Lesson 14–4

Example 1 Transform One Side of an Equation

Verify that $\sec^2\theta - \tan\theta\cot\theta = \tan^2\theta$ is an identity.

Transform the left side.

$\sec^2\theta - \tan\theta\cot\theta \stackrel{?}{=} \tan^2\theta$ Original equation

$\sec^2\theta - \tan\theta\left(\frac{1}{\tan\theta}\right) \stackrel{?}{=} \tan^2\theta$ $\cot\theta = \frac{1}{\tan\theta}$

$\sec^2\theta - 1 \stackrel{?}{=} \tan^2\theta$ Multiply.

$(\tan^2\theta + 1) - 1 \stackrel{?}{=} \tan^2\theta$ $\sec^2\theta = \tan^2\theta + 1$

$\tan^2\theta = \tan^2\theta$ Subtract.

Example 2 Find an Equivalent Expression

Multiple–Choice Test Item

$\tan\theta + \cot\theta =$

A. $\sec\theta + \csc\theta$ **B.** $\sec\theta\csc\theta$ **C.** $\sec^2\theta + \csc^2\theta$ **D.** $\frac{\sec\theta}{\csc\theta}$

Read the Test Item

Find an expression that is equal to the given expression.

Solve the Test Item

Write a trigonometric identity by using the basic trigonometric identities and the definitions of trigonometric functions to transform the given expression to match one of the choices.

$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin\theta(\sin\theta) + \cos\theta(\cos\theta)}{\cos\theta\sin\theta}$ Write the fractions with a common denominator.

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ Simplify.

$= \frac{1}{\cos\theta\sin\theta}$ $\sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$ Factor.

$= \sec\theta\csc\theta$ $\sec\theta = \frac{1}{\cos\theta}$ and $\csc\theta = \frac{1}{\sin\theta}$

Since $\tan\theta + \cot\theta = \sec\theta\csc\theta$, the answer is B.

Example 3 Verify by Transforming Both Sides

Verify that $\frac{\sin\theta}{\csc\theta}+\frac{\cos\theta}{\sec\theta} = \csc^2\theta - \cot^2\theta$ is an identity.

$\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} \stackrel{?}{=} \csc^2\theta - \cot^2\theta$ Original equation

$\frac{\sin\theta}{\frac{1}{\sin\theta}} + \frac{\cos\theta}{\frac{1}{\cos\theta}} \stackrel{?}{=} (1 + \cot^2\theta) - \cot^2\theta$ Express terms on each side with the same functions.

$\sin^2\theta + \cos^2\theta = 1$ Simplify both sides.