Lesson 14–3

Example 1 Find a Value of a Trigonometric Function

a. Find tan θ if cos $\theta = \frac{\sqrt{2}}{3}$ and $0^\circ < \theta < 90^\circ$.

First, you must find sin θ.

$\cos^2\theta + \sin^2\theta = 1$	Trigonometric identity
$\sin^2\theta = 1 - \cos^2\theta$	Subtract $\cos^2\theta$ from each side.
$\sin^2\theta = 1 - \left(\frac{\sqrt{2}}{3}\right)^2$	Substitute $\frac{\sqrt{2}}{3}$ for cos θ.
$\sin^2\theta = 1 - \frac{2}{9}$	Square $\frac{\sqrt{2}}{3}$.
$\sin^2\theta = \frac{7}{9}$	Subtract.
$\sin\theta = \pm\frac{\sqrt{7}}{3}$	Take the square root of each side.

Since θ is in the first quadrant, sin θ is positive. Now, find tan θ.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$	Trigonometric identity
$\tan\theta = \frac{\frac{\sqrt{7}}{3}}{\frac{\sqrt{2}}{3}}$	Substitute $\frac{\sqrt{7}}{3}$ for sin θ and $\frac{\sqrt{2}}{3}$ for cos θ.
$\tan\theta = \frac{\sqrt{7}}{\sqrt{2}}$ or $\frac{\sqrt{14}}{2}$	Simplify.

Therefore, $\tan\theta = \frac{\sqrt{14}}{2}$.

b. Find sec θ if cot $\theta = \frac{\sqrt{6}}{2}$ and $180^\circ < \theta < 270^\circ$.

If $\cot\theta = \frac{\sqrt{6}}{2}$, then $\tan\theta = \frac{2}{\sqrt{6}}$ or $\frac{\sqrt{6}}{3}$.

$\tan^2\theta + 1 = \sec^2\theta$	Trigonometric identity
$\left(\frac{\sqrt{6}}{3}\right)^2 + 1 = \sec^2\theta$	Substitute $\frac{\sqrt{6}}{3}$ for tan θ.
$\frac{2}{3} + 1 = \sec^2\theta$	Square $\frac{\sqrt{6}}{3}$.

$$\frac{5}{3} = \sec^2 \theta \qquad \text{Add.}$$

$$\pm\sqrt{\frac{5}{3}} = \sec \theta \qquad \text{Take the square root of each side.}$$

$$\pm\frac{\sqrt{15}}{3} = \sec \theta \qquad \text{Simplify.}$$

Since θ is in the third quadrant, $\sec \theta$ is negative. Thus, $\sec \theta = -\frac{\sqrt{15}}{3}$.

Example 2 Simplify an Expression

Simplify each expression.

a. $\sin \theta(1 + \cot^2 \theta)$

$$\sin \theta (1 + \cot^2 \theta) = \sin \theta (\csc^2 \theta) \qquad \cot^2 \theta + 1 = \csc^2 \theta$$

$$= \sin \theta \left(\frac{1}{\sin^2 \theta}\right) \qquad \csc^2 \theta = \frac{1}{\sin^2 \theta}$$

$$= \frac{1}{\sin \theta} \qquad \text{Multiply.}$$

$$= \csc \theta$$

b. $\csc^2 \theta - \frac{\cos^2 \theta}{\sin^2 \theta}$

$$\csc^2 \theta - \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} \qquad \csc^2 \theta = \frac{1}{\sin^2 \theta}$$

$$= \frac{1 - \cos^2 \theta}{\sin^2 \theta} \qquad \text{Subtract.}$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta} \qquad \cos^2 \theta + \sin^2 \theta = 1 \text{ or } 1 - \cos^2 \theta = \sin^2 \theta$$

$$= 1 \qquad \text{Simplify.}$$

Example 3 Simplify and Use an Expression

The amount of light that a source provides to a surface is called illuminance. The illuminance *E* in foot candles on a surface is related to the distance *R* in feet from the light source. The formula $\sec \theta = \frac{I}{ER^2}$, where *I* is the intensity of the light source measured in candles and θ is the angle between the light beam and a line perpendicular to the surface, can be used in situations in which lighting is important.

a. Solve the formula in terms of *R*.

$$\sec \theta = \frac{I}{ER^2} \qquad \text{Original equation}$$

$$ER^2 (\sec \theta) = I \qquad \text{Multiply each side by } ER^2.$$

$R^2 = \dfrac{I}{E \sec \theta}$ Divide each side by $E \sec \theta$.

$R^2 = \dfrac{I \cos \theta}{E}$ $\cos \theta = \dfrac{1}{\sec \theta}$

$R = \sqrt{\dfrac{I \cos \theta}{E}}$ Take the square root of each side. Solve for the positive root since distance must be positive.

b. As θ increases from 0˚ to 90˚, how does the value of *R* change?

If $I > E$, the value will decrease as θ increases from 0˚ to 90˚ since $\dfrac{I}{E} > 1$ and $\cos \theta$ decreases from 1 to 0.

If $I < E$, the value will also decrease as θ increases from 0˚ to 90˚ since $\dfrac{I}{E} < 1$ and $\cos \theta$ decreases from 1 to 0.

Therefore, the value of *R* will decrease since the square root of decreasing numbers will decrease.