Lesson 14–2

Example 1 Graph Horizontal Translations

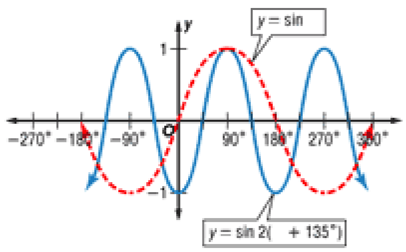
State the amplitude, period, and phase shift for each function. Then graph the function.

a. $y = \sin 2(\theta + 135^\circ)$

Since $a = 1$, the amplitude is the same as $y = \sin\theta$. However, $b = 2$, so the period is $\frac{360^\circ}{|2|}$ or 180°.

Because $h = -135^\circ$ and $h < 0$, the phase shift is 135° to the left.

To graph $y = \sin 2(\theta + 135^\circ)$, consider the graph of $y = \sin\theta$. Change the period of this graph to 180° and then shift the graph 135° to the left.



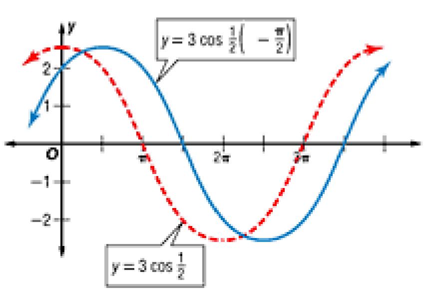
b. $y = 3\cos\frac{1}{2}\left(\theta - \frac{\pi}{2}\right)$

Amplitude: $a = |3|$ or 3

Period: $\frac{2\pi}{\frac{1}{2}}$ or 4π

Phase shift: $h = \frac{\pi}{2}$

The phase shift is to the right since $\frac{\pi}{2} > 0$.



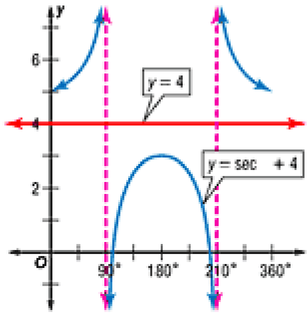
Example 2 Graph Vertical Translations

State the vertical shift, equation of the midline, amplitude, and period for each function. Then graph the function.

a. $y = \sec\theta + 4$

In $\sec\theta + 4$, $k = 4$, so the vertical shift is 4. Draw the midline, $y = 4$. The cosecant function has no amplitude and the period is 360˚.

Draw the graph of the function relative to the midline.



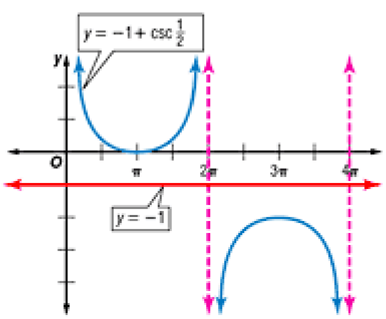
b. $y = -1 + \csc \frac{1}{2}\theta$

Vertical shift: $k = -1$, so the midline is the graph if $y = -1$.

Amplitude: none

Period: $\frac{2\pi}{\left|\frac{1}{2}\right|}$ or 4π

Draw the midline of graph. Then draw the graph of the function relative to the midline.



Example 3 Graph Transformations

State the vertical shift, amplitude, period, and phase shift of $y = 3 \sin [4(\theta + 45^\circ)] + 1$. Then graph the function.

The function is written in the form $y = a \sin [b(\theta - h)] + k$. Identify the values of k, a, b, and h.

$k = 1$, so the vertical shift is 1.

$a = 3$, so the amplitude is $| 3 |$ or 3.

$b = 4$, so the period is $\frac{360\text{Þ}}{| 4 |}$ or 90°.

$h = -45^\circ$, so the phase shift is 45° to the left.

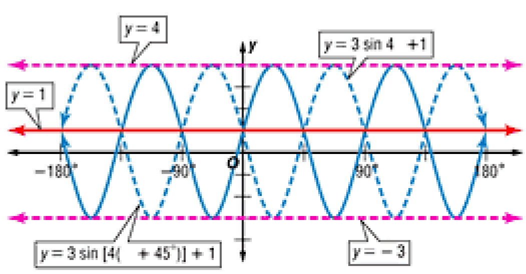
Then graph the function.

Step 1 The vertical shift is 1. Graph the midline $y = 1$.

Step 2 The amplitude is 3. Draw dashed lines 3 units above and below the midline at $y = 4$ and $y = -2$.

Step 3 The period is 90°, so the graph will have a shorter period that $y = \sin \theta$. Graph $y = 3 \sin 4\theta + 1$ using the midline as a reference.

Step 4 Shift the graph 45° to the left.



Example 4 Use Translations to Solve a Problem

WEATHER The monthly normal temperatures for San Diego, CA, can be approximately modeled by the sine function $y = 8 \sin \frac{\pi}{6}(x - 5) + 65$, where y is the temperature and x is the integer representing the month. The months are represented by the integers 1, 2, … , 12 and the temperature is in degrees Fahrenheit. Use a graph to estimate the highest monthly normal temperature for San Diego. During which month does the temperature occur?

Explore You know that the highest point of the graph of the function will show the highest temperature. Then, to find the month, locate the closest integer on the x–axis to that point.

Plan Graph the function and locate the highest point of the sine curve.

Solve The function is written in the form $y = a \sin [b(\theta - h)] + k$. Identify the values of k, a, b, and h.

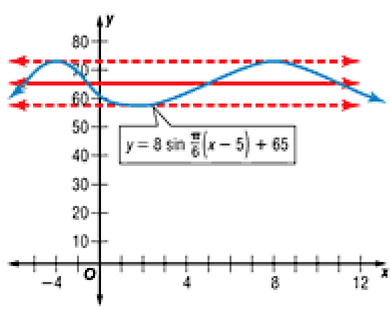
$k = 65$, so the vertical shift is 65.

$a = 8$, so the amplitude is | 8 | or 8.

$b = \frac{\pi}{6}$, so the period is $\frac{2\pi}{\left|\frac{\pi}{6}\right|}$ or 12.

$h = \frac{\pi}{6}(5)$ or 2.6, so the phase shift is 2.6 to the right.

Now graph the function.



Step 1 The vertical shift is 65. Graph the midline $y = 65$.

Step 2 The amplitude is 8. Draw dashed lines 8 units above and below the midline at $y = 73$ and $y = 57$.

Step 3 The period is 12.

Step 4 Shift the graph 2.6 units to the right.

Since the function approximates the temperatures for San Diego, an estimate for the highest temperature is 73° which occurs in the 8th month, or August.

Examine It is reasonable that the highest temperature would occur in August and since San Diego is on the coast, but in the southern U.S., 73˚ is a likely high temperature.