1-1 Lesson Reading Guide

Expressions and Formulas

Get Ready for the Lesson

Read the introduction to Lesson 1-1 in your textbook.

- Nurses use the formula $F = \frac{V \times d}{t}$ to control the flow rate for IVs. Name the quantity that each of the variables in this formula represents and the units in which each is measured.

  F represents the ________________ and is measured in ________________ per minute.

  V represents the ________________ of solution and is measured in ________________.

  d represents the ________________ and is measured in ________________ per milliliter.

  t represents ________________ and is measured in ________________.

- Write the expression that a nurse would use to calculate the flow rate of an IV if a doctor orders 1350 milliliters of IV saline to be given over 8 hours, with a drop factor of 20 drops per milliliter. Do not find the value of this expression.

Read the Lesson

1. There is a customary order for grouping symbols. Brackets are used outside of parentheses. Braces are used outside of brackets. Identify the innermost expression(s) in each of the following expressions.

a. $[(3 - 2^2) + 8] \div 4$

b. $9 - [5(8 - 6) + 2(10 + 7)]$

c. $\{14 - [8 + (3 - 12)^2]\} \div (6^3 - 100)$

2. Read the following instructions. Then use grouping symbols to show how the instructions can be put in the form of a mathematical expression.

Multiply the difference of 13 and 5 by the sum of 9 and 21. Add the result to 10. Then divide what you get by 2.

3. Why is it important for everyone to use the same order of operations for evaluating expressions?

Remember What You learned

4. Think of a phrase or sentence to help you remember the order of operations.

1-2 Lesson Reading Guide

Properties of Real Numbers

Get Ready for the Lesson

Read the introduction to Lesson 1-2 in your textbook.

- Why are all of the amounts listed on the register slip at the top of the page followed by negative signs?
- Describe two ways of calculating the amount of money you saved by using coupons if your register slip is the one shown on page 11.

Read the Lesson

1. Refer to the Key Concepts box on page 11. The numbers $2.\overline{57}$ and 0.010010001… both involve decimals that "go on forever." Explain why one of these numbers is rational and the other is irrational.

2. Write the Associative Property of Addition in symbols. Then illustrate this property by finding the sum $12 + 18 + 45$ in two different ways.

3. Consider the equations $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ and $(a \cdot b) \cdot c = c \cdot (a \cdot b)$. One of the equations uses the Associative Property of Multiplication and one uses the Commutative Property of Multiplication. How can you tell which property is being used in each equation?

Remember What You Learned

4. How can the meanings of the words *commuter* and *association* help you to remember the difference between the commutative and associative properties?

1-3 Lesson Reading Guide

Solving Equations

Get Ready for the Lesson

Read the introduction to Lesson 1-3 in your textbook.

- To find your target heart rate, what two pieces of information must you supply?
- Write an equation that shows how to calculate your target heart rate.

Read the Lesson

1. a. How are algebraic expressions and equations alike?

b. How are algebraic expressions and equations different?

c. How are algebraic expressions and equations related?

Read the following problem and then write an equation that you could use to solve it. Do not actually solve the equation. In your equation, let *m* be the number of miles driven.

2. When Louisa rented a moving truck, she agreed to pay $28 per day plus $0.42 per mile. If she kept the truck for 3 days and the rental charges (without tax) were $153.72, how many miles did Louisa drive the truck?

Remember What You Learned

3. How can the words *reflection* and *symmetry* help you remember and distinguish between the reflexive and symmetric properties of equality? Think about how these words are used in everyday life or in geometry.

NAME ____ DATE______________ PERIOD _____

1-4 Lesson Reading Guide

Solving Absolute Value Equations

Get Ready for the Lesson

Read the introduction to Lesson 1-4 in your textbook.

- What is a seismologist and what does magnitude of an earthquake mean?
- Why is an absolute value equation rather than an equation without absolute value used to find the extremes in the actual magnitude of an earthquake in relation to its measured value on the Richter scale?
- If the magnitude of an earthquake is estimated to be 6.9 on the Richter scale, it might actually have a magnitude as low as ________ or as high as ________.

Read the Lesson

1. Explain how $-a$ could represent a positive number. Give an example.

2. Explain why the absolute value of a number can never be negative.

3. What does the sentence $b \geq 0$ mean?

4. What does the symbol $\varnothing$ mean as a solution set?

Remember What You Learned

5. How can the number line model for absolute value that is shown on page 28 of your textbook help you remember that many absolute value equations have two solutions?

NAME ____ DATE______________ PERIOD _____

1-5 Lesson Reading Guide

Solving Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 1-5 in your textbook.

- Write an inequality comparing the number of minutes per month included in the two phone plans.
- Suppose that in one month you use 475 minutes of airtime on your wireless phone. Find your monthly cost with each plan.

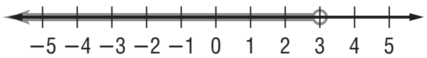
  Plan 1: ________ Plan 2: ________

  Which plan should you choose? ____________

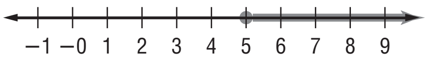
Read the Lesson

1. There are several different ways to write or show inequalities. Write each of the following in interval notation.

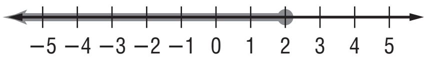
a.



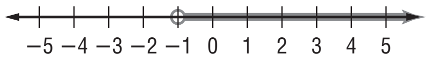
b.



c.



d.



2. Show how you can write an inequality symbol followed by a number to describe each of the following situations.

a. There are fewer than 600 students in the senior class.

b. A student may enroll in no more than six courses each semester.

c. To participate in a concert, you must be willing to attend at least ten rehearsals.

d. There is space for at most 165 students in the high school band.

Remember What You Learned

3. One way to remember something is to explain it to another person. A common student error in solving inequalities is forgetting to reverse the inequality symbol when multiplying or dividing both sides of an inequality by a negative number. Suppose that your classmate is having trouble remembering this rule. How could you explain this rule to your classmate?

1-6 Lesson Reading Guide

Solving Compound and Absolute Value Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 1-6 in your textbook.

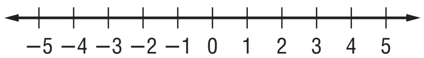
- Five patients arrive at a medical laboratory at 11:30 A.M. for a glucose tolerance test. Each of them is asked when they last had something to eat or drink. Some of the patients are given the test and others are told that they must come back another day. Each of the patients is listed below with the times when they started to fast. (The P.M. times refer to the night before.) Which of the patients were accepted for the test?

Ora	5:00 A.M.	Juanita	11:30 P.M.
Jason	1:30 A.M.	Samir	5:00 P.M.

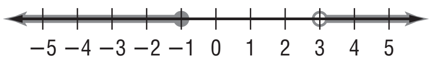
Read the Lesson

1. a. Write a compound inequality that says, “x is greater than -3 and x is less than or equal to 4.”

b. Graph the inequality that you wrote in part a on a number line.



2. Use a compound inequality and set-builder notation to describe the following graph.



3. Write a statement equivalent to $|4x - 5| > 2$ that does not use the absolute value symbol.

4. Write a statement equivalent to $|3x + 7| < 8$ that does not use the absolute value symbol.

Remember What You Learned

5. Many students have trouble knowing whether an absolute value inequality should be translated into an *and* or an *or* compound inequality. Describe a way to remember which of these applies to an absolute value inequality. Also describe how to recognize the difference from a number line graph.

2-1 Lesson Reading Guide

Relations and Functions

Get Ready for the Lesson

Read the introduction to Lesson 2-1 in your textbook.

- Refer to the table. What does the ordered pair (8, 20) tell you?

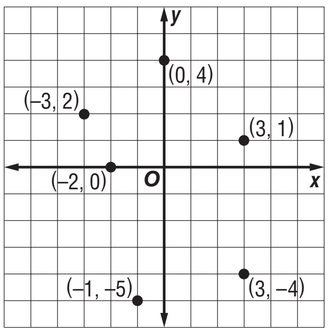
- Suppose that this table is extended to include more animals. Is it possible to have an ordered pair for the data in which the first number is larger than the second?

Read the Lesson

1. a. Explain the difference between a relation and a function.

b. Explain the difference between domain and range.

2. a. Write the domain and range of the relation shown in the graph.



b. Is this relation a function? Explain.

Remember What You Learned

3. Look up the words *dependent* and *independent* in a dictionary. How can the meaning of these words help you distinguish between independent and dependent variables in a function?

NAME ____ DATE______________ PERIOD _____

2-2 Lesson Reading Guide

Linear Equations

Get Ready for the Lesson

Read the introduction to Lesson 2-2 in your textbook.

- If Lolita spends $2\frac{1}{2}$ hours studying math, how many hours will she have to study chemistry?
- Suppose that Lolita decides to stay up one hour later so that she now has 5 hours to study and do homework. Write a linear equation that describes this situation.

Read the Lesson

1. Write *yes* or *no* to tell whether each linear equation is in standard form. If it is not, explain why it is not.

a. $-x + 2y = 5$

b. $9x - 12y = -5$

c. $5x - 7y = 3$

d. $2x - \frac{4}{7}y = 1$

e. $0x + 0y = 0$

f. $2x + 4y = 8$

2. How can you use the standard form of a linear equation to tell whether the graph is a horizontal line or a vertical line?

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that you are studying this lesson with a friend who thinks that she should let $x = 0$ to find the x-intercept and let $y = 0$ to find the y-intercept. How would you explain to her how to remember the correct way to find intercepts of a line?

2-3 Lesson Reading Guide

Slope

Get Ready for the Lesson

Read the introduction to Lesson 2-3 in your textbook.

- What is the grade of a road that rises 40 feet over a horizontal distance of 1000 feet?
- What is the grade of a road that rises 525 meters over a horizontal distance of 10 kilometers? (1 kilometer = 1000 meters)

Read the Lesson

1. Describe each type of slope and include a sketch.

Type of Slope	Description of Graph	Sketch
Positive		
Zero		
Negative		
Undefined		

2. a. How are the slopes of two nonvertical parallel lines related?

b. How are the slopes of two oblique perpendicular lines related?

Remember What You Learned

3. Look up the terms *grade*, *pitch*, *slant*, and *slope*. How can everyday meanings of these words help you remember the definition of slope?

NAME ____ DATE______________ PERIOD _____

2-4 Lesson Reading Guide

Writing Linear Equations

Get Ready for the Lesson

Read the introduction to Lesson 2-4 in your textbook.

- If the total cost of producing a product is given by the equation $y = 5400 + 1.37x$, what is the fixed cost? What is the variable cost (for each item produced)?
- Write a linear equation that describes the following situation:
  A company that manufactures computers has a fixed cost of $228,750 and a variable cost of $852 to produce each computer.

Read the Lesson

1. a. Write the slope-intercept form of the equation of a line. Then explain the meaning of each of the variables in the equation.

b. Write the point-slope form of the equation of a line. Then explain the meaning of each of the variables in the equation.

2. Suppose that your algebra teacher asks you to write the point-slope form of the equation of the line through the points $(-6, 7)$ and $(-3, -2)$. You write $y + 2 = -3(x + 3)$ and your classmate writes $y - 7 = -3(x + 6)$. Which of you is correct? Explain.

3. You are asked to write an equation of two lines that pass through $(3, -5)$, one of them parallel to and one of them perpendicular to the line whose equation is $y = -3x + 4$. The first step in finding these equations is to find their slopes. What is the slope of the parallel line? What is the slope of the perpendicular line?

Remember What You Learned

4. Many students have trouble remembering the point-slope form for a linear equation. How can you use the definition of slope to remember this form?

NAME ____ DATE______________ PERIOD _____

2-5 Lesson Reading Guide

Modeling Real-World Data: Using Scatter Plots

Get Ready for the Lesson

Read the introduction to Lesson 2-5 in your textbook.

- If a woman runs 5.5 miles per hour, about how many Calories will she burn in an hour?
- If a man runs 7.5 miles per hour, about how many Calories will he burn in half an hour?

Read the Lesson

1. Suppose that a set of data can be modeled by a linear equation. Explain the difference between a scatter plot of the data and a graph of the linear equation that models that data.

2. Suppose that tuition at a state college was $3800 per year in 2000 and has been increasing at a rate of $200 per year.

a. Write a prediction equation that expresses this information.

b. Explain the meaning of each variable in your prediction equation.

3. Use this model to predict the tuition at this college in 2007.

Remember What You Learned

4. Look up the word *scatter* in a dictionary. How can its definition help you to remember the meaning of the difference between a scatter plot and the graph of a linear equation?

2-6 Lesson Reading Guide

Special Functions

Get Ready for the Lesson

Read the introduction to Lesson 2-6 in your textbook.

- What is the cost of mailing a letter that weighs 0.5 ounce?
- Give three different weights of letters that would each cost 60 cents to mail.

Read the Lesson

1. Find the value of each expression.

a. $|-3| =$ ________ $[\![-3]\!] =$ ________

b. $|6.2| =$ ________ $[\![6.2]\!] =$ ________

c. $|-4.01| =$ ________ $[\![-4.01]\!] =$ ________

2. Tell how the name of each kind of function can help you remember what the graph looks like.

a. constant function

b. absolute value function

c. step function

d. identity function

Remember What You Learned

3. Many students find the greatest integer function confusing. Explain how you can use a number line to find the value of this function for any real number.

2-7 Lesson Reading Guide

Graphing Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 2-7 in your textbook.

- Which of the combinations of passing yards and touchdown passes listed would Dana consider a good game?
- Suppose that in one of the games Dana plays, Bledsoe passes 157 yards. What is the smallest number of touchdown passes he must get in order for Dana to consider this a good game?

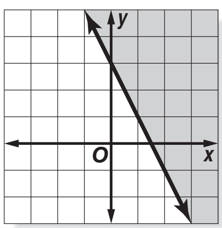
Read the Lesson

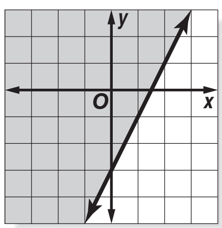
1. When graphing a linear inequality in two variables, how do you know whether to make the boundary a solid line or a dashed line?

2. How do you know which side of the boundary to shade?

3. Match each inequality with its graph.

a. $y > 2x - 3$ **b.** $y < -2x + 3$ **c.** $y \geq 2x - 3$ **d.** $y \geq -2x + 3$

i.


ii.


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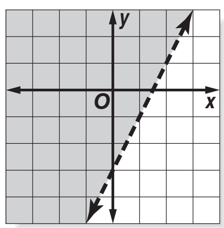

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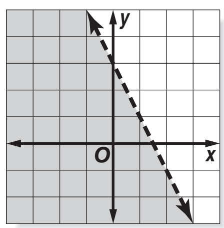


Remember What You Learned

4. Describe some ways in which graphing an inequality in one variable on a number line is similar to graphing an inequality in two variables in a coordinate plane. How can what you know about graphing inequalities on a number line help you to graph inequalities in a coordinate plane?

3-1 Lesson Reading Guide

Solving Systems of Equations by Graphing

Get Ready for the Lesson

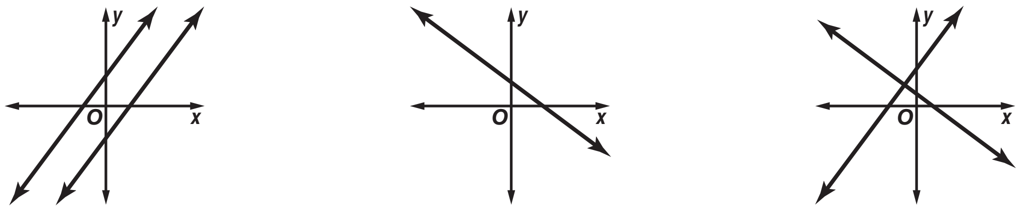
Read the introduction to Lesson 3-1 in your textbook.

- Which are growing faster, in-store sales or online sales?
- In what year is the in-store and online sales the same?

Read the Lesson

1. The Study Tip on page 117 of your textbook says that when you solve a system of equations by graphing and find a point of intersection of the two lines, you must always check the ordered pair in *both* of the original equations. Why is it not good enough to check the ordered pair in just one of the equations?

2. Under each system graphed below, write all of the following words that apply: *consistent*, *inconsistent*, *dependent*, and *independent*.



Remember What You Learned

3. Look up the words *consistent* and *inconsistent* in a dictionary. How can the meaning of these words help you distinguish between consistent and inconsistent systems of equations?

3-2 Lesson Reading Guide

Solving Systems of Equations Algebraically

Get Ready for the Lesson

Read the introduction to Lesson 3-2 in your textbook.

- How many more minutes of long distance time did Yolanda use in February than in January?
- How much more were the February charges than the January charges?
- Using your answers for the questions above, how can you find the rate per minute?

Read the Lesson

1. Suppose that you are asked to solve the system of equations at the right by the substitution method.

$4x - 5y = 7$
$3x + y = -9$

The first step is to solve one of the equations for one variable in terms of the other. To make your work as easy as possible, which equation would you solve for which variable? Explain.

2. Suppose that you are asked to solve the system of equations at the right by the elimination method.

$2x + 3y = -2$
$7x - y = 39$

To make your work as easy as possible, which variable would you eliminate? Describe how you would do this.

Remember What You Learned

3. The substitution method and elimination method for solving systems both have several steps, and it may be difficult to remember them. You may be able to remember them more easily if you notice what the methods have in common. What step is the same in both methods?

NAME _______________________________________ DATE____________ PERIOD _____

3-3 Lesson Reading Guide

Solving Systems of Inequalities by Graphing

Get Ready for the Lesson

Read the introduction to Lesson 3-3 in your textbook.

Satish is 37 years old. He has a blood pressure reading of 135/99. Is his blood pressure within the normal range? Explain.

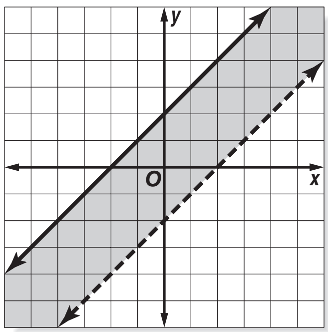
Read the Lesson

1. Without actually drawing the graph, describe the boundary lines for the system of inequalities shown at the right.

$$|x| < 3$$
$$|y| \leq 5$$

2. Think about how the graph would look for the system given above. What will be the shape of the shaded region? (It is not necessary to draw the graph. See if you can imagine it without drawing anything. If this is difficult to do, make a rough sketch to help you answer the question.)

3. Which system of inequalities matches the graph shown at the right?



A. $x - y \leq -2$
$x - y > 2$

B. $x - y \geq -2$
$x - y < 2$

C. $x + y \leq -2$
$x + y > 2$

D. $x - y > -2$
$x - y \leq 2$

Remember What You Learned

4. To graph a system of inequalities, you must graph two or more boundary lines. When you graph each of these lines, how can the inequality symbols help you remember whether to use a dashed or solid line?

3-4 Lesson Reading Guide

Linear Programming

Get Ready for the Lesson

Read the introduction to Lesson 3-4 in your textbook.

Name two or more facts that indicate that you will need to use inequalities to model this situation.

Read the Lesson

1. Complete each sentence.

a. When you find the feasible region for a linear programming problem, you are solving a system of linear ____________________ called ____________________. The points in the feasible region are ____________________ of the system.

b. The corner points of a polygonal region are the ____________________ of the feasible region.

2. A polygonal region always takes up only a limited part of the coordinate plane. One way to think of this is to imagine a circle or rectangle that the region would fit inside. In the case of a polygonal region, you can always find a circle or rectangle that is large enough to contain all the points of the polygonal region. What word is used to describe a region that can be enclosed in this way? What word is used to describe a region that is too large to be enclosed in this way?

3. How do you find the corner points of the polygonal region in a linear programming problem?

4. What are some everyday meanings of the word *feasible* that remind you of the mathematical meaning of the term *feasible region*?

Remember What You Learned

5. Look up the word *constraint* in a dictionary. If more than one definition is given, choose the one that seems closest to the idea of a *constraint* in a linear programming problem. How can this definition help you to remember the meaning of *constraint* as it is used in this lesson?

3-5 Lesson Reading Guide

Solving Systems of Equations in Three Variables

Get Ready for the Lesson

Read the introduction to Lesson 3-5 in your textbook.

At the 1960 Summer Olympics in Rome, Italy, the United States won 71 medals. The U.S. team won 13 more gold medals than silver and 5 fewer bronze medals than silver. Using the same variables as those in the introduction, write a system of equations that describes the medals won for the 1960 Olympics.

Read the Lesson

1. The planes for the equations in a system of three linear equations in three variables determine the number of solutions. Match each graph description below with the description of the number of solutions of the system. (Some of the items on the right may be used more than once, and not all possible types of graphs are listed.)

a. three parallel planes ________	**I.** one solution
b. three planes that intersect in a line ________	**II.** no solutions
c. three planes that intersect in one point ________	**III.** infinite solutions
d. one plane that represents all three equations ________	

2. Suppose that three classmates, Monique, Josh, and Lilly, are studying for a quiz on this lesson. They work together on solving a system of equations in three variables, x, y, and z, following the algebraic method shown in your textbook. They first find that $z = 3$, then that $y = -2$, and finally that $x = -1$. The students agree on these values, but disagree on how to write the solution. Here are their answers:

Monique: $(3, -2, -1)$ Josh: $(-2, -1, 3)$ Lilly: $(-1, -2, 3)$

a. How do you think each student decided on the order of the numbers in the ordered triple?

b. Which student is correct?

Remember What You Learned

3. How can you remember that obtaining the equation $0 = 0$ indicates a system with infinitely many solutions, while obtaining an equation such as $0 = 8$ indicates a system with no solutions?

4-1 Lesson Reading Guide

Introduction to Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-1 in your textbook.

a. What is the base price of a Mid-Size SUV?
b. What is the exterior length of a Compact SUV?

Read the Lesson

1. Give the dimensions of each matrix.

a. $\begin{bmatrix} 3 & 2 & 5 \\ -1 & 0 & 6 \end{bmatrix}$

b. $[1 \quad 4 \quad 0 \quad -8 \quad 2]$

2. Identify each matrix with as many of the following descriptions that apply: *row matrix*, *column matrix*, *square matrix*, *zero matrix*.

a. $[6 \quad 5 \quad 4 \quad 3]$

b. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

c. $[0]$

3. Write a system of equations that you could use to solve the following matrix equation for x, y, and z. (Do not actually solve the system.)

$$\begin{bmatrix} 3x \\ x + y \\ y - z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 6 \end{bmatrix}$$

Remember What You Learned

4. Some students have trouble remembering which number comes first in writing the dimensions of a matrix. Think of an easy way to remember this.

NAME ____ DATE______________ PERIOD _____

4-2 Lesson Reading Guide

Operations with Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-2 in your textbook.

- Write a sum that represents the total number of Calories in the patient's diet for Day 2. (Do not actually calculate the sum.)
- Write the sum that represents the total fat content in the patient's diet for Day 3. (Do not actually calculate the sum.)

Read the Lesson

1. For each pair of matrices, give the dimensions of the indicated sum, difference, or scalar product. If the indicated sum, difference, or scalar product does not exist, write *impossible*.

$A = \begin{bmatrix} 3 & 5 & 6 \\ -2 & 8 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$

$C = \begin{bmatrix} 5 & 10 \\ -3 & 6 \\ 4 & 12 \end{bmatrix}$ $\quad D = \begin{bmatrix} -3 & 6 & 0 \\ -8 & 4 & 0 \end{bmatrix}$

$A + D$: ____________ $C + D$: ____________ $5B$: ____________

$-4C$: ____________ $2D - 3A$: ____________

2. Suppose that M, N, and P are nonzero 2×4 matrices and k is a negative real number. Indicate whether each of the following statements is *true* or *false*.

a. $M + (N + P) = M + (P + N)$ **b.** $M - N = N - M$

c. $M - (N - P) = (M - N) - P$ **d.** $k(M - N) = kM - kN$

Remember What You Learned

3. The mathematical term *scalar* may be unfamiliar, but its meaning is related to the word *scale* as used in a *scale of miles* on a map. How can this usage of the word *scale* help you remember the meaning of *scalar*?

4-3 Lesson Reading Guide

Multiplying Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-3 in your textbook.

Write a sum that shows the total points scored by the Houston Texans during the 2004 season. (The sum will include multiplications. Do not actually calculate this sum.)

Read the Lesson

1. Determine whether each indicated matrix product is defined. If so, state the dimensions of the product. If not, write *undefined*.

   a. $M_{3 \times 2}$ and $N_{2 \times 3}$ MN: ______________ NM: ______________

   b. $M_{1 \times 2}$ and $N_{1 \times 2}$ MN: ______________ NM: ______________

   c. $M_{4 \times 1}$ and $N_{1 \times 4}$ MN: ______________ NM: ______________

   d. $M_{3 \times 4}$ and $N_{4 \times 4}$ MN: ______________ NM: ______________

2. The regional sales manager for a chain of computer stores wants to compare the revenue from sales of one model of notebook computer and one model of printer for three stores in his area. The notebook computer sells for $1850 and the printer for $175. The number of computers and printers sold at the three stores during September are shown in the following table.

Store	Computers	Printers
A	128	101
B	205	166
C	97	73

   Write a matrix product that the manager could use to find the total revenue for computers and printers for each of the three stores. (Do not calculate the product.)

Remember What You Learned

3. Many students find the procedure of matrix multiplication confusing at first because it is unfamiliar. Think of an easy way to use the letters R and C to remember how to multiply matrices and what the dimensions of the product will be.

Lesson 4-3

NAME ____ DATE______________ PERIOD _____

4-4 Lesson Reading Guide

Transformations with Matrices

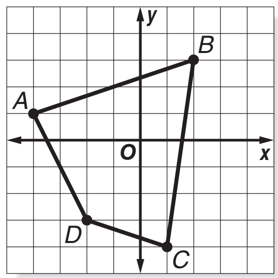
Get Ready for the Lesson

Read the introduction to Lesson 4-4 in your textbook.

Describe how you can change the orientation of a figure without changing its size or shape.

Read the Lesson

1. a. Write the vertex matrix for the quadrilateral *ABCD* shown in the graph at the right.



b. Write the vertex matrix that represents the position of the quadrilateral $A'B'C'D'$ that results when quadrilateral *ABCD* is translated 3 units to the right and 2 units down.

2. Describe the transformation that corresponds to each of the following matrices.

a. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 3 & 3 \\ -4 & -4 & -4 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Remember What You Learned

3. Describe a way to remember which of the reflection matrices corresponds to reflection over the *x*-axis.

4-5 Lesson Reading Guide

Determinants

Get Ready for the Lesson

Read the introduction to Lesson 4-5 in your textbook.

In this lesson, you will learn how to find the area of a triangle if you know the coordinates of its vertices using determinants. Describe a method you already know for finding the area of the Bermuda Triangle.

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.
   - **a.** Every matrix has a determinant.
   - **b.** If both rows of a 2×2 matrix are identical, the determinant of the matrix will be 0.
   - **c.** Every element of a 3×3 matrix has a minor.
   - **d.** In order to evaluate a third-order determinant by expansion by minors it is necessary to find the minor of every element of the matrix.
   - **e.** If you evaluate a third-order determinant by expansion about the second row, the position signs you will use are $- + -$.

2. Suppose that triangle RST has vertices $R(-2, 5)$, $S(4, 1)$, and $T(0, 6)$.
   - **a.** Write a determinant that could be used in finding the area of triangle RST.
   - **b.** Explain how you would use the determinant you wrote in part **a** to find the area of the triangle.

Remember What You Learned

3. A good way to remember a complicated procedure is to break it down into steps. Write a list of steps for evaluating a third-order determinant using expansion by minors.

4-6 Lesson Reading Guide

Cramer's Rule

Get Ready for the Lesson

Read the introduction to Lesson 4-6 in your textbook.

A triangle is bounded by the x-axis, the line $y = \frac{1}{2}x$, and the line $y = -2x + 10$. Write three systems of equations that you could use to find the three vertices of the triangle. (Do not actually find the vertices.)

Read the Lesson

1. Suppose that you are asked to solve the following system of equations by Cramer's Rule.

$3x + 2y = 7$
$2x - 3y = 22$

Without actually evaluating any determinants, indicate which of the following ratios of determinants gives the correct value for x.

A. $\dfrac{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}$ **B.** $\dfrac{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$ **C.** $\dfrac{\begin{vmatrix} 3 & 7 \\ 2 & 22 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$

2. In your textbook, the statements of Cramer's Rule for two variables and three variables specify that the determinant formed from the coefficients of the variables cannot be 0. If the determinant is zero, what do you know about the system and its solutions?

Remember What You Learned

3. Some students have trouble remembering how to arrange the determinants that are used in solving a system of two linear equations by Cramer's Rule. What is a good way to remember this?

4-7 Lesson Reading Guide

Identity and Inverse Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-7 in your textbook.

Refer to the code table given in the introduction to this lesson. Suppose that you receive a message coded by this system as follows:

16 12 5 1 19 5 2 5 13 25 6 18 9 5 14 4.

Decode the message.

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every element of an identity matrix is 1.

b. There is a 3×2 identity matrix.

c. Two matrices are inverses of each other if their product is the identity matrix.

d. If M is a matrix, M^{-1} represents the reciprocal of M.

e. No 3×2 matrix has an inverse.

f. Every square matrix has an inverse.

g. If the two columns of a 2×2 matrix are identical, the matrix does not have an inverse.

2. Explain how to find the inverse of a 2×2 matrix. Do not use any special mathematical symbols in your explanation.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that you are studying with a classmate who is having trouble remembering how to find the inverse of a 2×2 matrix. He remembers how to move elements and change signs in the matrix, but thinks that he should multiply by the determinant of the original matrix. How can you help him remember that he must multiply by the *reciprocal* of this determinant?

NAME ____ DATE______________ PERIOD _____

4-8 Lesson Reading Guide

Using Matrices to Solve Systems of Equations

Get Ready for the Lesson

Read the introduction to Lesson 4-8 in your textbook.

Write a 2×2 matrix that summarizes the information given in the introduction about the food and territory requirements for the two species.

Read the Lesson

1. a. Write a matrix equation for the following system of equations.

$3x + 5y = 10$
$2x - 4y = -7$

b. Explain how to use the matrix equation you wrote above to solve the system. Use as few mathematical symbols in your explanation as you can. Do not actually solve the system.

2. Write a system of equations that corresponds to the following matrix equation.

$$\begin{bmatrix} 3 & 2 & -4 \\ 2 & -1 & 0 \\ 0 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ -4 \end{bmatrix}$$

Remember What You Learned

3. Some students have trouble remembering how to set up a matrix equation to solve a system of linear equations. What is an easy way to remember the order in which to write the three matrices that make up the equation?

5-1 Lesson Reading Guide

Graphing Quadratic Functions

Get Ready for the Lesson

Read the introduction to Lesson 5-1 in your textbook.

- Based on the graph in your textbook, for what ticket price is the income the greatest?
- Use the graph to estimate the maximum income.

Read the Lesson

1. a. For the quadratic function $f(x) = 2x^2 + 5x + 3$, $2x^2$ is the ____________________ term, $5x$ is the ____________________ term, and 3 is the ____________________ term.

b. For the quadratic function $f(x) = -4 + x - 3x^2$, $a =$ _________, $b =$ _________, and $c =$ _________.

2. Consider the quadratic function $f(x) = ax^2 + bx + c$, where $a \neq 0$.

a. The graph of this function is a ______________________.

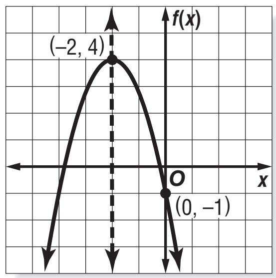
b. The y-intercept is ______________________.

c. The axis of symmetry is the line ______________________.

d. If $a > 0$, then the graph opens ______________________ and the function has a ______________________ value.

e. If $a < 0$, then the graph opens ______________________ and the function has a ______________________ value.

3. Refer to the graph at the right as you complete the following sentences.



a. The curve is called a ______________________.

b. The line $x = -2$ is called the ______________________.

c. The point $(-2, 4)$ is called the ______________________.

d. Because the graph contains the point $(0, -1)$, -1 is the ______________________.

Remember What You Learned

4. How can you remember the way to use the x^2 term of a quadratic function to tell whether the function has a maximum or a minimum value?

5-2 Lesson Reading Guide

Solving Quadratic Equations by Graphing

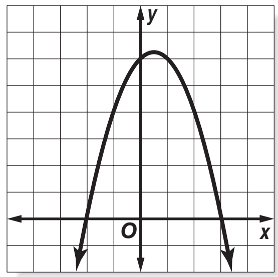
Get Ready for the Lesson

Read the introduction to Lesson 5-2 in your textbook.

Write a quadratic function that describes the height of a ball t seconds after it is dropped from a height of 125 feet.

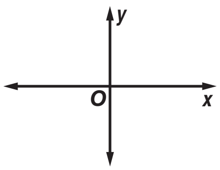
Read the Lesson

1. The graph of the quadratic function $f(x) = -x^2 + x + 6$ is shown at the right. Use the graph to find the solutions of the quadratic equation $-x^2 + x + 6 = 0$.

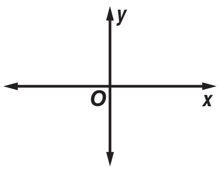


2. Sketch a graph to illustrate each situation.

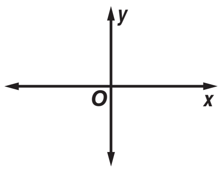
a. A parabola that opens downward and represents a quadratic function with two real zeros, both of which are negative numbers.



b. A parabola that opens upward and represents a quadratic function with exactly one real zero. The zero is a positive number.



c. A parabola that opens downward and represents a quadratic function with no real zeros.



Remember What You Learned

3. Think of a memory aid that can help you recall what is meant by the *zeros* of a quadratic function.

5-3 Lesson Reading Guide

Solving Quadratic Equations by Factoring

Get Ready for the Lesson

Read the introduction to Lesson 5-3 in your textbook.

Write two different quadratic equations in intercept form that have corresponding graphs with the same x-intercepts.

Read the Lesson

1. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution.

$x^2 - 10x = -21$	Original equation
$x^2 - 10x + 21 = 0$	______________________________
$(x - 3)(x - 7) = 0$	______________________________
$x - 3 = 0$ or $x - 7 = 0$	______________________________
$x = 3$ $\quad$ $x = 7$	______________________________

The solution set is ___________.

2. On an algebra quiz, students were asked to write a quadratic equation with −7 and 5 as its roots. The work that three students in the class wrote on their papers is shown below.

Marla	Rosa	Larry
$(x - 7)(x + 5) = 0$	$(x + 7)(x - 5) = 0$	$(x + 7)(x - 5) = 0$
$x^2 - 2x - 35 = 0$	$x^2 + 2x - 35 = 0$	$x^2 - 2x - 35 = 0$

Who is correct?

Explain the errors in the other two students' work.

Remember What You Learned

3. A good way to remember a concept is to represent it in more than one way. Describe an algebraic way and a graphical way to recognize a quadratic equation that has a double root.

5-4 Lesson Reading Guide

Complex Numbers

Get Ready for the Lesson

Read the introduction to Lesson 5-4 in your textbook.

Suppose the number i is defined such that $\boldsymbol{i}^2 = -1$. Complete each equation.

$2\boldsymbol{i}^2 =$ ________ $(2\boldsymbol{i})^2 =$ ________ $\boldsymbol{i}^4 =$ ________

Read the Lesson

1. Complete each statement.

a. The form $a + b\boldsymbol{i}$ is called the ____________________ of a complex number.

b. In the complex number $4 + 5\boldsymbol{i}$, the real part is ______ and the imaginary part is ______.

This is an example of a complex number that is also a(n) ________________ number.

c. In the complex number 3, the real part is ______ and the imaginary part is ______.

This is example of complex number that is also a(n) ________________ number.

d. In the complex number $7\boldsymbol{i}$, the real part is ______ and the imaginary part is ______.

This is an example of a complex number that is also a(n) ________________ number.

2. Give the complex conjugate of each number.

a. $3 + 7\boldsymbol{i}$ ____________

b. $2 - \boldsymbol{i}$ ____________

3. Why are complex conjugates used in dividing complex numbers?

4. Explain how you would use complex conjugates to find $(3 + 7\boldsymbol{i}) \div (2 - \boldsymbol{i})$.

Remember What You Learned

5. How can you use what you know about simplifying an expression such as $\frac{1 + \sqrt{3}}{2 - \sqrt{5}}$ to help you remember how to simplify fractions with imaginary numbers in the denominator?

5-5 Lesson Reading Guide

Completing the Square

Get Ready for the Lesson

Read the introduction to Lesson 5-5 in your textbook.

Explain what it means to say that the driver accelerates at a constant rate of 8 feet per second squared.

Read the Lesson

1. Give the reason for each step in the following solution of an equation by using the Square Root Property.

$x^2 - 12x + 36 = 81$	Original equation
$(x - 6)^2 = 81$	______________________
$x - 6 = \pm\sqrt{81}$	______________________
$x - 6 = \pm 9$	______________________
$x - 6 = 9$ or $x - 6 = -9$	______________________
$x = 15$ $x = -3$	______________________

2. Explain how to find the constant that must be added to make a binomial into a perfect square trinomial.

3. a. What is the first step in solving the equation $3x^2 + 6x = 5$ by completing the square?

b. What is the first step in solving the equation $x^2 + 5x - 12 = 0$ by completing the square?

Remember What You Learned

4. How can you use the rules for squaring a binomial to help you remember the procedure for changing a binomial into a perfect square trinomial?

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5-6 Lesson Reading Guide

The Quadratic Formula and the Discriminant

Lesson 5-6

Get Ready for the Lesson

Read the introduction to Lesson 5-6 in your textbook.

Describe how you would calculate the position of the diver after 1 second using the equation in your textbook.

Read the Lesson

1. a. Write the Quadratic Formula.

b. Identify the values of a, b, and c that you would use to solve $2x^2 - 5x = -7$, but do not actually solve the equation.

a = __________ b = __________ c = __________

2. Suppose that you are solving four quadratic equations with rational coefficients and have found the value of the discriminant for each equation. In each case, give the number of roots and describe the type of roots that the equation will have.

Value of Discriminant	Number of Roots	Type of Roots
64		
−8		
21		
0		

Remember What You Learned

3. How can looking at the Quadratic Formula help you remember the relationships between the value of the discriminant and the number of roots of a quadratic equation and whether the roots are real or complex?

5-7 Lesson Reading Guide

Analyzing Graphs of Quadratic Equations

Get Ready for the Lesson

Read the introduction to Lesson 5-7 in your textbook.

- What does adding a positive number to x^2 do to the graph of $y = x^2$?

- What does subtracting a positive number to x before squaring do to the graph of $y = x^2$?

Read the Lesson

1. Complete the following information about the graph of $y = a(x - h)^2 + k$.

a. What are the coordinates of the vertex?

b. What is the equation of the axis of symmetry?

c. In which direction does the graph open if $a > 0$? If $a < 0$?

d. What do you know about the graph if $|a| < 1$?

If $|a| > 1$?

2. Match each graph with the description of the constants in the equation in vertex form.

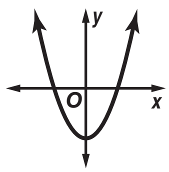
a. $a > 0, h > 0, k < 0$

b. $a < 0, h < 0, k < 0$

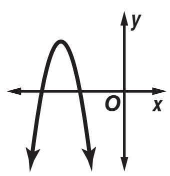
c. $a < 0, h < 0, k > 0$

d. $a > 0, h = 0, k < 0$

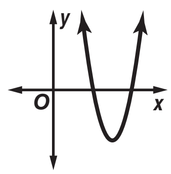
i.



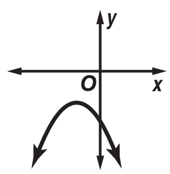
ii.



iii.



iv.



Remember What You Learned

3. When graphing quadratic functions such as $y = (x + 4)^2$ and $y = (x - 5)^2$, many students have trouble remembering which represents a translation of the graph of $y = x^2$ to the left and which represents a translation to the right. What is an easy way to remember this?

5-8 Lesson Reading Guide

Graphing and Solving Quadratic Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 5-8 in your textbook.

- How far above the ground is the trampoline surface?
- Using the quadratic function given in the introduction, write a quadratic inequality that describes the times at which the trampolinist is more than 20 feet above the ground.

Read the Lesson

1. Answer the following questions about how you would graph the inequality $y \geq x^2 + x - 6$.

a. What is the related quadratic equation?

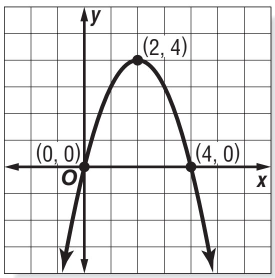
b. Should the parabola be solid or dashed? How do you know?

c. The point (0, 2) is inside the parabola. To use this as a test point, substitute __________ for x and __________ for y in the quadratic inequality.

d. Is the statement $2 \geq 0^2 + 0 - 6$ true or false?

e. Should the region inside or outside the parabola be shaded?

2. The graph of $y = -x^2 + 4x$ is shown at the right. Match each of the following related inequalities with its solution set.



a. $-x^2 + 4x > 0$ **i.** $\{x \mid x < 0 \text{ or } x > 4\}$

b. $-x^2 + 4x \leq 0$ **ii.** $\{x \mid 0 < x < 4\}$

c. $-x^2 + 4x \geq 0$ **iii.** $\{x \mid x \leq 0 \text{ or } x \geq 4\}$

d. $-x^2 + 4x < 0$ **iv.** $\{x \mid 0 \leq x \leq 4\}$

Remember What You Learned

3. A quadratic inequality in two variables may have the form $y > ax^2 + bx + c$, $y < ax^2 + bx + c$, $y \geq ax^2 + bx + c$, or $y \leq ax^2 + bx + c$. Describe a way to remember which region to shade by looking at the inequality symbol and without using a test point.

6-1 Lesson Reading Guide

Properties of Exponents

Get Ready for the Lesson

Read the introduction to Lesson 6-1 in your textbook.

Your textbook gives the U.S. public debt as an example from economics that involves large numbers that are difficult to work with when written in standard notation. Give an example from science that involves very large numbers and one that involves very small numbers.

Read the Lesson

1. Tell whether each expression is a monomial or not a monomial.

a. $3x^2$

b. $y^2 + 5y - 6$

c. $-73x$

d. $\frac{1}{z}$

2. Complete the following definitions of a negative exponent and a zero exponent.

For any real number $a \neq 0$ and any integer n, $a^{-n} =$ ___.

For any real number $a \neq 0$, $a^0 =$ _____.

3. Name the property or properties of exponents that you would use to simplify each expression. (Do not actually simplify.)

a. $\frac{m^8}{m^3}$

b. $y^6 \cdot y^9$

c. $(3r^2s)^4$

Remember What You Learned

4. When writing a number in scientific notation, some students have trouble remembering when to use positive exponents and when to use negative ones. What is an easy way to remember this?

NAME ____ DATE______________ PERIOD _____

6-2 Lesson Reading Guide

Operations with Polynomials

Get Ready for the Lesson

Read the introduction to Lesson 6-2 in your textbook.

Suppose that Shenequa decides to enroll in a five-year engineering program rather than a four-year program. Using the model given in your textbook, how could she describe the tuition for the fifth year of her program?

Read the Lesson

1. State whether each expression is a polynomial.

a. $2r^7 + r^5 - r$

b. $3x - \frac{1}{x^2}$

c. $4 - 9x^2$

d. x^2

2. State the degree of each polynomial.

a. $4r^4 - 2r^5 + 1$

b. $3x - 3x^2$

c. $6x^2 + 2x - 4$

d. $5x + 3$

3. State whether or not each polynomial is in simplified form.

a. $3x^2 + 3y^2$

b. $3x - 11x$

c. $6m^3 + m^2$

d. $r^2 - 2r$

Remember What You Learned

4. You can always find the degree of a polynomial by remembering to look at the monomial with the greatest degree. Write two polynomials of degree 3, two polynomials of degree 2, and two polynomials of degree 1.

6-3 Lesson Reading Guide

Dividing Polynomials

Get Ready for the Lesson

Read the introduction to Lesson 6-3 in your textbook.

Using the division symbol (÷), write the division problem that you would use to answer the question asked in the introduction. (Do not actually divide.)

Read the Lesson

1. a. Explain in words how to divide a polynomial by a monomial.

b. If you divide a trinomial by a monomial and get a polynomial, what kind of polynomial will the quotient be?

2. Look at the following division example that uses the division algorithm for polynomials.

$$\begin{array}{r} 2x \;+\; 4 \qquad \\ x-4\overline{)2x^2 - 4x + 7} \\ \underline{2x^2 - 8x} \qquad \\ 4x + 7 \\ \underline{4x - 16} \\ 23 \end{array}$$

Which of the following is the correct way to write the quotient?

A. $2x + 4$ **B.** $x - 4$ **C.** $2x + 4 + \frac{23}{x-4}$ **D.** $\frac{23}{x-4}$

3. If you use synthetic division to divide $x^3 + 3x^2 - 5x - 8$ by $x - 2$, the division will look like this:

2	1	3	−5	−8
		2	10	10
	1	5	5	2

Which of the following is the answer for this division problem?

A. $x^2 + 5x + 5$ **B.** $x^2 + 5x + 5 + \frac{2}{x-2}$

C. $x^3 + 5x^2 + 5x + \frac{2}{x-2}$ **D.** $x^3 + 5x^2 + 5x + 2$

Remember What You Learned

4. When you translate the numbers in the last row of a synthetic division into the quotient and remainder, what is an easy way to remember which exponents to use in writing the terms of the quotient?

Lesson 6-3

6-4 Lesson Reading Guide

Polynomial Functions

Get Ready for the Lesson

Read the introduction to Lesson 6-4 in your textbook.

- In the honeycomb cross section shown in your textbook, there is 1 hexagon in the center, 6 hexagons in the second ring, and 12 hexagons in the third ring. How many hexagons will there be in the fourth, fifth, and sixth rings?
- There is 1 hexagon in a honeycomb with 1 ring. There are 7 hexagons in a honeycomb with 2 rings. How many hexagons are there in honeycombs with 3 rings, 4 rings, 5 rings, and 6 rings?

Read the Lesson

1. Give the degree and leading coefficient of each polynomial in one variable.

	degree	**leading coefficient**
a. $10x^3 + 3x^2 - x + 7$	________	________
b. $7y^2 - 2y^5 + y - 4y^3$	________	________
c. 100	________	________

2. Match each description of a polynomial function from the list on the left with the corresponding end behavior from the list on the right.

a. even degree, negative leading coefficient	**i.** $f(x) \to +\infty$ as $x \to +\infty$; $f(x) \to +\infty$ as $x \to -\infty$
b. odd degree, positive leading coefficient	**ii.** $f(x) \to -\infty$ as $x \to +\infty$; $f(x) \to +\infty$ as $x \to -\infty$
c. odd degree, negative leading coefficient	**iii.** $f(x) \to -\infty$ as $x \to +\infty$; $f(x) \to -\infty$ as $x \to -\infty$
d. even degree, positive leading coefficient	**iv.** $f(x) \to +\infty$ as $x \to +\infty$; $f(x) \to -\infty$ as $x \to -\infty$

Remember What You Learned

3. What is an easy way to remember the difference between the end behavior of the graphs of even-degree and odd-degree polynomial functions?

6-5 Lesson Reading Guide

Analyze Graphs of Polynomial Functions

Get Ready for the Lesson

Read the introduction to Lesson 6-5 in your textbook.

Three points on the graph shown in your textbook are (0, 14), (70, 3.78), and (100, 9). Give the real-world meaning of the coordinates of these points.

Read the Lesson

1. Suppose that $f(x)$ is a third-degree polynomial function and that c and d are real numbers, with $d > c$. Indicate whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

a. If $f(c) > 0$ and $f(d) < 0$, there is exactly one real zero between c and d.

b. If $f(c) = f(d) \neq 0$, there are no real zeros between c and d.

c. If $f(c) < 0$ and $f(d) > 0$, there is at least one real zero between c and d.

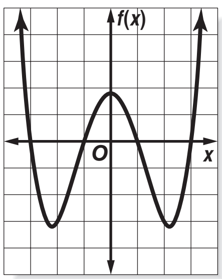
2. Match each graph with its description.

a. third-degree polynomial with one relative maximum and one relative minimum; leading coefficient negative

b. fourth-degree polynomial with two relative minima and one relative maximum

c. third-degree polynomial with one relative maximum and one relative minimum; leading coefficient positive

d. fourth-degree polynomial with two relative maxima and one relative minimum

i.


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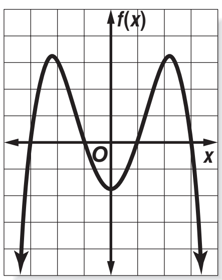

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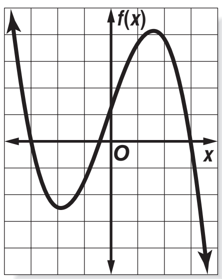

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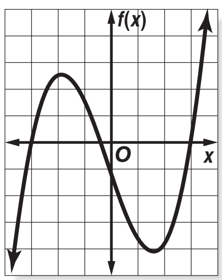


Remember What You Learned

3. The origins of words can help you to remember their meaning and to distinguish between similar words. Look up *maximum* and *minimum* in a dictionary and describe their origins (original language and meaning).

6-6 Lesson Reading Guide

Solving Polynomial Equations

Get Ready for the Lesson

Read the introduction to Lesson 6-6 in your textbook.

If a trinomial that represents the volume of a box is factored into three binomials, what might the three binomials represent?

Read the Lesson

1. Name three types of binomials that it is always possible to factor.

2. Name a type of trinomial that it is always possible to factor.

3. Complete: Since $x^2 + y^2$ cannot be factored, it is an example of a _____________ polynomial.

4. On an algebra quiz, Marlene needed to factor $2x^2 - 4x - 70$. She wrote the following answer: $(x + 5)(2x - 14)$. When she got her quiz back, Marlene found that she did not get full credit for her answer. She thought she should have gotten full credit because she checked her work by multiplication and showed that $(x + 5)(2x - 14) = 2x^2 - 4x - 70$.

a. If you were Marlene's teacher, how would you explain to her that her answer was not entirely correct?

b. What advice could Marlene's teacher give her to avoid making the same kind of error in factoring in the future?

Remember What You Learned

5. Some students have trouble remembering the correct signs in the formulas for the sum and difference of two cubes. What is an easy way to remember the correct signs?

6-7 Lesson Reading Guide

The Remainder and Factor Theorems

Get Ready for the Lesson

Read the introduction to Lesson 6-7 in your textbook.

Show how you would use the model in the introduction to estimate the number of international travelers (in millions) to the United States in the year 2000. (Show how you would substitute numbers, but do not actually calculate the result.)

Read the Lesson

1. Consider the following synthetic division.

1	3	2	−6	4
		3	5	−1
	3	5	−1	3

a. Using the division symbol ÷, write the division problem that is represented by this synthetic division. (Do not include the answer.)

b. Identify each of the following for this division.

dividend ________________________ divisor ________________________

quotient ________________________ remainder ________________________

c. If $f(x) = 3x^3 + 2x^2 - 6x + 4$, what is $f(1)$?

2. Consider the following synthetic division.

−3	1	0	0	27
		−3	9	−27
	1	−3	9	0

a. This division shows that ____________ is a factor of ________________.

b. The division shows that ________ is a zero of the polynomial function $f(x) =$ ________________.

c. The division shows that the point ____________ is on the graph of the polynomial function $f(x) =$ ________________.

Remember What You Learned

3. Think of a mnemonic for remembering the sentence, "Dividend equals quotient times divisor plus remainder."

6-8 Lesson Reading Guide

Roots and Zeros

Get Ready for the Lesson

Read the introduction to Lesson 6-8 in your textbook.

Using the model given in the introduction, write a polynomial equation with 0 on one side that can be solved to find the time or times at which there is 100 milligrams of medication in a patient's bloodstream.

Read the Lesson

1. Indicate whether each statement is *true* or *false*.

a. Every polynomial equation of degree greater than one has at least one root in the set of real numbers.

b. If c is a root of the polynomial equation $f(x) = 0$, then $(x - c)$ is a factor of the polynomial $f(x)$.

c. If $(x + c)$ is a factor of the polynomial $f(x)$, then c is a zero of the polynomial function f.

d. A polynomial function f of degree n has exactly $(n - 1)$ complex zeros.

2. Let $f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 + 6x - 7$.

a. What are the possible numbers of positive real zeros of f?

b. Write $f(-x)$ in simplified form (with no parentheses).

What are the possible numbers of negative real zeros of f?

c. Complete the following chart to show the possible combinations of positive real zeros, negative real zeros, and imaginary zeros of the polynomial function f.

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros

Remember What You Learned

3. It is easier to remember mathematical concepts and results if you relate them to each other. How can the Complex Conjugates Theorem help you remember the part of Descartes' Rule of Signs that says, "or is less than this number by an even number."

6-9 Lesson Reading Guide

Rational Zero Theorem

Get Ready for the Lesson

Read the introduction to Lesson 6-9 in your textbook.

Rewrite the polynomial equation $w(w + 8)(w - 5) = 2772$ in the form $f(x) = 0$, where $f(x)$ is a polynomial written in descending powers of x.

Read the Lesson

1. For each of the following polynomial functions, list all the possible values of p, all the possible values of q, and all the possible rational zeros $\frac{p}{q}$.

a. $f(x) = x^3 - 2x^2 - 11x + 12$

possible values of p:

possible values of q:

possible values of $\frac{p}{q}$:

b. $f(x) = 2x^4 + 9x^3 - 23x^2 - 81x + 45$

possible values of p:

possible values of q:

possible values of $\frac{p}{q}$:

2. Explain in your own words how Descartes' Rule of Signs, the Rational Zero Theorem, and synthetic division can be used together to find all of the rational zeros of a polynomial function with integer coefficients.

Remember What You Learned

3. Some students have trouble remembering which numbers go in the numerators and which go in the denominators when forming a list of possible rational zeros of a polynomial function. How can you use the linear polynomial equation $ax + b = 0$, where a and b are nonzero integers, to remember this?

NAME ____ DATE______________ PERIOD _____

7-1 Lesson Reading Guide

Operations on Functions

Get Ready for the Lesson

Read the introduction to Lesson 7-1 in your textbook.

Describe two ways to calculate Ms. Coffmon's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.)

Read the Lesson

1. Determine whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

a. If f and g are polynomial functions, then $f + g$ is a polynomial function.

b. If f and g are polynomial functions, then $\frac{f}{g}$ is a polynomial function.

c. If f and g are polynomial functions, the domain of the function $f \cdot g$ is the set of all real numbers.

d. If $f(x) = 3x + 2$ and $g(x) = x - 4$, the domain of the function $\frac{f}{g}$ is the set of all real numbers.

e. If f and g are polynomial functions, then $(f \circ g)(x) = (g \circ f)(x)$.

f. If f and g are polynomial functions, then $(f \cdot g)(x) = (g \cdot f)(x)$.

2. Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.

a. Explain in words how you would find $(f \circ g)(-3)$. (Do not actually do any calculations.)

b. Explain in words how you would find $(g \circ f)(-3)$. (Do not actually do any calculations.)

Remember What You Learned

3. Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word *closest* in the first sentence, the words *inside* and *outside* in the second, and the words *left* and *right* in the third.)

7-2 Lesson Reading Guide

Inverse Functions and Relations

Get Ready for the Lesson

Read the introduction to Lesson 7-2 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order?

Read the Lesson

1. Complete each statement.

a. If two relations are inverses, the domain of one relation is the ________________ of the other.

b. Suppose that g is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of g^{-1} is ________________.

c. The ________________ test can be used on the graph of a function to determine whether the function has an inverse function.

d. If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation ________________.

e. If f and g are inverse functions, then $(f \circ g)(x) =$ ________________ and $(g \circ f)(x) =$ ________________.

f. A function has an inverse that is also a function only if the given function is ________________.

g. Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) =$ ________________.

2. Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.

1. ____
2. ____
3. ____
4. ____

Remember What You Learned

3. A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related?

Lesson 7-2

NAME ____ DATE______________ PERIOD _____

7-3 Lesson Reading Guide

Square Root Functions

Get Ready for the Lesson

Read the introduction to Lesson 7-3 in your textbook.

If the weight to be supported by a steel cable is doubled, should the diameter of the support cable also be doubled? If not, by what number should the diameter be multiplied?

Read the Lesson

1. Match each square root function from the list on the left with its domain and range from the list on the right.

a. $y = \sqrt{x}$	**i.** domain: $x \geq 0$; range: $y \geq 3$
b. $y = \sqrt{x + 3}$	**ii.** domain: $x \geq 0$; range: $y \leq 0$
c. $y = \sqrt{x} + 3$	**iii.** domain: $x \geq 0$; range: $y \leq -3$
d. $y = \sqrt{x - 3}$	**iv.** domain: $x \geq 0$; range: $y \geq 0$
e. $y = -\sqrt{x}$	**v.** domain: $x \geq 3$; range: $y \geq 0$
f. $y = -\sqrt{x - 3}$	**vi.** domain: $x \leq 3$; range: $y \geq 3$
g. $y = \sqrt{3 - x} + 3$	**vii.** domain: $x \geq 3$; range: $y \leq 0$
h. $y = -\sqrt{x} - 3$	**viii.** domain: $x \geq -3$; range: $y \geq 0$

2. The graph of the inequality $y \leq \sqrt{3x + 6}$ is a shaded region. Which of the following points lie inside this region?

(3, 0) (2, 4) (5, 2) (4, −2) (6, 6)

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose you are studying this lesson with a classmate who thinks that you cannot have square root functions because every positive real number has two square roots. How would you explain the idea of square root functions to your classmate?

7-4 Lesson Reading Guide

*n*th *Roots*

Get Ready for the Lesson

Read the introduction to Lesson 7-4 in your textbook.

A basketball has a volume of about 382 cubic inches. Explain how you would find the radius of the basketball using a calculator. (Do not actually calculate the radius.)

Read the Lesson

1. For each radical below, identify the radicand and the index.

a. $\sqrt[3]{23}$ radicand: ____________ index: ____________

b. $\sqrt{15x^2}$ radicand: ____________ index: ____________

c. $\sqrt[5]{-343}$ radicand: ____________ index: ____________

2. Complete the following table. (Do not actually find any of the indicated roots.)

Number	Number of Positive Square Roots	Number of Negative Square Roots	Number of Positive Cube Roots	Number of Negative Cube Roots
27				
−16				

3. State whether each of the following is *true* or *false*.

a. A negative number has no real fourth roots.

b. $\pm\sqrt{121}$ represents both square roots of 121.

c. When you take the fifth root of x^5, you must take the absolute value of x to identify the principal fifth root.

Remember What You Learned

4. What is an easy way to remember that a negative number has no real square roots but has one real cube root?

7-5 Lesson Reading Guide

Operations with Radical Expressions

Get Ready for the Lesson

Read the introduction to Lesson 7-5 in your textbook.

Describe how you could use the golden ratio to find the height of a golden triangle if you knew its width.

Read the Lesson

1. Complete the conditions that must be met for a radical expression to be in simplified form.

- The ________________ n is as ________________ as possible.
- The ________________ contains no ________________ (other than 1) that are nth powers of a(n) ________________ or polynomial.
- The radicand contains no ________________.
- No ________________ appear in the ________________.

2. a. What are conjugates of radical expressions used for?

b. How would you use a conjugate to simplify the radical expression $\frac{1 + \sqrt{2}}{3 - \sqrt{2}}$?

c. In order to simplify the radical expression in part b, two multiplications are necessary. The multiplication in the numerator would be done by the ________________ method, and the multiplication in the denominator would be done by finding the ________________of ________________ ________________.

Remember What You Learned

3. One way to remember something is to explain it to another person. When rationalizing the denominator in the expression $\frac{1}{\sqrt[3]{2}}$, many students think they should multiply numerator and denominator by $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$. How would you explain to a classmate why this is incorrect and what he should do instead.

7-6 Lesson Reading Guide

Rational Exponents

Get Ready for the Lesson

Read the introduction to Lesson 7-6 in your textbook.

The formula in the introduction contains the exponent $\frac{2}{5}$. What do you think it might mean to raise a number to the $\frac{2}{5}$ power?

Read the Lesson

1. Complete the following definitions of rational exponents.

- For any real number b and for any positive integer n, $b^{\frac{1}{n}}$ = ________________ except when b ________________ and n is ________________.
- For any nonzero real number b, and any integers m and n, with n ________________, $b^{\frac{m}{n}}$ = ________________ = ________________, except when b ________________ and n is ________________.

2. Complete the conditions that must be met in order for an expression with rational exponents to be simplified.

- It has no ________________ exponents.
- It has no ________________ exponents in the ________________.
- It is not a ________________ fraction.
- The ________________ of any remaining ________________ is the ________________ number possible.

3. Margarita and Pierre were working together on their algebra homework. One exercise asked them to evaluate the expression $27^{\frac{4}{3}}$. Margarita thought that they should raise 27 to the fourth power first and then take the cube root of the result. Pierre thought that they should take the cube root of 27 first and then raise the result to the fourth power. Whose method is correct?

Remember What You Learned

4. Some students have trouble remembering which part of the fraction in a rational exponent gives the power and which part gives the root. How can your knowledge of integer exponents help you to keep this straight?

7-7 Lesson Reading Guide

Solving Radical Equations and Inequalities

Get Ready for the lesson

Read the introduction to Lesson 7-7 in your textbook.

Explain how you would use the formula in your textbook to find the cost of producing 125,000 computer chips. (Describe the steps of the calculation in the order in which you would perform them, but do not actually do the calculation.)

Read the Lesson

1. a. What is an *extraneous solution* of a radical equation?

b. Describe two ways you can check the proposed solutions of a radical equation in order to determine whether any of them are extraneous solutions.

2. Complete the steps that should be followed in order to solve a radical inequality.

Step 1 If the ________________ of the root is ________________, identify the values of the variable for which the ________________ is ________________.

Step 2 Solve the ________________ algebraically.

Step 3 Test ________________ to check your solution.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that your friend Leora thinks that she does not need to check her solutions to radical equations by substitution because she knows she is very careful and seldom makes mistakes in her work. How can you explain to her that she should nevertheless check every proposed solution in the original equation?

NAME ____ DATE______________ PERIOD _____

8-1 Lesson Reading Guide

Multiplying and Dividing Rational Expressions

Get Ready for the Lesson

Read the introduction to Lesson 8-1 in your textbook.

- Suppose that the Goodie Shoppe also sells a candy mixture made with 4 pounds of chocolate mints and 3 pounds of caramels, then _____________ of the mixture is mints and _____________ of the mixture is caramels.
- If the store manager adds another y pounds of mints to the mixture, what fraction of the mixture will be mints?

Read the Lesson

1. a. In order to simplify a rational number or rational expression, _________________ the numerator and _________________ and divide both of them by their _________________________________.

b. A rational expression is undefined when its _________________ is equal to ___________. To find the values that make the expression undefined, completely _________________ the original _________________ and set each factor equal to ___________.

2. a. To multiply two rational expressions, _________________ the _________________ and multiply the denominators.

b. To divide two rational expressions, _________________ by the _________________ of the _________________.

3. a. Which of the following expressions are complex fractions?

i. $\frac{7}{12}$ **ii.** $\frac{\frac{3}{8}}{\frac{5}{16}}$ **iii.** $\frac{r+5}{r-5}$ **iv.** $\frac{\frac{z+1}{z}}{z}$ **v.** $\frac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$

b. Does a complex fraction express a multiplication or division problem? How is multiplication used in simplifying a complex fraction?

Remember What You Learned

4. One way to remember something new is to see how it is similar to something you already know. How can your knowledge of division of fractions in arithmetic help you to understand how to divide rational expressions?

8-2 Lesson Reading Guide

Adding and Subtracting Rational Expressions

Get Ready for the Lesson

Read the introduction to Lesson 8-2 in your textbook.

A person is standing 5 feet from a camera that has a lens with a focal length of 3 feet. Write an equation that you could solve to find how far the film should be from the lens to get a perfectly focused photograph.

Read the Lesson

1. a. In work with rational expressions, LCD stands for ________________________ and LCM stands for ________________________. The LCD is the __________ of the denominators.

b. To find the LCM of two or more numbers or polynomials, ______________ each number or ______________. The LCM contains each ______________ the ______________ number of times it appears as a ______________.

2. To add $\frac{x^2 - 3}{x^2 - 5x + 6}$ and $\frac{x - 4}{x^3 - 4x^2 + 4x}$, you should first factor the ______________ of each fraction. Then use the factorizations to find the __________ of $x^2 - 5x + 6$ and $x^3 - 4x^2 + 4x$. This is the __________ for the two fractions.

3. When you add or subtract fractions, you often need to rewrite the fractions as equivalent fractions. You do this so that the resulting equivalent fractions will each have a denominator equal to the __________ of the original fractions.

4. To add or subtract two fractions that have the same denominator, you add or subtract their ______________ and keep the same ______________.

5. The sum or difference of two rational expressions should be written as a polynomial or as a fraction in ______________.

Remember What You Learned

6. Some students have trouble remembering whether a common denominator is needed to add and subtract rational expressions or to multiply and divide them. How can your knowledge of working with fractions in arithmetic help you remember this?

8-3 Lesson Reading Guide

Graphing Rational Functions

Get Ready for the Lesson

Read the introduction to Lesson 8-3 in your textbook.

- If 15 students contribute to the gift, how much would each of them pay?
- If each student pays \$5, how many students contributed?

Read the Lesson

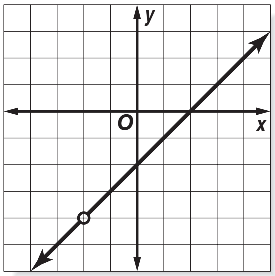
1. Which of the following are rational functions?

A. $f(x) = \dfrac{1}{x - 5}$ **B.** $g(x) = \sqrt{x}$ **C.** $h(x) = \dfrac{x^2 - 25}{x^2 + 6x + 9}$

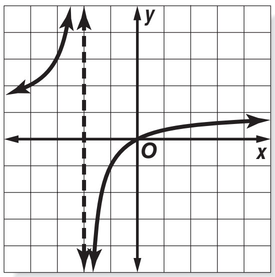
2. a. Graphs of rational functions may have breaks in _________________. These may occur as vertical _________________ or as point _________________. The _________________ of a rational function is limited to values for which the function is defined.

b. The graphs of two rational functions are shown below.

I.



II.



Graph I has a __________________________ at $x =$ _________.

Graph II has a __________________________ at $x =$ _________.

Match each function with its graph above.

$f(x) = \dfrac{x}{x + 2}$ $g(x) = \dfrac{x^2 - 4}{x + 2}$

Remember What You Learned

3. One way to remember something new is to see how it is related to something you already know. How can knowing that division by zero is undefined help you to remember how to find the places where a rational function has a point discontinuity or an asymptote?

Lesson 8-3

8-4 Lesson Reading Guide

Direct, Joint, and Inverse Variation

Get Ready for the Lesson

Read the introduction to Lesson 8-4 in your textbook.

- For each additional student who enrolls in a public college, the total high-tech spending will ________________ (increase/decrease) by ________________.
- For each decrease in enrollment of 100 students in a public college, the total high-tech spending will ________________ (increase/decrease) by ________________.

Read the Lesson

1. Write an equation to represent each of the following variation statements. Use k as the constant of variation.

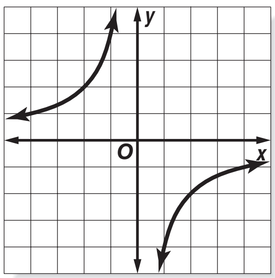
a. m varies inversely as n.

b. s varies directly as r.

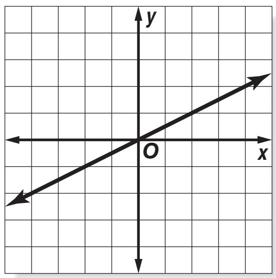
c. t varies jointly as p and q.

2. Which type of variation, direct or inverse, is represented by each graph?

a.



b.



Remember What You Learned

3. How can your knowledge of the equation of the slope-intercept form of the equation of a line help you remember the equation for direct variation?

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8-5 Lesson Reading Guide

Classes of Functions

Get Ready for the Lesson

Read the introduction to Lesson 8-5 in your textbook.

- Based on the graph, estimate the weight on Mars of a child who weighs 40 pounds on Earth.

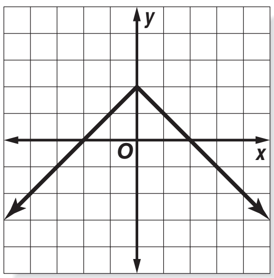
- Although the graph does not extend far enough to the right to read it directly from the graph, use the weight you found above and your knowledge that this graph represents direct variation to estimate the weight on Mars of a woman who weighs 120 pounds on Earth.

Read the Lesson

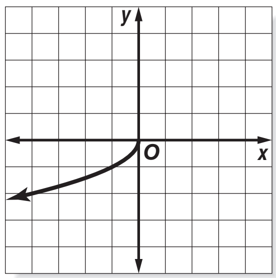
1. Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

I. square root **II.** quadratic **III.** absolute value **IV.** rational
V. greatest integer **VI.** constant **VII.** identity

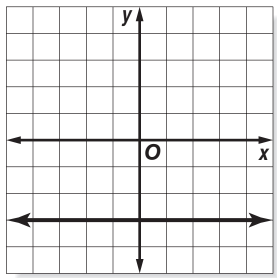
a.



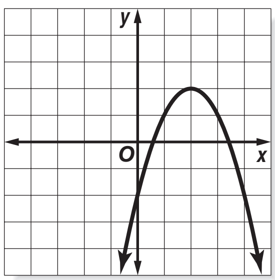
b.



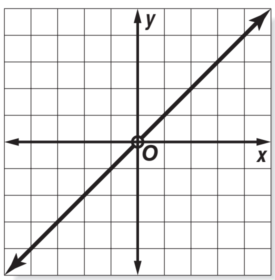
c.



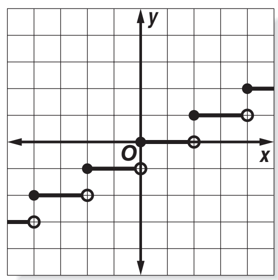
d.



e.



f.



Remember What You Learned

2. How can the symbolic definition of absolute value that you learned in Lesson 1-4 help you to remember the graph of the function $f(x) = |x|$?

8-6 Lesson Reading Guide

Solving Rational Equations and Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 8-6 in your textbook.

- If you increase the number of songs that you download, will your total bill increase or decrease?
- Will your actual cost per song increase or decrease?

Read the Lesson

1. When solving a rational equation, any possible solution that results in __________ in the denominator must be excluded from the list of solutions.

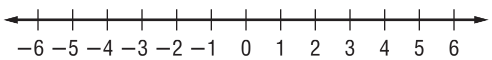
2. Suppose that on a quiz you are asked to solve the rational inequality $\frac{3}{z+2} - \frac{6}{z} > 0$. Complete the steps of the solution.

Step 1 The excluded values are __________ and __________.

Step 2 The related equation is ______________.

To solve this equation, multiply both sides by the LCD, which is ______________. Solving this equation will show that the only solution is -4.

Step 3 Divide a number line into __________ regions using the excluded values and the solution of the related equation. Draw dashed vertical lines on the number line below to show these regions.



Consider the following values of $\frac{3}{z+2} - \frac{6}{z}$ for various test values of z.

If $z = -5$, $\frac{3}{z+2} - \frac{6}{z} = 0.2$.

If $z = -3$, $\frac{3}{z+2} - \frac{6}{z} = -1$.

If $z = -1$, $\frac{3}{z+2} - \frac{6}{z} = 9$.

If $z = 1$, $\frac{3}{z+2} - \frac{6}{z} = -5$.

Using this information and your number line, write the solution of the inequality.

Remember What You Learned

3. How are the processes of adding rational expressions with different denominators and of solving rational expressions alike, and how are they different?

9-1 Lesson Reading Guide

Exponential Functions

Get Ready for the Lesson

Read the introduction to Lesson 9-1 in your textbook.

How many rounds of play would be needed for a tournament with 100 players?

Read the Lesson

1. Indicate whether each of the following statements about the exponential function $y = 10^x$ is *true* or *false*.

a. The domain is the set of all positive real numbers.

b. The y-intercept is 1.

c. The function is one-to-one.

d. The y-axis is an asymptote of the graph.

e. The range is the set of all real numbers.

2. Determine whether each function represents exponential *growth* or *decay*.

a. $y = 0.2(3)^x$ **b.** $y = 3\left(\frac{2}{5}\right)^x$ **c.** $y = 0.4(1.01)^x$

3. Supply the reason for each step in the following solution of an exponential equation.

$9^{2x-1} = 27x$	Original equation
$(3^2)^{2x-1} = (3^3)^x$	______________________
$3^{2(2x-1)} = 3^{3x}$	______________________
$2(2x - 1) = 3x$	______________________
$4x - 2 = 3x$	______________________
$x - 2 = 0$	______________________
$x = 2$	______________________

Remember What You Learned

4. One way to remember that polynomial functions and exponential functions are different is to contrast the polynomial function $y = x^2$ and the exponential function $y = 2^x$. Tell at least three ways they are different.

9-2 Reading to Learn Mathematics

Logarithms and Logarithmic Functions

Get Ready for the Lesson

Read the introduction to Lesson 9-2 in your textbook.

How many times louder than a whisper is normal conversation?

Read the Lesson

1. a. Write an exponential equation that is equivalent to $\log_3 81 = 4$.

b. Write a logarithmic equation that is equivalent to $25^{-\frac{1}{2}} = \frac{1}{5}$.

c. Write an exponential equation that is equivalent to $\log_4 1 = 0$.

d. Write a logarithmic equation that is equivalent to $10^{-3} = 0.001$.

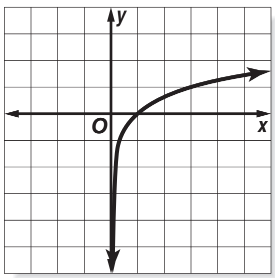
e. What is the inverse of the function $y = 5^x$?

f. What is the inverse of the function $y = \log_{10} x$?

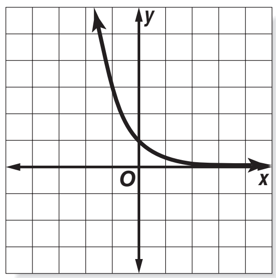
2. Match each function with its graph.

a. $y = 3^x$ **b.** $y = \log_3 x$ **c.** $y = \left(\frac{1}{3}\right)^x$

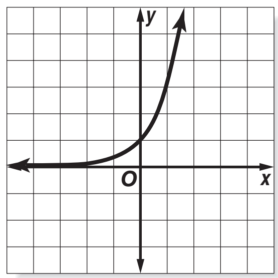
I.



II.



III.



3. Indicate whether each of the following statements about the exponential function $y = \log_5 x$ is *true* or *false*.

a. The y-axis is an asymptote of the graph.

b. The domain is the set of all real numbers.

c. The graph contains the point (5, 0).

d. The range is the set of all real numbers.

e. The y-intercept is 1.

Remember What You Learned

4. An important skill needed for working with logarithms is changing an equation between logarithmic and exponential forms. Using the words *base*, *exponent*, and *logarithm*, describe an easy way to remember and apply the part of the definition of logarithm that says, "$\log_b x = y$ if and only if $b^y = x$."

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9-3 Lesson Reading Guide

Properties of Logarithms

Get Ready for the Lesson

Read the introduction to Lesson 9-3 in your textbook.

1. Find the value of each of the following.

a. $\log_5 125$ **b.** $\log_5 5$ **c.** $\log_5 (125 \div 5)$

2. Which of the following statements is true?

A. $\log_5 (125 \div 5) = (\log_5 125) \div (\log_5 5)$

B. $\log_5 (125 \div 5) = \log_5 125 - \log_5 5$

Read the Lesson

1. Each of the properties of logarithms can be stated in words or in symbols. Complete the statements of these properties in words.

a. The logarithm of a quotient is the ________________ of the logarithms of the ________________ and the ________________.

b. The logarithm of a power is the ________________ of the logarithm of the base and the ________________.

c. The logarithm of a product is the ________________ of the logarithms of its ________________.

2. State whether each of the following equations is *true* or *false*. If the statement is true, name the property of logarithms that is illustrated.

a. $\log_3 10 = \log_3 30 - \log_3 3$

b. $\log_4 12 = \log_4 4 + \log_4 8$

c. $\log_2 81 = 2 \log_2 9$

d. $\log_8 30 = \log_8 5 \cdot \log_8 6$

3. The algebraic process of solving the equation $\log_2 x + \log_2 (x + 2) = 3$ leads to "$x = -4$ or $x = 2$." Does this mean that both -4 and 2 are solutions of the logarithmic equation? Explain your reasoning.

Remember What You Learned

4. A good way to remember something is to relate it something you already know. Use words to explain how the Product Property for exponents can help you remember the product property for logarithms.

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9-4 Lesson Reading Guide

Common Logarithms

Get Ready for the Lesson

Read the introduction to Lesson 9-4 in your textbook.

Which substance is more acidic, milk or tomatoes?

Read the Lesson

1. Rhonda used the following keystrokes to enter an expression on her graphing calculator:

LOG 17) ENTER

The calculator returned the result 1.230448921.
Which of the following conclusions are correct?

a. The base 10 logarithm of 17 is about 1.2304.

b. The base 17 logarithm of 10 is about 1.2304.

c. The common logarithm of 17 is about 1.230449.

d. $10^{1.230448921}$ is very close to 17.

e. The common logarithm of 17 is exactly 1.230448921.

2. Match each expression from the first column with an expression from the second column that has the same value.

a. $\log_2 2$	**i.** $\log_4 1$
b. $\log 12$	**ii.** $\log_2 8$
c. $\log_3 1$	**iii.** $\log_{10} 12$
d. $\log_5 \frac{1}{5}$	**iv.** $\log_5 5$
e. $\log 1000$	**v.** $\log 0.1$

3. Calculators do not have keys for finding base 8 logarithms directly. However, you can use a calculator to find $\log_8 20$ if you apply the __________________________ formula.
Which of the following expressions are equal to $\log_8 20$?

A. $\log_{20} 8$ **B.** $\frac{\log_{10} 20}{\log_{10} 8}$ **C.** $\frac{\log 20}{\log 8}$ **D.** $\frac{\log 8}{\log 20}$

Remember What You Learned

4. Sometimes it is easier to remember a formula if you can state it in words. State the change of base formula in words.

NAME ____ DATE______________ PERIOD _____

9-5 Lesson Reading Guide

Base e and Natural Logarithms

Get Ready for the Lesson

Read the introduction to Lesson 9-5 in your textbook.

Suppose that you deposit $675 in a savings account that pays an annual interest rate of 5%. In each case listed below, indicate which method of compounding would result in more money in your account at the end of one year.

a. annual compounding or monthly compounding

b. quarterly compounding or daily compounding

c. daily compounding or continuous compounding

Read the Lesson

1. Jagdish entered the following keystrokes in his calculator:

LN 5) ENTER

The calculator returned the result 1.609437912. Which of the following conclusions are correct?

a. The common logarithm of 5 is about 1.6094.

b. The natural logarithm of 5 is exactly 1.609437912.

c. The base 5 logarithm of e is about 1.6094.

d. The natural logarithm of 5 is about 1.609438.

e. $10^{1.609437912}$ is very close to 5.

f. $e^{1.609437912}$ is very close to 5.

2. Match each expression from the first column with its value in the second column. Some choices may be used more than once or not at all.

a. $e^{\ln 5}$	**I.** 1
b. $\ln 1$	**II.** 10
c. $e^{\ln e}$	**III.** -1
d. $\ln e^5$	**IV.** 5
e. $\ln e$	**V.** 0
f. $\ln \left(\frac{1}{e}\right)$	**VI.** e

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that you are studying with a classmate who is puzzled when asked to evaluate $\ln e^3$. How would you explain to him an easy way to figure this out?

9-6 Lesson Reading Guide

Exponential Growth and Decay

Get Ready for the Lesson

Read the introduction to Lesson 9-6 in your textbook.

- Between which two years shown in the table did the car depreciate by the greatest amount?
- Describe two ways to calculate the value of the car 6 years after it was purchased. (Do not actually calculate the value.)

Read the Lesson

1. State whether each situation is an example of exponential *growth* or *decay*.

a. A city had 42,000 residents in 1980 and 128,000 residents in 2000.

b. Raul compared the value of his car when he bought it new to the value when he traded it in six years later.

c. A paleontologist compared the amount of carbon-14 in the skeleton of an animal when it died to the amount 300 years later.

d. Maria deposited $750 in a savings account paying 4.5% annual interest compounded quarterly. She did not make any withdrawals or further deposits. She compared the balance in her passbook immediately after she opened the account to the balance 3 years later.

2. State whether each equation represents exponential growth or decay.

a. $y = 5e^{0.15t}$ **b.** $y = 1000(1 - 0.05)^t$

c. $y = 0.3e^{-1200t}$ **d.** $y = 2(1 + 0.0001)^t$

Remember What You Learned

3. Visualizing their graphs is often a good way to remember the difference between mathematical equations. How can your knowledge of the graphs of exponential equations from Lesson 9-1 help you to remember that equations of the form $y = a(1 + r)^t$ represent exponential growth, while equations of the form $y = a(1 - r)^t$ represent exponential decay?

10-1 Lesson Reading Guide

Midpoint and Distance Formulas

Get Ready for the Lesson

Read the introduction to Lesson 10-1 in your textbook.

How do you find distances on a road map?

Read the Lesson

1. a. Write the coordinates of the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2).

b. Explain how to find the midpoint of a segment if you know the coordinates of the endpoints. Do not use subscripts in your explanation.

2. a. Write an expression for the distance between two points with coordinates (x_1, y_1) and (x_2, y_2).

b. Explain how to find the distance between two points. Do not use subscripts in your explanation.

3. Consider the segment connecting the points $(-3, 5)$ and $(9, 11)$.

a. Find the midpoint of this segment.

b. Find the length of the segment. Write your answer in simplified radical form.

Remember What You Learned

4. How can the "mid" in *midpoint* help you remember the midpoint formula?

10-2 Lesson Reading Guide

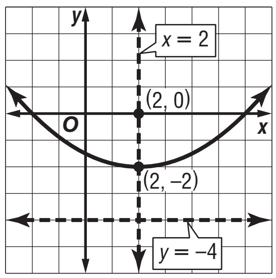
Parabolas

Get Ready for the Lesson

Read the introduction to Lesson 10-2 in your textbook.

Name at least two reflective objects that might have the shape of a parabola.

Read the Lesson



1. In the parabola shown in the graph, the point $(2, -2)$ is called the ______________________ and the point $(2, 0)$ is called the ______________________. The line $y = -4$ is called the ______________________, and the line $x = 2$ is called the ______________________.

2. a. Write the standard form of the equation of a parabola that opens upward or downward.

b. The parabola opens downward if ___________ and opens upward if ___________. The equation of the axis of symmetry is ___________, and the coordinates of the vertex are ___________.

3. A parabola has equation $x = -\frac{1}{8}(y - 2)^2 + 4$. This parabola opens to the ___________. It has vertex ___________ and focus ___________. The directrix is ___________. The length of the latus rectum is ___________ units.

Remember What You Learned

4. How can the way in which you plot points in a rectangular coordinate system help you to remember what the sign of a tells you about the direction in which a parabola opens?

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10-3 Lesson Reading Guide

Circles

Get Ready for the Lesson

Read the introduction to Lesson 10-3 in your textbook.

A large home improvement chain is planning to enter a new metropolitan area and needs to select locations for its stores. Market research has shown that potential customers are willing to travel up to 12 miles to shop at one of their stores. How can circles help the managers decide where to place their store?

Read the Lesson

1. a. Write the equation of the circle with center (h, k) and radius r.

b. Write the equation of the circle with center $(4, -3)$ and radius 5.

c. The circle with equation $(x + 8)^2 + y^2 = 121$ has center __________ and radius __________.

d. The circle with equation $(x - 10)^2 + (y + 10)^2 = 1$ has center __________ and radius __________.

2. a. In order to find center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$, it is necessary to ______________________________. Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

$$x^2 + y^2 + 4x - 6y = \_\_\_\_$$

$$x^2 + 4x + \_\_\_\_ + y^2 - 6y + \_\_\_\_ = \_\_\_\_ + \_\_\_\_ + \_\_\_\_$$

$$(x + \_\_\_\_)^2 + (y - \_\_\_\_)^2 = \_\_\_\_$$

b. This circle has radius 4 and center at __________.

Remember What You Learned

3. How can the distance formula help you to remember the equation of a circle?

NAME ____ DATE______________ PERIOD _____

10-4 Lesson Reading Guide

Ellipses

Get Ready for the Lesson

Read the introduction to Lesson 10-4 in your textbook.

Is the Earth always the same distance from the Sun? Explain your answer using the words *circle* and *ellipse*.

Read the Lesson

1. An ellipse is the set of all points in a plane such that the ________________ of the distances from two fixed points is ________________. The two fixed points are called the ________________ of the ellipse.

2. Consider the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

a. For this equation, $a =$ ________ and $b =$ ________.

b. Write an equation that relates the values of a, b, and c.

c. Find the value of c for this ellipse.

3. Consider the ellipses with equations $\frac{y^2}{25} + \frac{x^2}{16} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Complete the following table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis		
Direction of Minor Axis		
Foci		
Length of Major Axis		
Length of Minor Axis		

Remember What You Learned

4. Some students have trouble remembering the two standard forms for the equation of an ellipse. How can you remember which term comes first and where to place a and b in these equations?

10-5 Lesson Reading Guide

Hyperbolas

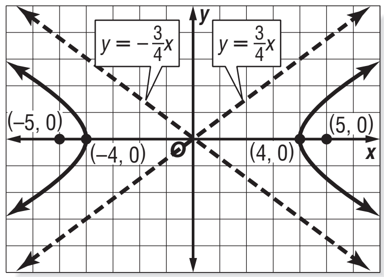
Get Ready for the Lesson

Read the introduction to Lesson 10-5 in your textbook.

Look at the sketch of a hyperbola in the introduction to this lesson. List three ways in which hyperbolas are different from parabolas.

Read the Lesson

1. The graph at the right shows the hyperbola whose equation in standard form is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.



The point (0, 0) is the _________________ of the hyperbola.

The points (4, 0) and (−4, 0) are the _________________ of the hyperbola.

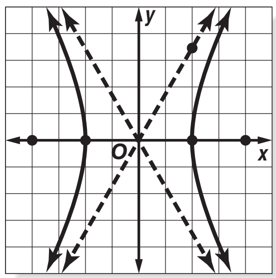
The points (5, 0) and (−5, 0) are the _________________ of the hyperbola.

The segment connecting (4, 0) and (−4, 0) is called the _________________ axis.

The segment connecting (0, 3) and (0, −3) is called the _________________ axis.

The lines $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$ are called the _________________.

2. Study the hyperbola graphed at the right.



The center is ________.

The value of a is ________.

The value of c is ________.

To find b^2, solve the equation ________ = ________ + ________.

The equation in standard form for this hyperbola is _________________.

Remember What You Learned

3. What is an easy way to remember the equation relating the values of a, b, and c for a hyperbola?

Lesson 10-5

NAME ____ DATE______________ PERIOD _____

10-6 Lesson Reading Guide

Conic Sections

Get Ready for the Lesson

Read the introduction to Lesson 10-6 in your textbook.

The figures in the introduction show how a plane can slice a double cone to form the conic sections. Name the conic section that is formed if the plane slices the double cone in each of the following ways:

- The plane is parallel to the base of the double cone and slices through one of the cones that form the double cone.
- The plane is perpendicular to the base of the double cone and slices through both of the cones that form the double cone.

Read the Lesson

1. Name the conic section that is the graph of each of the following equations. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.

a. $\frac{(x - 3)^2}{36} + \frac{(y + 5)^2}{15} = 1$

b. $x = -2(y + 1)^2 + 7$

c. $(x - 5)^2 - (y + 5)^2 = 1$

d. $(x + 6)^2 + (y - 2)^2 = 1$

2. Each of the following is the equation of a conic section. For each equation, identify the values of A and C. Then, without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle, ellipse*, or *hyperbola*.

a. $2x^2 + y^2 - 6x + 8y + 12 = 0$ A = _____; C = _____; type of graph: _____________

b. $2x^2 + 3x - 2y - 5 = 0$ A = _____; C = _____; type of graph: _____________

c. $5x^2 + 10x + 5y^2 - 20y + 1 = 0$ A = _____; C = _____; type of graph: _____________

d. $x^2 - y^2 + 4x + 2y - 5 = 0$ A = _____; C = _____; type of graph: _____________

Remember What You Learned

3. What is an easy way to recognize that an equation represents a parabola rather than one of the other conic sections?

10-7 Lesson Reading Guide

Solving Quadratic Systems

Get Ready for the Lesson

Read the introduction to Lesson 10-7 in your textbook.

The figure in your textbook shows that the spaceship hits the circular force field in two points. Is it possible for the spaceship to hit the force field in either fewer or more than two points? State all possibilities and explain how these could happen.

Read the Lesson

1. Draw a sketch to illustrate each of the following possibilities.

a. a parabola and a line that intersect in 2 points

b. an ellipse and a circle that intersect in 4 points

c. a hyperbola and a line that intersect in 1 point

2. Consider the following system of equations.

$x^2 = 3 + y^2$
$2x^2 + 3y^2 = 11$

a. What kind of conic section is the graph of the first equation?

b. What kind of conic section is the graph of the second equation?

c. Based on your answers to parts a and b, what are the possible numbers of solutions that this system could have?

Remember What You Learned

3. Suppose that the graph of a quadratic inequality is a region whose boundary is a circle. How can you remember whether to shade the interior or the exterior of the circle?

11-1 Lesson Reading Guide

Arithmetic Sequences

Get Ready for the Lesson

Read the introduction to Lesson 11-1 in your textbook.

Describe how you would find the number of shingles needed for the fifteenth row. (Do not actually calculate this number.) Explain why your method will give the correct answer.

Read the Lesson

1. Consider the formula $a_n = a_1 + (n - 1)d$.

a. What is this formula used to find?

b. What do each of the following represent?

a_n: ______________________________

a_1: ______________________________

n: ______________________________

d: ______________________________

2. Consider the equation $a_n = -3n + 5$.

a. What does this equation represent?

b. Is the graph of this equation a straight line? Explain your answer.

c. The functions represented by the equations $a_n = -3n + 5$ and $f(x) = -3x + 5$ are alike in that they have the same formula. How are they different?

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that your classmate Shala has trouble remembering the formula $a_n = a_1 + (n - 1)d$ correctly. She thinks that the formula should be $a_n = a_1 + nd$. How would you explain to her that she should use $(n - 1)d$ rather than nd in the formula?

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11-2 Lesson Reading Guide

Arithmetic Series

Get Ready for the Lesson

Read the introduction to Lesson 11-2 in your textbook.

Suppose that an amphitheater can seat 50 people in the first row and that each row thereafter can seat 9 more people than the previous row. Using the vocabulary of arithmetic sequences, describe how you would find the number of people who could be seated in the first 10 rows. (Do not actually calculate the sum.)

Read the Lesson

1. What is the relationship between an arithmetic sequence and the corresponding arithmetic series?

2. Consider the formula $S_n = \frac{n}{2}(a_1 + a_n)$. Explain the meaning of this formula in words.

3. a. What is the purpose of sigma notation?

b. Consider the expression $\sum_{i=2}^{12}(4i - 2)$.

This form of writing a sum is called ___________________________.

The variable i is called the ___________________________.

The first value of i is ___________.

The last value of i is ___________.

How would you read this expression?

Remember What You Learned

4. A good way to remember something is to relate it to something you already know. How can your knowledge of how to find the average of two numbers help you remember the formula $S_n = \frac{n}{2}(a_1 + a_n)$?

NAME ____ DATE______________ PERIOD _____

11-3 Lesson Reading Guide

Geometric Sequences

Get Ready for the Lesson

Read the introduction to Lesson 11-3 in your textbook.

Suppose that you drop a ball from a height of 4 feet, and that each time it falls, it bounces back to 74% of the height from which it fell. Describe how would you find the height of the third bounce. (Do not actually calculate the height of the bounce.)

Read the Lesson

1. Explain the difference between an arithmetic sequence and a geometric sequence.

2. Consider the formula $a_n = a_1 \cdot r^{n-1}$.

a. What is this formula used to find?

b. What do each of the following represent?

a_n: ____

a_1: ____

r: ____

n: ____

3. a. In the sequence 5, 8, 11, 14, 17, 20, the numbers 8, 11, 14, and 17 are ____________________ between 5 and 20.

b. In the sequence 12, 4, $\frac{4}{3}$, $\frac{4}{9}$, $\frac{4}{27}$, the numbers 4, $\frac{4}{3}$, and $\frac{4}{9}$ are ____________________ between 12 and $\frac{4}{27}$.

Remember What You Learned

4. Suppose that your classmate Ricardo has trouble remembering the formula $a_n = a_1 \cdot r^{n-1}$ correctly. He thinks that the formula should be $a_n = a_1 \cdot r^n$. How would you explain to him that he should use r^{n-1} rather than r^n in the formula?

11-4 Lesson Reading Guide

Geometric Series

Get Ready for the Lesson

Read the introduction to Lesson 11-4 in your textbook.

- Suppose that you e-mail the joke on Monday to five friends, rather than three, and that each of those friends e-mails it to five friends on Tuesday, and so on. Write a sum that shows that total number of people, including yourself, who will have read the joke by Thursday. (Write out the sum using plus signs rather than sigma notation. Do not actually find the sum.)
- Use exponents to rewrite the sum you found above. (Use an exponent in each term, and use the same base for all terms.)

Read the Lesson

1. Consider the formula $S_n = \frac{a_1(1 - r^n)}{1 - r}$.

a. What is this formula used to find?

b. What do each of the following represent?

S_n: ____

a_1: ____

r: ____

c. Suppose that you want to use the formula to evaluate $3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27}$. Indicate the values you would substitute into the formula in order to find S_n. (Do not actually calculate the sum.)

n = __________ a_1 = __________ r = __________ r^n = __________

d. Suppose that you want to use the formula to evaluate the sum $\sum_{n=1}^{6} 8(-2)^{n-1}$. Indicate the values you would substitute into the formula in order to find S_n. (Do not actually calculate the sum.)

n = __________ a_1 = __________ r = __________ r^n = __________

Remember What You Learned

2. This lesson includes three formulas for the sum of the first n terms of a geometric series. All of these formulas have the same denominator and have the restriction $r \neq 1$. How can this restriction help you to remember the denominator in the formulas?

Lesson 11-4

11-5 Lesson Reading Guide

Infinite Geometric Series

Get Ready for the Lesson

Read the introduction to Lesson 11-5 in your textbook.

Note the following powers of 0.6: $0.6^1 = 0.6$; $0.6^2 = 0.36$; $0.6^3 = 0.216$; $0.6^4 = 0.1296$; $0.6^5 = 0.07776$; $0.6^6 = 0.046656$; $0.6^7 = 0.0279936$. If a ball is dropped from a height of 10 feet and bounces back to 60% of its previous height on each bounce, after how many bounces will it bounce back to a height of less than 1 foot?

Read the Lesson

1. Consider the formula $S = \frac{a_1}{1 - r}$.

a. What is the formula used to find?

b. What do each of the following represent?

S: ______________________________

a_1: ______________________________

r: ______________________________

c. For what values of r does an infinite geometric sequence have a sum?

d. Rewrite your answer for part d as an absolute value inequality.

2. For each of the following geometric series, give the values of a_1 and r. Then state whether the sum of the series exists. (Do not actually find the sum.)

a. $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$ $a_1 =$ __________ $r =$ __________

Does the sum exist? __________

b. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \ldots$ $a_1 =$ __________ $r =$ __________

Does the sum exist? __________

c. $\sum_{i=1}^{\infty} 3^i$ $a_1 =$ __________ $r =$ __________

Does the sum exist? __________

Remember What You Learned

3. One good way to remember something is to relate it to something you already know. How can you use the formula $S_n = \frac{a_1(1 - r^n)}{1 - r}$ that you learned in Lesson 11-4 for finding the sum of a geometric series to help you remember the formula for finding the sum of an infinite geometric series?

11-6 Lesson Reading Guide

Recursion and Special Sequences

Get Ready for the Lesson

Read the introduction to Lesson 11-6 in your textbook.

What are the next three numbers in the sequence that gives the number of shoots corresponding to each month?

Read the Lesson

1. Consider the sequence in which $a_1 = 4$ and $a_n = 2a_{n-1} + 5$.

a. Explain why this is a *recursive* formula.

b. Explain in your own words how to find the first four terms of this sequence. (Do not actually find any terms after the first.)

c. What happens to the terms of this sequence as n increases?

2. Consider the function $f(x) = 3x - 1$ with an initial value of $x_0 = 2$.

a. What does it mean to *iterate* this function?

b. Fill in the blanks to find the first three iterates. The blanks that follow the letter x are for subscripts.

$x_1 = f(x\_) = f(\_\_\_\_) = 3(\_\_\_\_) - 1 = \_\_\_\_ - 1 = \_\_\_\_$

$x_2 = f(x\_) = f(\_\_\_\_) = 3(\_\_\_\_) - 1 = \_\_\_\_$

$x_3 = f(x\_) = f(\_\_\_\_) = 3(\_\_\_\_) - 1 = \_\_\_\_$

c. As this process continues, what happens to the values of the iterates?

Remember What You Learned

3. Use a dictionary to find the meanings of the words *recurrent* and *iterate*. How can the meanings of these words help you to remember the meaning of the mathematical terms *recursive* and *iteration*? How are these ideas related?

11-7 Lesson Reading Guide

The Binomial Theorem

Get Ready for the Lesson

Read the introduction to Lesson 11-7 in your textbook.

- If a family has four children, list the sequences of births of girls and boys that result in three girls and one boy.
- Describe a way to figure out how many such sequences there are without listing them.

Read the Lesson

1. Consider the expansion of $(w + z)^5$.

a. How many terms does this expansion have?

b. In the second term of the expansion, what is the exponent of w?

What is the exponent of z?

What is the coefficient of the second term?

c. In the fourth term of the expansion, what is the exponent of w?

What is the exponent of z?

What is the coefficient of the fourth term?

d. What is the last term of this expansion?

2. a. State the definition of a *factorial* in your own words. (Do not use mathematical symbols in your definition.)

b. Write out the product that you would use to calculate 10!. (Do not actually calculate the product.)

c. Write an expression involving factorials that could be used to find the coefficient of the third term of the expansion of $(m - n)^6$. (Do not actually calculate the coefficient.)

Remember What You Learned

3. Without using Pascal's triangle or factorials, what is an easy way to remember the first two and last two coefficients for the terms of the binomial expansion of $(a + b)^n$?

NAME ____ DATE______________ PERIOD _____

11-8 Lesson Reading Guide

Proof and Mathematical Induction

Get Ready for the Lesson

Read the introduction to Lesson 11-8 in your textbook.

Suppose there are 100 dominoes standing upright and closely together in a row. If any of the dominoes fall down, the domino next to it will also fall down. What will happen if you knock down the first domino in the row? How does this relate the mathematical induction?

Read the Lesson

1. Fill in the blanks to describe the three steps in a proof by mathematical induction.

Step 1 Show that the statement is _____________ for the number _________.

Step 2 Assume that the statement is _____________ for some positive _____________ k.

This assumption is called the __________________________.

Step 3 Show that the statement is _____________ for the next integer ___________.

2. Suppose that you wanted to prove that the following statement is true for all positive integers.

$$3 + 6 + 9 + \ldots + 3n = \frac{3n(n + 1)}{2}$$

a. Which of the following statements shows that the statement is true for $n = 1$?

i. $3 = \frac{3 \cdot 2 + 1}{2}$ **ii.** $3 = \frac{3 \cdot 1 \cdot 2}{2}$ **iii.** $3 = \frac{3 + 1 + 2}{2}$

b. Which of the following is the statement for $n = k + 1$?

i. $3 + 6 + 9 + \ldots + 3^k = \frac{3k(k + 1)}{2}$

ii. $3 + 6 + 9 + \ldots + 3^{k + 1} = \frac{3k(k + 1)}{2}$

iii. $3 + 6 + 9 + \ldots + 3^{k + 1} = 3(k + 1)(k + 2)$

iv. $3 + 6 + 9 + \ldots + 3(k + 1) = \frac{3(k + 1)(k + 2)}{2}$

Remember What You Learned

3. Many students confuse the roles of n and k in a proof by mathematical induction. What is a good way to remember the difference in the ways these variables are used in such a proof?

12-1 Lesson Reading Guide

The Counting Principle

Get Ready for the Lesson

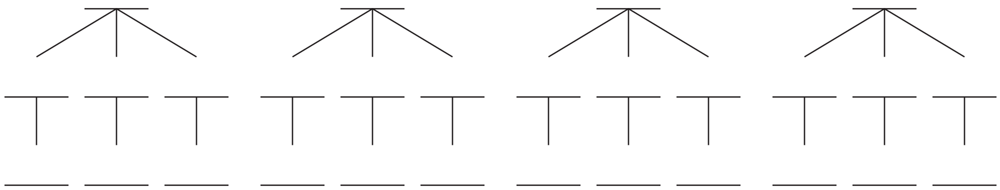
Read the introduction to Lesson 12-1 in your textbook.

Assume that all Florida license plates have three letters followed by three digits, and that there are no rules against using the same letter or number more than once. How many choices are there for each letter? for each digit?

Read the Lesson

1. Shamim is signing up for her classes. Most of her classes are required, but she has two electives. For her arts class, she can chose between Art, Band, Chorus, or Drama. For her language class, she can choose between French, German, and Spanish.

a. To organize her choices, Shamim decides to make a tree diagram. Let A, B, C, and D represent Art, Band, Chorus, and Drama, and F, G, and S represent French, German, and Spanish. Complete the following diagram.



b. How could Shamim have found the number of possible combinations without making a tree diagram?

2. A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.

a. A marble is drawn out of the jar and is not replaced. A second marble is drawn.

b. A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn.

Remember What You Learned

3. One definition of *independent* is “not determined or influenced by someone or something else.” How can this definition help you remember the difference between *independent* and *dependent* events?

12-2 Lesson Reading Guide

Permutations and Combinations

Get Ready for the Lesson

Read the introduction to Lesson 12-2 in your textbook.

Suppose that 20 students enter a math contest. In how many ways can first, second, and third places be awarded? (Write your answer as a product. Do not calculate the product.)

Read the Lesson

1. Indicate whether each situation involves a *permutation* or a *combination*.

a. choosing five students from a class to work on a special project

b. arranging five pictures in a row on a wall

c. drawing a hand of 13 cards from a 52-card deck

d. arranging the letters of the word *algebra*

2. Write an expression that can be used to calculate each of the following.

a. number of combinations of n distinct objects taken r at a time

b. number of permutations of n objects of which p are alike and q are alike

c. number of permutations of n distinct objects taken r at a time

3. Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades.

a. Which of the following would you use to solve this problem: *Fundamental Counting Principle*, *permutations*, or *combinations*? (More than one of these may apply.)

b. Write an expression that involves the notation $P(n, r)$ and/or $C(n, r)$ that you would use to solve this problem. (Do not do any calculations.)

Remember What You Learned

4. Many students have trouble knowing when to use permutations and when to use combinations to solve counting problems. How can the idea of *order* help you to remember the difference between permutations and combinations?

NAME ____ DATE______________ PERIOD _____

12-3 Lesson Reading Guide

Probability

Get Ready for the Lesson

Read the introduction to Lesson 12-3 in your textbook.

What is the probability that a person will *not* be struck by lightning in a given year?

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. If an event can never occur, its probability is a negative number.

b. If an event is certain to happen, its probability is 1.

c. If an event can succeed in s ways and fail in f ways, then the probability of success is $\frac{s}{f}$.

d. If an event can succeed in s ways and fail in f ways, then the odds against the event are $s:f$.

e. A probability distribution is a function in which the domain is the sample space of an experiment.

2. A weather forecast says that the chance of rain tomorrow is 40%.

a. Write the probability that it will rain tomorrow as a fraction in lowest terms.

b. Write the probability that it will not rain tomorrow as a fraction in lowest terms.

c. What are the odds in favor of rain?

d. What are the odds against rain?

3. Refer to the table in Example 4 on page 646 in your textbook.

a. What other sum has the same probability as a sum of 11?

b. What are the odds of rolling a sum of 8?

c. What are the odds against rolling a sum of 9?

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose that your friend Roberto is having trouble remembering the difference between probability and odds. What would you tell him to help him remember this easily?

12-4 Lesson Reading Guide

Multiplying Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-4 in your textbook.

Write the probability that Yao Ming made a field goal shot during the 2004–05 season as a fraction in lowest terms. (Your answer should not include a decimal.)

Read the Lesson

1. A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. *A* ball is drawn from the bag and is not replaced. *A* second ball is drawn.

a. Let Y be the event "first ball is yellow" and B be the event "second ball is black." Are these events *independent* or *dependent*?

b. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black.

A. $P(Y \text{ and } B) = \frac{P(Y)}{P(Y) + P(B)}$

B. $P(Y \text{ and } B) = P(Y) \cdot P(B)$

C. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$

c. Which equation shows the correct calculation of this probability?

A. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$ **B.** $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$

C. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ **D.** $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

d. Which equation shows the correct calculation of the probability that if three balls are drawn in succession without replacement, all three will be red?

A. $\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \frac{125}{1728}$ **B.** $\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \frac{1}{22}$

C. $\frac{5}{12} + \frac{4}{11} + \frac{3}{10} = \frac{713}{660}$

Remember What You Learned

2. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both independent and dependent events? Explain your reasoning.

NAME ____ DATE______________ PERIOD _____

12-5 Lesson Reading Guide

Adding Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-5 in your textbook.

Why do the percentages shown on the bar graph add up to more than 100%?

Read the Lesson

1. Indicate whether the events in each pair are *inclusive* or *mutually exclusive*.

   a. Q: drawing a queen from a standard deck of cards
   D: drawing a diamond from a standard deck of cards

   b. J: drawing a jack from a standard deck of cards
   K: drawing a king from a standard deck of cards

2. Marla took a quiz on this lesson that contained the following problem.

   Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?

   Here is Marla's work.

   $$P(\text{odd}) = \frac{13}{25} \qquad P(\text{multiple of 5}) = \frac{5}{25} \text{ or } \frac{1}{5}$$

   $$P(\text{odd or multiple of 5}) = P(\text{odd}) + P(\text{multiple of 5}) = \frac{13}{25} + \frac{5}{25} = \frac{18}{25}$$

   a. Why is Marla's work incorrect?

   b. Show the corrected work.

Remember What You Learned

3. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both mutually exclusive and inclusive events? Explain your reasoning.

12-6 Lesson Reading Guide

Statistical Measures

Get Ready for the Lesson

Read the introduction to Lesson 12-6 in your textbook.

There is more than one way to give an “average” score for this test. Three measures of central tendency for these scores are 94, 76.5 and 73.9. Can you tell which of these is the mean, the median, and the mode without doing any calculations? Explain your answer.

Read the Lesson

1. Match each measure with one of the six descriptions of how to find measures of central tendency and variation.

a. median **b.** mode **c.** range

d. variance **e.** mean **f.** standard deviation

i. Find the most commonly occurring values or values in a set of data.

ii. Add the data and divide by the number of items.

iii. Find the mean of the squares of the differences between each value in the set of data and the mean.

iv. Find the difference between the largest and smallest values in the set of data.

v. Take the positive square root of the variance.

vi. If there is an odd number of items in a set of data, take the middle one. If there is an even number of items, add the two middle items and divide by 2.

Remember What You Learned

2. It is usually easier to remember a complicated procedure if you break it down into steps. Write the procedure for finding the standard deviation for a set of data in a series of brief, numbered steps.

12-8 Lesson Reading Guide

Exponential and Binomial Distribution

Get Ready for the Lesson

Read the introduction to Lesson 12-8 in your textbook.

Is a randomly chosen student likely to be one that talks on the phone for a very short period of time or for a very long period of time?

Read the Lesson

1. Indicate whether each situation can be represented using an exponential distribution or a binomial distribution.

a. You are trying to predict how many times a coin will land with the tails side up if you flip it 50 times.

b. You would like to find the probability that there will be more than 5 pink gumballs in a bag of assorted color gumballs.

c. You are trying to predict how long your refrigerator will last.

d. You are calculating the probability that a person in your class will be taller than 5 feet, 5 inches

e. You would like to determine the percentage of cellular phones that will last longer than 7 years and the percentage that will last longer than 5 years.

f. You want to predict the probability that a person in your neighborhood is older than you are.

2. Write an equation that can be used to calculate each of the following:

a. The expected number of successes in a binomial distribution that has a 30% rate of success when there are 50 trials.

b. The probability that a randomly selected number from an exponential distribution will be greater than 4 if the mean is 1.5.

Remember What You Learned

3. In binomial distributions, the only possible outcomes are success and failure, but sometimes binomial experiments include events that can have several results. Explain how this is possible.

12-9 Lesson Reading Guide

Binomial Experiments

Get Ready for the Lesson

Read the introduction to Lesson 12-9 in your textbook.

Suppose you are taking a 50-question multiple-choice test in which there are 5 answer choices for each question. You are told that no points will be deducted for wrong answers. Should you guess the answers to the questions you do not know? Explain your reasoning.

Read the Lesson

1. Indicate whether each of the following is a *binomial experiment* or *not a binomial experiment*. If the experiment is not a binomial experiment, explain why.

a. A fair coin is tossed 10 times and "heads" or "tails" is recorded each time.

b. A pair of dice is thrown 5 times and the sum of the numbers that come up is recorded each time.

c. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is not put back in the bag. A second marble is drawn and its color recorded.

d. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is put back in the bag. A second marble is drawn and its color recorded.

2. Len randomly guesses the answers to all 6 multiple-choice questions on his chemistry test. Each question has 5 choices. Which of the following expressions gives the probability that he will get at least 4 of the answers correct?

A. $P(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + P(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + P(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

B. $C(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + C(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + C(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

C. $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 + C(6, 5)\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^5 + C(6, 6)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$

Remember What You Learned

3. Some students have trouble remembering how to calculate binomial probabilities. What is an easy way to remember which numbers to put into an expression like $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4$?

Lesson 12-9

12-10 Lesson Reading Guide

Sampling and Error

Get Ready for the Lesson

Read the introduction to Lesson 12-10 in your textbook.

Do you think the results of the survey show that more mothers spend \$249 or less than \$250–\$349? If there is not enough information given to determine this, list at least two questions you would ask about the survey that would help you determine the significance of the survey.

Read the Lesson

1. Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

   a. asking all the customers at five restaurants on the same evening how many times a month they eat dinner in restaurants to determine how often the average American eats dinner in a restaurants

   b. putting the names of all seniors at your high school in a hat and then drawing 20 names for a survey to find out where seniors would like to hold their prom

2. A survey determined that 58% of registered voters in the United States support increased federal spending for education. The margin of error for this survey is 4%. Explain in your own words what this tells you about the actual percentage of registered voters who support increased spending for education.

Remember What You Learned

3. The formula for margin of sampling error may be tricky to remember. A good way to start is to think about the variables that must be included in the formula. What are these variables, and what do they represent? What is an easy way to remember which variable goes in the denominator in the formula?

13-1 Lesson Reading Guide

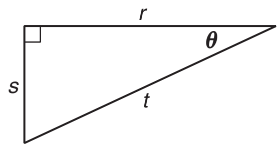
Right Triangle Trigonometry

Pre-Activity **How is trigonometry used in building construction?**

Read the introduction to Lesson 13-1 in your textbook. If a different ramp is built so that the angle shown in the figure has a tangent of $\frac{1}{14}$, will this ramp meet, exceed, or fail to meet ADA regulations?

Reading the Lesson

1. Refer to the triangle at the right. Match each trigonometric function with the correct ratio.



i. $\frac{r}{t}$ **ii.** $\frac{r}{s}$ **iii.** $\frac{t}{r}$ **iv.** $\frac{s}{t}$ **v.** $\frac{s}{r}$ **vi.** $\frac{t}{s}$

a. sin θ **b.** tan θ **c.** sec θ

d. cot θ **e.** cos θ **f.** csc θ

2. Refer to the Key Concept box on page 761 in your textbook. Use the drawings of the 30°-60°-90° triangle and 45°-45°-90° triangle and/or the table to complete the following.

a. The tangent of 45° and the _________________ of 45° are equal.

b. The sine of 30° is equal to the cosine of _________________.

c. The sine and _________________ of 45° are equal.

d. The reciprocal of the cosecant of 60° is the _________________ of 60°.

e. The reciprocal of the cosine of 30° is the _________________ of 60°.

f. The reciprocal of the tangent of 60° is the _________________ of 30°.

Helping You Remember

3. In studying trigonometry, it is important for you to know the relationships between the lengths of the sides of a 30°-60°-90° triangle. If you remember just one fact about this triangle, you will always be able to figure out the lengths of all the sides. What fact can you use, and why is it enough?

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13-2 Lesson Reading Guide

Angles and Angle Measure

Pre-Activity **How can angles be used to describe circular motion?**

Read the introduction to Lesson 13-2 in your textbook.

If a gondola revolves through a complete revolution in one minute, what is its angular velocity in degrees per second?

Reading the Lesson

1. Match each degree measure with the corresponding radian measure on the right.

a. $30°$	**i.** $\frac{2\pi}{3}$
b. $90°$	**ii.** $\frac{\pi}{2}$
c. $120°$	**iii.** $\frac{7\pi}{6}$
d. $135°$	**iv.** π
e. $180°$	**v.** $\frac{\pi}{6}$
f. $210°$	**vi.** $\frac{3\pi}{4}$

2. The sine of $30°$ is $\frac{1}{2}$ and the sine of $150°$ is also $\frac{1}{2}$. Does this mean that $30°$ and $150°$ are coterminal angles? Explain your reasoning.

3. Describe how to find two angles that are coterminal with an angle of $155°$, one with positive measure and one with negative measure. (Do not actually calculate these angles.)

4. Describe how to find two angles that are coterminal with an angle of $\frac{5\pi}{3}$, one positive and one negative. (Do not actually calculate these angles.)

Helping You Remember

5. How can you use what you know about the circumference of a circle to remember how to convert between radian and degree measure?

13-3 Lesson Reading Guide

Trigonometric Functions of General Angles

Pre-Activity **How can you model the position of riders on a skycoaster?**

Read the introduction to Lesson 13-3 in your textbook.

- What does $t = 0$ represent in this application?
- Do negative values of t make sense in this application? Explain your answer.

Reading the Lesson

1. Suppose θ is an angle in standard position, $P(x, y)$ is a point on the terminal side of θ, and the distance from the origin to P is r. Determine whether each of the following statements is *true* or *false*.

a. The value of r can be found by using either the Pythagorean Theorem or the distance formula.

b. $\cos \theta = \frac{x}{r}$

c. $\csc \theta$ is defined if $y \neq 0$.

d. $\tan \theta$ is undefined if $y = 0$.

e. $\sin \theta$ is defined for every value of θ.

2. Let θ be an angle measured in degrees. Match the quadrant of θ from the first column with the description of how to find the reference angle for θ from the second column.

a. Quadrant III

b. Quadrant IV

c. Quadrant II

d. Quadrant I

i. Subtract θ from 360°.

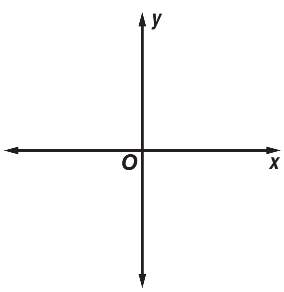
ii. Subtract 180° from θ.

iii. θ is its own reference angle.

iv. Subtract θ from 180°.

Helping You Remember

3. The chart on page 779 in your textbook summarizes the signs of the six trigonometric functions in the four quadrants. Since reciprocals always have the same sign, you only need to remember where the sine, cosine, and tangent are positive. How can you remember this with a simple diagram?



13-4 Lesson Reading Guide

Law of Sines

Pre-Activity **How can trigonometry be used to find the area of a triangle?**

Read the introduction to Lesson 13-4 in your textbook.

What happens when the formula Area $= \frac{1}{2}ab \sin C$ is applied to a right triangle in which C is the right angle?

Reading the Lesson

1. In each case below, the measures of three parts of a triangle are given. For each case, write the formula you would use to find the area of the triangle. Show the formulas with specific values substituted, but do not actually calculate the area. If there is not enough information provided to find the area of the triangle by using the area formulas on page 725 in your textbook and without finding other parts of the triangle first, explain why.

a. $A = 48°, b = 9, c = 5$

b. $a = 15, b = 15, C = 120°$

c. $b = 16, c = 10, B = 120°$

2. Tell whether the equation must be true based on the Law of Sines. Write *yes* or *no*.

a. $\frac{\sin A}{b} = \frac{\sin B}{a}$

b. $\frac{b}{\sin B} = \frac{c}{\sin C}$

c. $a \sin C = c \sin A$

d. $b = \frac{a \sin A}{\sin B}$

3. Determine whether $\triangle ABC$ has *no solution*, *one solution*, or *two solutions*. Do not try to solve the triangle.

a. $a = 20, A = 30°, B = 70°$

b. $A = 55°, b = 5, a = 3$ $(b \sin A \approx 4.1)$

c. $c = 12, A = 100°, a = 30$

d. $C = 27°, b = 23.5, c = 17.5$ $(b \sin C \approx 10.7)$

Helping You Remember

4. Suppose that you are taking a quiz and cannot remember whether the formula for the area of a triangle is Area $= \frac{1}{2}ab \cos C$ or Area $= \frac{1}{2}ab \sin C$. How can you quickly remember which of these is correct?

13-5 Lesson Reading Guide

Law of Cosines

Pre-Activity **How can you determine the angle at which to install a satellite dish?**

Read the introduction to Lesson 13-5 in your textbook.

One side of the triangle in the figure is not labeled with a length. What does the length of this side represent? Is this length greater than or less than the distance from the satellite to the equator?

Reading the Lesson

1. Each of the following equations can be changed into a correct statement of the Law of Cosines by making one change. In each case, indicate what change should be made to make the statement correct.

a. $b^2 = a^2 + c^2 + 2ac \cos B$

b. $a^2 = b^2 + c^2 - 2bc \sin A$

c. $c = a^2 + b^2 - 2ab \cos C$

d. $a^2 = b^2 - c^2 - 2bc \cos A$

2. Suppose that you are asked to solve $\triangle ABC$ given the following information about the sides and angles of the triangle. In each case, indicate whether you would begin by using the *Law of Sines* or the *Law of Cosines*.

a. $a = 8, b = 7, c = 6$

b. $b = 9.5, A = 72°, B = 39°$

c. $C = 123°, b = 22.95, a = 34.35$

Helping You Remember

3. It is often easier to remember a complicated procedure if you can break it down into small steps. Describe in your own words how to use the Law of Cosines to find the length of one side of a triangle if you know the lengths of the other two sides and the measure of the included angle. Use numbered steps. (You may use mathematical terms, but do not use any mathematical symbols.)

NAME ____ DATE______________ PERIOD _____

13-6 Lesson Reading Guide

Circular Functions

Pre-Activity **How can you model annual temperature fluctuations?**

Read the introduction to Lesson 13-6 in your textbook.

- If the graph in your textbook is continued, what month will $x = 17$ represent?
- About what do you expect the average high temperature to be for that month?
- Will this be exactly the average high temperature for that month? Explain your answer.

Reading the Lesson

1. Use the unit circle on page 740 in your textbook to find the exact values of each expression.

a. cos 45° **b.** sin 150° **c.** sin 240°

d. sin 315° **e.** cos 270° **f.** sin 210°

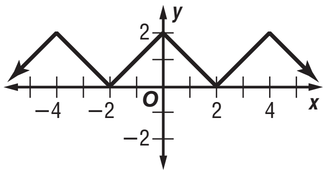
g. cos 0° **h.** sin 180° **i.** cos 330°

2. Tell whether each function is periodic. Write *yes* or *no*.

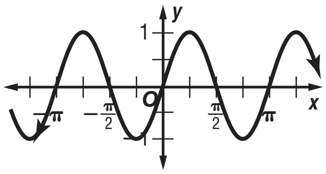
a. $y = 2x$ **b.** $y = x^2$ **c.** $y = \cos x$ **d.** $y = |x|$

3. Find the period of each function by examining its graph.

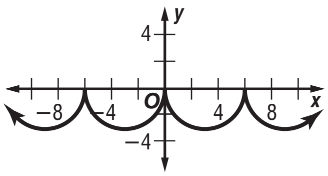
a.



b.



c.



Helping You Remember

4. What is an easy way to remember the periods of the sine and cosine functions in radian measure?

13-7 Lesson Reading Guide

Inverse Trigonometric Functions

Pre-Activity **How are inverse trigonometric functions used in road design?**

Read the introduction to Lesson 13-7 in your textbook.

Suppose you are given specific values for v and r. What feature of your graphing calculator could you use to find the approximate measure of the banking angle θ?

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.

a. The domain of the function $y = \sin x$ is the set of all real numbers.

b. The domain of the function $y = \text{Cos } x$ is $0 \leq x \leq \pi$.

c. The range of the function $y = \text{Tan } x$ is $-1 \leq y \leq 1$.

d. The domain of the function $y = \text{Cos}^{-1} x$ is $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

e. The domain of the function $y = \text{Tan}^{-1} x$ is the set of all real numbers.

f. The range of the function $y = \text{Arcsin } x$ is $0 \leq x \leq \pi$.

2. Answer each question in your own words.

a. What is the difference between the functions $y = \sin x$ and the function $y = \text{Sin } x$?

b. Why is it necessary to restrict the domains of the trigonometric functions in order to define their inverses?

Helping You Remember

3. What is a good way to remember the domains of the functions $y = \text{Sin } x$, $y = \text{Cos } x$, and $y = \text{Tan } x$, which are also the range of the functions $y = \text{Arcsin } x$, $y = \text{Arccos } x$, and $y = \text{Arctan } x$? (You may want to draw a diagram.)

14-1 Lesson Reading Guide

Graphing Trigonometric Functions

Pre-Activity **Why can you predict the behavior of tides?**

Read the introduction to Lesson 14-1 at the top of page 762 in your textbook.

Consider the tides of the Atlantic Ocean as a function of time. Approximately what is the period of this function?

Reading the Lesson

1. Determine whether each statement is *true* or *false*.

a. The period of a function is the distance between the maximum and minimum points.

b. The amplitude of a function is the difference between its maximum and minimum values.

c. The amplitude of the function $y = \sin\theta$ is 2π.

d. The function $y = \cot\theta$ has no amplitude.

e. The period of the function $y = \sec\theta$ is π.

f. The amplitude of the function $y = 2\cos\theta$ is 4.

g. The function $y = \sin 2\theta$ has a period of π.

h. The period of the function $y = \cot 3\theta$ is $\frac{\pi}{3}$.

i. The amplitude of the function $y = -5\sin\theta$ is -5.

j. The period of the function $y = \csc\frac{1}{4}\theta$ is 4π.

k. The graph of the function $y = \sin\theta$ has no asymptotes.

l. The graph of the function $y = \tan\theta$ has an asymptote at $\theta = 180°$.

m. When $\theta = 360°$, the values of $\cos\theta$ and $\sec\theta$ are equal.

n. When $\theta = 270°$, $\cot\theta$ is undefined.

o. When $\theta = 180°$, $\csc\theta$ is undefined.

Helping You Remember

2. What is an easy way to remember the periods of $y = a\sin b\theta$ and $y = a\cos b\theta$?

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14-2 Lesson Reading Guide

Translations of Trigonometric Graphs

Pre-Activity **How can translations of trigonometric graphs be used to show animal populations?**

Read the introduction to Lesson 14-2 at the top of page 769 in your textbook.

According to the model given in your textbook, what would be the estimated rabbit population for January 1, 2005?

Reading the Lesson

1. Determine whether the graph of each function represents a shift of the parent function *to the left*, *to the right*, *upward*, or *downward*. (Do not actually graph the functions.)

a. $y = \sin(\theta + 90°)$

b. $y = \sin\theta + 3$

c. $y = \cos\left(\theta - \frac{\pi}{3}\right)$

d. $y = \tan\theta - 4$

2. Determine whether the graph of each function has an *amplitude change*, *period change*, *phase shift*, or *vertical shift* compared to the graph of the parent function. (More than one of these may apply to each function. Do not actually graph the functions.)

a. $y = 3\sin\left(\theta + \frac{5\pi}{6}\right)$

b. $y = \cos(2\theta + 70°)$

c. $y = -4\cos 3\theta$

d. $y = \sec\frac{1}{2}\theta + 3$

e. $y = \tan\left(\theta - \frac{\pi}{4}\right) - 1$

f. $y = 2\sin\left(\frac{1}{3}\theta + \frac{\pi}{6}\right) - 4$

Helping You Remember

3. Many students have trouble remembering which of the functions $y = \sin(\theta + \alpha)$ and $y = \sin(\theta - \alpha)$ represents a shift to the left and which represents a shift to the right. Using $\alpha = 45°$, explain a good way to remember which is which.

14-3 Lesson Reading Guide

Trigonometric Identities

Pre-Activity **How can trigonometry be used to model the path of a baseball?**

Read the introduction to Lesson 14-3 at the top of page 777 in your textbook.

Suppose that a baseball is hit from home plate with an initial velocity of 58 feet per second at an angle of 36° with the horizontal from an initial height of 5 feet. Show the equation that you would use to find the height of the ball 10 seconds after the ball is hit. (Show the formula with the appropriate numbers substituted, but do not do any calculations.)

Reading the Lesson

1. Match each expression from the list on the left with an expression from the list on the right that is equal to it for all values for which each expression is defined. (Some of the expressions from the list on the right may be used more than once or not at all.)

a. $\sec^2 \theta - \tan^2 \theta$	**i.** $\frac{1}{\sin \theta}$
b. $\cot^2 \theta + 1$	**ii.** $\tan \theta$
c. $\frac{\sin \theta}{\cos \theta}$	**iii.** 1
d. $\sin^2 \theta + \cos^2 \theta$	**iv.** $\sec \theta$
e. $\csc \theta$	**v.** $\csc^2 \theta$
f. $\frac{1}{\cos \theta}$	**vi.** $\cot \theta$
g. $\frac{\cos \theta}{\sin \theta}$	

2. Write an identity that you could use to find each of the indicated trigonometric values and tell whether that value is positive or negative. (Do not actually find the values.)

a. $\tan \theta$, if $\sin \theta = -\frac{4}{5}$ and $180° < \theta < 270°$

b. $\sec \theta$, if $\tan \theta = -3$ and $90° < \theta < 180°$

Helping You Remember

3. A good way to remember something new is to relate it to something you already know. How can you use the unit circle definitions of the sine and cosine that you learned in Chapter 13 to help you remember the Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$?

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14-4 Lesson Reading Guide

Verifying Trigonometric Identities

Pre-Activity **How can you verify trigonometric identities?**

Read the introduction to Lesson 14-4 at the top of page 782 in your textbook.

For $\theta = -\pi$, 0, or π, $\sin \theta = \sin 2\theta$. Does this mean that $\sin \theta = \sin 2\theta$ is an identity? Explain your reasoning.

Reading the Lesson

1. Determine whether each equation is an *identity* or *not an identity*.

a. $\dfrac{1}{\sin^2 \theta} - \dfrac{1}{\tan^2 \theta} = 1$

b. $\dfrac{\cos \theta}{\sin \theta \tan \theta}$

c. $\dfrac{\sin \theta}{\cos \theta} + \dfrac{\cos \theta}{\sin \theta} = \cos \theta \sin \theta$

d. $\cos^2 \theta (\tan^2 \theta + 1) = 1$

e. $\dfrac{\sin^2 \theta}{\cos^2 \theta} + \sin \theta \csc \theta = \sec^2 \theta$

f. $\dfrac{1}{1 - \sin \theta} + \dfrac{1}{1 + \sin \theta} = 2 \cos^2 \theta$

g. $\tan^2 \theta \cos^2 \theta = \dfrac{1}{\csc^2 \theta}$

h. $\dfrac{\sin \theta}{\sec \theta} = \dfrac{1}{\tan \theta} + \dfrac{1}{\cot \theta}$

2. Which of the following is *not* permitted when verifying an identity?

A. simplifying one side of the identity to match the other side

B. cross multiplying if the identity is a proportion

C. simplifying each side of the identity separately to get the same expression on both sides

Helping You Remember

3. Many students have trouble knowing where to start in verifying a trigonometric identity. What is a simple rule that you can remember that you can always use if you don't see a quicker approach?

14-5 Lesson Reading Guide

Sum and Difference of Angles Formulas

Pre-Activity **How are the sum and difference formulas used to describe communication interference?**

Read the introduction to Lesson 14-5 at the top of page 786 in your textbook.

Consider the functions $y = \sin x$ and $y = 2 \sin x$. Do the graphs of these two functions have *constructive* interference or *destructive* interference?

Reading the Lesson

1. Match each expression from the list on the left with an expression from the list on the right that is equal to it for all values of the variables. (Some of the expressions from the list on the right may be used more than once or not at all.)

a. $\sin(\alpha - \beta)$	**i.** $\sin \beta$
b. $\cos(\alpha + \beta)$	**ii.** $\sin \alpha \cos \beta + \cos \alpha \sin \beta$
c. $\sin(180° + \beta)$	**iii.** $-\cos \beta$
d. $\sin(180° - \beta)$	**iv.** $\cos \alpha \cos \beta + \sin \alpha \sin \beta$
e. $\cos(180° + \beta)$	**v.** $\sin \alpha \cos \beta - \cos \alpha \sin \beta$
f. $\sin(\alpha + \beta)$	**vi.** $\cos \alpha \cos \beta - \sin \alpha \sin \beta$
g. $\cos(90° - \beta)$	**vii.** $-\sin \beta$
h. $\cos(\alpha - \beta)$	**viii.** $\cos \beta$

2. Which expressions are equal to sin 15°? (There may be more than one correct choice.)

A. $\sin 45° \cos 30° + \cos 45° \sin 30°$ **B.** $\sin 45° \cos 30° - \cos 45° \sin 30°$

C. $\sin 60° \cos 45° - \cos 60° \sin 45°$ **D.** $\cos 60° \cos 45° - \sin 60° \sin 45°$

Helping You Remember

3. Some students have trouble remembering which signs to use on the right-hand sides of the sum and difference of angle formulas. What is an easy way to remember this?

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14-6 Lesson Reading Guide

Double-Angle and Half-Angle Formulas

Pre-Activity **How can trigonometric functions be used to describe music?**

Read the introduction to Lesson 14-6 at the top of page 791 in your textbook.

Suppose that the equation for the second harmonic is $y = \sin a\theta$. Then what would be the equations for the fundamental tone (first harmonic), third harmonic, fourth harmonic, and fifth harmonic?

Reading the Lesson

1. Match each expression from the list on the left with *all* expressions from the list on the right that are equal to it for all values of β.

a. $\sin \frac{\beta}{2}$

b. $\cos 2\beta$

c. $\cos \frac{\beta}{2}$

d. $\sin 2\beta$

i. $2 \sin \beta \cos \beta$

ii. $1 - 2 \sin^2 \beta$

iii. $\cos^2 \beta - \sin^2 \beta$

iv. $\pm\sqrt{\frac{1 + \cos \beta}{2}}$

v. $\pm\sqrt{\frac{1 - \cos \beta}{2}}$

2. Determine whether you would use the *positive* or *negative* square root in the half-angle identities for $\sin \frac{\alpha}{2}$ and $\cos \frac{\alpha}{2}$ in each of the following situations. (Do not actually calculate $\sin \frac{\alpha}{2}$ and $\cos \frac{\alpha}{2}$.)

a. $\sin \frac{\alpha}{2}$, if $\cos \alpha = \frac{2}{5}$ and α is in Quadrant I

b. $\cos \frac{\alpha}{2}$, if $\cos \alpha = -0.9$ and α is in Quadrant II

c. $\cos \frac{\alpha}{2}$, if $\sin \alpha = -0.75$ and α is in Quadrant III

d. $\sin \frac{\alpha}{2}$, if $\sin \alpha = -0.8$ and α is in Quadrant IV

Helping You Remember

3. Many students find it difficult to remember a large number of identities. How can you obtain all three of the identities for $\cos 2\theta$ by remembering only one of them and using a Pythagorean identity?

14-7 Lesson Reading Guide

Solving Trigonometric Equations

Pre-Activity **How can trigonometric equations be used to predict temperature?**

Read the introduction to Lesson 14-7 at the top of page 799 in your textbook.

Describe how you could use a graphing calculator to determine the months in which the average daily high temperature is above 80°F. (Assume that $x = 1$ represents January.) Specify the graphing window that you would use.

Reading the Lesson

1. Identify which equations have no solution.

A. $\sin \theta = 1$ **B.** $\tan \theta = 0.001$ **C.** $\sec \theta = \frac{1}{2}$

D. $\csc \theta = -3$ **E.** $\cos \theta = 1.01$ **F.** $\cot \theta = -1000$

G. $\cos \theta + 2 = -1$ **H.** $\sec \theta - 1.5 = 0$ **I.** $\sin \theta - 0.009 = 0.99$

2. Use a trigonometric identity to write the first step in the solution of each trigonometric equation. (Do not complete the solution.)

a. $\tan \theta = \cos^2 \theta + \sin^2 \theta, 0 \le \theta < 2\pi$

b. $\sin^2 \theta - 2 \sin \theta + 1 = 0, 0° \le \theta < 360°$

c. $\cos 2\theta = \sin \theta, 0° \le \theta < 360°$

d. $\sin 2\theta = \cos \theta, 0 \le \theta < 2\pi$

e. $2 \cos 2\theta + 3 \cos \theta = -1, 0° \le \theta < 360°$

f. $3 \tan^2 \theta + 5 \tan \theta - 2 = 0$

Helping You Remember

3. A good way to remember something is to explain it to someone else. How would you explain to a friend the difference between verifying a trigonometric identity and solving a trigonometric equation.