Lesson 14–6

Example 1 Double–Angle Formulas

Find the exact value of each expression if cos $\theta = -\frac{2}{5}$ and θ is between 180° and 270°.

a. sin 2θ

Use the identity $\sin 2\theta = 2 \sin \theta \cos \theta$.
First, find the value of cos θ.

$\sin^2 \theta = 1 - \cos^2 \theta$	$\cos^2 \theta + \sin^2 \theta = 1$
$\sin^2 \theta = 1 - (-\frac{2}{5})^2$	$\cos \theta = -\frac{2}{5}$
$\sin^2 \theta = \frac{21}{25}$	Subtract.
$\sin \theta = \pm\frac{\sqrt{21}}{5}$	Take the square root of each side.

Since θ is in the third quadrant, sin is negative. Thus, $\sin \theta = -\frac{\sqrt{21}}{5}$.

Now find sin 2θ.

$\sin 2\theta = 2 \sin \theta \cos \theta$	Double–Angle Formula
$\sin 2\theta = 2(-\frac{\sqrt{21}}{5})(-\frac{2}{5})$	$\sin \theta = -\frac{\sqrt{21}}{5}$, $\cos \theta = -\frac{2}{5}$
$= \frac{4\sqrt{21}}{25}$	Multiply.

The value of sin 2θ is $\frac{4\sqrt{21}}{25}$.

b. cos 2θ

Use the identity $\cos 2\theta = 1 - 2\sin^2 \theta$.

$\cos 2\theta = 1 - 2\sin^2 \theta$	Double–Angle Formula
$= 1 - 2(-\frac{\sqrt{21}}{5})^2$	$\sin \theta = -\frac{\sqrt{21}}{5}$
$= -\frac{17}{25}$	Simplify.

Example 2 Half–Angle Formulas

Find $\sin \frac{\alpha}{2}$ if $\cos \alpha = -\frac{2}{5}$ and α is in the second quadrant.

$$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{2}}$$ Half–Angle Formula

$$= \pm\sqrt{\frac{1 - \left(-\frac{2}{5}\right)}{2}}$$ $\cos \alpha = -\frac{2}{5}$

$$= \pm\sqrt{\frac{7}{10}}$$ Simplify the radicand.

$$= \pm\frac{\sqrt{70}}{10}$$ Rationalize.

Since α is between 90˚ and 180˚, $\frac{\alpha}{2}$ is between 45˚ and 90˚. Thus, $\sin \frac{\alpha}{2}$ is positive and equals $\frac{\sqrt{70}}{10}$.

Example 3 Evaluate Using Half–Angle Formulas

Find the exact value of each expression by using the half–angle formulas.

a. cos 112.5˚

$$\cos 112.5 = \cos \frac{225}{2}$$

$$= -\sqrt{\frac{1+\cos 225^\circ}{2}}$$ $\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$; 112.5˚ is in the second quadrant where cosine is negative

$$= -\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$ $\cos 225^\circ = -\frac{\sqrt{2}}{2}$

$$= -\sqrt{\frac{2 - \sqrt{2}}{4}}$$ Simplify the radicand.

$$= -\frac{\sqrt{2 - \sqrt{2}}}{2}$$ Simplify the denominator.

b. $\mathbf{\sin \frac{5\pi}{8}}$

$$\sin \frac{5\pi}{8} = \sin \frac{\frac{5\pi}{4}}{2}$$

$$= \sqrt{\frac{1 - \cos\alpha}{2}}$$

$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$; $\frac{5\pi}{8}$ is in the second quadrant where sine is positive

$$= \sqrt{\frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

$$= \sqrt{\frac{2 + \sqrt{2}}{4}}$$

Simplify the radicand.

$$= \frac{\sqrt{2 + \sqrt{2}}}{2}$$

Simplify the denominator.

Example 4 **Verify Identities**

Verify that $\left(\sin \frac{x}{2}\right)\left(\cos \frac{x}{2}\right) = \frac{\sin x}{2}$ **is an identity.**

$$\left(\sin \frac{x}{2}\right)\left(\cos \frac{x}{2}\right) \stackrel{?}{=} \frac{\sin x}{2}$$

Original equation

$$\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2} \stackrel{?}{=} \frac{\sin x}{2}$$

Rewrite.

$$\frac{\sin 2\left(\frac{x}{2}\right)}{2} \stackrel{?}{=} \frac{\sin x}{2}$$

Simplify.

$$\frac{\sin x}{2} = \frac{\sin x}{2}$$

Multiply 2 and $\frac{x}{2}$.