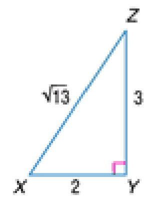
Lesson 13–1

Example 1 Find Trigonometric Values

Find the values of the six trigonometric functions for angle θ.



For this triangle, the side opposite θ is $\overline{YZ}$, and the side adjacent to θ is $\overline{XY}$. Recall that the hypotenuse is always the longest side of a right triangle, in this case $\overline{XZ}$.

Use opp = 3, adj = 2, and hyp = $\sqrt{13}$ to write each trigonometric ratio.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{13}}$ or $\frac{2\sqrt{13}}{13}$ $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{13}}{3}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{13}}{2}$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{3}$

Example 2 Use One Trigonometric Ratio to Find Another

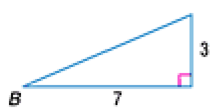
Multiple–Choice Test Item

If tan *B* = $\frac{3}{7}$, find the value of sin *B*.

A. $\frac{3\sqrt{10}}{20}$ **B.** $\frac{7\sqrt{10}}{20}$ **C.** $\frac{7\sqrt{58}}{58}$ **D.** $\frac{3\sqrt{58}}{58}$

Read the Test Item

Begin by drawing a right triangle and labeling one acute angle *B*. The simplest measures to use so that tan *B* = $\frac{3}{7}$ are to label the opposite side as 3 and the adjacent side as 7.



Solve the Test Item

Use the Pythagorean Theorem to find an expression for the measure of the hypotenuse of the triangle.

$a^2 + b^2 = c^2$ Pythagorean Theorem
$7^2 + 3^2 = c^2$ Replace *a* with 7 and *b* with 3.
$49 + 9 = c^2$ Simplify.
$58 = c^2$ Add.
$\sqrt{58} = c$ Take the square root of each side.

Now find sin *B*.

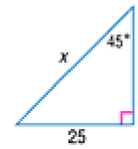
$\sin B = \frac{\text{opp}}{\text{hyp}}$ Sine ratio

$= \frac{3}{\sqrt{58}}$ or $\frac{3\sqrt{58}}{58}$ Replace opp with 3 and hyp with $\sqrt{58}$.

The answer is D.

Example 3 **Find the Missing Side Length of a Right Triangle**

Write an equation involving sin, cos, or tan that can be used to find the value of *x*. Then solve the equation. Round your answer to the nearest tenth.



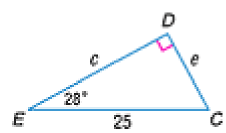
The measure of the side opposite the angle is 25, the given angle measures 45˚, and the missing length is the hypotenuse of the triangle. The trigonometric function relating the opposite side of a right triangle and the hypotenuse is the sine function.

$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	Sine ratio
$\sin 45^\circ = \frac{25}{x}$	Replace θ with 45˚, opp with 25, and hyp with x.
$\frac{\sqrt{2}}{2} = \frac{25}{x}$	$\sin 45^\circ = \frac{\sqrt{2}}{2}$
$\sqrt{2}\,x = 50$	Cross multiply.
$x = \frac{50}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$x = 25\sqrt{2}$	Simplify.

The value of x is $25\sqrt{2}$ or about 35.4.

Example 4 **Solve a Right Triangle**

Solve ΔCDE. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



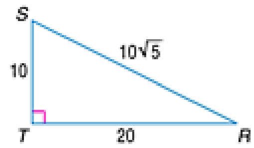
You know the measure of the hypotenuse, one acute angle, and the right angle. You need to find *c*, *e*, and *C*.

Find *c* and *e*.	$\sin 28^\circ = \frac{e}{25}$	$\cos 28^\circ = \frac{c}{25}$
	$25 \sin 28^\circ = e$	$25 \cos 28^\circ = c$
	$11.7 \approx e$	$22.1 \approx c$
Find *C*.	$28^\circ + C = 90^\circ$	Angles *C* and *E* are complementary.
	$C = 62^\circ$	Solve for *C*.

Therefore, $C = 62^\circ$, $c \approx 22.1$, and $e \approx 11.7$.

Example 5 Find Missing Angle Measures of Right Triangles

Solve ΔRST. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



You know the measures of the sides. You need to find *S* and *R*.

Find *S*. $\sin S = \frac{20}{10\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$ $\sin S = \frac{\text{opp}}{\text{hyp}}$

Use a calculator and the $\sin^{-1}$ function to find the angle whose sine is $\frac{2\sqrt{5}}{5}$.

KEYSTROKES: [2nd] [SIN^{-1}] 2 [×] [2nd] [√] 5 [)] [÷] 5 [)] [ENTER] 63.43494882

To the nearest degree, $S \approx 63^\circ$.

Find *R*. $63^\circ + R = 90^\circ$ Angles *R* and *S* are complementary.

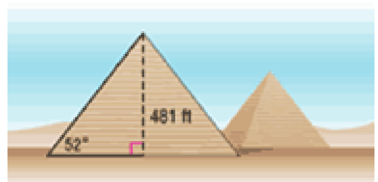
$R = 27^\circ$ Solve for *R*.

Therefore, $S \approx 63^\circ$ and $R \approx 27^\circ$.

Example 6 Indirect Measurement

HISTORY The Great Pyramid of Khufu in Egypt is one of the Seven Wonders of the World. Its original height was 481 feet. The angle that the sides make with the square base is about 52°. What is the length of one side of the base?

Make a drawing of the situation. The base will be twice the length of *a* shown in the diagram. Write an equation using a trigonometric function that involves the ratio of the length *a* and 481.



$\tan 52^\circ = \frac{481}{a}$ $\quad$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$a = \frac{481}{\tan 52^\circ}$ $\quad$ Solve for *a*.

$a \approx 375.8$ $\quad$ Use a calculator.

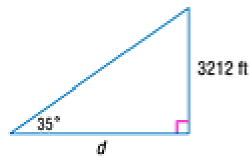
To find the length of a side of the base, find 2*a*.
$2a = 2(375.8) = 751.6$

The length of one side of the base of the pyramid is about 752 feet.

Example 7 **Use an Angle of Elevation**

GEOGRAPHY The world's highest waterfall is Angel Falls in Venezuela. It is 3,212 feet tall. If a tourist estimates the angle of elevation to the top of the falls to be 35°, what is the tourist's distance from the base of the falls?

Make a drawing of the situation. Let *d* be the tourist's distance from the base of the falls. Write an equation using a trigonometric function that involves the ratio of *d* and 3212.



$\tan 35^\circ = \dfrac{3212}{d}$ $\qquad$ $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$d = \dfrac{3212}{\tan 35^\circ}$ $\qquad$ Solve for *d*.

$d \approx 4587.2$ $\qquad$ Use a calculator.

The tourist's distance from the base of the waterfall is about 4587 feet.