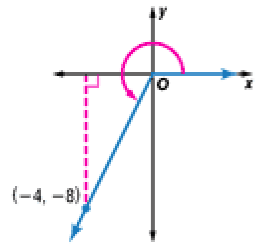
Lesson 13–3

Example 1 Evaluate Trigonometric Functions for a Given Point

Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point (–4, –8).



You know that $x = -4$ and $y = -8$. You need to find r.

$r = \sqrt{x^2 + y^2}$ — Pythagorean Theorem

$= \sqrt{(-4)^2 + (-8)^2}$ — Replace x with –4 and y with –8.

$= \sqrt{80}$ or $4\sqrt{5}$ — Simplify.

Now, use $x = -4$, $y = -8$, and $r = 4\sqrt{5}$ to write the ratios.

$\sin\theta = \frac{y}{r} = \frac{-8}{4\sqrt{5}}$ or $-\frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{x}{r} = \frac{-4}{4\sqrt{5}}$ or $-\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{y}{x} = \frac{-8}{-4}$ or 2

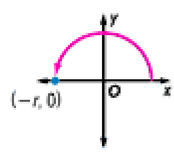
$\csc\theta = \frac{r}{y} = \frac{4\sqrt{5}}{-8}$ or $-\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{r}{x} = \frac{4\sqrt{5}}{-4}$ or $-\sqrt{5}$

$\cot\theta = \frac{x}{y} = \frac{-4}{-8}$ or $\frac{1}{2}$

Example 2 Quadrantal Angles

Find the values of the six trigonometric functions for an angle in standard position that measures 180˚.



When $\theta = 180^\circ$, $x = -r$ and $y = 0$.

$\sin\theta = \frac{y}{r} = \frac{0}{r}$ or 0

$\cos\theta = \frac{x}{r} = \frac{-r}{r}$ or -1

$\tan\theta = \frac{y}{x} = \frac{0}{-r}$ or 0

$\csc\theta = \frac{r}{y} = \frac{r}{0}$ or undefined

$\sec\theta = \frac{r}{x} = \frac{r}{-r}$ or -1

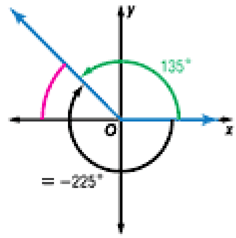
$\cot\theta = \frac{x}{y} = \frac{-r}{0}$ or undefined

Example 3 **Find the Reference Angle for a Given Angle**

Sketch each angle. Then find its reference angle.

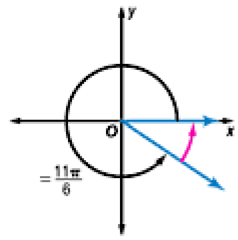
a. –225˚

A coterminal angle of –225˚ is –225˚ + 360˚ or 135˚. Because the terminal side of this angle lies in Quadrant II, the reference angle is 180˚ – 135˚ or 45˚.



b. $\mathbf{\frac{11\pi}{6}}$

Because the terminal side of $\frac{11\pi}{6}$ lies in Quadrant IV, the reference angle is $2\pi - \frac{11\pi}{6}$ or $\frac{\pi}{6}$.



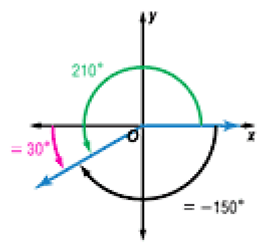
Example 4 **Use a Reference Angle to Find a Trigonometric Value**

Find the exact value of each trigonometric function.

a. sec (–150˚)

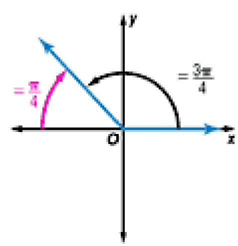
A coterminal angle of –150˚ is –150˚ + 360˚ or 210˚. Because the terminal side of the angle lies in Quadrant III, the reference angle θ' is 210˚ – 180˚ or 30˚. The secant function is negative in Quadrant III, so

$\sec(-150^\circ) = -\sec 30^\circ = -\frac{2\sqrt{3}}{3}$.



b. tan $\mathbf{\frac{3\pi}{4}}$

Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$. The tangent function is negative in Quadrant II.



$$\tan\frac{3\pi}{4} = -\tan\frac{\pi}{4}$$

$$= -\tan 45^\circ \qquad \frac{\pi}{4} \text{ radians} = 45^\circ$$

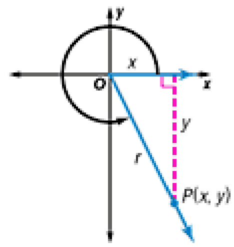
$$= -1 \qquad \tan 45^\circ = 1$$

Example 5 **Quadrant and One Trigonometric Value of θ**

Suppose θ is an angle in standard position whose terminal side is in Quadrant IV and tan $\theta = -\frac{9}{5}$.

Find the exact values of the remaining five trigonometric functions of θ.

Draw a diagram of this angle, labeling a point $P(x, y)$ on the terminal side of θ. Use the definition of tangent to find the values of x and y.



$$\tan\theta = \frac{y}{x} = -\frac{9}{5}$$

Since x is positive in Quadrant IV and y is negative, $x = 5$ and $y = –9$. Use these values and the Pythagorean Theorem to find r.

$x^2 + y^2 = r^2$	Pythagorean Theorem
$5^2 + (-9)^2 = r^2$	Replace x with 5 and y with –9.
$106 = r^2$	Simplify.
$\pm\sqrt{106} = r$	Take the square root of each side.
$\sqrt{106} = r$	The value of r is always positive.

Use $x = 5$, $y = –9$, and $r = \sqrt{106}$ to write the remaining trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{-9}{\sqrt{106}} \text{ or } -\frac{9\sqrt{106}}{106}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{106}} \text{ or } \frac{5\sqrt{106}}{106}$$

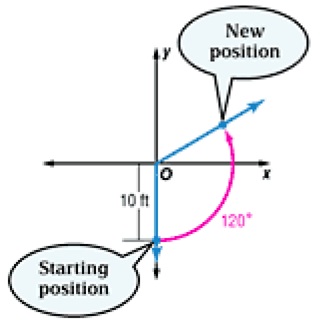
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{106}}{-9} \text{ or } -\frac{\sqrt{106}}{9}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{106}}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{5}{-9} \text{ or } -\frac{5}{9}$$

Example 6 **Find Coordinates Given a Radius and an Angle**

AMUSEMENT The diagram shows the position of a rider on a Ferris wheel at the beginning of the ride. The car is 10 feet from the center of the wheel. When the ride begins, the wheel moves the car 120˚ counterclockwise and stops the car to load another group of riders. What is new position of the car relative to the pivot point of the wheel?



The diagram shows the rider positioned on a coordinate system with the beginning position of the car at a distance of 10 feet from the origin and on the negative *y*-axis. The angle in standard position is 270˚. If the car rotates 120˚, the new position is 270˚ + 120˚ or 390˚. The reference angle for 390˚ is 30˚. The angle is in Quadrant I.

Let the new position of the car have coordinates (*x*, *y*). Then, use the definitions of sine and cosine to find the value of *x* and *y*. The value of *r* is the distance from the center of the wheel, 10 feet. Since the angle is in Quadrant I, all functions have positive values.

$\cos 30^\circ = \frac{x}{r}$	cosine ratio	$\sin 30^\circ = \frac{y}{r}$	sine ratio
$\frac{\sqrt{3}}{2} = \frac{x}{10}$	Substitute.	$\frac{1}{2} = \frac{y}{10}$	Substitute.
$x = 5\sqrt{3}$	Solve for *x*.	$y = 5$	Solve for *y*.

The exact coordinates of the car at its stopping position are $\left(5\sqrt{3},\ 5\right)$. Since $5\sqrt{3}$ is about 8.66, the car is 8.66 feet to the right of the center of the wheel and 5 feet above a horizontal line through the center of the wheel.