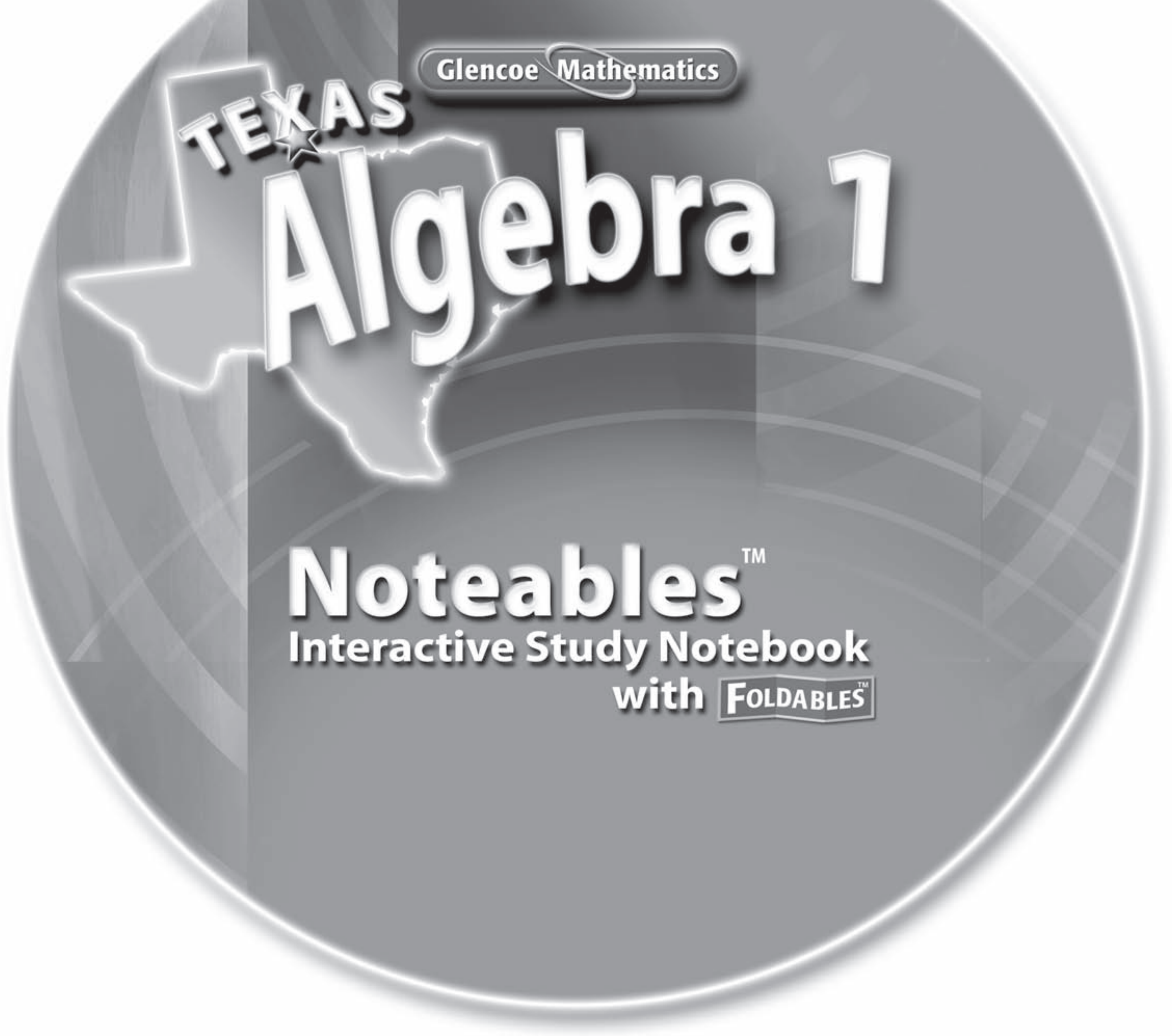

Glencoe Mathematics

TEXAS Algebra 1

Noteables™

Interactive Study Notebook with FOLDABLES™

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Algebra 1 (*Texas Student Edition*)
Noteables™: *Interactive Study Notebook with Foldables*™

1 2 3 4 5 6 7 8 9 10 047 11 10 09 08 07 06

Contents

CHAPTER 1

CHAPTER 2

CHAPTER 3

CHAPTER 4

CHAPTER 5

CHAPTER 6

Contents

CHAPTER 7

CHAPTER 8

CHAPTER 9

CHAPTER 10

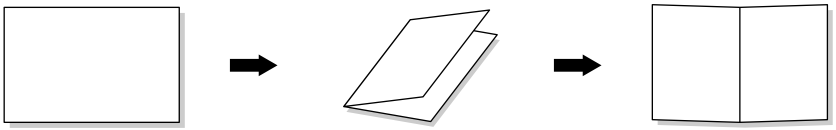
CHAPTER 11

CHAPTER 12

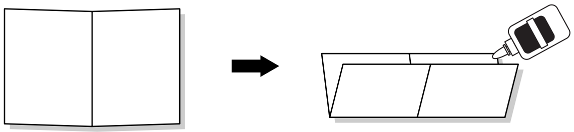
Organizing Your Foldables

Have students make this Foldable to help them organize and store their chapter Foldables. Begin with one sheet of 11" × 17" paper.

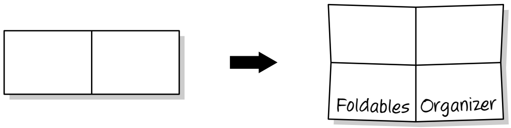
STEP 1 **Fold**
Fold the paper in half lengthwise. Then unfold.



STEP 2 **Fold and Glue**
Fold the paper in half widthwise and glue all of the edges.



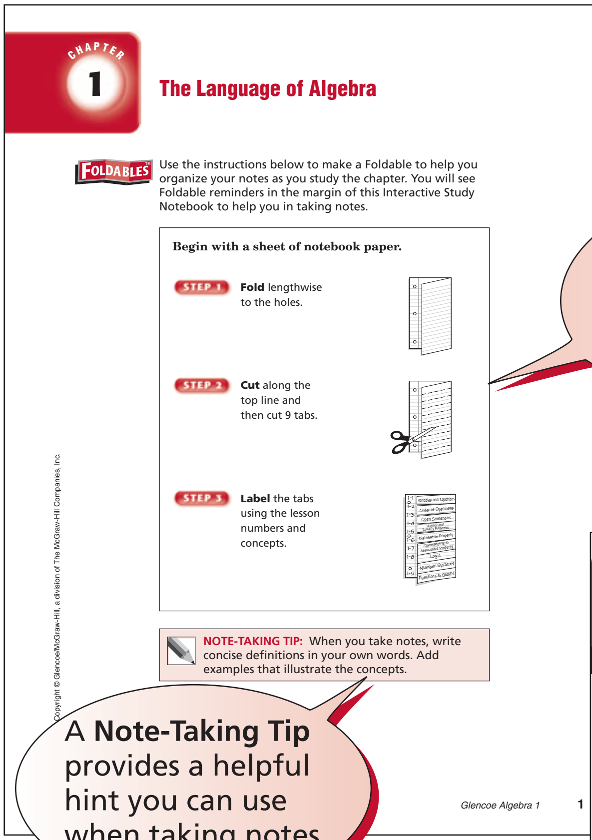
STEP 3 **Glue and Label**
Glue the left, right, and bottom edges of the Foldable to the inside back cover of your Noteables notebook.



Reading and Taking Notes As you read and study each chapter, record notes in your chapter Foldable. Then store your chapter Foldables inside this Foldable organizer.

Using Your Noteables™ with Foldables™ Interactive Study Notebook

This note-taking guide is designed to help you succeed in *Algebra 1*. Each chapter includes:



CHAPTER 1

The Language of Algebra

Chapter 1

FOLDABLES Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of notebook paper.

STEP 1 **Fold** lengthwise to the holes.

STEP 2 **Cut** along the top line and then cut 9 tabs.

STEP 3 **Label** the tabs using the lesson numbers and concepts.

NOTE-TAKING TIP: When you take notes, write concise definitions in your own words. Add examples that illustrate the concepts.

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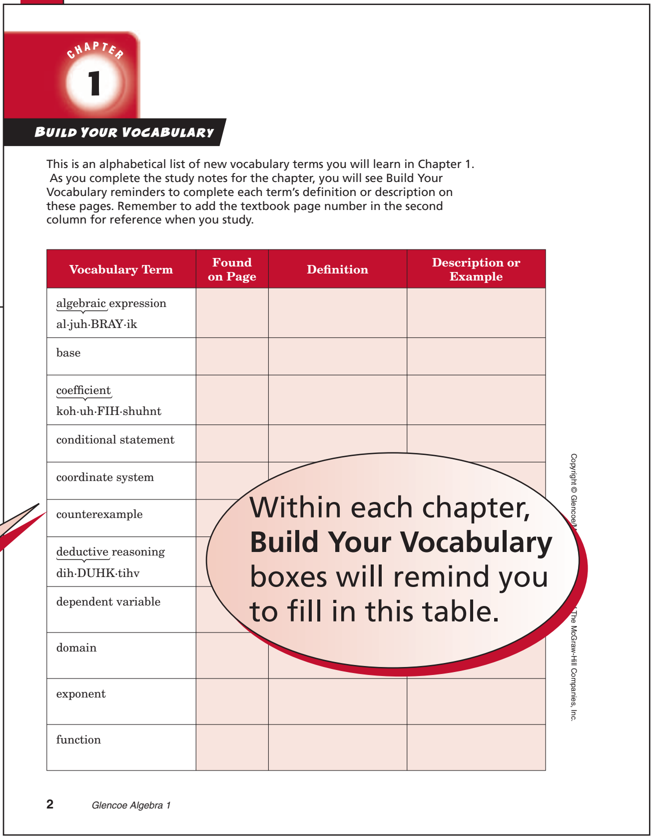
The **Chapter Opener** contains instructions and illustrations on how to make a Foldable that will help you to organize your notes.

A **Note-Taking Tip** provides a helpful hint you can use when taking notes.



CHAPTER 1

BUILD YOUR VOCABULARY

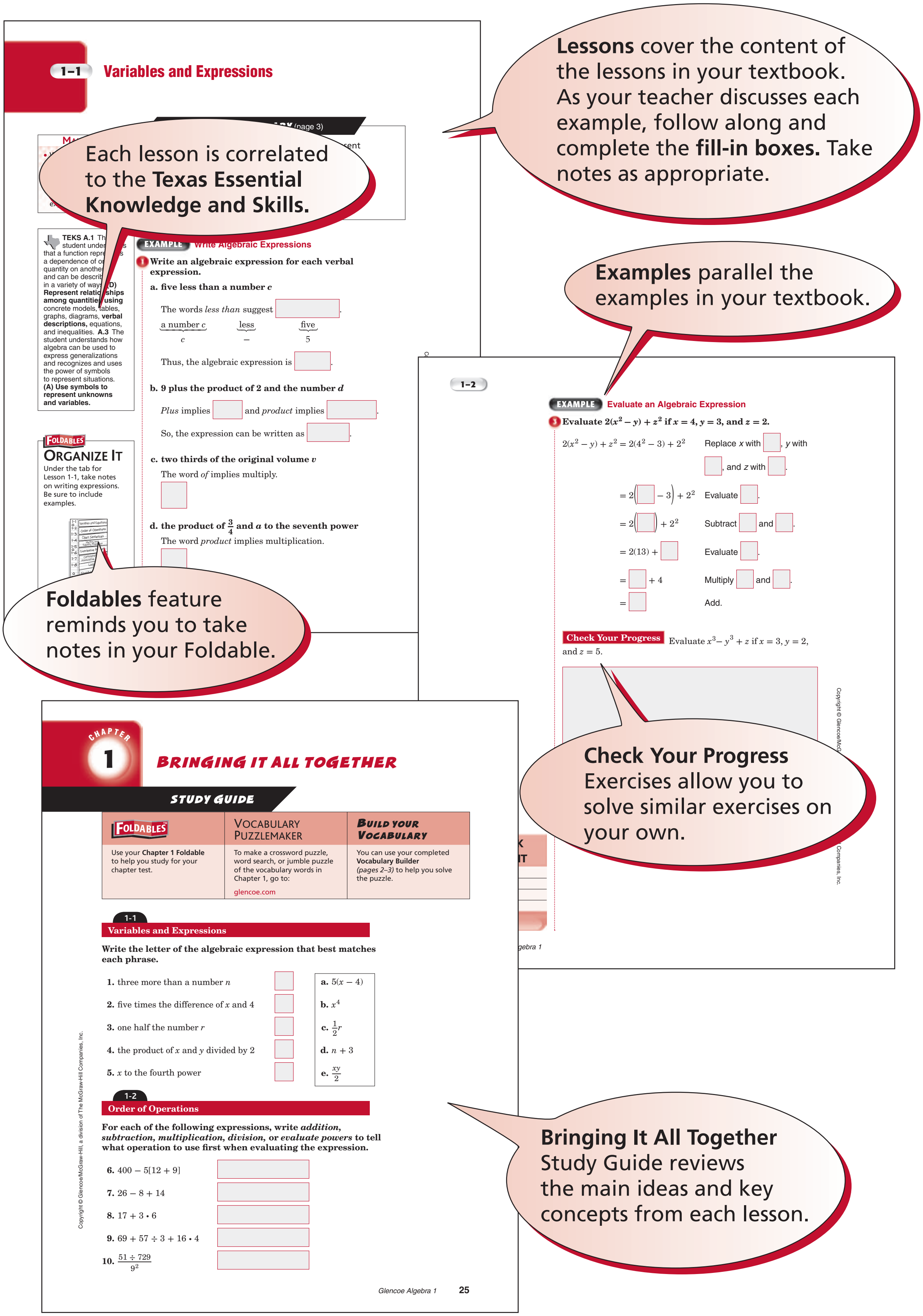
This is an alphabetical list of new vocabulary terms you will learn in Chapter 1. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
algebraic expression al·juh·BRAY·ik			
base			
coefficient koh·uh·FIH·shuhnt			
conditional statement			
coordinate system			
counterexample			
deductive reasoning dih·DUHK·tihv			
dependent variable			
domain			
exponent			
function			

2 Glencoe Algebra 1

The **Build Your Vocabulary** table allows you to write definitions and examples of important vocabulary terms together in one convenient place.

Within each chapter, **Build Your Vocabulary** boxes will remind you to fill in this table.



Lessons cover the content of the lessons in your textbook. As your teacher discusses each example, follow along and complete the **fill-in boxes.** Take notes as appropriate.

Each lesson is correlated to the **Texas Essential Knowledge and Skills.**

1–1 Variables and Expressions

TEKS A.1 The student under… that a function repre… a dependence of o… quantity on anothe… and can be describ… in a variety of ways. (D) **Represent relationships among quantities using** concrete models, tables, graphs, diagrams, **verbal descriptions,** equations, and inequalities. **A.3** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A) Use symbols to represent unknowns and variables.**

FOLDABLES
ORGANIZE IT
Under the tab for Lesson 1-1, take notes on writing expressions. Be sure to include examples.

EXAMPLE Write Algebraic Expressions

① **Write an algebraic expression for each verbal expression.**

a. five less than a number *c*

The words *less than* suggest ☐.

$\underbrace{\text{a number } c}_{c}\quad \underbrace{\text{less}}_{-}\quad \underbrace{\text{five}}_{5}$

Thus, the algebraic expression is ☐.

b. 9 plus the product of 2 and the number *d*

Plus implies ☐ and *product* implies ☐.

So, the expression can be written as ☐.

c. two thirds of the original volume *v*

The word *of* implies multiply.

☐

d. the product of $\frac{3}{4}$ and *a* to the seventh power

The word *product* implies multiplication.

☐

Foldables feature reminds you to take notes in your Foldable.

Examples parallel the examples in your textbook.

1–2

EXAMPLE Evaluate an Algebraic Expression

③ **Evaluate $2(x^2 - y) + z^2$ if $x = 4$, $y = 3$, and $z = 2$.**

$2(x^2 - y) + z^2 = 2(4^2 - 3) + 2^2$ Replace *x* with ☐, *y* with ☐, and *z* with ☐.

$= 2(\square - 3) + 2^2$ Evaluate ☐.

$= 2(\square) + 2^2$ Subtract ☐ and ☐.

$= 2(13) + \square$ Evaluate ☐.

$= \square + 4$ Multiply ☐ and ☐.

$= \square$ Add.

Check Your Progress Evaluate $x^3 - y^3 + z$ if $x = 3$, $y = 2$, and $z = 5$.

Check Your Progress Exercises allow you to solve similar exercises on your own.

CHAPTER 1

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 1 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 1, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 2–3)* to help you solve the puzzle.

1-1 Variables and Expressions

Write the letter of the algebraic expression that best matches each phrase.

1. three more than a number *n*	☐	**a.** $5(x - 4)$
2. five times the difference of *x* and 4	☐	**b.** x^4
3. one half the number *r*	☐	**c.** $\frac{1}{2}r$
4. the product of *x* and *y* divided by 2	☐	**d.** $n + 3$
5. *x* to the fourth power	☐	**e.** $\frac{xy}{2}$

1-2 Order of Operations

For each of the following expressions, write *addition, subtraction, multiplication, division,* or *evaluate powers* to tell what operation to use first when evaluating the expression.

6. $400 - 5[12 + 9]$ ☐

7. $26 - 8 + 14$ ☐

8. $17 + 3 \cdot 6$ ☐

9. $69 + 57 \div 3 + 16 \cdot 4$ ☐

10. $\frac{51 \div 729}{9^2}$ ☐

Glencoe Algebra 1 25

Bringing It All Together Study Guide reviews the main ideas and key concepts from each lesson.

NOTE-TAKING TIPS

Your notes are a reminder of what you learned in class. Taking good notes can help you succeed in mathematics. The following tips will help you take better classroom notes.

- Before class, ask what your teacher will be discussing in class. Review mentally what you already know about the concept.
- Be an active listener. Focus on what your teacher is saying. Listen for important concepts. Pay attention to words, examples, and/or diagrams your teacher emphasizes.
- Write your notes as clear and concise as possible. The following symbols and abbreviations may be helpful in your note-taking.

Word or Phrase	Symbol or Abbreviation	Word or Phrase	Symbol or Abbreviation
for example	e.g.	not equal	$\neq$
such as	i.e.	approximately	$\approx$
with	w/	therefore	$\therefore$
without	w/o	versus	vs
and	+	angle	$\angle$

- Use a symbol such as a star (★) or an asterisk (*) to emphasize important concepts. Place a question mark (?) next to anything that you do not understand.
- Ask questions and participate in class discussion.
- Draw and label pictures or diagrams to help clarify a concept.
- When working out an example, write what you are doing to solve the problem next to each step. Be sure to use your own words.
- Review your notes as soon as possible after class. During this time, organize and summarize new concepts and clarify misunderstandings.

Note-Taking Don'ts

- **Don't** write every word. Concentrate on the main ideas and concepts.
- **Don't** use someone else's notes as they may not make sense.
- **Don't** doodle. It distracts you from listening actively.
- **Don't** lose focus or you will become lost in your note-taking.

CHAPTER 1

The Language of Algebra

FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

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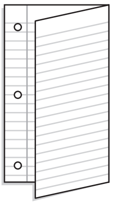
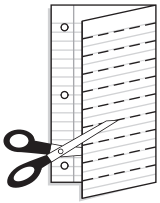
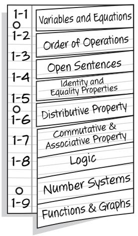


STEP 2 **Cut** along the top line and then cut 9 tabs.



STEP 3 **Label** the tabs using the lesson numbers and concepts.



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CHAPTER 1

BUILD YOUR VOCABULARY

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Vocabulary Term	Found on Page	Definition	Description or Example
algebraic expression al·juh·BRAY·ik			
base			
coefficient koh·uh·FIH·shuhnt			
conditional statement			
coordinate system			
counterexample			
deductive reasoning dih·DUHK·tihv			
dependent variable			
domain			
exponent			
function			

Vocabulary Term	Found on Page	Definition	Description or Example
hypothesis hy·PAH·thuh·suhs			
independent variable			
inequality			
integer			
irrational numbers			
like terms			
multiplicative inverses			
open sentence			
order of operations			
perfect square			
principal square root			
rational approximation			
real numbers			
reciprocal			
solution set			

1–1 Variables and Expressions

MAIN IDEAS

- Write mathematical expressions for verbal expressions.
- Write verbal expressions for mathematical expressions.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, **verbal descriptions,** equations, and inequalities. **A.3** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A) Use symbols to represent unknowns and variables.**

BUILD YOUR VOCABULARY (page 3)

In algebra, **variables** are symbols used to represent unspecified ______ or ______.

An expression like ______ is called a **power** and is read ______ power.

EXAMPLE Write Algebraic Expressions

① Write an algebraic expression for each verbal expression.

a. five less than a number *c*

The words *less than* suggest ______.

$$\underbrace{\text{a number } c}_{c} \qquad \underbrace{\text{less}}_{-} \qquad \underbrace{\text{five}}_{5}$$

Thus, the algebraic expression is ______.

b. 9 plus the product of 2 and the number *d*

Plus implies ______ and *product* implies ______.

So, the expression can be written as ______.

c. two thirds of the original volume *v*

The word *of* implies multiply.

______

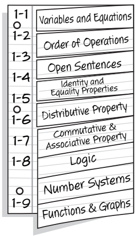
d. the product of $\frac{3}{4}$ and *a* to the seventh power

The word *product* implies multiplication.

______

FOLDABLES™ ORGANIZE IT

Under the tab for Lesson 1-1, take notes on writing expressions. Be sure to include examples.



Check Your Progress **Write an algebraic expression for each verbal expression.**

a. nine more than a number h

b. the difference of 6 and 4 times a number x

c. one half the size of the original perimeter p

d. the product of 6 and x to the fifth power

EXAMPLE Evaluate Powers

② **Evaluate 3^4.**

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ Use ☐ as a factor ☐ times.

$= \boxed{}$ Multiply.

Check Your Progress Evaluate the expression 5^4. ☐

EXAMPLE Write Verbal Expressions

③ **Write a verbal expression for each algebraic expression.**

a. $\frac{8x^2}{5}$

the quotient of 8 times ☐ and ☐

b. $y^5 - 16y$

the difference of ☐ and ☐

Check Your Progress **Write a verbal expression for each algebraic expression.**

a. $7a^4$ **b.** $x^2 + 3$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–2 Order of Operations

TEKS A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A) Use symbols to represent unknowns and variables.**

MAIN IDEAS

- Evaluate numerical expressions by using the order of operations.
- Evaluate algebraic expressions by using the order of operations.

EXAMPLE Evaluate Expressions

① **Evaluate $48 \div 2^3 \cdot 3 + 5$.**

$48 \div 2^3 \cdot 3 + 5 = 48 \div \boxed{} \cdot 3 + 5$	Evaluate powers.
$= \boxed{} \cdot 3 + 5$	Divide ☐ by ☐.
$= \boxed{} + 5$	Multiply ☐ and ☐.
$= \boxed{}$	Add ☐ and ☐.

Check Your Progress Evaluate $3 + 6^2 \div 4 - 5$.

☐

EXAMPLE Grouping Symbols

② **Evaluate each expression.**

a. $(8 - 3) \cdot 3(3 + 2)$

$(8 - 3) \cdot 3(3 + 2) = 5 \cdot 3(5)$	Evaluate inside grouping symbols.
$= \boxed{}(5)$	Multiply ☐ and ☐.
$= \boxed{}$	Multiply ☐ and ☐.

b. $4[12 \div (6 - 2)]^2$

$4[12 \div (6 - 2)]^2 = 4(12 \div 4)^2$	Evaluate innermost expression first.
$= 4\left(\boxed{}\right)^2$	Evaluate expression in grouping symbol.
$= 4\left(\boxed{}\right)$	Evaluate power.
$= \boxed{}$	Multiply.

KEY CONCEPT

Order of Operations

Step 1 Evaluate expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

FOLDABLES On the tab for Lesson 1–2, write the Order of Operations. Include examples.

c. $\frac{2^5 - 6 \cdot 2}{3^3 - 5 \cdot 3 - 2}$

$\frac{2^5 - 6 \cdot 2}{3^3 - 5 \cdot 3 - 2}$ means $(2^5 - 6 \cdot 2) \boxed{} (3^3 - 5 \cdot 3 - 2)$.

$\frac{2^5 - 6 \cdot 2}{3^3 - 5 \cdot 3 - 2} = \frac{\boxed{} - 6 \cdot 2}{3^3 - 5 \cdot 3 - 2}$ Evaluate the power in the numerator.

$= \frac{\boxed{} - \boxed{}}{3^3 - 5 \cdot 3 - 2}$ Multiply 6 and 2 in the numerator.

$= \frac{20}{3^3 - 5 \cdot 3 - 2}$ Subtract $\boxed{}$ and $\boxed{}$ in the numerator.

$= \frac{20}{\boxed{} - 5 \cdot 3 - 2}$ Evaluate the power in the denominator.

$= \frac{20}{\boxed{} - \boxed{} - 2}$ Multiply $\boxed{}$ and $\boxed{}$ in the denominator.

$= \frac{20}{\boxed{}}$ or $\boxed{}$ Subtract from left to right in the denominator. Then simplify.

Check Your Progress **Evaluate each expression.**

a. $2(4 + 7) \cdot (9 - 5)$

b. $3[5 - 2 \cdot 2]^2$

c. $\frac{3^3 - 4 \cdot 3}{2^5 - 5 \cdot 3 - 2}$

EXAMPLE

Evaluate an Algebraic Expression

③ Evaluate $2(x^2 - y) + z^2$ if $x = 4$, $y = 3$, and $z = 2$.

$2(x^2 - y) + z^2 = 2(4^2 - 3) + 2^2$ Replace *x* with ☐, *y* with ☐, and *z* with ☐.

$= 2(\square - 3) + 2^2$ Evaluate ☐.

$= 2(\square) + 2^2$ Subtract ☐ and ☐.

$= 2(13) + \square$ Evaluate ☐.

$= \square + 4$ Multiply ☐ and ☐.

$= \square$ Add.

Check Your Progress Evaluate $x^3 - y^3 + z$ if $x = 3$, $y = 2$, and $z = 5$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–3 Open Sentences

MAIN IDEAS

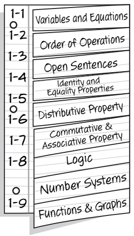
- Solve open sentence equations.
- Solve open sentence inequalities.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, **tables,** graphs, diagrams, verbal descriptions, equations, **and inequalities. A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values,** simplify polynomial expressions, transform **and solve equations,** and factor as necessary **in problem situations.**

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 1-3, explain how to solve open sentence equations and inequalities. Include examples.



BUILD YOUR VOCABULARY (pages 2–3)

The process of finding a value for a variable that results in a sentence is called **solving the open sentence.**

A sentence that contains an sign is called an **equation**.

A set of numbers from which replacements for a may be chosen is called a **replacement set.**

EXAMPLE Use a Replacement Set to Solve an Equation

① **Find the solution set for $3(8 - b) = 6$ if the replacement set is {2, 3, 4, 5, 6}.**

Replace b in $3(8 - b) = 6$ with each value in the replacement set.

a	$3(8 - b) = 6$	True or False?
2	$3(8 - 2) \stackrel{?}{=} 6 \rightarrow 18 \neq 6$	
3	$3(8 - 3) \stackrel{?}{=} 6 \rightarrow 15 \neq 6$	
4	$3(8 - 4) \stackrel{?}{=} 6 \rightarrow 12 \neq 6$	
5	$3(8 - 5) \stackrel{?}{=} 6 \rightarrow 9 \neq 6$	
6	$3(8 - 6) \stackrel{?}{=} 6 \rightarrow 6 = 6$	

The solution set is .

Check Your Progress Find the solution set for the equation $6c - 5 = 7$ if the replacement set is {0, 1, 2, 3, 4}.

BUILD YOUR VOCABULARY (page 3)

An open sentence that contains the symbol , , or is called an **inequality**.

EXAMPLE

② **FISHING** **Carlos needs $35 or more for a fishing trip. He already bought a ticket for the charter boat for $13. Does Carlos need to save $20, $21, $22, or $23 to have enough money for the fishing trip? Find the solution set for $s + 13 \geq 35$ if the replacement set is {20, 21, 22, 23}.**

Replace s in $s + 13 \geq 35$ with each value in the replacement set.

s	$s + 13 \geq 35$	True or False?
20	$20 + 13 \geq 35 \rightarrow 33 \geq 35?$	
21	$21 + 13 \geq 35 \rightarrow 34 \geq 35?$	
22	$22 + 13 \geq 35 \rightarrow 35 \geq 35$	
23	$23 + 13 \geq 35 \rightarrow 36 \geq 35$	

The solution set for $s + 13 \geq 35$ is . Carlos needs to save at least $22 or $23 for the fishing trip.

Check Your Progress **SHOPPING** Maleka needs $75 or more for a shopping trip. She already bought a sweater for $22. Does Maleka need to save $51, $52, $53, or $54 to have enough money for the shopping trip? Find the solution set for $s + 23 \geq 75$ if the replacement set is {51, 52, 53, 54}.

A {51, 52}; Maleka needs to save at least $51 or $52 for the shopping trip.

B {53, 54}; Maleka needs to save at least $53 or $54 for the shopping trip.

C {55}; Maleka needs to save at least $55 for the shopping trip.

D {35}; Maleka needs to save at least $35 for the shopping trip.

REVIEW IT

Write in words how each of the following symbols is read: >, <, ≥, ≤.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–4 Identity and Equality Properties

MAIN IDEAS

- Recognize the properties of identity and equality.
- Use the properties of identity and equality.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and **solve equations,** and factor as necessary **in problem situations.**

KEY CONCEPTS

Additive Identity For any number *a*, the sum of *a* and 0 is *a*.

Multiplicative Identity For any number *a*, the product of *a* and 1 is *a*.

Multiplicative Property of Zero For any number *a*, the product of *a* and 0 is *a*.

Multiplicative Inverse For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.

BUILD YOUR VOCABULARY (page 3)

Two numbers whose ____ is 1 are called **multiplicative inverses** or **reciprocals.**

EXAMPLE Identify Properties

① **Name the property used in each equation. Then find the value of *n*.**

a. $n \cdot 12 = 0$

Multiplicative Property of Zero

$n =$ ____, since ____ $\cdot\ 12 = 0$.

b. $n \cdot \frac{1}{5} = 1$

Multiplicative Inverse Property

$n =$ ____, since ____ $\cdot\ \frac{1}{5} = 1$.

Check Your Progress **Name the property used in each equation. Then find the value of *n*.**

a. $n \cdot \frac{1}{2} = 1$

b. $n \cdot 4 = 0$

KEY CONCEPTS

Reflexive Any quantity is equal to itself.

Symmetric If one quantity equals a second quantity, then the second quantity equals the first.

Transitive If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.

Substitution A quantity may be substituted for its equal in any expression.

FOLDABLES List the Identity and Equality Properties under the tab for Lesson 1–4. Include an example of each property.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Evaluate Using Properties

② **Evaluate $\frac{1}{4}(12 - 8) + 3(15 \div 5 - 2)$. Name the property used in each step.**

$\frac{1}{4}(12 - 8) + 3(15 \div 5 - 2)$

$= \frac{1}{4}(4) + 3(15 \div 5 - 2)$; $12 - 8 = 4$

$= \frac{1}{4}(4) + 3(3 - 2)$; $15 \div 5 = 3$

$= \frac{1}{4}(4) + 3(1)$; $3 - 2 = 1$

$= 1 + 3(1)$; $\frac{1}{4} \cdot 4 = 1$

$= 1 + 3$; $3 \cdot 1 = 3$

$= 4$; $1 + 3 = 4$

Check Your Progress Evaluate $\frac{1}{3}(10 - 7) + 4(18 \div 9 - 1)$. Name the property used in each step.

$\frac{1}{3}(10 - 7) + 4(18 \div 9 - 1)$

$= \frac{1}{3}(3) + 4(18 \div 9 - 1)$

$= \frac{1}{3}(3) + 4(2 - 1)$

$= \frac{1}{3}(3) + 4(1)$

$= 1 + 4(1)$

$= 1 + 4$

$= 5$

1–5 The Distributive Property

MAIN IDEAS

- Use the Distributive Property to evaluate expressions.
- Use the Distributive Property to simplify expressions.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.**

EXAMPLE Distribute Over Addition or Subtraction

① **EXERCISE Julia walks 5 days a week. She walks at a fast rate for 7 minutes and then cools down for 2 minutes. Rewrite 5(7 + 2) using the Distributive Property. Evaluate to find the total number of minutes Julia walks.**

$5(7 + 2) = 5 \cdot 7 + 5 \cdot 2$ Distributive Property

$= \boxed{} + \boxed{}$ Multiply.

$= \boxed{}$ Add.

Julia is on the treadmill for $\boxed{}$ minutes each week.

Check Your Progress **WALKING** Susanne walks to school and home from school 5 days each week. She walks to school in 15 minutes and then walks home in 10 minutes. Rewrite 5(15 + 10) using the Distributive Property. Then evaluate to find the total number of minutes Susanne spends walking to and home from school.

KEY CONCEPT

Distributive Property

For any numbers *a*, *b*, and *c*,

$a(b + c) = ab + ac$ and
$(b + c)a = ba + ca$ and
$a(b - c) = ab - ac$ and
$(b - c)a = ba - ca$

FOLDABLES™ Under the tab for Lesson 1–5, write the Distributive Property. Write a numeric and algebraic example of the property.

EXAMPLE The Distributive Property and Mental Math

② **Use the Distributive Property to find 12 • 82.**

$12 \cdot 82 = 12(80 + 2)$ Think: 82 = 80 + 2

$= 12\left(\boxed{}\right) + 12\left(\boxed{}\right)$ Distributive Property

$= 960 + 24$ Multiply.

$= \boxed{}$ Add.

Check Your Progress Use the Distributive Property to find 6 • 54.

BUILD YOUR VOCABULARY (pages 2–3)

Like terms are terms that contain the same variables, with corresponding variables having the same $\boxed{}$.

The **coefficient** of a term is the $\boxed{}$ factor.

EXAMPLE Algebraic Expressions

③ **Rewrite each product using the Distributive Property. Then simplify.**

a. $12(y + 3)$

$12(y + 3) = 12 \cdot y + 12 \cdot 3$ $\boxed{}$ Property

$= 12y + \boxed{}$ Multiply.

b. $4(y^2 + 8y + 2)$

$4(y^2 + 8y + 2) = \boxed{}(y^2) + \boxed{}(8y) + \boxed{}(2)$ Distributive Property

$= \boxed{} + \boxed{} + \boxed{}$ Multiply.

Check Your Progress Rewrite $3(x^3 + 2x^2 - 5x + 7)$ using the Distributive Property. Then simplify.

$\boxed{}$

EXAMPLE Combine Like Terms

④ **a. Simplify $17a + 21a$.**

$17a + 21a = (17 + 21)a$ Distributive Property

$= \boxed{}$ Substitution

b. Simplify $12b^2 - 8b^2 + 6b$.

$12b^2 - 8b^2 + 6b = \boxed{}b^2 + 6b$ Distributive Property

$= \boxed{}$ Substitution

Check Your Progress **Simplify each expression.**

a. $14x - 9x$ $\boxed{}$

b. $6n^2 + 7n + 8n$ $\boxed{}$

REMEMBER IT

When you simplify expressions, first identify like terms.

WRITE IT

Give an example of two like terms. Then give an example of two terms that are not like terms.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–6 Commutative and Associative Properties

MAIN IDEAS

- Recognize the Commutative and Associative Properties.
- Use the Commutative and Associative Properties to simplify expressions.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(B) Use the commutative, associative,** and distributive **properties to simplify algebraic expressions.**

EXAMPLE Use Addition Properties

① **TRANSPORTATION Refer to Example 1 on p. 35 of your book. Find the distance between Lakewood/ Ft. McPherson and Five Points. Explain how the Commutative Property makes calculating the answer unnecessary.**

Calculating the answer is actually unnecessary because the route is the opposite of the one in Example 1 of the textbook.

The ______ Property states that the ______ in which numbers are added does not matter.

The distance is ______ miles.

EXAMPLE Use Multiplication Properties

② **Evaluate $2 \cdot 8 \cdot 5 \cdot 7$ using properties of numbers. Name the property used in each step.**

You can rearrange and group the factors to make mental calculations easier.

$2 \cdot 8 \cdot 5 \cdot 7 = 2 \cdot 5 \cdot 8 \cdot 7$ ______ Property (×)

$=$ ______ $\cdot (8 \cdot 7)$ ______ Property (×)

$=$ ______ $\cdot$ ______ Multiply.

$=$ ______ Multiply.

KEY CONCEPTS

Commutative Property The order in which you add or multiply numbers does not change their sum or product.

Associative Property The way you group three or more numbers when adding or multiplying does not change their sum or product.

FOLDABLES List the properties on the tab for Lesson 1–6.

Check Your Progress

a. The distance from Five Points to Garnett is 0.4 mile. From Garnett, West End is 1.5 miles. From West End, Oakland City is 1.5 miles. Write an expression to find the distance from Five Points to Oakland City. Then write an expression to find the distance from Oakland City to Five Points.

b. Evaluate $3 \cdot 5 \cdot 3 \cdot 4$.

EXAMPLE Write and Simplify an Expression

3 **Use the expression *three times the sum of 3x and 2y added to five times the sum of x and 4y.***

a. Write an algebraic expression for the verbal expression.

Words	Three times the sum of $3x$ and $2y$ added to five times the sum of x and $4y$
Variables	Let x and y represent the numbers.
Expression	

b. Simplify the expression and indicate the properties used.

$3(3x + 2y) + 5(x + 4y)$

$= 3(3x) + 3(2y) + 5(x) + 5(4y)$	Distributive Property
$= 9x + 6y + 5x + 20y$	Multiply.
$= 9x + 5x + 6y + 20y$	Commutative (+)
$= (9x + 5x) + (6y + 20y)$	Associative (+)
$= (9 + 5)x + (6 +20)y$	Distributive Property
$=$	Substitution

Check Your Progress **Use the expression *five times the sum of 2x and 3y increased by 2 times the sum of x and 6y.***

a. Write an algebraic expression for the verbal expression.

b. Simplify the expression and indicate the properties used.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–7 Logical Reasoning and Counterexamples

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(E) Interpret and make** decisions, **predictions,** and critical judgments **from functional relationships.**

MAIN IDEAS

- Identify the hypothesis and conclusion in a conditional statement.
- Use a counterexample to show that an assertion is false.

BUILD YOUR VOCABULARY (pages 2–3)

Conditional statements can be written in the form

A B.

The part of the statement immediately after is called the **hypothesis**.

The part of the statement immediately after is called the **conclusion**.

EXAMPLE Identify Hyphothesis and Conclusion

① **Identify the hypothesis and conclusion of each statement.**

a. If it is raining, then Beau and Chloe will not play softball.

The hypothesis follows the word and the conclusion follows the word .

Hypothesis:

Conclusion:

b. If $7y + 5 \leq 26$, then $y \leq 3$.

Hypothesis:

Conclusion:

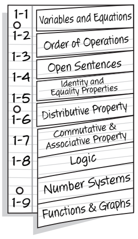
Check Your Progress **Identify the hypothesis and conclusion of each statement.**

a. If it is above 75°, then you can go swimming.

b. If $2x + 3 = 5$, then $x = 1$.

FOLDABLES™ ORGANIZE IT

On the tab for Lesson 1–7, write a conditional sentence and label the hypothesis and conclusion.



EXAMPLE Write a Conditional in If-Then Form

2 Identify the hypothesis and conclusion of the statement. Then write the statement in if-then form.

I eat light meals.

Hypothesis: I eat a meal

Conclusion: it is light

Check Your Progress Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

For a number x such that $11 + 5x < 21$, $x \leq 2$.

EXAMPLE

3 Determine a valid conclusion that follows from the statement, "*If one number is odd and another number is even, then their sum is odd*" for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

The two numbers are 5 and 12

5 is odd and 12 is even, so the hypothesis is true.

Check Your Progress Determine a valid conclusion that follows from the statement "*If the last digit in a number is 0, then the number is divisible by 10*" for the given conditions. If a valid conclusion does not follow, write *no valid conclusion*.

The number is 4005.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–8 Square Roots and Real Numbers

MAIN IDEAS

- Find square roots.
- Classify and order real numbers.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values,** simplify polynomial expressions, transform and solve equations, and factor as necessary **in problem situations.**

BUILD YOUR VOCABULARY (page 31)

A **square root** is one of two square ______ of a number.

A number whose square root is a ______ number is called a **perfect square.**

A radical sign is used to indicate the ______ or **principal square root** of the expression under the radical sign.

EXAMPLE Classify Real Numbers

① **Name the set or sets of numbers to which each number belongs.**

a. $\sqrt{17}$

Because $\sqrt{17}$ = ______, which is neither a repeating nor terminating decimal, this number is ______.

b. $\frac{1}{6}$

Because 1 and 6 are integers and $1 \div 6 = 0.1666\ldots$ is a repeating decimal, this number is a ______ number.

c. $\sqrt{169}$

Because $\sqrt{169}$ = ______, this number is a ______ number, a ______ number, an ______ and a ______ number.

Check Your Progress **Name the set or sets of numbers to which each real number belongs.**

a. $\frac{7}{9}$ ______

b. $\sqrt{36}$ ______

c. $\sqrt{45}$ ______

d. $-\frac{56}{7}$ ______

WRITE IT

Are the square roots for $\sqrt{-81}$ and $\sqrt{81}$ the same? Explain.

______

______

______

______

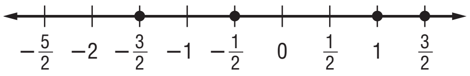
______

Key Concept

Real Numbers The set of real numbers consists of the set of rational numbers and the set of irrational numbers.

EXAMPLE

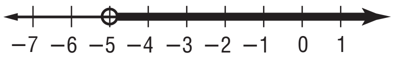
2 **a. Graph $\left\{\frac{3}{2}, -\frac{1}{2}, 1, \frac{3}{2}\right\}$.**



b. Graph $y \leq 8$.

The heavy arrow indicates that all numbers to the left of are included in the graph. The dot at indicates that *is* included in the graph.

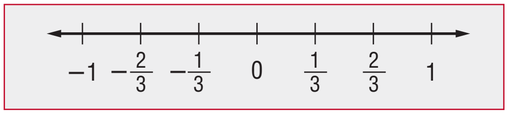
c. Graph $z > -5$.



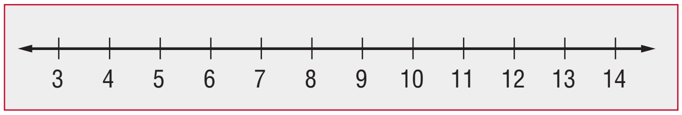
The heavy arrow indicates that all the points to the right of are included in the graph. The circle at indicates that is *not* included in the graph.

Check Your Progress **Graph each solution set.**

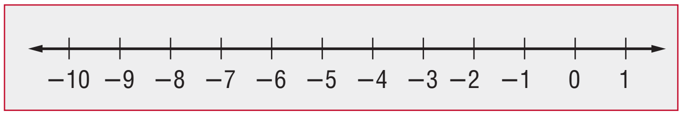
a. $\left\{-\frac{2}{3}, -\frac{1}{3}, 0, \frac{2}{3}\right\}$



b. $x \geq 6$



c. $x < -3$



EXAMPLE Find Square Roots

3 **Find $-\sqrt{\frac{16}{9}}$.**

$-\sqrt{\frac{16}{9}}$ represents the square root of $\frac{16}{9}$.

$\frac{16}{9} =$, so $-\sqrt{\frac{16}{9}} =$

Check Your Progress Find $\sqrt{\frac{64}{25}}$.

EXAMPLE

④ **SPORTS SCIENCE Refer to the application on p. 45 of your textbook. Find the surface area of an athlete whose height is 147 centimeters and whose weight is 48 kilograms.**

$$\sqrt{\frac{\text{height} \cdot \text{weight}}{3600}} = \sqrt{\frac{147 \cdot 48}{3600}}$$ $h =$, $w =$.

$$= \sqrt{\frac{7056}{3600}}$$ Simplify.

$=$ or Simplify.

The surface area of the athlete is .

Check Your Progress **SPORTS SCIENCE** Find the surface area of an athlete whose height is 152 centimeters and whose weight is 50 kilograms.

EXAMPLE Order Real Numbers

⑤ **Write $\frac{12}{5}$, $\sqrt{6}$, $2.\overline{4}$, and $\frac{61}{25}$ in order from least to greatest.**

Write each number as a decimal.

$\frac{12}{5} =$

$\sqrt{6} = 2.4494897\ldots$ or about 2.4495.

$2.\overline{4} = 2.444444\ldots$ or about 2.4444.

$\frac{61}{25} =$

Since $2.4 < 2.44 < 2.4444 < 2.4495$, the numbers arranged in order from least to greatest are .

Check Your Progress Write $\frac{5}{2}$, $\sqrt{5}$, $2.\overline{2}$, and $\frac{51}{20}$ in order from least to greatest.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–9 Functions and Graphs

MAIN IDEAS

- Interpret graphs of functions.
- Draw graphs of functions.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(A) Describe independent and dependent quantities in functional relationships.** **A.2** The student uses the properties and attributes of functions. **(B)** Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete. **(C) Interpret situations in terms of given graphs or create situations that fit given graphs.** *Also addresses TEKS A.1(D).*

BUILD YOUR VOCABULARY (page 2)

A **function** is a relationship between input and output, in which the ______ depends on the ______.

A **coordinate system** is used to graph ______.

In a function, the value of one quantity ______ on the ______ of the other. This ______ is called the **dependent variable.** The other quantity is called the **independent variable.** The set of values for the ______ variable is called the **domain.**

The set of values for the ______ variable is called the **range.**

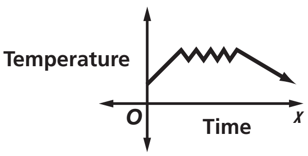
EXAMPLE Identify Coordinates

① **MEDICINE Refer to the graph on p. 51 of your book. Name the ordered pair at point E and explain what it represents.**

Point E is at 6 along the x-axis and about 100 along the y-axis. So, its ordered pair is ______. This represents about ______ normal blood flow 6 days after the injury.

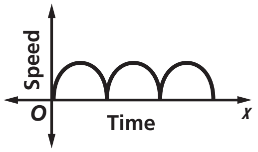
EXAMPLE Analyze Graphs

② **The graph represents the temperature in Ms. Ling's classroom on a winter school day. Describe what is happening in the graph.**



The ______ is low until the heat is turned on. Then the temperature fluctuates ______ and ______ because of the thermostat. Finally, the temperature drops when the heat is turned ______.

Check Your Progress The graph represents Macy's speed as she swims laps in a pool. Describe what is happening in the graph.



EXAMPLE Draw Graphs

3 **There are three lunch periods at a school cafeteria. During the first period, 352 students eat lunch. During the second period, 304 students eat lunch. During the third period, 391 students eat lunch.**

a. Make a table showing the number of students for each of the three lunch periods.

Period	1	2	3
Number of Students			

b. Write the data as a set of ordered pairs. Then graph the data.

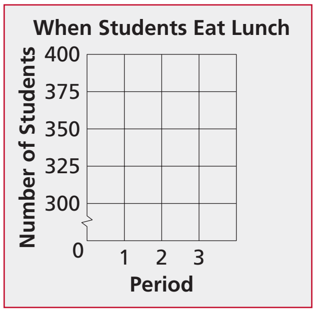
The period is the variable and the number of students is the variable.

The ordered pairs are

(1,),

(2,),

and (3,).



Check Your Progress **At a car dealership, a salesman worked for three days. On the first day, he sold 5 cars. On the second day, he sold 3 cars. On the third day he sold 8 cars.**

a. Make a table showing the number of cars sold for each day.

Day			
Number of Cars Sold			

WRITE IT

List three ways data can be represented.

REMEMBER IT

The *x*-axis is the horizontal axis and the *y*-axis is the vertical axis. The independent variable is graphed on the *x*-axis and the dependent variable is graphed on the *y*-axis.

b. Write the data as a set of ordered pairs.

c. Draw a graph that shows the relationship between the day and the number of cars sold.

EXAMPLE Domain and Range

⑤ Mr. Ohms tutors students. He works at most 120 hours for $4 per hour.

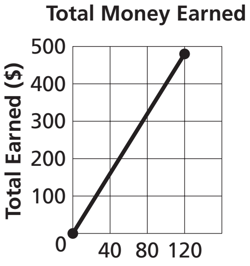
a. Identify a domain and range for this situation.

The domain contains the number of hours he works.

The domain is to hours. The range contains the amount he makes from $0. Thus, the range is to × $4 or .

b. Draw a graph that shows the relationship between the number of hours worked and the money Mr. Ohms makes.

Graph the ordered pairs and . Since any number of hours up to 120 can be worked, connect the two points with a line to include those points.



Check Your Progress **Prom tickets cost $25 per person. The prom is limited to 250 people.**

a. Identitfy a domain and range for the situation.

b. Draw a graph that shows the relationship between the number of persons attending the prom and total admission price.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 1

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 1 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 1, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 2–3)* to help you solve the puzzle.

1-1

Variables and Expressions

Write the letter of the algebraic expression that best matches each phrase.

1. three more than a number n

2. five times the difference of x and 4

3. one half the number r

4. the product of x and y divided by 2

5. x to the fourth power

a. $5(x - 4)$

b. x^4

c. $\frac{1}{2}r$

d. $n + 3$

e. $\frac{xy}{2}$

1-2

Order of Operations

For each of the following expressions, write *addition, subtraction, multiplication, division,* or *evaluate powers* to tell what operation to use first when evaluating the expression.

6. $400 - 5[12 + 9]$

7. $26 - 8 + 14$

8. $17 + 3 \cdot 6$

9. $69 + 57 \div 3 + 16 \cdot 4$

10. $\frac{51 \div 729}{9^2}$

Chapter 1 BRINGING IT ALL TOGETHER

1-3

Open Sentences

11. How would you read each inequality symbol in words?

Symbol	Words	Symbol	Words
$<$		$\leq$	
$>$		$\geq$	

1-4

Identity and Equality Properties

Name the property used in each statement.

12. $\frac{5}{7} \cdot \frac{7}{5} = 1$

13. $3 \cdot 1 = 3$

14. $6 + 0 = 6$

15. If $2 + 4 = 5 + 1$ and $5 + 1 = 6$, then $2 + 4 = 6$.

16. If $n = 2$, then $5n = 5 \cdot 2$.

1-5

The Distributive Property

Rewrite using the distributive property.

17. $5(6 - 4)$

18. $12m + 8m$

1-6

Commutative and Associative Properties

Write the letter of the term that best matches each equation.

19. $3 + 6 = 6 + 3$

20. $2 + (3 + 4) = (2 + 3) + 4$

21. $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$

22. $2 \cdot (3 \cdot 4) = 2 \cdot (4 \cdot 3)$

a. Associative Property of Addition

b. Associative Property of Multiplication

c. Commutative Property of Addition

d. Commutative Property of Multiplication

1-7

Logical Reasoning and Counterexamples

Write *hypothesis* or *conclusion* to tell which part of the if-then statement is underlined.

23. If it is Tuesday, then it is raining.

24. If $\underline{3x + 7 = 13}$, then $x = 2$.

1-8

Number Systems

Complete each statement.

25. The positive square root of a number is called the square root of the number.

26. A number whose positive square root is a rational number is a .

Write each of the following as a mathematical expression that uses the $\sqrt{\ }$ symbol. Then find each square root.

27. the positive square root of 1600

28. the negative square root of 729

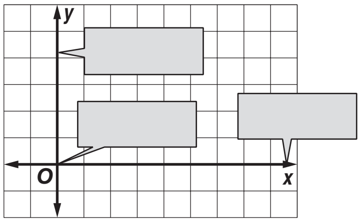
29. the principal square root of 3025

30. the irrational numbers and rational numbers together form the set of numbers

1-9

Functions and Graphs

31. Identify each part of the coordinate system.



CHAPTER 1

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 1.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 1 Practice Test on page 63 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 1 Study Guide and Review on pages 58–62 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice on page 63.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 1 Foldable.
- Then complete the Chapter 1 Study Guide and Review on pages 58–62 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 63.

Student Signature

Parent/Guardian Signature

Teacher Signature

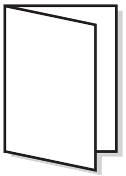
CHAPTER 2

Solving Linear Equations

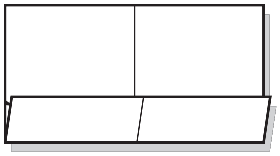
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with 4 sheets of plain $8\frac{1}{2}$" by 11" paper.

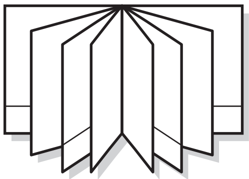
STEP 1 **Fold** in half along the width.



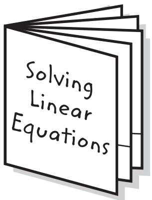
STEP 2 **Open and Fold** the bottom to form a pocket Glue edges.



STEP 3 **Repeat Steps 1 and 2** three times and glue all four pieces together.



STEP 4 **Label** each pocket. Place an index card in each pocket.



NOTE-TAKING TIP: When taking notes, write down a question mark next to anything you do not understand. Before your next quiz, ask your instructor to explain these sections.

CHAPTER 2

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 2. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
consecutive integers [kuhn·SEH·kyuh·tihv]			
defining a variable			
dimensional analysis [duh·MEHNCH·nuhl]			
equivalent equations [ih·KWIHV·luhnt]			
extremes			
formula			
four-step problem-solving plan			
identity			
means			
mixture problem			
multi-step equations			

Vocabulary Term	Found on Page	Definition	Description or Example
number theory			
percent of change			
percent of decrease			
percent of increase			
$\underbrace{\text{proportion}}$ [pruh·POHR·shun]			
ratio			
rate			
scale			
solve an equation			
uniform motion problem			
weighted average			

2–1 Writing Equations

MAIN IDEAS

- Translate verbal sentences into equations.
- Translate equations into verbal sentences.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(C) Describe functional relationships for given problem situations and write equations** or inequalities **to answer questions arising from the situations.** **(D) Represent relationships among quantities using** concrete models, tables, graphs, **diagrams, verbal descriptions, equations,** and inequalities. *Also addresses TEKS A.3(A).*

BUILD YOUR VOCABULARY (pages 30–31)

Choosing a variable to represent an unspecific in a problem is called **defining a variable**.

EXAMPLE Translate Sentences into Equations

1 Translate each sentence into an equation.

a. A number *b* divided by three is equal to six less than *c*.

$$\underbrace{b\text{ divided by three}}_{\frac{b}{3}} \qquad \underbrace{\text{is equal to}}_{=} \qquad \underbrace{\text{six less than } c.}_{}$$

The equation is .

b. Fifteen more than *z* times 6 is *y* times 2 minus eleven.

Check Your Progress Translate the sentence into an equation: *A number c multiplied by six is equal to two more than d.*

KEY CONCEPT

Four-Step Problem-Solving Plan

Step 1 Explore the problem.

Step 2 Plan the solution.

Step 3 Solve the problem.

Step 4 Examine the solution.

FOLDABLES Write the four-step problem-solving plan on an index card.

EXAMPLE Use the Four-Step Plan

2 JELLYBEANS **A jellybean manufacturer produces 1,250,000 jellybeans per hour. How many hours does it take them to produce 10,000,000 jellybeans?**

Write an equation. Let h represent the number of hours needed to produce the jellybeans.

$$\underbrace{1{,}250{,}000}_{1{,}250{,}000} \quad \underbrace{\text{times}}_{} \quad \underbrace{\text{hours}}_{h} \quad \underbrace{\text{equals}}_{} \quad \underbrace{10{,}000{,}000.}_{10{,}000{,}000}$$

$1{,}25\cancel{0{,}000}h = 10{,}00\cancel{0{,}000}$

Find h mentally by asking, “What number times 125 equals 1,000?”

$h = \boxed{}$

It will take $\boxed{}$ hours to produce 10,000,000 jellybeans.

Check Your Progress A person at the KeyTronic World Invitational Type-Off typed 148 words per minute. How many minutes would it take to type 3552 words?

BUILD YOUR VOCABULARY (pages 30–31)

A **formula** is an $\boxed{}$ that states a $\boxed{}$ for the relationship between certain quantities.

EXAMPLE Write a Formula

3 Translate the sentence into a formula.

The perimeter of a square equals four times the length of the side.

Words

Variable

Equation

Perimeter equals four times the length of the side.

Let P = perimeter and s = length of a side.

$$\underbrace{\text{Perimeter}}_{P} \quad \underbrace{\text{equals}}_{=} \quad \underbrace{\text{four times the length of a side.}}_{\boxed{}}$$

The formula is $\boxed{} = \boxed{}$

Check Your Progress Translate the sentence *The area of a circle equals the product of* π *and the square of the radius r* into a formula.

EXAMPLE Translate Equations into Sentences

④ **Translate each equation into a verbal sentence.**

a. $12 - 2x = -5$

$$\underbrace{12}_{\boxed{}}\quad\underbrace{-}_{\text{minus}}\quad\underbrace{2x}_{\boxed{}}\quad\underbrace{=}_{\text{equals}}\quad\underbrace{-5}_{\boxed{}}.$$

b. $a^2 + 3b = \frac{c}{6}$

$$\underbrace{a^2}_{\boxed{}}\quad\underbrace{+}_{\text{plus}}\quad\underbrace{3b}_{\boxed{}}\quad\underbrace{=}_{\text{equals}}\quad\underbrace{\frac{c}{6}}_{\boxed{}}.$$

Check Your Progress **Translate each equation into a verbal sentence.**

a. $\frac{12}{b} - 4 = -1$

b. $5a = b^2 + 1$

⑤ **Write a problem based on the given information.**

f = cost of fries $\qquad f + 1.50$ = cost of a burger

$4(f + 1.50) - f = 8.25$

The cost of a burger is $\boxed{}$ more than the cost of fries.

Four times the cost of a burger $\boxed{}$ the cost of fries equals $\boxed{}$. How much do fries cost?

Check Your Progress **Write a problem based on the given information.**

h = Tiana's height in inches

$h - 3$ = Consuelo's height in inches

$3h(h - 3) = 8262$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–2 Solving Equations by Using Addition and Subtraction

MAIN IDEAS

- Solve equations by using addition.
- Solve equations by using subtraction.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, verbal descriptions, **equations**, and inequalities. **A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values,** simplify polynomial expressions, **transform and solve equations**, and factor as necessary **in problem situations.** *Also addresses TEKS A.1(C).*

BUILD YOUR VOCABULARY (pages 30–31)

Equivalent equations have the $\boxed{}$ solution.

To **solve an equation** means to find all values of the $\boxed{}$ that make the equation a $\boxed{}$ statement.

EXAMPLE Solve by Adding

① Solve each equation. Check your solution.

a. $h - 12 = -27$

$h - 12 = -27$	Original equation
$h - 12 + \boxed{} = -27 + \boxed{}$	Add $\boxed{}$ to each side.
$h = \boxed{}$	$-12 + 12 = 0$ and $-27 + 12 = \boxed{}$.

b. $k + 63 = 92$

$k + 63 = 92$	Original equation
$k + 63 + \boxed{} = 92 + \boxed{}$	Add $\boxed{}$ to each side.
$k = \boxed{}$	$63 + (-63) = 0$ and $92 + (-63) = \boxed{}$.

EXAMPLE Solve by Subtracting

② Solve $c + 102 = 36$.

$c + 102 = 36$	Original equation
$c + 102 - \boxed{} = 36 - \boxed{}$	Subtract $\boxed{}$ from each side.
$c = \boxed{}$	$102 - 102 = 0$ and $36 - 102 = \boxed{}$

KEY CONCEPTS

Addition Property of Equality If an equation is true and the same number is added to each side, the resulting equation is true.

Subtraction Property of Equality If an equation is true and the same number is subtracted from each side, the resulting equation is true.

EXAMPLE Solve by Adding or Subtracting

③ **Solve $y + \frac{4}{5} = \frac{2}{3}$ in two ways.**

METHOD 1 Use the Subtraction Property of Equality.

$$y + \frac{4}{5} = \frac{2}{3}$$ Original equation

$$y + \frac{4}{5} - \square = \frac{2}{3} - \square$$ Subtract $\square$ from each side.

$$y = -\frac{2}{15}$$ Simplify.

METHOD 2 Use the Addition Property of Equality.

$$y + \frac{4}{5} = \frac{2}{3}$$ Original equation

$$y + \frac{4}{5} + \square = \frac{2}{3} + \square$$ Add $\square$ to each side.

$$y = -\frac{2}{15}$$ Simplify.

Check Your Progress **Solve each equation.**

a. $a - 24 = 16$

b. $t + 22 = -39$

c. $129 + k = -42$

d. $\frac{2}{3} + y = \frac{5}{6}$

EXAMPLE Write and Solve an Equation

④ **Write and solve an equation for the problem.**

Fourteen more than a number is equal to twenty-seven. Find this number.

$$\underbrace{\text{Fourteen}}_{14} \quad \underbrace{\text{more than}}_{+} \quad \underbrace{\text{a number}}_{n} \quad \underbrace{\text{is equal to}}_{=} \quad \underbrace{\text{twenty-seven.}}_{27}$$

$$14 + n = 27$$ Original equation

$$14 + n - \square = 27 - \square$$ Subtract $\square$ from each side.

$$n = \square$$ 14 – 14 = 0 and 27 – 14 = 13

Check Your Progress Twelve less than a number is equal to negative twenty-five. Find the number.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–3 Solving Equations by Using Multiplication and Division

MAIN IDEAS

- Solve equations by using multiplication.
- Solve equations by using division.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, verbal descriptions, **equations,** and inequalities. **A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values,** simplify polynomial expressions, **transform and solve equations,** and factor as necessary **in problem situations.** *Also addresses TEKS A.1(C).*

KEY CONCEPT

Multiplication Property of Equality If an equation is true, and each side is multiplied by the same number, the resulting equation is true.

FOLDABLES On an index card, write and solve an equation that uses the Multiplication Property of Equality.

EXAMPLE Solve Using Multiplication by a Positive Number

① **Solve $\frac{s}{12} = \frac{3}{4}$. Check your solution.**

$\frac{s}{12} = \frac{3}{4}$	Original equation
$\square\left(\frac{s}{12}\right) = \square\left(\frac{3}{4}\right)$	Multiply each side by $\square$.
$s = \square$	Simplify.

The solution is $\square$.

EXAMPLE Solve Using Multiplication by a Fraction

② **Solve each equation**

a. $\left(-1\frac{3}{8}\right)k = \frac{2}{3}$

$\left(-1\frac{3}{8}\right)k = \frac{2}{3}$	original equation
$\left(-\frac{11}{8}\right)k = \square$	Rewrite $-1\frac{3}{8}$ as an improper fraction.
$\square\left(-\frac{11}{8}\right)k = \square\left(\frac{2}{3}\right)$	Multiply by the reciprocal of $-\frac{11}{8}$
$k = \square$	Simplify.

The solution is $\square$.

b. $-75 = -15b$

$-75 = -15b$	Original equation
$\square(-75) = \square(-15b)$	Multiply each side by the reciprocal of -15.
$\square = b$	Simplify.

The solution is $\square$.

Check Your Progress **Solve each equation.**

a. $\frac{a}{18} = \frac{2}{3}$

b. $\left(4\frac{1}{3}\right)m = 5\frac{3}{7}$

c. $32 = -14c$

EXAMPLE Write and Solve an Equation Using Multiplication

③ **SPACE TRAVEL Using information from Example 3 in the Student Edition, what would be the weight of Neil Armstrong's suit and life-support backpack on Mars if three times the Mars weight equals the Earth weight?**

Words Three times the weight on Mars equals the weight on Earth.

Variable Let $w = \boxed{}$.

Equation

$$\underbrace{\text{Three}}_{3}\quad \underbrace{\text{times}}_{\boxed{}}\quad \underbrace{\text{the weight on mars}}_{w}\quad \underbrace{\text{equals}}_{=}\quad \underbrace{\text{the weight on Earth.}}_{\boxed{}}$$

$3w = 198$ Original equation

$\boxed{}(3w) = \boxed{}(198)$ Multiply each side by $\boxed{}$.

$w = \boxed{}$ $\frac{1}{3}(3) = 1$ and $\frac{1}{3}(198) = 66$

The weight of Neil Armstrong's suit and life-support backpacks on Mars would be $\boxed{}$ pounds.

Check Your Progress Refer to Example 3. If Neil Armstrong weighed 216 pounds on Earth, how much would he weigh on Mars?

KEY CONCEPT

Division Property of Equality If an equation is true, and each side is divided by the same nonzero number, the resulting equation is true.

EXAMPLE Solve Using Division

④ **Solve each equation. Check your solution.**

a. $11w = 143$

$11w = 143$ Original equation

$\frac{11w}{\boxed{\quad}} = \frac{143}{\boxed{\quad}}$ Divide each side by $\boxed{\quad}$.

$w = \boxed{\quad}$ $\frac{11w}{11} = w$ and $\frac{143}{11} = 13$

b. $-8x = 96$

$-8x = 96$ Original equation

$\frac{-8x}{\boxed{\quad}} = \frac{96}{\boxed{\quad}}$ Divide each side by $\boxed{\quad}$.

$x = \boxed{\quad}$ $\frac{-8x}{-8} = x$ and $\frac{96}{-8} = -12$

Check Your Progress **Solve each equation.**

a. $35t = 595$

$\boxed{\qquad\qquad}$

b. $-12b = 276$

$\boxed{\qquad\qquad}$

EXAMPLE

⑤ **Write an equation for the problem below. Then solve the equation.**

Negative fourteen times a number equals 224.

$$\underbrace{\text{Negative fourteen}}_{-14}\quad \underbrace{\text{times}}_{\times}\quad \underbrace{\text{a number}}_{n}\quad \underbrace{\text{equals}}_{=}\quad \underbrace{224.}_{224}$$

$-14n = \boxed{\quad}$ Original equation

$\frac{-14n}{-14} = \frac{224}{-14}$ Divide each side by $\boxed{\quad}$.

$\boxed{\qquad}$ Check this result.

Check Your Progress Negative thirty-four times a number equals 578. Find the number.

$\boxed{\qquad\qquad\qquad\qquad}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–4 Solving Multi-Step Equations

MAIN IDEAS

- Solve problems by working backward.
- Solve equations involving more than one operation.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, verbal descriptions, **equations,** and inequalities. **A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values,** simplify polynomial expressions, **transform and solve equations,** and factor as necessary **in problem situations.** *Also addresses TEKS A.1(C) and A.7(B).*

BUILD YOUR VOCABULARY (pages 30–31)

Work backward is one of the many problem-solving strategies that you can use to solve multi-step equations.

To solve equations with more than one operation, often called **multi-step equations,** $\boxed{\quad\quad}$ operations by working backward.

EXAMPLE Solve Using Addition and Division

1 **Solve $5q - 13 = 37$.**

$5q - 13 = 37$	Original equation
$5q - 13 + \boxed{\quad} = 37 + \boxed{\quad}$	Add $\boxed{\quad}$ to each side.
$\boxed{\quad} = \boxed{\quad}$	Simplify.
$\dfrac{\boxed{\quad}}{\boxed{\quad}} = \dfrac{\boxed{\quad}}{\boxed{\quad}}$	Divide each side by $\boxed{\quad}$.
$q = \boxed{\quad}$	Simplify.

EXAMPLE Solve Using Subtraction and Multiplication

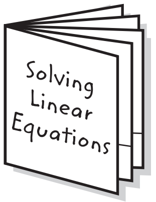
2 **Solve $\frac{s}{12} + 9 = -11$.**

$\frac{s}{12} + 9 = -11$	Original equation
$\frac{s}{12} + 9 - \boxed{\quad} = -11 - \boxed{\quad}$	Subtract $\boxed{\quad}$ from each side.
$\boxed{\quad} = \boxed{\quad}$	$\boxed{\quad\quad\quad}$.
$\boxed{\quad}\left(\frac{s}{12}\right) = \boxed{\quad}(-20)$	Multiply each side by $\boxed{\quad}$.
$s = \boxed{\quad}$	Simplify.

FOLDABLES™

ORGANIZE IT

Explain how to solve multi-step equations on an index card. Include an example.



Check Your Progress Solve each equation.

a. $4 + \frac{a}{11} = 37$

b. $4a - 42 = 14$

BUILD YOUR VOCABULARY (pages 30–31)

Consecutive integers are integers in order, such as 7, , and 9.

The study of and the relationships between them is called **number theory.**

EXAMPLE

③ **SHOPPING** **Susan had a $10 coupon for the purchase of any item. She bought a coat that was $\frac{1}{2}$ its original price. After using the coupon, Susan paid $125 for the coat before taxes. What was the original price of the coat? Write an equation for the problem. Then solve the equation.**

Words	One-half of the price minus ten dollars is 125.
Variables	Let $p =$ the original price.
Equation	$\frac{1}{2}p - 10 = 125$

$\frac{1}{2}p - 10 =$	Original equation
$\frac{1}{2}p - 10 + 10 = 125 + 10$	Add to each side.
$= 135$	Simplify.
$2\left(\frac{1}{2}p\right) = 2(135)$	Multiply each side by .
$p = 270$	Simplify.

The solution is .

Check Your Progress Three-fourths of seven subtracted from a number is negative fifteen. What is the number?

WRITE IT

What is meant by undoing an equation?

____

____

____

____

EXAMPLE

④ **NUMBER THEORY** **Write an equation for the problem below. Then solve the equation and answer the problem.**

Find three consecutive odd integers whose sum is 57.

Let n = the last odd integer.

Let $n + 2$ = the next greater odd integer.

Let $n + 4$ = the greatest of the three odd integers.

$$\underbrace{\text{The sum of three consecutive odd integers}}_{n + (n + 2) + (n + 4)} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{57.}}_{57}$$

$n + (n + 2) + (n + 4) = 57$	Original equation
$\boxed{} + 6 = 57$	Simplify.
$3n + 6 - 6 = 57 - 6$	Subtract $\boxed{}$ from each side.
$3n = \boxed{}$	Simiplify.
$\dfrac{3n}{\boxed{}} = \dfrac{57}{\boxed{}}$	Divide each side by 3.
$n = 17$	Simplify.

$n + 2 = 17 + 2$ or $\boxed{}$

$n + 4 = 17 + 4$ or $\boxed{}$

The consecutive odd integers are $\boxed{}$.

Check Your Progress Find three consecutive even integers whose sum is 84.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–5 Solving Equations with the Variable on Each Side

MAIN IDEAS

- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, verbal descriptions, **equations,** and inequalities. **A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values,** simplify polynomial expressions, **transform and solve equations,** and factor as necessary **in problem situations.** *Also addresses TEKS A.1(C) and A.7(B).*

EXAMPLE Solve an Equation with Variables on Each Side

① **Solve $8 + 5s = 7s - 2$. Check your solution.**

$8 + 5s = 7s - 2$	Original equation
$8 + 5s - \boxed{} = 7s - 2 - \boxed{}$	Subtract $\boxed{}$ from each side.
$\boxed{} = -2$	Simplify.
$8 - 2s - \boxed{} = -2 - \boxed{}$	Subtract $\boxed{}$ from each side.
$-2s = \boxed{}$	Simplify.
$\dfrac{-2s}{\boxed{}} = \dfrac{-10}{\boxed{}}$	Divide each side by $\boxed{}$.
$s = 5$	Simplify.

EXAMPLE Solve an Equation with Grouping Symbols

② **Solve $\frac{1}{3}(18 + 12q) = 6(2q - 7)$. Check your solution.**

$\frac{1}{3}(18 + 12q) = 6(2q - 7)$	Original equation
$6 + 4q = \boxed{}$	Distributive Property
$6 + 4q - \boxed{} = 12q - 42 - \boxed{}$	Subtract $\boxed{}$ from each side.
$\boxed{} = -42$	Simplify.
$6 - 8q - \boxed{} = -42 - \boxed{}$	Subtract $\boxed{}$ from each side.
$\boxed{} = -48$	Simplify.
$\dfrac{-8q}{\boxed{}} = \dfrac{-48}{\boxed{}}$	Divide each side by $\boxed{}$
$q = \boxed{}$	Simplify.

REVIEW IT

Give an example of the Distributive Property. *(Lesson 1–5)*

EXAMPLE No Solutions or Identity

3 Solve each equation.

a. 8(5c − 2) = 10(32 + 4c)

$8(5c - 2) = 10(32 + 4c)$	Original equation
$\boxed{\quad} = 320 + 40c$	Distributive Property
$40c - 16 - \boxed{\quad} = 320 + 40c - \boxed{\quad}$	Subtract.
$0c - 16 = 320$	This statement is false.

There must be at least one c to represent the variable. This equation has no solution.

b. $\mathbf{4(t + 20) = \frac{1}{5}(20t + 400)}$

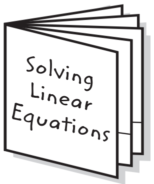
$4(t + 20) = \frac{1}{5}(20t + 400)$	Original equation
$\boxed{\quad} = \boxed{\quad}$	Distributive Property

Since the expression on each side of the equation is the $\boxed{\quad}$, this equation is an identity. The statement $4t + 80 = 4t + 80$ is $\boxed{\quad}$ for all values of t.

FOLDABLES™

ORGANIZE IT

On an index card, write an equation that has no solution.



Check Your Progress **Solve each equation.**

a. $9f - 6 = 3f + 7$

b. $6(3r - 4) = \frac{3}{8}(46r + 8)$

c. $2(4a + 8) = \left(3\frac{8a}{3} - 10\right)$

d. $\frac{1}{7}(21c - 56) = 3\left(c - \frac{8}{3}\right)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–6 Ratios and Proportions

MAIN IDEAS

- Determine whether two ratios form a proportion.
- Solve proportions.

REMEMBER IT

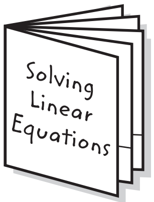
A whole number can be written as a ratio with a denominator of 1.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(G)** Relate direct variation to linear functions and **solve problems involving proportional change.**

FOLDABLES™

ORGANIZE IT

On an index card, explain the difference between a ratio and a proportion.



BUILD YOUR VOCABULARY (pages 30–31)

A **ratio** is a comparison of two numbers by .

An equation stating that two ratios are is called a **proportion.**

In the proportion $\frac{0.4}{0.8} = \frac{0.7}{1.4}$, 0.4 and 1.4 are called the **extremes** and 0.8 and 0.7 are called the **means.**

EXAMPLE Determine Whether Ratios Form a Proportion

① **Determine whether the ratios $\frac{4}{8}$ and $\frac{49}{56}$ form a proportion.**

$$\frac{7}{8} = \frac{7}{8} \qquad\qquad \frac{49}{56} = \frac{7}{8}$$

The ratios are . Therefore, they form a proportion.

Check Your Progress Do the ratios $\frac{5}{6}$ and $\frac{40}{49}$ form a proportion?

EXAMPLE Use Cross Products

② **Use cross products to determine whether the pair of ratios below forms a proportion.**

a. $\frac{0.25}{0.6}, \frac{1.25}{2}$

$\frac{0.25}{0.6} \quad \frac{1.25}{2}$ Write the equation.

$(2) = 0.6$ Find the cross products.

$0.5 \neq 0.75$ Simplify.

The cross products are not equal, so $\frac{0.25}{0.6} \neq \frac{1.25}{2}$.

The ratios form a proportion.

b. $\frac{4}{5}, \frac{16}{20}$

$\frac{4}{5} \times \frac{16}{20}$ Write the equation.

$\square = 5(16)$ Find the cross products.

$\square = \square$ Simplify.

The cross products are $\square$, so $\frac{4}{5} = \frac{16}{20}$.

Since the ratios are equal, they form a $\square$.

Check Your Progress **Use cross products to determine whether each pair of ratios below forms a proportion.**

a. $\frac{0.5}{1.3}, \frac{0.45}{1.17}$

b. $\frac{5}{6}, \frac{12}{15}$

EXAMPLE Solve a Proportion

③ **Solve the proportion $\frac{n}{12} = \frac{3}{8}$.**

$\frac{n}{12} \times \frac{3}{8}$ Original equation

$8(n) = 12(3)$ Find the cross products.

$\square = \square$ Simplify.

$\frac{\square}{\square} = \frac{\square}{\square}$ Divide each side by $\square$.

$n = \square$ $\square$.

KEY CONCEPT

Means-Extremes Property of Proportion
In a proportion, the product of the extremes is equal to the product of the means.

Check Your Progress Solve the proportion $\frac{r}{9} = \frac{7}{10}$.

BUILD YOUR VOCABULARY (pages 30–31)

The of two measurements having units of measure is called a **rate**.

EXAMPLE Use Rates

③ **BICYCLING** **The gear on a bicycle is 8:5. This means that for every eight turns of the pedals, the wheel turns five times. Suppose the bicycle wheel turns about 2435 times during a trip. How many times would you have to turn the pedals during the trip?**

$\frac{8}{5} = \frac{p}{2435}$ Original proportion

$= 5(p)$ Find the .

$= 5p$.

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$ Divide each side by .

$= p$ Simplify.

Your Turn Before 1980, Disney created animated movies using cels. These hand drawn cels (pictures) of the characters and scenery represented the action taking place, one step at a time. For the movie *Snow White*, it took 24 cels per second to have the characters move smoothly. The movie is around 42 minutes long. About how many cels were drawn to produce *Snow White*?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–7 Percent of Change

MAIN IDEAS

- Find percents of increase and decrease.
- Solve problems involving percents of change.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(G)** Relate direct variation to linear functions and **solve problems involving proportional change.**

BUILD YOUR VOCABULARY (pages 30–31)

When an ____ or ____ is expressed as a percent, the percent is called the **percent of change.**

If the new number is ____ than the original number, the percent of change is a **percent of increase.**

If the new number is ____ than the original, the percent of change is a **percent of decrease.**

EXAMPLE Find Percent of Change

① **State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change.**

original: 32
new: 40

Find the *amount* of change. Since the new amount is greater than the original, the percent of change is a percent of

____.

____ $= 8$

Find the percent using the original number, 32, as the base.

$$\begin{array}{r} \text{change} \rightarrow \\ \text{original amount} \rightarrow \end{array} \frac{8}{32} = \frac{r}{100}$$

____ $= 32(r)$ Cross products

$800 =$ ____ Simplify.

$\dfrac{800}{\text{____}} = \dfrac{32r}{\text{____}}$ Divide each side by ____.

____ $= r$ Simplify.

The percent of increase is ____.

Check Your Progress **State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change.**

a. original: 20
new: 18

b. original: 12
new: 48

EXAMPLE Find the Missing Value

② **SALES** **The price a used-book store pays to buy a book is $5. The store sells the book for 28% above the price that it pays for the book. What is the selling price of the $5 book?**

Let s = the selling price of the book. Since 28% is the percent of increase, the amount the used-book store pays to buy a book is less than the selling price. Therefore, $s - 5$ represents the amount of change.

$$\begin{array}{r} \text{change} \longrightarrow \\ \text{book store cost} \longrightarrow \end{array} \frac{s-5}{5} = \frac{28}{100}$$

$(s - 5)(100) = 5(28)$ Cross products

$100s - 500 = 140$ Distributive Property

$100s - 500 + \boxed{} = 140 + \boxed{}$ Add $\boxed{}$ to each side.

$100s = 640$ Simplify.

$s = \boxed{}$ Divide each side by 100.

The selling price of the $5 book is $\boxed{}$.

Check Your Progress At one store the price of a pair of jeans is $26.00. At another store the same pair of jeans has a price that is 22% higher. What is the price of jeans at the second store?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–8 Solving Equations and Formulas

MAIN IDEAS

- Solve equations for given variables.
- Use formulas to solve real-world problems.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, verbal descriptions, **equations,** and inequalities. **A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(B) Use the commutative, associative, and distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.4(A) and A.7(B).*

EXAMPLES Solve an Equation for a Specific Variable

① Solve $5b + 12c = 9$ for b.

$5b + 12c = 9$ Original equation

$5b + 12c - \square = 9 - \square$ Subtract.

$5b = 9 - 12c$ Simplify.

$\dfrac{5b}{\square} = \dfrac{9 - 12c}{\square}$ Divide each side by ☐.

$b = \dfrac{9 - 12c}{5}$ Simplify.

or $\dfrac{-12c + 9}{5}$

The value of b is ☐.

② Solve $7x - 2z = 4 - xy$ for x.

$7x - 2z = 4 - xy$ Original equation

$7x - 2z + \square = 4 - xy + \square$ Add ☐ to each side.

$7x - 2z + xy = 4$ Simplify.

$7x - 2z + xy + \square = 4 + \square$ Add ☐ to each side.

$\square = 4 + 2z$ Simplify.

$\square = 4 + 2z$ Use the Distributive Property.

$\square = \dfrac{4 + 2z}{7 + y}$ Divide each side by ☐.

The value of x is ☐. Since division by 0 is undefined, ☐ $\neq 0$, or $y \neq$ ☐.

Check Your Progress

a. Solve $2x - 17y = 13$ for y.

b. Solve $12a + 3c = 2ab + 6$ for a.

EXAMPLE Use a Formula to Solve Problems

③ **a. FUEL ECONOMY A car's fuel economy E (miles per gallon) is given by the formula $E = \frac{m}{g}$, where m is the number of miles driven and g is the number of gallons of fuel used. Solve the formula for m.**

$E = \frac{m}{g}$ Formula for fuel economy

$E\boxed{} = \frac{m}{g}\boxed{}$ Multiply each side by $\boxed{}$.

$\boxed{} = \boxed{}$ Simplify.

b. FUEL ECONOMY If Claudia's car has an average fuel consumption of 30 miles per gallon and she used 9.5 gallons, how far did she drive?

$Eg = m$ Formula for how many miles driven

$30\boxed{} = m$ $E = 30$ mpg and $g = 9.5$ gallons

$\boxed{} = m$ Multiply.

She drove $\boxed{}$ miles.

Check Your Progress

a. Refer to Example 3. Solve the formula for g.

b. If Claudia drove 1477 miles and her pickup has an average fuel consumption of 19 miles per gallon, how many gallons of fuel did she use?

BUILD YOUR VOCABULARY (pages 30–31)

Dimensional analysis is the process of carrying

throughout a .

EXAMPLE Use Dimensional Analysis

3 **a. GEOMETRY The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height. Solve the formula for h.**

$V = \pi r^2 h$ Original formula

$\frac{V}{\boxed{}} = \frac{\pi r^2 h}{\boxed{}}$ Divide each side by $\boxed{}$

$\boxed{} = h$

b. What is the height of a cylindrical swimming pool that has a radius of 12 feet and a volume of 1810 cubic feet?

$\boxed{} = h$ Formula for h

$\frac{1810}{\pi 12^2} = h$ $V = \boxed{}$ and $r = \boxed{}$

$\boxed{} = h$ Use a calculator.

The height of the cylindrical swimming pool is about .

Check Your Progress

a. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height. Solve the formula for r.

b. What is the radius of a cylindrical swimming pool if the volume is 2010 cubic feet and a height of 6 feet?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–9 Weighted Averages

MAIN IDEAS

- Solve mixture problems.
- Solve uniform motion problems.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, graphs, diagrams, verbal descriptions, **equations**, and inequalities. **A.4** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(B) Use the commutative,** associative**, and distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.4(A).*

BUILD YOUR VOCABULARY (pages 30–31)

The **weighted average** M of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

EXAMPLE Solve a Mixture Problem with Prices

① **PETS** **Jeri likes to feed her cat gourmet cat food that costs $1.75 per pound. However, food at that price is too expensive so she combines it with cheaper cat food that costs $0.50 per pound. How many pounds of cheaper food should Jeri buy to go with 5 pounds of gourmet food, if she wants the price to be $1.00 per pound?**

Let w = the number of pounds of cheaper cat food.

Type of Cat Food	Units (lb)	Price per Unit	Price
Gourmet cat food			
Cheaper cat food	w	$0.50	$0.5w$
Mixed cat food		$1.00	

$$\overbrace{8.75}^{\text{Price of gourmet cat food}} \overbrace{+}^{\text{plus}} \overbrace{0.5w}^{\text{price of cheaper cat food}} \overbrace{=}^{\text{equals}} \overbrace{1.00(5 + w)}^{\text{price of mixed cat food.}}$$

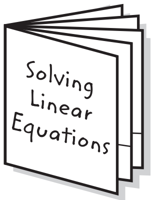
$$\begin{array}{rl}
8.75 + 0.5w = 1.00(5 + w) & \text{Original equation} \\
8.75 + 0.5w = \boxed{} & \text{Distributive Property} \\
8.75 + 0.5w - \boxed{} = 5.0 + 1w - \boxed{} & \text{Subtract.} \\
8.75 = 5.0 + 0.5w & \text{Simplify.} \\
8.75 - 5.0 = 5.0 + 0.5w - 5.0 & \text{Subtract.} \\
3.75 = 0.5w & \text{Simplify.} \\
7.5 = w & \text{Divide.}
\end{array}$$

Check Your Progress A recipe calls for mixed nuts with 50% peanuts. $\frac{1}{2}$ pound of 15% peanuts has already been used. How many pounds of 75% peanuts needs to be added to obtain the required 50% mix?

FOLDABLES™

ORGANIZE IT

On an index card, take notes on mixture problems and uniform motion problems.



EXAMPLE Solve for an Average Speed

② **AIR TRAVEL Mirasol took a non-stop flight from Newark to Austin to visit her grandmother. The 1500-mile trip took three hours and 45 minutes. Because of bad weather, the return trip took four hours and 45 minutes. What was her average speed for the round trip?**

To find the average speed for each leg of the trip, rewrite $d = rt$ as $r = \frac{d}{t}$.

Going

$r = \frac{d}{t} = \frac{1500 \text{ miles}}{\boxed{} \text{ hours}}$ or $\boxed{}$ miles per hour

Returning

$r = \frac{d}{t} = \frac{1500 \text{ miles}}{\boxed{} \text{ hours}}$ or $\boxed{}$ miles per hour

Round trip

$M = \frac{400(1) + 315.79(2)}{1 + 2}$ Definition of weighted average

$= \boxed{}$ or $\boxed{}$ Simplify.

The average speed was about $\boxed{}$ miles per hour.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress In the morning, when traffic is light, it takes 30 minutes to get to work. The trip is 15 miles through towns. In the afternoon when traffic is a little heavier, it takes 45 minutes. What is the average speed for the round trip?

CHAPTER 2

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 2 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 2, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 30–31)* to help you solve the puzzle.

2-1 Writing Equations

Translate each sentence into an equation.

1. Two times the sum of x and three minus four equals four times x

2. The difference of k and 3 is two times k divided by five.

2-2 Solving Equations by Using Addition and Subtraction

Complete each sentence.

3. To solve $y - 9 = -30$ using the Addition Property of Equality, you would add ____ to each side.

4. Write an equation that you could solve by subtracting 32 from each side.

2-3 Solving Equations by Using Multiplication or Division

Complete the sentence after each equation to tell how you would solve the equation.

5. $\frac{x}{7} = 16$ ____ each side by ____.

6. $5x = 125$ ____ each side by ____, or multiply each side by ____.

2-4

Solving Multi-Step Equations

Suppose you want to solve $\frac{x+3}{5} = 6$.

7. What is the grouping symbol in the equation $\frac{x+3}{5} = 6$?

8. What is the first step in solving the equation?

9. What is the next step in solving the equation?

2-5

Solving Equations with the Variable on Each Side

10. When solving $2(3x - 4) = 3(x + 5)$, why is it helpful first to use the Distributive Property to remove the grouping symbols?

The solutions of three equations are shown in Exercises 11–13. Write a sentence to describe each solution.

11. $x = 24$

12. $6m = 6m$

13. $12 = 37$

2-6

Ratios and Proportions

14. A jet flying at a steady speed traveled 825 miles in 2 hours. If you solved the proportion $\frac{825}{2} = \frac{x}{1.5}$, what would the answer tell you about the jet?

Solve each proportion.

15. $\frac{10}{a} = \frac{60}{108}$

16. $\frac{b}{32} = \frac{12}{8}$

17. $\frac{3}{7} = \frac{x-2}{6}$

2-7

Percent of Change

Match the problem on the left with its answer on the right.

18. Original Amount = 10
New Amount = 13

19. Original Amount = 10
New Amount = 7

20. Original Amount = 50
New Amount = 42

21. Original Amount = 50
New Amount = 58

a. 30% increase
b. 50% decrease
c. 16% increase
d. 30% decrease
e. 16% decrease

2-8

Solving Equations and Formulas

Solve each equation or formula for the variable specified.

22. $7f + g = 5$ for f

23. $\frac{rx + 9}{5} = h$ for r

24. $3y + w = 5 + 5y$ for y

2-9

Weighted Averages

25. Suppose Clint drives at 50 miles per hour for 2 hours. Then he drives at 60 miles per hour for 3 hours. Write his speed for each hour of the trip.

Speed					
Hour	1	2	3	4	5

26. What is his average speed?

27. How many grams of sugar must be added to 60 grams of a solution that is 32% sugar to obtain a solution that is 50% sugar?

CHAPTER

2

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 2.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 2 Practice Test on page 133 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 2 Study Guide and Review on pages 128–132 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 135.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 2 Foldable.
- Then complete the Chapter 2 Study Guide and Review on pages 128–132 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 133.

Student Signature

Parent/Guardian Signature

Teacher Signature

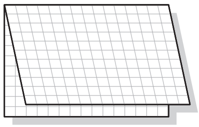
CHAPTER
3

Functions and Patterns

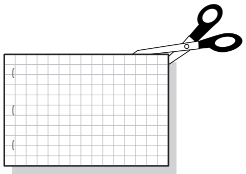
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with four sheets of grid paper.

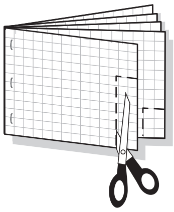
STEP 1 **Fold** each sheet of grid paper in half from top to bottom.



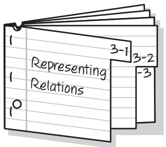
STEP 2 **Cut** along fold. Staple the eight half-sheets together to form a booklet.



STEP 3 **Cut** tabs into margin. The top tab is 4 lines wide, the next tab is 8 lines wide, and so on.



STEP 4 **Label** each of the tabs with a lesson number.



NOTE-TAKING TIP: When you take notes, be sure to listen actively. Always think before you write, but don't get behind in your note-taking. Remember to enter your notes legibly.

CHAPTER

3

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
arithmetic sequence			
common difference			
function			
function notation			
function value			
inverse			
linear equation			
mapping			

Vocabulary Term	Found on Page	Definition	Description or Example
sequence			
standard form			
terms			
vertical line test			
x-intercept			
y-intercept			
zero			

3–1 Representing Relations

MAIN IDEAS

- Represent relations as sets of ordered pairs, tables, mappings, and graphs.
- Find the inverse of a relation.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, **tables, graphs, diagrams**, verbal descriptions, equations, and inequalities. **A.2** The student uses the properties and attributes of functions. **(B) Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.**

BUILD YOUR VOCABULARY (pages 60–61)

A **mapping** illustrates how each element of the ☐ is paired with an element in the ☐.

The **inverse** of any relation is obtained by switching the ☐ in each ☐.

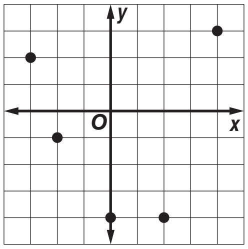
EXAMPLE Represent a Relation

① **Express the relation {(4, 3), (−2, −1), (−3, 2), (2, −4), (0, −4)} as a table, a graph, and a mapping.**

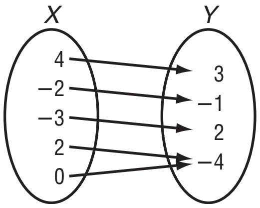
x	*y*
☐	☐
☐	☐
☐	☐
☐	☐
☐	☐

Table
List the set of x-coordinates in the first column and the corresponding y-coordinates in the second column.

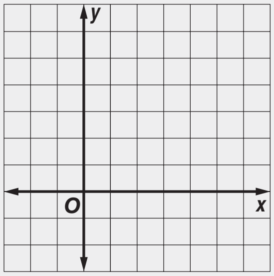
Graph
Graph each ordered pair on a coordinate plane.

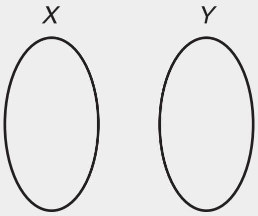


Mapping
List the x values in set X and the y values in set Y. Draw an arrow from each x value in X to the corresponding y value in Y.



Check Your Progress Express the relation {(3, −2), (4, 6), (5, 2), (−1, 3)} as a table, a graph, and a mapping.





REVIEW IT

Explain how to add a decimal and a whole number. *(Prerequisite Skill)*

EXAMPLE Use a Relation

2 OPINION POLLS **The table shows the percent of people satisfied with the way things were going in the U.S. at the time of the survey.**

Year	1992	1995	1998	2001
Percent Satisfied	21	32	60	51

a. Determine the domain and range of the relation.

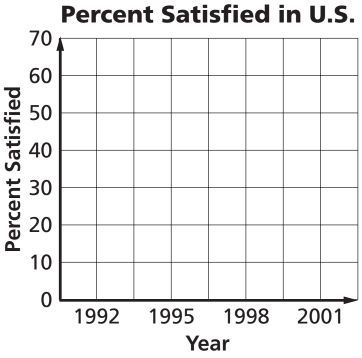
The domain is ________.

The range is ________.

b. Graph the data.

The values of the x-axis need to go from 1992 to 2001. Begin at 1992 and extend to 2001 to include all of the data. The units can be 1 unit per grid square.

The values on the y-axis need to go from 21 to 60. Begin at 0 and extend to 70. You can use units of 10.



c. What conclusions might you make from the graph of the data?

Americans became more satisfied with the country

from ________, but the percentage dropped from

________.

Check Your Progress **The table shows the approximate world population of the Indian Rhinoceros from 1982 to 1998.**

Indian Rhinoceros Population					
Year	1982	1986	1990	1994	1998
Population	1000	1700	1700	1900	2100

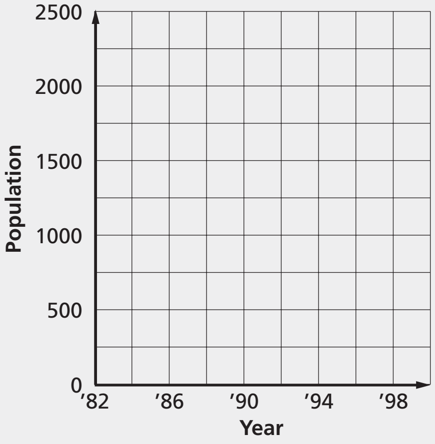
a. Determine the domain and range of the relation.

KEY CONCEPT

Inverse of a Relation Relation *Q* is the inverse of relation *S* if and only if for every ordered pair (*a*, *b*) in *S*, there is an ordered pair (*b*, *a*) in *Q*.

FOLDABLES™ Under the tab for Lesson 3–1. Write a relation with four ordered pairs. Then find the inverse of the relation.

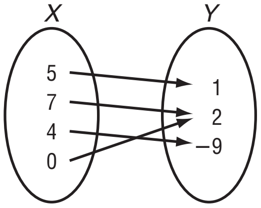
b. Graph the data.



c. What conclusions might you make from the graph of the data?

EXAMPLE Inverse Relation

3 **Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.**



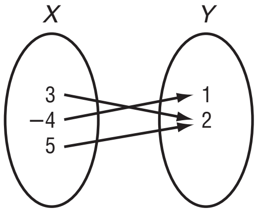
Relation Notice that both 7 and 0 in the domain are paired with 2 in the range.

{(5, 1), (7, 2), }

Inverse Exchange X and Y in each ordered pair to write the inverse relation.

{(1, 5), }

Check Your Progress Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3-2 Representing Functions

MAIN IDEAS

- Determine whether a relation is a function.
- Find function values.

KEY CONCEPT

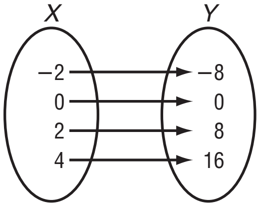
A **function** is a relation in which each element of the domain is paired with exactly one element of the range.

FOLDABLES Use the tab for Lesson 3–2. Explain two ways to determine whether a relation is a function.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Find specific function values**, simplify polynomial expressions, transform and solve equations, and factor as necessary **in problem situations. (C) Connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$.** **A.5** The student understands that linear functions can be represented in different ways and translates among their various representations. **(C) Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.**

EXAMPLE Identify Functions

1 **a. Determine whether each relation is a function. Explain.**



This is a function because the mapping shows each element of the ____ paired with exactly one member of the ____.

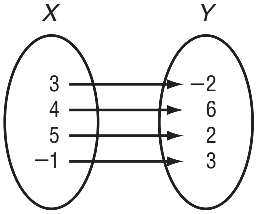
b.

x	y
−7	−12
−4	−9
2	−3
5	0

This table represents a function because the table shows each element of the domain paired with ____ element of the range.

Check Your Progress **Determine whether each relation is a function. Explain.**

a.



b.

x	y
3	2
1	−2
2	−4
3	−1

EXAMPLE Function Values

② **a. If $f(x) = 3x - 4$, find $f(4)$.**

$f\boxed{\quad} = 3\boxed{\quad} - 4$ Replace x with 4.

$= 12 - 4$ Multiply.

$= \boxed{\quad}$ Subtract.

b. If $f(x) = 3x - 4$, find $f(-5)$.

$f\left(\boxed{\quad}\right) = 3\left(\boxed{\quad}\right) - 4$ Replace x with -5.

$= \boxed{\quad} - 4$ Multiply.

$= -19$ Subtract.

Check Your Progress **If $f(x) = 2x + 5$, find each value.**

a. $f(-8)$ $\boxed{\qquad\qquad}$

b. $f(x + 3)$ $\boxed{\qquad\qquad}$

EXAMPLE

③ **PHYSICS** **The function $h(t) = 160t + 16t^2$ represents the height of an object ejected downward from an airplane at a rate of 160 feet per second.**

a. Find the value $h(3)$.

$h\left(\boxed{\quad}\right) = 160\left(\boxed{\quad}\right) + 16\left(\boxed{\quad}\right)^2$ Replace t with $\boxed{\quad}$.

$= 480 + 144$ Multiply.

$= \boxed{\quad}$ Simplify.

b. Find the value $h(2z)$.

$h(2z) = 160(2z) + 16(2z)^2$ Replace t with $\boxed{\quad}$.

$= \boxed{\quad} + \boxed{\quad}$ Multiply.

Check Your Progress **The function $h(t) = 180 - 16t^2$ represents the height of a ball thrown from a cliff that is 180 feet above the ground. Find each value.**

a. $h(2)$ $\boxed{\qquad\qquad}$

b. $h(3z)$ $\boxed{\qquad\qquad}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–3 Linear Functions

TEKS A.5 The student understands that linear functions can be represented in different ways and translates among their various representations. **(B) Determine the domain and range for linear functions in given situations.**

MAIN IDEAS

- Determine whether an equation is linear.
- Graph linear equations.

KEY CONCEPT

Standard Form of a Linear Equation The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers whose greatest common factor is 1.

FOLDABLES™ On the Lesson 3–3 tab, write an example of a linear equation and one that is not linear. Draw a graph of the linear equation.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(B) Interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs. (E) Determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations.** *Also addresses TEKS A.5(A), A.5(C), A.7(B), and A.7(C).*

BUILD YOUR VOCABULARY (pages 60–61)

A **linear equation** is the equation of a line. When an equation is written in the form $Ax + By = C$, it is said to be in **standard form**.

EXAMPLE Identifying Linear Equations

① **Determine whether each equation is a linear equation. If so, write the equation in standard form.**

a. $5x + 3y = z + 2$

Rewrite the equation with the variables on one side.

$5x + 3y = z + 2$	Original equation
$5x + 3y - z = z + 2 - z$	Subtract.
$5x + 3y - z = 2$	Simplify.

Since there are ______ different variables on the left side of the equation, it ______ be written in the form $Ax + By = C$. This is not a ______.

b. $\frac{3}{4}x = y + 8$

Rewrite the equation with the variables on one side.

$\frac{3}{4}x = y + 8$	Original equation
$\frac{3}{4}x - y = y + 8 - y$	Subtract y from each side.
$\frac{3}{4}x - y = 8$	Simplify.

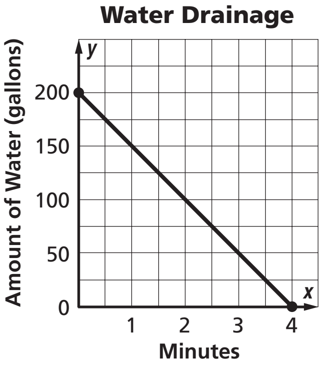
Write the equation with integer coefficients.

$\frac{3}{4}x - y = 8$	
$4\left(\frac{3}{4}x\right) - 4(y) = 8(4)$	Multiply each side by 4.
$3x - 4y = 32$	Simplify.

The equation is now in standard form where $A =$ ______, $B =$ ______, and $C =$ ______. This is a ______ equation.

EXAMPLE

2 WATER STORAGE A storage tank contains water. A valve is opened, and the water is drained, as shown in the graph.



a. Determine the x-intercept, y-intercept, and zero.

The x-intercept is because it is the coordinate where the line crosses the x-axis. The zero of the function is also 4.

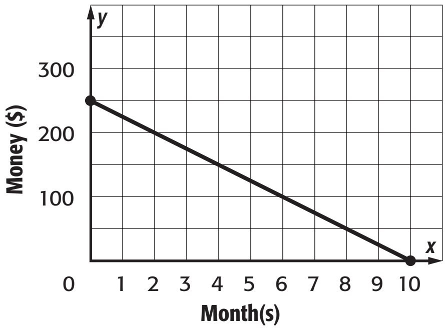
The y-intercept is because it is the coordinate of the point where the line crosses the y-axis.

b. Describe what the intercepts mean.

The x-intercept 4 means that after 4 minutes, there are gallons of water left in the tank.

The y-intercept of 200 means that at time 0, or before any water was drained, there were gallons of water in the tank.

Check Your Progress **BANKING Janine has money in a checking account. She begins withdrawing a constant amount of money each month as shown in the graph.**



a. Determine the x-intercept, y-intercept, and zero.

b. Describe what the intercepts mean.

EXAMPLE Graph Using Intercepts

③ Graph $4x - y = 4$ using the x-intercept and y-intercept.

To find the x-intercept, let $y = 0$.

$$4x - y = 4$$

$$4x - \boxed{} = 4$$

$$4x = 4$$

$$x = \boxed{}$$

To find the y-intercept, let $x = 0$.

$$4x - y = 4$$

$$4\boxed{} - y = 4$$

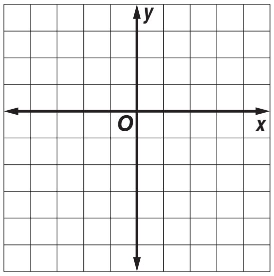
$$-y = 4$$

$$y = \boxed{}$$

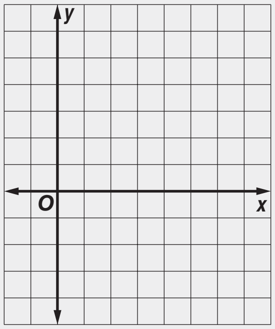
The x-intercept is 1, so the graph intersects the x-axis at $\boxed{}$.

The y-intercept is -4, so the graph intersects the y-axis at $\boxed{}$.

Plot these points. Then draw a line that connects them.



Check Your Progress Graph $2x + 5y = 10$ using the x-intercept and y-intercept.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–4 Arithmetic Sequences

TEKS A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(B) Look for patterns and represent generalizations algebraically.**

EXAMPLES Identify Arithmetic Sequences

MAIN IDEAS

- Recognize arithmetic sequences.
- Extend and write formulas for arithmetic sequences.

① Determine whether each sequence is arithmetic. Explain.

a. $-15, -13, -11, -9, \ldots$

$-15 \quad -13 \quad -11 \quad -9$

$+2$

This an arithmetic sequence because the difference between terms is .

b. $\frac{7}{8}, \frac{5}{8}, \frac{1}{8}, -\frac{5}{8}, \ldots$

This an arithmetic sequence because the difference is .

KEY CONCEPT

Arithmetic Sequence
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.

② MONEY The arithmetic sequence 1, 10, 19, 28, … represents the total number of dollars Erin has in her account after her weekly allowance is added.

a. Write an equation for the nth term of the sequence.
In this sequence, the first term, a_1, is 2. Find the common difference.

$1 \quad 10 \quad 19 \quad 28$

$+9 \quad +9 \quad +9$

The common difference is .

Use the formula for the nth term to write an equation.

$a_n = a_1 + (n - 1)d$	Formula for the nth term
$a_n = 1 + (n - 1)(9)$	$a_1 =$, $d =$
$a_n = 1 + 9n - 9$	Distributive Property
$a_n =$	Simplify.

KEY CONCEPT

***n*th Term of an Arithmetic Sequence** The *n*th term a_n of an arithmetic sequence with first term a_1 and common difference *d* is given by $a_n = a_1 + (n - 1)d$, when *n* is a positive integer.

FOLDABLES™ Use the tab for Lesson 3–4. Write the general form for an arithmetic sequence. Explain what each of the variables means.

KEY CONCEPT

Writing Arithmetic Sequences Each term of an arithmetic sequence after the first term can be found by adding the common difference to the preceding term.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check: For $n = 1$, $9(1) - 8 =$.

For $n = 2$, $9(2) - 8 =$.

For $n = 3$, $9(3) - 8 =$, and so on.

b. Find the 12th term in the sequence.
Replace *n* with 12 in the equation written in part **a**.

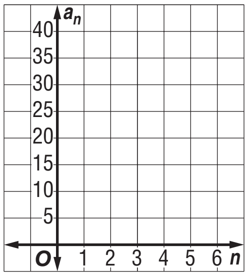
$a_n = 9n - 8$ Equation for the *n*th term

$a_{25} = 9(12) - 8$ Replace *n* with .

$a_{25} =$ Simplify.

c. Graph the first five terms of the sequence.

n	9*n* – 8	a_n	(n, a_n)
1	9(1) – 8	1	(1, 1)
2	9(2) – 8	10	(2, 10)
3	9(3) – 8	19	(3, 19)
4	9(4) – 8	28	(4, 28)
5	9(5) – 8	37	(5, 37)



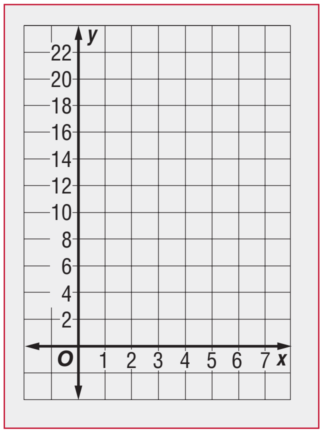
The points fall on a line. The graph of an arithmetic sequence is .

Check Your Progress **MONEY The arithmetic sequence 2, 7, 12, 17... represents the total number of pencils Claire has in her collection after she goes to her school store each week.**

a. Write an equation for the *n*th term of the sequence.

b. Find the 12th term in the sequence

c. Graph the first five terms of the sequence.



3–5 Proportional and Nonproportional Relationships

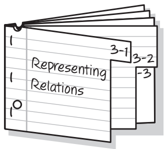
MAIN IDEAS

- Look for a pattern.
- Write an equation given some of the solutions.

TEKS A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(B) Look for patterns and represent generalizations algebraically. A.5** The student understands that linear functions can be represented in different ways and translates among their various representations. **(C) Use, translate, and make connections among algebraic, tabular, graphical,** or verbal **descriptions of linear functions.**

FOLDABLES™ ORGANIZE IT

On the tab for Lesson 3–5, write two ways you can decide what a pattern is in a sequence.



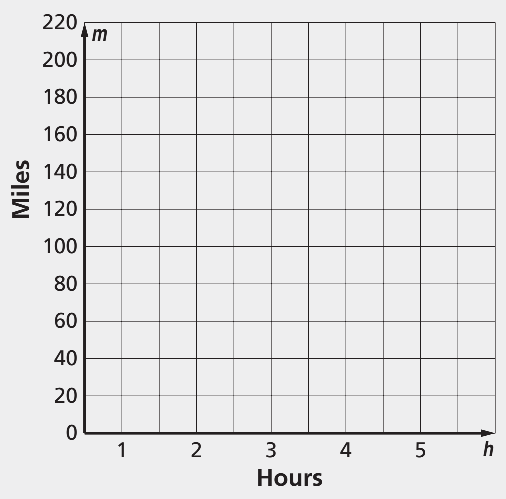
EXAMPLE Proportional Relationships

① **ENERGY The table shows the number of miles driven for each hour of driving.**

Hours	1	2	3	4
Miles	50	100	150	200

a. Graph the data. What conclusion can you make about the relationship between the number of hours driving and the number of miles driven?

This graph shows a ______ relationship between the number of hours h and the number of miles driven m.



b. Write an equation to describe this relationship.

Look at the relationship between the domain and the range to find a pattern that can be described as an equation.

+1 +1 ______

Hours	1	2	3	4
Miles	50	100	150	200

______ +50 +50

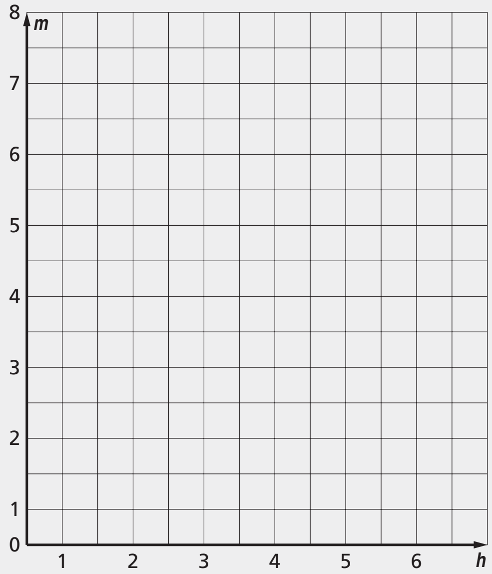
The difference of the values for h is ______ and the difference of the values for m is ______. This suggests that $m =$ ______.

Since the relation is also a ______, we can write the equation as $f(h) =$ ______, where $f(h)$ represents the number of ______.

Check Your Progress **The table below shows the number of miles walked for each hour of walking.**

Hours	1	2	3	4	5
Miles	1.5	3	4.5	6	7.5

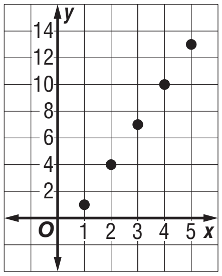
a. Graph the data. What conclusion can you make about the relationship between the number of miles and the time spent walking?



b. Write an equation to describe the relationship.

EXAMPLE

② **Write an equation in function notation for the relation graphed below.**



Make a table of ordered pairs for several points of the graph.

+1 +1 +1 +1

x	1	2	3	4	5
y	1	4	7	10	13

+3 +3 +3 +3

(continued on the next page)

The difference in the x values is , and the difference in the y values is . The difference in y values is the difference of the x values. This suggests that . Check this equation.

Check If $x = 1$, then $y =$ or 3. But the y value for $x = 1$ is 1. This is a difference of . Try some other values in the domain to see if the same difference occurs.

x	1	2	3	4	5
3x	3			12	
y		4	7		13

y is always less than $3x$.

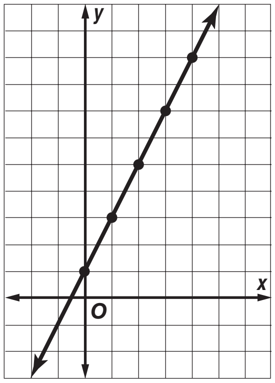
This pattern suggests that 2 should be from one side of the equation in order to correctly describe the relation. Check $y = 3x - 2$.

If $x = 2$, then $y =$.

If $x = 3$, then $y =$.

correctly describes this relation. Since this relation is also a , we can write the equation in function notation as .

Check Your Progress Write an equation in function notation for the relation graphed below.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 3

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 3 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 3, go to: glencoe.com	You can use your completed **Vocabulary Builder** (*pages 60–61*) to help you solve the puzzle.

3-1

Representing Relations

1. Write the relation shown in the table.

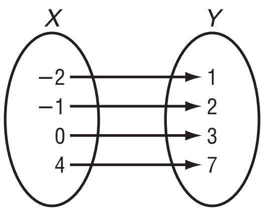
x	*y*
0	–2
1	4
–3	5
–2	0

2. Write the inverse of the relation {(1, 2), (2, 4), (3, 6), (4, 8)}.

3-2

Representing Functions

3. Describe how the mapping shows that the relation represented is a function.



3-3

Linear Functions

Determine whether each equation is a linear equation. If so, write the equation in standard form.

	Equation	Linear or nonlinear?	Standard Form
4.	$4xy + 2y = 7$		
5.	$\frac{x}{5} - \frac{4y}{3} = 2$		

3-4

Arithmetic Sequences

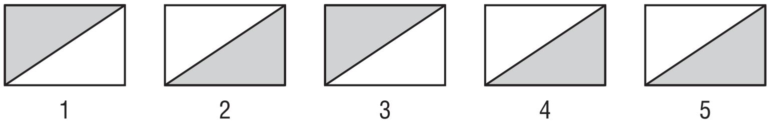
Complete the table.

	Pattern	Is the sequence increasing or decreasing?	Is there common difference? If so, what is it?
6.	55, 50, 45, 40, …		
7.	1, 2, 4, 9, 16, …		
8.	$\frac{1}{2}, 0, -\frac{1}{2}, -1, \ldots$		

3-5

Proportional and Nonproportional Relationships

9. Explain why Figure 5 does not follow the pattern below.



10. Write the next 3 terms of the sequence 1, 5, 25, 125, … .

CHAPTER 3

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 3.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 3 Practice Test on page 179 of your textbook as a final check.

☐ **I used my Foldables or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 3 Study Guide and Review on pages 175–178 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 179.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 3 Foldable.
- Then complete the Chapter 3 Study Guide and Review on pages 175–178 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 179.

Student Signature

Parent/Guardian Signature

Teacher Signature

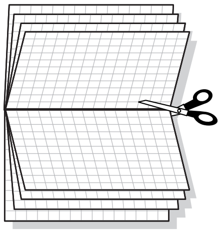
CHAPTER 4

Analyzing Linear Equations

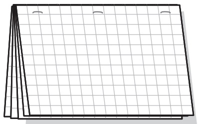
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with ten sheets of grid paper.

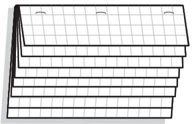
STEP 1 **Fold** each sheet of grid paper in half along the width. Then cut along the crease.



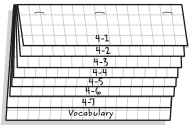
STEP 2 **Staple** the eight half-sheets together to form a booklet.



STEP 3 **Cut** seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.



STEP 4 **Label** each of the tabs with a lesson number. The last tab is for the vocabulary.



NOTE-TAKING TIP: When you take notes, circle, underline, or star anything the teacher emphasizes. When your teacher emphasizes a concept, it will usually appear on a test, so make an effort to include it in your notes.

CHAPTER 4

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
best-fit line			
constant of variation			
direct variation			
family of graphs			
line of fit			
linear extrapolation [ihk·stra·puh·LAY·shun]			
linear intrapolation [ihn·tuhr·puh·LAY·shun]			
negative correlation [kawr·uh·LAY·shun]			
parallel lines			
parent graph			

Vocabulary Term	Found on Page	Definition	Description or Example
perpendicular lines [puhr·puhn·DIH·kyuh·luhr]			
point-slope form			
positive correlation			
rate of change			
scatter plot			
slope			
slope-intercept form [IHN·tuhr·sehpt]			

4–1 Rate of Change and Slope

MAIN IDEAS

- Use rate of change to solve problems.
- Find the slope of a line.

BUILD YOUR VOCABULARY (pages 80–81)

The **rate of change** tells, on average, how a quantity is changing over time.

The **slope** of a line is a number determined by any two points on the line.

KEY CONCEPT

Slope of a Line The slope of a line is the ratio of the rise to the run.

FOLDABLES™ Write the formula for finding the slope of a line under the tab for Lesson 4-1.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(A) Develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations. (B) Interpret the meaning of slope** and intercepts **in situations using data, symbolic representations, or graphs.** *Also addresses TEKS A.3(B).*

EXAMPLE

① **DRIVING TIME The table shows how the distance traveled changes with the number of hours driven. Use the table to find the rate of change. Explain the meaning if the rate of change.**

Time Driving (h) x	Distance Traveled (mi) y
2	76
4	152
6	228

Each time x increases by ____ hours, y increases by ____ miles.

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{change in } \_\_\_\_}{\text{change in } \_\_\_\_}$$

$$= \frac{152 - 76}{4 - 2}$$

$$= \_\_\_\_ \text{ or } \_\_\_\_$$

The rate of change is $\frac{38}{1}$. This means the speed traveled is ____.

Check Your Progress **CELL PHONE** The table shows how the cost changes with the number of minutes used. Use the table to find the rate of change. Explain the meaning of the rate of change.

Minutes Used x	Cost ($) y
20	1
40	2
60	3

A rate of change is $\frac{.05}{1}$; This means that it costs $0.05 per minute to use the cell phone.

B rate of change is $\frac{5}{1}$; This means that it costs $5 per minute to use the cell phone.

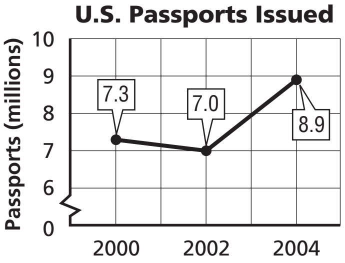
C rate of change is $\frac{.05}{1}$; This means that it costs $0.50 per minute to use the cell phone.

D rate of change is $\frac{.20}{1}$; This means that it costs $0.20 per minute to use the cell phone.

$\boxed{}$

EXAMPLE

② TRAVEL The graph to the right shows the number of U.S. passports issued in 2000, 2002, and 2004.



a. Find the rates of change for 2000–2002 and 2002–2004.

Use the formula for slope.

$$\frac{\boxed{}}{\boxed{}} = \frac{\text{change in quantity}}{\text{change in time}} \begin{array}{l} \leftarrow \text{millions of passports} \\ \leftarrow \text{years} \end{array}$$

2000–2002:

$$\frac{\text{change in quantity}}{\text{change in time}} = \frac{\boxed{} - \boxed{}}{2002 - 2000} \qquad \text{Substitute.}$$

$$= \frac{-.3}{2} \text{ or } \boxed{} \qquad \text{Simplify.}$$

The number of passports issued decreased by 1 million in a 2-year period for a rate of change of $\boxed{}$ per year.

2002–2004:

$$\frac{\text{change in quantity}}{\text{change in time}} = \frac{\square - \square}{2004 - 2002} \qquad \text{Substitute.}$$

$$= \frac{1.8}{2} \text{ or } \square \qquad \text{Simplify.}$$

Over this 2-year period, the number of U.S. passports issued ______ by 1.8 million for a rate of change of ______ per year.

b. Explain the meaning of the rate of change in each case.

For 2000–2002, on average, ______ fewer passports were issued each year than the last. For 2002–2004, on average, ______ more passports were issued each year than the last.

c. How are the different rates of change shown on the graph?

The first rate of change is ______, and the line goes ______ on the graph; the second rate of change is ______, and the graph goes ______.

REVIEW IT

Describe how you find cross products.
(Lesson 2-6)

______

______

______

______

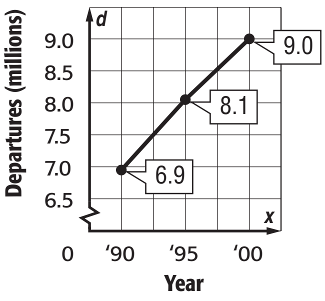
______

______

Check Your Progress

AIRLINES The graph shows the number of airplane departures in the United States in recent years.

U.S. Airline Departures



a. Find the rates of change for 1990–1995 and 1995–2000.

______

b. Explain the meaning of the slope in each case.

______

c. How are the different rates of changes shown on the graph?

EXAMPLE Finding Slope

3 **Find the slope of the line that passes through (–3, –4) and (–2, –8).**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ rise/run

$= \dfrac{\square - \square}{-2 - (-3)}$ $(-3, -4) = (x_1, y_1)$ and $(-2, -8) = (x_2, y_2)$

$= \square$ or $\square$ Simplify.

Check Your Progress Find the slope of the line that passes through (–3, 4) and (4, 4).

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–2 Slope and Direct Variation

MAIN IDEAS

- Write and graph direct variation equations.
- Solve problems involving direct variation.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(F) Interpret and predict the effects of changing slope and *y*-intercept in applied situations. (G) Relate direct variation to linear functions and solve problems involving proportional change.** *Also addresses TEKS A.5(C) and A.7(A).*

BUILD YOUR VOCABULARY (pages 80–81)

A **direct variation** is described by an equation of the form ☐, where k ≠ 0.

In the equation $y = kx$, ☐ is the **constant of variation.**

A **family of graphs** includes graphs and equations of graphs that have at least ☐ characteristic in common.

EXAMPLE Graph a Direct Variation

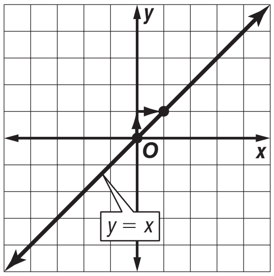
① Graph each equation.

a. $y = x$

Recall that the slope of the graph of $y = kx$ is k.

Step 1 Write the slope as a ratio.

$1 =$ ☐ $\frac{\text{rise}}{\text{run}}$



Step 2 Graph (0, 0).

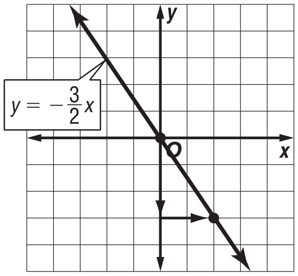
Step 3 From the point (0, 0), move up ☐ unit and right ☐ unit. Draw a dot.

Step 4 Draw a line containing the points.

b. $y = -\frac{3}{2}x$

Step 1 Write the slope as a ratio.

$-\frac{3}{2} =$ ☐ $\frac{\text{rise}}{\text{run}}$



Step 2 Graph (0, 0).

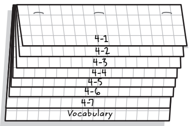
Step 3 From the point (0, 0), move ☐ 3 units and right ☐ units. Draw a dot.

Step 4 Draw a line containing the points.

FOLDABLES™

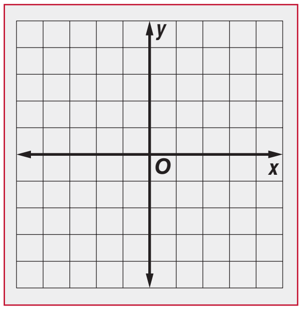
ORGANIZE IT

Under the tab for Lesson 4-2, give an example of a direct variation equation and its graph.

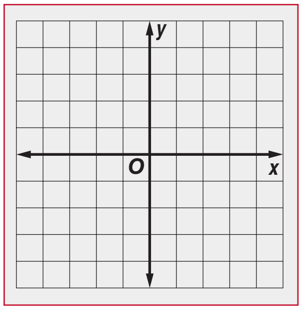


Check Your Progress **Graph each equation.**

a. $y = 2x$



b. $y = -\frac{2}{3}x$



EXAMPLE Write and Solve a Direct Variation Equation

② **Suppose y varies directly as x, and $y = 9$ when $x = -3$.**

a. Write a direct variation equation that relates x and y.

$y = kx$	Direct variation formula
$\boxed{} = k\boxed{}$	Replace y with $\boxed{}$ and x with $\boxed{}$.
$\frac{9}{-3} = \frac{k(-3)}{-3}$	Divide each side by $\boxed{}$.
$\boxed{} = k$	Simplify.

Therefore $y = \boxed{}$.

b. Use the direct variation equation to find x when $y = 15$.

$y = \boxed{}$	Direct variation formula
$\boxed{} = \boxed{}$	Replace y with $\boxed{}$.
$\frac{\boxed{}}{-3} = \frac{\boxed{}}{-3}$	Divide each side by -3.
$\boxed{} = x$	Simplify.

Therefore, $x = \boxed{}$ when $y = \boxed{}$.

Check Your Progress **Suppose y varies directly as x, and $y = 15$ when $x = 5$.**

a. Write a direct variation equation that relates x and y.

$\boxed{}$

b. Use the direct variation equation to find x when $y = -45$.

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–3 Graphing Equations in Slope-Intercept Form

MAIN IDEAS

- Write and graph linear equations in slope-intercept form.
- Model real-world data with an equation in slope-intercept form.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, **tables, graphs,** diagrams, **verbal descriptions, equations,** and inequalities. **A.6** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(D)** Graph and **write equations of lines given characteristics such as** two points, a point and a slope, or **a slope and *y*-intercept.** *Also addresses TEKS A.5(C), A.7(A), A.7(B), and A.7(C)*

KEY CONCEPT

Slope-Intercept Form
The linear equation $y = mx + b$ is written in slope-intercept form, where m is the slope and b is the y-intercept.

BUILD YOUR VOCABULARY (pages 80–81)

An equation of the form $\boxed{}$ is in the **slope-intercept form.**

EXAMPLE Write an Equation Given Slope and y-intercept

① **Write an equation of the line whose slope is $\frac{1}{4}$ and whose y-intercept is –6.**

$y = mx + b$ Slope-intercept form

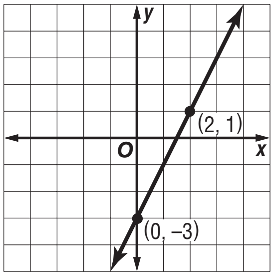
$y = \frac{1}{4}x - 6$ Replace m with $\boxed{}$ and b with $\boxed{}$.

Check Your Progress Write an equation of the line whose slope is 4 and whose y-intercept is 3.

$\boxed{}$

EXAMPLE Write an Equation from a Graph

② **Write an equation in slope-intercept form of the line shown in the graph.**



Step 1 You know the coordinates of two points on the line. Find the slope. Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (2, 1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{\text{rise}}{\text{run}}$

$m = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}}$ $x_1 = 0, x_2 = 2$
$y_1 = -3, y_2 = 1$

$m = \boxed{}$ or $\boxed{}$ Simplify.

Step 2 The line crosses the y-axis at $\boxed{}$.

So, the y-intercept is $\boxed{}$.

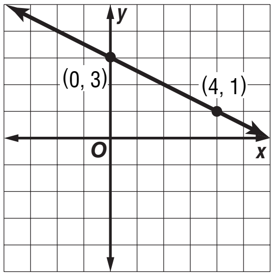
Step 3 Finally, write the equation.

$y = mx + b$ Slope-intercept form

$y = 2x - 3$ Replace m with and b with .

The equation of the line is $y =$.

Check Your Progress Write an equation of the line shown in the graph.



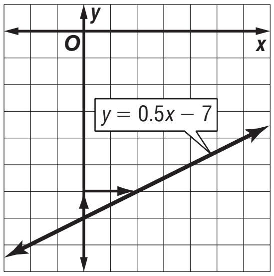
WRITE IT

Which type of lines, vertical or horizontal, can be written in slope-intercept form?

EXAMPLE Graph Equations

③ **Graph each equation.**

a. $y = 0.5x - 7$



Step 1 The y-intercept is .

So graph .

Step 2 The slope is 0.5 or .

From (0, −7), move up

unit and right units.

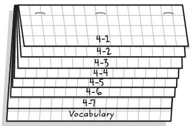
Draw a dot.

Step 3 Draw a line connecting the points.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 4–3, write the slope-intercept form of a linear equation. Under the tab, describe how to use the slope and intercept to graph of $y = 4x + 3$.



b. $\mathbf{5x + 4y = 8}$

Step 1 Solve for y to find the slope-intercept form.

$5x + 4y = 8$ Original Equation

$5x + 4y - \boxed{} = 8 - \boxed{}$ Subtract $\boxed{}$ from each side.

$4y = 8 - 5x$ Simplify.

$4y = -5x + 8$ $8 - 5x = 8 + (-5x)$ or $-5x + 8$

$\dfrac{4y}{\boxed{}} = \dfrac{-5x + 8}{\boxed{}}$ Divide each side by $\boxed{}$.

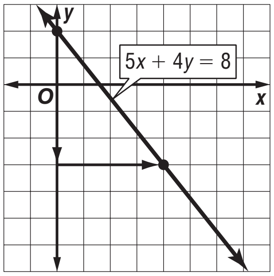
$\dfrac{4y}{4} = \dfrac{-5x}{4} + \dfrac{8}{4}$ Divide each term in the numerator by 4.

$y = \boxed{}$

Step 2 The y-intercept of

$y = -\frac{5}{4}x + 2$ is $\boxed{}$.

So graph $\boxed{}$.

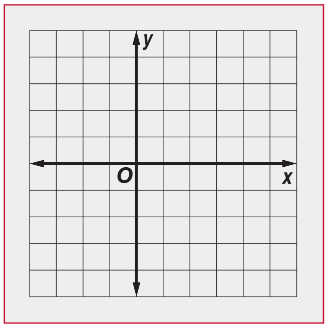


Step 3 The slope is $\boxed{}$. From (0, 2), move $\boxed{}$ 5 units and $\boxed{}$ 4 units. Draw a dot.

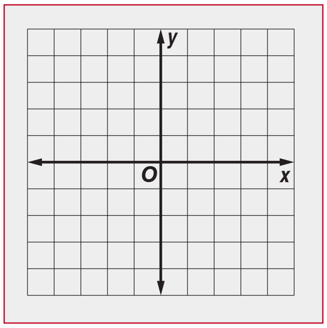
Step 4 Draw a line connecting the points.

Check Your Progress **Graph each equation.**

a. $y = 2x - 4$



b. $3x + 2y = 6$



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–4 Writing Equations in Slope-Intercept Form

MAIN IDEAS

- Write an equation of a line given the slope and one point on a line.
- Write an equation of a line given two points on the line.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(D) Represent relationships among quantities using** concrete models, tables, **graphs,** diagrams, **verbal descriptions,** equations, and inequalities. **A.6** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(D)** Graph and **write equations of lines given characteristics such as two points, a point and a slope,** or a slope and *y*-intercept. *Also addresses TEKS A.7(B).*

BUILD YOUR VOCABULARY (pages 80–81)

When you use a linear equation to $\boxed{}$ values that are beyond the range of the data, you are using **linear extrapolation.**

EXAMPLE Write an Equation Given Slope and One Point

① **Write an equation of a line that passes through (2, –3) with slope $\frac{1}{2}$.**

Step 1 The line has slope $\frac{1}{2}$. To find the *y*-intercept, replace *m* with $\frac{1}{2}$ and (*x*, *y*) with (2, –3) in the slope-intercept form. Then, solve for *b*.

$y = mx + b$ Slope-intercept form

$-3 = \frac{1}{2}(2) + b$ Replace *m* with $\frac{1}{2}$, *y* with –3, and *x* with 2.

$-3 = \boxed{} + b$ $\boxed{}$.

$-3 - \boxed{} = 1 + b - \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = b$ Simplify.

Step 2 Write the slope-intercept form using $m = \boxed{}$ and *b* with $\boxed{}$.

$y = mx + b$ Slope-intercept form

$y = \boxed{}$ Replace *m* with $\boxed{}$ and *b* with $\boxed{}$.

The equation is $y = \boxed{}$.

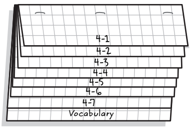
Check Your Progress Write an equation of a line that passes through (1, 4) and has a slope of –3.

$\boxed{}$

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-4, explain how to write an equation given slope and one point and given two points. Include examples.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Write an Equation Given Two Points

② **The table of ordered pairs shows the coordinates of two points on the graph of a function. Write an equation that describes that function.**

x	y
-3	-4
-2	-8

The table represents the ordered pairs $\boxed{}$ and $\boxed{}$.

Step 1 Find the slope of the line containing the points.

Let $(x_1, y_1) = (-3, -4)$ and $(x_2, y_2) = (-2, -8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \boxed{}$$

$$m = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}}$$

$x_1 = -3$, $x_2 = -2$,
$y_1 = -4$, $y_2 = -8$

$m = \boxed{}$ or $\boxed{}$ Simplify.

Step 2 You know the slope and two points. Choose one point and find the y-intercept. In this case, we chose $(-3, -4)$.

$y = mx + b$ Slope-intercept form

$-4 = -4(-3) + b$ Replace m with $\boxed{}$, x with $\boxed{}$ and y with $\boxed{}$.

$-4 = 12 + b$ Multiply.

$-4 - \boxed{} = 12 + b - \boxed{}$ Subtract.

$\boxed{} = b$ Simplify.

Step 3 Write the slope-intercept form using

$m = \boxed{}$ and $b = \boxed{}$.

$y = mx + b$ Slope-intercept form

$y = \boxed{}$ Replace m with -4 and b with -16.

The equation is $y = \boxed{}$.

Check Your Progress The table of ordered pairs shows the coordinates of two points on the graph of a function. Write an equation that describes the function.

x	y
-1	4
2	6

$\boxed{}$

4–5 Writing Equations in Point-Slope Form

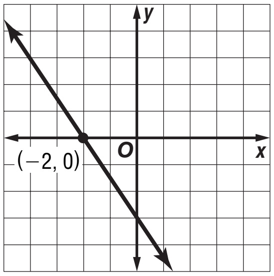
MAIN IDEAS

- Write the equation of a line in point-slope form.
- Write linear equations in different forms.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(A)** Develop the concept of slope as rate of change and **determine slopes from graphs,** tables, and algebraic representations. **A.7** The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear equations** and inequalities **using** concrete models, graphs, and **the properties of equality,** select a method, and solve the equations and inequalities.

EXAMPLE Write an Equation Given Slope and a Point

① **Write the point-slope form of an equation for a line that passes through $(-2, 0)$ with slope $-\frac{3}{2}$.**



$y - y_1 = m(x - x_1)$ Point-slope form

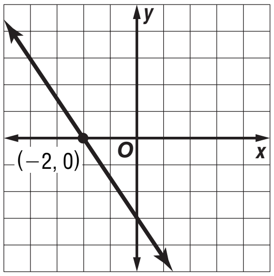
$y - 0 = -\frac{3}{2}[x - (-2)]$ $(x_1, y_1) = (-2, 0)$

$y =$ Simplify.

The equation is $y =$.

EXAMPLE Write an Equation Given Slope and a Point

② **Write the point-slope form of an equation for a horizontal line that passes through $(0, 5)$.**



$y - y_1 = m(x - x_1)$ Point-slope form

$=$ $(x_1, y_1) = (0, 5)$

$= 0$ Simplify.

The equation is .

Check Your Progress

a. Write the point-slope form of an equation for a line that passes through (4, −3) with slope −2.

b. Write the point-slope form of an equation for a horizontal line that passes through (−3, −4).

EXAMPLE Write an Equation in Standard Form

③ Write $y = \frac{3}{4}x - 5$ in standard form.

In standard form, the variables are on the left side of the equation. A, B, and C are all integers.

$y = \frac{3}{4}x - 5$	Original equation
$\boxed{}(y) = \boxed{}\left(\frac{3}{4}x - 5\right)$	Multiply each side by $\boxed{}$ to eliminate the fraction.
$\boxed{} = \boxed{} - \boxed{}$	Distributive Property
$4y - \boxed{} = 3x - 20 - \boxed{}$	Subtract $\boxed{}$ from each side.
$3x - 4y = 20$	Simplify.

Check Your Progress Write $y - 3 = 2(x + 4)$ in standard form.

$\boxed{}$

EXAMPLE Write an Equation in Slope-Intercept Form

④ Write $y - 5 = \frac{3}{4}(x - 3)$ in slope-intercept form.

In slope-intercept form, y is on the left side of the equation. The constant and x are on the right side.

$y - 5 = \frac{4}{3}(x - 3)$	Original equation
$y - 5 = \boxed{}$	Distributive Property
$y - 5 + \boxed{} = \frac{4}{3}x - 4 + \boxed{}$	Add $\boxed{}$ to each side.
$y = \boxed{}$	Simplify.

Check Your Progress Write $3x + 2y = 6$ in slope-intercept form.

$\boxed{}$

KEY CONCEPT

Point-Slope Form The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

FOLDABLES™ Under the tab for Lesson 4-5, draw a graph that goes through (3, −3) and has slope of −5. Explain how to find the equation of this line.

REVIEW IT

Write the standard form of a linear equation. *(Lesson 3-3)*

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–6 Statistics: Scatter Plots and Lines of Fit

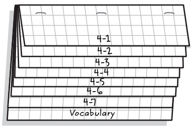
MAIN IDEAS

- Interpret points on a scatter plot.
- Write equations for lines of fit.

TEKS A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. **(E) Interpret and make decisions, predictions, and critical judgments from functional relationships.**
A.2 The student uses the properties and attributes of functions. **(D) Collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.** *Also addresses TEKS A.3(B) and A.6(D).*

FOLDABLES™ ORGANIZE IT

Write the definitions of the vocabulary builder words under the vocabulary tab.



BUILD YOUR VOCABULARY (pages 80–81)

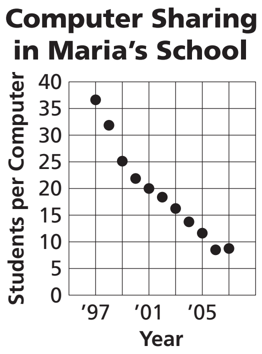
A **scatter plot** is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane.

A ______ correlation exists when as *x* increases, *y* increases. A ______ correlation exists when as *x* increases, *y* decreases.

A **line of fit** describes the trend of the data.

EXAMPLE

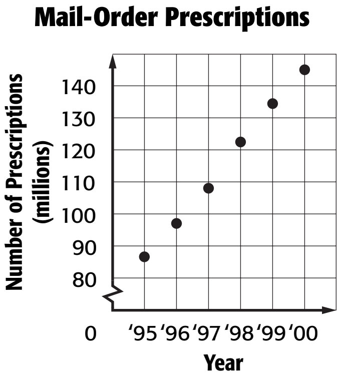
1 TECHNOLOGY Determine whether the graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe it.



The graph shows a ______ correlation. With each year, ______ computers are in Maria's school, making the students per computer rate ______.

Check Your Progress

Determine whether the graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe it.



REMEMBER IT

When graphing, the line of fit is only an approximation.

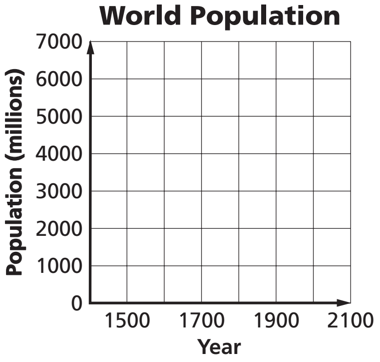
EXAMPLE Make and Evaluate Predictions

2 **The table shows the world's population growing at a rapid rate.**

Year	Population (millions)
1650	500
1850	1000
1930	2000
1975	4000
1998	5900

a. Draw a scatter plot and determine what relationship exists, if any, in the data.

Let the independent variable *x* be the year and let the dependent variable *y* be the population (in millions).



The scatter plot seems to indicate that as the year, the population. There is a correlation between the two variables.

b. Draw a line of fit for the scatter plot.

No one line will pass through all of the data points. Draw a that passes to the points. A line is shown in the scatter plot.

c. Write the slope intercept form of an equation for the equation for the line of fit.

The line of fit shown passes through the data points (1850, 1000) and (1998, 5900).

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m =$ Let $(x_1, y_1) = (1850, 1000)$ and $(x_2, y_2) = (1998, 5900)$.

$m =$ or ≈ 33.1 Simplify.

Step 2 Use $m = 33.1$ and either the point-slope form or the slope-intercept form to write the equation. You can use either data point. We chose (1850, 1000).

Point-slope form	**Slope-intercept form**
$y - y_1 = m(x - x_1)$	$y = mx + b$
$y - 1000 \approx 33.1(x - 1850)$	$1000 \approx 33.1(1850) + b$
$y - 1000 \approx 33.1x - 61{,}235$	$1000 \approx 61{,}235 + b$
$y \approx$	$-60{,}235 \approx b$
	$y \approx$

The equation of the line is $y \approx$.

Check Your Progress **The table shows the number of bachelor's degrees received since 1988.**

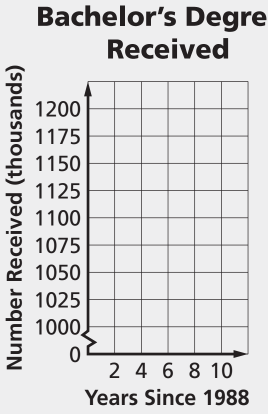
Years since 1998	2	4	6	8	10
Bachelor's Degrees Received (thousands)	1051	1136	1169	1165	1184

Source: *National Center for Education Statistics*

a. Draw a scatter plot and determine what relationship exists, if any, in the data.

b. Draw a line of best fit for the scatter plot.

c. Write the slope-intercept form of an equation for the line of fit.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–7 Geometry: Parallel and Perpendicular Lines

MAIN IDEAS

- Write an equation of the line that passes through a given point, parallel to a given line.
- Write an equation of the line that passes through a given point, perpendicular to a given line.

KEY CONCEPT

Parallel Lines in a Coordinate Plane Two nonvertical lines are parallel if they have the same slope. All vertical lines are parallel.

TEKS A.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. **(F) Interpret and predict the effects of changing slope** and *y*-intercept **in applied situations.**

BUILD YOUR VOCABULARY (pages 80–81)

Lines in the same plane that do not $\boxed{}$ are called **parallel lines.**

Lines that intersect at $\boxed{}$ are called **perpendicular lines.**

EXAMPLE Parallel Line Through a Given Point

① Write the slope-intercept form of an equation for the line that passes through (4, –2) and is parallel to the graph of $y = \frac{1}{2}x - 7$.

The line parallel to $y = x - 7$ has the same slope, $\frac{1}{2}$. Replace m with $\frac{1}{2}$ and (x, y) with $(4, -2)$ in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = \frac{1}{2}(x - 4)$	Replace m with $\frac{1}{2}$, y with –2, and x with 4.
$\boxed{} = \frac{1}{2}(x - 4)$	Simplify.
$y + 2 = \boxed{\phantom{\frac{1}{2}x-2}}$	Distributive Property
$y + 2 - \boxed{} = \frac{1}{2}x - 2 - \boxed{}$	Subtract $\boxed{}$ from each side.
$y = \boxed{\phantom{\frac{1}{2}x-4}}$	Write the equation in slope-intercept form.

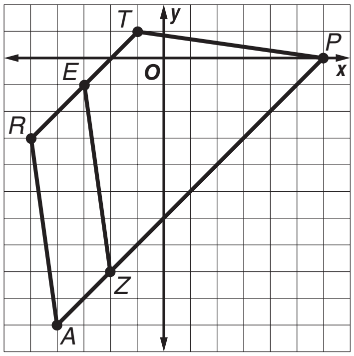
Check Your Progress Write the slope-intercept form of an equation for the line that passes through (2, 3) and is parallel to the graph of $y = \frac{1}{2}x - 1$.

KEY CONCEPT

Perpendicular Lines in a Coordinate Plane Two nonvertical lines are perpendicular if the product of their slopes is −1. That is, the slopes are *opposite reciprocals* of each other. Vertical lines and horizontal lines are also perpendicular.

EXAMPLE Determine Whether Lines are Perpendicular

② **GEOMETRY The height of a trapezoid is measured on a segment that is perpendicular to a base. In a trapezoid $ARTP$, $\overline{RT}$ and $\overline{AP}$ are bases. Can $\overline{EZ}$ be used to measure the height of the trapezoid? Explain.**



Find the slope of each segment.

Slope of $\overline{RT}$: $m = \frac{-3 - 1}{-5 - (-1)}$ or ____

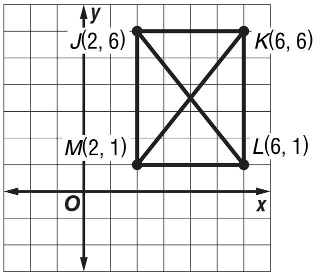
Slope of $\overline{AP}$: $m =$ ____ or ____

Slope of $\overline{EZ}$: $m =$ ____ or ____

The slope of $\overline{RT}$ and $\overline{AP}$ is ____ and the slope of $\overline{EZ}$ is ____.

$-7 \cdot 1 \neq$ ____. $\overline{EZ}$ is not ____ to $\overline{RT}$ and $\overline{AP}$, so it cannot be used to measure height.

Check Your Progress The graph shows the diagonals of a rectangle. Determine whether $\overline{JL}$ is perpendicular to $\overline{KM}$.



EXAMPLE Perpendicular Line Through a Given Point

③ **Write the slope-intercept form for an equation of a line that passes through (4, –1) and is perpendicular to the graph of $7x - 2y = 3$.**

Step 1 Find the slope of the given line.

$7x - 2y = 3$	Original equation
$7x - 2y - 7x = 3 - 7x$	Subtract $\boxed{}$ from each side.
$\boxed{} = -7 + 3$	Simplify.
$\dfrac{-2y}{-2} = \dfrac{-7x + 3}{-2}$	Divide each side by $\boxed{}$.
$y = \boxed{}$	Simplify.

Step 2 The slope of the given line is $\boxed{}$. So, the slope of the line perpendicular to this line is the opposite reciprocal of $\frac{7}{2}$, or $-\frac{2}{7}$.

Step 3 Use the point-slope form to find the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-1) = -\frac{2}{7}(x - 4)$	$(x_1, y_1) = (4, -1)$, $m = \boxed{}$
$\boxed{} = -\frac{2}{7}(x - 4)$	Simplify.
$y + 1 = -\frac{2}{7}x + \frac{8}{7}$	Distributive Property
$y + 1 \boxed{} = -\frac{2}{7}x + \frac{8}{7} \boxed{}$	Subtract.
$y = \boxed{}$	Simplify.

Check Your Progress Write the slope-intercept form for an equation of a line that passes through (–3, 6) and is perpendicular to the graph of $3x + 2y = 6$.

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 4

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	**BUILD YOUR VOCABULARY**
Use your **Chapter 4 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 4, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 80–81)* to help you solve the puzzle.

4-1

Rate of Change and Slope

Describe each type of slope.

	Type of Slope	Description of Graph
1.	positive	
2.	negative	
3.	zero	

4-2

Slope and Direct Variation

For each situation, write an equation with the proper constant of variation.

4. The distance d varies directly as time t, and a cheetah can travel 88 feet in 1 second.

5. The perimeter p of a pentagon with all sides of equal length varies directly as the length s of a side of the pentagon. A pentagon has 5 sides.

4-3

Graphing Equations in Slope-Intercept Form

6. Fill in the boxes with the correct words to describe what *m* and *b* represent.

$$y = mx + b$$

$$\boxed{} \qquad \boxed{}$$

7. What are the slope and *y*-intercept of a vertical line?

8. What are the slope and *y*-intercept of a horizontal line?

4-4

Writing Equations in Slope-Intercept Form

9. Suppose you are given that a line goes through (2, 5) and has a slope of −2. Use this information to complete the following equation.

$$y \quad = \quad mx \quad + \quad b$$

$$\boxed{} = \boxed{} \cdot \boxed{} + \boxed{}$$

10. What must you first do if you are not given the slope in the problem?

Write an equation of the line that passes through each pair of points.

11. (−5, 4), $m = -3$

12. (−2, −3), (4, 5)

4-5

Writing Equations in Point-Slope Form

13. In the formula $y - y_1 = m(x - x_1)$, what do x_1 and y_1 represent?

Complete the chart.

	Form of Equation	Formula	Example
14.	slope-intercept		$y = 3x + 2$
15.	point-slope		$y - 2 = 4(x + 3)$
16.	standard		$3x - 5y = 15$

4-6

Statistics: Scatter Plots and Lines of Fit

17. What is a *line of fit*? How many data points fall on the line of fit?

4-7

Geometry: Parallel and Perpendicular Lines

Write the slope-intercept form for an equation of the line that passes through the given point and is either parallel or perpendicular to the graph of the equation.

18. $(-2, 2)$, $y = 4x - 2$ (parallel)

19. $(4, 2)$, $y = \frac{1}{2}x + 1$ (perpendicular)

CHAPTER 4

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 4.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want take the Chapter 4 Practice Test on page 245 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 4 Study Guide and Review on pages 240–244 of your textbook.
- If you are unsure of any concepts or skills, refer to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 245.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 4 Foldable.
- Then complete the Chapter 4 Study Guide and Review on pages 240–244 of your textbook.
- If you are unsure of any concepts or skills, refer to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 245.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER

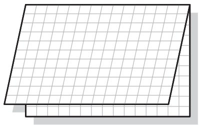
5

Solving Systems of Linear Equations and Inequalities

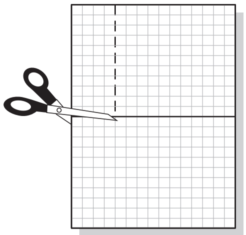
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of grid paper.

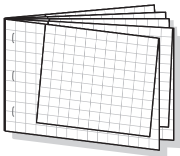
STEP 1 **Fold** each sheet in half along the width.



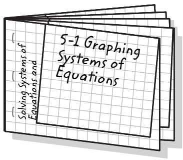
STEP 2 **Unfold** and cut four rows from left side of each sheet, from the top to the crease.



STEP 3 **Stack** the sheets and staple to form a booklet.



STEP 4 **Label** each page with a lesson number and title.



NOTE-TAKING TIP: Before going to class, look over your notes from the previous class, especially if the day's topic builds from the last one.

CHAPTER 5

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
consistent [kuhn·SIHS·tuhnt]			
dependent			
elimination [ih·LIH·muh·NAY·shuhn]			

Vocabulary Term	Found on Page	Definition	Description or Example
independent			
inconsistent			
$\underbrace{\text{substitution}}$ [SUHB·stuh·TOO·shuhn]			
system of equations			

5–1 Graphing Systems of Equations

MAIN IDEAS

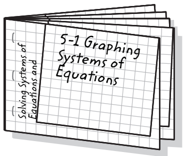
- Determine whether a system of linear equations has 0, 1, or infinitely many solutions.
- Solve systems of equations by graphing.

TEKS A.8 The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Analyze situations and formulate systems of linear equations in two unknowns to solve problems.** **(B) Solve systems of linear equations using concrete models, graphs,** tables, and algebraic methods. **(C) Interpret and determine the reasonableness of solutions to systems of linear equations.**

FOLDABLES™

ORGANIZE IT

In Lesson 5–1 of your booklet, draw the graph of a system of equations that has no solutions.



BUILD YOUR VOCABULARY (pages 106–107)

Two equations together are called a **system of equations.**

If the graphs intersect or coincide, the system of equations is said to be **consistent.**

If the graphs are , the system of equations is said to be **inconsistent.**

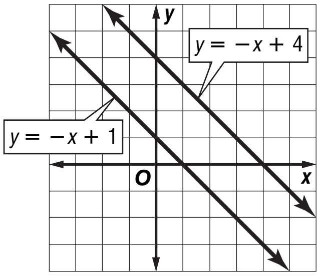
If a system has exactly solution, it is **independent.**

If the system has an number of solutions, it is **dependent.**

EXAMPLE Number of Solutions

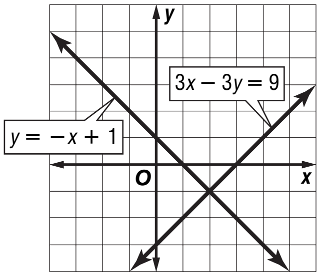
① **Use each graph to determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions.**

a. $y = -x + 1$
$y = -x + 4$



Since the graphs of $y = -x + 1$ and $y = -x + 4$ are , there are solutions.

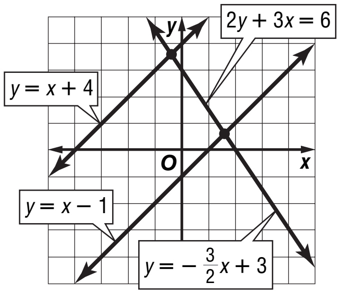
b. $3x - 3y = 9$
$y = -x + 1$



Since the graphs of $3x - 3y = 9$ and $y = -x + 1$ are lines, there is solution.

Check Your Progress **Use the graph to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.**

a. $2y + 3x = 6$
$y = x - 1$



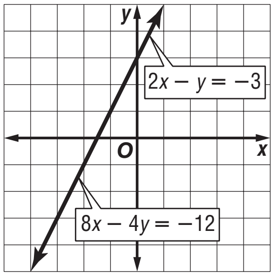
b. $y = x + 4$
$y = x - 1$

c. $y = -\frac{3}{2}x + 3$
$2y + 3x = 6$

EXAMPLE Solve a System of Equations

2 **Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.**

$\mathbf{2x - y = -3}$
$\mathbf{8x - 4y = -12}$



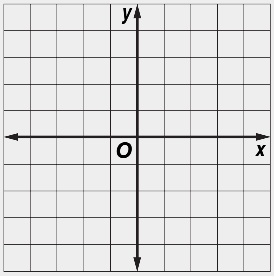
The graphs of the equations .

There are solutions of this system of equations.

Check Your Progress Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

$y = 2x + 3$

$y = \frac{1}{2}x + 3$



REVIEW IT

Describe the graph of a linear equation. *(Lesson 3-3)*

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–2 Substitution

MAIN IDEAS

- Solve systems of equations by using substitution.
- Solve real-world problems involving systems of equations.

TEKS A.8 The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Analyze situations and formulate systems of linear equations in two unknowns to solve problems. (B) Solve systems of linear equations using** concrete models, graphs, tables, and **algebraic methods. (C) Interpret** and determine the reasonableness of **solutions to systems of linear equations.**

BUILD YOUR VOCABULARY (pages 106–107)

The $\boxed{}$ solution of a system of equations can be found by using algebraic methods. One such method is called **substitution.**

EXAMPLE Solve Using Substitution

① **Use substitution to solve each system of equations.**

a. $x = 4y$
$4x - y = 75$

Since $x = 4y$, substitute $4y$ for x in the second equation.

$4x - y = 75$ Second equation

$4\left(\boxed{}\right) - y = 75$ $x = 4y$

$\boxed{} = 75$ Simplify.

$15y = 75$ Combine like terms.

$\frac{15y}{\boxed{}} = \frac{75}{\boxed{}}$ Divide each side by $\boxed{}$.

$y = \boxed{}$ Simplify.

Use $x = 4y$ to find the value of x.

$x = 4y$ First equation

$x = 4\left(\boxed{}\right)$ $y = \boxed{}$

$x = \boxed{}$ Simplify.

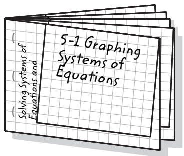
The solution is $\boxed{}$.

b. $4x + y = 12$
$-2x - 3y = 14$

Solve the first equation for y since the coefficient of y is 1.

FOLDABLES™ ORGANIZE IT

In Lesson 5-2 of your booklet, explain why it might be easier to solve a system of equations using substitution rather than graphing.



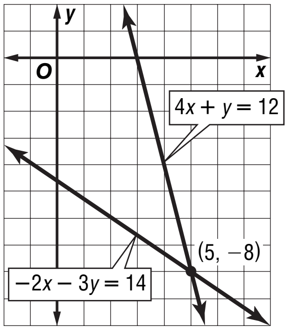
$4x + y = 12$	First equation
$4x + y \boxed{} = 12 \boxed{}$	Subtract $4x$ from each side.
$y = \boxed{}$	Simplify.

Find the value of x by substituting $12 - 4x$ for y in the second equation.

$-2x - 3y = 14$	Second equation
$-2x - 3\boxed{} = 14$	$y = 12 - 4x$
$-2x \boxed{} + \boxed{} = 14$	Distributive Property
$10x - 36 = 14$	$\boxed{}$
$10x - 36 + 36 = 14 + 36$	Add 36 to each side.
$\boxed{} = 50$	Simplify.
$\frac{10x}{10} = \frac{50}{10}$	Divide each side by $\boxed{}$.
$x = \boxed{}$	Simplify.

Substitute 5 for x in either equation to find the value of y.

$4x + y = 12$	First equation
$4\left(\boxed{}\right) + y = 12$	$x = \boxed{}$
$\boxed{} + y = 12$	Simplify.
$y = \boxed{}$	Subtract.



The solution is $\boxed{}$.

The graph verifies the solution.

Check Your Progress **Use substitution to solve each system of equations.**

a. $y = 2x$ and $3x + 4y = 11$

$\boxed{}$

b. $x + 2y = 1$ and $5x - 4y = -23$

$\boxed{}$

EXAMPLE Infinitely Many or No Solutions

② $2x + 2y = 8$
$x + y = -2$

Solve the second equation for y.

$x + y = -2$ Second equation

$x + y$ ☐ $= -2$ ☐ Subtract x from each side.

$y =$ ☐ Simplify.

Substitute ☐ for y in the first equation.

$2x + 2y = 8$ First equation

$2x + 2($☐$) = 8$ $y = -2 - x$

$2x - 4 - 2x = 8$ Distributive Property

☐ $= 8$ Simplify.

The statement ☐ = 8 is ☐. This means there are ☐ solutions of the system of equations. The graphs of the lines are ☐.

Check Your Progress Use substitution to solve the system of equations $3x - 2y = 3$ and $-6x + 4y = -6$.

REVIEW IT

Describe the first step when using the Distributive Property. *(Lesson 1-5)*

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–3 Elimination Using Addition and Subtraction

MAIN IDEAS

- Solve systems of equations by using elimination with addition.
- Solve systems of equations by using elimination with subtraction.

BUILD YOUR VOCABULARY (pages 106–107)

Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination.**

TEKS A.8 The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Analyze situations and formulate systems of linear equations in two unknowns to solve problems. (B) Solve systems of linear equations using** concrete models, graphs, tables, and **algebraic methods. (C) Interpret and determine the reasonableness of solutions to systems of linear equations.**

EXAMPLE Elimination Using Addition

① **Use elimination to solve the system of equations.**

$$-3x + 4y = 12$$
$$3x - 6y = 18$$

Since the coefficients of the x terms, -3 and 3, are additive inverses, you can eliminate the x terms by adding the equations.

$-3x + 4y = 12$ $\underline{(+)\ 3x - 6y = 18}$	Write the equation in column form and add.
$-2y = 30$	Notice that the $\boxed{}$ value is eliminated.
$\frac{-2y}{\boxed{}} = \frac{30}{\boxed{}}$	Divide each side by $\boxed{}$.
$y = \boxed{}$	Simplify.

Now substitute $\boxed{}$ for y in either equation to find x.

$-3x + 4y = 12$	First equation
$-3x + 4\left(\boxed{}\right) = 12$	Replace y with $\boxed{}$.
$-3x - \boxed{} = 12$	Simplify.
$-3x - 60 + \boxed{} = 12 + \boxed{}$	Add $\boxed{}$ to each side.
$-3x = 72$	Divide each side by $\boxed{}$.
$x = \boxed{}$	Simplify.

The solution is $\boxed{}$.

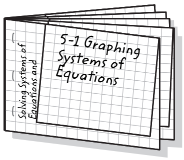
Check Your Progress Use elimination to solve $3x - 5y = 1$ and $2x + 5y = 9$.

$\boxed{}$

FOLDABLES™

ORGANIZE IT

In Lesson 5-3 of your booklet, write an example of a system that can be solved by subtracting the equations. Then solve your system.



EXAMPLE **Elimination Using Subtraction**

② **Use elimination to solve the system of equations.**

$\mathbf{4x + 2y = 28}$
$\mathbf{4x + 3y = 18}$

Since the coefficients of the x terms, 4 and 4, are the $\boxed{}$, you can eliminate the x terms by subtracting the equations.

$4x + 2y = 28$ $(-)\ 4x - 3y = 18$	Write the equation in column form and subtract.
$\boxed{} = 10$	Notice that the x value is eliminated.
$\dfrac{5y}{\boxed{}} = \dfrac{10}{\boxed{}}$	Divide each side by $\boxed{}$.
$y = \boxed{}$	Simplify.

Now substitute $\boxed{}$ for y in either equation.

$4x - 3y = 18$	Second equation
$4x - 3\left(\boxed{}\right) = 18$	Replace y with $\boxed{}$.
$4x - 6 = 18$	Simplify.
$4x - 6 + 6 = 18 + 6$	Add 6 to each side.
$4x = 24$	Simplify.
$\boxed{} = \boxed{}$	Divide each side by 4. Simplify.

The solution is $\boxed{}$.

Check Your Progress **Use elimination to solve each system of equations.**

a. $3x - 5y = 1$
$2x - 5y = 8$

$\boxed{}$

b. $9x - 2y = 30$
$x - 2y = 14$

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–4 Elimination Using Multiplication

MAIN IDEAS

- Solve systems of equations by using elimination with multiplication.
- Solve real-world problems involving systems of equations.

TEKS A.8 The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Analyze situations and formulate systems of linear equations in two unknowns to solve problems. (B) Solve systems of linear equations using** concrete models, graphs, tables, and **algebraic methods**. **(C) Interpret** and determine the reasonableness of **solutions to systems of linear equations.**

EXAMPLE Multiply One Equation to Eliminate

① **Use elimination to solve the system of equations.**

$\mathbf{2x + y = 23}$
$\mathbf{3x + 2y = 37}$

Multiply the first equation by $\boxed{}$ so the coefficients of the y terms are additive inverses. Then add the equations.

$2x + y = 23 \rightarrow \boxed{} - 2y = \boxed{}$ Multiply by $\boxed{}$.

$3x + 2y = 37 \quad \underline{(+)\ 3x + 2y = 37}$

$\boxed{} = \boxed{}$ Add the equations.

$\dfrac{x}{\boxed{}} = \dfrac{-9}{\boxed{}}$ Divide.

$x = \boxed{}$ Simplify.

Now substitute $\boxed{}$ for x in either equation to find the value of y.

$2x + y = 23$ First equation

$2\left(\boxed{}\right) + y = 23$ $x = \boxed{}$

$\boxed{} + y = 23$ Simplify.

$18 + y - \boxed{} = 23 - \boxed{}$ Subtract $\boxed{}$ from each side.

$y = 5$ Simplify.

The solution is $\boxed{}$.

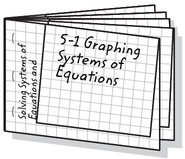
Check Your Progress Use elimination to solve $x + 7y = 12$ and $3x - 5y = 10$.

$\boxed{}$

FOLDABLES™

ORGANIZE IT

In Lesson 5-4 of your booklet, list the 5 different methods for solving a system of equations. Be sure to tell when it is best to use each one.



REMEMBER IT

When solving a system of equations by elimination, you can choose to eliminate either variable. See Example 2 on page 271 of your textbook.

EXAMPLE Multiply Both Equations to Eliminate

② **Use elimination to solve the system of equations.**

$\mathbf{4x + 3y = 8}$
$\mathbf{3x - 5y = -23}$

Choose either variable to eliminate. Let's eliminate x.

$4x + 3y = 8 \rightarrow$ ☐ $+ 9y = 24$	Multiply by ☐.
$3x - 5y = -23 \rightarrow \underline{(+)-12x +}$ ☐ $\underline{= 92}$	Multiply by ☐.
☐ $=$ ☐	Add the equations.
$\frac{29y}{29} = \frac{166}{29}$	Divide each side by ☐.
$y =$ ☐	Simplify.

Now substitute ☐ for y in either equation to find x.

$4x + 3y = 8$	First equation
$4x + 3$☐ $= 8$	$y =$ ☐
☐ $+$ ☐ $= 8$	Simplify.
$4x + 12 -$ ☐ $= 8 -$ ☐	Subtract ☐ from each side.
$4x =$ ☐	Simplify.
$\frac{4x}{4} = \frac{-4}{4}$	Divide each side by 4.
$x =$ ☐	Simplify.

The solution is ☐.

Check Your Progress Use elimination to solve $3x + 2y = 10$ and $2x + 5y = 3$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–5 Applying Systems of Linear Equations

MAIN IDEAS

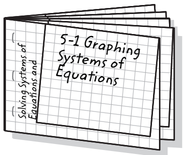
- Determine the best method for solving systems of equations.
- Apply systems of linear equations.

TEKS A.8 The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Analyze situations and formulate systems of linear equations in two unknowns to solve problems. (B) Solve systems of linear equations using** concrete models, graphs, tables, and **algebraic methods. (C) Interpret and determine the reasonableness of solutions to systems of linear equations.**

FOLDABLES™

ORGANIZE IT

In Lesson 5-5 of your booklet, explain when graphing would be the method that should be used to solve a system of equations.



EXAMPLE Determine the Best Method

① **FUND-RAISING At a Boy Scout fund-raising dinner, Mr. Jones bought 2 adult meals and 3 child meals for $23. Mrs. Gomez bought 4 adult meals and 2 child meals for $34. All adult meals are the same price and all child meals are the same price. The following system can be used to represent this situation. Determine the best method to solve the system of equations. Then solve the system.**

$$\begin{aligned} \mathbf{2x + 3y} &\mathbf{= 23} \\ \mathbf{4x + 2y} &\mathbf{= 34} \end{aligned}$$

- For an exact solution, an algebraic method is best.
- Since neither the coefficients of ______ nor the coefficients of ______ are 1 or –1, you cannot use the substitution method.
- Since the coefficients are not the same for either x or y, you will need to use ______ with multiplication.

Multiply the first equation by –2 so the coefficients of the x-terms are additive inverses. Then add the equations.

$$\begin{array}{rll} 2x + 3y = 23 & & \text{Multiply by } \underline{\qquad}. \\ -4x - 6y = -46 & & \\ (+)\ 4x + 2y = 34 & & \text{Add the equations.} \\ \hline -4y = -12 & & \\ y = \underline{\qquad} & & \text{Divide each side by } \underline{\qquad}. \end{array}$$

Now substitute 3 for y in either equation to find the value of x.

$$\begin{array}{rll} 4x + 2y = 34 & & \text{Second equation} \\ 4x + 2(\underline{\qquad}) = 34 & & y = 3 \\ 4x + \underline{\qquad} = 34 & & \text{Simplify.} \\ 4x = 28 & & \text{Subtract 6 from each side.} \\ x = 7 & & \text{Divide each side by } \underline{\qquad}. \end{array}$$

The solution is ______. So, adult meals cost $7 and child meals cost $3.

Check Your Progress **POOL PARTY** At the school pool party, Mr. Lewis bought 1 adult ticket and 2 child tickets for \$10. Mrs. Vroom bought 2 adult tickets and 3 child tickets for \$17. All adult tickets are the same price and all child tickets are the same price. The following system can be used to represent this situation. Determine the best method to solve the system of equations. Then solve the system.

$x + 2y = 10$

$2x + 3y = 17$

EXAMPLE Solve Systems of Equations to Solve Problems

2 **CAR RENTAL** **Ace Car Rental rents a car for \$45 a day and \$0.25 per mile. Star Car Rental rents a car for \$35 per day and \$0.30 per mile. After how many miles will the cost of renting a car at Ace Car Rental be the same as the cost of renting a car at Start Car Rental?**

EXPLORE You know the cost to rent a car for each company.

PLAN Write an equation to represent the

. Then solve.

SOLVE Let x = the number of and y = the of renting a car.

Ace Car Rental: $y = 45 + 0.25x$
Star Car Rental: $y = 35 + 0.30x$

You can use elimination to solve.

$$\begin{aligned} y &= 45 + 0.25x \\ (-)\ y &= 35 + 0.30x \\ \hline 0 &= 10 - x \\ -10 &= -0.05x \\ &= x \end{aligned}$$

This means that after 200 miles, the cost will be

.

CHECK Check by sketching a graph of the equations. The graphs appear to intersect at which verifies that x = .

Check Your Progress **VIDEO GAMES** The cost to rent a video game from Action Video is $2 plus $0.50 per day. The cost to rent a video game at TeeVee Rentals is $1 plus $0.75 per day. After how many days will the cost of renting a video game at Action Video be the same as the cost of renting a video game at TeeVee Rentals?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 5

BRINGING IT ALL TOGETHER

STUDY GUIDE

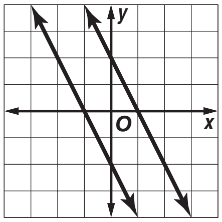
FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 5 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to: glencoe.com	You can use your completed **Vocabulary Builder** (*pages 106–107*) to help you solve the puzzle.

5-1

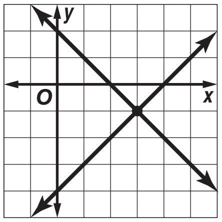
Graphing Systems of Equations

Each figure shows the graph of a system of two equations. Write the letter(s) of the figures that illustrate each statement

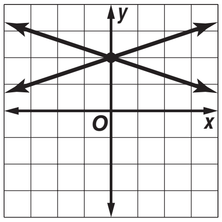
A.



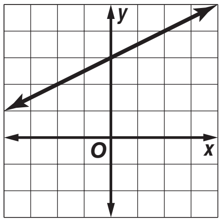
B.



C.



D.



1. A system of two linear equations can have an infinite number of solutions.

2. If two graphs are parallel, there are no ordered pairs that satisfy both equations.

3. If a system of equations has exactly one solution, it is independent.

4. If a system of equations has an infinite number of solutions, it is dependent.

5-2

Substitution

Solve each system using substitution.

5. $y = -2x$
$x + 3y = 15$

6. $3x - 2y = 12$
$x = 2y$

7. $-3x + 5y = 81$
$2x + y = 24$

5-3

Elimination Using Addition and Subtraction

Write *addition* or *subtraction* to tell which operation it would be easiest to use to eliminate a variable of the system. Explain your choice.

	System of Equations	Operation	Explanation
8.	$3x + 5y = 12$ $-3x + 2y = 6$		
9.	$3x + 5y = 7$ $3x - 2y = 8$		

Use elimination to solve each system of equations.

10. $7x + 2y = 10$
$-7x + y = -16$

11. $2x + 5y = -22$
$10x + 3y = 22$

5-4

Elimination Using Multiplication

Three methods for solving systems of linear equations are summarized below. Complete the table.

	Method	The Best Time to Use
12.	Graphing	to ______ the solution, since graphing usually does not give an ______ solution
13.		if one of the variables in either equation has a coefficient of 1 or ______
14.	Elimination Using Multiplication	if none of the coefficients are ______ or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations

5-5

Applying Systems of Linear Equations

Determine the best method to solve each system of equations. Then solve the system.

1. $-2x + 3y = 0$
$-1x + 5y = 7$

2. $-3x - 4y = -65$
$3x + 2y = 43$

3. $6x - 2y = 22$
$4x + 1y = 24$

CHAPTER 5

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 5.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 5 Practice Test on page 287 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 5 Study Guide and Review on pages 283–286 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 287.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 5 Foldable.
- Then complete the Chapter 5 Study Guide and Review on pages 283–286 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 287.

Student Signature

Parent/Guardian Signature

Teacher Signature

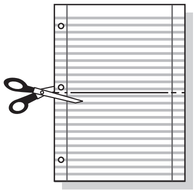
CHAPTER 6

Solving Linear Inequalities

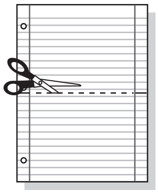
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with two sheets of notebook paper.

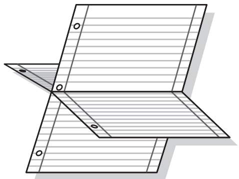
STEP 1 **Fold** one sheet in half along the width. Cut along the fold from the edges to the margins.



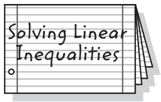
STEP 2 **Fold** the second sheet in half along the width. Cut along the fold between the margins.



STEP 3 **Insert** the first sheet through the second sheet and align the folds.



STEP 4 **Label** each page with a lesson number and title.



NOTE-TAKING TIP: When you take notes, write down the math problem and each step in the solution using math symbols. Next to each step, write down, in your own words, exactly what you are doing.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 6. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
absolute value			
boundary			
compound inequality			
half-plane			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
intersection			
set-builder notation			
system of inequalities			
union			

6–1 Solving Inequalities by Addition and Subtraction

MAIN IDEAS

- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

KEY CONCEPT

Addition Property of Inequalities If any number is added to each side of a true inequality, the resulting inequality is also true.

TEKS A.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear** equations and **inequalities using** concrete models, graphs, and **the properties of equality,** select a method, and solve the equations and inequalities. *Also addresses TEKS A.1(C), A.1(D), and A.7(C).*

EXAMPLE Solve by Adding

① **Solve $s - 12 > 65$. Then check your solution.**

$s - 12 > 65$ Original inequality

$s - 12 + \quad > 65 + \quad$ Add 12 to each side.

$s >$ All numbers greater than

Check Substitute 77, a number less than 77, and a number greater than 77.

Let $s = 77$.	Let $s = 64$.	Let $s = 80$.
$77 - 12 \overset{?}{>} 65$	$64 - 12 \overset{?}{>} 65$	$80 - 12 \overset{?}{>} 65$
$65 \not> 65$	$52 \not> 65$	$68 > 65$ ✔

The solution is the set .

Check Your Progress Solve $k - 4 < 10$. Check your solution.

EXAMPLE Solve by Subtracting

② **TEMPERATURE By 5:00 P.M. the temperature in Fairbanks had risen 23 degrees to a temperature of 14°F. What was the temperature at the beginning of the day?**

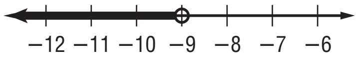
Solve $q + 24 < 14$. Then graph the solution.

$q + 23 < 14$ Original inequality

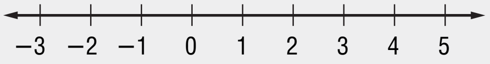
$q + 23 \quad < 14 \quad$ Subtract from each side.

$q < -9$ Simplify.

The solution set is .



Check Your Progress The temperature at the end of the day in Cleveland had risen 15° F to a temperature of 13° F. What was the temperature at the beginning of the day? Solve $m + 15 > 13$. Then graph the solution



KEY CONCEPT

Subtraction Property of Inequalities If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

FOLDABLES™ Include the Addition and Subtraction Properties of Inequalities in your Foldable. Be sure to show examples.

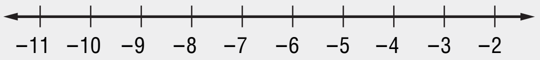
EXAMPLE Variables on Each Side

3 Solve $12n - 4 \le 13n$. Then graph the solution.

$12n - 4 \le 13n$	Original inequality
$12n - 4 - \boxed{} \le 13n - \boxed{}$	Subtract.
$\boxed{} \le n$	Simplify.

Since $-4 \le n$ is the same as $n \ge -4$, the solution set is $\boxed{}$.

Check Your Progress Solve $3p - 6 \ge 4p$. Graph the solution.



4 ENTERTAINMENT Alicia wants to buy season passes to two theme parks. If one season pass cost $54.99, and Alicia has $100 to spend on passes, the second season pass must cost no more than what amount?

Words The total cost of the two passes must be less than or equal to $100.

Variable Let s = the cost of the second pass.

Inequality

$$\underbrace{\text{The total cost}}_{54.99 + s} \quad \underbrace{\text{is less than or equal to}}_{\le} \quad \underbrace{\$100.}_{100}$$

Solve the inequality.

$54.99 + s \le 100$	Original inequality
$54.99 + s \boxed{} \le 100 \boxed{}$	Subtract $\boxed{}$ from each side.
$s \le 45.01$	Simplify.

The second pass must cost no more than $\boxed{}$.

Check Your Progress Michael scored 30 points in the four rounds of the free throw contest. Randy scored 11 points in the first round, 6 points in the second round, and 8 in the third round. How many points must he score in the final round to surpass Michael's score?

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–2 Solving Inequalities by Multiplication and Division

MAIN IDEAS

- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

KEY CONCEPTS

Multiplying by a Positive Number If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.

Multiplying by a Negative Number If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is also true.

TEKS A.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear** equations and **inequalities using** concrete models, **the properties of equality,** select a method, and solve the equations and inequalities. *Also addresses TEKS A.1(C), A.1(D), and A.7(C).*

EXAMPLE Write and Solve an Inequality

① **HIKING** **Bob is walking at a rate of $\frac{3}{4}$ mile per hour. He knows that it is at least 9 miles to Onyx Lake. How long will it take Bob to get there? Write and solve an inequality to find the length of time.**

$\frac{3}{4}t \ge 9$	Original inequality
$\boxed{}\frac{3}{4}t \ge \boxed{}9$	Multiply each side by $\boxed{}$.
$t \ge 12$	Simplify

The solution set is $\boxed{}$.

Check Your Progress **SCHOOL** At Midpark High School, $\frac{2}{3}$ of the junior class attended the dance. There were at least 200 juniors at the dance. How many students are in the junior class?

$\boxed{}$

EXAMPLE Multiply by a Negative Number

② **Solve $-\frac{3}{5}d \ge 6$.**

$-\frac{3}{5}d \ge 6$	Original inequality
$\boxed{}\left(-\frac{3d}{5}\right)\boxed{}\boxed{}(6)$	Multiply each side by $\boxed{}$ and change $\ge$ to $\boxed{}$.
$d \le -10$	Simplify.

The solution set is $\boxed{}$.

Check Your Progress Solve $-\frac{1}{3}x > 10$.

$\boxed{}$

EXAMPLE Divide to Solve an Inequality

3 **Solve each inequality.**

a. $12s \geq 60$

$12s \geq 60$	Original inequality
$\frac{12s}{\square} \geq \frac{60}{\square}$	Divide each side by $\square$ and do not change the direction of the inequality sign.
$s \geq \square$	Simplify.

The solution set is $\square$.

b. $-8q < 136$

$-8q < 136$	Original inequality
$\frac{-8q}{\square} > \frac{136}{\square}$	Divide each side by $\square$ and change $<$ to $>$.
$q > \square$	Simplify.

The solution set is $\square$.

Check Your Progress **Solve each inequality.**

a. $15p < 60$

b. $-4z > 64$

KEY CONCEPTS

Dividing by a Positive Number If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.

Dividing by a Negative Number If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

FOLDABLES™ Be sure to write the Multiplication and Division Properties of Inequalities in your Foldable.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–3 Solving Multi-Step Inequalities

MAIN IDEAS

- Solve linear inequalities involving more than one operation.
- Solve linear inequalities involving the Distributive Property.

TEKS A.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear** equations and **inequalities using** concrete models, **the properties of equality,** select a method, and solve the equations and inequalities. *Also addresses TEKS A.1(C), A.1(D), and A.7(C).*

REMEMBER IT

You only change the direction of the inequality sign when multiplying or dividing both sides by a negative number.

EXAMPLE Multi-Step Inequality

① **SCIENCE The inequality $F > 212$ represents the temperature in degrees Fahrenheit for which water is a gas (steam). Similarly, the inequality $\frac{9}{5}C + 32 > 212$ represents the temperature in degrees Celsius for which water is a gas. Find the temperature in degrees Celsius for which water is a gas.**

$\frac{9}{5}C + 32 > 212$	Original inequality
$\frac{9}{5}C + 32 - \boxed{} > 212 - \boxed{}$	Subtract $\boxed{}$ from each side.
$\frac{9}{5}C > \boxed{}$	Simplify.
$\boxed{}\frac{9}{5}C > \boxed{}180$	Multiply each side by $\boxed{}$.
$C > \boxed{}$	Simplify.

Water will be a gas for all temperatures greater than 100°C.

EXAMPLE Inequality Involving a Negative Coefficient

② **Solve $13 - 11d \geq 79$.**

$13 - 11d \geq 79$	Original inequality
$13 - 11d - \boxed{} \geq 79 - \boxed{}$	Subtract $\boxed{}$ from each side.
$\boxed{} \geq \boxed{}$	Simplify.
$\boxed{} \leq \boxed{}$	Divide each side by $\boxed{}$ and change $\geq$ to $\leq$.
$d \leq \boxed{}$	Simplify.

The solution set is $\boxed{}$

Check Your Progress

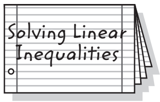
a. The boiling point of helium is −452°F. Solve $\frac{9}{5}C + 32 > -452$ to find the temperatures in degrees Celsius for which helium is a gas.

b. Solve $-8y + 3 > -5$

FOLDABLES™

ORGANIZE IT

In Lesson 6-3 of your Foldable, explain how solving an inequality is different from solving an equation.



3 Define a variable, write an inequality, and solve the problem below. Check your solution.

Four times a number plus twelve is less than a number minus three.

Four times a number	plus	twelve	is less than	a number minus three.
$4n$	$+$	12	$<$	$n - 3$

$4n + 12 < n - 3$	Original inequality
$4n + 12 - \boxed{} < n - 3 - \boxed{}$	Subtract $\boxed{}$ from each side.
$3n + 12 < -3$	Simplify.
$3n + 12 - \boxed{} < -3 - \boxed{}$	Subtract $\boxed{}$ from each side.
$3n < -15$	Simplify.
$\frac{3n}{\boxed{}} < \frac{-15}{\boxed{}}$	Divide each side by $\boxed{}$.
$n < -5$	Simplify.

The solution set is $\boxed{}$.

REMEMBER IT

If solving an inequality results in a statement that is

- true, the solution is all real numbers.
- false, the solution is the empty set, Ø.

Check Your Progress Write an inequality for the sentence below. Then solve the inequality.

6 times a number is greater than 4 times the number minus 2

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–4 Solving Compound Inequalities

MAIN IDEAS

- Solve compound inequalities containing the word *and* and graph their solution sets.
- Solve compound inequalities containing the word *or* and graph their solution sets.

BUILD YOUR VOCABULARY (pages 125–126)

Two or more inequalities connected by the words or are a **compound inequality.**

The graph of a compound inequality containing is the **intersection** of the graphs of the two inequalities.

The graph of a compound inequality containing is the **union** of the graphs of the two inequalities.

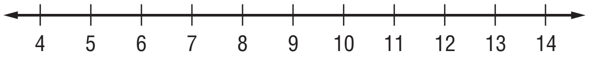
WRITE IT

Use words to describe the compound inequality 25 < *x* > 30.

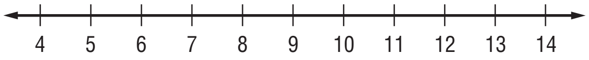
EXAMPLE Graph an Intersection

① Graph the solution set of $y \geq 5$ and $y < 12$.

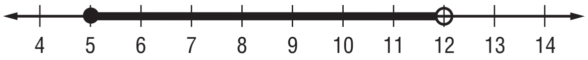
Graph $y \geq 5$.



Graph $y < 12$.

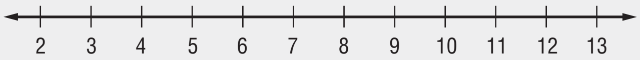


Find the .



The solution set is . Note that the graph of $y \geq 5$ includes the point 5. The graph of $y < 12$ does *not* include .

Check Your Progress Graph the solution set of $y > 6$ and $y \leq 10$.

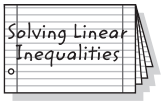


TEKS A.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear** equations and **inequalities using** concrete models, **the properties of equality,** select a method, and solve the equations and inequalities. *Also addresses TEKS A.1(C), A.1(D), and A.7(C).*

FOLDABLES™

ORGANIZE IT

In Lesson 6-4 of your Foldable, explain how the solution of an intersection is different from the solution of a union.



EXAMPLE Solve and Graph an Intersection

② **Solve $7 < z + 2 \leq 11$. Then graph the solution set.**

First express $7 < z + 2 \leq 11$ using *and*. Then solve each inequality.

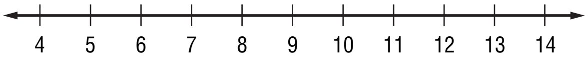
$7 < z + 2$ and $z + 2 \leq 11$

$7 \boxed{} < z + 2 \boxed{}$ $z + 2 \boxed{} \leq 11 \boxed{}$

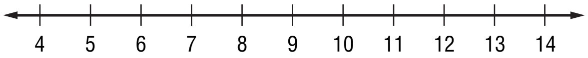
$\boxed{} < z$ $z \leq \boxed{}$

The solution set is the $\boxed{}$ of the two graphs.

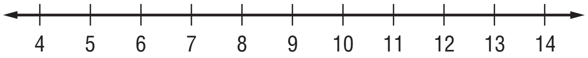
Graph $5 < z$ or $z > 5$



Graph $z \leq 9$



Find the $\boxed{}$.



The solution set is $\boxed{}$.

Check Your Progress Solve $-3 < x - 2 < 5$. Then graph the solution set.

EXAMPLE **Solve and Graph a Union**

③ **Solve $4k - 7 \leq 25$ or $12 - 9k \geq 30$. Then graph the solution set.**

$4k - 7 \leq 25$ or $12 - 9k \geq 30$

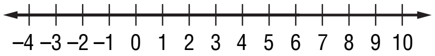
$4k - 7$ ______ ≤ 25 ______ $12 - 9k$ ______ ≥ 30 ______

$4k \leq 32$ $-9k \geq 18$

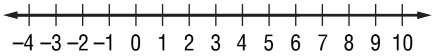
$\frac{4k}{4} \leq \frac{32}{4}$ $\frac{-9k}{-9} \leq \frac{18}{-9}$

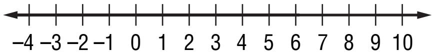
$k \leq$ ______ $k \leq$ ______

Graph $k \leq 8$.



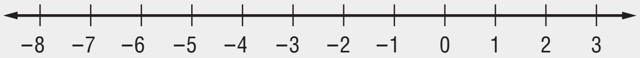
Graph $k \geq -2$.





Notice that the graph of $k \leq 8$ contains ______ point in the graph of k ≤ -2. So, the ______ is the graph of $k \leq 8$. The solution set is ______.

Check Your Progress Solve $-2x + 5 < 15$ *or* $5x + 15 > 20$. Then graph the solution set.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–5 Solving Open Sentences Involving Absolute Value

MAIN IDEAS

- Solve absolute value equations.
- Solve absolute value inequalities.

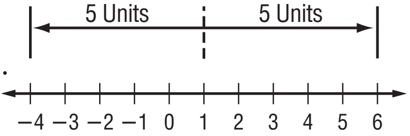
EXAMPLE Solve an Absolute Value Equation

① **a. WEATHER The average January temperature in a northern Canadian city is 1 degree Fahrenheit. The actual January temperature for that city may be about 5 degrees Fahrenheit warmer or colder. Solve $|t - 1| = 5$ to find the range of temperatures.**

METHOD 1 Graphing

$|t - 1| = 5$ means that the distance between t and 1 is 5 units. To find t on the number line, start at 1 and move 5 units in either direction.

The distance from 1 to 6 is 5 units.
The distance from 1 to −4 is 5 units.
The solution set is {−4, 6}.



METHOD 2 Compound Sentence

Write $|t - 1| = 5$ as $t - 1 = 5$ or $t - 1 = -5$.

Case 1		Case 2
$t - 1 = 5$		$t - 1 = -5$
$t - 1 + 1 = 5 + 1$	Add 1 to each side.	$t - 1 + 1 = -5 + 1$
$t =$	Simplify.	$t =$

The solution set is . The range of temperatures is −4°F to 6°F.

b. Solve $|x + 2| = -1$.

$|x + 2| = -1$ means that the distance between x and -2 is . Since distance cannot be negative, the solution is .

Check Your Progress

a. WEATHER The average temperature for Columbus on Tuesday was 45°F. The actual temperature for anytime during the day may have actually varied from the average temperature by 15°F. Solve $|t - 45| = 15$ to find the range of temperatures.

b. Solve $|x - 3| = -5$.

REVIEW IT

Why is the absolute value of a number always greater than or equal to zero? *(Lesson 2-1).*

TEKS A.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear** equations and **inequalities using** concrete models, **the properties of equality,** select a method, and solve the equations and inequalities. *Also addresses TEKS A.1(C) and A.1(D).*

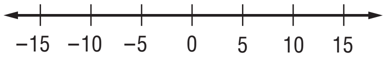
EXAMPLE Solve an Absolute Value Inequality (<)

2 Solve $|s - 3| \leq 12$. Then graph the solution set.

Write $|s - 3| \leq 12$ as $s - 3 \leq 12$ and $s - 3 \geq -12$.

Case 1		Case 2
$s - 3 \leq 12$	Original inequality	$s - 3 \geq -12$
$s - 3 + 3 \leq 12 + 3$	Add 3 to each side.	$s - 3 + 3 \geq -12 + 3$
$s \leq$	Simplify.	$s \geq$

The solution set is .



WRITE IT

Will the solution to $|x + 7| < 11$ require finding the intersection or union of the two cases? Explain.

EXAMPLE Solve an Absolute Value Inequality (>)

3 Solve $|3y - 3| > 9$.

Write $|3y - 3| > 9$ as $3y - 3 > 9$ or $3y - 3 < -9$.

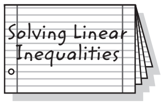
Case 1		Case 2
$3y - 3 > 9$	Original inequality	$3y - 3 < -9$
$3y - 3 + 3 > 9 + 3$	Add 3 to each side.	$3y - 3 + 3 < -9 + 3$
$>$	Simplify.	$<$
$\frac{3y}{3} > \frac{12}{3}$	Divide each side by 3.	$\frac{3y}{3} < \frac{-6}{3}$
$y >$	Simplify.	$y <$

The solution set is .

FOLDABLES™

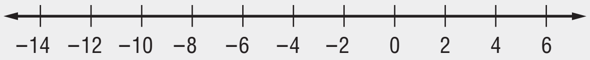
ORGANIZE IT

In Lesson 6-5 of your Foldable, write your own absolute value inequality. Then solve and graph it. Explain the steps you use.



Check Your Progress **Solve each open sentence. Then graph the solution set.**

a. $|p + 4| < 6$



b. $|2m - 2| > 6$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–6 Graphing Inequalities in Two Variables

MAIN IDEAS

- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.

KEY CONCEPT

Half-Planes and Boundaries Any line in the plane divides the plane into two regions called half-planes. The line is called the boundary of each of the two half-planes.

TEKS A.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and **formulate linear** equations or **inequalities to solve problems. (B) Investigate methods for solving linear** equations and **inequalities using** concrete models, **the properties of equality,** select a method, and solve the equations and inequalities. *Also addresses TEKS A.1(C), A.1(D), and A.7(C).*

BUILD YOUR VOCABULARY (pages 125–126)

The region of the graph of an inequality on one side of the ____ is called a **half-plane.**

An ____ defines the **boundary** or edge for each half-plane.

EXAMPLE Graph an Inequality

① **Graph $2y - 4x > 6$.**

Step 1 Solve for y in terms of x.

$2y - 4x > 6$ Original Inequality

$2y - 4x + \text{____} > \text{____} + 6$ Add ____ to each side.

$2y > 4x + 6$ Simplify.

$\frac{2y}{2} > \frac{4x + 6}{2}$ Divide each side by 2.

$y > \text{____}$ Simplify.

Step 2 Graph $y = 2x + 3$.

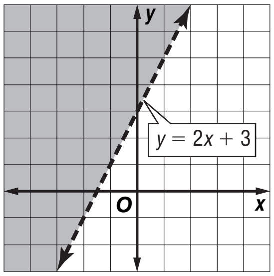
Since $y > 2x + 3$ does not include values when $y = 2x + 3$, the boundary is ____ in the solution set. The boundary should be drawn as a ____.

Step 3 Select a point in one of the half-planes and test it. Let's use (0, 0).

$y > 2x + 3$ Original inequality

$0 > 2(0) + 3$ $x = 0, y = 0$

$0 > 3$ False



Since the statement is false, the ____ containing the origin is ____ part of the solution. Shade the other half-plane.

Check Test a point in the other half-plane, for example, (–3, 1).

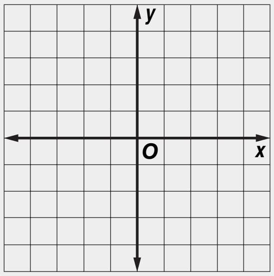
$y > 2x + 3$	Original inequality
$1 > 2(-3) + 3$	$x = -3$, $y = 1$
$1 > -3$ ✓	

Since the statement is true, the half-plane containing (–3, 1) should be ☐.

REMEMBER IT

A dashed line indicates that the boundary is *not* part of the solution set. A solid line indicates that the boundary line *is* part of the solution set.

Check Your Progress Graph $y - 3x < 2$.



EXAMPLE Write and Solve an Inequality

2 JOURNALISM **Lee Cooper writes and edits short articles for a local newspaper. It generally takes her an hour to write an article and about a half-hour to edit an article. If Lee works up to 8 hours a day, how many articles can she write and edit in one day?**

Step 1 Let x equal the number of articles Lee can write. Let y equal the number of articles that Lee can edit. Write an open sentence representing the situation.

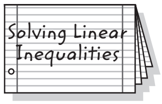
Number of articles she can write	plus	$\frac{1}{2}$ hour	times	number of articles she can edit	is up to	8 hours.
☐	+	☐	×	☐	☐	8

Step 2 Solve for y in terms of x.

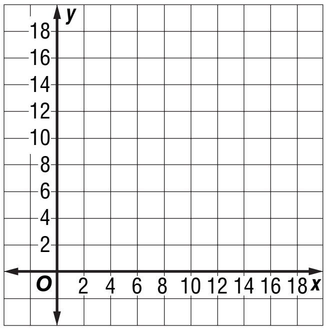
$x + \frac{1}{2}y \leq 8$	Original inequality
$x + \frac{1}{2}y - \square \leq \square + 8$	Subtract ☐ from each side.
$\square \leq -x + 8$	Simplify.
$(2)\,\frac{1}{2}y \leq 2(-x + 8)$	Multiply each side by 2.
$y \leq \square$	Simplify.

FOLDABLES™ ORGANIZE IT

In Lesson 6-6 of your Foldable, explain how to check the solution to an inequality in two variables.



Step 3 Since the open sentence includes the equation, graph $y = -2x + 16$ as a ______ line. Test a ______ in one of the half-planes, for example, (0, 0). Shade the half-plane containing (0, 0) since $0 \leq -2(0) + 16$ is true.



Step 4 Examine the situation

- Lee cannot work a negative number of hours. Therefore, the domain and range contain only ______ numbers.
- Lee only wants to count articles that are completely written or completely edited. Thus, only points in the half-plane whose x- and y-coordinates are ______ numbers are possible solutions.
- One solution is (2, 3). This represents ______ written articles and ______ edited articles.

Check Your Progress You offer to go to the local deli and pick up sandwiches for lunch. You have \$30 to spend. Chicken sandwiches cost \$3.00 and tuna sandwiches are \$1.50 each. How many sandwiches can you purchase for \$30?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–7 Graphing Systems of Inequalities

MAIN IDEAS

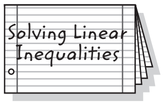
- Solve systems of inequalities by graphing.
- Solve real-world problems involving systems of inequalities.

Preparation for TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of** equations in two or more unknowns or **inequalities in two unknowns to solve problems. (B) Use** algebraic methods, **graphs**, tables, or matrices, **to solve systems of equations or inequalities. (C) Interpret** and determine the reasonableness of **solutions to systems of** equations or **inequalities for given contexts.**

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 6–7, write a description of how to graph the solution of a system of inequalities.



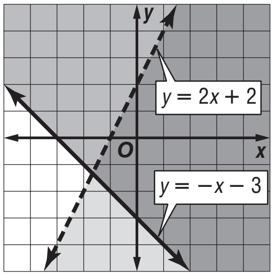
BUILD YOUR VOCABULARY (pages 123–124)

To solve a **system of inequalities,** you need to find ______ that satisfy ______ the inequalities involved.

EXAMPLE Solve by Graphing

1 **Solve the system of inequalities by graphing.**

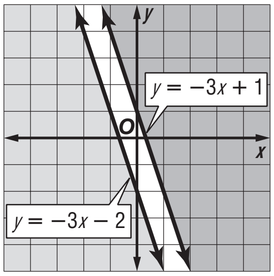
a. $y < 2x + 2$
$y \geq -x - 3$



The solution includes the ordered pairs in the intersection of the graphs of $y < 2x + 2$ and $y \geq -x - 3$. The region is shaded in dark grey. The graphs $y = 2x + 2$ and $y = -x - 3$ are ______ of this region.

The graph ______ is dashed and is not ______ in the graph of $y < 2x + 2$. The graph of $y = -x - 3$ is included in the graph of $y \geq -x - 3$.

b. $y \geq -3x + 1$
$y \leq -3x - 2$

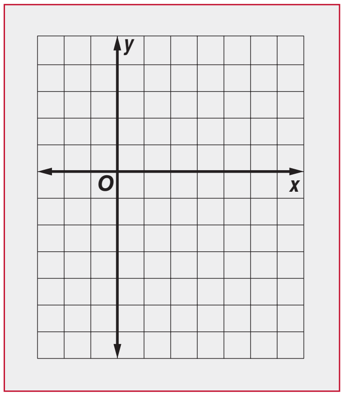


The graphs of $y = -3x + 1$ and $y = -3x - 2$ are ______ lines.

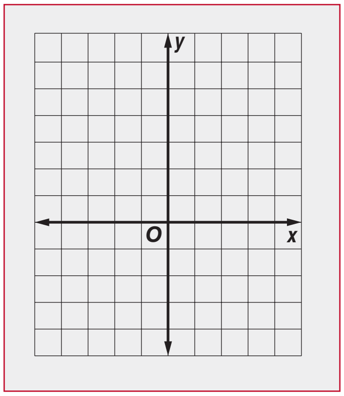
Because the two regions have no points in common, the system of inequalities has ______.

Check Your Progress **Solve each system of inequalities by graphing.**

a. $2x + y \leq 4$
$x + 2y < -4$



b. $y > 4x$
$y < 4x - 3$



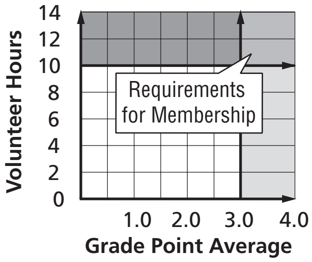
EXAMPLE Use a System of Inequalities to Solve a Problem

2 **SERVICE** **A college service organization require that its members maintain at least a 3.0 grade point average, and volunteer at least 10 hours a week. Graph these requirements.**

If g = the grade point average and v = the number of volunteers, the following inequalities represent these requirements.

The grade point average is at least 3.0.

The number of volunteer hours is at least 10.

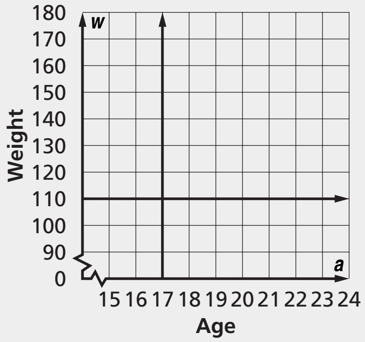


The solution is the set of ordered pairs whose graphs are in the of the graphs of these inequalities.

WRITE IT

Describe the graph of a system of inequalities that has no solution.

Check Your Progress The senior class is sponsoring a blood drive. Anyone who wishes to give blood must be at least 17 years old and weigh at least 110 pounds. Graph these requirements.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 6

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 6 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 6, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 125–126)* to help you solve the puzzle.

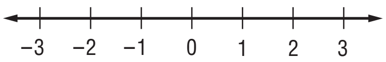
6-1
Solving Inequalities by Addition and Subtraction

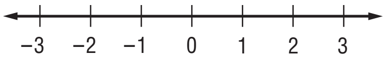
Write the letter of the graph that matches each inequality.

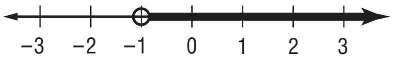
1. $x \leq -1$ ______

2. $x > -1$ ______

3. $x < -1$ ______

a.


b.


c.


4. According to the Subtraction Property of Inequalities, if any number is ______ from each side of a ______ inequality, the resulting inequality is also ______.

6-2
Solving Inequalities by Multiplication and Division

Write an inequality that describes each situation.

5. A number n divided by 8 is greater than 5. ______

6. Twelve times a number k is at least 7. ______

Use words to tell what each inequality says.

7. $12 < 6n$ ______

8. $\frac{t}{-3} \geq 14$ ______

6-3

Solving Multi-Step Inequalities

Solve each inequality. Then check your solution.

9. $5 \leq 11 + 3h$

10. $5 - 2n \leq 3 - n$

Define a variable, write an inequality, and solve each problem. Then check your solution.

11. Six plus four times a number is no more than the number.

12. Three times a number plus eight is at least ten less than four times the number.

13. Six times a number is greater than twelve less than 8 times the number.

6-4

Solving Compound Inequalities

14. When is a compound inequality containing *and* true?

15. The graph of a compound inequality containing *and* is the ☐ of the graphs of the two inequalities.

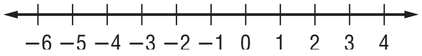
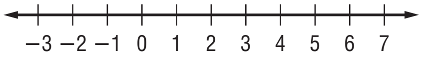
16. When is a compound inequality containing *or* true?

17. The graph of a compound inequality containing *or* is the ☐ of the graphs of the two inequalities.

6-5

Solving Open Sentences Involving Absolute Value

Complete each compound sentence by writing *and* or *or* in the blank. Use the result to help you graph the absolute value sentence.

	Absolute Value Sentence	Compound Sentence	Graph
18.	$\|2x + 2\| = 8$	$2x + 2 = 8$ ☐ $2x + 2 = -8$	
19.	$\|x - 5\| \leq 4$	$x - 5 \leq 4$ ☐ $x - 5 \geq -4$	
20.	$\|2x - 3\| > 5$	$2x - 3 > 5$ ☐ $2x - 3 < -5$	

21. A thermometer is guaranteed to give a temperature no more than 2.1°F from the actual temperature. If the thermometer reads 58°F, what is the range for the actual temperature?

6-6

Graphing Inequalities in Two Variables

22. Complete the chart to show which type of line is needed for each symbol.

Symbol	Type of Line	Boundary Part of Solution?
$<$		
$>$		
$\geq$		
$\leq$		

23. If a test point results in a false statement, what do you know about the graph?

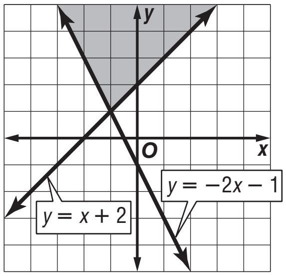
24. If a test point results in a true statement, what do you know about the graph?

6-7

Graphing Systems of Inequalities

Write the inequality symbols that you need to get a system whose graph looks like the one shown. Use <, ≤, >, or ≥.

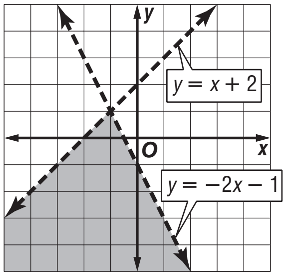
25.



$y \quad x + 2$

$y \quad -2x - 1$

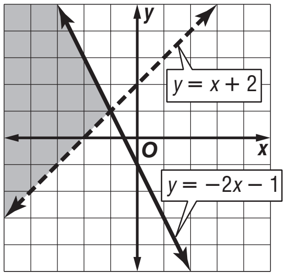
26.



$y \quad x + 2$

$y \quad -2x - 1$

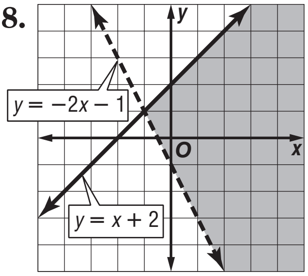
27.



$y \quad x + 2$

$y \quad -2x - 1$

28.



$y \quad x + 2$

$y \quad -2x - 1$

29. The solution of a is the set of all ordered pairs that satisfy both inequalities. If you graph the inequalities in the same coordinate plane, the is the region where the graphs .

30. Describe how you would explain the process of using a graph to solve a system of inequalities to a friend who missed Lesson 6-7.

CHAPTER 6

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 6.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want take the Chapter 6 Practice Test on page 345 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 6 Study Guide and Review on pages 340–344 of your textbook.
- If you are unsure of any concepts or skills, refer to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 345.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 6 Foldable.
- Then complete the Chapter 6 Study Guide and Review on pages 340–344 of your textbook.
- If you are unsure of any concepts or skills, refer to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 345.

Student Signature

Parent/Guardian Signature

Teacher Signature

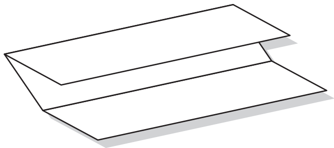
CHAPTER 7

Polynomials

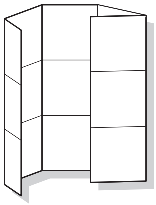
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11" by 17" paper.

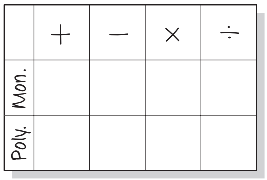
STEP 1 **Fold** in thirds lengthwise.



STEP 2 **Open** and fold a 2" tab along the width. Then fold the rest in fourths.



STEP 3 **Draw** lines along folds and label as shown.



NOTE-TAKING TIP: It is helpful to read through your notes before beginning your homework. Look over any page referenced material.

CHAPTER

7

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 7. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
$\underbrace{\text{binomial}}$ [by·NOH·mee·uhl]			
constant			
degree of monomial			
degree of polynomial			
difference of squares			

Vocabulary Term	Found on Page	Definition	Description or Example
FOIL method			
monomial [mah·NOH·mee·uhl]			
negative exponent			
polynomial [PAH·luh·NOH-mee·uhl]			
trinomial [try·NOH·mee-uhl]			
zero exponent			

7–1 Multiply Monomials

MAIN IDEAS

- Multiply monomials.
- Simplify expressions involving powers of monomials.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **A.11** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(A)** Use patterns to generate the laws of exponents and **apply them in problem-solving situations.**

BUILD YOUR VOCABULARY (pages 150–151)

A **monomial** is a number, a ____, or a product of a number and one or more variables.

Monomials that are ____ numbers are called **constants**.

EXAMPLES Identify Monomials

1 Determine whether each expression is a monomial. Explain your reasoning.

	Expression	Monomial?	Reason
a.	$17 - s$	no	The expression involves subtraction, not the product, of two variables.
b.	$8f^2g$		The expression is the product of a number and two variables.
c.	$\frac{3}{4}$	yes	$\frac{3}{4}$ is a real number and an example of a constant.
d.	xy		The expression is the product of two variables.

Check Your Progress Determine whether each expression is a monomial. Explain your reasoning.

	Expression	Monomial?	Reason
a.	x^5		
b.	$3p - 1$		
c.	$\frac{9x}{y}$		
d.	$\frac{cd}{8}$		

EXAMPLE Product of Powers

2 **a. Simplify $(r^4)(-12r^7)$.**

$(r^4)(-12r^7) = (1)(-12)(r^4)(r^7)$	Commutative and Associative Properties
$= \boxed{}$	Product of Powers
$= \boxed{}$	Simplify.

b. Simplify $(6cd^5)(5c^5d^2)$.

$(6cd^5)(5c^5d^2) = (6)(5)(c \bullet c^5)(d^5 \bullet d^2)$	Commutative and Associative Properties
$= 30(c^{\boxed{}})(d^{\boxed{}})$	Product of Powers
$= \boxed{}$	Simplify.

Check Your Progress **Simplify each expression.**

a. $(5x^2)(4x^3)$

b. $3xy^2(-2x^2y^3)$

EXAMPLE Power of a Power

3 **Simplify $((2^3)^3)^2$.**

$((2^3)^3)^2 = (2^{3 \bullet 3})^2$	$\boxed{}$ of a Power
$= \boxed{}$	Simplify.
$= 2^{\boxed{}}$	$\boxed{}$ of a Power
$= 2^{\boxed{}}$ or $\boxed{}$	Simplify.

Check Your Progress Simplify $((4^2)^2)^3$.

KEY CONCEPTS

Product of Powers
To multiply two powers that have the same base, add the exponents.

Power of a Power
To find the power of a power, multiply the exponents.

Power of a Product
To find the power of a product, find the power of each factor and multiply.

EXAMPLE Power of a Product

4 GEOMETRY **Find the volume of a cube with a side length $s = 5xyz$.**

Volume $= s^3$ Formula for volume of a cube

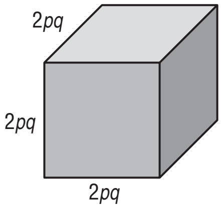
$= (5xyz)^3$ $s = \boxed{}$

$= 5^3x^3y^3z^3$ Power of a Product

$= \boxed{}$ Simplify.

Check Your Progress Express the surface area of the cube as a monomial.

$\boxed{}$



EXAMPLE Simplify Expressions

5 **Simplify $[(8g^3h^4)^2]^2(2gh^5)^4$.**

$[(8g^3h^4)^2]^2(2gh^5)^4$

$= (8g^3h^4)^{\boxed{}}(2gh^5)^4$ Power of a Power

$= (8^4)(g^3)^4(h^4)^4(2)^4g^4(h^5)^4$ Power of a Product

$= 4096g^{\boxed{}}h^{\boxed{}}(16)g^{\boxed{}}h^{\boxed{}}$ Power of a Power

$= 4096(16)g^{12}\cdot g^4\cdot h^{16}\cdot h^{20}$ Commutative Property

$= \boxed{}$ Power of Powers

Check Your Progress Simplify $[(2c^2d^3)^2]^3(3c^5d^2)^3$.

$\boxed{}$

FOLDABLES™

ORGANIZE IT

In your Foldable, in the box for monomial multiplication, write the name of each exponent rule and an example illustrating the rule.

	+	−	×	÷
Mon.				
Poly.				

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–2 Dividing Monomials

MAIN IDEAS

- Simplify expressions involving the quotient of monomials.
- Simplify expressions containing negative exponents.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **A.11** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(A)** Use patterns to generate the laws of exponents and **apply them in problem-solving situations.**

KEY CONCEPTS

Quotient of Powers To divide two powers that have the same base, subtract the exponents.

Power of a Quotient To find the power of a quotient, find the power of the numerator and the power of the denominator.

Zero Exponent Any nonzero number raised to the zero power is 1.

EXAMPLE Quotient of Powers

1 Simplify $\frac{x^7y^{12}}{x^6y^3}$. Assume that no denominator is equal to zero.

$\frac{x^7y^{12}}{x^6y^3} = \left(\frac{x^7}{x^6}\right)\left(\frac{y^{12}}{y^3}\right)$	Group powers that have the same base.
$= (x^{7-6})(y^{12-3})$	Quotient of Powers
$= x^{\square}y^{\square}$	Simplify.

EXAMPLE Power of a Quotient

2 Simplify $\left(\frac{4c^3d^2}{5e^4f^7}\right)^3$. Assume that no denominator is equal to zero.

$\left(\frac{4c^3d^2}{5e^4f^7}\right)^3 = \frac{(4c^3d^2)^3}{(5e^4f^7)^3}$	Power of a
$= \frac{4^3(c^3)^3(d^2)^3}{5^3(e^4)^3(f^7)^3}$	Power of a
$= \frac{64c^{\square}d^{\square}}{125e^{\square}f^{\square}}$	Power of a

Check Your Progress **Simplify each expression. Assume that *a*, *b*, *p* and *q* are not equal to zero.**

a. $\frac{a^3b^9}{ab^2}$

b. $\left(\frac{3m^3n^2}{4p^5q}\right)^3$

EXAMPLE Zero Exponent

3 Simplify $\left(\frac{12m^8n^7}{8m^5n^{10}}\right)^0$. Assume that no denominator is equal to zero.

$\left(\frac{12m^8n^7}{8m^5n^{10}}\right)^0 = 1$ $\quad a^0 = 1$

KEY CONCEPT

Negative Exponent For any nonzero number *a* and any integer *n*, a^{-n} is the reciprocal of a^n. In addition, the reciprocal of a^{-n} is a^n.

FOLDABLES In your Foldable, in the monomial division box, write the name of each exponent rule in the lesson and an example illustrating the rule.

EXAMPLE Negative Exponent

4 **a. Simplify $\frac{x^{-4}y^9}{z^{-6}}$. Assume that no denominator is equal to zero.**

$$\frac{x^{-4}y^9}{z^{-6}} = \left(\frac{x^{-4}}{1}\right)\left(\frac{y^9}{1}\right)\left(\frac{1}{z^{-6}}\right)$$

Write as a product of fractions.

$$= \left(\frac{1}{x^4}\right)\left(\frac{y^9}{1}\right)\left(\frac{z^6}{1}\right)$$

$$= \frac{y^{\boxed{}}z^6}{x^{\boxed{}}}$$

Multiply fractions.

b. Simplify $\frac{75p^3q^{-5}}{15p^5q^{-4}r^{-8}}$. Assume that no denominator is not equal to zero.

$$\frac{75p^3q^{-5}}{15p^5q^{-4}r^{-8}} = \left(\frac{75}{15}\right)\left(\frac{p^3}{p^5}\right)\left(\frac{q^{-5}}{q^{-4}}\right)\left(\frac{1}{r^{-8}}\right)$$

Group powers with the same base.

$$= \boxed{}$$

Quotient of Powers and Negative Exponent Properties

$$= 5p^{\boxed{}}q^{\boxed{}}r^{\boxed{}}$$

Simplify.

$$= 5\left(\frac{1}{p^2}\right)q^{-1}r^8$$

Negative Exponent Property

$$= \boxed{}$$

Multiply fractions.

Check Your Progress **Simplify each expression. Assume that no denominator is equal to zero.**

a. $\left(\frac{3x^2y^9}{5z^{12}}\right)^0$

b. $\frac{x^0k^5}{k^3}$

c. $\frac{a^{-2}b^3}{c^{-5}}$

d. $\frac{36x^5y^8z^2}{9x^4y^2z^6}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–3 Polynomials

MAIN IDEAS

- Find the degree of a polynomial.
- Arrange the terms of a polynomial in ascending or descending order.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **A.11** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(A)** Use patterns to generate the laws of exponents and **apply them in problem-solving situations.**

REVIEW IT

Define like terms.
(Lesson 1-5)

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BUILD YOUR VOCABULARY (pages 150–151)

A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of ______ monomials, and a **trinomial** is the sum of ______ monomials.

The **degree of a monomial** is the ______ of the exponents of all its variables. The **degree of a polynomial** is the greatest ______ of any term in the polynomial.

EXAMPLE Identify Polynomials

① State whether each expression is a polynomial. If it is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.

a. $6 - 4$

Yes, $6 - 4$ is the difference of two real numbers.

It is a ______.

b. $x^2 + 2xy - 7$

Yes, $x^2 + 2xy - 7$ is the sum and difference of three monomials.

It is a ______.

c. $\dfrac{14d + 19c^2}{5d^4}$

No, $\frac{14d}{5d^4}$ and $\frac{19c^2}{5d^4}$ are not monomials.

Check Your Progress **State whether each expression is a polynomial. If it is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.**

a. $3x^2 + 2y + z$ ______

b. $4a^2 - b^{-2}$ ______

c. $8r - 5s$ ______

EXAMPLE Degree of a Polynomial

2 **Find the degree of each polynomial.**

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	$12 + 5b + 6bc + 8bc^2$	12, $5b$, $6bc$, $8bc^2$	0, 1, 2, 3	
b.	$9x^2 - 2x - 4$	$9x^2$, $-2x$, -4		
c.	$14g^2h^5i$			

Check Your Progress **Find the degree of each polynomial.**

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	$11ab + 6b + 2ac^2 - 7$			
b.	$3r^3 + 5r^2s^2 - s^3$			
c.	$2x^5yz - x^2yz^2$			

EXAMPLE Arrange Polynomials in Ascending Order

3 **Arrange the terms of each polynomial so that the powers of x are in ascending order.**

a. $16 + 14x^3 + 2x - x^2$

$= 16x^0 + \square + \square - \square$ $x^0 = \square$

$= \square$

b. $7y^2 + 4x^3 + 2xy^3 - x^2y^2$

$= 7y^2 + 4y^4 + 2x^1y^2 - x^2y^2$ $x = x^1$

$= \square$

EXAMPLE Arrange Polynomials in Descending Order

④ Arrange the terms of each polynomial so that the powers of *x* are in descending order.

a. $\mathbf{8 + 7x^2 - 12xy^3 - 4x^3y}$

$= 8x^0 + 7x^2 - 12x^1y^3 - 4x^3y$ $\quad$ $x^0 = 1$ and $x = x^1$

$=$

b. $\mathbf{a^4 + ax^2 - 2a^3xy^3 - 9x^4y}$

$= a^4x^0 + a^1x^2 - 2a^3x^1y^3 - 9x^4y^1$ $\quad$ $x^0 = 1$ and $x = x^1$

$=$

Check Your Progress **Arrange the terms of each polynomial so that the powers of *x* are in descending order.**

a. $6x^2 - 3x^4 - 2x + 1$

b. $3 - 2xy^4 + 4x^3yz - x^2$

c. $3x^3 + 4x^4 - x^2 + 2$

d. $2y^5 - 7y^3x^2 - 8x^3y^2 - 3x^5$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–4 Adding and Subtracting Polynomials

MAIN IDEAS

- Add polynomials.
- Subtract polynomials.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **(B) Use the commutative, associative**, and distributive **properties to simplify algebraic expressions.** *Also addresses TEKS A.11(A).*

FOLDABLES™

ORGANIZE IT

In your Foldable, write examples that involve adding and subtracting polynomials.

	+	−	×	÷
Mon.				
Poly.				

EXAMPLE Add Polynomials

1 Find $(7y^2 + 2y - 3) + (2 - 4y + 5y^2)$.

METHOD 1 Horizontal
Group like terms together.

$(7y^2 + 2y - 3) + (2 - 4y + 5y^2)$

$= (7y^2 + 5y^2) + \boxed{} + [(-3) + 2]$ Associative and Commutative Properties.

$= \boxed{}$ Add like terms.

METHOD 2 Vertical
Align the like terms in columns and add.

$$\begin{array}{rcccc} & 7y^2 & + \boxed{} & - & 3 \\ (+) & \boxed{} & - \; 4y & + & \boxed{} \\ \hline & \multicolumn{4}{l}{\boxed{}} \end{array}$$

Notice that terms are in descending order with like terms aligned.

EXAMPLE Subtract Polynomials

2 Find $(6y^2 + 8y^4 - 5y) - (9y^4 - 7y + 2y^2)$.

METHOD 1 Horizontal
Subtract $9y^4 - 7y + 2y^2$ by adding its additive inverse.

$(6y^2 + 8y^4 - 5y) - (9y^4 - 7y + 2y^2)$

$= (6y^2 + 8y^4 - 5y) + \boxed{}$

The additive inverse of $9y^4 - 7y + 2y^2$ is $\boxed{}$

$= [8y^4 + (-9y^4)] + \boxed{} + (-5y + 7y)$ Group like terms.

$= \boxed{}$ Add like terms.

METHOD 2 Vertical
Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 6y^2 + 8y^4 - 5y \\ (-) & 2y^2 + 9y^4 - 7y \\ \hline \end{array}$$

Add the opposite.

$$\begin{array}{rrrrrr} & 6y^2 & + & 8y^4 & - & 5y \\ (+) & \square\ 2y^2 & \square & 9y^4 & \square & 7y \\ \hline & & & & & \square \end{array}$$

$\square$ or $\square$

Check Your Progress

a. Find $(3x^2 + 2x - 1) + (-5x^2 + 3x + 4)$.

b. Find $(3x^3 + 2x^2 - x^4) - (x^2 + 5x^3 - 2x^4)$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–5 Multiplying a Polynomial by a Monomial

MAIN IDEAS

- Find the product of a monomial and a polynomial.
- Solve equations involving polynomials.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **(B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.11(A).*

EXAMPLE Multiply a Polynomial by a Monomial

1 Find $6y(4y^2 - 9y - 7)$.

$6y(4y^2 - 9y - 7)$

$= 6y(\square) - 6y(\square) - 6y(\square)$ Distributive Property

$= \square$ Multiply.

Check Your Progress Find $3x(2x^2 + 3x + 5)$.

EXAMPLE Simplify Expressions

2 Simplify $3(2t^2 - 4t - 15) + 6t(5t + 2)$.

$3(2t^2 - 4t - 15) + 6t(5t + 2)$

$= 3(2t^2) - 3(4t) - 3(15) + 6t(5t) + 6t(2)$ Distributive Property

$= \square$ Product of Powers

$= (6t^2 + 30t^2) + [(-12t) + 12t] - 45$ Commutative and Associative Properties

$= \square$ Combine like terms.

Check Your Progress Simplify $5(4y^2 + 5y - 2) + 2y(4y + 3)$.

REVIEW IT

Explain how to write an expression in simplest form. (*Lesson 1-5*)

____

____

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____

EXAMPLE Polynomials on Both Sides

③ Solve $b(12 + b) - 7 = 2b + b(-4 + b)$.

$b(12 + b) - 7 = 2b + b(-4 + b)$	Original equation
$12b + b^2 - 7 =$	Distributive Property
$12b + b^2 - 7 =$	Combine like terms.
$=$	Subtract b^2 from each side.
$12b = -2b + 7$	Add to each side.
$=$	Add to each side.
$b =$	Divide each side by .

Check Your Progress Solve $x(x + 2) + 2x(x - 3) + 7 = 3x(x - 5) - 12$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–6 Multiplying Polynomials

MAIN IDEAS

- Multiply two binomials by using the FOIL method.
- Multiply two polynomials by using the Distributive Property.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **(B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.11(A).*

EXAMPLE The Distributive Property

① Find $(y + 8)(y - 4)$.

$(y + 8)(y - 4) = y(y - 4) + 8(y - 4)$ Distributive Property

$= \boxed{}$ Distributive Property

$= \boxed{}$ Multiply.

$= \boxed{}$ Combine like terms.

Check Your Progress Find $(c + 2)(c - 4)$.

$\boxed{}$

BUILD YOUR VOCABULARY (pages 150–151)

The shortcut of the Distributive Property is called the **FOIL method**, which can be used when multiplying two binomials.

KEY CONCEPT

FOIL Method for Multiplying Binomials

To multiply two binomials, find the sum of the products of

F the *First* terms,

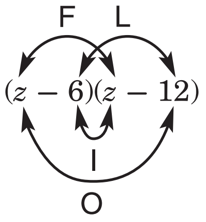
O the *Outer* terms,

I the *Inner* terms, and

L the *Last* terms.

EXAMPLE FOIL Method

② a. Find $(z - 6)(z - 12)$.



F O I L

$= \boxed{} + \boxed{} + \boxed{}\,z + \boxed{}$

$= \boxed{}$ Multiply.

$= z^2 - \boxed{}\,z + 72$ Combine like terms.

REMEMBER IT

When multiplying binomials, you can check your answer by reworking the problem using the Distributive Property.

b. Find $(5x - 4)(2x + 8)$.

$(5x - 4)(2x + 8)$

F O I L

$=$ + + +

$=$ Multiply

$=$ Combine like terms.

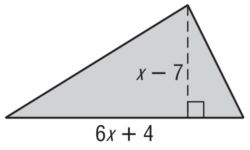
Check Your Progress **Find each product.**

a. $(x + 2)(x - 3)$

b. $(3x + 5)(2x - 6)$

EXAMPLE Foil Method

3 GEOMETRY **The area A of a triangle is one-half the height h times the base b. Write an expression for the area of the triangle.**



The height is $x - 7$ and the base is $6x + 4$. Write and apply the formula.

$A = \frac{1}{2}hb$ Original formula

$A = \frac{1}{2}$ Substitution

$A = \frac{1}{2}[x(6x) + x(4) - 7(6x) - 7(4)]$ FOIL method

$A = \frac{1}{2}$ Multiply.

$A = \frac{1}{2}$ Combine like terms.

$A =$ Distributive Property

The area of the triangle is square units.

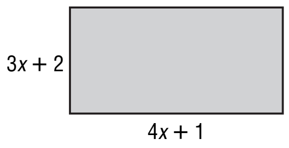
FOLDABLES™

ORGANIZE IT

In your Foldable, in the box for polynomial multiplication, write examples of multiplying binomials using the Distributive Property and the FOIL method.

	+	−	×	÷
Mon.				
Poly.				

Check Your Progress The area of a rectangle is the measure of the base times the height. Write an expression for the area of the rectangle.



EXAMPLE The Distributive Property

4 **Find $(3a + 4)(a^2 - 12a + 1)$.**

$(3a + 4)(a^2 - 12a + 1)$

$= 3a \qquad + 4$	Distributive Property
$= 3a^3 - \qquad + $	Distributive Property
$=$	Combine like terms.

Check Your Progress **Find each product.**

a. $(3z + 2)(4z^2 + 3z + 5)$

b. $(3x^2 + 2x + 1)(4x^2 - 3x - 2)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–7 Special Products

MAIN IDEAS

- Find squares of sums and differences.
- Find the product of a sum and a difference.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, **simplify polynomial expressions**, transform and solve equations, and factor as necessary in problem situations. **(B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.11(A).*

KEY CONCEPTS

Square of a Sum The square of $a + b$ is the square of a plus twice the product of a and b plus the square of b.

Square of a Difference The square of $a - b$ is the square of a minus twice the product of a and b plus the square of b.

EXAMPLE Square of a Sum

① **Find each product.**

a. $(7z + 2)^2$

$(a + b)^2 =$	Square of a Sum
$(7z + 2)^2 =$	$a = 7z$ and $b = 2$
$=$	Simplify.

EXAMPLE Square of a Difference

② **Find each product.**

a. $(3c - 4)^2$

$(a - b)^2 =$	Square of a Difference
$(3c - 4)^2 = \quad - \quad +$	$a = 3c$ and $b = 4$
$=$	Simplify.

Check Your Progress **Find each sum.**

a. $(3x + 2)^2$

b. $(4x + 2y)^2$

c. $(3m - 2)^2$

d. $(2p - 2q)^2$

EXAMPLE Product of a Sum and a Difference

3 Find each product.

a. (9*d* − 4)(9*d* + 4)

$(a + b)(a - b) = a^2 - b^2$	Product of a Sum and a Difference
$(9d - 4)(9d + 4) =$	$a = 9d$ and $b = 4$
$=$	Simplify.

b. (10*g* + 13*h*³)(10*g* − 13*h*³)

$(a + b)(a - b) = a^2 - b^2$	Product of a Sum and a Difference
$(10g + 13h^3)(10g - 13h^3) = (10g)^2 - (13h^3)^2$	$a = 10g$ and $b = 13h^3$
$=$	Simplify.

KEY CONCEPTS

Product of a Sum and a Difference The product of $a + b$ and $a - b$ is the square of a minus the square of b.

Special Products
Square of a Sum
$(a + b)^2 = a^2 + 2ab + b^2$

Square of a Difference
$(a + b)^2 = a^2 + 2ab + b^2$

Product of a Sum and a Difference
$(a - b)(a + b) = a^2 - b^2$

Check Your Progress **Find each product.**

e. $(3y + 2)(3y - 2)$

f. $(4y^2 + 5z)(4y^2 - 5z)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 7

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 7 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 7, go to: glencoe.com	You can use your completed **Vocabulary Builder** (*pages 150–151*) to help you solve the puzzle.

7-1

Multiplying Monomials

Simplify.

1. $3^5 \cdot 3^2$

2. $(a^3)^4$

3. $(-4xy)^5$

4. $y(y^3)(y^5)$

5. $(3c^2d^5)(cd^2)$

6. $(3m^5n^3)^2$

7-2

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

7. $\frac{12x^5}{36x}$

8. $\frac{y^4}{y^{-8}}$

9. $\frac{-5w^2}{25w^7}$

10. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$

11. $\left(\frac{x^{-5}y^4}{5^{-2}}\right)$

12. $\frac{(3q)^3}{q^4}$

7-3

Polynomials

13. Complete the table.

	monomial	binomial	trinomial	polynomial with more than three terms
Example	$3r^2t$	$2x^2 + 3x$	$5x^2 + 3x + 2$	$7s^2 + s^4 + 2s^3 - s + 5$
Number of Terms				

14. What is the degree of the polynomial $4x^4 + 2x^3y^3 + y^2 + 14$? Explain how you found your answer.

15. Use a dictionary to find the meaning of the terms *ascending* and *descending*. Write their meanings and then describe a situation in your everyday life that relates to them.

7-4

Adding and Subtracting Polynomials

Find each sum or difference.

16. $(3k - 8) + (7k + 12)$

17. $(w^2 + w - 4) + (7w^2 - 4w + 8)$

18. $(7h^2 + 4h - 8) - (3h^2 - 2h + 10)$

19. $(17n^4 + 2n^3) - (10n^4 + n^3)$

7-5

Multiplying a Polynomial by a Monomial

Find each product.

20. $2y^2(3y^2 + 2y - 7)$

21. $-3x^3(x^3 - 2x^2 + 3)$

22. Let n be an integer. What is the product of five times the integer added to two times the next consecutive integer?

7-6

Multiplying Polynomials

Find each product.

23. $(x + 5)(x - 3)$

24. $(3y + 6)(y - 2)$

25. $(7x - 4)(7x + 4)$

26. $(3x - 4)(2x^2 + 3x + 3)$

7-7

Special Products

Find each product. Then identify the special product.

27. $(x - 4)^2$

28. $(x + 11)(x - 11)$

CHAPTER 7

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 7.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 7 Practice Test on page 409 of your textbook as a final check.

☐ **I used my Foldables or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 7 Study Guide and Review on pages 404–408 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may want to take the Chapter 7 Practice Test on page 409.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 7 Foldable.
- Then complete the Chapter 7 Study Guide and Review on pages 404–408 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 409.

Student Signature

Parent/Guardian Signature

Teacher Signature

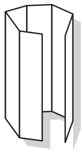
CHAPTER 8

Factoring

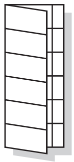
Use the instructions below to make a Foldable to help you organize your note as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

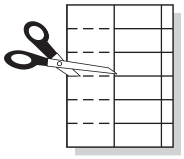
STEP 1 **Fold** in thirds and then in half along the width.



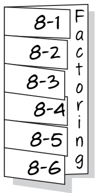
STEP 2 **Open.** Fold lengthwise, leave a $\frac{1}{2}$" tab on the right.



STEP 3 **Open.** Cut short side along folds to make tabs.



STEP 4 **Label** each tab as shown.



NOTE-TAKING TIP: As soon as possible, go over your notes. Clarify any ideas that were not complete.

CHAPTER

8

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
composite number [kahm·PAH·zeht]			
factored form			
factoring			
factoring by grouping			
greatest common factor (GCF)			

Vocabulary Term	Found on Page	Definition	Description or Example
perfect square $\underbrace{\text{trinomial}}_{\text{try·NOH·mee·uhl}}$			
prime $\underbrace{\text{factorization}}_{\text{FAK·tuh·ruh·ZAY·shuhn}}$			
prime number			
prime polynomial			
roots			
Zero Products Property			

8–1 Monomials and Factoring

MAIN IDEAS

- Find prime factorizations of integers and monomials.
- Find the greatest common factors of integers and monomials.

KEY CONCEPT

Prime and Composite Numbers A whole number greater than 1 whose only factors are 1 and itself is called a **prime number.**

A whole number, greater than 1 that has more than two factors is called a **composite number.**

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and solve equations, and **factor as necessary in problem situations.**

BUILD YOUR VOCABULARY (pages 174–175)

When a whole number is expressed as a product of that are all numbers, the expression is called the **prime factorization** of the number.

A monomial is in **factored form** when it is expressed as the product of numbers and and no variable has an exponent greater than 1.

EXAMPLE Prime Factorization of a Monomial

① **Factor $18x^3y^3$ completely.**

$18x^3y^3 = 2 \cdot 9 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$ $18 = 2 \cdot 9$, $x^3 = x \cdot x \cdot x$, and $y^3 = y \cdot y \cdot y$

$= 2 \cdot \quad x \cdot x \cdot x \cdot y \cdot y \cdot y$ $9 =$

in factored form is $2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$.

Check Your Progress **Factor each monomial completely.**

a. $15a^3b^2$

b. $-45xy^2$

KEY CONCEPT

Greatest Common Factor (GCF)

- The GCF of two or more monomials is the product of their common factors when each monomial is in factored form.
- If two or more integers or monomials have a GCF of 1, then the integers or monomials are said to be *relatively prime*.

BUILD YOUR VOCABULARY (pages 174–175)

The **greatest common factor** ( ) of two or more integers is the product of the common to the integers.

EXAMPLE Finding GCF

② **GEOMETRY The sides of a trianlge are $12wz^2$, $8wz$, and $16w^2z$. Find the GCF of the three sides.**

Find the factors of $12wz^2$, $8wz$, and $16w^2z$.

The factors of $12wz^2$, are .

The factors of $8wz$ are .

The factors of $16w^2z$ are .

So, the GCF is .

EXAMPLE GCF of a set of Monomials

③ **Find the GCF of $27a^2b$ and $15ab^2c$.**

$27a^2b = \textcircled{3} \cdot 3 \cdot 3 \cdot \textcircled{a} \cdot a \cdot \textcircled{b}$ Factor each number.

$15ab^2c = \textcircled{3} \cdot 5 \cdot \textcircled{a} \cdot \textcircled{b} \cdot b \cdot c$ Circle the common prime factors.

The GCF of $27a^2b$ and $15ab^2c$ is .

Check Your Progress **Find the GCF of each set of monomials.**

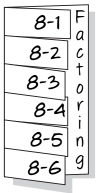
a. 15 and 35

b. $39x^2y^3$ and $26xy^4$

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 8–1, write a monomial that can be factored. Then factor the monomial.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–2

Factoring Using the Distributive Property

MAIN IDEAS

- Factor polynomials by using the Distributive Property.
- Solve quadratic equations of the form $ax^2 + bx = 0$.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and **solve equations, and factor as necessary in problem situations. (B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.10(A).*

KEY CONCEPT

Factoring by Grouping A polynomial can be factored by grouping if all the following situations exist.

- There are four or more terms.
- Terms with common factors can be grouped together.
- The two common factors are identical or are additive inverses of each other.

FOLDABLES Under the tab for Lesson 8–2, list the steps to factor a polynomial grouping.

BUILD YOUR VOCABULARY (pages 174–175)

Factoring a polynomial means to find its $\boxed{}$ factored form. The $\boxed{}$ Property can also be used to factor some polynomials having $\boxed{}$ or more terms. This method is called **factoring by grouping.**

EXAMPLE Use the Distributive Property

① Use the Distributive Property to factor $15x + 25x^2$.

First, find the GCF of $15x$ and $25x^2$.

$15x = 3 \cdot \textcircled{5} \cdot \textcircled{x}$ Factor each number.

$25x^2 = \textcircled{5} \cdot 5 \cdot \textcircled{x} \cdot x$ Circle the common prime factors.

GCF: $5 \cdot x$ or $\boxed{}$

Then use the Distributive Property to factor out the GCF.

$15x + 25x^2 = 5x(3) + 5x(5 \cdot x)$ Rewrite each term using the GCF.

$= 5x\left(\boxed{}\right) + 5x\left(\boxed{}\right)$ Simplify remaining factors.

$= \boxed{}(3 + 5x)$ Distributive Property

EXAMPLE Use Grouping

① Factor $2xy + 7x - 2y - 7$.

$2xy + 7x - 2y - 7$

$= \boxed{} + 7x - 7$ Group terms with common factors.

$= 2y\boxed{} + 7\boxed{}$ Factor the GCF from each grouping.

$= (x - 1)\boxed{}$ Distributive Property

EXAMPLE Use the Additive Inverse Property

③ $15a - 3ab + 4b - 20$

$15a - 3ab + 4b - 20$

$= \boxed{} + \boxed{}$	Group terms with common factors.
$= \boxed{}(5 - b) + \boxed{}(b - 5)$	Factor GCF from each grouping.
$= 3a(-1)\boxed{} + 4\boxed{}$	$(5 - b) = -1(b - 5)$
$= \boxed{}(b - 5) + \boxed{}(b - 5)$	$3a(-1) = \boxed{}$
$= (b - 5)\boxed{}$	Distributive Property

Check Your Progress **Factor each polynomial.**

a. $3x^2y + 12xy^2$ $\boxed{}$

b. $6ab^2 + 15a^2b^2 + 27ab^3$ $\boxed{}$

EXAMPLE Solve an Equation

④ $4y = 12y^2$

Write the equation so that it is of the form $ab = 0$.

$4y = 12y^2$	Original equation
$4y - \boxed{} = 0$	Subtract $\boxed{}$ from each side.
$4y\boxed{} = 0$	Factor the GCF of $4y$ and $12y^2$, which is $4y$.
$4y = 0$ or $1 - 3y = 0$	Zero Product Property
$y = \boxed{}$ $y = \boxed{}$	Solve each equation.

The solution set is $\boxed{}$.

Check Your Progress **Solve each equation. Then check the solutions.**

a. $(s - 3)(3s + 6) = 0$ $\boxed{}$

b. $5x - 40x^2 = 0$ $\boxed{}$

KEY CONCEPT

Zero Product Property If the product of two factors is 0, then at least one of the factors must be 0.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–3 Factoring Trinomials: $x^2 + bx + c$

MAIN IDEAS

- Factor trinomials of the form $x^2 + bx + c$.
- Solve equations of the form $x^2 + bx + c = 0$.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and **solve equations, and factor as necessary in problem situations. (B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.10(A).*

KEY CONCEPT

Factoring $x^2 + bx + c$
To factor quadratic trinomials of the form $x + bx + c$, find two integers, m and n, whose sum is equal to b and whose product is equal to c. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

FOLDABLES Take notes explaining how to factor trinomials in the form $x^2 + bx + c$. Include examples.

EXAMPLE *b* is Negative and *c* is Negative

① **Factor $x^2 - 12x + 27$.**

In this trinomial, $b =$ and c = . This means $m + n$ is negative and mn is positive. So m and n must both be negative. Make a list of the negative factors of , and look for the pair whose sum is .

Factors of 27	Sum of Factors
−1, −27	
−3, −9	

The correct factors are and .

$x^2 - 12x + 27 = (x + m)(x + n)$ Write the pattern

$=$ $m =$ and $n =$

EXAMPLE *c* is Negative

② **a. Factor $x^2 + 3x - 18$.**

In this trinomial, $b = 3$ and $c = -18$. This means $m + n$ is positive and mn is negative, so either m or n is negative, but not both. Make a list of the factors of −18. Look for the pair of factors whose sum is 3.

Factors of 27		Sum of Factors
1,		−17
−1,		17
2,		−7
−2,	9	
3,	−6	
−3,	6	

The correct factors are and .

$x^2 + 3x - 18 = (x + m)(x + n)$ Write the pattern.

$=$ $m =$ and $n =$

REMEMBER IT

Before factoring, rewrite the equation so that one side equals 0.

b. Factor $x^2 - x - 20$.

Since $b =$ and $c =$, $m + n$ is negative and mn is negative. So either m or n is negative, but not both.

Factors of –20	Sum of Factors
1, –20	
–1, 20	
2,	–8
, 10	8
, –5	–1

The correct factors are 4 and –5.

$x^2 - x - 20 = (x + m)(x + n)$ Write the pattern.

$=$ $m = 4$ and $n = -5$

Check Your Progress **Factor each trinomial.**

a. $x^2 + 3x + 2$

b. $x^2 - 10x + 16$

c. $x^2 + 4x - 5$

d. $x^2 - 5x - 24$

EXAMPLE Solve an Equation by Factoring

3 Solve $x^2 + 2x - 15 = 0$. Check your solutions.

$x^2 + 2x = 15$ Original equation

$= 0$ Subtract 15 from each side.

$= 0$ Factor.

$= 0$ or $= 0$ Zero Product Property

$x =$ $x =$ Solve each equation.

The solution is .

Check Your Progress Solve $x^2 - 20 = x$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–4 Factoring Trinomials: $ax^2 + bx + c$

MAIN IDEAS

- Factor trinomials of the form $ax^2 + bx + c$.
- Solve equations of the form $ax^2 + bx + c = 0$.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and **solve equations, and factor as necessary in problem situations. (B) Use the** commutative, associative, and **distributive properties to simplify algebraic expressions.** *Also addresses TEKS A.10(A).*

EXAMPLE Factor $ax^2 + bx + c$

① **a. Factor $5x^2 + 27x + 10$.**

In this trinomial, $a =$ ____, $b =$ ____, and $c =$ ____. Find two numbers whose sum is 27 and whose product is 5 • 10 or 50. Make a list of factors of 50 and look for a pair of factors whose sum is 27.

Factors of 50	Sum of Factors
1, 50	____
2, 25	____

The correct factors are 2 and 25.

$5x^2 + 27x + 10 = 5x^2 + mx + nx + 10$ Write the pattern.

$= (5x^2 + 2x) + (25x + 10)$ $m =$ ____ and $n =$ ____

$=$ ____ $+$ ____ Group terms with common factors

$=$ ____$(5x + 2) +$ ____$(5x + 2)$ Factor the GCF from each grouping

$=$ ____ Distributive Property

b. Factor $24x^2 - 22x + 3$.

In this trinomial, $a =$ ____, $b =$ ____, and $c =$ ____.

Since b is negative, $m + n$ is negative. Since c is positive, mn is positive. So m and n must both be negative. Make a list of the negative factors of 24 • 3 or 72, and look for the pair of factors whose sum is −22.

Factors of −20	Sum of Factors
−1, ____	−73
−2, ____	−38
____, −24	−27
____, −18	−22

The correct factors are −4, −18

REVIEW IT

Which property is applied when factoring by grouping? *(Lesson 8-2)*

____

____

____

____

____

$24x^2 - 22x + 3 = 24^2 + mx + nx + 3$ Write the pattern

$= \boxed{}$ $m = \boxed{}$ and $n = \boxed{}$

$= \left(\boxed{}\right) + (-18x + 3)$ Group terms with common factors.

$= \boxed{}(6x - 1) + \left(\boxed{}\right)(6x - 1)$ Factor the GCF from each grouping.

$= \boxed{}$ Distributive Property

c. Factor $4x^2 + 24x + 32$.

Notice that the GCF of the terms $4x^2$, $24x$, and 32 is $\boxed{}$. When the GCF of the terms of a trinomial is an integer other than 1, you should first factor out this GCF.

$4x^2 + 24x + 32 = \boxed{}(x^2 + 6x + 8)$ Distributive Property

Now factor $x^2 + 6x + 8$. Since the lead coefficient is 1, find the two factors of 8 whose sum is 6.

Factors of 8	Sum of Factors
$\boxed{}$	9
$\boxed{}$	6

The correct factors are $\boxed{}$ and $\boxed{}$.

So, $x^2 + 6x + 4 = (x + 2)(x + 4)$. Thus, the complete factorization of $4x^2 + 24x + 32$ is $\boxed{}$.

WRITE IT

Before trying to factor a trinomial, what should you check for?

____

____

____

____

Check Your Progress **Factor each trinomial.**

a. $3x^2 + 26x + 35$

$\boxed{}$

b. $3x^2 - 17x + 10$

$\boxed{}$

BUILD YOUR VOCABULARY (pages 174–175)

A polynomial that cannot be written as a product of two polynomials with $\boxed{}$ coefficients is called a **prime polynomial.**

EXAMPLE Determine Whether a Polynomial is Prime

② **Factor $3x^2 + 7x - 5$.**

In this trinomial, $a = 3$, $b = 7$, and $c = -5$. Since b is positive, $m + n$ is positive. Since c is negative, mn is negative, so either m or n is negative, but not both. Make a list of all the factors of $3(-5)$ or -15. Look for the pair of factors whose sum is 7.

Factors of –15	Sum of Factors	Factors of –15	Sum of Factors
$-1, 15$		$-3, 5$	
$1, -15$		$3, -5$	

There are no integral factors whose sum is 7. Therefore,

$3x^2 + 7x - 5$ is a .

Check Your Progress **Factor each trinomial. If the trinomial cannot be factored using integers, write *prime*.**

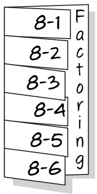
a. $2x^2 + 14x + 20$

b. $3x^2 - 5x + 3$

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 8-4, list the steps you use to solve equations by factoring.



EXAMPLE Solve Equations by Factoring

③ **Solve $18b^2 - 19b - 8 = 3b^2 - 5b$.**

$18b^2 - 19b - 8 = 3b^2 - 5b$ Original equation

$= 0$ Rewrite so one side equals 0.

$(3b - 4) = 0$ Factor the left side.

$= 0$ or $= 0$ Zero Product Property

$5b =$ $3b =$ Solve each equation.

$b =$ $b =$

The solution set is .

Check Your Progress Solve $12x^2 + 19x + 5 = 0$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–5 Factoring Differences of Squares

MAIN IDEAS

- Factor binomials that are the difference of squares.
- Solve equations involving the differences of squares.

KEY CONCEPT

Difference of Squares
$a^2 - b^2 = (a + b)(a - b)$
or $(a - b)(a + b)$

FOLDABLES™ Under the tab for Lesson 8-5, write a binomial that is the difference of squares. Then factor the binomial.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and **solve equations, and factor as necessary in problem situations. A.10** The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. **(A) Solve quadratic equations using** concrete models, tables, graphs, and **algebraic methods.**

EXAMPLE Factor the Difference of Squares

① Factor each binomial.

a. $16y^2 - 81z^2$

Write in the form $a^2 - b^2$.

$16y^2 - 81z^2 = \boxed{}^2 - \boxed{}^2$ $\quad 16y^2 = 4y \cdot 4y$ and $81z^2 = 9z \cdot 9z$

$= \boxed{}\boxed{}$ $\quad$ Factor the difference of squares.

b. $3b^2 - 27b$

$3b^2 - 27b = 3b(b^2 - 9)$ $\quad$ The GCF is $3b$.

$= 3b(b^2 - 3^2)$ $\quad$ $b^2 = b \cdot b$ and $9 = 3 \cdot 3$.

$= \boxed{}$ $\quad$ Factor the difference of squares.

EXAMPLE Apply a Factoring Technique More Than Once

② Factor $y^4 - 625$.

$y^4 - 625 = \left(\boxed{}\right) - \boxed{}$ $\quad y^4 = y^2 \cdot y^2$ and $625 = 25 \cdot 25$

$= \boxed{}\boxed{}$ $\quad$ Factor the difference of squares.

$= (y^2 + 25)(y^2 - 5^2)$ $\quad y^2 = y \cdot y$ and $25 = 5 \cdot 5$

$= \boxed{}$ $\quad$ Factor the difference of squares.

Check Your Progress **Factor each polynomial.**

a. $b^2 - 9$

$\boxed{}$

b. $36k^2 - 144m^2$

$\boxed{}$

c. $5x^3 - 20x$

$\boxed{}$

d. $3y^4 - 48$

$\boxed{}$

EXAMPLE **Apply Several Different Factoring Techniques**

3 **Factor $6x^3 + 30x^2 - 24x - 120$.**

$6x^3 + 30x^2 - 24x - 120$ Original polynomial

$= \boxed{\quad}(x^3 + 5x^2 - 4x - 20)$ Factor out the GCF.

$= \boxed{\quad}\,\boxed{\qquad\qquad} + (5x^2 - 20)$ Group terms with common factors.

$= \boxed{\quad}\left[\boxed{\qquad\qquad} + \boxed{\qquad\qquad}\right]$ Factor each grouping.

$= 6(x^2 - 4)(x - 5)$ $x^2 - 4$ is the common factor.

$= \boxed{\qquad\qquad\qquad\qquad}$ Factor the difference of squares.

Check Your Progress Factor $5x^3 + 25x^2 - 45x - 225$.

$\boxed{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

EXAMPLE

4 **TEST EXAMPLE** **In the equation $q^2 - \frac{4}{25} = y$, which is a value of q when $y = 0$?**

A $\frac{2}{25}$ **B** $\frac{4}{25}$ **C** 0 **D** $\frac{-2}{5}$

$q^2 - \frac{4}{25} = \boxed{\quad}$ Replace y wtih $\boxed{\quad}$.

$q^2 - \left(\frac{2}{5}\right)^2 = 0$ $q^2 = q \cdot q$ and $\frac{4}{25} = \frac{2}{5} \cdot \frac{2}{5}$

$\left(q + \boxed{\quad}\right)\left(q - \boxed{\quad}\right) = 0$ Factor the difference of squares.

$q + \frac{2}{5} = 0$ or $q - \frac{2}{5} = 0$ Zero Product Property

$q = \boxed{\quad}$ $q = \boxed{\quad}$ Solve each equation.

The roots are $\boxed{\quad}$ and $\boxed{\quad}$. The correct answer is $\boxed{\quad}$.

Check Your Progress In the equation $m^2 - 81 = y$, which is a value of m when $y = 0$?

A 0 **B** $\frac{1}{9}$ **C** –9 **D** 81

$\boxed{\quad}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–6 Perfect Squares and Factoring

MAIN IDEAS

- Factor perfect square trinomials.
- Solve equations involving perfect squares.

BUILD YOUR VOCABULARY (pages 174–175)

Perfect Square Trinomials are trinomials that are the ____ of a ____.

KEY CONCEPT

Factoring Perfect Square Trinomials If a trinomial can be written in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, then it can be factored as $(a + b)^2$ or as $(a - b)^2$, respectively.

EXAMPLE Factor Perfect Square Trinomials

① **Determine whether each trinomial is a perfect square trinomial. If so, factor it.**

a. $25x^2 - 30x + 9$

1. Is the first term a perfect square? ____, $25x^2 = (5x)^2$.
2. Is the last term a perfect square? ____, $9 = 3^2$.
3. Is the middle term equal to ____ $(5x)(3)$? Yes, ____.

$25x^2 - 30x + 9$ ____ a perfect square trinomial.

$25x^2 - 30x + 9 = (5x)^2 - 2(5x)(3) + 3^2$ Write as $a^2 - 2ab + b^2$

= ____ Factor using the pattern.

b. $49y^2 + 42y + 36$

1. Is the first term a perfect square? ____, $49y^2 = (7y)^2$.
2. Is the last term a perfect square? ____, $36 = 6^2$
3. Is the middle term equal to $2(7y)(6)$? ____, $42y \neq 2(7y)(6)$

$49y^2 + 42y + 36$ ____ a perfect square trinomial.

TEKS A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A)** Find specific function values, simplify polynomial expressions, transform and **solve equations, and factor as necessary in problem situations.** **A.10** The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. **(A) Solve quadratic equations using concrete models, tables, graphs, and algebraic methods.**

Check Your Progress **Determine whether each trinomial is a perfect square trinomial. If so, factor it.**

a. $9x^2 - 12x + 16$ $\boxed{}$

b. $16x^2 + 16x + 4$ $\boxed{}$

EXAMPLE **Factor Completely**

② **Factor each polynomial.**

a. $6x^2 - 96$

First check for a GCF. Then, since the polynomial has two terms, check for the difference of squares.

$6x^2 - 96 = 6\boxed{}$ 6 is the GCF.

$= 6\boxed{}$ $x^2 = x \cdot x$ and $16 = 4 \cdot 4$

$= \boxed{}$ Factor the difference of squares.

b. $16y^2 + 8y - 15$

This polynomial has three terms that have a GCF of 1.

While the first term is a perfect square, $16y^2 = \boxed{}$, the last term is not. Therefore, this is not a perfect square trinomial.

This trinomial is in the form $ax^2 + bx + c$. Are there two numbers m and n whose product is $16 \cdot -15$ or -240 and whose sum is 8? Yes, the product of $\boxed{}$ and $\boxed{}$ is -240 and their sum is 8.

$16y^2 + 8y - 15 = 16y^2 + mx + nx - 15$ Write the pattern.

$= 16y^2\boxed{} - 15$

$= \boxed{} + \boxed{}$

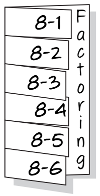
$= \boxed{}(4y + 5) - \boxed{}(4y + 5)$

$= (4y + 5)\boxed{}$

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 8-6, use your own words to explain why equations with perfect square trinomials only have one solution.



Check Your Progress **Factor each polynomial.**

a. $3x^2 - 3$

b. $4x^2 + 10x + 6$

EXAMPLE Solve Equations with Repeated Factors

3 **Solve $4x^2 + 36x + 81 = 0$.**

$4x^2 + 36x + 81 = 0$ Original equation

$\left(\boxed{}\right)^2 + 2\boxed{} + \boxed{}^2 = 0$ Recognize $4x^2 + 36x + 81$ as a perfect square trinomial.

$\boxed{} = 0$ Factor the perfect square trinomial.

$\boxed{} = 0$ Set the repeated factor equal to zero.

$x = \boxed{}$ Solve for x.

Thus, the solution set is $\boxed{}$.

Check Your Progress Solve $9x^2 - 30x + 25 = 0$.

BUILD YOUR VOCABULARY (pages 174–175)

The **square root property** states that for any number $n > 0$, if $x^2 = n$, then $x = \boxed{}\sqrt{n}$.

EXAMPLE Use the Square Root Property To Solve Equations

④ **a. Solve $(b - 7)^2 = 36$.**

$(b - 7)^2 = 36$ Original equation

$b - 7 =$ Square Root Property

$b - 7 = \pm 6$ $36 =$

$b = 7 \pm 6$ Add to each side.

$b = 7 + 6$ or $b = 7 - 6$ Separate into two equations.

$=$ $=$ Simplify.

The roots are and . Check each solution in the original equation.

b. Solve $(x + 9)^2 = 8$.

$(x + 9)^2 = 8$ Original equation

$=$ Square Root Property

$x =$ Subtract 9 from each side.

Since 8 is not a perfect square, the solution set is . Using a calculator, the approximate solutions are $-9 + \sqrt{8}$ or about and $-9 - \sqrt{8}$ or about .

Check Your Progress **Solve each equation. Check your solutions.**

a. $(x - 4)^2 = 25$

b. $y^2 + 6y + 9 = 64$

c. $(x - 5)^2 = 15$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 8

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	Build your Vocabulary
Use your **Chapter 8 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 8, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 174–175)* to help you solve the puzzle.

8-1

Factors and Greatest Common Factors

Choose the letter of the term that best matches each phrase.

1. the number 14

2. a monomial that is expressed as the product of prime numbers and variables, with no variable having an exponent greater than 1

3. the number 5

a. composite number
b. prime number
c. factored form

Find the GCF of each set of monomials.

4. 12, 30, 114

5. $6a^2$, $8a$

6. $24xy^5$, $56x^3y$

8-2

Factoring Using the Distributive Property

7. Complete.

$d^2 = -2d$ Original equation

$d^2 + 2d = 0$ 2*d* to each side.

$(d + 2) = 0$ Factor the GCF.

$d =$ or $d + 2 = 0$ Product Property

$d =$ Solve each equation.

The solution set is

8-3

Factoring Trinomials: $x^2 + bx + c$

Tell what sum and product m and n must have in order to use the pattern $(x + m)(x + n)$ to factor the given trinomial.

8. $x^2 + 10x + 24$ sum: ☐ product: ☐

9. $x^2 - 12x + 20$ sum: ☐ product: ☐

10. $x^2 - 4x - 21$ sum: ☐ product: ☐

11. $x^2 + 6x - 16$ sum: ☐ product: ☐

12. Find two consecutive even integers whose product is 168.

8-4

Factoring Trinomials: $ax^2 + bx + c$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

13. $2b^2 + 10b + 12$

14. $4y^2 + 4y - 3$

15. $12x^2 - 4y - 5$

16. $10x^2 - 9x + 6$

17. Explain how you know that the trinomial $2x^2 - 7x + 4$ is a prime polynomial.

8-5

Factoring Differences of Squares

Factor each polynomial.

18. $4x^2 - 25$

19. $49a^2 - 64b^2$

20. Explain what is done in each step to factor $4x^4 - 64$.

$4x^4 - 64$

$= 4(x^4 - 16)$

$= 4[(x^2)^2 - 4^2)]$

$= 4(x^2 + 4)(x^2 - 4)$

$= 4(x^2 + 4)(x^2 - 2^2)$

$= 4(x^2 + 4)(x + 2)(x - 2)$

8-6

Perfect Squares and Factoring

Match each polynomial from the first column with a factoring technique in the second column. Some of the techniqes may be used more than once. If none of the techniques can be used to factor the polynomial, write *none*.

21. $9x^2 - 64$ ☐ **a.** factor as $x^2 + bx + c$

22. $9x^2 + 12x + 4$ ☐ **b.** factor as $ax^2 + bx + c$

23. $x^2 - 5x + 6$ ☐ **c.** difference of squares

24. $4x^2 + 13x + 9$ ☐ **d.** perfect square trinomial

25. The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. If increasing the radius of a circle by 3 inches gives the resulting circle an area of 81π square inches, what is the radius of the original circle?

CHAPTER

8

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 8.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want take the Chapter 8 Practice Test on page 459 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 8 Study Guide and Review on pages 455–458 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 459.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 8 Foldable.
- Then complete the Chapter 8 Study Guide and Review on pages 455–458 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 459.

Student Signature

Parent/Guardian Signature

Teacher Signature

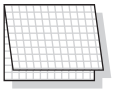
CHAPTER 9

Quadratic and Exponential Functions

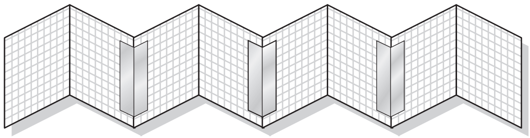
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with ten sheets of notebook paper.

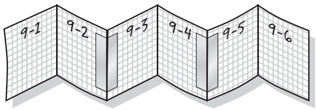
STEP 1 **Fold** each sheet in half along the width.



STEP 2 **Unfold** each sheet and tape to form one long piece.



STEP 3 **Label** each page with the lesson number as shown. Refold to form a booklet.



NOTE-TAKING TIP: If you find it difficult to write and pay attention at the same time, ask your instructor if you may record the classes with a tape recorder.

CHAPTER 9

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
axis of symmetry [SIH·muh·tree]			
completing the square			
compound interest			
discriminant			
double root			
exponential decay [EHK·spuh·NEHN·chuhl]			
exponential function			
exponential growth			

Vocabulary Term	Found on Page	Definition	Description or Example
maximum			
minimum			
parabola [puh·RA·buh·lh]			
quadratic equation [kwah·dra·tihk]			
Quadratic Formula			
quadratic function			
roots			
symmetry			
vertex			
zeros			

9–1 Graphing Quadratic Functions

MAIN IDEAS

- Graph quadratic functions.
- Find the equation of the axis of symmetry and the coordinates of the vertex of a parabola.

BUILD YOUR VOCABULARY (pages 196–197)

The graph of a ______ function is called a **parabola**.

When graphing a parabola the ______ point is called the **minimum** and the ______ point is called the **maximum**.

The ______ or ______ point of a parabola is called the **vertex**.

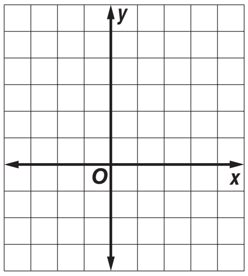
TEKS A.2 The student uses the properties and attributes of functions. **(A) Identify and sketch the general forms of** linear ($y = x$) and **quadratic ($y = x^2$) parent functions. A.9** The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions. **(A)** Determine the domain and range for quadratic functions in given situations. **(D) Analyze graphs of quadratic functions and draw conclusions.**

EXAMPLE Graph Opens Upward

① **Use a table of values to graph $y = x^2 - x - 2$.**

Graph these ordered pairs and connect them with a smooth curve.

x	y
−2	
−1	0
0	
1	
2	0
3	

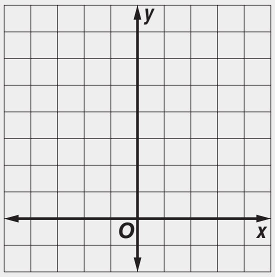


KEY CONCEPT

Quadratic Function A **quadratic function** can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.

FOLDABLES On the page for Lesson 9-1, write an example of a quadratic function that opens upward. Then write an example of a quadratic function that opens downward.

Check Your Progress Use a table of values to graph $y = x^2 + 2x + 3$.



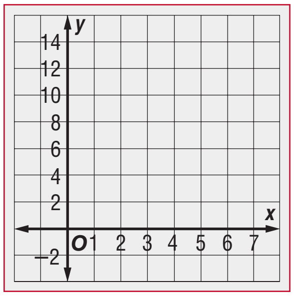
EXAMPLE Graph Opens Downward

② **ARCHERY The equation $y = -x^2 + 6x + 4$ represents the height y of an arrow x seconds after it is shot into the area.**

a. Use a table of values to graph $y = -x^2 + 6x + 4$.

x	y
–2	–12
–1	–3
0	
1	9
2	12
3	13
4	
5	

Graph these ordered pairs and connect them with a smooth curve.



b. What are the mathematical domain and range of the function? Describe reasonable domain and range values for this situation.

D: $\{x \mid x \text{ is a real number}\}$

R: $\{y \mid y \leq 13\}$

The arrow is in the air for about seconds, so a reasonable domain is D: . The height of the arrow ranges from 0 to feet, so a reasonable range is R: .

KEY CONCEPT

Equation of the Axis of Symmetry of a Parabola
The equation of the axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

BUILD YOUR VOCABULARY (pages 196–197)

Parabolas possess a geometric property called **symmetry**.

The axis of symmetry $\boxed{}$ the parabola into two $\boxed{}$ halves.

EXAMPLE Vertex and Axis of Symmetry

3 Consider the graph of $y = -2x^2 - 8x - 2$.

a. Write the equation of the axis of symmetry.

In $y = -2x^2 - 8x - 2$, $a = \boxed{}$ and $b = \boxed{}$.

$x = -\frac{b}{2a}$ Equation for the axis of symmetry of a parabola

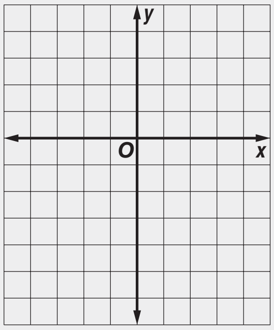
$x = -\frac{\boxed{}}{2\boxed{}}$ $a = \boxed{}$ and $b = \boxed{}$

$= \boxed{}$

The equation of the axis of symmetry is $x = \boxed{}$.

Check Your Progress **The equation $y = -x^2 + 4$ represents the shape of a jump rope when it is positioned above the jumper's head.**

a. Use a table of values to graph $y = -x^2 + 4$.



b. What are the domain and range of this function?

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -2$ and the vertex lies on the axis, the x-coordinate for the vertex is -2.

$y = -2x^2 - 8x - 2$ Original Equation

$y = -2(\_\_\_\_)^2 - 8(\_\_\_\_) - 2$ $x = -2$

$y = \_\_\_\_$ Simplify.

$y = \_\_\_\_$ Add.

The vertex is at ____.

c. Identify the vertex as a maximum or minimum.

Since the coefficient of the x^2 term is ____, the parabola opens ____ and the vertex is a ____ point.

REMEMBER IT

Functions can be graphed using the symmetry of the parabola. See page 467 of your textbook.

d. Graph the function.

You can use the symmetry of the parabola to help you draw its graph. On a coordinate plane, graph the vertex and the axis of symmetry.

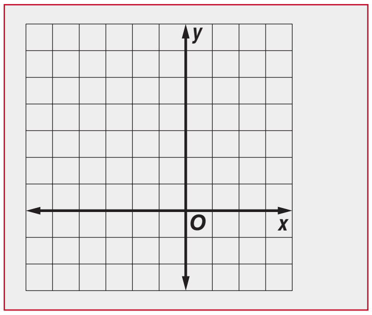
Choose a value for x other than -2. For example, choose -1 and find the y-coordinate that satisfies the equation.

$y = -2x^2 - 8x - 2$ Original equation

$y = -2(\_\_\_\_)^2 - 8(\_\_\_\_) - 2$ $x = \_\_\_\_$

$y = 4$ Simplify.

Since the graph is symmetrical about its axis of symmetry $x = \_\_\_\_$, you can find another point on the other side of the axis of symmetry. The point at ____ is 1 unit to the right of the axis. Go 1 unit to the right of the axis. Go 2 unit to the left of the axis and plot the point ____.



Repeat this for several other points. Then sketch the parabola.

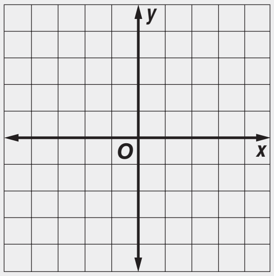
Check Your Progress **Consider the graph of $y = 3x^2 - 6x + 1$.**

a. Write the equation of the axis of symmetry.

b. Find the coordinates of the vertex.

c. Identify the vertex as a maximum or minimum.

d. Graph the function.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–2

Solving Quadratic Equations by Graphing

MAIN IDEAS

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

TEKS A.10 The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. **(A) Solve quadratic equations using** concrete models, **tables, graphs,** and algebraic methods. **(B) Make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.**

BUILD YOUR VOCABULARY (pages 196–197)

In a **quadratic equation,** the value of the related quadratic function is .

The of a quadratic equation are called the **roots** of the equation. They can be found by the or **zeros** of the related quadratic function.

EXAMPLE Two Roots

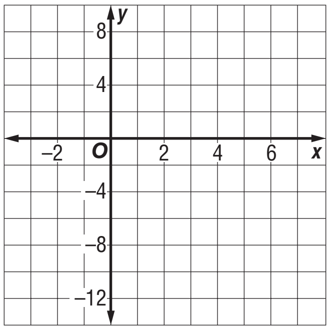
① **Solve $x^2 - 3x - 10 = 0$ by graphing.**

Graph the related function $f(x) = x^2 - 3x - 10$.

The equation of the axis of symmetry is $x = -\frac{-3}{2(1)}$ or $x = \frac{3}{2}$.

When $x = \frac{3}{2}$, $f(x)$ equals $\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10$ or . So the coordinates of the vertex are .

Make a table of values to find other points to sketch the graph.



x	y
−3	8
−1	
0	−10
1	−12
2	
3	−10
4	
6	

To solve $x^2 - 3x - 10 = 0$, you need to know where the value of $f(x)$ is . This occurs at the x-intercepts. The x-intercepts of the parabola appear to be and .

EXAMPLE A Double Root

2 $x^2 - 6x = -9$

First rewrite the equation so one side is equal to zero.

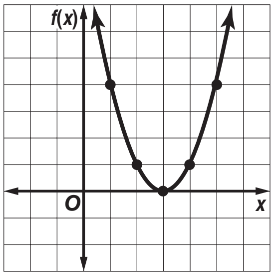
$x^2 - 6x = -9$ Original equation

$x^2 - 6x \boxed{} = -9 \boxed{}$ Add $\boxed{}$ to each side.

$x^2 - 6x + 9 = \boxed{}$ Simplify.

Graph the related function $f(x) = x^2 - 6x + 9$.

x	*f*(*x*)
1	4
2	
3	
4	
5	

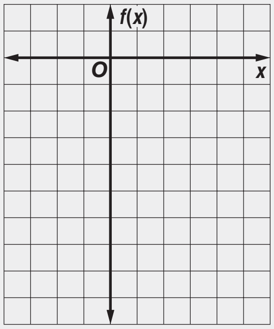


Notice that the vertex of the parabola is the *x*-intercept.

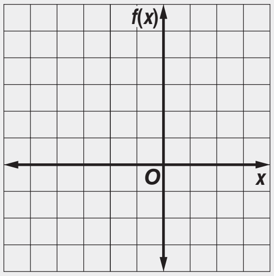
Thus, one solution is $\boxed{}$. What is the other solution?

Check Your Progress **Solve each equation by graphing.**

a. $x^2 - 2x - 8 = 0$



b. $x^2 + 2x = -1$

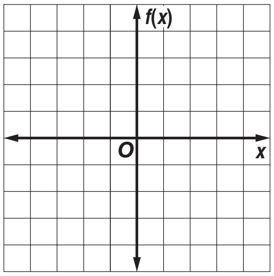


EXAMPLE Rational Roots

3 **Solve $x^2 - 4x + 2 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(x) = x^2 - 4x + 2$.

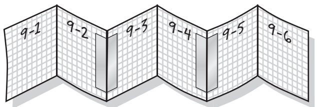
x	*f*(*x*)
0	2
1	
2	
3	
4	2



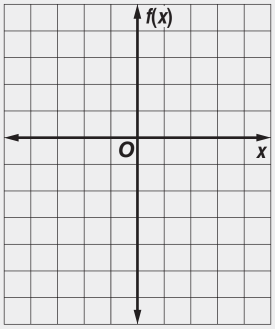
The x-intercepts of the graph are between 0 and 1 and between 3 and 4. One root is between and , and the other root is between and .

FOLDABLES™ ORGANIZE IT

On the page for Lesson 9-2, write how you solve a quadratic equation by graphing.



Check Your Progress Solve $x^2 - 2x - 5$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–3 Solving Quadratic Equations by Completing the Square

MAIN IDEAS

- Solve quadratic equations by finding the square root.
- Solve quadratic equations by completing the square.

TEKS A.10 The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. **(A) Solve quadratic equations using concrete models,** tables, graphs, **and algebraic methods.**

REMEMBER IT

When taking a square root of a positive number, there are two roots, one positive and one negative.

EXAMPLE Irrational Roots

① **Solve $x^2 + 6x + 9 = 5$ by taking the square roots of each side. Round to the nearest tenth if necessary.**

$x^2 + 6x + 9 = 5$	Original equation
$(x + 3)^2 = 5$	$x^2 + 6x + 9$ is a $\boxed{}$ $\boxed{}$ trinomial.
$\boxed{} = \boxed{}$	Take the square root of each side.
$\|x + 3\| = \sqrt{5}$	Simplify.
$x + 3 = \boxed{}$	Definition of absolute value
$x + 3 - \boxed{} = \pm\sqrt{5} - \boxed{}$	Subtract $\boxed{}$ from each side.
$x = \boxed{}$	Simplify.

Use a calculator to evaluate each value of x.

$x = -3 + \sqrt{5}$ or $x = -3 - \sqrt{5}$

$\approx \boxed{}$ $\approx \boxed{}$

The solution set is $\boxed{}$.

Check Your Progress Solve $x^2 + 8x + 16 = 3$ by taking the square root of each side. Round to the nearest tenth if necessary.

$\boxed{}$

BUILD YOUR VOCABULARY (pages 196–197)

To add a $\boxed{}$ term to a binomial of the form $ax^2 + bx$ so that the resulting trinomial is a $\boxed{}$ is referred to as **completing the square.**

KEY CONCEPT

Completing the Square To complete the square for a quadratic expression of the form $x^2 + bx$, you can follow the steps below.

Step 1 Find $\frac{1}{2}$ of b, the coefficient of x.

Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$, the original expression.

FOLDABLES On the page for Lesson 9-3, write the steps for completing the square.

EXAMPLE Complete the Square

② **Find the value of c that makes $x^2 + 12x + c$ a perfect square.**

Complete the square.

Step 1 Find $\frac{1}{2}$ of -12. $-\frac{12}{2} = \boxed{}$

Step 2 Square the result of Step 1. $\left(\boxed{}\right)^2 = \boxed{}$

Step 3 Add the result of Step 2 to $x^2 - 12x$. $x^2 - 12x\ \boxed{}$

Thus, $c = \boxed{}$. Notice that $x^2 - 12x + 36 = \boxed{}$.

Check Your Progress Find the value of c that makes $x^2 + 14x + c$ a perfect square.

$\boxed{}$

EXAMPLE Solve an Equation by Completing the Square

③ **Solve $x^2 - 18x + 5 = -12$ by completing the square.**

Step 1 Isolate the x^2 and x terms.

$x^2 - 18x + 5 = -12$ Original equation

$x^2 - 18x + 5 - \boxed{} = -12 - \boxed{}$ Subtract.

$x^2 - 18x = \boxed{}$ Simplify.

Step 2 Complete the square and solve.

$x^2 - 18x + \boxed{} = -17 + \boxed{}$ Add $\boxed{}$ to each side.

$\boxed{} = 64$ Factor $x^2 - 18x + 81$.

$\boxed{} = \boxed{}$ Take the square root of each side.

$x - 9 + \boxed{} = \pm 8 + \boxed{}$ Add $\boxed{}$ to each side.

$x = \boxed{}$ Simplify.

$x = \boxed{} + \boxed{}$ or $x = \boxed{} - \boxed{}$

$= \boxed{}$ $= \boxed{}$

Check Substitute each value for x in the original equation.

$x^2 - 18x + 5 = -12$ $x^2 - 18x + 5 = -12$

$(17)^2 - 18(17) + 5 \stackrel{?}{=} -12$ $(1)^2 - 18(1) + 5 \stackrel{?}{=} -12$

$\boxed{} - \boxed{} + 5 \stackrel{?}{=} -12$ $\boxed{} - \boxed{} + 5 \stackrel{?}{=} -12$

$\boxed{} = -12$ ✓ $\boxed{} = -12$ ✓

The solution set is $\boxed{}$.

Check Your Progress Solve $x^2 - 8x + 10 = 30$.

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–4 Solving Quadratic Equations by Using the Quadratic Formula

MAIN IDEAS

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number of solutions for a quadratic equation.

BUILD YOUR VOCABULARY (pages 196–197)

When solving the standard form of the $\boxed{}$ equation for $\boxed{}$, the result produces the **Quadratic Formula.**

EXAMPLE Solve Quadratic Equations

① **Solve each equation. Round to the nearest tenth if necessary.**

a. Solve $x^2 - 2x - 35 = 0$

For this equation, $a = 1$, $b = -2$, and $c = -35$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-35)}}{2(1)} \qquad a = 1,\ b = -2,\ \text{and } c = -35$$

$$= \frac{2 \pm \boxed{}}{2} \qquad \text{Multiply.}$$

$$= \frac{2 \pm \sqrt{\boxed{}}}{2} \qquad \text{Add.}$$

$$= \frac{2 \pm \boxed{}}{2} \qquad \text{Simplify.}$$

$$x = \boxed{} \quad \text{or } x = \boxed{}$$

$$= \boxed{} \qquad = \boxed{}$$

The solution set is $\boxed{}$.

KEY CONCEPT

The Quadratic Formula The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \pm 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

FOLDABLES Write this formula in your Foldable. Be sure to explain the formula.

TEKS A.10 The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. **(A) Solve quadratic equations using** concrete models, tables, graphs, and **algebraic methods. (B) Make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (*x*-intercepts) of the graph of the function.**

b. $15x^2 - 8x = 4$

Step 1 Rewrite the equation in standard form.

$15x^2 - 8x = 4$ Original equation

$15x^2 - 8x - \boxed{} = 4 - \boxed{}$ Subtract $\boxed{}$ from each side.

$15x^2 - 8x - 4 = \boxed{}$ Simplify.

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-\left(\boxed{}\right) \pm \sqrt{\left(\boxed{}\right)^2 - 4\left(\boxed{}\right)\left(\boxed{}\right)}}{2(15)}$$ $a = 15$, $b = -8$, and $c = -4$

$$= \boxed{}$$ Multiply.

$$= \frac{8 \pm \sqrt{\boxed{}}}{30}$$ Add.

$$x = \boxed{} \quad \text{or } x = \boxed{}$$

$$\approx \boxed{} \quad \approx \boxed{}$$

The approximate solution set is $\boxed{}$.

Check Your Progress **Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.**

a. $x^2 + x - 30 = 0$

b. $20x^2 - 4x = 8$

EXAMPLE Use the Discriminant

2 State the value of the discriminant. Then determine the number of real roots of the equation.

a. $3x^2 + 10x = 12$

Step 1 Rewrite the equation in standard form.

$3x^2 + 10x = 12$	Original equation
$3x^2 + 10x \quad = 12 \quad$	Subtract from each side.
$3x^2 + 10x - 12 = 0$	Simplify.

Step 2 Find the discriminant.

$b^2 - 4ac = (10)^2 - 4(3)(-12)$	$a = 3$, $b = 10$, and $c = -12$
$=$	Simplify.

The discriminant is . Since the discriminant is positive, the equation has real roots.

KEY CONCEPT

Using the Discriminant

Negative Discriminant: There are no real roots since no real number can be the square root of a negative number.

Zero Discriminant: There is a double root.

Positive Discriminant: There are two roots.

b. $4x^2 - 2x + 14 = 0$

$b^2 - 4ac = (-2)^2 - 4(4)(14)$	$a = 4$, $b = -2$ and $c = 14$
$=$	Simplify.

The discriminant is . Since the discriminant is , the equation has real roots.

Check Your Progress **State the value of the discriminant for each equation. Then determine the number of real roots for the equation.**

a. $x^2 + 2x + 2 = 0$

b. $-5x^2 + 10x = -1$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–5 Exponential Functions

TEKS A.11 The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(C) Analyze data and represent situations involving exponential growth and decay using** concrete models, **tables, graphs, or algebraic methods.**

MAIN IDEAS

- Graph exponential functions.
- Identify data that displays exponential behavior.

BUILD YOUR VOCABULARY (pages 196–197)

The type of function in which the is the is called an **exponential function.**

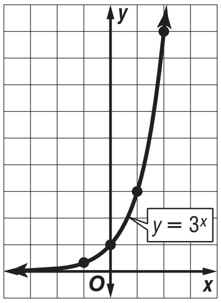
KEY CONCEPT

Exponential Function An exponential function is a function that can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

EXAMPLE Graph an Exponential Function with $a > 1$

① **a. Graph $y = 3^x$. State the y-intercept.**

x	3^x	y
-1		
0		
1		
2		



Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is .

b. Use the graph to determine the approximate value of $3^{1.5}$.

The graph represents all real values of x and their corresponding values of y for $y = 3^x$. The value of y is about when $x =$.

Use a calculator to confirm this value. $3^{1.5} \approx 5.196$

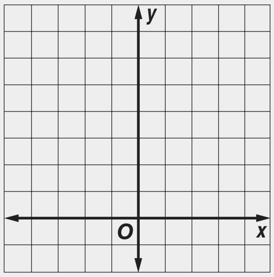
REMEMBER IT

The graph of $y = a^x$, where $a > 0$ and $a \neq 1$ never has an x-intercept.

Check Your Progress

a. Graph $y = 5^x$. State the y-intercept.

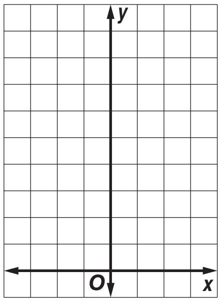
b. Use the graph to determine the approximate value of $5^{0.25}$.



EXAMPLE Graph Exponential Functions with $0 < a < 1$

② **a. Graph $y = \left(\frac{1}{4}\right)^x$. State the y-intercept.**

x	$\left(\frac{1}{4}\right)^x$	y
-1	$\left(\frac{1}{4}\right)^{-1}$	
0		
1		



Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is .

b. Use the graph to determine the approximate value of $\left(\frac{1}{4}\right)^{-1.5}$.

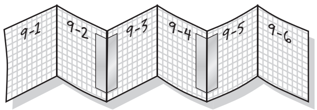
The value of y is about when $x =$.

Use a calculator to confirm this value. $\left(\frac{1}{4}\right)^{-1.5} = 8$

FOLDABLES™

ORGANIZE IT

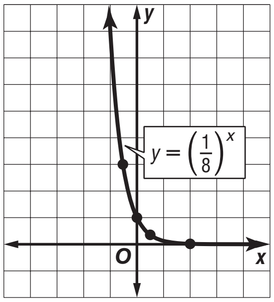
On the page for Lesson 9-5, sketch a graph of an exponential function when $a > 1$. Then sketch a graph of an exponential function when $0 < a < 1$.



Check Your Progress

a. Graph $y = \left(\frac{1}{8}\right)^x$. State the y-intercept.

b. Use the graph to determine the approximate value of $\left(\frac{1}{8}\right)^{-0.5}$.



EXAMPLE Use Exponential Functions to Solve Problems

③ **The function $V = 25{,}000 \bullet 0.82^t$ models the depreciation of the value of a new car that originally cost \$25,000. V represents the value of the car and t represents the time in years from the time the car was purchased.**

a. What values of V and t are meaningful in the function?

Only the values of $V \leq$ and $t \leq$ are meaningful in the context of the problem.

b. What is the value of the car after one year?

$V = 25{,}000 \cdot 0.82^t$ Original equation

$V = 25{,}000 \cdot 0.82^1$ $t = 1$

$V =$ Use a calculator.

After one year, the car's value is about .

Check Your Progress **The function $V = 22{,}000 \cdot 0.82^t$ models the depreciation of the value of a new car that originally cost $22,000. V represents the value of the car and t represents the time in years from the time the car was purchased.**

a. What values of V and t are meaningful in the function?

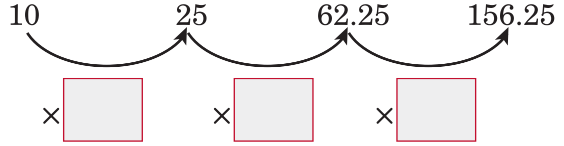
b. What is the value of the car after one year?

EXAMPLE Identify the Exponential Behavior

④ **Determine whether the set of data displays exponential behavior.**

x	0	10	20	30
y	10	25	62.5	156.25

Look for a Pattern The domain values are at regular intervals of 10. Look for a common factor among the range of values.



Since the domain values are at regular intervals and the range values have a common factor, the data are probably exponential. The equation for the data may involve ^{x}.

Check Your Progress Determine whether the set of data displays exponential behavior.

x	0	10	20	30
y	100	50	25	12.5

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–6

Growth and Decay

TEKS A.11 The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(C) Analyze data and represent situations involving exponential growth and decay using** concrete models, tables, graphs, or **algebraic methods.**

MAIN IDEAS

- Solve problems involving exponential growth.
- Solve problems involving exponential decay.

KEY CONCEPT

General Equation for Exponential Growth
The general equation for exponential growth is $y = C(1 + r)^t$ where y represents the final amount, C represents the initial amount, r represents the rate of change expressed as a decimal, and t represents time.

EXAMPLE Exponential Growth

① **POPULATION** **In 2005 the town of Flat Creek had a population of about 280,000 and a growth rate of 0.85% per year.**

a. Write an equation to represent the population of Flat Creek since 2005.

The rate 0.85% can be written as 0.0085.

$y = C(1 + r)^t$	General equation for exponential growth
$y = 280{,}000(1 + 0.0085)^t$	$C = 280{,}000$ and $r = 0.0085$
$y = 280{,}000(1.0085)^t$	Simplify.

An equation to represent the population of Flat Creek is ☐, where ☐ is the population and ☐ is the number of years since 2005.

b. According to the equation, what will be the population of Flat Creek in the year 2015?

In 2015, t will equal 2015 – 2005 or 10.

$y = 280{,}000(1.0085)^t$	Equation for population of Flat Creek
$y = 280{,}000(1.0085)^{\square}$	$t =$ ☐
$y \approx 304{,}731$	Use a calculator.

In 2015, there will be about ☐ people in Flat Creek.

Check Your Progress **In 2005, Scioto School District had a student population of about 4500 students, and a growth rate of about 0.15% per year.**

a. Write an equation to represent the student population of the Scioto School District since the year 2005.

☐

b. According to the equation, what will be the student population of the Scioto School District in the year 2011?

☐

BUILD YOUR VOCABULARY (pages 196–197)

The equation $A = P\left(1 + \frac{r}{n}\right)^{nt}$ is used to find **compound interest** which is an application of $\boxed{}$ growth.

EXAMPLE Compound Interest

② **SAVINGS** **When Jing May was born, her grandparents invested $1000 in a fixed rate savings account at a rate of 7% compounded annually. The money will go to Jing May when she turns 18 to help with her college expenses. What amount of money will Jing May receive from the investment?**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$A = \boxed{}\left(1 + \frac{\boxed{}}{\boxed{}}\right)^{\boxed{}}$ $P = 1000$, $r = 7\%$ or 0.07, $n = 1$, and $t = 18$

$A = \boxed{}$ Compound interest equation

$A = \boxed{}$ Simplify.

She will receive about $\boxed{}$.

Check Your Progress When Lucy was 10 years old, her father invested $2500 in a fixed rate savings account at a rate of 8% compounded semiannually. When Lucy turns 18, the money will help to buy her a car. What amount of money will Lucy receive from the investment?

KEY CONCEPT

General Equation for Exponential Decay
The general equation for exponential decay is $y = C(1 - r)^t$ where y represents the final amount, C represents the initial amount, r represents the rate of decay expressed as a decimal, and t represents time.

FOLDABLES™ Write the equations for exponential growth and decay in your Foldable.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Exponential Decay

③ **CHARITY** **During an economic recession, a charitable organization found that its donations dropped by 1.1% per year. Before the recession, its donations were $390,000.**

a. Write an equation to represent the charity's donations since the beginning of the recession.

$y = C(1 - r)^t$ General equation for exponential decay

$y = \boxed{}\left(1 - \boxed{}\right)^t$ C = 390,000 and r = 1.1% or 0.011

$y = \boxed{}$ Simplify.

b. Estimate the amount of the donations 5 years after the start of the recession.

$y = 390{,}000(0.989)^t$ Equation for the amount of donations

$y = 390{,}000(0.989)^{\boxed{}}$ $t = \boxed{}$

$y = \boxed{}$

The amount of donations should be about $\boxed{}$.

Check Your Progress **A charitable organization found that the value of its clothing donations dropped by 2.5% per year. Before this downturn in donations, the organization received clothing valued at $24,000.**

a. Write an equation to represent the value of the charity's clothing donations since the beginning of the downturn.

$\boxed{}$

b. Estimate the value of the clothing donations 3 years after the start of the downturn.

$\boxed{}$

CHAPTER

9

BRINGING IT ALL TOGETHER

STUDY GUIDE

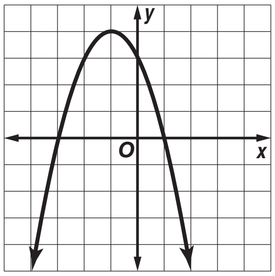
FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 9 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 9, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 196–197)* to help you solve the puzzle.

9-1

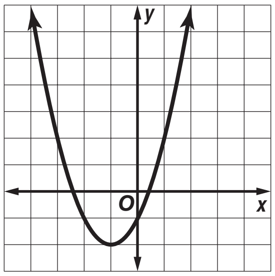
Graphing Quadratic Functions

The graphs of two quadratic functions are shown below. Complete each statement about the graphs.

A.



B.



1. Each graph is a curve called a ______.

2. The highest point of graph A is located at ______.

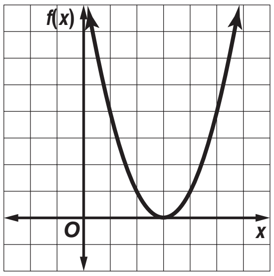
3. The lowest point of graph B is located at ______.

4. The maximum or minimum point of a parabola is called the ______ of the parabola.

9-2

Solving Quadratic Equations by Graphing

Refer to the graph shown at the right to answer the questions about the related equation $f(x) = x^2 - 6x + 9$.



5. The related quadratic equation is ______.

6. How many real number solutions are there? ______

7. Name one solution.

9-3

Solving Quadratic Equations by Completing the Square

8. Draw a line under each quadratic equation that you could solve by taking the square root of each side.

$x^2 + 6x + 9 = 100$ $x^2 - 14x + 40 = 25$ $x^2 - 16x + 64 = 26$

$x^2 - 20x + 80 = 16$ $x^2 + 10x + 36 = 49$ $x^2 - 12x + 36 = 6$

9-4

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by completing the square.

9. $x^2 + 18x + 50 = 9$

10. $3x^2 + 15x - 3 = 0$

11. What is the quadratic formula?

Solve each equation by using the quadratic formula. Round to the nearest tenth if necessary.

12. $2a^2 - 3a = -1$

13. $3w^2 - 1 = 8w$

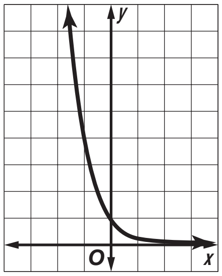
14. You can use the discriminant to determine the number of real roots for a quadratic equation. What is the discriminant?

9-5

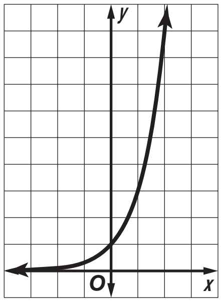
Exponential Functions

The graphs of two exponential functions of the form $y = a^x$ are shown below.

A.



B.



15. In graph A, the value of a is greater than 0 and less than ☐.

The y values decrease as the x values ☐.

16. In graph B, the value of a is greater than ☐. The y values increase as the x values ☐.

9-6

Growth and Decay

Match an equation to each solution, and then indicate whether the situation is an example of exponential growth or decay.

17. A coin had a value of $1.17 in 1995. Its value has been increasing at a rate of 9% per year.

A. $y = 1.17(1.09)^t$ ☐

B. $y = 1.17(0.91)^t$

18. A business owner has just paid $6000 for a computer. It depreciates at a rate of 22% per year. How much will it be worth in 5 years?

A. $A = 6000(1.22)^5$ ☐

B. $A = 6000(0.78)^5$

CHAPTER 9

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 9.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want take the Chapter 9 Practice Test on page 513 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 9 Study Guide and Review on pages 509–512 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 513.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 9 Foldable.
- Then complete the Chapter 9 Study Guide and Review on pages 509–512 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 513.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER

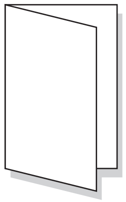
10

Radical Expressions and Triangles

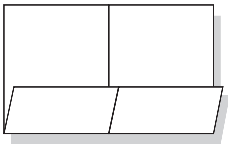
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

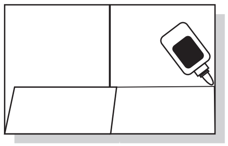
STEP 1 **Fold** in half matching the short sides.



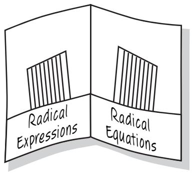
STEP 2 **Unfold** and fold the long side up 2 inches to form a pocket.



STEP 3 **Staple** or glue the outer edges to complete the pocket.



STEP 4 **Label** each side as shown. Use index cards to record examples.



NOTE-TAKING TIP: Remember to study your notes daily. Reviewing small amounts at a time will help you retain the information.

CHAPTER

10

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 10. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
conjugate [KAHN·jih·guht]			
converse			
Distance Formula			
extraneous solution [ehk·STRAY·nee·uhs]			
hypotenuse [hy·PAH·tn·oos]			
legs			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
Pythagorean triple puh·THA·guh·REE·uhn			
radical equation			
radical expression			
radicand RA·duh·KAND			
rationalizing the denominator			
similar triangles			

10–1 Simplifying Radical Expressions

Preparation for TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

MAIN IDEAS

- Simplify radical expressions using the Product Property of Square Roots.
- Simplify radical expressions using the Quotient Property of Square Roots.

KEY CONCEPT

Product Property of Square Roots For any numbers *a* and *b*, where $a \geq 0$ and $b \geq 0$, the square root of the product *ab* is equal to the product of each square root.

BUILD YOUR VOCABULARY (pages 223–224)

A **radical expression** is an expression that contains a ☐ root.

A **radicand** is the expression under the ☐ sign.

EXAMPLE Simplify Square Roots

① Simplify $\sqrt{52}$.

$\sqrt{52} =$ ☐	Prime factorization of 52
$= \sqrt{2^2} \cdot \sqrt{13}$	Product Property of Square Roots
$=$ ☐	Simplify.

EXAMPLE Multiply Square Roots

② Find $\sqrt{2} \cdot \sqrt{24}$.

$\sqrt{2} \cdot \sqrt{24} = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{3}$	Product Property
$=$ ☐ $\cdot$ ☐ $\cdot \sqrt{3}$	Product Property
$=$ ☐	Simplify.

Check Your Progress **Simplify.**

a. $\sqrt{45}$ ☐

b. $\sqrt{60}$ ☐

c. $\sqrt{5} \cdot \sqrt{35}$ ☐

EXAMPLE Simplify a Square Root with Variables

③ **Simplify $\sqrt{45a^4b^5c^6}$.**

$\sqrt{45a^4b^5c^6}$

$= \boxed{}$ Prime factorization

$= \boxed{} \cdot \sqrt{5} \cdot \boxed{} \cdot \sqrt{b^4} \cdot \boxed{} \cdot \sqrt{c^6}$ Product Property

$= \boxed{} \cdot \sqrt{b} \cdot |c^3|$ Simplify.

$= \boxed{}$ The absolute value of $|c^3|$ ensures a nonnegative result.

Check Your Progress Simplify $\sqrt{32m^2n^3c}$.

$\boxed{}$

BUILD YOUR VOCABULARY (pages 223–224)

Rationalizing the denominator of a radical expression is a method used to eliminate $\boxed{}$ from the $\boxed{}$ of a fraction.

KEY CONCEPT

Quotient Property of Square Roots For any numbers a and b, where $a \geq 0$ and $b > 0$, the square root of the quotient $\frac{a}{b}$ is equal to the quotient of each square root.

EXAMPLE Rationalizing the Denominator

④ **Simplify each quotient.**

a. $\dfrac{\sqrt{12}}{\sqrt{5}}$

$\dfrac{\sqrt{12}}{\sqrt{5}} = \dfrac{\sqrt{12}}{\sqrt{5}} \cdot \boxed{}$ Multiply by $\boxed{}$.

$= \dfrac{\boxed{}}{5}$ Product Property of Square Roots

$= \dfrac{2\sqrt{15}}{5}$ Simplify.

b. $\dfrac{\sqrt{3}}{\sqrt{8n}}$

$\dfrac{\sqrt{3}}{\sqrt{8n}} = \dfrac{\sqrt{3}}{\sqrt{8n}} \cdot \boxed{}$ Multiply by $\boxed{}$.

$= \dfrac{\sqrt{24n}}{\boxed{}}$ Product Property of Square Roots

$= \dfrac{\sqrt{2 \cdot 2 \cdot 2 \cdot 3n}}{8n}$ Prime factorization

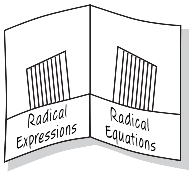
$= \boxed{}$ $\sqrt{2^2} = 2$

$= \boxed{}$ Divide the numerator and denominator by 2.

FOLDABLES™

ORGANIZE IT

On an index card, write the three steps that must be met for a radical expression to be in simplest form. Place it in the pocket for Radical Expressions.



Check Your Progress **Simplify.**

a. $\dfrac{\sqrt{5}}{\sqrt{2}}$

b. $\dfrac{\sqrt{18y}}{\sqrt{8}}$

c. $\dfrac{\sqrt{2}}{\sqrt{27}}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–2 Operations with Radical Expressions

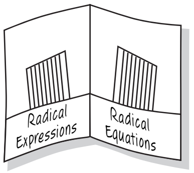
Preparation for TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

MAIN IDEAS

- Add and subtract radical expressions.
- Multiply radical expressions.

FOLDABLES™ ORGANIZE IT

On an index card, write how simplifying like radicands is similar to simplifying like terms. Place it in the pocket for Radical Expressions.



EXAMPLE Expressions with Like Radicands

① Simplify $7\sqrt{2} + 8\sqrt{11} - 4\sqrt{11} - 6\sqrt{2}$.

$7\sqrt{2} + 8\sqrt{11} - 4\sqrt{11} - 6\sqrt{2}$

$= \boxed{\quad} - 6\sqrt{2} + \boxed{\quad} - 4\sqrt{11}$ Commutative Property

$= \boxed{\quad}\sqrt{2} + \boxed{\quad}\sqrt{11}$ Distributive Property

$= \boxed{\quad}$ Simplify.

Check Your Progress Simplify $4\sqrt{2} + 8\sqrt{3} - 5\sqrt{3} + 2\sqrt{2}$.

$\boxed{\quad}$

EXAMPLE Expressions with Unlike Radicands

② Simplify $6\sqrt{27} + 8\sqrt{12} + 2\sqrt{75}$.

$6\sqrt{27} + 8\sqrt{12} + 2\sqrt{75}$

$= 6\sqrt{3^2 \cdot 3} + \boxed{\quad} + 2\sqrt{5^2 \cdot 3}$

$= 6\boxed{\quad} + 8\boxed{\quad} + 2\left(\sqrt{5^2} \cdot \sqrt{3}\right)$

$= 6(3\sqrt{3}) + 8(2\sqrt{3}) + 2(5\sqrt{3})$

$= \boxed{\quad}$

$= \boxed{\quad}$

The simplified form is $\boxed{\quad}$.

Check Your Progress Simplify $6\sqrt{245} + 3\sqrt{125} + \sqrt{80}$.

$\boxed{\quad}$

EXAMPLE Multiply Radical Expressions

3 Find the area of a rectangle with a width of $4\sqrt{6} - 2\sqrt{10}$ and a length of $5\sqrt{3} + 7\sqrt{5}$.

To find the area of the rectangle multiply the measures of the length and width.

$$(4\sqrt{6} - 2\sqrt{10})(5\sqrt{3} + 7\sqrt{5})$$

$$= \overbrace{(4\sqrt{6})\boxed{}}^{\textit{First terms}} + \overbrace{(4\sqrt{6})\boxed{}}^{\textit{Outer terms}} +$$

$$\overbrace{(-2\sqrt{10})\boxed{}}^{\textit{Inner terms}} + \overbrace{(-2\sqrt{10})\boxed{}}^{\textit{Last terms}}$$

$$= 20\sqrt{18} + \boxed{} - 10\sqrt{30} - \boxed{}$$ Multiply.

$$= 20\boxed{} + 28\sqrt{30} - \boxed{} - 14\sqrt{5^2 \cdot 2}$$ Prime factorization

$$= 60\sqrt{2} + \boxed{} - 10\sqrt{30} - \boxed{}$$ Simplify.

$$= \boxed{}$$ Combine like terms.

The area of the rectangle is $\boxed{}$ square units.

Check Your Progress Find the area of a rectangle with a width of $3\sqrt{3} + 5\sqrt{7}$ and a length of $2\sqrt{14} + 4\sqrt{6}$.

REVIEW IT

Give an example of two binomials. Then explain how you multiply them using FOIL method. (*Lesson 7–6*)

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–3 Radical Equations

Preparation for TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

MAIN IDEAS

- Solve radical equations.
- Solve radical equations with extraneous solutions.

BUILD YOUR VOCABULARY (pages 223–224)

Equations that contain radicals with variables in the $\boxed{}$ are called **radical equations.**

EXAMPLE Variable in Radical

① **FREE-FALL HEIGHT** **An object is dropped from an unknown height and reaches the ground in 5 seconds. From what height is it dropped?**

Use the equation $t = \frac{\sqrt{h}}{4}$ to replace t with $\boxed{}$.

$t = \frac{\sqrt{h}}{4}$ Original equation

$\boxed{} = \frac{\sqrt{h}}{4}$ Replace t with 5.

$\boxed{} = \sqrt{h}$ Multiply each side by 4.

$\boxed{} = \left(\sqrt{h}\right)^2$ Square each side.

$\boxed{} = h$ Simplify.

The object is dropped from $\boxed{}$ feet.

REMEMBER IT

Substitute your result into the *original* equation to check your solution.

EXAMPLE Radical Equation with an Expression

② **Solve $\sqrt{x-3} + 8 = 15$.**

$\sqrt{x-3} + 8 = 15$ Original equation

$\sqrt{x-3} = \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = 7^2$ Square each side.

$\boxed{} = 49$ $\left(\sqrt{x-3}\right)^2 = x - 3$

$x = \boxed{}$ Add 3 to each side.

The solution is $\boxed{}$.

REVIEW IT

Explain the Zero Product Property in your own words. (*Lesson 8–2*)

Check Your Progress

a. Refer to Example 1. If an unknown object reaches the ground in 7 seconds, from what height is it dropped?

$\boxed{}$

b. Solve $\sqrt{x+4}+6=14$.

$\boxed{}$

BUILD YOUR VOCABULARY (pages 223–224)

An **extraneous solution** is a solution derived from an equation that is $\boxed{}$ a solution of the $\boxed{}$ equation.

EXAMPLE Variable on Each Side

③ **Solve $\sqrt{2-y}=y$.**

$\sqrt{2-y}=y$	Original equation
$\left(\sqrt{2-y}\right)^2=y^2$	$\boxed{}$
$\boxed{}=\boxed{}$	Simplify.
$0=y^2+y-2$	Subtract $\boxed{}$ and add $\boxed{}$ to each side.
$0=\boxed{}$	Factor.
$\boxed{}=0$ or $\boxed{}=0$	Zero Product Property
$y=\boxed{}$ $\quad y=\boxed{}$	Solve.

Check

$\sqrt{2-y}=y$	$\sqrt{2-y}=y$
$\sqrt{2-(-2)}\overset{?}{=}-2$	$\sqrt{2-1}\overset{?}{=}1$
$\sqrt{4}\overset{?}{=}-2$	$\sqrt{1}\overset{?}{=}1$
$2\neq-2$ ✗	$1=1$ ✓

Since $\boxed{}$ does not satisfy the original equation, $\boxed{}$ is the only solution.

Check Your Progress

Solve each equation.

a. $\sqrt{x+3}-1=8$ $\boxed{}$

b. $y=\sqrt{2y+3}$ $\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–4 The Pythagorean Theorem

MAIN IDEAS

- Solve problems by using the Pythagorean Theorem.
- Determine whether a triangle is a right triangle.

KEY CONCEPT

The Pythagorean Theorem If *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Preparation for TEKS G.5 The student uses a variety of representations to describe geometric relationships and solve problems. **(D)** Identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples. **G.8** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. **(C)** Derive, extend, and use the Pythagorean Theorem.

BUILD YOUR VOCABULARY (pages 223–224)

In a right triangle, the side opposite the angle is called the **hypotenuse**. The other two are called the **legs** of the triangle.

Whole numbers that satisfy the are called **Pythagorean triples.**

EXAMPLE Find the Length of the Hypotenuse

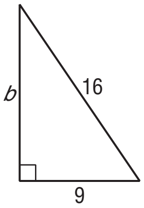
① Find the length of the hypotenuse of a right triangle if $a = 18$ and $b = 24$.

	Pythagorean Theorem
$c^2 = 18^2 + 24^2$	$a =$ and $b =$
$c^2 =$	Simplify.
$\sqrt{c^2} =$	Take the square root of each side.
$c =$	Use the positive value.

The length of the hypotenuse is units.

Check Your Progress Find the length of the hypotenuse of a right triangle if $a = 25$ and $b = 60$.

EXAMPLE Find the Length of a Side



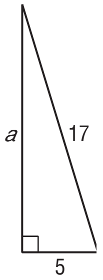
② Find the length of the missing side.

In the triangle, $c = \boxed{}$ and $a = \boxed{}$ units.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$\boxed{} = \boxed{} + b^2$	$a = 9$ and $c = 16$
$\boxed{} = \boxed{} + b^2$	Evaluate squares.
$\boxed{} = b^2$	Subtract 81 from each side.
$\pm\sqrt{175} = b$	Take the square root of each side.
$\boxed{} \approx b$	Use the positive value. Use a calculator to evaluate $\sqrt{175}$.

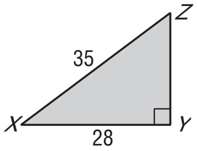
To the nearest hundredth, the length of the leg is $\boxed{}$ units.

Check Your Progress Find the length of the missing side.



EXAMPLE Pythagorean Triples

③ What is the area of triangle *XYZ*?



Use the measures of the hypotenuse and the base to find the height of the triangle.

Step 1 Check to see if the measurements of this triangle are a multiple of a common Pythagorean triple. The hypotenuse is $7 \cdot 5$ units and the leg is $7 \cdot 4$ units. This triangle is a multiple of a (3, 4, 5) triangle.

$\boxed{} = 21$

$\boxed{} = 28$

$\boxed{} = 35$

The height of the triangle is $\boxed{}$ units.

WRITE IT

Do the numbers 6, 8, and 10 represent a Pythagorean triple? Explain.

Step 2 Find the area of the triangle.

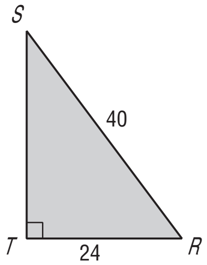
$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}$ $b = 28$ and $h = 21$

$A =$ Simplify.

The area of the triangle is 294 square units.

Check Your Progress What is the area of triangle *RST*?



KEY CONCEPT

Corollary to the Pythagorean Theorem If *a* and *b* are measures of the shorter sides of a triangle, *c* is the measure of the longest side, and $c^2 = a^2 + b^2$, then the triangle is a right triangle. If $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle.

BUILD YOUR VOCABULARY (pages 223–224)

A statement that can easily be proved using a is often called a **corollary**.

EXAMPLE Check for Right Triangles

④ **Determine whether the side measures of 27, 36, and 45 form a right triangle.**

Since the measure of the longest side is 45, let $c =$, $a =$, and $b =$. Then determine whether $c^2 = a^2 + b^2$.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$^2 = ^2 + 36^2$ $a = 27$, $b = 36$, and $c = 45$

$2025 = 729 +$ Multiply.

$=$ Add.

Since $c^2 = a^2 + b^2$, the triangle a right triangle.

Check Your Progress Determine whether the side measures 12, 22, and 40 form a right triangle.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–5 The Distance Formula

Preparation for TEKS G.7 The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. **(C)** Derive and use formulas involving length, slope, and midpoint.

MAIN IDEAS

- Find the distance between two points on the coordinate plane.
- Find a point that is a given distance from a second point in a plane.

KEY CONCEPT

The Distance Formula
The distance *d* between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

EXAMPLE Distance Between Two Points

1 Find the distance between the points at (1, 2) and (–3, 0).

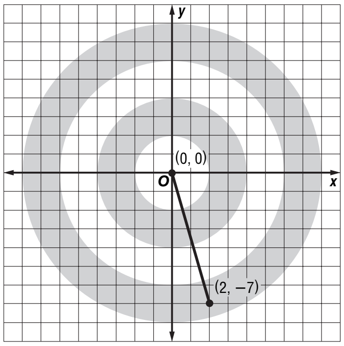
$d = \boxed{}$	Distance Formula
$= \sqrt{\boxed{}^2 + \boxed{}^2}$	$(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-3, 0)$
$= \sqrt{\boxed{}^2 + (-2)^2}$	Simplify.
$= \boxed{}$	Evaluate squares and simplify.
$= \boxed{}$ or about 4.47 units	

Check Your Progress Find the distance between the points at (5, 4) and (0, 22).

$\boxed{}$

EXAMPLE Use the Distance Formula

2 BIATHLON Julianne is sighting her rifle for an upcoming biathlon competition. Her first shot is 2 inches to the right and 7 inches below the bull's-eye. What is the distance between the bull's-eye and where her first shot hit the target?



If the bull's-eye is at (0, 0), then the location of the first shot is $\boxed{}$. Use the Distance Formula.

$d = \boxed{}$	Distance Formula
$= \sqrt{\boxed{}^2 + (-7 - 0)^2}$	$(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, -7)$
$= \sqrt{2^2 + (-7)}$	Simplify.
$= \boxed{}$ or about 7.28 inches	

The distance is $\boxed{}$ or about $\boxed{}$ inches.

REMEMBER IT

You can choose either point to be (x_1, y_1) when using the Distance Formula.

Check Your Progress Marcy is pitching a horseshoe in her local park. Her first pitch is 9 inches to the left and 3 inches below the pin. What is the distance between the horseshoe and the pin?

EXAMPLE Find a Missing Coordinate

3 **Find the value of *a* if the distance between the points at (2, –1) and (*a*, –4) is 5 units.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$\boxed{} = \sqrt{\boxed{}^2 + (-4 - (-1))^2}$	Let $d = 5$, $x_2 = a$, $x_1 = 2$, $y_2 = -4$, and $y_1 = -1$.
$5 = \sqrt{\boxed{}^2 + \boxed{}^2}$	Simplify.
$5 = \sqrt{a^2 - 4a + 4 + 9}$	Evaluate squares.
$5 = \sqrt{\boxed{}}$	Simplify.
$25 = a^2 - 4a + 13$	Square each side.
$0 = \boxed{}$	Subtract $\boxed{}$ from each side.
$0 = \boxed{}(a + 2)$	Factor.
$\boxed{} = 0$ or $\boxed{} = 0$	Zero Product Property
$a = \boxed{}$ $a = \boxed{}$	Solve.

The value of a is $\boxed{}$ or $\boxed{}$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress Find the value of a if the distance between the points at (2, 3) and $(a, 2)$ is $\sqrt{37}$ units.

10–6 Similar Triangles

Preparation for TEKS G.11 The student applies the concepts of similarity to justify properties of figures and solve problems. **(B)** Use ratios to solve problems involving similar figures. **(C)** Develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods.

MAIN IDEAS

- Determine whether two triangles are similar.
- Find the unknown measures of sides of two similar triangles.

KEY CONCEPT

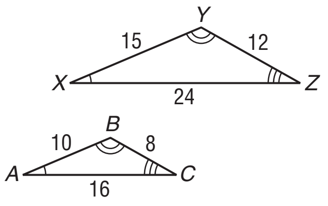
Similar Triangles If two triangles are similar, then the measures of their corresponding sides are proportional, and the measures of their corresponding angles are equal.

BUILD YOUR VOCABULARY (pages 223–224)

Similar triangles have the same ______, but not necessarily the same ______.

EXAMPLE Determine Whether Two Triangles Are Similar

① **Determine whether the pair of triangles is similar. Justify your answer.**



The ratio of sides $\overline{XY}$ to $\overline{AB}$

is ______ or ______.

The ratio of sides $\overline{YZ}$ to $\overline{BC}$

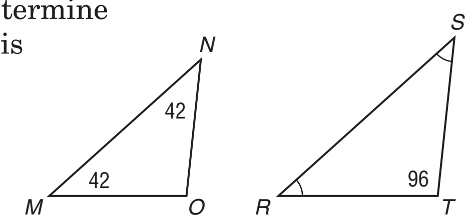
is ______ or ______.

The ratio of sides $\overline{XZ}$ to $\overline{AC}$

is ______ or ______.

Since the measures of the corresponding sides are ______ and the measures of the corresponding angles are equal, triangle ______ is similar to triangle ______.

Check Your Progress Determine whether the pair of triangles is similar. Justify your answer.

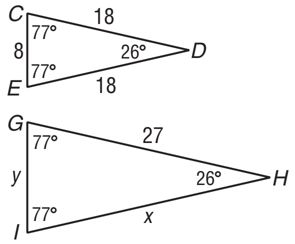


WRITE IT

Find the word *corresponding* in a dictionary, and write its definition below.

EXAMPLE **Find Missing Measures**

② **a. Find the missing measures if the pair of triangles is similar.**



Since the corresponding angles have equal measures, □. The lengths of the corresponding sides are proportional.

$\frac{CE}{\square} = \frac{\square}{GH}$ Corresponding sides of similar triangles are proportional.

$\frac{\square}{y} = \frac{18}{\square}$ $CE = 8$, $GI = y$, $CD = 18$, and $GH = 27$

$\square = 18y$ Find the cross products.

$\square = y$ Divide each side by 18.

$\frac{\square}{GH} = \frac{\square}{IH}$ Corresponding sides of similar triangles are proportional.

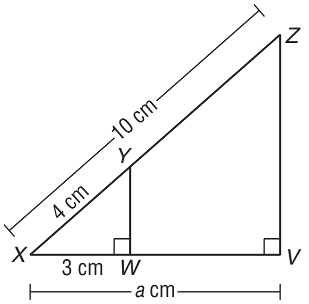
$\frac{\square}{27} = \frac{\square}{x}$ $CD = 18$, $GH = 27$, $ED = 18$, and $IH = x$

$\square = 486$ Find the cross products.

$x = \square$ Divide each side by □.

The missing measures are □ and □.

b. Find the missing measures if the pair of triangles is similar.



$\triangle XYW \sim \square$

$\frac{\square}{XZ} = \frac{\square}{XV}$ Corresponding sides of similar triangles are proportional.

$\square = \frac{3}{a}$ $XY = \square$, $XZ = \square$, $XW = 3$, and $XV = a$

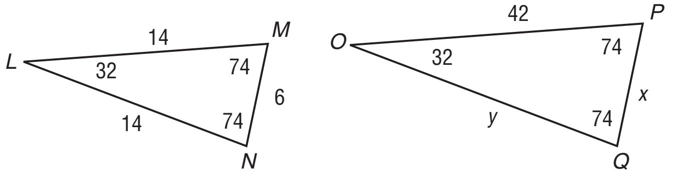
$\square = 30$ Find the cross products.

$a = \square$ Divide each side by 4.

The missing measure is $\square$.

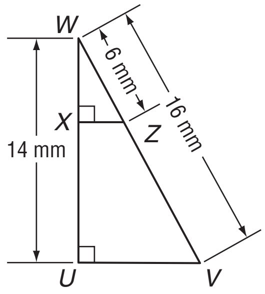
Check Your Progress **Find the missing measures if each pair of triangles is similar.**

a.



$\square$

b.



$\triangle WXZ \sim \triangle WUV$

$\square$

Homework Assignment

Page(s):

Exercises:

CHAPTER

10

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 10 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 10, go to: glencoe.com	You can use your completed **Vocabulary Builder** (*pages 223–224*) to help you solve the puzzle.

10-1

Simplifying Radical Expressions

Simplify.

1. $\sqrt{28x^2y^4}$ **2.** $\sqrt{\frac{5}{32}}$ **3.** $\frac{8}{3+\sqrt{3}}$

4. What should you remember to check for when you want to determine if a radical expression is in simplest form?

Check radicands for and ,

and check fractions for in the .

10-2

Operations with Radical Expressions

Simplify each expression.

5. $6\sqrt{3} - \sqrt{12}$

6. $2\sqrt{12} - 7\sqrt{3}$

7. $3\sqrt{2}\left(\sqrt{8} + \sqrt{24}\right)$

8. $\left(2\sqrt{5} - 2\sqrt{3}\right)\left(\sqrt{10} + \sqrt{6}\right)$

9. $\sqrt{27} + \sqrt{18} + \sqrt{300}$

10. Below the words **F**irst terms, **O**uter terms, **I**nner terms, and **L**ast terms, write the products you would use to simplify the expression $\left(2\sqrt{15} + 3\sqrt{15}\right)\left(6\sqrt{3} - 5\sqrt{2}\right)$.

First terms		**O**uter terms		**I**nner terms		**L**ast terms
	+		+		+	

10-3
Radical Equations

11. To solve a radical equation, you first isolate the radical on one side of the equation. Why do you then square each side of the equation?

12. Provide the reason for each step in the solution of the given radical equation.

$\sqrt{5x - 1} - 4 = x - 3$	Original equation
$\sqrt{5x - 1} = x + 1$	
$\left(\sqrt{5x - 1}\right)^2 = (x + 1)^2$	
$5x - 1 = x^2 + 2x + 1$	
$0 = x^2 - 3x + 2$	
$0 = (x - 1)(x - 2)$	
$x - 1 = 0$ or $x - 2 = 0$	
$x = 1$ $\quad$ $x = 2$	

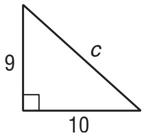
13. To be sure that 1 and 2 are the correct solutions, into which equation should you substitute to check?

14. A computer screen measures 12 inches high and 17 inches wide. What is the length of the screen's diagonal? Round your answer to the nearest whole number.

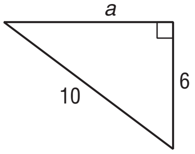
10-4
The Pythagorean Theorem

Write an equation that you could solve to find the missing side length of each right triangle. Then solve.

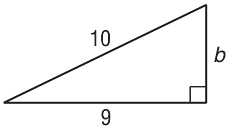
15.



16.



17.



10-5

The Distance Formula

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

18. (6, 4), (2, 1)

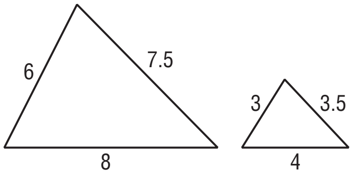
19. (3, 7), (9, –2)

10-6

Similar Triangles

Determine whether each pair of triangles is similar. Explain how you would know that your answer is correct.

20.



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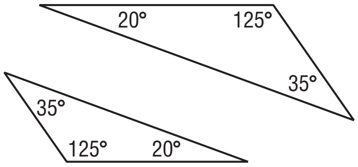


CHAPTER 10

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 10.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 10 Practice Test on page 563 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 10 Study Guide and Review on pages 559–562 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 563.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 10 Foldable.
- Then complete the Chapter 10 Study Guide and Review on pages 559–562 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 563.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER

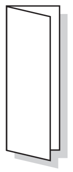
11

Rational Expressions and Equations

FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

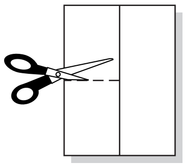
Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

STEP 1 **Fold** in half lengthwise.

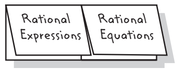


STEP 2 **Fold** the top to the bottom.

STEP 3 **Open.** Cut along the second fold to make two tabs.



STEP 4 **Label** each tab as shown.



NOTE-TAKING TIP: When you take notes, it may be helpful to sit as close as possible to the front of the class. There are fewer distractions and it is easier to hear.

CHAPTER 11

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 11. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
complex fraction			
excluded values			
extraneous solutions [ehk·STRAY·nee·uhs]			
inverse variation [ihn·VUHRS]			
least common multiple			
least common denominator			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
mixed expression			
product rule			
rate problems			
rational equations			
rational expression			
work problems			

11–1 Inverse Variation

TEKS A.11 The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. **(B) Analyze data and represent situations involving inverse variation using** concrete models, **tables, graphs, or algebraic methods.**

MAIN IDEAS

- Graph inverse variations.
- Solve problems involving inverse variations.

KEY CONCEPT

Inverse Variation
y varies inversely as *x* if there is some nonzero constant *k* such that $xy = k$.

BUILD YOUR VOCABULARY (pages 245–246)

When the product of two values remains $\boxed{}$ the relationship forms an **inverse variation.**

EXAMPLE Graph an Inverse Variation

① **MANUFACTURING The time t in hours that it takes to build a particular model of computer varies inversely with the number of people p working on the computer. The equation $pt = 12$ can be used to represent the people building a computer. Draw a graph of the relation.**

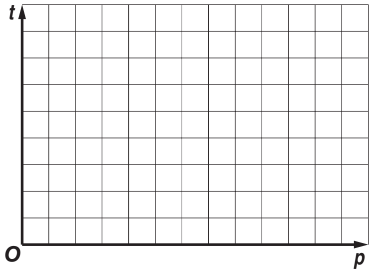
Solve for $p = 2$.

$pt = 12$	Original equation
$\boxed{}\,t = 12$	Replace *p* with $\boxed{}$.
$t = \dfrac{12}{\boxed{}}$	Divide each side by $\boxed{}$.
$t = \boxed{}$	Simplify.

Solve the equation for the other values of *p*.

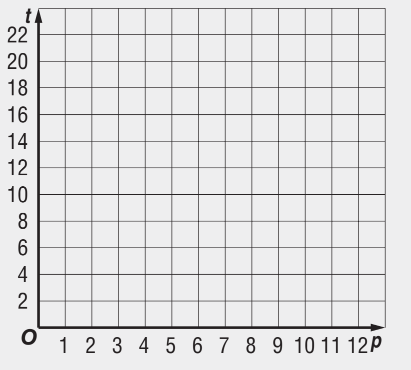
P	2	4	6	8	10	12
t						

Graph the ordered pairs. As the number of people *p* increases, the time *t* it takes to build a computer decreases.



Check Your Progress

The time *t* in hours that it takes to prepare packages for delivery varies inversely with the number of people *p* that are preparing them. The equation $pt = 36$ can be used to represent the people preparing the packages. Draw a graph of the relation.



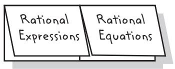
BUILD YOUR VOCABULARY (pages 245–246)

The equation $x_1y_1 = x_2y_2$ is called the **product rule** for $\boxed{}$ variations.

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Expressions, write the general form for inverse variation. Then give an example of an inverse variation equation.



EXAMPLE Solve for *x*

② **If y varies inversely as x and $y = 5$ when $x = 12$, find x when $y = 15$.**

Let $x_1 = 12$, $y_1 = 5$, and $y_2 = 15$. Solve for x_2.

METHOD 1 Use the product rule.

$x_1y_1 = x_2y_2$ Product rule for inverse variations

$\boxed{} \cdot \boxed{} = x_2 \cdot \boxed{}$ $x_1 = 12$, $y_1 = 5$, $y_2 = 15$

$\boxed{} = x_2$ Divide each side by $\boxed{}$.

$\boxed{} = x_2$ Simplify.

METHOD 2 Use a proportion.

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$ Proportion rule for inverse variations

$\frac{\boxed{}}{x_2} = \frac{\boxed{}}{\boxed{}}$ $x_1 = 12$, $y_1 = 5$, $y_2 = 15$

$\boxed{} = \boxed{}$ Cross multiply.

$\boxed{} = x_2$ Divide each side by 15.

Both methods show that $x = \boxed{}$ when $y = \boxed{}$.

Check Your Progress

a. If y varies inversely as x and $y = 6$ when $x = 40$, find x when $y = 30$.

b. If y varies inversely as x and $y = -5$ when $x = 15$, find y when $x = 3$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–2

Rational Expressions

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B)** Analyze various representations of rational functions with respect to problem situations.

MAIN IDEAS

- Identify values excluded from the domain of a rational expression.
- Simplify rational expressions.

BUILD YOUR VOCABULARY (pages 245–246)

A **rational expression** is an algebraic fraction whose ____ and ____ are polynomials.

Any values of a variable that result in a denominator of ____ must be excluded from the ____ of the variable and are called **excluded values** of the rational expression.

EXAMPLE Excluded Values

① **State the excluded value for each rational expression.**

a. $\dfrac{\mathbf{3b - 2}}{\mathbf{b + 7}}$

Exclude the values for which $b + 7 = 0$.

$b + 7 = 0$ The denominator cannot equal ____.

$b = \boxed{}$ Subtract ____ from each side.

So, b cannot equal ____.

b. $\dfrac{\mathbf{5a^2 + 2}}{\mathbf{a^2 - a - 12}}$

Exclude the values for which $a^2 - a - 12 = \boxed{}$.

$a^2 - a - 12 = 0$ The denominator cannot equal ____.

$(\boxed{})(\boxed{}) = 0$ Factor.

Use the Zero Product Property to solve for a.

____ or ____

$a = \boxed{}$ $a = \boxed{}$

So, a cannot equal ____ or ____.

Check Your Progress **State the excluded value for each rational expression.**

a. $\dfrac{2y-7}{y+3}$ $\boxed{}$

b. $\dfrac{x^2+1}{x^2-5x+6}$ $\boxed{}$

EXAMPLE Expressions Involving Monomials

2 **Simplify** $\dfrac{32x^5y^2}{4xy^7}$**.**

$$\frac{32x^5y^2}{4xy^7} = \frac{(4xy^2)(8x^4)}{(4xy^2)(y^5)}$$

The GCF of the numerator and denominator is $\boxed{}$.

$$= \frac{(\overset{1}{\cancel{4xy^2}})(8x^4)}{(\underset{1}{\cancel{4xy^2}})(y^5)}$$

Divide the numerator and denominator by $\boxed{}$.

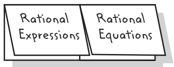
$$= \boxed{}$$

Simplify.

FOLDABLES™

ORGANIZE IT

Take notes on how to simplify a rational expression.



WRITE IT

How do you know that a rational expression is in simplest form?

EXAMPLE Excluded Values

3 **Simplify** $\dfrac{4x+16}{x^2-5x-36}$**. State the excluded values of x.**

$$\frac{4x+16}{x^2-5x-36} = \frac{4(x+4)}{(x-9)(x+4)}$$

Factor.

$$= \frac{4(\overset{1}{\cancel{x+4}})}{(x-9)(\underset{1}{\cancel{x+4}})}$$

Divide the numerator and denominator by the GCF, $x + 4$.

$$= \boxed{}$$

Simplify.

Exclude the values for which $x^2 - 5x - 36$ equals 0.

$x^2 - 5x - 36 = 0$ The denominator cannot equal zero.

$(x - 9)(x + 4) = 0$ Factor.

$x = \boxed{}$ or $x = \boxed{}$ Zero Product Property

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress Simplify $\dfrac{5w-10}{w^2+6w-16}$. State the excluded values of w.

11–3

Multiplying Rational Expressions

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B)** Analyze various representations of rational functions with respect to problem situations.

MAIN IDEAS

- Multiply rational expressions.
- Use dimensional analysis with multiplication.

EXAMPLE Expressions Involving Monomials

1 **Find** $\frac{7x^2y}{12z^3} \cdot \frac{14z}{49xy^4}$.

METHOD 1

Divide by the greatest common factor after multiplying.

$$\frac{7x^2y}{12z^3} \cdot \frac{14z}{49xy^4} = \frac{\boxed{}}{\boxed{}}$$

← Multiplying the numerators.

← Multiplying the denominators.

$$= \frac{\cancel{98xyz}(\boxed{})}{\cancel{98xyz}(\boxed{})}$$

The GCF is $\boxed{}$.

$$= \boxed{}$$

Simplify.

METHOD 2

Divide the common factors before multiplying.

$$\frac{7x^2y}{12z^3} \cdot \frac{14z}{49xy^4} = \frac{\overset{1}{\cancel{7}}\overset{x}{\cancel{x^2}}\overset{1}{y}}{\underset{6}{\cancel{12}}\underset{z^2}{\cancel{z^3}}} \cdot \frac{\overset{1}{\cancel{14}}\overset{1}{\cancel{z}}}{\underset{1}{\cancel{49}}\underset{1}{\cancel{x}}\underset{y^3}{\cancel{y^4}}}$$

Divide by common factors $\boxed{}$ and $\boxed{}$.

$$= \boxed{}$$

Multiply.

EXAMPLE Expressions Involving Polynomials

2 **Find** $\frac{b+3}{4b-12} \cdot \frac{b^2-4b+3}{b^2-7b-30}$.

$$\frac{b+3}{4b-12} \cdot \frac{b^2-4b+3}{b^2-7b-30} = \frac{b+3}{4(b-3)} \cdot \frac{(b-3)(b-1)}{(b-10)(b+3)}$$

Factor.

$$= \frac{\overset{1}{\cancel{(b+3)}}\overset{1}{\cancel{(b-3)}}(b-1)}{4\underset{1}{\cancel{(b-3)}}(b-10)\underset{1}{\cancel{(b+3)}}}$$

The GCF is $(b+3)(b-3)$.

$$= \frac{b-1}{4(b-10)}$$

Multiply.

$$= \boxed{}$$

Simplify.

Check Your Progress **Find each product.**

a. $\dfrac{6mn^2}{11m^3p^4} \cdot \dfrac{22p^3}{3n}$

b. $\dfrac{14k^3w^2}{5s^2t^2} \cdot \dfrac{10s^4t}{21kw^5}$

c. $\dfrac{y+1}{y} \cdot \dfrac{y^2}{y^2+8y+7}$

d. $\dfrac{13c-39}{c-4} \cdot \dfrac{c^2-16}{c^2+3c-18}$

REVIEW IT

When do you need to use dimensional analysis in a word problem? *(Lesson 2–8)*

EXAMPLE Dimensional analysis

3 **SPACE** **The velocity that a spacecraft must have in order to escape Earth's gravitational pull is called the escape velocity. The escape velocity for a spacecraft leaving Earth is about 40,320 kilometers per hour. What is this speed in meters per second?**

$$\frac{40{,}320 \text{ kilometers}}{\text{hour}} \cdot \frac{100 \text{ meters}}{1 \text{ kilometer}} \cdot \frac{1 \text{ hour}}{\boxed{}} \cdot \frac{\boxed{} \text{ minute}}{\boxed{} \text{ seconds}}$$

$$= \frac{40,320 \cancel{\text{kilometers}}}{\cancel{\text{hour}}} \cdot \frac{1000 \text{ meters}}{1 \cancel{\text{kilometer}}} \cdot \frac{1 \cancel{\text{hour}}}{60 \cancel{\text{minutes}}} \cdot \frac{1 \cancel{\text{minute}}}{60 \text{ seconds}}$$

$$= \frac{\overset{1120}{\cancel{40{,}320}} \cdot \overset{10}{\cancel{1000}} \cdot 1 \cdot 1 \text{ meters}}{1 \cdot 1 \cdot \underset{1}{\cancel{60}} \cdot \underset{1}{\cancel{60}}}$$

$$= \boxed{} \quad \text{Simplify.}$$

$$= \boxed{} \quad \text{Multiply.}$$

The escape velocity is ☐ meters per second.

Check Your Progress The speed of sound, or Mach 1, is approximately 330 meters per second at sea level. What is the speed of sound in kilometers per hour?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11-4 Dividing Rational Expressions

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B)** Analyze various representations of rational functions with respect to problem situations.

MAIN IDEAS

- Divide rational expressions.
- Use dimensional analysis with division.

EXAMPLE Divide by Fractions

1 Find each quotient.

a. $\dfrac{64x^4}{5} \div \dfrac{24x}{75}$

$\dfrac{6x^4}{5} \div \dfrac{24x}{75} = \dfrac{6x^4}{5} \cdot \boxed{}$ Multiply by $\boxed{}$, the reciprocal of $\dfrac{24x}{75}$.

$= \dfrac{\overset{1x^3}{\cancel{6x^4}}}{\underset{1}{\cancel{5}}} \cdot \dfrac{\overset{15}{\cancel{75}}}{\underset{4}{\cancel{24}}\,\underset{1}{\cancel{x}}}$ Divide by common factors $\boxed{}$, $\boxed{}$, and $\boxed{}$.

$= \boxed{}$ Simplify.

b. $\dfrac{3m + 12}{m + 5} \div \dfrac{m + 4}{m - 2}$

$\dfrac{3m + 12}{m + 5} \div \dfrac{m + 4}{m - 2}$

$= \dfrac{3m + 12}{m + 5} \cdot \dfrac{m - 2}{m + 4}$ Multiply by $\dfrac{m - 2}{m + 4}$, the reciprocal of $\boxed{}$.

$= \dfrac{3(m + 4)}{m + 5} \cdot \dfrac{m - 2}{m + 4}$ Factor $\boxed{}$.

$= \dfrac{3\overset{1}{\cancel{(m + 4)}}}{m + 5} \cdot \dfrac{m - 2}{\underset{1}{\cancel{m + 4}}}$ The GCF is $\boxed{}$.

$= \boxed{}$ or $\boxed{}$ Simplify.

Check Your Progress **Find each quotient.**

a. $\dfrac{3a}{7} \div \dfrac{9a^5}{14}$

b. $\dfrac{n - 8}{n - 12} \div \dfrac{6n - 48}{n + 3}$

EXAMPLE Expression Involving Polynomials

2 **Find $\dfrac{q^2 - 11q - 26}{7} \div \dfrac{q - 13}{q + 7}$.**

$$\frac{q^2 - 11q - 26}{7} \div \frac{q - 13}{q + 7}$$

$$= \frac{q^2 - 11q - 26}{7} \cdot \boxed{}$$

Multiply by the reciprocal, $\boxed{}$.

$$= \frac{(\boxed{})(\boxed{})}{7} \cdot \frac{q + 7}{q - 13}$$

Factor $q^2 - 11q - 26$.

$$= \frac{\overset{1}{\cancel{(q - 13)}}(q + 2)}{7} \cdot \frac{q + 7}{\underset{1}{\cancel{q - 13}}}$$

The GCF is $\boxed{}$.

$$= \boxed{} \text{ or } \boxed{}$$

Simplify.

REMEMBER IT

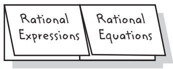
When you are dividing rational expressions, always multiply by the reciprocal.

Check Your Progress **Find the quotient.**

$$\frac{k^2 - 13k + 30}{10} \div \frac{k - 3}{k - 2}$$

FOLDABLES™ ORGANIZE IT

Under the tab for Rational Expressions, write the question and answer to Example 2. Then label the quotient, dividend, and divisor.



EXAMPLE Dimensional Analysis

3 AVIATION **In 1986, an experimental aircraft named Voyager was piloted by Jenna Yeager and Dick Rutan around the world non-stop, without refueling. The trip took exactly 9 days and covered a distance of 25,012 miles. What was the speed of the aircraft in miles per hour? Round to the nearest miles per hour.**

Use the formula for rate, time, and distance.

$rt = d$ rate • time = distance

$r \bullet 9 \text{ days} = 25{,}012 \text{ mi}$ $t = \boxed{}$ days,

$d = \boxed{}$ mi

$r = \dfrac{25{,}012 \text{ mi}}{\boxed{}}$ Divide each side by $\boxed{}$.

$= \dfrac{25{,}012 \text{ mi}}{9\ \cancel{\text{days}}} \bullet \boxed{}$ Convert days to hours.

$= \dfrac{25{,}012 \text{ mi}}{\boxed{}}$ or about $\dfrac{\boxed{}}{1 \text{ hour}}$

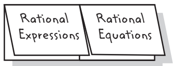
The speed of the aircraft was about $\boxed{}$ miles per hour.

Check Your Progress Suppose that Jenna Yeager and Dick Rutan wanted to complete the trip in exactly 7 days. What would be their average speed in miles per hour for the 25,012 mile trip?

FOLDABLES™

ORGANIZE IT

Write how to divide rational expressions in your Foldable.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–5 Dividing Polynomials

MAIN IDEAS

- Divide a polynomial by a monomial.
- Divide a polynomial by a binomial.

Preparation for TEKS 2A.10
The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior.

EXAMPLE Divide Polynomials by Monomials

① **a. Find $(4x^2 - 18x) \div 2x$.**

$$(4x^2 - 18x) \div 2x = \frac{4x^2 - 18x}{2x}$$

Write as a rational expression.

$$= \frac{4x^2}{\boxed{}} - \frac{18x}{\boxed{}}$$

Divide each term by $\boxed{}$.

$$= \frac{\overset{2x}{\cancel{4x^2}}}{\underset{1}{\cancel{2x}}} - \frac{\overset{9}{\cancel{18x}}}{\underset{1}{\cancel{2x}}}$$

Simplify each term.

$$= \boxed{}$$

Simplify.

b. Find $(2y^2 - 3y - 9) \div 3y$.

$$(2y^2 - 3y - 9) \div 3y = \boxed{}$$

Write as a rational expression.

$$= \frac{2y^2}{3y} - \frac{3y}{3y} - \frac{9}{3y}$$

Divide each term by $\boxed{}$.

$$= \frac{\overset{2y}{\cancel{2y^2}}}{\underset{3}{\cancel{3y}}} - \frac{\overset{1}{\cancel{3y}}}{\underset{1}{\cancel{3y}}} - \frac{\overset{3}{\cancel{9}}}{\underset{y}{\cancel{3y}}}$$

Simplify each term.

$$= \boxed{}$$

Simplify.

EXAMPLE Divide a Polynomial by a Binomial

② **Find $(2r^2 + 5r - 3) \div (r + 3)$.**

$$(2r^2 + 5r - 3) \div (r + 3) = \boxed{}$$

Write as a rational expression.

$$= \frac{(\boxed{})(\boxed{})}{r + 3}$$

Factor the numerator.

$$= \frac{(2r - 1)\overset{1}{\cancel{(r + 3)}}}{\underset{1}{\cancel{r + 3}}}$$

Divide by the GCF.

$$= \boxed{}$$

Simplify.

EXAMPLE Long Division

3 Find $(x^2 + 7x - 15) \div (x - 2)$.

Step 1 Divide the first term of the dividend, x^2, by the first term of the divisor, x.

$$\begin{array}{r} x \\ x-2\,\overline{)\,x^2 + 7x - 15} \end{array}$$

$x^2 \div x = x$.

Multiply x and $x - 2$.

Subtract.

Step 2 Divide the first term of the partial dividend, $9x - 15$, by the first term of the divisor, x.

$$\begin{array}{r} x + 9 \\ x-2\,\overline{)\,x^2 + 7x - 15} \end{array}$$

$9x \div x = 9$

Subtract and bring down .

$9x - 15$

Multiply 9 and $x - 2$.

Subtract.

The quotient of $(x^2 + 7x - 15) \div (x - 2)$ is with a remainder of , which can be written as .

WRITE IT

Explain how dividing polynomials is similar to dividing whole numbers.

Check Your Progress **Find each quotient.**

a. $(48z^2 + 18z) \div 6z$

b. $(-8x^2 + 6x - 28) \div 4x$

c. $(2c^2 - 3c - 9) \div (c - 3)$

d. $(y^2 - 4y + 5) \div (y - 3)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–6 Rational Expression with Like Denominators

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B)** Analyze various representations of rational functions with respect to problem situations.

MAIN IDEAS

- Add rational expressions with like denominators.
- Subtract rational expressions with like denominators.

EXAMPLE Numbers in Denominator

① **Find $\frac{4b}{15} + \frac{16b}{15}$.**

$\frac{4b}{15} + \frac{16b}{15} = \boxed{}$ The common denominator is $\boxed{}$.

$= \frac{\boxed{}}{\boxed{}}$ Add the numerators.

$= \frac{\overset{4}{\cancel{20b}}}{\underset{3}{\cancel{15}}}$ or $\frac{4b}{3}$ Divide by the common factor $\boxed{}$ and simplify.

REMEMBER IT You must have like denominators before adding or subtracting rational expressions.

EXAMPLE Binomials in Denominator

② **Find $\frac{6c}{c+2} + \frac{12}{c+2}$.**

$\frac{6c}{c+2} + \frac{12}{c+2} = \frac{6c+12}{c+2}$ The common denominator is $\boxed{}$.

$= \frac{6(\boxed{})}{c+2}$ Factor the numerator.

$= \frac{6(\overset{1}{\cancel{c+2}})}{\underset{1}{\cancel{c+2}}}$ or 6 Divide by the common factor, $c + 2$ and simplify.

EXAMPLE Subtract Rational Expressions

③ **Find $\frac{7x+9}{x} - \frac{x-5}{x-3}$.**

$\frac{7x+9}{x-3} - \frac{x-5}{x-3}$

$= \boxed{}$ The common denominator is $\boxed{}$.

$= \frac{(7x+9) = [-(x-5)]}{x-3}$ The additive inverse of $(x - 5)$ is $\boxed{}$.

$= \boxed{}$ Distributive Property

$= \boxed{}$ or $\boxed{}$ Simplify.

EXAMPLE Inverse Denominators

4 **Find $\frac{3s}{11 - s} + \frac{-5s}{s - 11}$.**

Rewrite the second expression so that it has the same denominator as the first.

$\frac{3s}{11 - s} + \frac{-5s}{s - 11}$

$=$ ☐ $s - 11 =$ ☐

$=$ ☐ Rewrite using common denominators.

$= \frac{3s + 5s}{11 - s}$ The common denominator is ☐.

$=$ ☐ Simplify.

Check Your Progress **Find each sum or difference.**

a. $\frac{7k}{9} + \frac{17k}{9}$

b. $\frac{5y}{y + 4} + \frac{20}{y + 4}$

c. $\frac{11y - 3}{y + 1} - \frac{5y + 6}{y + 1}$

d. $\frac{8n}{n - 4} + \frac{n}{4 - n}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–7 Rational Expression with Unlike Denominators

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B)** Analyze various representations of rational functions with respect to problem situations.

MAIN IDEAS

- Add rational expressions with unlike denominators.
- Subtract rational expressions with unlike denominators.

KEY CONCEPT

Add Rational Expressions Use the following steps to add rational expressions with unlike denominators.

Step 1 Find the LCD.

Step 2 Change each rational expression into an equivalent expression with the LCD as the denominator.

Step 3 Add just as with rational expressions with like denominators.

Step 4 Simplify if necessary.

BUILD YOUR VOCABULARY (pages 245–246)

The **least common multiple** is the ______ number that is a ______ multiple of two or more numbers.

The least common multiple of the ______ of two or more ______ is known as the **least common denominator.**

EXAMPLE LCM of Polynomials

1 Find the LCM of each pair of polynomials.

a. $12b^4c^5$ and $32bc^2$

Find the prime factors of each coefficient and variable expression.

$12b^4c^5 =$ ______

$32bc^2 =$ ______

Use each prime factor the greatest number of times it appears in any of the factorizations.

LCM = ______

b. $x^2 - 3x - 28$ and $x^2 - 8x + 7$

Express each polynomial in factored form.

$x^2 - 3x - 28 = ($ ______ $)($ ______ $)$

$x^2 - 8x + 7 = ($ ______ $)($ ______ $)$

Use each factor the greatest number of times it appears.

LCM = ______.

Check Your Progress **Find the LCM for each pair of expressions.**

a. $21a^2b^4$ and $35a^3b^2$

$\boxed{}$

b. $y^2 + 12y + 36$ and $y^2 + 2y - 24$

$\boxed{}$

EXAMPLE Polynomial Denominators

2 Find each sum.

a. $\frac{z+2}{5z} + \frac{z-6}{z}$

Factor each denominator and find the LCD. The LCD is $\boxed{}$.

$$\frac{z+2}{5z} + \frac{z-6}{z}$$

$$= \frac{z+2}{5z} + \boxed{} \qquad \text{Rename } \frac{z-6}{z}.$$

$$= \frac{z+2}{5z} + \frac{\boxed{}}{5z} \qquad \text{Distributive Property}$$

$$= \frac{\boxed{}}{5z} \qquad \text{Add the numerators.}$$

$$= \boxed{} \text{ or } \boxed{} \qquad \text{Simplify.}$$

b. $\frac{x+7}{x^2 - 6x + 9} + \frac{x+3}{x-3}$

$$\frac{x+7}{x^2 - 6x + 9} + \frac{x+3}{x-3}$$

$$= \frac{x+7}{\left(\boxed{}\right)^2} + \frac{x+3}{x-3} \qquad \text{Factor the denominators.}$$

$$= \frac{x+7}{(x-3)^2} + \frac{x+3}{x-3} \cdot \boxed{} \qquad \text{The LCD is } \boxed{}.$$

$$= \frac{x+7}{(x-3)^2} + \frac{x^2-9}{(x-3)^2} \qquad (x+3)(x-3) = \boxed{}.$$

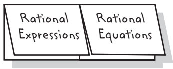
$$= \frac{\boxed{}}{(x-3)^2} \qquad \text{Add the numerators.}$$

$$= \boxed{} \text{ or } \boxed{} \qquad \text{Simplify.}$$

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Equations, write each new Vocabulary Builder word. Then give an example of each word.



EXAMPLE Polynomials in Denominators

③ **Find $\frac{c}{20 + 4c} - \frac{6}{5 - c}$.**

$$\frac{c}{20 + 4c} - \frac{6}{5 - c}$$

$$= \frac{c}{4\left(\boxed{}\right)} - \frac{6}{(5 - c)}$$ Factor.

$$= \frac{\boxed{}}{4(5 + c)(5 - c)} - \frac{\boxed{}}{4(5 - c)(5 + c)}$$ The LCD is $\boxed{}$.

$$= \boxed{\phantom{\frac{c(5 - c) - 24(5 + c)}{4(5 + c)(5 - c)}}}$$ Add the numerators.

$$= \frac{\boxed{}}{4(5 + c)(5 - c)}$$ Multiply.

$$= \frac{-c^2 - 19c - 120}{4(5 + c)(5 - c)} \text{ or } \boxed{\phantom{\frac{c^2 + 19c + 120}{4(c + 5)(c - 5)}}}$$ Simplify.

Check Your Progress **Find each sum or difference.**

a. $\frac{b - 2}{4b} + \frac{b - 7}{b}$ $\boxed{}$

b. $\frac{y - 14}{y^2 - 8y + 16} + \frac{y + 4}{y - 4}$ $\boxed{}$

c. $\frac{3}{b + 1} - \frac{b}{4b - 4}$

$\boxed{}$

d. $\frac{n + 3}{n^2 + 10 + 25} - \frac{n - 7}{n^2 + 2n - 15}$

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–8 Mixed Expression and Complex Fractions

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B)** Analyze various representations of rational functions with respect to problem situations.

MAIN IDEAS

- Simplify mixed expressions.
- Simplify complex fractions.

BUILD YOUR VOCABULARY (pages 245–246)

An expression that contains the sum of a ______ and a rational expression is known as a **mixed expression.**

If a fraction has ______ or more fractions in the numerator or denominator, it is called a **complex fraction.**

EXAMPLE Mixed Expression to Rational Expression

① Simplify $3 + \frac{7}{x-2}$.

$3 + \frac{7}{x-2} = \frac{3(x-2)}{x-2} + \frac{7}{x-2}$ The LCD is $x - 2$.

$= \frac{3(x-2)+7}{x-2}$ Add the numerators.

$= \boxed{}$ Distributive Property

$= \boxed{}$ Simplify.

EXAMPLE Complex Fraction Involving Monomials

② Simplify $\dfrac{\frac{a^5b}{c^2}}{\frac{ab^4}{c^4}}$.

$\dfrac{\frac{a^5b}{c^2}}{\frac{ab^4}{c^4}} = \frac{a^5b}{c^2} \div \frac{ab^4}{c^4}$ Rewrite as a ______ sentence.

$= \frac{a^5b}{c^2} \cdot \boxed{}$ Rewrite as multiplication by the reciprocal.

$= \dfrac{\overset{a^4}{\cancel{a^5}}\,\overset{1}{\cancel{b}}}{\underset{1}{\cancel{c^2}}} \cdot \dfrac{\overset{c^2}{\cancel{c^4}}}{\underset{1}{\cancel{a}}\,\underset{b^3}{\cancel{b^4}}}$ Divide by common factors.

$= \boxed{}$ Simplify.

KEY CONCEPT

Simplifying a Complex Fraction Any complex fraction $\dfrac{\frac{a}{b}}{\frac{c}{d}}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$, can be expressed as $\frac{ad}{bc}$.

EXAMPLE Complex Fraction Involving Polynomials

3 Simplify $\dfrac{b+\frac{2}{b+3}}{b-4}$.

The numerator contains a mixed expression. Rewrite it as a rational expression first.

$$\frac{b+\frac{2}{b+3}}{b-4}=\frac{\frac{b(b+3)}{b+3}+\frac{2}{b+3}}{b-4}$$

The LCD of the fractions in the numerator is $\boxed{}$.

$$=\frac{\boxed{}}{b-4}$$

Simplify the numerator.

$$=\frac{\boxed{}\boxed{}}{\frac{b+3}{b-4}}$$

Factor.

$$=\frac{(b+1)(b+2)}{b+3}\div(b-4)$$

Rewrite as a division sentence.

$$=\frac{(b+1)(b+2)}{b+3}\cdot\boxed{}$$

Multiply by the reciprocal of $b-4$.

$$=\boxed{}$$

Simplify.

Check Your Progress **Simplify each expression.**

a. $\dfrac{\frac{p^2q^3}{r^2}}{\frac{p^2q}{r^5}}$

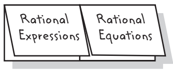
b. $\dfrac{c-\frac{5}{c-4}}{c-5}$

c. $5+\dfrac{2}{y-4}$

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Equations, write why the fraction bar in a complex fraction is considered a grouping symbol.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–9 Solving Rational Equations

Preparation for TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(D)** Determine the solutions of rational equations using graphs, tables, and algebraic methods.

MAIN IDEAS

- Solve rational equations.
- Eliminate extraneous solutions.

BUILD YOUR VOCABULARY (pages 245–246)

A **rational equation** is an equation that contains rational expressions.

EXAMPLE Use Cross Products

1 Solve $\frac{8}{x+3}=\frac{2}{x-6}$.

$\frac{8}{x+3}=\frac{2}{x-6}$ Original equation

$8\left(\boxed{}\right)=2\left(\boxed{}\right)$ Cross multiply.

$8x-48=2x+6$ Distributive Property

$6x=\boxed{}$ Add $\boxed{}$ and $\boxed{}$ to each side.

$x=\boxed{}$ Divide each side by 6.

EXAMPLE Use the LCD

2 Solve each equation.

a. $\frac{5}{x+1}+\frac{1}{x}=\frac{2}{x^2+x}$

$\frac{5}{x+1}+\frac{1}{x}=\frac{2}{x^2+x}$ Original equation

$\boxed{}\left(\frac{5}{x+1}+\frac{1}{x}\right)=\boxed{}\left(\frac{2}{x^2+x}\right)$ The LCD is $\boxed{}$.

$$\left(\frac{x(\overset{1}{\cancel{x+1}})}{1}\cdot\frac{5}{\underset{1}{\cancel{(x+1)}}}\right)+\left(\frac{\overset{1}{\cancel{x}}(x+1)}{1}\cdot\frac{1}{\underset{1}{\cancel{x}}}\right)=\frac{x(\overset{1}{\cancel{x+1}})}{1}\cdot\frac{2}{\underset{1}{\cancel{x^2+x}}}$$

Distributive Property

$\boxed{}=2$ Simplify.

$\boxed{}=2$ Add.

$6x=1$ Subtract.

$x=\frac{1}{6}$ Divide.

b. $a + \frac{a^2 - 5}{a^2 - 1} = \frac{a^2 + a + 2}{a + 1}$

$$a + \frac{a^2 - 5}{a^2 - 1} = \frac{a^2 + a + 2}{a + 1}$$ Original equation

$$\boxed{}\left(a + \frac{a^2 - 5}{a^2 - 1}\right) = \boxed{}\left(\frac{a^2 + a + 2}{a + 1}\right)$$

$$(a^2 - 1)a + \left(\frac{\overset{1}{\cancel{a^2 - 1}}}{1} \cdot \frac{a^2 - 5}{\underset{1}{\cancel{a^2 - 1}}}\right) = (\overset{a-1}{\cancel{a^2 - 1}})\left(\frac{a^2 + a + 2}{\underset{1}{\cancel{a + 1}}}\right)$$

$$\boxed{} = a^3 + a - 2$$ Simplify.

$$a^2 - 2a - 3 = 0$$

$$(\boxed{})(\boxed{}) = 0$$

$$\boxed{} = 0 \text{ or } \boxed{} = 0$$

$$a = \boxed{} \qquad a = \boxed{}$$

The number $\boxed{}$ is an $\boxed{}$ value for x.

Thus, the solution is $\boxed{}$.

REVIEW IT

When checking solutions to equations, why do you check both solutions in the original equation? *(Lesson 10–3)*

____

____

____

____

____

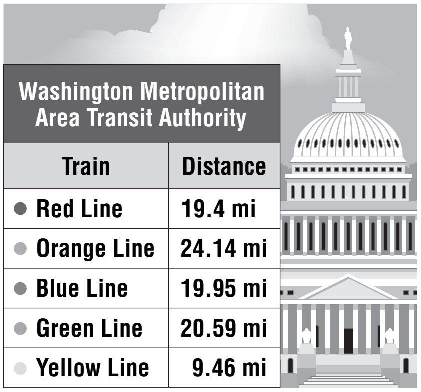
Check Your Progress **Solve each equation.**

a. $\frac{2}{x - 6} = \frac{4}{5x - 3}$ $\boxed{}$

b. $\frac{3}{m - 2} = \frac{m + 1}{m} - \frac{1}{m}$ $\boxed{}$

EXAMPLE Rate Problem

3 TRANSPORTATION The schedule for the Washington, D.C., Metrorail is shown to the right. Suppose two Red Line trains leave their stations at opposite ends of the line at exactly 2:00 P.M. One train travels between the two stations in 48 minutes and the other train takes 54 minutes. At what time do the two trains pass each other?



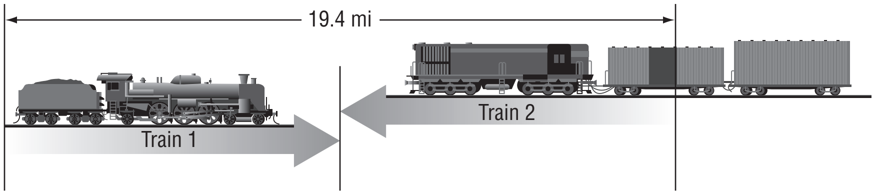
Washington Metropolitan Area Transit Authority

Train	Distance
● Red Line	19.4 mi
● Orange Line	24.14 mi
● Blue Line	19.95 mi
● Green Line	20.59 mi
● Yellow Line	9.46 mi

Determine the rates of both trains. The total distance is 19.4 miles.

Train 1 $\frac{\boxed{}\text{ mi}}{\boxed{}\text{ min}}$ **Train 2** $\frac{\boxed{}\text{ mi}}{\boxed{}\text{ min}}$

Next, since both trains left at the same time, the time both have traveled when they pass will be the same. And since they started at opposite ends of the route, the sum of their distances is equal to the total route, $\boxed{}$ miles.



	r	t	$d = r \cdot t$
Train 1	$\frac{19.4}{48}$	t min	$\frac{19.4t}{48}$
Train 2	$\frac{19.4}{54}$	t min	$\frac{19.4t}{54}$

$$\frac{19.4t}{48} + \frac{19.4t}{54} = 19.4$$
The sum of the distances is 19.4.

$$\boxed{}\left(\frac{19.4t}{48} + \frac{19.4t}{54}\right) = \boxed{} \cdot 19.4$$
The LCD is 432.

$$\frac{\overset{9}{\cancel{432}}}{1} \cdot \frac{19.4t}{\underset{1}{\cancel{48}}} + \frac{\overset{8}{\cancel{432}}}{1} \cdot \frac{19.4t}{\underset{1}{\cancel{54}}} = \boxed{}$$
Distributive Property

$$174.6t + 155.2t = 8380.8$$
Simplify.

$$329.8t = 8380.8$$
Add.

$$t = \boxed{}$$
Divide each side by 329.8.

The trains passed each other at about $\boxed{}$ minutes after they left their stations, at $\boxed{}$.

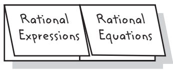
Check Your Progress **TRANSPORTATION** Two cyclists are riding on a 5-mile circular bike trail. They both leave the bike trail entrance at 3:00 P.M. traveling in opposite directions. It usually takes the first cyclist one hour to complete the trail and it takes the second cyclist 50 minutes. At what time will they pass each other?

$\boxed{}$

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Equations, write the definition of an extraneous solution in your own words.



EXAMPLE Extraneous Solutions

④ **Solve $\frac{x^2}{x-2} = \frac{4}{x-2}$.**

$$\frac{x^2}{x-2} = \frac{4}{x-2}$$ Original equation

$$\boxed{}\left(\frac{x^2}{x-2}\right) = \boxed{}\left(\frac{4}{x-2}\right)$$ The LCD is $\boxed{}$.

$$\overset{1}{\cancel{(x-2)}}\left(\frac{x^2}{\underset{1}{\cancel{x-2}}}\right) = \overset{1}{\cancel{(x-2)}}\left(\frac{4}{\underset{1}{\cancel{x-2}}}\right)$$ Distributive Property

$$x^2 = 4$$ Simplify.

$$x^2 - \boxed{} = 0$$ Subtract.

$$(x-2)(x+2) = 0$$ Factor.

$$x - 2 = 0 \text{ or } x + 2 = 0$$ Zero Product Property

$$x = \boxed{} \qquad x = \boxed{}$$

The number 2 is an extraneous solution, since 2 is an excluded value for x. Thus, −2 is the solution of the equation.

Check Your Progress **Solve each equation. State any extraneous solutions.**

a. $\frac{9y}{y+2} - \frac{5y-8}{y+2} = 3$

$\boxed{}$

b. $\frac{3w}{w-2} - 2 = \frac{5w+14}{w^2-4}$

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 11

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 11** Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 11, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 245–246)* to help you solve the puzzle.

11-1

Inverse Variation

Write *direct variation, inverse variation,* or *neither* to describe the relationship between x and y described by each equation.

1. $y = 3x$

2. $xy = 5$

3. $y = -8x$

4. $y = \frac{2}{x}$

5. $x = \frac{10}{y}$

6. $y = 7x - 1$

For each problem, assume that y varies inversely as x. Use the Product Rule to write an equation you could use to solve the problem. Then write a proportion and solve the problem.

	Problem	Product Rule	Proportion	Solve
7.	If $y = 8$ when $x = 12$, find y when $x = 4$.			
8.	If $x = 50$ when $y = 6$, find x when $y = 30$			

11-2

Rational Expressions

Simplify each expression. State the excluded values of the variables.

9. $\frac{21bc}{28bc^2}$

10. $\frac{2x + 10}{x^2 - 2x - 35}$

11. $\frac{2y^2 + 9 + 4}{4y^2 - 4y - 3}$

11-3

Multiplying Rational Expressions

Find each product.

12. $\frac{18a^2}{10b^2} \cdot \frac{15b^2}{24a}$

13. $\frac{x - 4}{x^2 - x - 12} \cdot \frac{x + 3}{x - 6}$

14. $\frac{y^2 + 5y + 4}{y^2 - 36} \cdot \frac{y^2 + 5y - 6}{y^2 + 2y - 8}$

15. The number of calories used to play basketball depends on your weight and how long you play. Playing basketball expends about 3.8 calories per hour per pound of weight. If you weigh 140 pounds, how many calories do you lose in 1.25 hours?

11-4

Dividing Rational Equations

State the reciprocal of the divisor in each of the following.

16. $\frac{3b + 15}{b + 1} \div (b - 2)$

17. $\frac{2c^2}{d} \div \frac{c}{3d}$

18. Supply the reason for the steps below.

$\frac{y + 1}{y^2 + 5y + 6} \div \frac{1}{y + 3}$	Original Expression
$= \frac{y + 1}{y^2 + 5y + 6} \cdot \frac{y + 3}{1}$	Multiply by the .
$= \frac{y + 1}{(y + 2)(y + 3)} \cdot \frac{y + 3}{1}$	$y^2 + 5y + 6$.
$= \frac{y + 1}{(y + 2)\cancel{(y + 3)}} \cdot \frac{\cancel{y + 3}}{1}$	Divide by the .
$= \frac{y + 1}{y + 2}$	.

11-5

Dividing Polynomials

Find each quotient.

19. $(20y^2 + 12y) \div 4y$

20. $\frac{2x^2 - 5x - 3}{2x + 1}$

21. $\frac{6a^3 + a^2 - 2a + 17}{2a + 3}$

11-6

Rational Expressions with Like Denominators

For each addition or subtraction problem, write the needed expression in each box on the right side of the equation.

22. $\frac{5n}{7} + \frac{8}{7} = \frac{5n + ■}{7}$

23. $\frac{d - c}{c + 2d} - \frac{c - d}{c + 2d} = \frac{■ - (c - d)}{c + 2d}$

24. $\frac{8}{6x - 1} + \frac{9}{1 - 6x} = \frac{8 + (■)}{6x - 1}$

11-7

Rational Expressions with Unlike Denominators

25. What is the LCM of $49k^2n^2$ and $21kn^5$?

Find each sum or difference.

26. $\frac{3}{y} + \frac{4}{y^2}$

27. $\frac{5x}{3y^2} - \frac{2x}{9y}$

28. $\frac{a}{a-5} + \frac{a-1}{a+5}$

29. $\frac{a+3}{a^2-3a-10} - \frac{4a-8}{a^2-10a+25}$

11-8

Mixed Expressions and Complex Fractions

Tell whether each expression is a mixed expression or complex fraction. Write M for mixed expression and C for complex fraction.

30. $7x + \frac{x+2}{x-5}$

31. $\frac{5 + \frac{2}{s-1}}{s^2}$

32. $(b-6) + \frac{b+3}{b+2}$

33. Simplify $\frac{\frac{x+4}{x}}{\frac{x^2-16}{x}}$.

11-9

Solving Rational Equations

34. Is $\frac{\sqrt{x-3}}{4} = \frac{3}{x}$ a rational equation? Explain.

Solve each equation. State any extraneous solutions.

35. $\frac{5}{x+2} = \frac{7}{x+6}$

36. $\frac{-2}{w+1} + \frac{2}{w} = 1$

37. $\frac{3}{2t} + \frac{2t}{t-3} = 2$

CHAPTER 11

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 11.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want take the Chapter 11 Practice Test on page 629 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 11 Study Guide and Review on pages 625–628 of your textbook.
- If you are unsure of any concepts or skills, refer to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 629.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 11 Foldable.
- Then complete the Chapter 11 Study Guide and Review on pages 625–628 of your textbook.
- If you are unsure of any concepts or skills, refer to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 629.

Student Signature

Parent/Guardian Signature

Teacher Signature

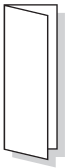
CHAPTER 12

Statistics and Probability

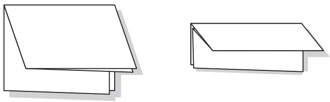
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

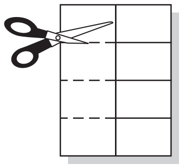
STEP 1 **Fold** in half lengthwise.



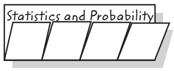
STEP 2 **Fold** the top to the bottom twice.



STEP 3 **Cut** along the second fold to make four tabs.



STEP 4 **Label** as shown.



NOTE-TAKING TIP: If your instructor points out definitions or procedures from your text, write a reference page in your notes. You can then write these referenced items in their proper place in your notes after class.

CHAPTER

12

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 12. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
biased sample			
combination			
complements			
compound event			
convenience sample			
dependent events			
empirical study			
event			
experimental probability			
factorial			

Vocabulary Term	Found on Page	Definition	Description or Example
inclusive			
independent events			
population			
random sample			
sample			
simple random sample			
stratified random sample			
systematic random sample SIHS·tuh·MA·tihk			
voluntary response sample			

12–1 Sampling and Bias

Reinforcement of TEKS 8.13 The student evaluates predictions and conclusions based on statistical data. **(A)** Evaluate methods of sampling to determine validity of an inference made from a set of data.

MAIN IDEAS

- Identify various sampling techniques.
- Recognize a biased sample.

BUILD YOUR VOCABULARY (pages 276–277)

A **sample** is some portion of a group, called the **population**, selected to represent that group. If all of the within a population are included, it is called a **census**.

In a **biased sample**, one or more parts of a population are over others.

KEY CONCEPTS

Simple Random Sample: A sample that is as likely to be chosen as any other from the population.

Stratified Random Sample In a stratified random sample, the population is first divided into similar, nonoverlapping groups. A simple random sample is then selected from each group.

Systematic Random Sample In a systematic random sample, the items are selected according to a specified time or item interval.

EXAMPLE Classify a Random Sample

① **a.** RETAIL **Each day, a department store chain selects one male and one female shopper randomly from each of their 57 stores, and asks them survey questions about their shopping habits.**

Identify the sample and suggest a population from which it was selected.

The sample is male and female shoppers. The population is .

b. Classify the sample as *simple*, *stratified*, or *systematic*.

The population is divided into similar, nonoverlapping groups. This is a sample.

Check Your Progress **At an automobile factory, every tenth item is checked for quality controls.**

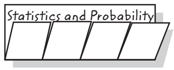
a. Identify the sample and suggest a population from which it was selected.

b. Classify the sample as *simple*, *stratified*, or *systematic*.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 12-1, write your own example of a biased sample.



EXAMPLE Identify Sample as Biased or Unbiased

② **STUDENT COUNCIL The student council surveys the students in one classroom to decide the theme for the spring dance. Identify the sample as *biased* or *unbiased*. Explain your reasoning.**

The sample includes only students in one classroom.

The sample is .

Check Your Progress Identify the sample as *biased* or *unbiased*. Explain your reasoning.

A local news station interviews one person on every street in Los Angeles to give their opinion on their mayor.

EXAMPLE Identify and Classify a Biased Sample

③ **a. COMMUNITY The residents of a neighborhood are to be surveyed to find out when to hold a neighborhood clean up day. The neighborhood chairperson decides to ask her immediate neighbors and the neighbors in the houses directly across the street from her house.**

Identify the sample, and suggest a population from which it was selected.

The sample is the

and the neighbors across the street. The

is the residents of the neighborhood.

b. Classify the sample as a *convenience sample*, or a *voluntary response sample*.

This is a sample because the chairperson asked only her closest neighbors.

Check Your Progress Mark wanted to find out what the average student in the United States does on the weekend. He decides to interview people in his dorm. Identify the sample, and suggest a population from which it was selected. Then classify the sample as a *convenience sample,* or a *voluntary response sample.*

KEY CONCEPTS

Biased Samples

A **convenience sample** includes members of a population that are easily accessed.

A **voluntary response sample** involves only those who want to participate in the sampling.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–2 Counting Outcomes

Reinforcement of TEKS 8.13 The student evaluates predictions and conclusions based on statistical data. **(B)** Recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

MAIN IDEAS

- Count outcomes using a tree diagram.
- Count outcomes using the Fundamental Counting Principle.

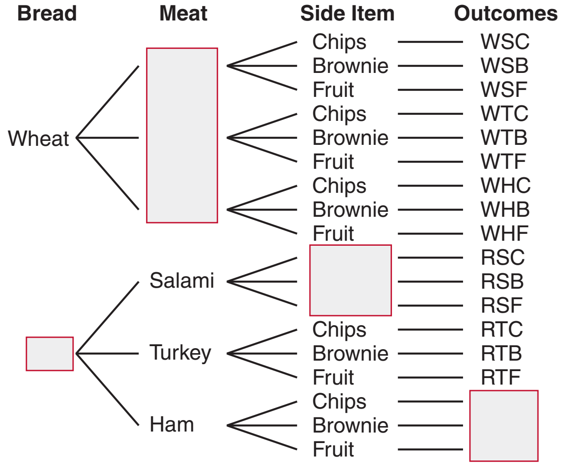
BUILD YOUR VOCABULARY (pages 276–277)

One method used for counting the number of possible is to draw a **tree diagram**.

The list of all possible is called the **sample space.** An **event** is any collection of one or more outcomes in the sample space.

EXAMPLE Tree Diagram

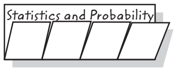
① **At football games, a concession stand sells sandwiches on either wheat or rye bread. The sandwiches come with salami, turkey, or ham, and either chips, a brownie, or fruit. Use a tree diagram to determine the number of possible sandwich combinations.**



There are possible combinations.

FOLDABLES™ ORGANIZE IT

Under the tab for Outcomes, explain how to use a tree diagram to show the number of possible outcomes.



Check Your Progress A buffet offers a combination of a meat, a vegetable, and a drink. The choices of meat are chicken or pork; the choices of vegetable are carrots, broccoli, green beans, or potatoes; and the choices of drink are milk, lemonade, or a soft drink. Use a tree diagram to determine the number of possible combinations.

KEY CONCEPT

Fundamental Counting Principle If an event *M* can occur in *m* ways and is followed by an event *N* that can occur in *n* ways, then the event *M* followed by event *N* can occur in *m* • *n* ways.

EXAMPLE Fundamental Counting Principle

2 The Best Deal computer company sells custom made personal computers. Customers have a choice of 11 different hard drives, 6 different keyboards, 4 different mice, and 4 different monitors. How many different custom computers can you order?

Multiply to find the number of custom computers.

$$\overbrace{\boxed{}}^{\text{hard drive choices}} \cdot \overbrace{\boxed{}}^{\text{keyboard choices}} \cdot \overbrace{\boxed{}}^{\text{mice choices}} \cdot \overbrace{\boxed{}}^{\text{monitor choices}} = \overbrace{\boxed{}}^{\text{custom computers}}$$

The number of different custom computers is $\boxed{}$.

Check Your Progress A baseball team is organizing their draft. In the first five rounds, they want a pitcher, a catcher, a first baseman, a third baseman, and an outfielder. They are considering 7 pitchers, 9 catchers, 3 first baseman, 4 third baseman, and 12 outfielders. How many top picks are there to choose from?

KEY CONCEPT

Factorial The expression *n*!, read *n* factorial, where *n* is greater than zero, is the product of all positive integers beginning with *n* and counting backward to 1.

EXAMPLE Factorial

3 Find the value of 9!.

$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= \boxed{}$ Simplify.

Check Your Progress **Find the value of each expression.**

a. 7!

b. 0!

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–3 Permutations and Combinations

Reinforcement of TEKS 8.11 The student applies concepts of theoretical and experimental probability to make predictions. **(B)** Use theoretical probabilities and experimental results to make predictions and decisions.

MAIN IDEAS

- Determine probabilities using permutations.
- Determine probabilities using combinations.

BUILD YOUR VOCABULARY (pages 276–277)

An arrangement or listing in which order or placement is important is called a **permutation**.

An arrangement or listing in which order is not important is called a **combination**.

KEY CONCEPT

Permutation The number of permutations of *n* objects taken *r* at a time is the quotient of *n*! and (*n* – *r*)!.

FOLDABLES Under the permutation tab, record this definition in words and in symbols.

EXAMPLE Permutation and Probability

① **Shaquille has a 5-digit code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.**

a. How many different pass codes could Shaquille have?

This situation is a permutation of 5 digits taken 5 at a time.

$${}_nP_t = \frac{n!}{(n-r)!}$$ Definition of permutation

$${}_5P_5 = \frac{5!}{(5-5)!}$$ $n = 5$, $r = 5$

$${}_5P_5 = \frac{\boxed{}}{1} \text{ or } \boxed{}$$ Definition of factorial

There are $\boxed{}$ possible pass codes.

b. What is the probability that the first two digits of his code are both greater than 5?

There are $\boxed{}$ digits greater than 5 and $\boxed{}$ digits less than 5. The number of choices for the first two digits is $\boxed{}$. The number of choices for the remaining digits is $\boxed{}$. So, the number of favorable outcomes is $2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$ or $\boxed{}$.

$$P(\text{first 2 digits} > 5) = \boxed{} \begin{array}{l} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{array}$$

$$= \boxed{} \text{ or } 10\%$$ Simplify.

Check Your Progress Bridget and Brittany are trying to find a house, but they cannot remember the address. They can remember only that the digits used are 1, 2, 5, and 8, and that no digit is used twice. Find the number of possible addresses. Then find the probability that the first two numbers are odd.

EXAMPLE Combinations and Probability

② Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag. What is the probability that she will pull two pennies and two nickels out of the bag?

The number of combinations of 22 coins taken 4 at a time is

$${}_{22}C_4 = \frac{22!}{(22-4)!4!} \text{ or } \boxed{}.$$

Using the Fundamental Counting Principle, the answer can be determined with the product of the two combinations.

$$({}_{10}C_2)({}_{6}C_2) = \frac{10!}{(10-2)!2!} \cdot \frac{6!}{(6-2)!2!}$$ Definition of combination

$$= \boxed{} \cdot \frac{6!}{4!2!}$$ Simplify.

$$= \frac{10 \cdot 9}{2!} \cdot \frac{6 \cdot 5}{2!}$$ Divide the first term by its GCF and the second term by its GCF.

$$= \boxed{}$$

There are $\boxed{}$ ways to choose this particular combination out of 7315 possible combinations.

$$P(2 \text{ pennies, } 2 \text{ nickels}) = \boxed{} \begin{matrix} \leftarrow \\ \leftarrow \end{matrix} \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$= \boxed{}$$ Simplify.

KEY CONCEPT

Combination The number of combinations of *n* objects taken *r* at a time is the quotient of *n*! and (*n* – *r*!)*r*!.

FOLDABLES Under the combination tab, record this definition in words and in symbols.

Check Your Progress At a factory, there are 10 union workers, 12 engineers, and 5 foremen. The company needs 6 of these workers to attend a national conference. If the workers are chosen randomly, what is the probability that 3 union workers, 2 engineers, and 1 foreman are selected?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–4 Probability of Compound Events

Reinforcement of TEKS 8.11 The student applies concepts of theoretical and experimental probability to make predictions. **(A)** Find the probabilities of dependent and independent events.

MAIN IDEAS

- Find the probability of two independent events or dependent events.
- Find the probability of two mutually exclusive events or inclusive events.

BUILD YOUR VOCABULARY (pages 276–277)

A **compound event** is made up of ______ or more ______ events.

Independent events are events in which the outcome of one event does not ______ the outcome of the other. When the outcome of one event ______ the outcome of another event, the events are dependent events.

KEY CONCEPTS

Probability of Independent Events If two events, *A* and *B*, are independent, then the probability of both events occuring is the product of the probability of *A* and the probability of *B*.

Probability of Dependent Events If two events, *A* and *B*, are dependent, then the probability of both events occuring is the product of the probability of *A* and the probability of *B* after *A* occurs.

EXAMPLE Independent Events

① **Rae is flying from Birmingham to Chicago. She has to fly from Birmingham to Houston on the first leg of her trip. In Houston she changes planes and heads to Chicago. The airline reports that the flight from Birmingham to Houston has a 90% on time record, and the flight from Houston to Chicago has a 50% on time record. What is the probability that both flights will be on time?**

$P(A \text{ and } B) = P(A) \bullet P(B)$ Definition of independent events

$P(\text{B–H on time and H–C on time})$

$$= \underbrace{P(\text{B–H on time})}_{\boxed{}} \bullet \underbrace{p(\text{H–C on time})}_{\boxed{}}$$

$= \boxed{}$ or 45% Multiply.

Check Your Progress Two cities, Fairfield and Madison, lie on different faults. There is a 60% chance that Fairfield will experience an earthquake by the year 2010 and a 40% chance that Madison will experience an earthquake by 2010. Find the probability that both cities will experience an earthquake by 2010.

EXAMPLE Dependent Events

② **At the school carnival, winners in the ring-toss game are randomly given a prize from a bag that contains 4 sunglasses, 6 hairbrushes, and 5 key chains. Three prizes are randomly drawn from the bag and not replaced. Find *P*(sunglasses, hairbrush, key chain).**

The selection of the first prize affects the selection of the next prize since there is one less prize from which to choose. So, the events are dependent.

1st prize: $P(\text{sunglasses}) = \boxed{}$ ← number of sunglasses / ← total number of prizes

2nd prize: $P(\text{hairbrush}) = \boxed{}$ or $\frac{3}{7}$ ← number of hairbrushes / ← total number of prizes

3rd prize: $P(\text{key chain}) = \boxed{}$ ← number of key chains / ← total number of prizes

$P(\text{sunglasses, hairbrush, key chain})$

$$= \boxed{} \cdot \boxed{} \cdot \boxed{}$$

$$= \boxed{} \text{ or } \frac{4}{91}$$

Check Your Progress **A gumball machine contains 16 red gumballs, 10 blue gumballs, and 18 green gumballs. Once a gumball is removed from the machine, it is not replaced. Find each probability if the gumballs are removed in the order indicated.**

a. *P*(red, green, blue) ☐

b. *P*(green, blue, not red) ☐

BUILD YOUR VOCABULARY (pages 276–277)

The events for drawing a marble that is green and for drawing a marble that is ☐ green are called **complements**.

Events that cannot occur at the ☐ time are called **mutually exclusive.**

Two events that ☐ occur at the same time are called **inclusive events.**

KEY CONCEPTS

Mutually Exclusive Events If two events, *A* and *B*, are mutually exlcusive, then the probability that either *A* or *B* occurs is the sum of their probabilities.

Probability of Inclusive Events If two events, *A* and *B*, are inclusive, then the probability that either *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occuring.

FOLDABLES™ Take notes on how to find the probability of compound events.

EXAMPLE Mutually Exclusive Events

③ **Alfred is going to the Lakeshore Animal Shelter to pick a new pet. Today, the shelter has 8 dogs, 7 cats, and 5 rabbits available for adoption. If Alfred randomly picks an animal to adopt, what is the probability that the animal would be a cat or a dog?**

$P(\text{cat}) = \boxed{}$ $P(\text{dog}) = \boxed{}$

$P(\text{cat or dog}) = \underbrace{P(\text{cat})} + \underbrace{P(\text{dog})}$ Mutually exclusive events

$= \boxed{} + \boxed{}$ Substitution

$= \frac{15}{20} \text{ or } \frac{3}{4}$ Add.

EXAMPLE Inclusive Events

④ **A dog has just given birth to a litter of 9 puppies. There are 3 brown females, 2 brown males, 1 mixed-color female, and 3 mixed-color males. If you choose a puppy at random from the litter, what is the probability that the puppy will be male or mixed-color?**

These events are inclusive.

$P(\text{male or mixed-color})$

$= \underbrace{P(\text{male})} + \underbrace{P(\text{mixed-color})} - \underbrace{P(\text{male and mixed-color})}$

$= \boxed{} + \boxed{} - \boxed{}$

$= \boxed{} \text{ or } \frac{2}{3} \approx 67\%$ Simplify.

Check Your Progress

a. The French Club has 16 seniors, 12 juniors, 15 sophomores, and 21 freshmen as members. What is the probability that a member chosen at random is a junior or a senior?

b. In Mrs. Kline's class, 7 boys have brown eyes and 5 boys have blue eyes. Out of the girls, 6 have brown eyes and 8 have blue eyes. If a student is chosen at random from the class, what is the probability that the student will be a boy or have brown eyes?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–5

Probability Distributions

Reinforcement of TEKS 8.11 The student applies concepts of theoretical and experimental probability to make predictions. **(B)** Use theoretical probabilities and experimental results to make predictions and decisions.

MAIN IDEAS

- Use random variables to compute probability.
- Use probability distributions to solve real-world predictions.

EXAMPLE Random Variable

① **The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table.**

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9

a. Find the probability that a randomly chosen customer has at most 2 pets.

There are 3 + 37 + 33 or outcomes in which a customer owns at most 2 pets. There are survey results.

$P(X \leq 2) =$ or

b. Find the probability that a randomly chosen customer has 2 or 3 pets.

There are + or outcomes in which a customer owns 2 or 3 pets.

$P(X = 2 \text{ or } 3) =$ or

Check Your Progress **A survey was conducted concerning the number of movies people watch at the theater per month. The results of this survey are shown in the table.**

Movies (per month)	Numbers of People
0	7
1	23
2	30
3	29
4	11

a. Find the probability that a randomly chosen person watched at most 1 movie per month.

b. Find the probability that a randomly chosen person watches 0 or 4 movies per month.

BUILD YOUR VOCABULARY (pages 276–277)

The probability of every possible ______ of the random variable X is called a **probability distribution.**

The ______ distribution for a random variable can be given in a table or in a **probability histogram.**

KEY CONCEPT

Properties of Probability Distributions

1. The probability of each value of X is greater than or equal to 0 and less than or equal to 1.
2. The probabilities of all the values of X add up to 1.

EXAMPLE Probability Distribution

② **The table shows the probability distribution of the number of students in each grade at Sunnybrook High School. If a student is chosen at random, what is the probability that he or she is in grade 11 or above?**

X = grade	$P(X)$
9	0.29
10	0.26
11	0.25
12	0.2

The probability of a student being in grade 11 or above is the sum of the probability of grade 11 and the probability of grade 12.

$P(X \geq 11) = P(X = 11) + P(X = 12)$ Sum of individual probabilities

$= \boxed{} + \boxed{}$ or $\boxed{}$ $P(X = 11) = \boxed{}$

$P(X = 12) = \boxed{}$

The probability is ______.

Check Your Progress The table shows the probability distribution of the number of children per family in the city of Maplewood. If a family was chosen at random, what is the probability that they have at least 2 children?

X = Number of Children	$P(X)$
0	0.11
1	0.23
2	0.32
3	0.26
4	0.08

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–6 Probability Simulations

Reinforcement of TEKS 8.11 The student applies concepts of theoretical and experimental probability to make predictions. **(C)** Select and use different models to simulate an event.

MAIN IDEAS

- Use theoretical and experimental probability to represent and solve problems involving uncertainty.
- Perform probability simulations to model real-world situations involving uncertainty.

BUILD YOUR VOCABULARY (pages 276–277)

Theoretical probabilities are determined ☐ and describe what should happen.

Experimental probability is determined using data from tests or ☐.

Experimental probability is the ☐ of the number of times an outcome occurred to the ☐ number of events or trials. This ratio is also known as the **relative frequency.**

WRITE IT

Where is data obtained for experimental probability?

EXAMPLE Experimental Probability

① **Miguel shot 50 free throws in the gym and found that his experimental probability of making a free throw was 40%. How many free throws did Miguel make?**

Miguel made ☐ out of every 100 free throws.

experimental probability = 40% or $\frac{\square}{\square}$ ← number of success ← total number of free throws

Miguel shot 50 free throws. Write and solve a proportion.

experimental successes → ← Miguel's successes

experimental total free throws → ← Miguel's total free throws

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$$50(40) = 100(x)$$ Find the cross products.

$$\square = \square$$ Simplify.

$$x = \square$$ Divide each side by 100.

Miguel made ☐ free throws.

Check Your Progress Nancy was testing her serving accuracy in volleyball. She served 80 balls and found that the experimental probability of keeping it in bounds was 60%. How many serves did she keep in bounds?

BUILD YOUR VOCABULARY (pages 276–277)

When you ______ an experiment repeatedly, collect and combine the ______, and ______ the results, this is known as an **empirical study.**

A **simulation** allows you to use objects to act out an ______ that would be difficult or impractical to perform.

EXAMPLE Empirical Study

② **A pharmaceutical company performs three clinical studies to test the effectiveness of a new medication. Each study involved 100 volunteers. The results of the studies are shown in the table.**

Result	Study 1	Study 2	Study 3
Expected Success Rate	70%	70%	70%
Condition Improved	61%	74%	67%
No improvement	39%	25%	33%
Condition Worsened	0%	1%	0%

What is the experimental probability that the drug showed no improvement in patients for all three studies?

The number of outcomes with no improvement for the three studies was 39 + 25 + 33 or ______ out of the 300 total patients.

experimental probability = ______ or about ______

Check Your Progress A new study is being developed to analyze the relationship between heart rate and watching scary movies. A researcher performs three studies, each with 100 volunteers. Based on similar studies, the researcher expects that 80% of the subjects will experience a significant increase in heart rate. The table shows the results of the study. What is the experimental probability that the movie would cause a significant increase in heart rate for all three studies?

Result	Study 1	Study 2	Study 3
Expected Success Rate	80%	80%	80%
Rate increased significantly	83%	75%	78%
Littler or no increase	16%	24%	19%
Rate decreased	1%	0%	0%

EXAMPLE Simulation

③ **In the last 30 school days, Bobbie's older brother has given her a ride to school 5 times.**

a. What could be used to simulate whether Bobbie's brother will give her a ride to school?

Bobbie got a ride to school $\frac{3}{50}$ or $\frac{1}{6}$ days. Since a die has ______ sides, you could use one side of a die to represent a ride to school.

b. Describe a way to simulate whether Bobbie's brother will give her a ride to school in the next 20 school days.

Choose the side of the die that will be used to represent a ride to school. Let the 1-side of the die equal a ride to school.

Toss the die ______ times and record each result.

REMEMBER IT

A spinner should simulate the possible outcomes of the event.

Check Your Progress In the last 52 days, it has rained 4 times. What could be used to simulate whether it will rain on a given day? Describe a way to simulate whether it will rain in the next 15 days.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 12

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™

Use your **Chapter 12** Foldable to help you study for your chapter test.

VOCABULARY PUZZLEMAKER

To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 12, go to:

glencoe.com

BUILD YOUR VOCABULARY

You can use your completed **Vocabulary Builder** *(pages 276–277)* to help you solve the puzzle.

12-1

Sampling and Bias

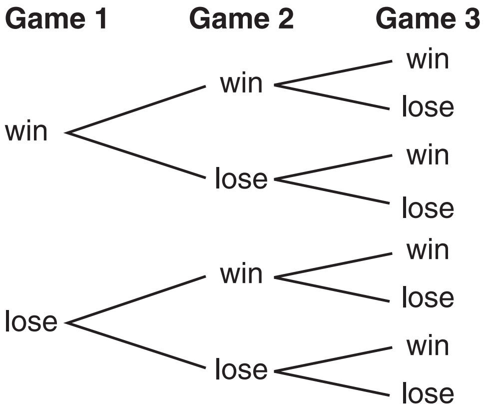
Suppose the principal at a school wants to use Saturdays as make-up days when school is closed due to weather. The principal selects and then polls a group of students to see if the student body supports the idea. Complete the sentences.

1. The student body is the ____ from which a ____ of students is selected to be polled. If all the students are polled, it is called a ____.

12-2

Counting Outcomes

Use the tree diagram for Exercises 2–4.



2. What is the sample space?

____

3. Name two different outcomes.

____

4. Use the Fundamental Counting Principle to find the possible outcomes shown above.

	Game 1		Game 2		Game 3		Number of Outcomes
Number of choices		•		•		=	

12-3

Permutations and Combinations

5. Three of seven students are chosen to go to a job fair. How many different groups of students could be selected?

12-4

Probability of Compound Events

A die is rolled and a card is drawn from a standard deck of 52 cards. Find each probability.

6. *P*(6 and queen)

7. *P*(greater than 1 and red ace)

12-5

Probability Distributions

The table shows the probability of various family sizes in the United States.

8. For each value of X, is the probability greater than or equal to 0 and less than or equal to 1?

9. What is the sum of the probabilities?

10. Is the probability distribution valid?

Family Size (United States)	
X = Size of Family	Probability
2	0.42
3	0.23
4	0.21
5	0.10
6	0.03
7	0.01

12-6

Probability Simulations

11. Choose the manipulative you would use to simulate the problem. Explain your choice.

Situation	Simulation method
58% of drivers (commercial and private vehicles) have a cell phone in their car. Simulate whether or not the next 10 drivers you meet on the road will have a cell phone.	• die • coins • marbles • spinner

CHAPTER 12
Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 12.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 12 Practice Test on page 681 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 12 Study Guide and Review on pages 676–680 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 681.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 12 Foldable.
- Then complete the Chapter 12 Study Guide and Review on pages 676–680 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 681.

Student Signature

Parent/Guardian Signature

Teacher Signature