surface area of the cylinder

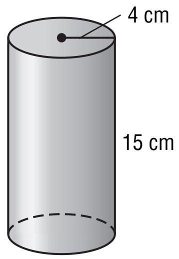
S.A. $= 2\pi r^2 + 2\pi rh$

$= 2\pi\ (4 \text{ cm})^2 + 2\pi(4 \text{ cm})(15 \text{ cm})$

$= 32\pi \text{ cm}^2 + 120\pi \text{ cm}^2$

$= 152\pi \text{ cm}^2$

$= 477.52 \text{ cm}^2$



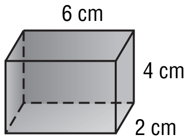
Example 2 Find the surface area of the rectangle.

S.A. $= 2ab + 2ac + 2bc$

$= 2(6 \text{ cm})(4 \text{ cm}) + 2(6 \text{ cm})(2 \text{ cm}) + 2(4 \text{ cm})(2 \text{ cm})$

$= 48 \text{ cm}^2 + 24 \text{ cm}^2 + 16 \text{ cm}^2$

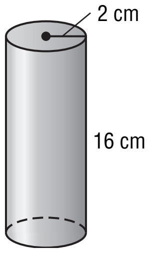
$= 88 \text{ cm}^2$



Exercises

Find the surface area of each solid.

1.



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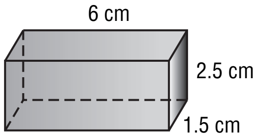

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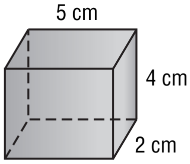

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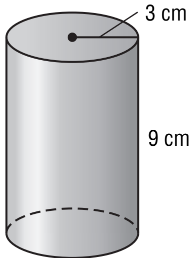

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12-1 Enrichment

U.S. Presidents

The political parties in our country have changed over time. At the time of our nation's founding, the Federalist and Democratic-Republican parties were the nationally prominent parties. In recent years, all U.S. Presidents have been from either the Republican or Democratic Party.

1. Make a table to display the data for the number of U.S. Presidents from each political party.

Republican	**Democrat**	**Federalist**
Abraham Lincoln	Andrew Jackson	George Washington
Ulysses S. Grant	Martin Van Buren	John Adams
Rutherford B. Hayes	James K. Polk	
James A. Garfield	Franklin Pierce	**Democratic-Republican**
Chester A. Arthur	James Buchanan	Thomas Jefferson
Benjamin Harrison	Grover Cleveland	James Madison
William McKinley	Woodrow Wilson	James Monroe
Theodore Roosevelt	Franklin D. Roosevelt	John Quincy Adams
William Howard Taft	Harry S Truman	
Warren G. Harding	John F. Kennedy	**Whig**
Calvin Coolidge	Lyndon B. Johnson	William Henry Harrison
Herbert Hoover	Jimmy Carter	John Tyler
Dwight D. Eisenhower	Bill Clinton	Zachary Taylor
Richard Nixon		Millard Fillmore
Gerald Ford		
Ronald Reagan		**Union**
George Bush		Andrew Johnson
George W. Bush		

2. Display the data from the table in a stem-and-leaf plot.

3. What is the difference in the number of presidents from the party with the most presidents and the party with the fewest presidents?

4. What is another type of information about U.S. Presidents that could be displayed using a stem-and-leaf plot?

12-2 Enrichment

Variance

Another way to measure the variation of a set of data is by computing the **variance**. The higher the variance is for a group of numbers, the more “spread out” the data will be.

The table below shows the price of the stock for two companies during one week.

	Monday	Tuesday	Wednesday	Thursday	Friday
Acme Computer Systems	$10	$7	$3	$8	$12
Baker Pencil Company	$7	$8	$7	$9	$9

Computing the variance will show which company’s stock has the greater variation. To compute the variance, follow these steps:

Step 1 Subtract the mean from each number in the set.

Step 2 Multiply each difference in step 1 by itself.

Step 3 Add these differences.

Step 4 Divide the total by the number of members of the set.

Example **Find the variance for Acme Computer Systems.**

The mean average price for the week for each company is $8

$(10 - 8) \times (10 - 8) + (7 - 8) \times (7 - 8) + (3 - 8) \times (3 - 8) + (8 - 8) \times (8 - 8) + (12 - 8) \times (12 - 8)$

$4 + 1 + 25 + 0 + 16 = 46$

The variance is 46 ÷ 5, or 9.2.

Exercises

Solve.

1. Do you think the variance for Baker Pencil Company will be higher than the variance for Acme Computer Systems? Why? Compute the variance for Baker Pencil Company to see whether you are correct.

2. Consolidated Airlines also had an average price last week of $8 per share, but its variance was 10.8. Indicate five stock prices that could produce this variance. (*Hint:* Change only the Monday and Tuesday prices for Acme.)

3. Sleepy Mattress Company’s stock had an average price last week of $8 per share and a variance of 0. What was the price of shares each day last week?

4. Are there any values that the variance cannot equal? If so, what are these values?

Lesson 12-2

12-3 Enrichment

Traffic Safety Facts

Primary enforcement seatbelt laws allow law enforcement officers to pull over drivers and ticket them for not wearing a seatbelt as they would for any other violation. Secondary seatbelt laws allow the driver to be ticketed for not wearing a seatbelt only if they are stopped for another violation of the law. The table to the right lists several states with seat belt laws and the estimated seatbelt use rates.

Type of Safety Belt Law	State	Seatbelt Use Rate (%)
Primary	Texas	76.1
Primary	Oklahoma	67.9
Secondary	Arkansas	54.5
Primary	Louisiana	68.1
Secondary	Nevada	74.5
Primary	California	91.1
Primary	Hawaii	82.5

Source: National Highway Traffic Safety Administration

1. Make a box-and-whisker plot of the seatbelt use rates.

a.What are the upper and lower quartiles?

b.What is the median seatbelt use rate?

2. Make a back-to-back stem-and-leaf plot to compare the seatbelt use rates in states with primary seatbelt laws to those states with secondary seat belt laws.

3. What conclusion can you make from the plot in Exercise 2?

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12-4 Enrichment

Displaying Real-World Data

The Big D Marathon is an annual event that takes place in Dallas, Texas, in the spring. Marathons are 26.2 miles in length and require intense preparation and endurance. Here are the results from the 50-54 age group from the April 3, 2005, Big D Marathon. 3:58:08 means 3 hours, 58 minutes, 8 seconds.

Place	Name	Age	Time (h:min:s)
1	Kathy Johnson	50	3:58:08
2	Lana Parks	51	4:17:16
3	Adrienne Gabriel	50	4:55:33
4	Sarah Gordon	50	5:07:45
5	Teresa Lynd	52	5:22:03
6	Margaret Darneille	53	5:25:56
7	Deborah Kerr-Leathem	51	5:34:36
8	Veleria Cowsen	54	6:03:29
9	Kathy Davidson	50	6:15:50

Source: www.texasmarathon.com

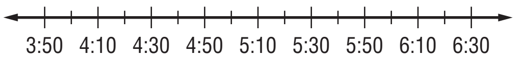
1. Use the data to complete the frequency table below. Then draw a histogram to display the data. Number of Runners in Each Time Group

Time (in hours)	Tally	Frequency
3:00 – 3:59		
4:00 – 4:59		
5:00 – 5:59		
6:00 – 6:59		

2. Draw a stem-and-leaf plot to display the data for the times at which the runners completed the marathon.

Hours	Minutes

3. Draw a box-and-whiskers plot to display the times data.



4. What is the median time that this group of runners finished the marathon?

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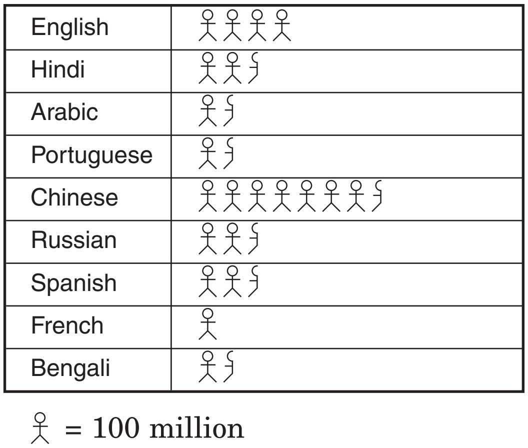
12-5 Enrichment

Statistical Graphs

Bar graphs and pictographs are used to compare quantities. Line graphs are used to show changes. Circle graphs compare parts to parts, or parts to the whole.

Solve. Use the pictograph.

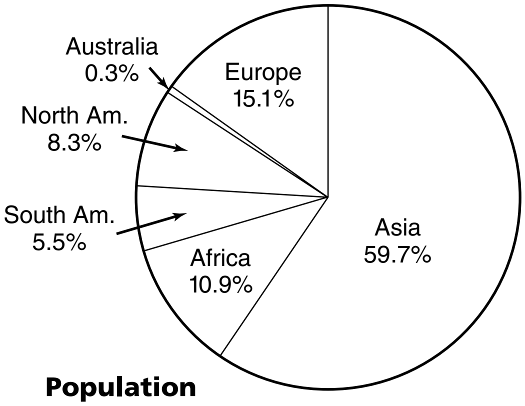
Principal Languages of the World
(to nearest fifty million)



1. How many people speak Portuguese?
2. What is the ratio of people who speak Spanish to those who speak Russian?
3. What three languages are each spoken by about 150 million people?
4. How many fewer people speak Arabic than Hindi?

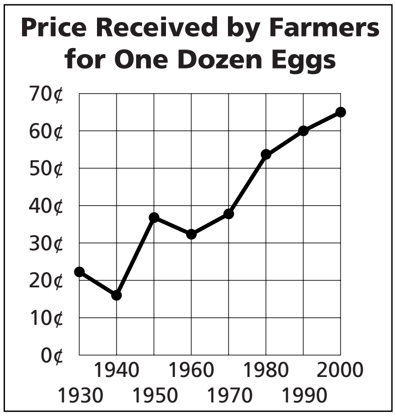
Solve. Use the circle graph.

5. Which continent has the smallest population?
6. How does the population of South America compare to that of Africa?
7. What is the population of Australia if the world's population is about 6 billion?



Population by Continent

Solve. Use the line graph.



8. During which ten-year period was the increase in the price of eggs greatest?
9. What was the price of a dozen eggs in 1940?
10. What was the percent of increase in the price of eggs from 1940 to 1950?
11. What was the increase in cents from 1930 to 1980?

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12-6 Enrichment

Using and Misusing Graphs

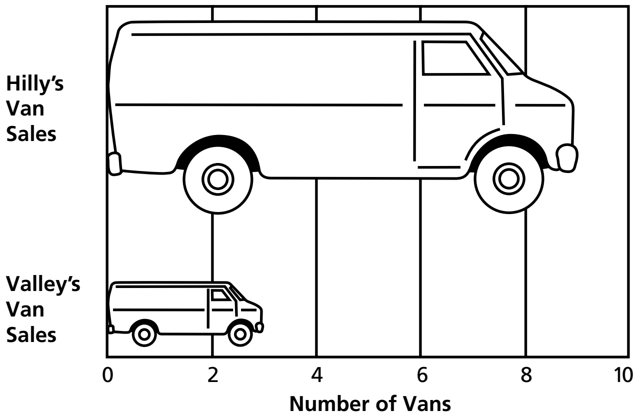
Refer to the graphs at the right.

1. Do Graphs A and B give the same information on sales?
2. Find the ratio of Hilly's sales to Valley's sales.
3. In Graph A, the Hilly van is about 2.5 cm high by 6 cm long. What is its area?
4. The Valley van is about 0.75 cm high by 2 cm long. What is its area?
5. In Graph B, both vans are about 1.5 cm high. The Hilly van is about 6 cm long. What is its area?
6. The Valley van is about 2 cm long. What is its area?
7. Compute the following ratios.

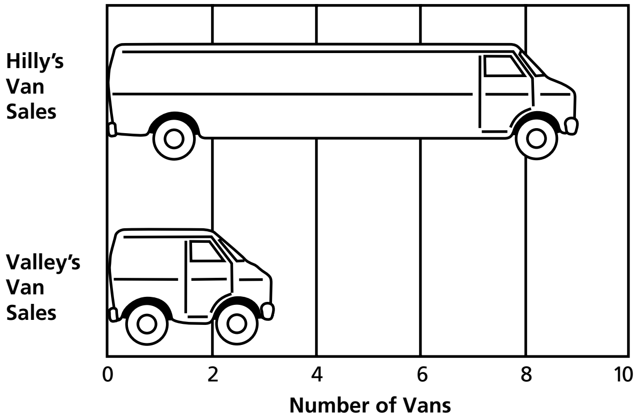
   Graph A: $\dfrac{\text{Area of Hilly}}{\text{Area of Valley}}$

   Graph B: $\dfrac{\text{Area of Hilly}}{\text{Area of Valley}}$
8. Compare the results of Exercises 2 and 7. Which graph is misleading? Explain your answer.

Graph A: Vans Sold In June



Graph B: Vans Sold In June



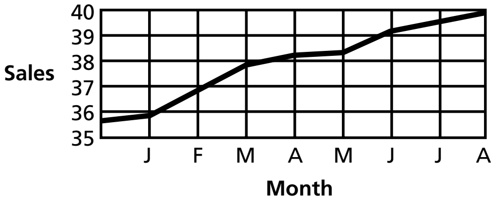
Use Graphs C and D to answer each question.

9. Which graph is easier to read?

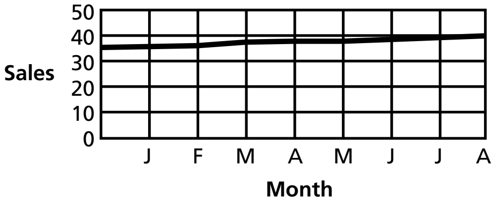
10. Compare the vertical scales. How do they differ?

11. Which graph gives a better impression of the trend in sales? Explain.

Graph C: Total Sales



Graph D: Total Sales



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12-7 Enrichment

Probability and Tables

SCHOOL In Rockville High School there are

400 freshmen—60 have A averages and 90 have B averages.

300 sophomores—40 have A averages and 60 have B averages.

200 juniors—10 have A averages and 30 have B averages.

100 seniors—20 have A averages and 60 have B averages.

1. Use the information above to complete the table below. Then use the table to answer Exercises 2–11.

Grade Class	A	B	Below B	Total
Freshmen				
Sophomores				
Juniors				
Seniors				
Total				

Suppose a student is selected at random from Rockville High School. Find the probability of selecting each of the following.

2. a freshman

3. a senior

4. an A student

5. a student whose grade is below B

6. a sophomore B student

7. a junior A student or a senior A student

8. a student who is neither a junior A student nor a senior A student

9. a B student who is *not* a junior

10. If selecting only from the juniors, what is the probability of picking an A student?

11. If selecting only from the students who are neither A nor B students, what is the probability of picking a senior?

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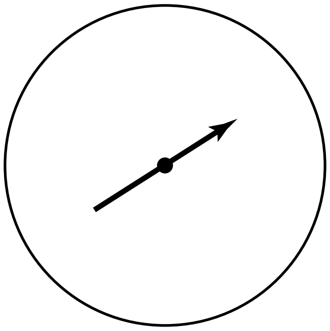
12-8 Enrichment

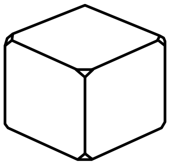
Outcomes

Complete.

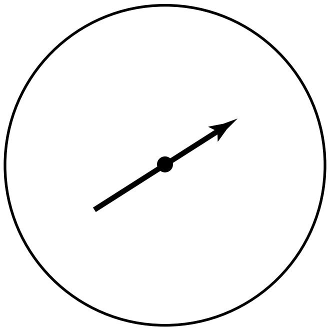
1. Complete the spinner so that it will have six different possible outcomes.



2. List the numbers that could be placed on the die to provide only four different possible outcomes.



3. Complete the spinner so that it is more likely to land on red than blue.



4. List the months in which you could choose a date and have 30 possible outcomes.

5. There are white, green, and blue marbles in a bag. What is the minimum number of each so that it is twice as likely that you draw a green one as a white one, and three times as likely that you draw a blue one as a green one?

6. A year between 1950 and 2001 is chosen at random. How many possible outcomes are there where the year is a leap year? List them.

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12-9 Enrichment

Permutations and Combinations

An arrangement of objects *in a given order* is called a **permutation** of the objects. A symbol for the number of permutations is $P(n, x)$, where x represents the number of objects to be arranged in order and n reminds us that these objects are chosen from an original set of n objects.

$$P(n, x) = \frac{n!}{(n - x)!}$$

Example 1 If gold, silver, and bronze medals are to be awarded to the first 3 finishers in an 8-person race, in how many ways can the medals be awarded?

$$\begin{aligned} P(8, 3) &= \frac{8!}{5!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 8 \cdot 7 \cdot 6 \\ &= 336 \text{ ways in which the medals may be awarded} \end{aligned}$$

A selection of x objects taken from a set of n objects *without regard for order* of the selection is called a **combination**. A symbol for the number of combinations is $C(n, x)$.

$$C(n, x) = \frac{n!}{x!(n - x)!}$$

Example 2 In how many ways can you choose 3 people from a group of 12 without regard for order?

$$\begin{aligned} C(12, 3) &= \frac{12!}{3!9!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} \\ &= \frac{12 \cdot 11 \cdot 10}{3 \cdot 2} \\ &= 220 \text{ possible groups of 3 people} \end{aligned}$$

Find each value.

1. $P(7, 2)$

2. $P(7, 5)$

3. $C(7, 2)$

4. $C(7, 5)$

5. $P(13, 2)$

6. $C(13, 11)$

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12-10 Enrichment

Probability of Dependent Events

Look at the letters in the word MATHEMATICAL. If these letters were placed in a hat, what would be the probability of drawing a vowel and then, without replacing the vowel, drawing a consonant? These are **dependent events** since the letter selected on the first draw affects the probability for the second draw.

$$P(\text{vowel, then consonant}) = \frac{5}{12} \cdot \frac{7}{11} = \frac{35}{132}$$

Find the probability of drawing each of the following from the letters in MATHEMATICAL if the letters are not replaced.

1. two Ms

2. two As

3. three As

4. three vowels

5. five consonants

6. the letters MATH in that order

Now, think of using variables instead of numbers. This is very useful, since this is the way formulas are developed. Once a formula is found, it can be used for any numbers. Begin by examining the following example.

Example Three of 10 socks in a box are blue. If socks are drawn without looking and not replaced, what is the probability of picking 3 blue socks in 3 drawings?

$$\frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} = \frac{6}{720}, \text{ or } \frac{1}{120}$$

7. If box containing n socks has k blue ones, what is the probability of picking 3 blue socks in 3 drawings?

8. If a box containing n socks has k blue ones, what is the probability of picking x blue socks in x drawings?

9. Use your formula from Exercise 7 to find the probability of picking 3 blue socks in 3 drawings from a box containing 6 socks, 4 of them blue.

10. Use your formula from Exercise 8 to find the probability of picking 4 blue socks in 4 drawings from a box containing 6 socks, 5 of them blue.

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13-1 Enrichment

A Cross-Number Puzzle

Use the clues at the bottom of the page to complete the puzzle. Write one digit in each box.

A		▨	B		▨	C	D	▨	▨	E	F
	▨	▨		▨	▨	▨	G	H	▨	▨	
I	J	▨	▨	K			▨		▨	L	
▨		▨	M		▨	▨	N	▨	▨		▨
▨	O			▨	P	▨	Q	R		▨	S
T		▨	▨	U		▨	V		▨	W	

Across

A $x^2 - 4$ for $x = 5$

B $3xy^2$ for $x = 4$ and $y = -1$

C $(2x + 50) + (x - 15)$ for $x = 0$

E $x^2 - 4x - y^2$ for $x = 10$ and $y = 5$

G x^2y for $x = 3$ and $y = 7$

I $10w + 5y$ for $w = 6$ and $y = 1$

K $3x^2 + 5x + 8$ for $x = -10$

L $(y - 8) + (10 - 4y)$ for $y = -6$

M $23x - 16x$ for $x = 11$

O $7x + 100y$ for $x = 5$ and $y = 6$

Q $(6x^2 - 2) + (4x^2 - 3)$ for $x = -7$

T $(x^2 - x + 7) + (x^2 - 2)$ for $x = 3$

U x^2y for $x = -2$ and $y = 8$

V $7y - 12y - 2$ for $y = -10$

W $w^2 - w - 7$ for $w = 9$

Down

A $(6x^2 - 1) + (4x^2 - 3)$ for $x = 5$

B $7y + 8y - 2$ for $y = 1$

D $x + x^2y^2$ for $x = 7$ and $y = 1$

F $5(7w + 3w)$ for $w = 10$

H $(z^2 + 2z + 1) + (z^2 - 2z - 2)$ for $z = 4$

J $6xy^2 - xy + 60$ for $x = 10$ and $y = 10$

K $w^2 - w - 3$ for $w = 6$

L $(3y - 20) + (45 - 3y)$ for $y = 16$

M $11x^2 - 8x^2$ for $x = -5$

N $x^2 - 2x + y^2$ for $x = 10$ and $y = 8$

P $(2x + 52) + (x - 11)$ for $x = -3$

R $2x^2 - 5x - 140$ for $x = 12$

S $(y - 75) + (120 + 4y)$ for $y = -6$

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13-2 Enrichment

Adding Polynomials

Can you make a sentence using these words?

A FRUIT TIME LIKE AN BUT FLIES BANANA ARROW LIKE FLIES

Add the polynomials. Then find the word in the table at the right that corresponds to the sum. Read the words in order down the column to discover the hidden saying.

Word

1. $(2x^2 + 3x^2) + (5x^2 + x^2)$

2. $(2x^2 + 3x^3) + (5x^2 + x^2)$

3. $(2x^2 + x) + (xy + x)$

4. $(x^3 + 2x^2) + (5x^3 + x)$

5. $(x + xy) + (x^2 + xy)$

6. $(5x^2 + x) + (x + 2x^4)$

7. $(xy + y^2 + x^2) + (2xy + x^2)$

8. $(3x^2 + 2x^3) + (x^3 + x)$

9. $(x + x^2) + x^3$

10. $(x^3 + x^3) + (x^3 + x^3)$

11. $2x^{12} + 2x^{12}$

$4x^3$	A
$2x^2 + 3xy + y^2$	FRUIT
$11x^2$	TIME
$x^3 + x^2 + x$	LIKE
$6x^3 + 2x^2 + x$	AN
$2x^4 + 5x^2 + 2x$	BUT
$3x^3 + 8x^2$	FLIES
$4x^{12}$	BANANA
$x^2 + 2xy + x$	ARROW
$2x^2 + 2x + xy$	LIKE
$3x^3 + 3x^2 + x$	FLIES

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13-3 Enrichment

Polynomials with Fractional Coefficients

Polynomials may have fractional coefficients in some or all of the terms. Computation with these types of polynomials is done in the same way as with whole-number coefficients.

Add or subtract. Write all coefficients as fractions.

1. Add $\frac{3}{4}x^2 + \frac{2}{5}y^2$ and $\frac{1}{6}x^2 - \frac{4}{3}y^2$.

2. From $\frac{1}{2}x^2 - \frac{1}{3}xy^2 + \frac{1}{4}y^2$, take $\frac{1}{3}x^2 - \frac{1}{2}xy + \frac{5}{6}y^2$.

3. Add $\frac{3}{2}x - \frac{4}{3}y$, $-\frac{7}{8}x - \frac{6}{7}y$, and $y - \frac{1}{4}x$.

4. Subtract $\frac{1}{6}x^2 + \frac{1}{8}x - \frac{1}{4}$ from $\frac{2}{3}x^2 + \frac{5}{8}x + \frac{1}{2}$.

5. Add $\frac{1}{3}xy + \frac{11}{12}y^2$ to $\frac{4}{9}xy - \frac{1}{6}y^2$.

6. Add $\frac{1}{5}x^2 - \frac{1}{8}x - \frac{1}{3}$ and $\frac{3}{10}x^2 + \frac{5}{8}x + \frac{1}{9}$.

7. From $\frac{1}{2} + \frac{2}{3}y + \frac{3}{4}y^2$, take $\frac{1}{8} + \frac{1}{6}y - \frac{5}{6}y^2$.

8. Subtract $\frac{7}{12}x - \frac{1}{4}$ from $\frac{3}{4}x - \frac{1}{3}$.

9. Add $\frac{3}{8}x^2 - \frac{1}{3}xy + \frac{5}{9}y^2$ and $\frac{1}{2}x^2 - \frac{1}{2}xy - \frac{1}{3}y^2$.

10. Subtract $\frac{3}{4}y^2 + \frac{1}{2}y$ from $\frac{4}{3}y^2 + \frac{7}{8}y$.

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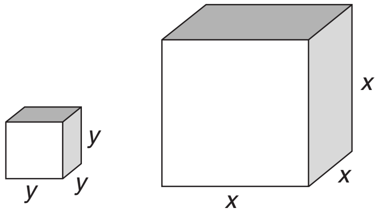
13-4 Enrichment

Polynomials and Volume

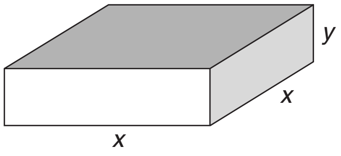
The volume of a rectangular prism can be written as the product of three polynomials. Recall that the volume equals the length times the width times the height.

The two prisms at the right represent the cube of y and the cube of x.

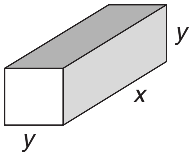


Multiply to find the volume of each prism. Write each answer as an algebraic expression.

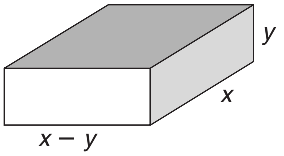
1.



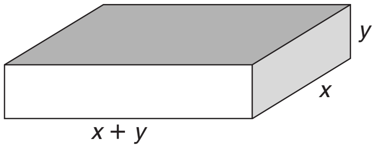
2.



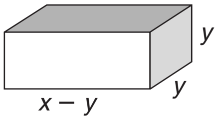
3.



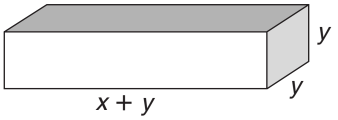
4.



5.

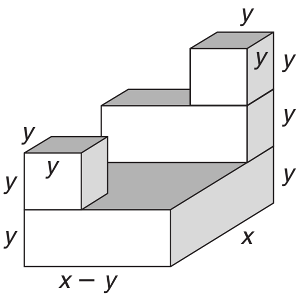


6.



Multiply, then add to find each volume. Write each answer as an algebraic expression.

7.



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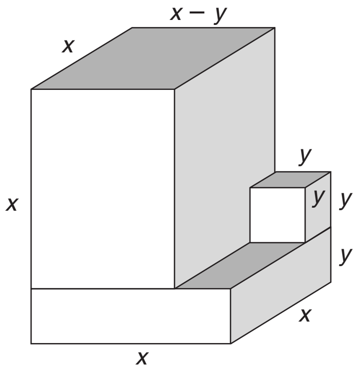

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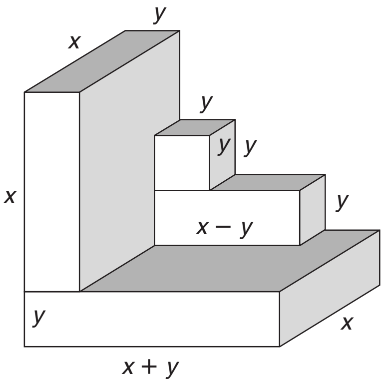

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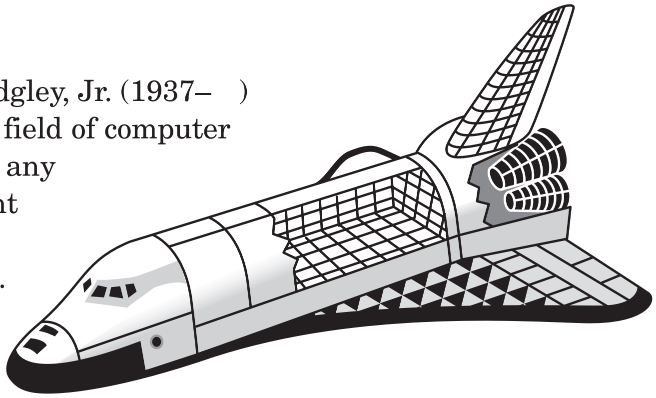
13-5 Enrichment

David R. Hedgley



African-American mathematician David R. Hedgley, Jr. (1937–) solved one of the most difficult problems in the field of computer graphics—how to program a computer to show any three-dimensional object from a given viewpoint just as the eye would see it. Hedgley's solution helped researchers in aircraft experimentation. Hedgley received an M.S. in Mathematics from California State University in 1970 and a Ph.D. in Computer Science from Somerset University in England in 1988. Hedgley has received numerous national achievement awards.

Polynomials in three variables are needed to describe some three-dimensional objects. Each variable represents one of the three dimensions: height, width, and depth.

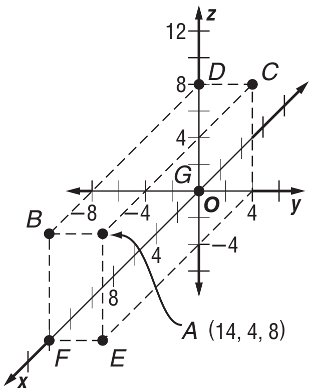
P_1: $x^2 + y^2 + z^2 + 10x + 4y + 2z - 19$

P_2: $2x^2 + 2y^2 + 2z^2 - 2x - 3y + 5z - 2$

1. Add the polynomials P_1 and P_2.

2. Subtract the polynomials, P_1 from P_2.

If the polynomials above were each set equal to zero, they would form equations describing two different spheres in three-dimensional space, or *3-space*. The coordinate plane you studied in Chapter 2 represents *two-space*. You described most lines in that plane by an equation in two variables. Each point on a line could be written as an ordered pair of numbers (x, y). Each point on any figure in 3-space can be written as an *ordered triple* of numbers (x, y, z).



3. What are the values of x, y, and z for point A in the diagram?

4. Give the ordered triple representing each of the points B through G in the diagram.

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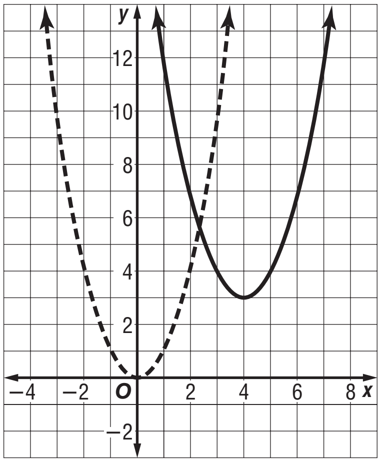
13-6 Enrichment

Translating Quadratic Graphs



When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been **translated** to the new position.

The graph of a quadratic equation in the form $y = (x - b)^2 + c$ is a translation of the graph of $y = x^2$.

Start with a graph of $y = x^2$.

Slide to the right 4 units.

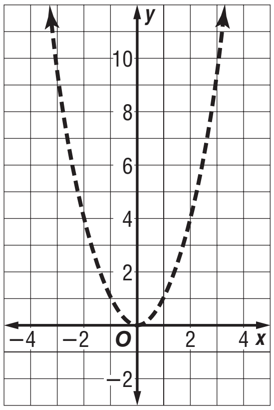
$$y = (x - 4)^2$$

Then slide up 3 units.

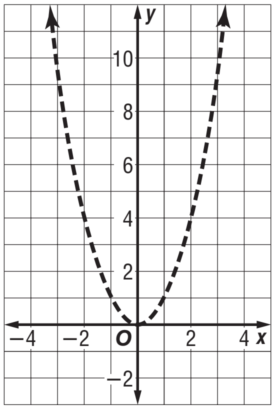
$$y = (x - 4)^2 + 3$$

The following equations are in the form $y = x^2 + c$. Graph each equation.

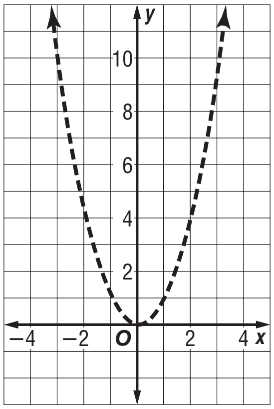
1. $y = x^2 + 1$



2. $y = x^2 + 2$

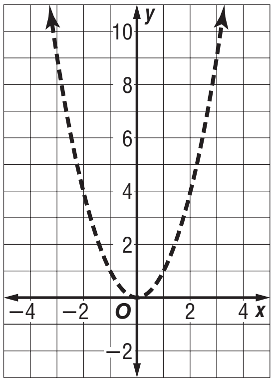


3. $y = x^2 - 2$

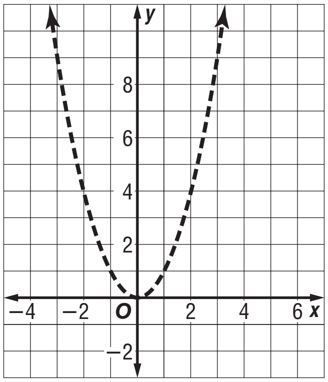


The following equations are in the form $y = (x - b)^2$. Graph each equation.

4. $y = (x - 1)^2$



5. $y = (x - 3)^2$



6. $y = (x + 2)^2$