Glencoe Mathematics

Algebra 2

Noteables™
Interactive Study Notebook
with Foldables™

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Foldables™

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Algebra 2 Noteables™: Interactive Study Notebook with Foldables™

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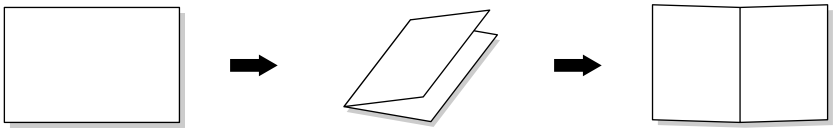
Organizing Your Foldables

FOLDABLES™

Make this Foldable to help you organize and store your chapter Foldables. Begin with one sheet of 11” × 17” paper.

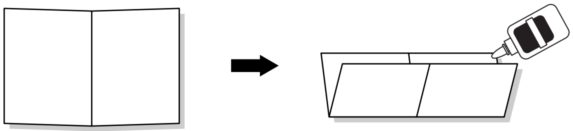
STEP 1

Fold
Fold the paper in half lengthwise. Then unfold.



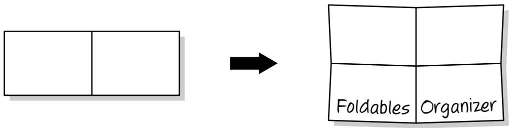
STEP 2

Fold and Glue
Fold the paper in half widthwise and glue all of the edges.



STEP 3

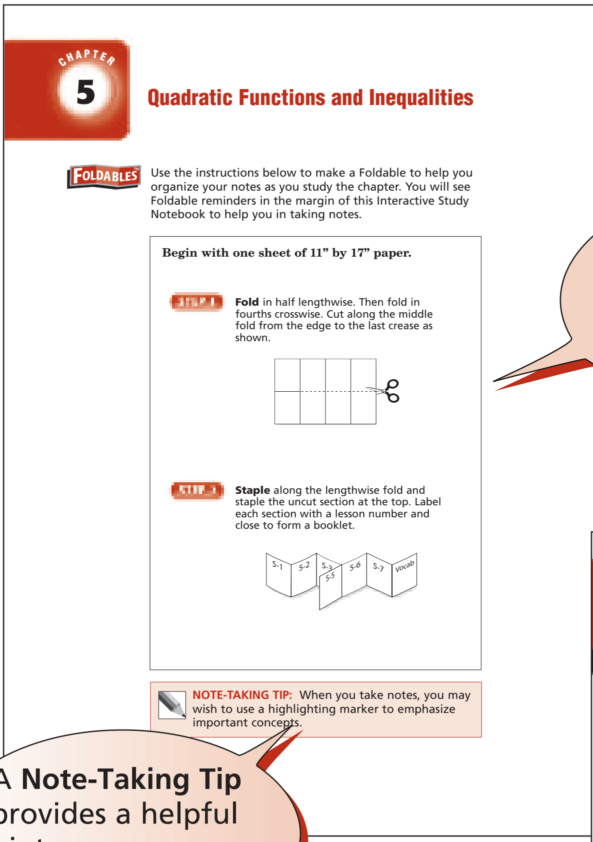
Glue and Label
Glue the left, right, and bottom edges of the Foldable to the inside back cover of your Noteables notebook.



Reading and Taking Notes As you read and study each chapter, record notes in your chapter Foldable. Then store your chapter Foldables inside this Foldable organizer.

Using Your Noteables™ with Foldables™ Interactive Study Notebook

This note-taking guide is designed to help you succeed in *Algebra 2*. Each chapter includes:



CHAPTER 5 Quadratic Functions and Inequalities

FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" by 17" paper.

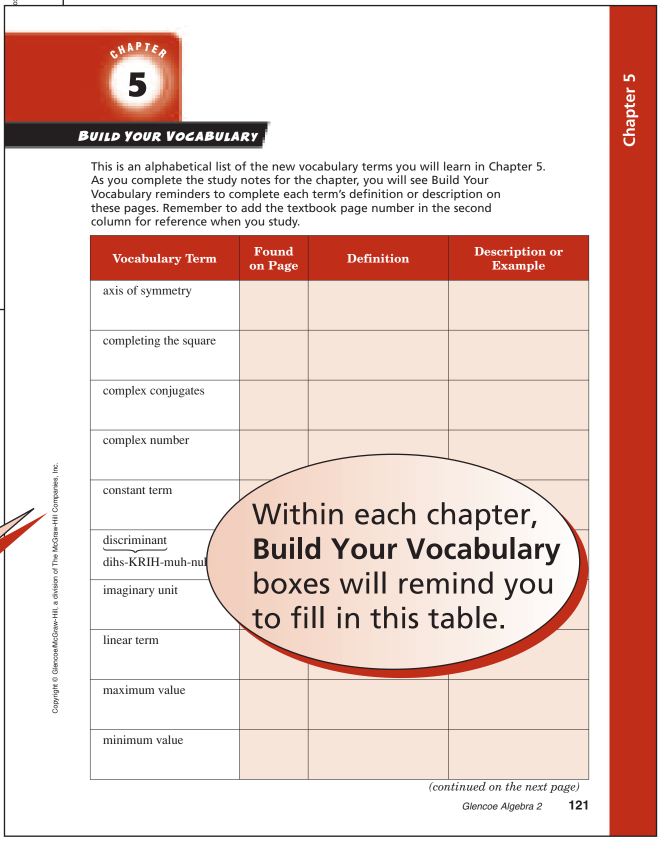
Fold in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.

Staple along the lengthwise fold and staple the uncut section at the top. Label each section with a lesson number and close to form a booklet.

NOTE-TAKING TIP: When you take notes, you may wish to use a highlighting marker to emphasize important concepts.

The **Chapter Opener** contains instructions and illustrations on how to make a Foldable that will help you to organize your notes.

A **Note-Taking Tip** provides a helpful hint you can use when taking notes.



CHAPTER 5

BUILD YOUR VOCABULARY

This is an alphabetical list of the new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
axis of symmetry			
completing the square			
complex conjugates			
complex number			
constant term			
discriminant dihs-KRIH-muh-nul			
imaginary unit			
linear term			
maximum value			
minimum value			

(continued on the next page)

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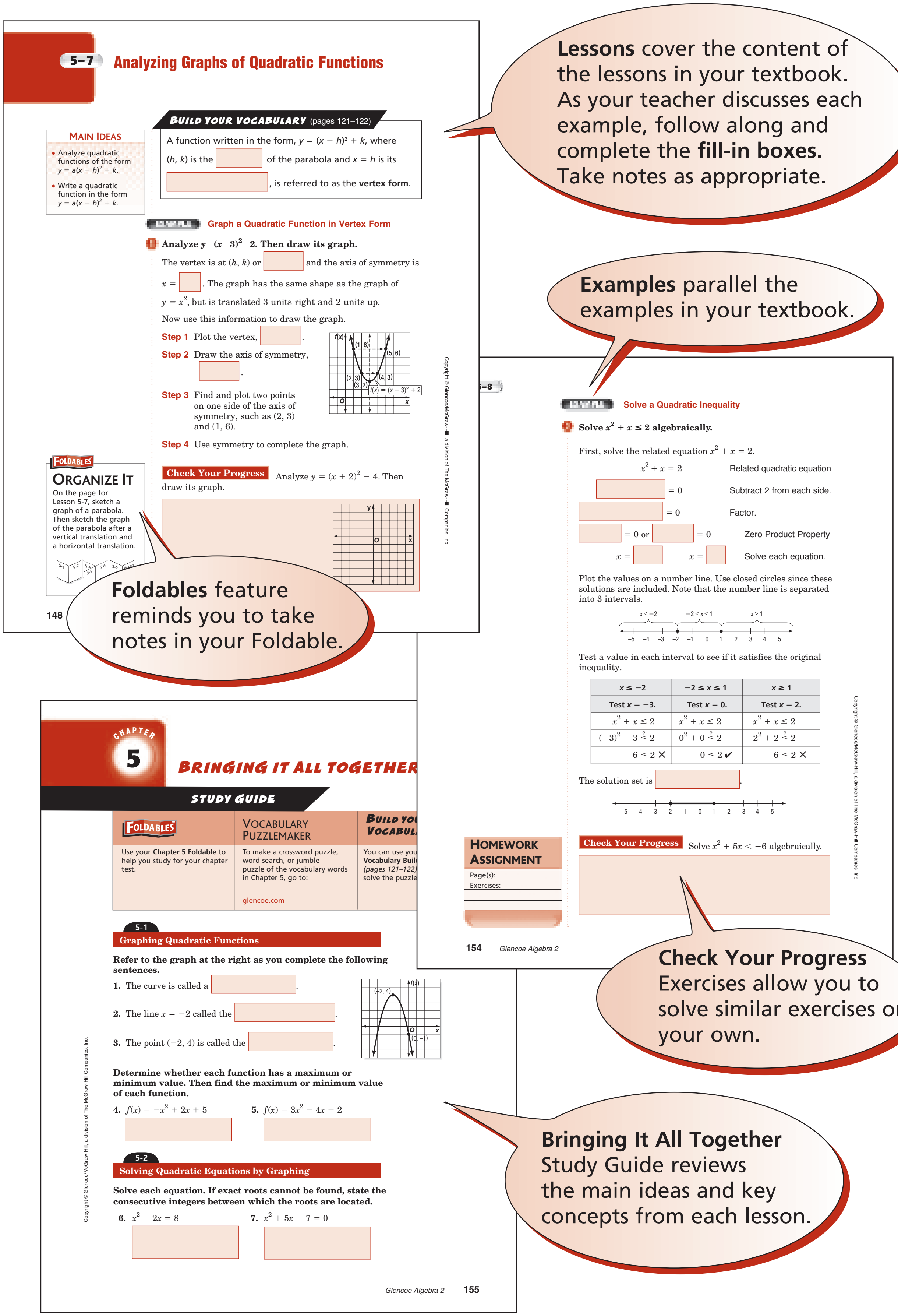
Chapter 5

Within each chapter, **Build Your Vocabulary** boxes will remind you to fill in this table.

The **Build Your Vocabulary** table allows you to write definitions and examples of important vocabulary terms together in one convenient place.



5–7 Analyzing Graphs of Quadratic Functions

MAIN IDEAS

- Analyze quadratic functions of the form $y = a(x - h)^2 + k$.
- Write a quadratic function in the form $y = a(x - h)^2 + k$.

BUILD YOUR VOCABULARY (pages 121–122)

A function written in the form, $y = (x - h)^2 + k$, where (h, k) is the of the parabola and $x = h$ is its , is referred to as the **vertex form**.

Graph a Quadratic Function in Vertex Form

Analyze $y \quad (x \quad 3)^2 \quad 2$. Then draw its graph.

The vertex is at (h, k) or and the axis of symmetry is $x =$. The graph has the same shape as the graph of $y = x^2$, but is translated 3 units right and 2 units up.

Now use this information to draw the graph.

Step 1 Plot the vertex, .

Step 2 Draw the axis of symmetry, .

Step 3 Find and plot two points on one side of the axis of symmetry, such as (2, 3) and (1, 6).

Step 4 Use symmetry to complete the graph.

FOLDABLES

ORGANIZE IT

On the page for Lesson 5-7, sketch a graph of a parabola. Then sketch the graph of the parabola after a vertical translation and a horizontal translation.

Check Your Progress Analyze $y = (x + 2)^2 - 4$. Then draw its graph.

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Lessons cover the content of the lessons in your textbook. As your teacher discusses each example, follow along and complete the **fill-in boxes.** Take notes as appropriate.

Foldables feature reminds you to take notes in your Foldable.

Examples parallel the examples in your textbook.

5–8

Solve a Quadratic Inequality

Solve $x^2 + x \leq 2$ algebraically.

First, solve the related equation $x^2 + x = 2$.

$x^2 + x = 2$ Related quadratic equation

$= 0$ Subtract 2 from each side.

$= 0$ Factor.

$= 0$ or $= 0$ Zero Product Property

$x =$ $x =$ Solve each equation.

Plot the values on a number line. Use closed circles since these solutions are included. Note that the number line is separated into 3 intervals.

Test a value in each interval to see if it satisfies the original inequality.

$x \leq -2$	$-2 \leq x \leq 1$	$x \geq 1$
Test $x = -3$.	Test $x = 0$.	Test $x = 2$.
$x^2 + x \leq 2$	$x^2 + x \leq 2$	$x^2 + x \leq 2$
$(-3)^2 - 3 \overset{?}{\leq} 2$	$0^2 + 0 \overset{?}{\leq} 2$	$2^2 + 2 \overset{?}{\leq} 2$
$6 \leq 2$ ✗	$0 \leq 2$ ✓	$6 \leq 2$ ✗

The solution set is .

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress Solve $x^2 + 5x < -6$ algebraically.

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Check Your Progress Exercises allow you to solve similar exercises on your own.

CHAPTER 5

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES	VOCABULARY PUZZLEMAKER	BUILD YOU VOCABUL
Use your **Chapter 5 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to: glencoe.com	You can use you **Vocabulary Buil** *(pages 121–122)* solve the puzzle

5-1

Graphing Quadratic Functions

Refer to the graph at the right as you complete the following sentences.

1. The curve is called a .
2. The line $x = -2$ called the .
3. The point (−2, 4) is called the .

Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.

4. $f(x) = -x^2 + 2x + 5$
5. $f(x) = 3x^2 - 4x - 2$

5-2

Solving Quadratic Equations by Graphing

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

6. $x^2 - 2x = 8$
7. $x^2 + 5x - 7 = 0$

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Bringing It All Together Study Guide reviews the main ideas and key concepts from each lesson.

NOTE-TAKING TIPS

Your notes are a reminder of what you learned in class. Taking good notes can help you succeed in mathematics. The following tips will help you take better classroom notes.

- Before class, ask what your teacher will be discussing in class. Review mentally what you already know about the concept.
- Be an active listener. Focus on what your teacher is saying. Listen for important concepts. Pay attention to words, examples, and/or diagrams your teacher emphasizes.
- Write your notes as clear and concise as possible. The following symbols and abbreviations may be helpful in your note-taking.

Word or Phrase	Symbol or Abbreviation	Word or Phrase	Symbol or Abbreviation
for example	e.g.	not equal	$\neq$
such as	i.e.	approximately	$\approx$
with	w/	therefore	$\therefore$
without	w/o	versus	vs
and	+	angle	$\angle$

- Use a symbol such as a star (★) or an asterisk (*) to emphasize important concepts. Place a question mark (?) next to anything that you do not understand.
- Ask questions and participate in class discussion.
- Draw and label pictures or diagrams to help clarify a concept.
- When working out an example, write what you are doing to solve the problem next to each step. Be sure to use your own words.
- Review your notes as soon as possible after class. During this time, organize and summarize new concepts and clarify misunderstandings.

Note-Taking Don'ts

- **Don't** write every word. Concentrate on the main ideas and concepts.
- **Don't** use someone else's notes as they may not make sense.
- **Don't** doodle. It distracts you from listening actively.
- **Don't** lose focus or you will become lost in your note-taking.

CHAPTER 1

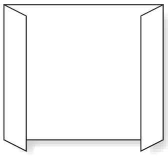
Solving Equations and Inequalities

FOLDABLES™

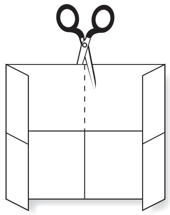
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of notebook paper.

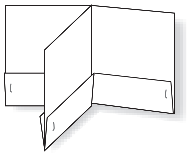
STEP 1 **Fold** 2″ tabs on each of the short sides.



STEP 2 Then **fold** in half in both directions and cut as shown.



STEP 3 **Refold** along the width. Staple each pocket. Label pockets as *Algebraic Expressions, Properties of Real Numbers, Solving Equations and Absolute Value Equations*, and *Solve and Graph Inequalities*.



NOTE-TAKING TIP: When you take notes, it is often a good idea to use symbols to emphasize important concepts.

CHAPTER

1

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 1. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
absolute value			
algebraic expression			
coefficient KOH-uh-FIH-shuhnt			
compound inequality			
constant			
degree			
empty set			
equation			
formula			
intersection			
irrational numbers			

Vocabulary Term	Found on Page	Definition	Description or Example
like terms			
monomial			
open sentence			
order of operations			
polynomial			
power			
rational numbers			
real numbers			
set-builder notation			
solution			
term			
trinomial			
union			
variable			

1–1 Expressions and Formulas

MAIN IDEAS

- Use the order of operations to evaluate expressions.
- Use formulas.

BUILD YOUR VOCABULARY (page 2)

Expressions that contain at least one variable are called **algebraic expressions**.

EXAMPLES Evaluate Algebraic Expressions

① **a. Evaluate $(x - y)^3 + 3$ if $x = 1$ and $y = 4$.**

$(x - y)^3 + 3 = (1 - 4)^3 + 3$ $x = \boxed{}$ and $y = \boxed{}$.

$= \boxed{} + 3$ Find -3^3.

$= \boxed{}$ Add -27 and 3.

KEY CONCEPT

Order of Operations

1. Evaluate expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars, as in $\frac{5 + 7}{2}$.
2. Evaluate all powers.
3. Do all multiplications and/or divisions from left to right.
4. Do all additions and/or subtractions from left to right.

FOLDABLES Write the order of operations in the Algebraic Expressions pocket of your Foldable.

b. Evaluate $s - t(s^2 - t)$ if $s = 2$ and $t = 3.4$.

$s - t(s^2 - t) = 2 - 3.4(2^2 - 3.4)$ Replace s with 2 and t with 3.4.

$= 2 - 3.4(4 - 3.4)$ Find 2^2.

$= 2 - 3.4\left(\boxed{}\right)$ Subtract 3.4 from 4.

$= 2 - \boxed{}$ Multiply 3.4 and 0.6.

$= \boxed{}$ Subtract 2.04 from 2.

c. Evaluate $\frac{8xy + z^3}{y^2 + 5}$ if $x = 5$, $y = -2$, and $z = -1$.

$\frac{8xy + z^3}{y^2 + 5} = \frac{8(5)(-2) + (-1)^3}{(-2)^2 + 5}$ $x = 5$, $y = -2$, and $z = -1$

$= \frac{\boxed{}(-2) + (-1)}{\boxed{} + 5}$ Evaluate the numerator and the denominator separately.

$= \frac{\boxed{} - 1}{4 + 5}$ Multiply 40 by -2.

$= \frac{-81}{9}$ or $\boxed{}$ Simplify the numerator and the denominator. Then divide.

WRITE IT

Why is it important to follow the order of operations when evaluating expressions?

____

____

____

____

____

____

Check Your Progress

a. Evaluate $x - y^2(x + 5)$ if $x = 2$ and $y = 4$.

b. Evaluate $\frac{3ab + c^2}{b - c}$ if $a = 5$, $b = 2$, and $c = 4$.

EXAMPLE Use a Formula

② **Find the area of a trapezoid with base lengths of 13 meters and 25 meters and a height of 8 meters.**

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(8)(13 + 25)$ Replace h with 8, b_1 with 13, and b_2 with 25.

$=$ Add 13 and 25.

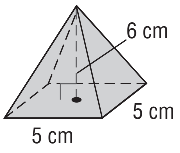
$=$ Multiply 8 by $\frac{1}{2}$.

$=$ Multiply 4 and 38.

The area of the trapezoid is ______ square meters.

Check Your Progress

The formula for the volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid. Find the volume of the pyramid shown.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–2 Properties of Real Numbers

MAIN IDEAS

- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

KEY CONCEPTS

Real Numbers

Rational Numbers A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal.

Irrational Numbers A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.

EXAMPLE Classify Numbers

① **Name the sets of numbers to which each number belongs.**

a. $\sqrt{6}$

$\sqrt{6}$ lies between 2 and 3 so it is not a whole number.

b. 5

c. $-\frac{2}{3}$

d. -43

e. -23.3

Check Your Progress **Name the sets of numbers to which each number belongs.**

a. $\frac{3}{5}$

b. $-2.\overline{52}$

c. $\sqrt{5}$

d. $\sqrt{121}$

KEY CONCEPT

Real Number Properties
For any real numbers *a*, *b*, and *c*:

Commuative
$a + b = b + a$ and
$a \cdot b = b \cdot a$

Associative
$(a + b) + c = a + (b + c)$
and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Identity
$a + 0 = a = 0 + a$ and
$a \cdot 1 = a = 1 \cdot a$

Inverse
$a + (-a) = 0 = (-a) + a$
If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.

Distributive
$a(b + c) = ab + ac$ and
$(b + c)a = ba + ca$

EXAMPLE Identify Properties of Real Numbers

2 Name the property illustrated by the equation $(-8 + 8) + 15 = 0 + 15$.

The ______ Property says that a number plus its opposite is 0.

Check Your Progress **Name the property illustrated by each equation.**

a. $3 + 0 = 3$ ______

b. $5 \cdot \frac{1}{5} = 1$ ______

EXAMPLE Additive and Multiplicative Inverses

3 Identify the additive and multiplicative inverse of −7.

Since $-7 + 7 = 0$, the additive inverse is ______. Since $(-7)\left(-\frac{1}{7}\right) = 1$, the multiplicative inverse is ______.

Check Your Progress **Identify the additive inverse and multiplicative inverse for each number.**

a. 5 ______

b. $-\frac{2}{3}$ ______

EXAMPLE Use the Distributive Property to Solve a Problem

4 POSTAGE **Audrey went to the post office and bought eight 39-cent stamps and eight 24-cent postcard stamps. How much did Audrey spend altogether on stamps?**

To find the total amount spent on stamps, multiply the price of each type of stamp by 8 and then add.

$S = 8(0.39) + 8(0.24)$

$= $ ______ $+$ ______ or ______

So, Audrey spent ______ stamps.

Check Your Progress Joel went to the grocery store and bought 3 plain chocolate candy bars for $0.69 each and 3 chocolate-peanut butter candy bars for $0.79 each. How much did Joel spend altogether on candy bars?

$\boxed{}$

EXAMPLE Simplify an Expression

⑤ **Simplify $4(3a - b) + 2(b + 3a)$.**

$4(3a - b) + 2(b + 3a)$

$= 4(\boxed{}) - 4(\boxed{}) + 2(\boxed{}) + 2(\boxed{})$	Distributive Property
$= 12a - 4b + 2b + 6a$	Multiply.
$= 12a + 6a - 4b + 2b$	$\boxed{}$ Property (+)
$= (\boxed{})a + (\boxed{})b$	Distributive Property
$= 18a - 2b$	Simplify.

Check Your Progress Simplify $2(3x - y) + 4(2x + 3y)$.

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–3 Solving Equations

MAIN IDEAS

- Translate verbal expressions into algebraic expressions and equations, and vice versa.
- Solve equations using the properties of equality.

EXAMPLE Verbal to Algebraic Expression

1 Write an algebraic expression to represent

a. 7 less than a number

b. the square of a number decreased by the product of 5 and the number

Check Your Progress **Write an algebraic expression to represent each verbal expression.**

a. 2 less than the cube of a number

b. 10 decreased by the product of a number and 2

BUILD YOUR VOCABULARY (page 3)

A mathematical sentence containing one or more is called an **open sentence.**

EXAMPLE Algebraic to Verbal Sentence

2 Write a verbal sentence to represent each equation.

a. $6 = -5 + x$ is equal to plus a number.

b. $7y - 2 = 19$ times a number minus is .

Check Your Progress **Write a verbal sentence to represent each equation.**

a. $5 = 2 + x$

b. $3a + 2 = 11$

KEY CONCEPT

Properties of Equality

Reflexive For any real number a, $a = a$.

Symmetric For all real numbers a and b, if $a = b$, then $b = a$.

Transitive For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Substitution If $a = b$, then a may be replaced by b and b may be replaced by a.

EXAMPLE Identify Properties of Equality

③ **Name the property illustrated by each statement.**

a. $\boldsymbol{a - 2.03 = a - 2.03}$

b. **If $\boldsymbol{9 = x}$, then $\boldsymbol{x = 9}$.**

Check Your Progress **Name the property illustrated by each statement.**

a. If $x + 4 = 3$, then $3 = x + 4$.

b. If $3 = x$ and $x = y$, then $3 = y$.

EXAMPLE Solve One-Step Equations

④ **Solve each equation.**

a. $\boldsymbol{s - 5.48 = 0.02}$

$s - 5.48 = 0.02$ Original equation

$s - 5.48 + \boxed{} = 0.02 + \boxed{}$ Add 5.48 to each side.

$s = \boxed{}$ Simplify.

b. $\boldsymbol{18 = \frac{1}{2}t}$

$18 = \frac{1}{2}t$ Original equation

$\boxed{}\,18 = \frac{1}{2}t\,\boxed{}$ Multiply each side by the multiplicative inverse of $\frac{1}{2}$.

$\boxed{} = t$ Simplify.

Check Your Progress **Solve each equation.**

a. $x + 5 = 3$

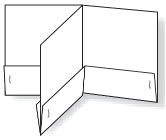
b. $\frac{2}{3}x = 10$

EXAMPLE Solve a Multi-Step Equation

FOLDABLES™

ORGANIZE IT

Write a multi-step equation in the Equations column of your Foldable. Then solve the equation, justifying each step with one of the properties you have learned.



⑤ **Solve $53 = 3(y - 2) - 2(3y - 1)$.**

$53 = 3(y - 2) - 2(3y - 1)$	Original equation
$53 = 3y - 6 - 6y + 2$	Distributive and Substitution Properties
$53 =$	Commutative, Distributive, and Substitution Properties
$=$	Addition and Substitution Properties
$= y$	Division and Substitution Properties

Check Your Progress Solve $25 = 3(2x + 2) - 5(2x + 1)$.

EXAMPLE Solve for a Variable

⑥ **GEOMETRY** **The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, where A is the area, b_1 is the length of the base, b_2 is the length of the other base, and h is the height of the trapezoid. Solve the formula for h.**

$A = \frac{1}{2}(b_1 + b_2)h$	Area of a trapezoid
$A = \cdot \frac{1}{2}(b_1 + b_2)h$	Multiply each side by 2.
$2A = (b_1 + b_2)h$	Simplify.
$\frac{2A}{\quad} = \frac{(b_1 + b_2)h}{\quad}$	Divide each side by $(b_1 + b_2)$.
$= h$	Simplify.

EXAMPLE Apply Properties of Equality

⑦ **TEST EXAMPLE** **If $4g + 5 = \frac{4}{9}$, what is the value of $4g - 2$?**

A $-\frac{41}{36}$

B $-\frac{41}{9}$

C $-\frac{59}{9}$

D $-\frac{67}{7}$

You are asked to find the value of the expression $\boxed{}$.

Your first thought might be to find the value of $\boxed{}$ and then evaluate the expression using this value. Notice that you are *not* required to find the value of *g*. Instead, you can use the $\boxed{}$.

Solve the Test Item.

$4g + 5 = \frac{4}{9}$ Original equation

$4g + 5 - \boxed{} = \frac{4}{9} - \boxed{}$ Subtract $\boxed{}$ from each side.

$4g - 2 = -\frac{59}{9}$ $\frac{4}{9} - 7 = \frac{4}{9} - \frac{63}{9} = \frac{-59}{9}$

The answer is $\boxed{}$.

EXAMPLE Write an Equation

8 **HOME IMPROVEMENT** **Carl wants to replace 5 windows in his home. His neighbor Will is a carpenter and he has agreed to help install them for $250. If Carl has budgeted $1000 for the total cost, what is the maximum amount he can spend on each window?**

Let *c* represent the cost of each window. Write and solve an equation to find the value of *c*.

$$\underbrace{\text{Number of windows}}_{5}\quad\underbrace{\text{times}}_{\cdot}\quad\underbrace{\text{cost of each window}}_{c}\quad\underbrace{\text{plus}}_{+}\quad\underbrace{\text{installation}}_{250}\quad\underbrace{\text{equals}}_{=}\quad\underbrace{\text{total cost.}}_{1000}$$

$5c + 250 = 1000$ Original equation

$5c + 250 - \boxed{} = 1000 - \boxed{}$ Subtract $\boxed{}$ from each side.

$5c = 750$ Simplify.

$c = \boxed{}$ Divide each side by 5.

Carl can afford to spend $\boxed{}$ on each window.

Check Your Progress Kelly wants to repair the siding on her house. Her contractor will charge her $300 plus $1.50 per square foot of siding. How much siding can she repair for $1500?

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–4 Solving Absolute Value Equations

MAIN IDEAS

- Evaluate expressions involving absolute values.
- Solve absolute value equations.

BUILD YOUR VOCABULARY (page 2)

The **absolute value** of a number is its distance from on the number line.

The solution set for an equation that has no solution is the **empty set**, symbolized by or .

EXAMPLE Evaluate an Expression with Absolute Value

① **Evaluate $2.7 + |6 - 2x|$ if $x = 4$.**

$2.7 + |6 - 2x| = 2.7 + |6 - 2(4)|$ Replace x with 4.

$=$ Simplify $-2(4)$ first.

$=$ Subtract 8 from 6.

$=$ $|-2| = 2$

$=$ Add.

Check Your Progress Evaluate $2.3 - |3y - 10|$ if $y = -2$.

REVIEW IT

What is the difference between an algebraic expression and an equation?
(Lessons 1-1, 1-3)

EXAMPLE Solve an Absolute Value Equation

② **Solve $|y + 3| = 8$.**

Case 1 $a = b$

$y + 3 = 8$

$y + 3 -$ $= 8 -$

$y =$

Case 2 $a = -b$

$y + 3 = -8$

$y + 3 -$ $= -8 -$

$y =$

The solutions are .

Thus, the solution set is .

EXAMPLE No Solution

3 **Solve $|6-4t|+5=0$.**

$|6-4t|+5=0$ Original equation

$|6-4t| = \boxed{}$ Subtract $\boxed{}$ from each side.

This sentence is *never* true. So, the solution set is $\boxed{}$.

REMEMBER IT

Determine if your solutions are reasonable before checking them. In Example 3, there is no need to consider the two cases because absolute value is never negative.

EXAMPLE One Solution

4 **Solve $|8+y| = 2y-3$. Check your solutions.**

Case 1 $a = b$

$8+y = 2y-3$

$\boxed{} = \boxed{}$

$\boxed{} = y$

Case 2 $a = -b$

$8+y = -(2y-3)$

$8+y = -2y+3$

$\boxed{} = 3$

$\boxed{} = \boxed{}$

$y = \boxed{}$

There appear to be two solutions.

Check:

$|8+y| = 2y-3$

$|8+\boxed{}| = 2\left(\boxed{}\right)-3$

$\boxed{} \overset{?}{=} 19$

$19 = \boxed{}$

$|8+y| = 2y-3$

$\left|8+\boxed{}\right| \overset{?}{=} 2\left(\boxed{}\right)-3$

$\left|\frac{19}{13}\right| \overset{?}{=} \boxed{} - 3$

$\frac{19}{13} = \boxed{}$

Since $\frac{19}{3} \neq -\frac{19}{3}$, the only solution is $\boxed{}$.

Check Your Progress **Solve each equation. Check your solutions.**

a. $|2x+5| = 15$

$\boxed{}$

b. $\left|5x-\frac{2}{3}\right| + 7 = 0$

$\boxed{}$

c. $|3x-5| = -4x+37$

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–5 Solving Inequalities

MAIN IDEAS

- Solve inequalities.
- Solve real-world problems involving inequalities.

BUILD YOUR VOCABULARY (page 3)

The **Trichotomy Property** says that, for any two real numbers, *a* and *b*, exactly $\boxed{}$ of the following statements is true.

$a < b \qquad a = b \qquad a > b$

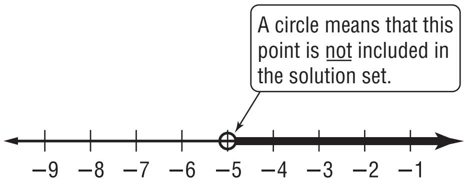
The $\boxed{}$ of an inequality can be expressed by using **set-builder notation**, for example, $\{x \mid x > 9\}$.

EXAMPLE Solve an Inequality Using Addition or Subtraction

① **Solve $4y - 3 < 5y + 2$. Graph the solution set on a number line.**

$4y - 3 < 5y + 2$	Original inequality
$4y - 3 - \boxed{} < 5y + 2 - \boxed{}$	Subtract 4*y* from each side.
$\boxed{} < \boxed{}$	Simplify.
$\boxed{} < \boxed{}$	Subtract from each side.
$\boxed{} < y \text{ or } y > \boxed{}$	Simplify.

Any real number greater than -5 is a solution of this inequality.


A circle means that this point is not included in the solution set.

Check Your Progress Solve $6x - 2 < 5x + 7$. Graph the solution on a number line.

KEY CONCEPTS

Properties of Inequality

Addition Property of Inequality For any real numbers *a*, *b*, and *c*:

If $a > b$, then $a + c > b + c$.

If $a < b$, then $a + c < b + c$.

Subtraction Property of Inequality For any real numbers *a*, *b*, and *c*:

If $a > b$, then $a - c > b - c$.

If $a < b$, then $a - c < b - c$.

EXAMPLE Solve an Inequality Using Multiplication or Division

KEY CONCEPTS

Properties of Inequality

Multiplication Property of Inequality For any real numbers *a*, *b*, and *c*, where *c* is positive:

if $a > b$, then $ac > bc$.

if $a < b$, then $ac < bc$.

c is negative:

if $a > b$, then $ac < bc$.

if $a < b$, then $ac > bc$.

Division Property of Inequality For any real numbers *a*, *b*, and *c*, where *c* is positive:

if $a > b$, then $\frac{a}{c} > \frac{b}{c}$.

if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

c is negative:

if $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

② **Solve $12 \geq -0.3p$. Graph the solution set on a number line.**

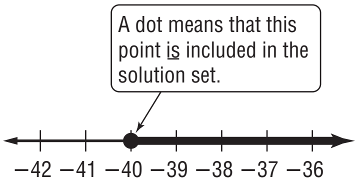
$12 \geq -0.3p$ Original inequality

$\frac{12}{\boxed{\quad}} \leq \frac{-0.3p}{\boxed{\quad}}$ Divide each side by $\boxed{\quad}$, reversing the inequality symbol.

$\boxed{\quad} \leq p$ Simplify.

$p \geq \boxed{\quad}$ Rewrite with *p* first.

The solution set is $\{p \mid p \geq -40]$.


A dot means that this point is included in the solution set.

EXAMPLE Solve a Multi-Step Inequality

③ **Solve $-x > \frac{x-7}{2}$. Graph the solution set on a number line.**

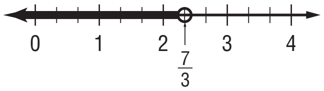
$-x > \frac{x-7}{2}$ Original inequality

$-2x > x - 7$ Multiply each side by 2.

$\boxed{\quad} > \boxed{\quad}$ Add $-x$ to each side.

$x < \boxed{\quad}$ Divide each side by −3, reversing the inequality symbol.

The solution set is $\left(-\infty, \frac{7}{3}\right)$ and is graphed below.



REMEMBER IT

The symbol ∞ represents *infinity*.

Check Your Progress **Solve each inequality. Graph each solution on a number line.**

a. $-3x \geq 21$

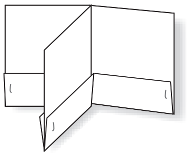
b. $-2x > \frac{x + 5}{3}$

EXAMPLE Write an Inequality

FOLDABLES™

ORGANIZE IT

Write the Properties of Inequality in your notes.



④ **CONSUMER COSTS Alida has at most $15.00 to spend today. She buys a bag of potato chips and a can of soda for $1.59. If gasoline at this store costs $2.89 per gallon, how many gallons of gasoline, to the nearest tenth of a gallon, can Alida buy for her car?**

Let g = the gallons of gasoline Alida can buy for her car. Write and solve an inequality.

$1.59 + 2.89g \leq 15.00$	Original inequality
$1.59 + 2.89g +$ $\leq 15.00 -$	Subtract from each side.
$\leq$	Simplify.
$\leq$	Divide.
$g \leq$	Simplify.

Alida can buy up to gallons of gasoline for her car.

Check Your Progress Jeb wants to rent a car for his vacation. Value Cars rents cars for $25 per day plus $0.25 per mile. How far can he drive for one day if he wants to spend no more than $200 on car rental?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–6

Solving Compound and Absolute Value Inequalities

MAIN IDEAS

- Solve compound inequalities.
- Solve absolute value inequalities.

BUILD YOUR VOCABULARY (page 2)

A **compound inequality** consists of two inequalities joined by the word $\boxed{}$ or the word $\boxed{}$.

The graph of a compound inequality containing *and* is the **intersection** of the solution sets of the two inequalities.

KEY CONCEPT

"And" Compound Inequalities
A compound inequality containing the word *and* is true if and only if *both* inequalities are true.

FOLDABLES Write this concept in your notes.

EXAMPLE Solve an "and" Compound Inequality

① **Solve $10 \leq 3y - 2 < 19$. Graph the solution set.**

METHOD 1 Write the compound inequality using the word *and*. Then solve each inequality.

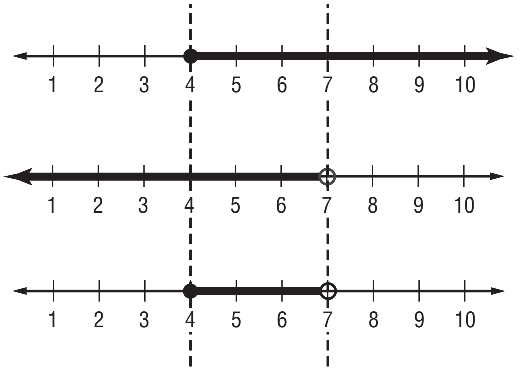
$$\begin{array}{rcl} 10 \leq 3y - 2 & \text{and} & 3y - 2 < 19 \\ \boxed{} \leq 3y & & 3y < \boxed{} \\ \boxed{} \leq y & & y < \boxed{} \end{array}$$

$$\boxed{} \leq y < \boxed{}$$

METHOD 2 Solve both parts at the same time by adding 2 to each part. Then divide each part by 3.

$$\begin{array}{ccccc} \boxed{} & \leq & 3y - 2 & < & \boxed{} \\ \boxed{} & \leq & 3y & < & \boxed{} \\ \boxed{} & \leq & y & < & \boxed{} \end{array}$$

Graph each solution set. Then find their intersection.



$y \geq \boxed{}$

$y < \boxed{}$

$\boxed{} \leq y < \boxed{}$

The solution set is $\{y | 4 \leq y < 7\}$.

Check Your Progress Solve $11 \leq 2x + 5 < 17$. Graph the solution set.

BUILD YOUR VOCABULARY (page 3)

The graph of a compound inequality containing

is the **union** of the solution sets of the two inequalities.

KEY CONCEPT

"Or" Compound Inequalities
A compound inequality containing the word *or* is true if one or more of the inequalities is true.

FOLDABLES Write this concept in your notes.

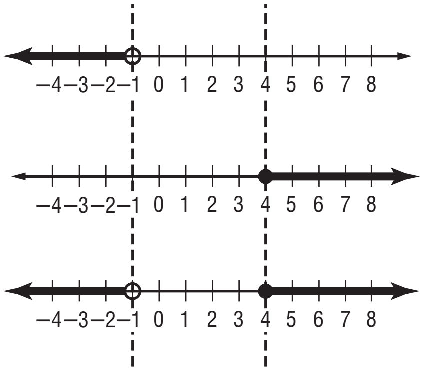
EXAMPLE Solve an "or" Compound Inequality

2 Solve $x + 3 < 2$ or $-x \leq -4$. Graph the solution set.

Solve each inequality separately.

$x + 3 < 2$ or $-x \leq -4$

$x <$ $x \geq$



$x <$

$x \geq$

$x <$ or $x \geq$

The solution set is $\{x \mid x < -1 \text{ or } x \geq 4\}$.

Check Your Progress Solve $x + 5 < 1$ or $-2x \leq -6$. Graph the solution set.

REMEMBER IT

Compound inequalities containing *and* are *conjunctions.* Compound inequalities containing *or* are *disjunctions.*

EXAMPLES Solve an Absolute Value Inequality (<)

3 Solve $3 > |d|$. Graph the solution set on a number line.

You can interpret $3 > |d|$ to mean that the distance between d and 0 on a number line is less than units.

To make $3 > |d|$ true, you must substitute numbers for d that are fewer than units from .

Notice that the graph of $3 > |d|$ is the same as the graph of $d >$ and $d <$.

All of the numbers *not* at or between −3 and 3 are less than units from 0. The solution set is .

Check Your Progress Solve $|x| < 5$. Graph the solution.

EXAMPLE Solve a Multi-Step Absolute Value Inequality

④ **Solve $|2x - 2| \geq 4$. Graph the solution set.**

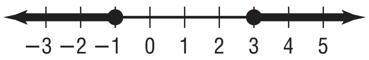
$|2x - 2| \geq 4$ is equivalent to $2x - 2 \geq 4$ or $2x - 2 \leq -4$. Solve each inequality.

$2x - 2 \geq 4$ or $2x - 2 \leq -4$

$\geq$ $\leq$

$x \geq$ $x \leq$

The solution set is .



Check Your Progress Solve $|3x - 3| > 9$. Graph the solution set.

EXAMPLE Write an Absolute Value Inequality

⑤ **HOUSING** **According to a recent survey, the average monthly rent for a one-bedroom apartment in one city is $750. However, the actual rent for any given one-bedroom apartment might vary as much as $250 from the average.**

a. Write an absolute value inequality to describe this situation.

Let r = the actual monthly rent

$$\underbrace{\boxed{}}_{\text{The rent for an apartment can differ from the average}} \quad \underbrace{\boxed{}}_{\text{by as much as}} \quad \underbrace{\boxed{}}_{\$250.}$$

b. Solve the inequality $|750 - r| \le 250$ to find the range of monthly rent. Rewrite the absolute value inequality as a compound inequality. Then solve for r.

$$-250 \le 750 - r \le 250$$

$$-250\ \boxed{} \le -r \le 250\ \boxed{}$$

$$-1000 \le -r \le -500$$

$$\boxed{} \ge r \ge \boxed{}$$

The solution set is $\boxed{}$. The actual rent falls between $\boxed{}$ and $\boxed{}$, inclusive.

Check Your Progress **The average birth weight of a newborn baby is 7 pounds. However, this weight can vary by as much as 4.5 pounds.**

a. What is an absolute value inequality to describe this situation?

b. What is the range of birth weights for newborn babies?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 1

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 1 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 1, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 2–3)* to help you solve the puzzle.

1-1

Expressions and Formulas

1. Find the value of $30 - 4^2 \div 2 \cdot 4$.

2. Evaluate $2x^2 - 3xy$ if $x = -4$ and $y = 5$.

3. Why is it important for everyone to use the same order of operations for evaluating expressions?

1-2

Properties of Real Numbers

4. Name the sets of numbers to which $-\frac{7}{8}$ belongs.

5. Write the Associative Property of Addition in symbols. Then illustrate this property by finding the sum $12 + 18 + 45$.

Complete each sentence.

6. The ______ Property of Addition says that adding 0 to any number does not change its value.

7. The ______ numbers can be written as ratios of two integers, with the integer in the denominator not being 0.

1-3

Solving Equations

Solve each equation. Check your solution.

8. $3 - 5y = 4y + 6$

9. $\frac{2}{3} - \frac{1}{2}x = \frac{1}{6}$

10. Solve $A = \frac{1}{2}h(a + b)$ for h.

Read the following problem and then write an equation that you could use to solve it. Do not actually solve the equation. In your equation, let *m* be the number of miles driven.

11. When Louisa rented a moving truck, she agreed to pay \$28 per day plus \$0.42 per mile. If she kept the truck for 3 days and the rental charges (without tax) were \$153.72, how many miles did Louisa drive the truck?

1-4

Solving Absolute Value Equations

12. Evaluate $|m - 5n|$ if $m = -3$ and $n = 2$.

Solve each equation.

13. $-2|4x - 5| = -46$

14. $|7 + 3x| = x - 5$

15. Explain why the absolute value of a number can never be negative.

1-5

Solving Inequalities

There are several different ways to write or show solution sets of inequalities. Write each of the following in interval notation.

16. $\{x \mid x < -3\}$

17.

Solve each inequality. Graph the solution set.

18. $5y + 9 > 34$

19. $-1 - 5x \leq 4(x + 2)$

1-6

Solving Compound and Absolute Value Inequalities

Complete each sentence.

20. Two inequalities combined by the word *and* or the word *or* form a .

21. The graph of a compound inequality containing the word *and* is the of the graphs of the two separate inequalities.

Solve each inequality. Graph the solution set.

22. $-11 < 3m - 2 < 22$

23. $|x + 3| \geq 1$

CHAPTER 1

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 1.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 1 Practice Test on page 53 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 1 Study Guide and Review on pages 49–52 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 53 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 1 Foldable.
- Then complete the Chapter 1 Study Guide and Review on pages 49–52 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 53 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

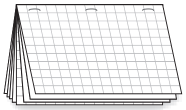
CHAPTER 2

Linear Relations and Functions

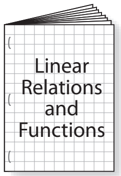
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with two sheets of grid paper.

STEP 1 **Fold** in half along the width and staple along the fold.



STEP 2 **Turn** the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.



NOTE-TAKING TIP: When taking notes, make annotations. Annotations are usually notes taken in the margins of books you own to organize the text for review or study.

CHAPTER

2

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 2. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
absolute value function			
boundary			
Cartesian coordinate plane			
constant function			
family of graphs			
function			
greatest integer function			
identity function			
linear function			
line of fit			

(continued on the next page)

Chapter 2 BUILDING YOUR VOCABULARY

Vocabulary Term	Found on Page	Definition	Description or Example
parent graph			
piecewise function PEES-WYZ			
point-slope form			
positive correlation			
prediction equation pree-DIHK-shuhn			
relation			
scatter plot			
slope			
slope-intercept form IHN-tuhr-SEHPT			
standard form			
step function			

2–1 Relations and Functions

MAIN IDEAS

- Analyze and graph relations.
- Find functional values.

BUILD YOUR VOCABULARY (pages 27–28)

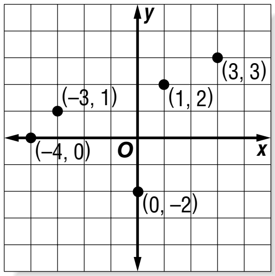
A **relation** is a set of ______.

A **function** is a special type of relation in which each element of the domain is paired with ______ element of the range.

A function where each element of the ______ is paired with exactly one element of the ______ is called a **one-to-one function.**

EXAMPLE Domain and Range

① **State the domain and range of the relation shown in the graph. Is the relation a function?**



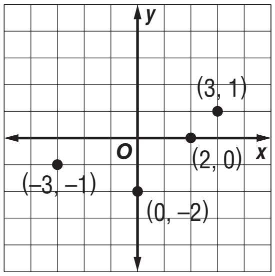
The relation is {(1, 2), (3, 3), (0, −2), (−4, 0), (−3, 1)}.

The domain is ______.

The range is ______.

Each member of the domain is paired with exactly one member of the range, so this relation is a ______.

Check Your Progress State the domain and range of the relation shown in the graph. Is the relation a function?



EXAMPLE Vertical Line Test

① **TRANSPORTATION The table shows the average fuel efficiency in miles per gallon for light trucks for several years. Graph this information and determine whether it represents a function. Is this relation *discrete* or *continuous*?**

The year values correspond to the values, and the Fuel Efficiency values correspond to the values.

Year	Fuel Efficiency (mi/gal)
1995	20.5
1996	20.8
1997	20.6
1998	20.9
1999	20.5
2000	20.5
2001	20.4

Source: U.S. Environmental Protection Agency

Graph the ordered pairs on a coordinate grid.

Use the vertical line test. Notice that no vertical line can be drawn that contains more than one data point

This relation is a function, and it is .

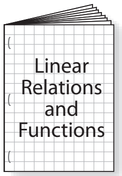
Check Your Progress **HEALTH** The table shows the average weight of a baby for several months during the first year. Graph this information and determine whether it represents a function.

Age (months)	Weight (pounds)
1	12.5
2	16
4	22
6	24
9	25
12	26

FOLDABLES™

ORGANIZE IT

On the page labeled Relations and Functions, define function and relation. Then draw a graph of a function and a relation.



EXAMPLE Graph a Relation

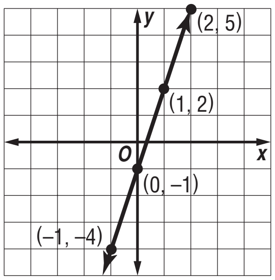
KEY CONCEPT

Vertical Line Test If no vertical line intersects a graph in more than one point, the graph represents a function. If some vertical line intersects a graph in two or more points, the graph does not represent a function.

1 **a. Graph the relation represented by $y = 3x - 1$.**

Make a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y. Then graph the ordered pairs.

x	y
-1	
0	
1	
2	



b. Find the domain and range.

The domain and range are both all numbers.

c. Determine whether the equation is a function and state whether it is *discrete* or *continuous*.

This graph passes the vertical line test. For each

-value, there is exactly one -value.

The equation represents a .

All (x, y) values that satisfy the equation lie on a ,

so the relation is .

Check Your Progress

a. Graph $y = 2x + 5$.

b. Find the domain and range.

c. Determine whether the equation is a function and state whether it is.

EXAMPLE **Evaluate a Function**

④ **Given $f(x) = x^3 - 3$, find each value.**

a. $f(-2)$

$f(x) = x^3 - 3$	Original function
$f(-2) =$	Substitute.
$=$ or	Simplify.

b. $f(2t)$

$f(x) = x^3 - 3$	Original function
$f(2t) =$	Substitute.
$=$	$(ab)^2 = a^2b^2$

Check Your Progress **Given $f(x) = x^2 + 5$ and $h(x) = 0.5x^2 + 2x + 2.5$, find each value.**

a. $f(-1)$

b. $h(1.5)$

c. $f(3a)$

REVIEW IT

Explain what it means to evaluate an expression. *(Lesson 1-1)*

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–2 Linear Equations

MAIN IDEAS

- Identify linear equations and functions.
- Write linear equations in standard form and graph them.

BUILD YOUR VOCABULARY (page 27)

An equation such as $x + y = 4$ is called a linear equation.

A **linear equation** has no operations other than , , and of a variable by a constant.

WRITE IT

Give an example of a linear function and a nonlinear function. Explain how you can tell the difference between the two functions.

EXAMPLE Identify Linear Functions

① **State whether each function is a linear function. Explain.**

a. $g(x) = 2x - 5$

This is a function because it is in the form $g(x) = mx + b$. $m =$, $b =$

b. $p(x) = x^3 + 2$

This is not a linear function because x has an exponent other than .

c. $t(x) = 4 + 7x$

This is a function because it is in the form $t(x) = mx + b$. $m =$, $b =$

Check Your Progress **State whether each function is a linear function. Explain.**

a. $h(x) = 3x - 2$

b. $g(x, y) = 3xy$

EXAMPLE Evaluate a Linear Function

1 **The linear function $f(C) = 1.8C + 32$ can be used to find the number of degrees Fahrenheit f that are equivalent to a given number of degrees Celsius C.**

a. On the Celsius scale, normal body temperature is 37°C. What is it in degrees Fahrenheit?

$f(C) = 1.8C + 32$ Original function

$f\left(\boxed{\quad}\right) = 1.8\left(\boxed{\quad}\right) + 32$ Substitute.

$= \boxed{\quad}$ Simplify.

b. There are 100 Celsius degrees between the freezing and boiling points of water and 180 Fahrenheit degrees between these two points. How many Fahrenheit degrees equal 1 Celsius degree?

$100°\text{C} = \boxed{\quad}°\text{F}$ Given

$\dfrac{100°\text{C}}{\boxed{\quad}} = \dfrac{\boxed{\quad}°\text{F}}{\boxed{\quad}}$ Divide each side by $\boxed{\quad}$.

$1°\text{C} = \boxed{\quad}$ Simplify.

Check Your Progress **The linear function $d(s) = \frac{1}{5}s$ can be used to find the distance d in miles from a storm, based on the number of seconds s that it takes to hear thunder after seeing lightning.**

a. If you hear thunder 10 seconds after seeing lightning, how far away is the storm?

b. If the storm is 3 miles away, how long will it take to hear thunder after seeing lightning?

KEY CONCEPT

Standard Form of a Linear Equation The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero.

EXAMPLE Standard Form

3 **Write each equation in standard form. Identify A, B, and C.**

a. $y = 3x - 9$

$y = 3x - 9$ Original equation

= Subtract $3x$ from each side.

= Multiply each side by -1 so that $A \geq 0$.

So, $A =$, $B =$, and $C =$.

b. $-\frac{2}{3}x = 2y - 1$

$-\frac{2}{3}x = 2y - 1$ Original equation

= Subtract $2y$ from each side.

= Multiply each side by -3 so that the coefficients are all integers.

So, $A =$, $B =$, and $C =$.

Check Your Progress **Write each equation in standard form. Identify A, B, and C.**

a. $y = -2x + 5$

b. $\frac{3}{5}x = -3y + 2$

c. $3x - 9y + 6 = 0$

REMEMBER IT

An equation such as $x = 3$ represents a vertical line, and only has an x-intercept.

An equation such as $y = 5$ represents a horizontal line, and only has a y-intercept.

EXAMPLE Use Intercepts to Graph a Line

④ **Find the x-intercept and the y-intercept of the graph of $-2x + y - 4 = 0$. Then graph the equation.**

The x-intercept is the value of x when $y = 0$.

$-2x + y - 4 = 0$ Original equation

$-2x + \boxed{} - 4 = 0$ Substitute 0 for y.

$\boxed{} = \boxed{}$ Add 4 to each side.

$x = \boxed{}$ Divide each side by −2.

The x-intercept is −2. The graph crosses the x-axis at $\boxed{}$.

Likewise, the y-intercept is the value of y when $x = 0$.

$-2x + y - 4 = 0$ Original equation

$-2\left(\boxed{}\right) + y - 4 = 0$ Substitute 0 for x.

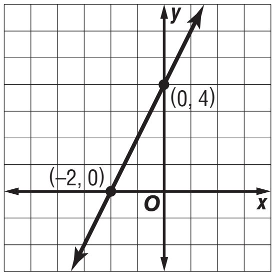
$y = \boxed{}$ Add 4 to each side.

The y-intercept is 4. The graph crosses the y-axis at $\boxed{}$.

Use the ordered pairs to graph this equation.

The x-intercept is $\boxed{}$,

and the y-intercept is $\boxed{}$.



Check Your Progress Find the x-intercept and the y-intercept of the graph of $3x - y + 6 = 0$. Then graph the equation.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–3 Slope

MAIN IDEAS

- Find and use the slope of a line.
- Graph parallel and perpendicular lines.

KEY CONCEPT

Slope of a Line The slope of a line is the ratio of the change in *y*-coordinates to the change in *x*-coordinates.

EXAMPLE Find Slope and Use Slope to Graph

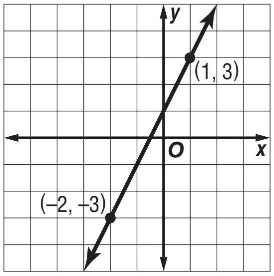
① **Find the slope of the line that passes through (1, 3) and (−2, −3). Then graph the line.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$=$ $(x_1, y_1) = (1, 3), (x_2, y_2) = (-2, -3)$

$=$ or Simplify.

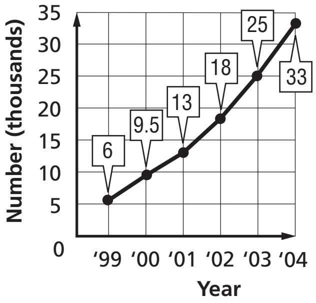
Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any on the line. Then go up units and right unit or go 2 units and left 1 unit. This point should also be on the line.



Check Your Progress Find the slope of the line that passes through (2, 3) and (−1, 5). Then graph the line.

EXAMPLE Find Rate of Change

2 **BUSINESS** **Refer to the graph at the right, which shows data on the fastest-growing restaurant chain in the U.S. during the time period of the graph. Find the rate of change of the number of stores from 1999 to 2004.**



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}}$ Substitute.

$= 5.4$ Simplify.

Between 1999 and 2004, the number of stores in the U.S. increases at an average rate of ____ or ____ stores per year

Check Your Progress Refer to the graph in Example 2. Find the rate of change of the number of stores from 2001 to 2004.

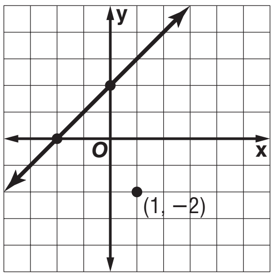
KEY CONCEPT

Parallel Lines In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.

EXAMPLE Parallel Lines

3 **Graph the line through (1, −2) that is parallel to the line with the equation $x - y = -2$.**

The x-intercept is ____, and the y-intercept is ____.



Use the intercepts to graph $x - y = -2$.

The line rises 1 unit for every 1 unit it moves to the right, so the slope is ____.

Now, use the slope and the point at (1, −2) to graph the line parallel to $x - y = -2$.

Check Your Progress Graph the line through (2, 3) that is parallel to the line with the equation $3x + y = 6$.

EXAMPLE Perpendicular Lines

④ **Graph the line through (2, 1) that is perpendicular to the line with the equation $2x - 3y = 3$.**

The x-intercept is or , and the y-intercept is

. Use the intercepts to graph $2x - 3y = 3$.

The line rises 1 unit for every 1.5 units it moves to the right,

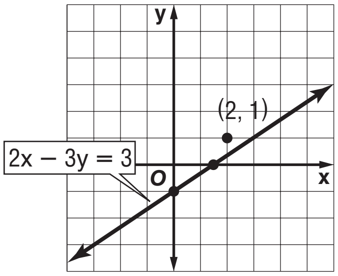
so the slope is or .

The slope of the line perpendicular

is the opposite reciprocal of

or . Start at (2, 1) and go

down units and right units.



Use this point and (2, 1) to graph the line.

KEY CONCEPT

Perpendicular Lines In a plane, two oblique lines are perpendicular if and only if the product of their slopes is −1.

FOLDABLES™ On the page opposite the page labeled Slope, graph two lines that are parallel. Then graph two lines that are perpendicular.

Check Your Progress Graph the line through (−3, 1) that is perpendicular to the line with the equation $5x - 10y = -20$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–4 Writing Linear Equations

MAIN IDEAS

- Write an equation of a line given the slope and a point on the line.
- Write an equation of a line parallel or perpendicular to a given line.

KEY CONCEPTS

Slope-Intercept Form of a Linear Equation The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

Point-Slope Form of a Linear Equation The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

EXAMPLE Write an Equation Given Slope and a Point

① **Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{5}$ and passes through (5, −2).**

$y = mx + b$	Slope-intercept form
$-2 = -\frac{3}{5}(5) + b$	$(x, y) = (5, -2)$, $m = -\frac{3}{5}$
$\boxed{} = \boxed{}$	Simplify.
$\boxed{} = \boxed{}$	Add 3 to each side.

The y-intercept is $\boxed{}$. So the equation in slope-intercept form is $\boxed{}$.

Check Your Progress Write an equation in slope-intercept form for the line that has a slope of $\frac{2}{3}$ and passes through $(-3, -1)$.

$\boxed{}$

EXAMPLE Write an Equation Given Two Points

② **What is an equation of the line through (2, −3) and (−3, 7)?**

A $y = -2x - 1$	**C** $y = \frac{1}{2}x + 1$
B $y = -\frac{1}{2}x + 1$	**D** $y = -2x + 1$

First, find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{\boxed{} - \left(\boxed{}\right)}{\boxed{} - \boxed{}}$	$(x_1, y_1) = (2, -3)$, $(x_2, y_2) = (-7)$
$= \boxed{}$ or $\boxed{}$	Simplify.

The slope is −2. That eliminates choices B and C.

Now use the point-slope formula to find an equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-3) = -2(x - 2)$	$m = -2$; you can use either point for (x_1, y_1).
$\boxed{} = \boxed{}$	Distributive Property
$y = \boxed{}$	Subtract 3 from each side.

The answer is $\boxed{}$.

Check Your Progress What is an equation of the line through (2, 5) and (−1, 3)?

A $y = \frac{2}{3}x + \frac{11}{3}$ **C** $y = \frac{-2}{3}x + \frac{19}{3}$

B $y = \frac{3}{2}x + \frac{9}{2}$ **D** $y = \frac{-3}{2}x + 8$

$\boxed{}$

EXAMPLE Write an Equation of a Perpendicular Line

3 **Write an equation for the line that passes through (3, −2) and is perpendicular to the line whose equation is $y = -5x + 1$.**

The slope of the given line is $\boxed{}$. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\boxed{}$.

Use the point-slope form and the ordered pair $\boxed{}$ to write the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - \left(\boxed{}\right) = \boxed{}\left(x - \boxed{}\right)$	$(x_1, y_1) = (3, -2)$, $m = \frac{1}{5}$
$\boxed{} = \boxed{}$	Distributive Property
$y = \boxed{}$	Subtract 2 from each side.

An equation of the line is $\boxed{}$.

REVIEW IT

Name the property you would use to simplify $8(z + 4)$. *(Lesson 1-2)*

____

____

Check Your Progress Write an equation for the line that passes through (3, 5) and is perpendicular to the line with equation $y = 3x - 2$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–5 Statistics: Using Scatter Plots

MAIN IDEAS

- Draw scatter plots.
- Find and use prediction equations.

EXAMPLE Find and use a Prediction Equation

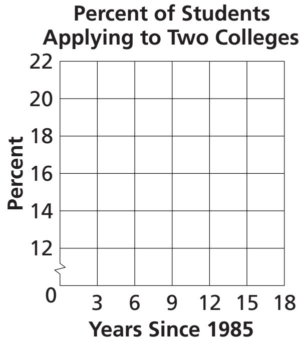
① **EDUCATION** **The table below shows the approximate percent of students who sent applications to two colleges in various years since 1985.**

Years Since 1985	0	3	6	9	12	15
Percent	20	18	15	15	14	13

Source: *U.S. News & World Report*

a. Draw a scatterplot and a line of fit for the data.

Graph the data as ordered pairs, with the number of years since 1985 on the ________ axis and the percentage on the ________ axis.



b. Find a prediction equation. What do the slope and *y*-intercept indicate?

Find an equation of the line through (3, 18) and (15, 13). Begin by finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{13 - 18}{15 - 3}$$ Substitute.

$$= \boxed{} \approx \boxed{}$$ Simplify.

(continued on the next page)

WRITE IT

An outlier is a data point that does not appear to belong with the rest of the set. Should you include an outlier when finding a line of fit? Explain.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - \boxed{\quad} = \boxed{\quad}\left(x - \boxed{\quad}\right)$	$m = -0.42$, $(x_1, y_1) = (3, 18)$
$\boxed{\quad} = \boxed{\quad}$	Distributive Property
$y = \boxed{\quad}$	Add 18 to each side.

The slope indicates that the percent of students sending applications to two colleges is falling at about $\boxed{\quad}$ each year. The y-intercept indicates that the percent in 1985 should have been about $\boxed{\quad}$.

c. Predict the percent in 2010.

The year 2010 is 25 years after 1985, so use $x = 25$ to find y.

$y = -0.42x + 19.26$	Prediction equation
$= -0.42\left(\boxed{\quad}\right) + 19.26$	$x = \boxed{\quad}$
$= \boxed{\quad}$	Simplify.

In 2010 the percent should be about $\boxed{\quad}$.

d. How accurate is the prediction?

The fit is only $\boxed{\quad}$, so the prediction may not be very accurate.

Check Your Progress **The table below shows the approximate percent of drivers who wear seat belts in various years since 1994.**

Years Since 1994	0	1	2	3	4	5	6	7
Percent	57	58	61	64	69	68	71	73

Source: National Highway Traffic Safety Administration

a. Make a scatter plot of the data and draw a line of best fit.

b. Find a prediction equation. What do the slope and y-intercept indicate?

c. Predict the percent of drivers who will be wearing seat belts in 2005.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–6 Special Functions

MAIN IDEAS

- Identify and graph step, constant, and identity functions.
- Identify and graph absolute value and piecewise functions.

BUILD YOUR VOCABULARY (pages 27–28)

A function whose graph is a series of line segments is called a **step function**.

The **greatest integer function**, written $f(x) = [[x]]$, is an example of a step function.

$f(x) = b$ is called a **constant function**.

When a function does not change the value, $f(x) = x$ is called the **identity function**.

Another special function is the **absolute value function**, $f(x) = |x|$.

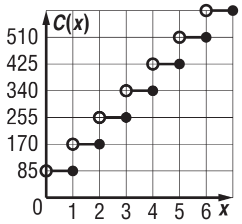
A that is written using two or more expressions is called a **piecewise function**.

EXAMPLE Step Function

① **PSYCHOLOGY One psychologist charges for counseling sessions at the rate of $85 per hour or any fraction thereof. Draw a graph that represents this situation.**

Use the pattern of times and costs to make a table, where x is the number of hours of the session and $C(x)$ is the total cost. Then draw the graph.

x	$C(x)$
$0 < x \leq 1$	
$1 < x \leq 2$	
$2 < x \leq 3$	
$3 < x \leq 4$	
$4 < x \leq 5$	



EXAMPLE Absolute Value Functions

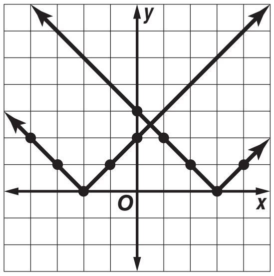
2 **Graph $f(x) = |x - 3|$ and $g(x) = |x + 2|$ on the same coordinate plane. Determine the similarities and differences in the two graphs.**

Find several ordered pairs for each function.

x	$\lvert x - 3\rvert$
0	3
1	
2	
3	
4	

x	$\lvert x + 2\rvert$
−4	2
−3	
−2	
−1	
0	

Graph the points and connect them.



- The domain of both graphs is all ______ numbers.
- The range of both graphs is $\{y \mid y \geq \ \_\_\_ \}$.
- The graphs have the same shape but different ______.
- The graph of $g(x)$ is the graph of $f(x)$ translated left ______ units.

Check Your Progress

a. Graph $f(x) = |x| - 2$ and $g(x) = |x| + 3$ on the same coordinate plane. Determine the similarities and differences in the two graphs.

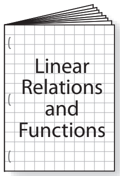
WRITE IT

Consider the general form of an absolute value function, $f(x) = |x|$. Explain why the range of the function only includes nonnegative numbers.

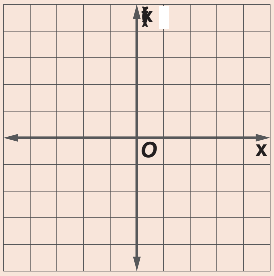
FOLDABLES™

ORGANIZE IT

Under the tab for Graphing Linear Functions, graph an example of each of the following functions: step, constant, absolute value, and piecewise.

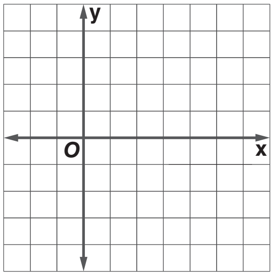


b. Graph the piecewise function $f(x) = \left\{\begin{matrix} 2x + 1 \text{ if } x > -1 \\ -3 \text{ if } x \leq -1 \end{matrix}\right\}$. Identify the domain and range.



EXAMPLE Piecewise Function

③ **Graph $f(x) = \left\{\begin{matrix} x - 1 \text{ if } x \leq 3 \\ -1 \text{ if } x > 3 \end{matrix}\right\}$. Identify the domain and range.**



First graph the linear function $f(x) = x - 1$ for $x \leq 3$. Since 3 satisfies this inequality, begin

with a closed circle at ______.

Then graph the function $f(x) = -1$ for $x > 3$. Since 3 does not satisfy this inequality, begin with an

______ cirlce at ______.

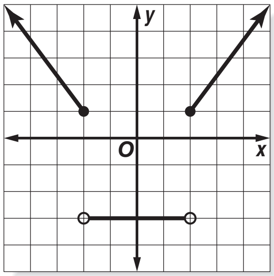
The ______ is all real numbers. The ______ is $\{y|y \leq 2\}$.

Check Your Progress Graph $f(x) = \left\{\begin{matrix} 2x + 1 \text{ if } x > -1 \\ -3 \text{ if } x \leq -1 \end{matrix}\right\}$. Identify the domain and range.

EXAMPLE Indentify Functions

④ **Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.**

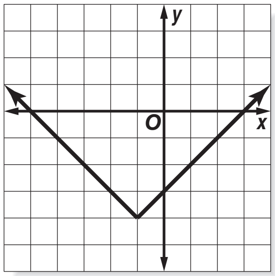
a.



Since this graph consists of different rays and segments, it is a

function.

b.

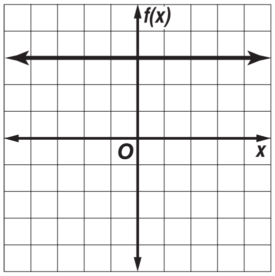


Since this graph is V-shaped, it is an

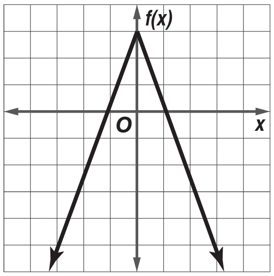
function.

Check Your Progress **Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.**

a.



b.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–7 Graphing Inequalities

MAIN IDEAS

- Graph linear inequalities.
- Graph absolute value inequalities.

BUILD YOUR VOCABULARY (page 27)

When graphing inequalities, the graph of the line is the **boundary** of each region.

EXAMPLE Dashed Boundary

① **Graph $x - 2y < 4$.**

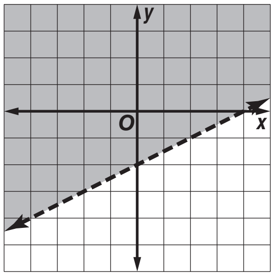
The boundary is the graph of $x - 2y = 4$. Since the inequality symbol is $<$, the boundary will be dashed. Use the slope-intercept form, $y = \frac{1}{2}x - 2$. Test (0, 0).

$x - 2y < 4$ Original inequality

$\overset{?}{<}$ $(x, y) = (0, 0)$

$0 < 4$ true

Shade the region that contains (0, 0)



EXAMPLE Solid Boundary

② **EDUCATION** **One tutoring company advertises that it specializes in helping students who have a combined SAT verbal and math score of 900 or less.**

a. Write an inequality to describe the combined scores of students who are prospective tutoring clients. Let x represent the verbal score and y the math score.

$$\underbrace{\text{verbal score}}_{x}\quad \underbrace{\text{and}}_{+}\quad \underbrace{\text{math score}}_{y}\quad \underbrace{\text{are at most}}_{\leq}\quad \underbrace{\text{nine hundred}}_{900}$$

The inequality is .

b. Does a student with a verbal score of 480 and a math score of 410 fit the tutoring company's guidelines?

$x + y \leq 900$ Original inequality

$480 + 410 \leq 900$ $(x, y) = (480, 410)$

$890 \leq 900$ Simplify

So, a student with a verbal score of 480 and a math score of 410 the tutoring company's guidelines.

EXAMPLE Absolute Value Inequality

3 **Graph $y \geq |x| - 2$.**

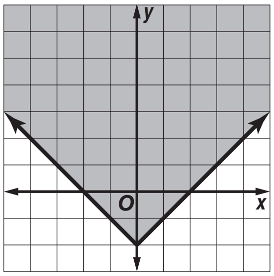
Since the inequality symbol is ≥, the graph of the related equation $y = |x| - 2$ is solid. Graph the equation. Test (0, 0).

$y \geq |x| - 2$

$0 \overset{?}{\geq} |0| - 2$

$0 \geq 0 - 2$ or ☐ ≥ ☐

Shade the region that contains ☐.



Check Your Progress **Graph each inequality.**

a. $2x + 5y > 10$

b. $y < |x| + 3$

c. **CLASS TRIP** **Two social studies classes are going on a field trip. The teachers have asked for parent volunteers to go on the trip as chaperones. However, there is only enough seating for 60 people on the bus.**

Write an inequality to describe the number of students and chaperones that can ride on the bus. Can 45 students and 10 chaperones go on the trip?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 2

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 2 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 2, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 27–28)* to help you solve the puzzle.

2-1

Relations and Functions

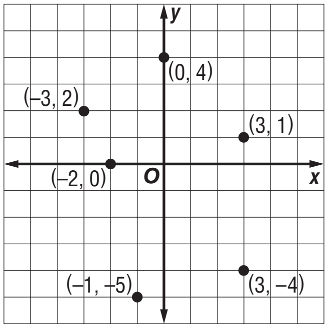
For Exercises 1 and 2, refer to the graph shown at the right.

1. Write the domain and range of the relation.

D:

R:

2. Is this relation a function? Explain.



2-2

Linear Equations

3. Write $x - 2 = \frac{1}{6}y$ in standard form. Identify A, B, and C.

Write *yes* or *no* to tell whether each linear equation is in standard form. If it is not, explain why it is not.

4. $-x + 2y = 5$

5. $5x - 7y = 3$

2-3

Slope

6. How are the slopes of two nonvertical parallel lines related?

7. How are the slopes of two oblique perpendicular lines related?

Find the slope of the line that passes through each pair of points.

8. (3, 7), (8, −1)

9. $\left(8, -\frac{1}{4}\right), \left(0, -\frac{1}{4}\right)$

2-4

Writing Linear Equations

Write the equation in slope-intercept form for the line that satisfies each set of conditions.

10. slope 4, passes through (0, 3)

11. passes through (5, −6) and (3, 2)

12. Write an equation for the line that passes through (8, −5) and is perpendicular to the line whose equation is $y = \frac{1}{2}x - 8$.

2-5

Statistics: Using Scatter Plots

13. Draw a scatter plot for the data. Then state which of the data points is an outlier.

x	2	6	10	14	20	24
y	15	20	30	16	40	50

2-6

Special Functions

Write the letter of the term that best describes each function.

14. $f(x) = |4x + 3|$ ☐

15. $f(x) = [[x]] + 1$ ☐

16. $f(x) = 6$ ☐

17. $f(x) = \begin{cases} x + 3 \text{ if } x < 0 \\ 2 - x \text{ if } x \geq 0 \end{cases}$ ☐

a. constant function
b. absolute value function
c. piecewise function
d. step function

2-7

Graphing Inequalities

18. When graphing a linear inequality in two variables, how do you know whether to make the boundary a solid line or a dashed line?

19. Graph the inequality $10 - 5y < 2x$.

CHAPTER 2

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 2.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 2 Practice Test on page 111 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 2 Study Guide and Review on pages 106–110 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 111 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 2 Foldable.
- Then complete the Chapter 2 Study Guide and Review on pages 106–110 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 111 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

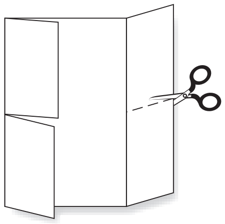
CHAPTER 3

Systems of Equations and Inequalities

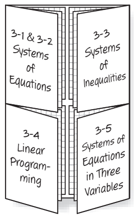
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11” × 17” paper and four sheets of grid paper.

STEP 1 **Fold** the short sides of the 11″ × 17″ paper to meet in the middle. Cut each table in half as shown.



STEP 2 **Cut** 4 sheets of grid paper in half and fold the half-sheets in half. Insert two folded halfsheets under each of the four tabs and staple along the fold. Label each tab as shown.



NOTE-TAKING TIP: When taking notes, summarize the main ideas presented in the lesson. Summaries are useful for condensing data and realizing what is important.

CHAPTER 3

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
bounded			
consistent			
constraints kuhn-STRAYNTS			
dependent			
elimination method			
feasible region FEE-zuh-buhl			
inconsistent ihn-kuhn-SIHS-tuhnt			
independent			

(*continued on the next page*)

Vocabulary Term	Found on Page	Definition	Description or Example
linear programming			
ordered triple			
substitution method			
system of equations			
system of inequalities			
unbounded			
vertex			

3–1

Solving Systems of Equations by Graphing

MAIN IDEAS

- Solve systems of linear equations by graphing.
- Determine whether a system of linear equations is consistent and independent, consistent and dependent, or inconsistent.

BUILD YOUR VOCABULARY (pages 57–58)

A **system of equations** is or more equations with the same variables.

A system of equations is **consistent** if it has at least solution and **inconsistent** if it has solutions.

A consistent system is **independent** if it has exactly solution or **dependent** if it has an number of solutions.

EXAMPLE Solve the System of Equations by Completing a Table

① **Solve the system of equations by completing a table.**

$x + y = 3$

$-2x + y = -6$

Write each equation in slope-intercept form.

$x + y = 3 \longrightarrow$

$-2x + y = -6 \longrightarrow$

Use a table to find the solution that satisfies both equations.

x	$y_1 = -x + 3$	y_1	$y_2 = 2x - 6$	y_2	(x_1, y_1)	(x_2, y_2)
0	$y_1 = (0) + 3$	3	$y_2 = 2(0) - 6$	−6	(0, 3)	(0, −6)
1	$y_1 = -(1) + 3$	2	$y_2 = 2(0) - 6$	−4	(1, 2)	(1, −4)
2	$y_1 = -(2) + 3$	1	$y_2 = 2(0) - 6$	−2	(2, 1)	(2, −2)
3	$y_1 = -(3) + 3$	0	$y_2 = 2(0) - 6$	0	(3, 0)	(3, 0)

The solution of the system of equations is .

EXAMPLE Solve by Graphing

1 Solve the system of equations by graphing.

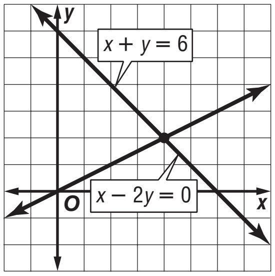
$\boldsymbol{x - 2y = 0}$

$\boldsymbol{x + y = 6}$

Write each equation in slope-intercept form.

$x - 2y = 0 \longrightarrow$

$x + \ \ y = 6 \longrightarrow$



The graphs appear to intersect at .

Check: Substitute the coordinates into each equation.

$x - 2y = 0$	$x + y = 6$	Original equations.
$4 - 2(2) \overset{?}{=}$	$4 + 2 \overset{?}{=} 6$	Replace *x* and *y*.
$= 0$	$= 6$	Simplify.

So, the solution of the system is .

REMEMBER IT

When solving a system of equations by graphing, you should always check the ordered pair in *each* of the original equations.

Check Your Progress

a. Solve the system by completing a table.

$x + y = 2$

$x - 3y = -6$

b. Solve the system by graphing.

$x + 3y = 7$

$x - y = 3$

REVIEW IT

Explain the steps you would use to write $2x + 5y = 10$ in slope-intercept form. *(Lesson 1-3)*

____

____

____

____

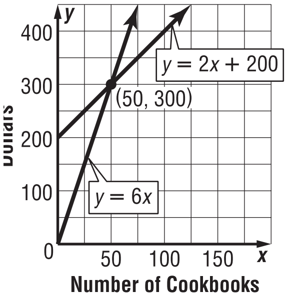
EXAMPLE Break-Even Point Analysis

③ **SALES A service club is selling copies of its holiday cookbook to raise funds for a project. The printer's set-up charge is $200, and each book costs $2 to print. The cookbooks will sell for $6 each. How many must the club sell before it makes a profit?**

$$\underbrace{\text{Income from books}}_{y}\quad\underbrace{\text{is}}_{=}\quad\underbrace{\text{price per book}}_{6}\quad\underbrace{\text{times}}_{\cdot}\quad\underbrace{\text{number of books.}}_{x}$$

The graphs interect at ______.

This is the break-even point. If the group sells less than 50 books, they will ______ money. If the group sells more than 50 books, they will make a ______.



Check Your Progress The student government is selling candy bars. It costs $1 for each candy bar plus a $60 set-up fee. The group will sell the candy bars for $2.50 each. How many do they need to sell to break even?

______

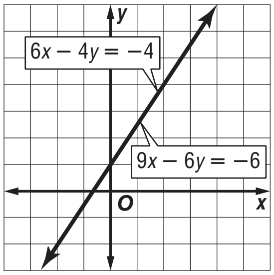
EXAMPLE Same Line

④ **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

$$\mathbf{9x - 6y = -6}$$
$$\mathbf{6x - 4y = -4}$$

$9x - 6y = -6 \longrightarrow$ ______

$6x - 4y = -4 \longrightarrow$ ______



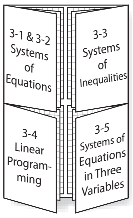
Since the equations are equivalent, their graphs are the ______ line. There are ______ many solutions.

This system is ______ and ______.

FOLDABLES™

ORGANIZE IT

Under the Systems of Equations tab, graph the three possible relationships between a system of equations and the number of solutions. Write the number of solutions below each graph.



Check Your Progress **Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

a. $x + y = 5$
$2x = y - 11$

b. $x + y = 3$
$2x = -2y + 6$

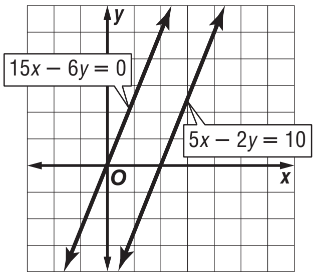
EXAMPLE Parallel Lines

⑤ **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

$$15x - 6y = 0$$
$$5x - 2y = 10$$

$15x - 6y = 0 \longrightarrow$

$5x - 2y = 10 \longrightarrow$



The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system is ______.

Check Your Progress Graph the system of equations $y = 3x + 2$ and $-6x + 2y = 10$ and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–2 Solving Systems of Equations Algebraically

MAIN IDEAS

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

BUILD YOUR VOCABULARY (pages 57–58)

Using the **substitution method**, one equation is solved for one ____ in terms of ____. Then, this expression is substituted for the variable in the other equation. Using the **elimination method**, you eliminate one of the variables by ____ or ____ the equations.

REMEMBER IT

In Example 1, you can substitute the value for *y* in either of the original equations. Choose the equation that is easiest to solve.

EXAMPLE Solve by Using Substitution

① **Use substitution to solve $x + 4y = 26$ and $x - 5y = -10$.**

Solve the first equation for x in terms of y.

$x + 4y = 26$ First equation

$x =$ ____ Subtract 4*y* from each side.

Substitute $26 - 4y$ for x in the second equation and solve for y.

$x - 5y = -10$ Second equation

____ $- 5y = -10$ Substitute for *x*.

____ $=$ ____ Subtract from each side.

$y =$ ____ Divide each side.

Now substitute the value for y in either of the original equations and solve for x.

$x + 4y = 26$ First equation

$x + 4($____$) = 26$ Replace *y* with 4.

$x +$ ____ $= 26$ Simplify.

$x =$ ____ Subtract ____ from each side.

The solution of the system is ____.

Check Your Progress Use Substitution to solve the system of equations.

$x - 3y = 2$

$x + 7y = 12$

EXAMPLE

Solve by Substitution

1 **FURNITURE** **Lancaster Woodworkers Furniture Store builds two types of wooden outdoor chairs. A rocking chair sells for $265 and an Adirondack chair with footstool sells for $320. The books show that last month, the business earned $13,930 for the 48 outdoor chairs sold. How many chairs were sold?**

Read the Item

You are asked to find the number of each type of chair sold.

Solve the Item

Step 1 Define variables and write the system of equations.
Let x represent the number of rocking chairs sold and y represent the number of Adirondack chairs.

$x + y =$ The total number of chairs sold

$265x + 320y =$ The total earned.

Step 2 Solve one of the equations for one of the variables in terms of the other. Since the coefficient of x is , solve the first equation for x in terms of y.

$x + y = 48$ First equation

$x = 48 - y$ Subtract from each side.

Step 3 Substitute $48 - y$ for x in the second equation.

$265x + 320y = 13,930$ Second equation

$265(\quad) + 320y = 13,930$ Substitute for x.

$\quad - \quad + 320y = 13,930$ Distributive Property

$55y = 1210$ Simplify.

$y =$ Divide each side by .

Step 4 Now find the value of x. Substitute the value for y into either equation.

$x + y = 48$	First equation
$x + \boxed{} = 48$	Replace y with $\boxed{}$.
$x = 26$	Subtract $\boxed{}$ from each side.

They sold $\boxed{}$ rocking chairs and $\boxed{}$ Adirondack chairs

Check Your Progress At Amy's Amusement Park, tickets sell for \$24.50 for adults and \$16.50 for children. On one day the amusement park took in \$6405 from selling 330 tickets. How many of each kind of ticket was sold?

$\boxed{}$

EXAMPLE Multiply, Then Use Elimination

③ **Use the elimination method to solve $2x + 3y = 12$ and $5x - 2y = 11$.**

Multiply the first equation by 2 and the second equation by 3. Then add the equations to eliminate one of the variables.

$$\begin{array}{lcrcrcr}
2x + 3y = 12 & \xrightarrow{\text{Multiply by 2.}} & & \boxed{} & + \boxed{} & = & \boxed{} \\
5x - 2y = 11 & \xrightarrow{\text{Multiply by 3.}} & (+) & \boxed{} & - \boxed{} & = & \boxed{} \\
\hline
 & & & 19x & & = & 57 \\
 & & & & x & = & \boxed{}
\end{array}$$

Replace x and solve for y.

$2x + 3y = 12$	First equation
$2(\boxed{}) + 3y = 12$	Replace x with $\boxed{}$.
$\boxed{} + 3y = 12$	Multiply.
$3y = 6$	Subtract 6 from each side.
$y = 2$	Divide each side by 3.

The solution is $\boxed{}$.

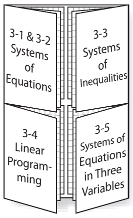
WRITE IT

When solving a system of equations, how do you choose whether to use the substitution method or the elimination method?

FOLDABLES™

ORGANIZE IT

Under the Systems of Equations tab, write how you recognize an inconsistent system of equations. Then write how you recognize a consistent system of equations.



EXAMPLE Inconsistent System

④ **Use the elimination method to solve $-3x + 5y = 12$ and $6x - 10y = -21$.**

Use multiplication to eliminate x.

$$\begin{array}{rcl} -3x + 5y = 12 & \xrightarrow{\text{Multiply by 2.}} & \boxed{} + \boxed{} = \boxed{} \\ 6x - 10y = \boxed{} & & (+)\ \boxed{} - \boxed{} = \boxed{} \\ \hline & & 0 = \boxed{} \end{array}$$

Since there are no values of x and y that will make the equation $\boxed{} = \boxed{}$ true, there are no solutions for the system of equations.

Check Your Progress **Use the elimination method to solve each system of equations.**

a. $x + 3y = 7$
$2x + 5y = 10$

d. $2x + 3y = 11$
$-4x - 6y = 20$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–3 Solving Systems of Inequalities by Graphing

MAIN IDEAS

- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

BUILD YOUR VOCABULARY (page 58)

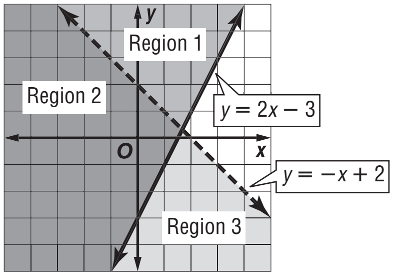
To solve a **system of inequalities**, find the pairs that satisfy of the inequalities in the system.

EXAMPLE Intersecting Regions

① **Solve each system of inequalities by graphing.**

a. $y \geq 2x - 3$
$y < -x + 2$

Graph each inequality.



solution of $y \geq 2x - 3$ ⟶ Regions

solution of $y < -x + 2$ ⟶ Regions

The intersection of these regions is Region , which is the solution of the system of inequalities. Notice that the solution is a region containing an number of ordered pairs.

b. $y \leq -x + 1$
$|x + 1| < 3$

The inequality $|x + 1| < 3$ can be written as $x + 1$ 3 and $x + 1$ -3

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.

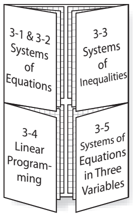
FOLDABLES™

ORGANIZE IT

Under the tab for Systems of Inequalities, solve the following by graphing.

$y \leq 3x - 6$

$y > -4x + 2$



Check Your Progress **Solve each system of inequalities by graphing.**

a. $y \geq 3x - 3$
$y > x + 1$

b. $y \geq -2x - 3$
$|x + 2| < 1$

EXAMPLE **Separate Regions**

1 **Solve the system of inequalities by graphing.**

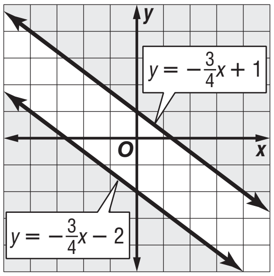
$$\boldsymbol{y \geq -\frac{3}{4}x + 1}$$

$$\boldsymbol{y \leq -\frac{3}{4}x - 2}$$

Graph both inequalities.

The graphs do not overlap, so the solutions have ____ points in common.

The solution set is ____.



REMEMBER IT

You can indicate that a system of inequalities has no solutions in the following ways: empty set, null set, ∅, or {}.

Check Your Progress Solve the system of inequalities $y < \frac{1}{2}x + 2$ and $y > \frac{1}{2}x + 4$ by graphing.

EXAMPLE Write and Use a System of Inequalities

③ **MEDICINE Medical Professionals recommend that patients have a cholesterol level below 200 milligrams per deciliter (mg/dL) of blood and a triglyceride levels below 150 mg/dL. Write and graph a system of inequalities that represents the range of cholesterol level and triglyceride levels for patients.**

Source: American Heart Association.

Let c represent the cholesterol levels in mg/dL. It must be less than mg/dL. Since cholesterol levels cannot be negative, we can write this as $0 \leq c <$.

Let t represent the triglyceride levels in mg/dL. It must be less than mg/dL. Since triglyceride levels also cannot be negative, we can write this as $0 \leq t <$.

Graph both inequalities. Any ordered pair in the intersection of the graphs is a of the system.

Check Your Progress The speed limits while driving on the highway are different for trucks and cars. Cars must drive between 45 and 65 miles per hour, inclusive. Trucks are required to drive between 40 and 55 miles per hour, inclusive. Let c represent the speed range for cars and t represent the speed range for trucks. Write and graph a system of inequalities to represent this situation.

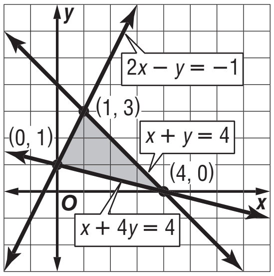
EXAMPLE Find Vertices

④ **Find the coordinates of the vertices of the figure formed by $2x - y \geq -1$, $x + y \leq 4$, and $x + 4y \geq 4$.**

Graph each inequality.

The of the graphs forms a triangular region.

The vertices of the triangle are at , , and .



Check Your Progress Find the coordinates of the vertices of the figure formed by $x + 2y \geq 1$, $x + y \leq 3$, and $-2x + y \leq 3$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–4 Linear Programming

MAIN IDEAS

- Find the maximum and minimum values of a function over a region.
- Solve real-world problems using linear programming.

BUILD YOUR VOCABULARY (pages 57–58)

In a graph of a system of inequalities, the are called the **constraints**.

The intersection of the graphs is called the **feasible region**.

When the graph of a system of constraints is a polygonal region, we say that the region is **bounded**.

The maximum or minimum value of a related function occurs at one of the **vertices** of the feasible region.

When a system of inequalities forms a region that is , the region is said to be **unbounded**.

The process of finding or values of a function for a region defined by inequalities is called **linear programming**.

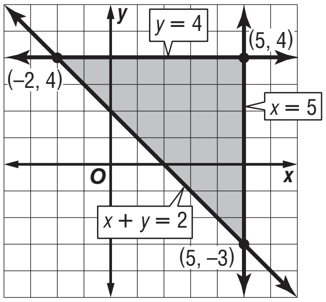
EXAMPLE Bounded Region

① **Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x - 2y$ for this region.**

$x \leq 5$

$y \leq 4$

$x + y \geq 2$



First, find the vertices of the region. Graph the inequalities.

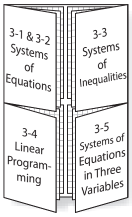
The polygon formed is a triangle with vertices at

, , and .

FOLDABLES™

ORGANIZE IT

Under the Linear Programming tab, sketch a graph of a system of inequalities like the one shown in Example 1. Then label the constraints, feasible region, and vertices of the graph.



Next, use a table to find the maximum and minimum values of $f(x, y)$. Substitute the coordinates of the vertices into the function.

(x, y)	$3x - 2y$	$f(x, y)$	
$(-2, 4)$			← minimum
$(5, -3)$			← maximum
$(5, 4)$			

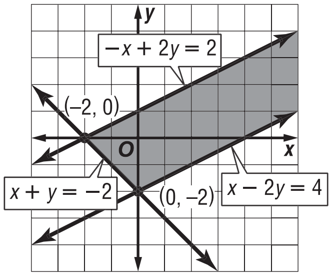
The vertices of the feasible region are , , and . The maximum value is at .

The minimum value is at .

EXAMPLE Unbounded Region

2 **Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 2x + 3y$ for this region.**

$-x + 2y \leq 2$
$x - 2y \leq 4$
$x + y \geq -2$



Graph the system of inequalities. there are only two points of intersection, and .

(x, y)	$2x + 3y$	$f(x, y)$
$(-2, 0)$		
$(0, -2)$		

WRITE IT

Explain how you recognize the unbounded region of a system of inequalities.

The minimum value is ☐ at (0, −2). Although $f(-2, 0)$ is ☐, it is not the maximum value since there are other points that produce greater values. For example, $f(2, 1)$ is ☐ and $f(3, 1)$ is ☐. It appears that because the region is unbounded, $f(x, y)$ has no maximum value.

Check Your Progress **Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.**

a. $x \leq 4$

$y \leq 5$

$x + y \geq 6$

$f(x, y) = 4x - 3y$

b. $x + 3y \leq 6$

$-x - 3y \leq 9$

$2y - x \geq -6$

$f(x, y) = x + 2y$

KEY CONCEPT

Linear Programming Procedure

Step 1 Define the variables.

Step 2 Write a system of inequalities.

Step 3 Graph the system of inequalities.

Step 4 Find the coordinates of the vertices of the feasible region.

Step 5 Write a function to be maximized or minimized.

Step 6 Substitute the coordinates of the vertices into the function.

Step 7 Select the greatest or least result. Answer the problem.

EXAMPLE Linear Programming

③ **LANDSCAPING A landscaping company has crews who mow lawns and prune shrubbery. The company schedules 1 hour for mowing jobs and 3 hours for pruning jobs. Each crew is scheduled for no more than 2 pruning jobs per day. Each crew's schedule is set up for a maximum of 9 hours per day. On the average, the charge for mowing a lawn is $40 and the charge for pruning shrubbery is $120. Find a combination of mowing lawns and pruning shrubs that will maximize the income the company receives per day from one of its crews.**

Step 1 Define the variables.

m = the number of mowing jobs
p = the number of pruning jobs

Step 2 Write a system of inequalities.

Since the number of jobs cannot be negative, m and p must be nonnegative numbers.

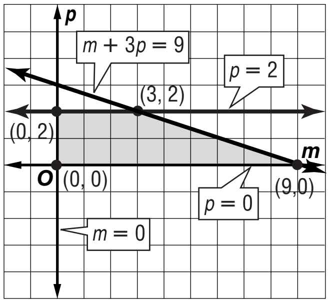
$m \geq \boxed{\quad}$, $p \geq \boxed{\quad}$

Mowing jobs take $\boxed{\quad}$ hour. Pruning jobs take $\boxed{\quad}$ hours. There are $\boxed{\quad}$ hours to do the jobs.

$\boxed{\quad} + \boxed{\quad} \leq 9$

There are no more than 2 pruning jobs a day. $\boxed{\quad} \leq 2$

Step 3 Graph the system of inequalities.



Step 4 Find the coordinates of the vertices of the feasible region.

From the graph, the vertices are at $\boxed{\quad}$, $\boxed{\quad}$, $\boxed{\quad}$, and $\boxed{\quad}$.

Step 5 Write the function to be maximized.

The function that describes the income is $f(m, p) = 40m + 120p$. We want to find the $\boxed{\quad}$ value for this function.

Step 6 Substitute the coordinates of the vertices into the function.

(*m*, *p*)	40*m* + 120*p*	*f*(*m*, *p*)
(0, 2)		
(3, 2)		
(9, 0)		
(0, 0)		

Step 7 Select the ____ amount. Answer the problem.

The maximum values are ____ at ____ and ____ at ____. This means that the company receives the most money with ____ mows and ____ prunings or ____ mows and ____ prunings.

Check Your Progress A landscaping company has crews who rake leaves and mulch. The company schedules 2 hours for mulching jobs and 4 hours for raking jobs. Each crew is scheduled for no more than 2 raking jobs per day. Each crew's schedule is set up for a maximum of 8 hours per day. On the average, the charge for raking a lawn is $50 and the charge for mulching is $30. Find a combination of raking leaves and mulching that will maximize the income the company receives per day from one of its crews.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–5 Solving Systems of Equations in Three Variables

MAIN IDEAS

- Solve systems of linear equations in three variables.
- Solve real-world problems using systems of linear equations in three variables.

KEY CONCEPT

System of Equations in Three Variables

One solution
- planes intersect in one point

Infinite Solutions
- planes intersect in a line
- planes intersect in the same plane

No solution
- planes have no point in common

BUILD YOUR VOCABULARY (page 58)

The solution of a system of equations in three variables, *x*, *y*, and *z* is called an **ordered triple** and is written as (*x*, *y*, *z*).

EXAMPLE One Solution

1 **Solve the system of equations.**

$$\begin{aligned} 5x + 3y + 2z &= 2 \\ 2x + y - z &= 5 \\ x + 4y + 2z &= 16 \end{aligned}$$

Use elimination to make a system of two equations in two variables. First, eliminate z in the first and second equations.

$$\begin{array}{r} 5x + 3y + 2z = 2 \\ 2x + y - z = 5 \end{array} \xrightarrow{(\times 2)} \begin{array}{r} 5x + 3y + 2z = 2 \\ (+)\ 4x + 2y - 2z = 10 \\ \hline \boxed{} = \boxed{} \end{array}$$

Eliminate z in the first and third equations.

$$\begin{array}{r} 5x + 3y + 2z = 2 \\ (-)x + 4y + 2z = 16 \\ \hline \boxed{} = \boxed{} \end{array}$$

Solve the system of two equations. Eliminate y.

$$\begin{array}{r} 9x + 5y = 12 \\ 4x - y = -14 \end{array} \xrightarrow{(\times 5)} \begin{array}{r} 9x + 5y = 12 \\ (+)\ \boxed{} = \boxed{} \\ \hline \boxed{} = \boxed{} \\ x = \boxed{} \end{array}$$

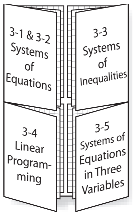
Substitute $\boxed{}$ for x in one of the two equations with two variables and solve for y.

$4x - y = -14$	Equation with two variables
$4\left(\boxed{}\right) - y = -14$	Replace *x*.
$\boxed{} - y = -14$	Multiply
$y = \boxed{}$	Simplify.

FOLDABLES™

ORGANIZE IT

Under the Systems of Equations in Three Variables tab, sketch a graph of a system with (a) one solution, (b) infinite soultions, (c) no solutions.



Substitute ____ for x and y in one of the original equations with three variables.

$2x + y - z = 5$ Equation with three variables

$2(\ \_\_\_\_\ ) + 6 - z = 5$ Replace x and y.

$\_\_\_\_ + 6 - z = 5$ Multiply.

$z = \_\_\_\_$ Simplify.

The solution is ____. You can check this solution in the other two original equations.

EXAMPLE Infinite Solutions

1 **Solve the system of equations.**

$\mathbf{2x + y - 3z = 5}$
$\mathbf{x + 2y - 4z = 7}$
$\mathbf{6x + 3y - 9z = 15}$

Eliminate y in the first and third equations.

$2x + y - 3z = 5 \quad \xrightarrow{(\times 3)} \quad 6x + 3y - 9z = 15$
$6x + 3y - 9z = 15 \qquad\qquad (-)6x + 3y - 9z = 15$

$\_\_\_\_ = \_\_\_\_$

The equation $0 = 0$ is always ____. This indicates that the first and third equations represent the same plane. Check to see if this plane intersects the second plane

$x + 2y - 4z = 7 \quad \xrightarrow{(\times 6)} \quad 6x + 12y - 24z = 42$
$6x + 3y - 9z = 15 \qquad\qquad (-)6x + 3y - 9z = 15$

$\_\_\_\_ = \_\_\_\_$

$\_\_\_\_ = \_\_\_\_$

The planes intersect in a ____. So, there are an ____ of solutions.

EXAMPLE No Solution

③ **Solve the system of equations.**

$$\begin{aligned} 3x - y - 2z &= 4 \\ 6x + 4y + 8z &= 11 \\ 9x + 6y + 12z &= -3 \end{aligned}$$

Eliminate x in the second two equations.

$$\begin{aligned} 6x + 4y + 8z = 11 &\xrightarrow{(\times 3)} \boxed{} + 12y + \boxed{} = \boxed{} \\ 9x + 6y + 12z = -3 &\xrightarrow{(\times 2)} (-)18x + \boxed{} + 24z = \boxed{} \\ &\qquad \boxed{} = \boxed{} \end{aligned}$$

The equation $\boxed{} = \boxed{}$ is never true. So, there is no solution of this system.

Check Your Progress **Solve each system of equations.**

a. $2x + 3y - 3z = 16$
$x + y + z = -3$
$x - 2y - z = -1$

b. $x + y - 2z = 3$
$-3x - 3y + 6z = -9$
$2x + y - z = 6$

c. $3x + y - z = 5$
$-15x - 5y + 5z = 11$
$x + y + z = 2$

EXAMPLE Write and Solve a System of Equations

4 **SPORTS There are 49,000 seats in a sports stadium. Tickets for the seats in the upper level sell for $25, the ones in the middle level cost $30, and the ones in the bottom level are $35 each. The number of seats in the middle and bottom levels together equals the number of seats in the upper level. When all of the seats are sold for an event, the total revenue is $1,419,500. How many seats are there in each level?**

Explore Read the problem and define the ______.

u = number of seats in the upper level
m = number of seats in the middle level
b = number of seats in the bottom level

Plan There are ______ seats.

$u + m + b = 49{,}000$

When all the seats are sold, the revenue is ______.

Seats cost ______, ______, and ______.

$25u + 30m + 35b = 1{,}419{,}500$

The number of seats in the middle and bottom levels together equal the number of seats in the upper level.

$m + b = \boxed{}$

Solve Substitute $m + b$ for u in each of the first two equations.

$u + m + b = 49{,}000$

$\left(\boxed{}\right) + m + b = 49{,}000$ Replace u with ______.

$\boxed{} + \boxed{} = 49{,}000$ Simplify.

$m + b = 24{,}500$ Divide by 2.

$25u + 30m + 35b = 1{,}419{,}500$

$25\left(\boxed{}\right) + 30m + 35b = 1{,}419{,}500$ Replace u with $m + b$.

$\boxed{} - 30m - 35b = 1{,}419{,}500$ Distributive Property

$\boxed{} + \boxed{} = 1{,}419{,}500$ Simplify.

Now, solve the system of two equations in two variables

$$\begin{array}{rcl} m + b = 24{,}500 & \xrightarrow{(\times\, 55)} & \boxed{\quad} + \boxed{\quad} = \boxed{\qquad} \\ 55m + 60b = 1{,}419{,}500 & & \underline{(-)55m + 60b = 1{,}419{,}500} \\ & & \boxed{\quad} = \boxed{\qquad} \\ & & b = 14{,}400 \end{array}$$

Substitute 14,400 for b in the one of the equations with two variables and solve for m.

$m + b = 24{,}500$ Equation with two variables

$m + \boxed{\qquad} = 24{,}500$ $b = 14{,}400$

$m = 10{,}100$ Subtract $\boxed{\qquad}$ from each side.

Substitute 14,400 for b and 10,100 for m in one of the original equation with three variables.

$m + b = u$ Equation with three variables

$\boxed{\qquad} + \boxed{\qquad} = u$ $m = 10{,}100$, $b = 14{,}400$

$\boxed{\qquad} = u$ Add.

There are $\boxed{\qquad}$ upper level, $\boxed{\qquad}$ middle level, and $\boxed{\qquad}$ bottom level seats.

Check Check to see if all the criteria are met.

There are $\boxed{\qquad}$ seats in the stadium.

$24{,}500 + 10{,}100 + 14{,}400 = 49{,}000$ ✓

The number of seats in the middle and bottom levels equals the number of seats in the upper level.

$10{,}100 + 14{,}400 = 24{,}500$ ✓

When all of the seats are sold, the revenue is

$\boxed{\qquad}$.

$24{,}500(\$25) + 10{,}100(\$30) + 14{,}400(\$35)$
$= 1{,}419{,}500$ ✓

Check Your Progress The school store sells pens, pencils, and paper. The pens are \$1.25 each, the pencils are \$0.50 each, and the paper is \$2 per pack. Yesterday the store sold 25 items and earned \$32. The number of pens sold equaled the number of pencils sold plus the number of packs of paper sold minus 5. How many of each item did the store sell?

$\boxed{\qquad\qquad\qquad\qquad}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 3

BRINGING IT ALL TOGETHER

STUDY GUIDE

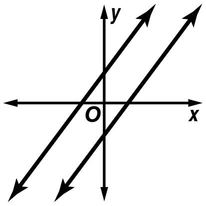
FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 3 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 3, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 57–58)* to help you solve the puzzle.

3-1

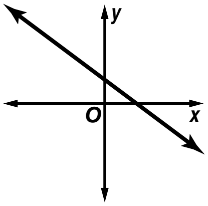
Solving Systems of Equations by Graphing

Under each system graphed below, write all of the following words that apply: *consistent*, *inconsistent*, *dependent*, and *independent*.

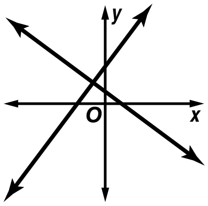
1.



2.



3.



4. Solve the system $x + y = 3$ and $3x - y = 1$ by graphing.

3-2

Solving Systems of Equations Algebraically

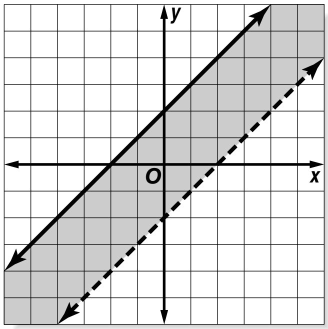
5. Solve $x = 4y + 3$ and $x - 2y = 9$ by using substitution.

6. Solve $-2x + 5y = 7$ and $2x + 4y = 11$ by using elimination.

3-3

Solving Systems of Inequalities by Graphing

7. Which system of inequalities matches the graph shown at the right?



a. $x - y \leq -2$
$x - y > 2$

b. $x - y \geq -2$
$x - y < 2$

c. $x + y \leq -2$
$x + y > 2$

d. $x - y > -2$
$x - y \leq 2$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

8. $x + y \geq 8$
$y \geq 5$
$x \geq 0$

9. $x + y \geq 6$
$x \leq 8$
$y \leq 5$

3-4

Linear Programming

10. A polygonal feasible region has vertices at (4, −1), (3, 0), (1, −6), and (−2, 2). Find the maximum and minimum of the function $f(x, y) = -2x + y$ over this region.

3-5

Solving Systems of Equations in Three Variables

Solve each system of equations.

11. $5x + y + 4z = 9$
$-5x + 3y + z = -15$
$15x + 5y + 7z = 9$

12. $2x + y - 4z = -3$
$-6x - 3y + 12z = 7$
$x - 9y + 4z = 1$

13. The sum of three numbers is 22. The sum of the first and second numbers is 19, and the first number is 5 times the third number. Find the numbers.

CHAPTER 3

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 3.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 3 Practice Test on page 157 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 3 Study Guide and Review on pages 153–156 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 157 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 3 Foldable.
- Then complete the Chapter 3 Study Guide and Review on pages 153–156 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 157 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

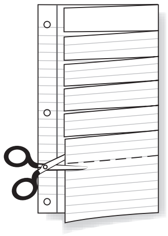
CHAPTER 4

Matrices

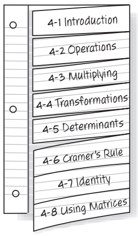
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of notebook paper.

STEP 1 **Fold** lengthwise to the holes. Cut eight tabs in the top sheet.



STEP 2 **Label** each tab with a lesson number and title.



NOTE-TAKING TIP: When you take notes, write descriptive paragraphs about your learning experiences.

CHAPTER 4

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
Cramer's Rule [KRAY-muhrs]			
determinant			
dilation [dy-LAY-shuhn]			
dimension			
element			
equal matrices			
expansion by minors			
identity matrix			
image			
inverse			
isometry eye-SAH-muh-tree			

(continued on the next page)

Chapter 4 BUILDING YOUR VOCABULARY

Vocabulary Term	Found on Page	Definition	Description or Example
matrix MAY-trihks			
matrix equation			
minor			
preimage			
reflection			
rotation			
scalar multiplication SKAY-luhr			
square matrix			
transformation			
translation			
vertex matrix			
zero matrix			

4–1 Introduction to Matrices

MAIN IDEAS

- Organize data in matrices.
- Solve equations involving matrices.

BUILD YOUR VOCABULARY (pages 85–86)

A **matrix** is a rectangular array of variables or constants in and vertical columns, usually enclosed in brackets.

Each in the matrix is called an **element.**

A matrix that has the same number of rows and is called a **square matrix.**

Another special type of matrix is the **zero matrix,** in which every element is .

EXAMPLE Organize Data into a Matrix

REMEMBER IT

An element of a matrix can be represented by the notation a_{ij}. This refers to the element in row i, column j.

① **COLLEGE Kaitlin wants to attend one of three Iowa universities next year. She has gathered information about tuition (T), room and board (R/B), and enrollment (E) for the universities. Use a matrix to organize the information. Which university's total cost is lowest?**

Iowa State University:

T - \$5426	R/B - \$5958	E - 26,380

University of Iowa:

T - \$5612	R/B - \$6560	E - 28,442

University of Northern Iowa:

T - \$5387	R/B - \$5261	E - 12,927

Organize the data into labeled columns and rows.

$$\begin{array}{c|ccc} & T & R/B & E \\ \hline ISU & & & \\ UI & & & \\ UNI & & & \end{array}$$

The University of has the lowest total cost.

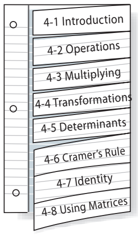
Check Your Progress Justin is going out for lunch. The information he has gathered from the two fast-food restaurants is listed below. Use a matrix to organize the information. When is each restaurant's total cost less expensive?

Burger Complex		Lunch Express	
Hamburger Meal	\$3.39	Hamburger Meal	\$3.49
Cheeseburger Meal	\$3.59	Cheeseburger Meal	\$3.79
Chicken Sandwich Meal	\$4.99	Chicken Sandwich Meal	\$4.89

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-1, tell how to find the dimensions of a matrix.



EXAMPLE Dimensions of a Matrix

① **State the dimensions of matrix G if**

$$G = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 1 & 5 & -3 & -1 \end{bmatrix}.$$

$$G = \underbrace{\begin{bmatrix} 2 & -1 & 0 & 3 \\ 1 & 5 & -3 & -1 \end{bmatrix}}_{\boxed{}\ \text{columns}} \Big\} \quad \boxed{}\ \text{rows}$$

Since matrix G has $\boxed{}$ rows and $\boxed{}$ columns, the dimensions of matrix G are $\boxed{}$.

Check Your Progress State the dimensions of matrix G

if $G = \begin{bmatrix} 2 & 3 \\ 0 & 4 \\ -1 & 4 \end{bmatrix}$.

EXAMPLE Solve an Equation Involving Matrices

③ **Solve $\begin{bmatrix} y \\ 3 \end{bmatrix} = \begin{bmatrix} 3x - 2 \\ 2y + x \end{bmatrix}$ for x and y.**

Since the matrices are equal, the corresponding elements are equal. When you write the sentences to solve this equation, two linear equations are formed.

$y = 3x - 2$

$3 = 2y + x$

This system can be solved using substitution.

$3 = 2y + x$	Second equation
$\boxed{} = 2\left(\boxed{}\right) + x$	Substitute $\boxed{}$ for y.
$\boxed{} = \boxed{} + x$	Distributive Property
$\boxed{} = \boxed{}$	Add 4 to each side.
$\boxed{} = x$	Divide each side by 7.

To find the value for y, substitute 1 for x in either equation.

$y = 3x - 2$	First equation
$y = 3\left(\boxed{}\right) - 2$	Substitute $\boxed{}$ for x.
$y = \boxed{}$	Simplify.

The solution is $\boxed{}$.

Check Your Progress Solve $\begin{bmatrix} y \\ 2x \end{bmatrix} = \begin{bmatrix} 3x - 1 \\ y - 2 \end{bmatrix}$ for x and y.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–2

Operations with Matrices

MAIN IDEAS

- Add and subtract matrices.
- Multiply by a matrix scalar.

KEY CONCEPTS

Addition of Matrices
If A and B are two $m \times n$ matrices, then $A + B$ is an $m \times n$ matrix in which each element is the sum of the corresponding elements of A and B.

Subtraction of Matrices
If A and B are two $m \times n$ matrices, then $A - B$ is an $m \times n$ matrix in which each element is the difference of the corresponding elements of A and B.

EXAMPLE Add Matrices

① **a. Find $A + B$ if $A = \begin{bmatrix} 6 & 4 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix}$.**

$$A + B = \begin{bmatrix} 6 & 4 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix}$$

Definition of matrix addition

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Add corresponding elements.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Simplify.

b. Find $A + B$ if $A = \begin{bmatrix} 4 & -2 & 0 \\ 1 & 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 7 \\ -9 & 3 \end{bmatrix}$.

Since their dimensions are $\boxed{}$, these matrices $\boxed{}$ be added.

EXAMPLE Subtract Matrices

② **Find $A - B$ if $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$.**

$$A - B = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$$

Definition of matrix subtraction

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Subtract corresponding elements.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Simplify.

Check Your Progress

a. Find $A + B$ if $A = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 5 \\ 3 & 0 \end{bmatrix}$.

b. Find $A - B$ if $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$.

EXAMPLE Solve Problems

③ **SCHOOL ATHLETES** **The table below shows the total number of student athletes and the number of female athletes in three high schools. Use matrices to find the number of male athletes in each school.**

School	Total Number of Athletes	Female Athletes
Jefferson	863	281
South	472	186
Ferguson	1023	346

The data in the table can be organized in matrices. Find the difference of the matrix that represents total number of and the matrix that represents the number of athletes.

$$\begin{array}{ccccc} \text{Total} & & \text{Female} & & \text{Male} \\ \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} & - & \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} & = & \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \end{array}$$

Subtract corresponding elements

$$= \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

There are male athletes at Jefferson, male athletes at South, and male athletes at Ferguson.

Check Your Progress The table shows the percent of students at Clark High School who passed the 9th and 10th grade proficiency tests in 2005 and 2006. Use matrices to find how the percent of passing students changed from 2005 to 2006.

Proficiency Tests Passing Percentages

Year	2001		2002	
Grade	9	10	9	10
Math	86.2	87.3	88.4	89.6
Reading	83.5	90.1	81.9	91.2
Science	79.6	89.7	85.0	89.9
Citizenship	86.1	87.4	86.4	85.7

BUILD YOUR VOCABULARY (page 85–86)

You can multiply any matrix by a constant called a

______. This operation is called **scalar multiplication.**

EXAMPLE Multiply a Matrix by a Scalar

④ **If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$, find $2A$.**

$2A = 2\begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$ Substitution

$= \begin{bmatrix} \boxed{} & 1(2) \\ -1(2) & \boxed{} \\ \boxed{} & 5(2) \end{bmatrix}$ Multiply each element by 2.

$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$ Simplify.

KEY CONCEPT

Scalar Multiplication The product of a scalar *k* and an *m* × *n* matrix is the matrix in which each element equals *k* times the corresponding elements of the original matrix.

FOLDABLES™ Under the tab for Lesson 4-2, write your own example that involves scalar multiplication. Then perform the multiplication.

EXAMPLE **Combination of Matrix Operations**

⑤ **If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$, find $4A - 3B$.**

Perform the scalar multiplication first. Then subtract the matrices.

$4A - 3B$

$= 4\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 3\begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$	Substitution
$= \begin{bmatrix} 4(2) & 4(3) \\ 4(-1) & 4(0) \end{bmatrix} - \begin{bmatrix} 3(-2) & 3(1) \\ 3(0) & 3(-1) \end{bmatrix}$	Multiply each element in the first matrix by 4 and multipy each element in the second matrix by 3.
$= \begin{bmatrix} \quad \end{bmatrix} - \begin{bmatrix} \quad \end{bmatrix}$	Simplify.
$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$	Subtract corresponding elements.
$= \begin{bmatrix} 14 & 9 \\ -4 & 3 \end{bmatrix}$	Simplify.

Check Your Progress

a. If $A = \begin{bmatrix} 2 & 3 & 0 \\ -1 & -5 & 6 \end{bmatrix}$ find $4A$.

b. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$, find $5A - 2B$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–3 Multiplying Matrices

MAIN IDEAS

- Multiply matrices.
- Use the properties of matrix multiplication.

KEY CONCEPT

Multiplying Matrices
The element a_{ij} of *AB* is the sum of the products of the corresponding elements in row *i* of *A* and column *j* of *B*.

FOLDABLES Under the tab for Lesson 4-3, write an example of multiplying square matrices.

EXAMPLE Dimensions of Matrix Products

① **a. Determine if the product of $A_{3 \times 4}$ and $B_{4 \times 2}$ is defined. If so, state the dimensions of the product.**

$$\begin{array}{ccccc} A & \cdot & B & = & AB \\ 3 \times 4 & & 4 \times 2 & & \boxed{} \end{array}$$

The inner dimensions are $\boxed{}$ so the matrix product is $\boxed{}$. The dimensions of the product are $\boxed{}$.

b. Determine whether the product of $A_{3 \times 2}$ and $B_{4 \times 3}$ is defined. If so, state the dimensions of the product.

$$\begin{array}{ccc} A & \cdot & B \\ 3 \times 2 & & 4 \times 3 \end{array}$$

The inner dimensions are $\boxed{}$, so the matrix product is $\boxed{}$.

Check Your Progress Determine if the product of $A_{2 \times 3}$ and $B_{2 \times 3}$ is defined. If so, state the dimensions of the product.

$\boxed{}$

EXAMPLE Multiply Square Matrices

② **Find *RS* if $R = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $S = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$.**

$$RS = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{} + \boxed{} & 3(1) + 2(-1) \\ -1(-2) + 0(1) & \boxed{} + \boxed{} \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Check Your Progress Find RS if $R = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$ and $S = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}$.

EXAMPLE Multiply Matrices with Different Dimensions

WRITE IT

Is multiplication of matrices commutative? Explain.

③ **CHESS Three teams competed in the final round of the Chess Club's championships. For each win, a team was awarded 3 points and for each draw a team received 1 point. Which team won the tournament?**

Team	Wins	Draws
Blue	5	4
Red	6	3
Green	4	5

$$RP = \begin{bmatrix} 5 & 4 \\ 6 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$ Write an equation.

$$= \begin{bmatrix} \boxed{} + \boxed{} \\ \boxed{} + \boxed{} \\ \boxed{} + \boxed{} \end{bmatrix}$$ Multiply columns by rows.

$$= \begin{bmatrix} \boxed{} \end{bmatrix}$$ Simplify.

The labels for the product matrix are shown. The red team won the championship with a total of $\boxed{}$ points.

$$\begin{array}{l} \text{Total Points} \\ \text{Blue} \\ \text{Red} \\ \text{Green} \end{array} \begin{bmatrix} \boxed{} \end{bmatrix}$$

Check Your Progress Three players made the points listed below. They scored 1 point for the free-throws, 2 points for the 2-point shots, and 3 points for the 3-point shots. How many points did each player score and who scored the most points?

Player	Free-throws	2-point	3-point
Warton	2	3	2
Bryant	5	1	0
Chris	2	4	5

EXAMPLE Commutative Property

④ **Find each product if $K = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ and $L = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}$.**

a. $KL = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}$ Substitution

$= \begin{bmatrix} -3 + 8 + 0 & \boxed{} \\ \boxed{} & 2 - 6 + 0 \end{bmatrix}$ Multiply.

$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$ Simplify.

b. $LK = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ Substitution

$= \begin{bmatrix} -3 + 2 & 2 + 4 & \boxed{} \\ \boxed{} & 8 - 6 & 8 - 0 \\ 0 + 1 & \boxed{} & 0 - 0 \end{bmatrix}$ Multiply.

$= \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}$ Simplify.

Check Your Progress Find each product if $A = \begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}$.

a. AB

b. BA

EXAMPLE Distributive Property

5 **Find each product if $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix}$.**

a. $A(B + C) = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix} \right)$

$= \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 2 & -2 \end{bmatrix}$ Add.

$= \begin{bmatrix} \boxed{} & -1 - 4 \\ \boxed{} & 0 - 2 \end{bmatrix}$ or $\begin{bmatrix} 6 & \boxed{} \\ 2 & \boxed{} \end{bmatrix}$ Multiply.

b. $AB + AC = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$ Multiply.

$= \begin{bmatrix} 5 + 1 & \boxed{} \\ 3 - 1 & \boxed{} \end{bmatrix}$ or $\begin{bmatrix} 6 & \boxed{} \\ 2 & \boxed{} \end{bmatrix}$ Add.

Check Your Progress Find $AB + AC$ if $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -3 & 3 \end{bmatrix}$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–4 Transformations with Matrices

MAIN IDEAS

- Use matrices to determine the coordinates of a translated or dilated figure.
- Use matrix multiplication to find the coordinates of a reflected or rotated figure.

BUILD YOUR VOCABULARY (pages 85–86)

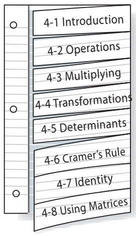
Transformations are functions that map points of a **preimage** onto its **image**.

A **translation** occurs when a figure is moved from one location to another without changing its , , or orientation.

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-4, write each new Vocabulary Builder word. Then give an example of each word.



EXAMPLE Translate a Figure

① **Find the coordinates of the vertices of the image of quadrilateral *ABCD* with *A*(−5, −1), *B*(−2, −1), *C*(−1, −4), and *D*(−3, −5) if it is moved 3 units to the right and 4 units up.**

Write the vertex matrix for quadrilateral *ABCD*.

$$\begin{bmatrix} & & -1 & \\ -1 & -1 & & -5 \end{bmatrix}$$

To translate the quadrilateral 3 units to the right, add 3 to each x-coordinate. To translate the figure 4 units up, add 4 to each y-coordinate. This can be done by adding the translation matrix to the vertex matrix.

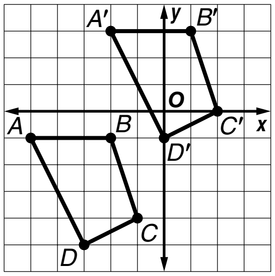
$$\begin{array}{ccc} \text{Vertex Matrix of } ABCD & & \text{Translation Matrix} \\ \begin{bmatrix} -5 & -2 & -1 & -3 \\ -1 & -1 & -4 & -5 \end{bmatrix} & + & \begin{bmatrix} \qquad\qquad\qquad \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} \qquad\qquad\qquad \end{bmatrix}$$

The coordinates of $A'B'C'D'$ are

A' , B' ,

C' , D' .



By graphing the preimage and the image, you find that the coordinates of $A'B'C'D'$ are correct.

Check Your Progress Find the coordinates of the vertices of the image of quadrilateral *HIJK* with $H(2, 3)$, $I(3, -1)$, $J(-1, -3)$, and $K(-2, 5)$ if it is moved 2 units to the left and 2 units up.

BUILD YOUR VOCABULARY (pages 85–86)

A **reflection** occurs when every point of a figure is mapped to a corresponding image across using a *reflection matrix*.

A **rotation** occurs when a figure is moved around a , usually the .

EXAMPLE Find a Translation Matrix

1 **TEST EXAMPLE** **Rectangle $E'F'G'H'$ is the result of a translation of rectangle *EFGH*. A table of the vertices of each rectangle is shown. Find the coordinates of G'.**

Rectangle *EFGH*	Rectangle *E′F′G′H′*
$E(-2, 2)$	$E'(-5, 0)$
$F(4, 2)$	$F'(1, 0)$
$G(4, -2)$	$G'(\quad,\quad)$
$H(-2, -2)$	$H'(-5, -4)$

A (1, 0)

B (1, −4)

C (7, 0)

D (7, −4)

Read the Item

You are given the coordinates of the preimage and image of points *E*, *F*, and *H*. Use this information to find the translation matrix. Then you can use the translation matrix to find the coordinates of *G*.

Solve the Item

Step 1 Write a matrix equation. Let (c, d) represent the coordinates of G.

$$\begin{bmatrix} -2 & 4 & 4 & -2 \\ 2 & 2 & -2 & -2 \end{bmatrix} + \begin{bmatrix} x & x & x & x \\ y & y & y & y \end{bmatrix} = \begin{bmatrix} -5 & 1 & c & -5 \\ 0 & 0 & d & -4 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{} \end{bmatrix} = \begin{bmatrix} -5 & 1 & c & -5 \\ 0 & 0 & d & -4 \end{bmatrix}$$

Step 2 The matrices are equal, so corresponding elements are equal.

$-2 + x = -5$ Solve for *x*. $2 + y = 0$ Solve for *y*.

$x = \boxed{}$ $y = \boxed{}$

Step 3 Use the values for x and y to find the values for $G'(c, d)$.

$4 + (-3) = c$ $-2 + (-2) = d$

$\boxed{} = c$ $\boxed{} = d$

So, the coordinates for G are $\boxed{}$, and the answer is $\boxed{}$.

EXAMPLE Dilation

③ **Dilate $\triangle XYZ$ with $X(1, 2)$, $Y(3, -1)$, and $Z(-1, -2)$ so that its perimeter is twice the original perimeter. Find the coordinates of the vertices of $\triangle X'Y'Z'$.**

If the perimeter of a figure is twice the original perimeter, then the lengths of the sides of the figure will be $\boxed{}$ the measure of the original lengths. Multiply the vertex matrix by the scale factor of $\boxed{}$.

$$2\begin{bmatrix} \boxed{} \end{bmatrix} = \begin{bmatrix} \boxed{} \end{bmatrix}$$

The coordinates of the vertices of $\triangle XYZ$ are $X'\boxed{}$, $Y'\boxed{}$, and $Z'\boxed{}$.

Check Your Progress

a. Rectangle $A'B'C'D'$ is the translation of rectangle $ABCD$ with vertices $A(-4, 5)$, $B(-1, 5)$, $C(-1, 0)$, and $D(-4, 0)$. What are the coordinates of A'?

A $(-13, 10)$ **B** $(5, 10)$ **C** $(5, 0)$ **D** $(-13, 0)$

b. $\triangle ABC$ has vertices $A(2, 1)$, $B(-3, -2)$, and $C(1, 4)$. If $\triangle ABC$ is dilated so its perimeter is 4 times the original perimeter, what are the coordinates of the vertices of $\triangle A'B'C'$?

EXAMPLE Reflection

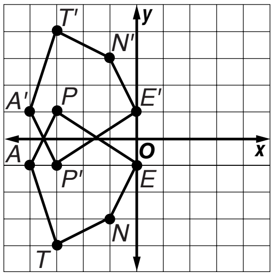
④ **Find the coordinates of the vertices of the image of pentagon $PENTA$ with $P(-3, 1)$, $E(0, -1)$, $N(-1, -3)$, $T(-3, -4)$, and $A(-4, -1)$ after a reflection across the x-axis.**

Write the ordered pairs as a vertex matrix. Then the vertex matrix by the matrix for the x-axis.

$$\begin{bmatrix} \boxed{} \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 & -1 & -3 & -4 \\ 1 & -1 & -3 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & \boxed{} & -1 & \boxed{} & \boxed{} \\ \boxed{} & 1 & \boxed{} & 4 & 1 \end{bmatrix}$$

The coordinates of the vertices of $P'E'N'T'A'$ are $P'(-3, -1)$, $E'(0, 1)$, $N'(-1, 3)$, $T'(-3, 4)$, and $A'(-4, 1)$. The graph of the preimage and image shows that the coordinates of $P'E'N'T'A'$ are correct.



Check Your Progress Find the coordinates of the vertices of the image of pentagon $PENTA$ with $P(-5, 0)$, $E(-3, 3)$, $N(1, 2)$, $T(1, -1)$, and $A(-4, -2)$ after a reflection across the line $y = x$.

KEY CONCEPT

Reflection Matrices

x-axis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

y-axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

line $y = x$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

KEY CONCEPT

Reflection Matrices

For a counterclockwise rotation about the origin of:

90° $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

180° $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

270° $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

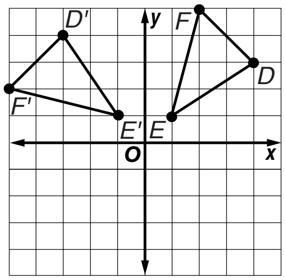
EXAMPLE Rotation

⑤ **Find the coordinates of the vertices of the image of $\triangle DEF$ with $D(4, 3)$, $E(1, 1)$, and $F(2, 5)$ after it is rotated 90° counterclockwise about the origin.**

Write the ordered pairs in a vertex matrix. Then multiply the vertex matrix by the rotation matrix.

$$\begin{bmatrix} \boxed{} \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} \boxed{} & \boxed{} & -5 \\ 4 & 1 & \boxed{} \end{bmatrix}$$

The coordinates of the vertices of triangle $D'E'F'$ are D' $\boxed{}$, E' $\boxed{}$, F' $\boxed{}$.



The graph of the preimage and image show that the coordinates of $D'E'F'$ are correct.

Check Your Progress Find the coordinates of the vertices of the image of $\triangle TRI$ with $T(-1, 2)$, $R(-3, 0)$ and $I(-2, -2)$ after it is rotated 180° counterclockwise about the origin.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–5 Determinants

MAIN IDEAS

- Evaluate the determinant of a 2 × 2 matrix.
- Evaluate the determinant of a 3 × 3 matrix.

KEY CONCEPT

Second-Order Determinant The value of a second-order determinant is found by calculating the difference of the products of the two diagonals.

EXAMPLE Second-Order Determinant

① **Find the value of the determinant $\begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix}$.**

$$\begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix} = \boxed{} - \boxed{}$$ Definition of determinant

$$= \boxed{} + \boxed{}$$ Multiply.

$$= \boxed{}$$ Simplify.

EXAMPLE Expansion by Minors

② **Evaluate $\begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 4 & -2 & -3 \end{vmatrix}$ using expansion by minors.**

Decide which row of elements to use for the expansion. For this example, let's use the first row.

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 4 & -2 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} -3 & \boxed{} \\ \boxed{} & -3 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ 4 & -3 \end{vmatrix} + (-1)\begin{vmatrix} \boxed{} & -1 \\ 4 & \boxed{} \end{vmatrix}$$

$$= 1\left(3 - \left(\boxed{}\right)\right) - 0\left(\boxed{} - 12\right) - 1\left(-4 - \left(\boxed{}\right)\right)$$

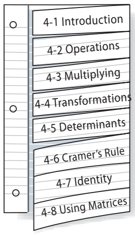
$$= 1\left(\boxed{}\right) - \boxed{}\Big) 1\left(\boxed{}\right)$$

$$= \boxed{}$$

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-5, write your own 3 × 3 matrix. Then evaluate your matrix. Include the steps shown in Example 3.



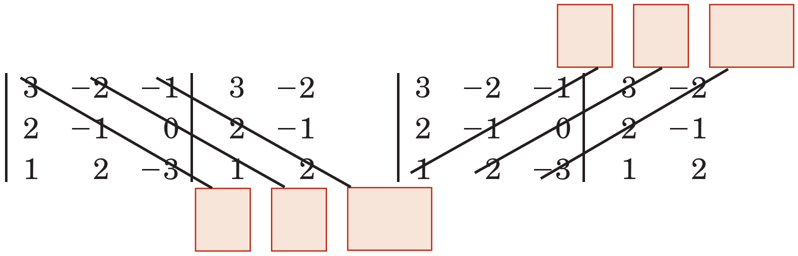
EXAMPLE Use Diagonals

③ **Evaluate** $\begin{vmatrix} 3 & -2 & -1 \\ 2 & -1 & 0 \\ 1 & 2 & -3 \end{vmatrix}$ **using diagonals.**

Step 1 Rewrite the first 2 columns to the right of the determinant.

$$\begin{vmatrix} 3 & -2 & -1 \\ 2 & -1 & 0 \\ 1 & 2 & -3 \end{vmatrix}$$

Step 2 Find the product of the elements of the diagonals.



Step 3 Add the bottom products and subtract the top products.

$$=$$

The value of the determinant is .

Check Your Progress **Evaluate each determinant.**

a. $\begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix}$

b. $\begin{bmatrix} -3 & 2 & 0 \\ 1 & 4 & 1 \\ 2 & 3 & 6 \end{bmatrix}$

c. $\begin{vmatrix} 2 & -3 & -1 \\ 5 & 0 & -2 \\ 1 & 2 & 5 \end{vmatrix}$

EXAMPLE

4 **SURVEYING** **A survey crew located three points on a map that formed the vertices of a triangular area. A coordinate grid in which one unit equals 10 inches is placed over the map so that the vertices are located at (0, −1), (−2, −6), and (3, −2). Use a determinant to find the area of the triangle.**

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Area Formula

$$= \frac{1}{2}\begin{vmatrix} 0 & -1 & 1 \\ -2 & -6 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

$(a, b) = \square$,

$(c, d) = \square$,

$(e, f) = \square$

$$= \frac{1}{2}\left[0\begin{vmatrix} -6 & \square \\ \square & 1 \end{vmatrix} - (-1)\begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} \square & -6 \\ 3 & \square \end{vmatrix}\right]$$

Expansion by minors

$$= \frac{1}{2}\left[0\left(-6 - \left(\square\right)\right) - (-1)\left(-2 - \square\right) + 1\left(4 - \left(\square\right)\right)\right]$$

Evaluate 2 × 2 determinants.

$$= \frac{1}{2}\left(\square - \square + \square\right)$$

Multiply.

$$= \frac{1}{2}\left(\square\right) \text{ or } \square$$

Simplify.

Remember that 1 unit equals 10 inches, so 1 square unit = 10 × 10 or 100 square inches. Thus, the area is $\square$ × 100 or $\square$ square inches.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–6 Cramer's Rule

MAIN IDEAS

- Solve systems of two linear equations by using Cramer's Rule.
- Solve systems of three linear equations by using Cramer's Rule.

KEY CONCEPT

Cramer's Rule for Two Variables The solution of the system of linear equations $ax + by = e$ and $cx + dy = f$ is (x, y), where

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}},\ y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}},$$

and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.

FOLDABLES™ Under the tab for Lesson 4-6, write this rule.

BUILD YOUR VOCABULARY (page 85)

Cramer's Rule uses determinants to solve systems of equations.

EXAMPLE System of Two Equations

① **Use Cramer's Rule to solve the system of equations $5x + 4y = 28$ and $3x - 2y = 8$.**

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad \text{Cramer's Rule} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} \boxed{} & \boxed{} \\ 8 & \boxed{} \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 3 & \boxed{} \end{vmatrix}} \qquad a = 5,\ b = 4,\ c = 3,\ d = -2,\ e = 28, \text{ and } f = 8 \qquad = \frac{\begin{vmatrix} \boxed{} & 28 \\ 3 & \boxed{} \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 3 & \boxed{} \end{vmatrix}}$$

$$= \frac{\boxed{} - \boxed{}}{5(-2) - 3(4)} \qquad \text{Evaluate.} \qquad = \frac{5(8) - 3(28)}{5(-2) - 3(4)}$$

$$= \boxed{} \text{ or } \boxed{} \qquad \text{Simplify.} \qquad = \boxed{} \text{ or } \boxed{}$$

The solution is $\boxed{}$.

Check Your Progress Use Cramer's Rule to solve the system of equations $3x + 2y = 1$ and $2x - 5y = -12$.

$\boxed{}$

KEY CONCEPT

The solution of the system whose equations are

$ax + by + cz = j$
$dx + ey + fz = k$
$gx + hy + iz = l$

is (x, y, z), where

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}},$$

$$y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}},$$

$$z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \text{ and}$$

and $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0$.

EXAMPLE System of Three Equations

② **Use Cramer's Rule to solve the system of equations.**

$\mathbf{2x - 3y + z = 5}$
$\mathbf{x + 2y + z = -1}$
$\mathbf{x - 3y + 2z = 1}$

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ \ell & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} = \frac{\begin{vmatrix} 5 & -3 & 1 \\ -1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & \ell & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} = \frac{\begin{vmatrix} 2 & 5 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}$$

$$z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & \ell \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} = \frac{\begin{vmatrix} 2 & -3 & 5 \\ 1 & 2 & -1 \\ 1 & -3 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}$$

Use a calculator to evaluate each determinant.

$x = \frac{\boxed{}}{\boxed{}}$ or $\frac{9}{4}$ $\quad y = \frac{-9}{12}$ or $-\frac{\boxed{}}{\boxed{}}$ $\quad z = \frac{\boxed{}}{\boxed{}}$ or $-\frac{7}{4}$

The solution is $\boxed{}$.

Check Your Progress Use Cramer's Rule to solve the system of equations.

$2x + y + z = -3$
$-3x + 2y - z = 5$
$x - y + 3z = 1$

HOMEWORK ASSIGNMENT

Page(s):
Exercises:

4–7 Identify and Inverse Matrices

MAIN IDEAS

- Determine whether two matrices are inverses.
- Find the inverse of a 2 × 2 matrix.

BUILD YOUR VOCABULARY (page 85)

The **identity matrix** is a square matrix that, when multiplied by another matrix, equals that same matrix.

Two $n \times n$ matrices are **inverses** of each other if their ________ is the ________ .

KEY CONCEPTS

Identity Matrix for Multiplication The identity matrix for multiplication I is a square matrix with 1 for every element of the main diagonal, from upper left to lower right, and 0 in all other positions. For any square matrix A of the same dimensions as I, $A \cdot I = I \cdot A = A$.

Inverse of a 2 × 2 matrix The inverse of Matrix $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$, where $ad - bc \neq 0$.

EXAMPLE Verify Inverse Matrices

1 Determine whether each pair of matrices are inverses.

$$X = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Find $X \cdot Y$.

$$X \cdot Y = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{} & 6 - 6 \\ -1 + 1 & \boxed{} \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Find $Y \cdot X$.

$$Y \cdot X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 2 & \boxed{} \\ 3 - 3 & \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Since $X \cdot Y = Y \cdot X = I$, X and Y are inverses.

Check Your Progress **Determine whether each pair of matrices are inverses.**

a. $A = \begin{bmatrix} -2 & 3 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 2 \\ 1 & 1 \end{bmatrix}$

b. $C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

EXAMPLE Find the Inverse of a Matrix

2 **Find the inverse of each matrix, if it exists.**

a. $S = \begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix}$

Find the value of the determinant.

$$\begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix} = 2 - \boxed{} = \boxed{}$$

Since the determinant is not equal to 0, S^{-1} exists.

$$S^{-1} = \frac{1}{ad - \text{bc}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{-1(-2) - \boxed{}(0)}\begin{bmatrix} \boxed{} & -1 \\ -8 & \boxed{} \end{bmatrix}$$

$$= \frac{1}{\boxed{}}\begin{bmatrix} -2 & 0 \\ -8 & -1 \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & -\frac{1}{2} \end{bmatrix}$$

b. $T = \begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix}$

Find the value of the determinant.

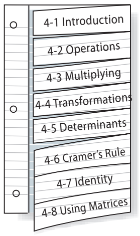
$$\begin{vmatrix} -4 & 6 \\ -2 & 3 \end{vmatrix} = \boxed{} + \boxed{} = \boxed{}$$

Since the determinant equals 0, T^{-1} does not exist.

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-7, write your own 2 × 2 matrix. Then find the inverse of the matrix, if it exists.



Check Your Progress **Find the inverse of each matrix if it exists.**

a. $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

b. $B = \begin{bmatrix} 6 & -2 \\ 7 & -2 \end{bmatrix}$

EXAMPLE

3 **a.** **CRYPTOGRAPHY** **Use the table at the beginning of the lesson to assign a number to each letter in the message ALWAYS_SMILE. Then code the message with the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. Convert the message to numbers using the table.**

A	L	W	A	Y	S	_	S	M	I	L	E
1	12	23	1	25	19	0	19	13	9	12	5

Write the message in matrix form. Arrange the numbers in a matrix with 2 columns and as many rows as are needed. Then multiply the message matrix B by the coding matrix A.

$$BA = \begin{bmatrix} 1 & 12 \\ 23 & 1 \\ 25 & 19 \\ 0 & 19 \\ 13 & 9 \\ 12 & 5 \end{bmatrix} \cdot \begin{bmatrix} \boxed{} \end{bmatrix}$$

Write an equation.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Multiply the matrices.

$$= \begin{bmatrix} \boxed{} \end{bmatrix}$$

Simplify.

The coded message is

$\boxed{}$.

b. Use the inverse matrix A^{-1} to decode the message in part a. First find the inverse matrix of $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

$$A^{-1} = \frac{1}{ad - \mathrm{bc}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ Definition of inverse

$$= \frac{1}{1(3) - 2(1)}\begin{bmatrix} \boxed{} \end{bmatrix}$$ $a = \boxed{}$, $b = \boxed{}$, $c = \boxed{}$, $d = \boxed{}$

$$= \boxed{}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$ Simplify.

$$= \begin{bmatrix} \boxed{} \end{bmatrix}$$ Simplify.

Next, decode the message by multiplying the coded matrix C by A^{-1}.

$$CA^{-1} = \begin{bmatrix} 13 & 38 \\ 24 & 49 \\ 44 & 107 \\ 19 & 57 \\ 22 & 53 \\ 17 & 39 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$ Write an equation.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$ Multiply the matrices.

$$= \begin{bmatrix} \boxed{} \end{bmatrix}$$ Write an equation.

Use the table again to convert the numbers to letters. You can now read the message.

A	L	W	A	Y	S	_	S	M	I	L	E
1	12	23	1	25	19	0	19	13	9	12	5

Check Your Progress

a. Use the table to assign a number to each letter in the message FUN_MATH. Then code the matrix $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$.

b. Use the inverse matrix of $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ to decode the message 12 | 63 | 28 | 14 | 26 | 16 | 40 | 44.

Code					
_	0	**I**	9	**R**	18
A	1	**J**	10	**S**	19
B	2	**K**	11	**T**	20
C	3	**L**	12	**U**	21
D	4	**M**	13	**V**	22
E	5	**N**	14	**W**	23
F	6	**O**	15	**X**	24
G	7	**P**	16	**Y**	25
H	8	**Q**	17	**Z**	26

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–8 Using Matrices to Solve Systems of Equations

MAIN IDEAS

- Write matrix equations for systems of equations.
- Solve systems of equations using matrix equations.

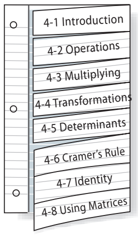
BUILD YOUR VOCABULARY (page 86)

A system of equations can be written with ☐ expressing the system of equations as a **matrix equation**.

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-8, write a matrix equation for this system of equations.
$x + 2y = 5$
$3x + 5y = 14$



EXAMPLE Two-Variable Matrix Equation

① **Write a matrix equation for the system of equations.**

$$\begin{aligned} x + 3y &= 3 \\ x + 2y &= 7 \end{aligned}$$

Determine the coefficient, variable, and constant matrices.

$$\begin{aligned} x + 3y &= 3 \\ x + 2y &= 7 \end{aligned} \quad \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

Write the matrix equation.

$$\begin{array}{ccccc} A & \cdot & X & = & B \\ \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} & \cdot & \begin{bmatrix} \square \\ \square \end{bmatrix} & = & \begin{bmatrix} \square \\ \square \end{bmatrix} \end{array}$$

EXAMPLE Problem Solve with Matrix Equations

② **FABRICS** **The table below shows the composition of three types of fabric.**

Type	Wool	Silk	Cotton	Cost
R	10%	20%	70%	\$7
S	20%	30%	50%	\$8
T	20%	50%	30%	\$10

a. Write a system of equations that represents the total cost for each of the three fabric components.

Let w represent the cost of wool.
Let s represent the cost of silk.
Let c represent the cost of cotton.

$$\begin{aligned} 0.1w + 0.2s + 0.7c &= 7 \\ 0.2w + 0.3s + 0.5c &= 8 \\ 0.2w + 0.5s + 0.3c &= 10 \end{aligned}$$

b. Write a matrix equation for the system of equations. Determine the coefficient, variable, and constant matrices. Then write the matrix equation.

$$\begin{aligned} 0.1w + 0.2s + 0.7c &= 7 \\ 0.2w + 0.3s + 0.5c &= 8 \\ 0.2w + 0.5s + 0.3c &= 10 \end{aligned} \longrightarrow \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \cdot \begin{bmatrix} w \\ s \\ c \end{bmatrix}$$

$$= \begin{bmatrix} \square \end{bmatrix}$$

EXAMPLE Solve a System of Equations

1 **Use a matrix equation to solve the system of equations.**

a. $\mathbf{5x + 3y = 13}$
$\mathbf{4x + 7y = -8}$

The matrix equation is $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$

when $A = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$, $X = \begin{bmatrix} \square \\ \square \end{bmatrix}$, and $B = \begin{bmatrix} \square \\ \square \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{\square}{\square - \square} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \text{ or } \square \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix}$$

Step 2 Multiply each side of the matrix equation by the inverse matrix.

$$\frac{1}{23}\begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{23}\begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} \square \end{bmatrix} \cdot \begin{bmatrix} \square \end{bmatrix} = \frac{1}{23}\begin{bmatrix} \square \end{bmatrix}$$

$$\begin{bmatrix} \square \end{bmatrix} = \begin{bmatrix} \square \end{bmatrix}$$

The solution is $\square$.

b. $\mathbf{10x + 5y = 15}$
$\mathbf{6x + 3y = -6}$

The matrix equation is $\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$

when $A = \begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$.

Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{30 - 30}\left[\boxed{}\right] \text{ or } \boxed{} \begin{bmatrix} 3 & -5 \\ -6 & 10 \end{bmatrix}$$

The determinant of the coefficient matrix $\begin{bmatrix} 3 & -5 \\ -6 & 10 \end{bmatrix}$ is

$\boxed{}$, so A^{-1} does not exist. The system is $\boxed{}$.

Check Your Progress

a. Write a matrix equation for the system of equations.
$x - 2y = 6$ and $3x + 4y = 7$.

b. Use a matrix equation to solve the system of equations.
$3x + 4y = 0$ and $x - 2y = 10$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 4

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 4 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 4, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 85–86)* to help you solve the puzzle.

4-1

Introduction to Matrices

Match each matrix with its dimensions.

1. $\begin{bmatrix} 3 & 2 & 5 \\ -1 & 0 & 6 \end{bmatrix}$

2. $[30 \quad -84]$

3. $\begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 4 & 0 \\ -1 & 2 \\ 6 & 1 \end{bmatrix}$

a. 3×2

b. 2×3

c. 2×2

d. 1×2

5. Write a system of equations that you could use to solve the following matrix equation for x, y, z.

$$\begin{bmatrix} 3x \\ x + y \\ y - z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 6 \end{bmatrix}$$

4-2

Operations with Matrices

6. Use $M = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -1 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} -2 & 5 & -4 \\ 3 & 1 & 0 \end{bmatrix}$ to find $2M + 3N$.

4-3

Multiplying Matrices

Determine whether each indicated matrix product is defined. If so, state the dimensions of the product. If not, write *undefined*.

7. $M_{3 \times 2}$ and $N_{2 \times 3}$ MN:

8. $M_{1 \times 2}$ and $N_{1 \times 2}$ MN:

9. $M_{4 \times 1}$ and $N_{1 \times 4}$ MN:

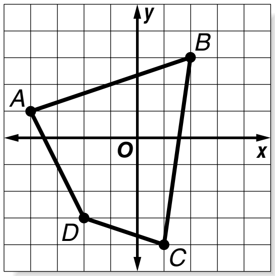
10. Find the product, if possible.

$$\begin{bmatrix} 2 & 0 & 3 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 5 & 2 \\ -3 & 0 \end{bmatrix}$$

4-4

Transformations with Matrices

Refer to quadrilateral *ABCD* shown.



11. Write the vertex matrix for the quadrilateral *ABCD*.

12. Write the vertex matrix that represents the position of the quadrilateral $A'B'C'D'$ that results when quadrilateral *ABCD* is translated 3 units to the right 2 units down.

4-5

Determinants

13. Find the value of $\begin{vmatrix} 8 & 3 \\ 2 & -1 \end{vmatrix}$.

14. Evaluate $\begin{vmatrix} 3 & 12 & -1 \\ 10 & 9 & 0 \\ -5 & 6 & -2 \end{vmatrix}$ using expansion by minors.

4-6

Cramer´s Rule

15. The two sides of an angle are contained in the lines whose equations are $3x + y = 5$ and $2x + 3y = 8$. Find the coordinates of the vertex of the angle.

16. Use Cramer's Rule to solve the system of equations.

$$\begin{aligned} 2x + 5y + 3z &= 10 \\ 3x - y + 4z &= 8 \\ 5x - 2y + 7z &= 12 \end{aligned}$$

4-7

Identity and Inverse Matrices

Indicate whether each of the following statements is *true* or *false*.

17. Every element of an identity matrix is 1.

18. There is a 3 × 2 identity matrix.

19. If M is a matrix, M^{-1} represents the reciprocal of M.

20. Every square matrix has an inverse.

21. Determine whether $A = \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$ are inverses.

4-8

Using Matrices to Solve Systems of Equations

22. Write a matrix equation for the following system equations.

$$\begin{aligned} 3x + 5y &= 10 \\ 2x - 4y &= -7 \end{aligned}$$

23. Solve the system of equations $4x - 5y = 4$ and $2x - y = 8$ using inverse matrices.

CHAPTER
4
Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 4.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 4 Practice Test on page 229 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 4 Study Guide and Review on pages 224–228 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 229.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 4 Foldable.
- Then complete the Chapter 4 Study Guide and Review on pages 224–228 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 229 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

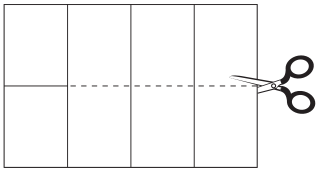
CHAPTER 5

Quadratic Functions and Inequalities

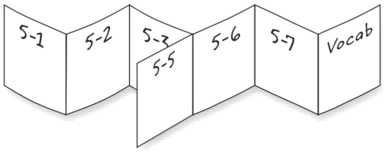
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11” by 17” paper.

STEP 1 **Fold** in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.



STEP 2 **Staple** along the lengthwise fold and staple the uncut section at the top. Label each section with a lesson number and close to form a booklet.



NOTE-TAKING TIP: When you take notes, you may wish to use a highlighting marker to emphasize important concepts.

CHAPTER 5

BUILD YOUR VOCABULARY

This is an alphabetical list of the new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
axis of symmetry			
completing the square			
complex conjugates			
complex number			
constant term			
discriminant dihs-KRIH-muh-nuhnt			
imaginary unit			
linear term			
maximum value			
minimum value			

(continued on the next page)

Chapter 5 BUILDING YOUR VOCABULARY

Vocabulary Term	Found on Page	Definition	Description or Example
parabola puh-RA-buh-luh			
pure imaginary number			
quadratic equation kwah-DRA-tihk			
quadratic function			
quadratic inequality			
quadratic term			
root			
square root			
vertex			
vertex form			
zero			

5–1 Graphing Quadratic Functions

MAIN IDEAS

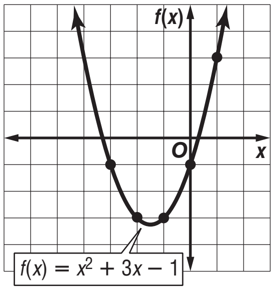
- Graph quadratic functions.
- Find and interpret the maximum and minimum values of a quadratic function.

EXAMPLE Graph a Quadratic Function

① **Graph $f(x) = x^2 + 3x - 1$ by making a table of values.**

First, choose integer values for x. Then, evaluate the function for each x value. Graph the function.

x	$x^2 + 3x - 1$	$f(x)$	(x, y)
-3	$(-3)^2 + 3(-3) - 1$		
-2	$(-2)^2 + 3(-2) - 1$		
-1	$(-1)^2 + 3(-1) - 1$		
0	$(0)^2 + 3(0) - 1$		
1	$(1)^2 + 3(1) - 1$		



Check Your Progress Graph $f(x) = 2x^2 + 3x + 2$.

EXAMPLE Axis of Symmetry, *y*-intercept, and Vertex

② **Consider the quadratic function $f(x) = 2 - 4x + x^2$.**

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

Begin by rearranging the terms of the function. Then identify a, b, and c.

$$f(x) = ax^2 + bx + c$$

$$f(x) = 2 - 4x + x^2 \longrightarrow f(x) = 1x^2 - 4x + 2$$

So, $a =$, $b =$, and $c =$.

KEY CONCEPT

Graph of a Quadratic Function Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The y-intercept is $a(0)^2 + b(0) + c$ or c.
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.

WRITE IT

Why is it helpful to know the axis of symmetry when graphing a quadratic function?

The y-intercept is 2. You can find the equation of the axis of symmetry by using a and b.

$x = -\frac{b}{2a}$	Equation of the axis of symmetry
$x =$	$a = 1$, $b = -4$
$x =$	Simplify.

The y-intercept is . The equation of the axis of symmetry is $x =$. Therefore, the x-coordinate of the vertex is .

b. Make a table of values that includes the vertex.

Choose some values for x that are less than 2 and some that are greater than 2.

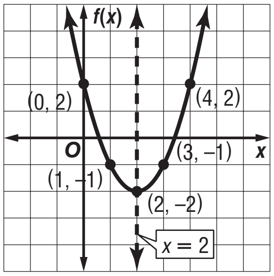
x	$x^2 - 4x + 2$	$f(x)$	$(x, f(x))$
0	$0^2 - 4(0) + 2$		
1	$1^2 - 4(1) + 2$		
2	$2^2 - 4(2) + 2$		
3	$3^2 - 4(3) + 2$		
4	$4^2 - 4(4) + 2$		

c. Use this information to graph the function.

Graph the vertex and y-intercept.

Then graph the points from your table and the y-intercept, connecting them with a smooth curve.

As a check, draw the axis of symmetry, $x = 2$, as a dashed line.



The graph of the function should be symmetric about this line.

Check Your Progress **Consider the quadratic function $f(x) = 3 - 6x + x^2$.**

a. Find the y-intercept, the equation of the axis of symmetry and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

KEY CONCEPTS

Maximum and Minimum Value The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.

FOLDABLES™ On the page for Lesson 5-1, sketch a parabola. Then label the axis of symmetry, vertex, maximum value, and minimum value.

EXAMPLE Maximum or Minimum Value

3 Consider the function $f(x) = -x^2 + 2x + 3$.

a. Determine whether the function has a maximum or a minimum value.

For this function, $a =$, $b =$, and $c =$.

Since a 0, the graph opens and the

function has a .

b. State the maximum or minimum value of the function.

The maximum value of this function is the y-coordinate of the vertex.

The x-coordinate of the vertex is or .

Find the y-coordinate of the vertex by evaluating the function for $x = 1$.

$f(x) = -x^2 + 2x + 3$ Original function

$f(1) =$ or $x = 1$

The maximum value of the function is .

c. State the domain and range of the function.

The domain is all real numbers. The range is all reals less than or equal to the value. That is

.

Check Your Progress **Consider the function $f(x) = x^2 + 4x - 1$.**

a. Determine whether the function has a maximum or a minimum value.

b. State the maximum or minimum value of the function.

c. State the domain and range of the function.

EXAMPLE

4 **ECONOMICS** **A souvenir shop sells about 200 coffee mugs each month for \$6 each. The shop owner estimates that for each \$0.50 increase in the price, he will sell about 10 fewer coffee mugs per month.**

a. How much should the owner charge for each mug in order to maximize the monthly income from their sales?

Words	The income is the number of mugs multiplied by the cost per mug.
Variables	Let x = the number of \$0.50 price increases. Then $6 + 0.50x$ = the price per mug and $200 - 10x$ = the number of mugs sold. Let $I(x)$ = income as a function of x.

The income is the number of mugs multiplied by the $\boxed{\quad\quad}$ per mug.

Equation

$$I(x) = (200 - 10x) \cdot (6 + 0.50x)$$

$$= 200(6) + 200(0.50x) - 10x(6) - 10x(0.50x)$$

$$= 1200 + 100x - 60x - 5x^2 \qquad \text{Multiply.}$$

$$= \boxed{\quad\quad} + \boxed{\quad\quad} - \boxed{\quad\quad} \qquad \text{Simplify.}$$

$$= -5x^2 + 40x + 1200 \qquad \text{Rewrite in } ax^2 + bx + c \text{ form.}$$

$I(x)$ is a quadratic function with $a = -5$, $b = 40$, and $c = 1200$. Since $a < 0$, the function has a maximum value at the ____ of the graph. Use the formula to find the x-coordinate of the vertex.

$x\text{-coordinate of the vertex} = -\frac{b}{2a}$	Formula for the x-coordinate of the vertex.
$= -\frac{40}{2(-5)}$	$a =$ ____, $b =$ ____
$=$ ____	Simplify.

This means the souvenir shop should make 4 price increases of \$0.50 to maximize its income. Thus, the price of a mug should be ____ or ____.

b. What is the maximum monthly income the owner can expect to make from the mugs?

To determine maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 4$.

$I(x) = -5x^2 + 40x + 1200$	Income function
$I(\underline{\quad}) = -5(\underline{\quad})^2 + 40(\underline{\quad}) + 1200$	$x =$ ____
$=$ ____	Use a calculator.

Thus, the maximum income the souvenir shop can expect is ____.

Check Your Progress **ECONOMICS A sports team sells about 100 coupon books for \$30 each during its annual fund-raiser. They estimate that for each \$0.50 decrease in the price, they will sell about 10 more coupon books.**

a. How much should they charge for each book in order to maximize the income from their sales?

b. What is the maximum monthly income the team can expect to make from these items?

HOMEWORK ASSIGNMENT

Page(s):
Exercises:

5–2 Solving Quadratic Equations by Graphing

MAIN IDEAS

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

BUILD YOUR VOCABULARY (pages 121–122)

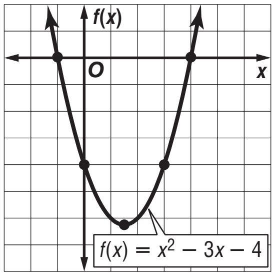
A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

The of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic is to find the **zeros** of the related quadratic .

EXAMPLE Two Real Solutions

① **Solve $x^2 - 3x - 4 = 0$ by graphing.**

Graph the related quadratic function $f(x) = x^2 - 3x - 4$. The equation of the axis of symmetry is $x = -\frac{-3}{2(1)}$ or $\frac{3}{2}$. Make a table using x-values around $\frac{3}{2}$. Then graph each point.



x	−1	0	1	2	3	4
$f(x)$						

From the table and the graph, we can see that the zeros of the function are −1 and 4. The solutions of the equation are and .

Check Your Progress Solve $x^2 + 2x - 3 = 0$ by graphing.

KEY CONCEPT

Solutions of a Quadratic Equation A quadratic equation can have one real solution, two real solutions, or no real solution.

EXAMPLE One Real Solution

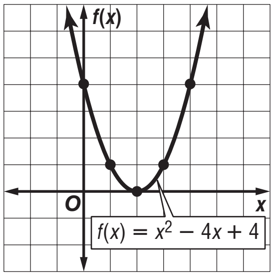
② **Solve $x^2 - 4x = -4$ by graphing.**

Write the equation in $ax^2 + bx + c = 0$ form.

$x^2 - 4x = -4 \longrightarrow$ ______ $= 0$ Add 4 to each side.

Graph the related quadratic function $f(x) = x^2 - 4x + 4$.

x	0	1	2	3	4
f(x)					



Notice that the graph has only one x-intercept, 2.

Check Your Progress Solve $x^2 - 6x = -9$ by graphing.

EXAMPLE No Real Solution

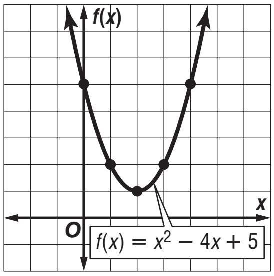
③ **NUMBER THEORY Find two real numbers whose sum is 4 and whose product is 5 or show that no such numbers exist.**

Let x = one of the numbers. Then $4 - x$ = the other number.

Since the product is 5, you know that $x(4 - x) = 5$ or $-x^2 + 4x - 5 = 0$.

You can solve $x^2 - 4x + 5 = 0$ by graphing the related function $f(x) = x^2 - 4x + 5$.

x	0	1	2	3	4
f(x)					



Notice that the graph has no x-intercepts. This means that the original equation has no real solution.

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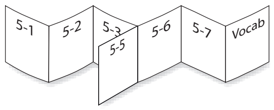
ORGANIZE IT

On the page for Lesson 5-2, sketch a graph of a parabola that has one real solution, two real solutions, and no real solution.



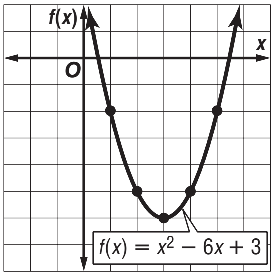
Check Your Progress Find two real numbers whose sum is 7 and whose product is 14 or show that no such numbers exist.

EXAMPLE Estimate Roots

④ **Solve $x^2 - 6x + 3 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

The equation of the axis of symmetry of the related function is

$x =$ ☐ .



x	0	1	2	3	4	5	6
f(x)							

The x-intercepts of the graph are between ☐ and between ☐.

Check Your Progress Solve $x^2 - 4x + 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

EXAMPLE

5 **ROYAL GORGE BRIDGE** **The highest bridge in the U.S. is the Royal Gorge Bridge in Colorado. The deck is 1053 feet above the river. Suppose a marble is dropped over the railing from a height of 3 feet above the bridge deck. How long will it take the marble to reach the surface of the water, assuming there is no air resistance? Use the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and h_0 is the initial height above the water in feet.**

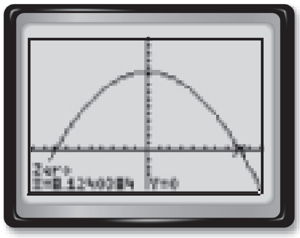
We need to find t when h_0 = ______ and $h(t)$ = ______. Solve $0 = -16t^2 + 1056$.

Graph the related function $y = -16t^2 + 1056$ on a graphing calculator.

Use the ZERO feature, 2nd [CALC], to find the positive zero of the function, since time cannot be ______. Use the arrow keys to locate a left bound and press ENTER. Then, locate a right bound and press ENTER twice.

The positive zero of the function is approximately ______.

The marble would fall for about ______ seconds.



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Check Your Progress One of the larger dams in the United States is the Hoover Dam on the Colorado River, which was built during the Great Depression. The dam is 726.4 feet tall. Suppose a marble is dropped over the railing from a height of 6 feet above the top of the dam. How long will it take the marble to reach the surface of the water, assuming there is no air resistance? Use the formula from Example 5.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–3 Solving Quadratic Equations by Factoring

MAIN IDEAS

- Write quadratic equations in intercept form.
- Solve quadratic equations by factoring

EXAMPLE Write an Equation Given Roots

1 **Write a quadratic equation with $-\frac{2}{3}$ and 6 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.**

$(x - p)(x - q) = 0$ Write the pattern.

$\left[x - \left(-\frac{2}{3}\right)\right](x - 6) = 0$ Replace p with $-\frac{2}{3}$ and q with 6.

$\boxed{} = 0$ Simplify.

$\boxed{} = 0$ Use FOIL.

$\boxed{} = 0$ Multiply each side by 3 so that b is an integer.

Check Your Progress Write a quadratic equation with $-\frac{3}{4}$ and 5 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

EXAMPLE Two or Three Terms

2 **Factor each polynomial.**

a. $2y^2 - 3y - 5$

The coefficient of the y terms must be $\boxed{}$ and $\boxed{}$ since $2(-5) = -10$ and $2 + (-5) = -3$. Rewrite the expression using $-5y$ and $2y$ in place of $-3y$ and factor by grouping.

$2y^2 - 3y - 5 = 2y^2 - 5y + 2y - 5$ Replace $-3y$.

$= (2y^2 - 5y) + (2y - 5)$ Associative Property

$= \boxed{}(2y - 5) + \boxed{}(2y - 5)$ Factor.

$= \boxed{}(2y - 5)$ Distributive Property

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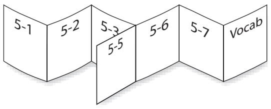
On the page for Lesson 5-3, write your own roots to a quadratic equation. Then find a quadratic equation with those roots. Check your result by graphing the related function.



b. $8x^3 - y^3$

$8x^3 - y^3 = \boxed{}^3 - y^3$ This is the difference of two cubes.

$= \boxed{}[(2x)^2 + (2x)y + y^2]$ Factor.

$= \boxed{}(4x^2 + 2xy + y^2)$ Simplify.

Check Your Progress **Factor each polynomial.**

a. $2x^2 + x - 3$

$\boxed{}$

b. $3x^3 - 12x$

$\boxed{}$

c. $a^3b^3 - 27$

$\boxed{}$

KEY CONCEPT

Zero Product Property For any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0, *b* = 0, or both *a* and *b* equal zero.

EXAMPLE **Two Roots**

3 **Solve $x^2 = 4x$ by factoring.**

$x^2 = 4x$ Original equation

$\boxed{} = 0$ Add 4*x* to each side.

$\boxed{} = 0$ Factor the binomial.

$\boxed{} = 0$ or $\boxed{} = 0$ Zero Product Property

$x = \boxed{}$ Solve the second equation.

The solution set is $\boxed{}$.

Check Substitute $\boxed{}$ and $\boxed{}$ for x in the original equation.

$x^2 = 4x$ $x^2 = 4x$

$\boxed{}^2 \overset{?}{=} 4\boxed{}$ $\boxed{}^2 = 4\boxed{}$

$\boxed{} = \boxed{}$ ✔ $\boxed{} = \boxed{}$ ✔

Check Your Progress **Solve each equation by factoring.**

a. $x^2 = 3x$

b. $6x^2 + 11x = -4$

EXAMPLE

Double Root

4 **Solve $x^2 - 6x = -9$ by factoring.**

$x^2 - 6x = -9$ Original equation

$= 0$ Add 9 to each side.

$= 0$ Factor.

$= 0$ or $= 0$ Zero Product Property

$x =$ $x =$ Solve each equation.

The solution set is .

Check Your Progress Solve $x^2 + 10x = -25$ by factoring.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–4 Complex Numbers

MAIN IDEAS

- Find square roots and perform operations with pure imaginary numbers.
- Perform operations with complex numbers

BUILD YOUR VOCABULARY (pages 121–122)

i is called the **imaginary unit** ($i = \sqrt{-1}$).

Pure imaginary numbers are square roots of $\boxed{}$ real numbers.

A **complex number**, such as the expression $5 + 2i$, is a complex number since it has a real number (5) and a pure imaginary number ($2i$).

Two complex numbers of the form $a + bi$ and $\boxed{}$ are called **complex conjugates**.

EXAMPLE Properties of Square Roots

① **Simplify.**

a. $\sqrt{18}$

$$= \sqrt{9 \cdot 2}$$
$$= \sqrt{9} \cdot \sqrt{2}$$
$$= \boxed{}$$

b. $\sqrt{\frac{10}{81}}$

$$= \frac{\sqrt{10}}{\sqrt{81}}$$
$$= \boxed{}$$

Check Your Progress **Simplify.**

a. $\sqrt{75}$

$\boxed{}$

b. $\sqrt{\frac{7}{36}}$

$\boxed{}$

EXAMPLE Square Roots of Negative Numbers

② **Simplify $\sqrt{-32y^3}$.**

$$\sqrt{-32y^3} = \sqrt{-1 \cdot 4^2 \cdot 2 \cdot y^2 \cdot y}$$
$$= \sqrt{-1} \cdot \sqrt{4^2} \cdot \sqrt{2} \cdot \sqrt{y^2} \cdot \sqrt{y}$$
$$= \boxed{} \cdot 4\,|y|\,\sqrt{2} \cdot \boxed{}$$
$$= \boxed{}$$

REMEMBER IT

You can write i to the left or right of the radical symbol. However, i is usually written to the left so it is clear that it is not under the radical.

Check Your Progress **Simplify.**

a. $\sqrt{-32}$ $\boxed{}$

b. $\sqrt{-50x^5}$ $\boxed{}$

EXAMPLE Multiply Pure Imaginary Numbers

3 **Simplify.**

a. $\boldsymbol{-3i \cdot 2i}$

$$-3i \cdot 2i = -6i^2$$
$$= -6\left(\boxed{}\right) \qquad i^2 = \boxed{}$$
$$= \boxed{}$$

b. $\boldsymbol{\sqrt{-12} \cdot \sqrt{-2}}$

$$\sqrt{-12} \cdot \sqrt{-2} = \boxed{} \cdot \boxed{}$$
$$= i^2\sqrt{24}$$
$$= \boxed{} \text{ or } \boxed{}$$

c. $\boldsymbol{i^{35}}$

$i^{35} = i \cdot i^{34}$ Multiplying powers

$= i \cdot \boxed{}^{17}$ Power of a Power

$= i \cdot \boxed{}^{17}$ $i^2 = \boxed{}$

$= i \cdot \boxed{}$ or $\boxed{}$ $(-1)^{17} = \boxed{}$

Check Your Progress **Simplify.**

a. $3i \cdot 5i$

$\boxed{}$

b. $\sqrt{-2} \cdot \sqrt{-6}$

$\boxed{}$

c. i^{57}

$\boxed{}$

KEY CONCEPT

Complex Numbers A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.

EXAMPLE Equation with Pure Imaginary Solutions

④ **Solve $5y^2 + 20 = 0$.**

$5y^2 + 20 = 0$	Original equation
$5y^2 = -$	Subtract from each side.
$y^2 = -4$	Divide each side by .
$y = \pm\sqrt{-4}$	Square Root Property
$y =$	$\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1}$

Check Your Progress Solve $2x^2 + 50 = 0$.

EXAMPLE Equate Complex Numbers

⑤ **Find the values of x and y that make the equation $2x + yi = -14 - 3i$ true.**

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x = -14$ Real parts

$x = -7$ Divide each side by . $y =$ Imaginary parts

$x =$, $y =$

Check Your Progress Find the value of x and y that make the equation $3x - yi = 15 + 2i$ true.

EXAMPLE Add and Subtract Complex Numbers

⑥ **Simplify.**

a. $(3 + 5i) + (2 - 4i)$

$(3 + 5i) + (2 - 4i)$	
$= (3 + 2) + (5 - 4)i$	Commutative and Associative Properties
$=$	Simplify.

b. $(4 - 6\boldsymbol{i}) - (3 - 7\boldsymbol{i})$

$(4 - 6\boldsymbol{i}) - (3 - 7\boldsymbol{i})$

$= (4 - 3) + [-6 - (-7)]\boldsymbol{i}$ Commutative and Associative Properties

$=$ ☐ Simplify.

Check Your Progress **Simplify.**

a. $(2 + 6\boldsymbol{i}) + (3 + 4\boldsymbol{i})$

b. $(3 + 2\boldsymbol{i}) - (-2 + 5\boldsymbol{i})$

EXAMPLE **Multiply Complex Numbers**

7 **ELECTRICITY In an AC circuit, the voltage E, current I, and impedance Z are related to the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 4j$ amps and impedance $3 - 6j$ ohms.**

$E = I \cdot Z$ Electricity formula

$= (1 + 4\boldsymbol{j})(3 - 6\boldsymbol{j})$ $I = 1 + 4\boldsymbol{j}, Z = 3 - 6\boldsymbol{j}$

$= 1(3) + 1(-6\boldsymbol{j}) + 4\boldsymbol{j}(3) + 4\boldsymbol{j}(-6\boldsymbol{j})$ FOIL

$= 3 -$ ☐ $+ 12\boldsymbol{j} -$ ☐ Multiply.

$=$ ☐ $\boldsymbol{j}^2 = -1$

$=$ ☐ volts Add.

Check Your Progress Refer to Example 5. Find the voltage in a circuit with current $1 - 3\boldsymbol{j}$ amps and impedance $3 + 2\boldsymbol{j}$ ohms.

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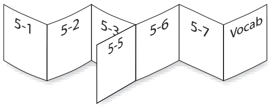
On the page for Lesson 5-4, write your own example of complex conjugates. Then explain why the product of complex conjugates is always a real number.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Divide Complex Numbers

⑧ **Simplify.**

a. $\dfrac{5i}{3 + 2i}$

$\dfrac{5i}{3 + 2i} = \dfrac{5i}{3 + 2i} \cdot \boxed{}$ 3 − 2*i* and 3 + 2*i* are conjugates.

$= \dfrac{15i + 10i^2}{9 - 4i^2}$ Multiply.

$= \dfrac{15i + 10}{\boxed{}}$ $i^2 = -1$

$= \boxed{} + \boxed{}$ Standard form

b. $\dfrac{4 - i}{5i}$

$\dfrac{4 - i}{5i} = \dfrac{4 - i}{5i} \cdot \dfrac{i}{i}$

$= \dfrac{4i - i^2}{-5i^2}$ Multiply.

$= \dfrac{4i + 1}{\boxed{}}$ $i^2 = -1$

$= \boxed{}$ Standard form

Check Your Progress **Simplify.**

a. $\dfrac{3i}{1 + i}$

$\boxed{}$

b. $\dfrac{3 + 2i}{2i}$

$\boxed{}$

5–5 Completing the Square

MAIN IDEAS

- Solve quadratic equations by using the Square Root Property.
- Solve quadratic equations by completing the square.

KEY CONCEPT

Square Root Property

For any real number *n*, if $x^2 = n$, then $x = \pm\sqrt{n}$.

EXAMPLE Equation with Rational Roots

① **Solve $x^2 + 14x + 49 = 64$ by using the Square Root Property.**

$x^2 + 14x + 49 = 64$	Original equation
$\boxed{\qquad\quad} = 64$	Factor the trinomial.
$\boxed{\qquad\quad} = \pm\boxed{\qquad}$	Square Root Property
$\boxed{\qquad\quad} = \boxed{\qquad}$	$\sqrt{64} = 8$
$x = -7 \pm 8$	Subtract 7 from each side.
$x = -7 + 8$ or $x = -7 - 8$	Write as two equations.
$x = \boxed{\quad}$ $\quad x = \boxed{\qquad}$	Solve each equation.

EXAMPLE Equation with Irrational Roots

② **Solve $x^2 - 10x + 25 = 12$ by using the Square Root Property.**

$x^2 - 10x + 25 = 12$	Original equation
$(x - 5)^2 = 12$	Factor the trinomial.
$x - 5 = \boxed{\qquad\quad}$	Square Root Property
$x = \boxed{\qquad\quad}$	Add 5 to each side. $\sqrt{12} = 2\sqrt{3}$
$x = 5 + \boxed{\qquad}$ or $x = 5 - \boxed{\qquad}$	Write as two equations.
$x \approx \boxed{\qquad}$ $\quad x \approx \boxed{\qquad}$	Use a calculator.

Check Your Progress **Solve by using the Square Root Property.**

a. $x^2 - 16x + 64 = 25$

$\boxed{\qquad\qquad\qquad}$

b. $x^2 - 4x + 4 = 8$

$\boxed{\qquad\qquad\qquad}$

EXAMPLE Complete the Square

KEY CONCEPT

Completing the Square To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

③ **Find the value of c that makes $x^2 + 16x + c$ a perfect square. Then write the trinomial as a perfect square.**

Step 1 Find one half of 16.

Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + 16x$.

The trinomial can be written as .

Check Your Progress Find the value of c that makes $x^2 + 6x + c$ a perfect square. Then write the trinomial as a perfect square.

REMEMBER IT

Be sure to add the same constant to both sides of the equation when solving equations by completing the square.

EXAMPLE Solve an Equation by Completing the Square

④ **Solve $x^2 + 4x - 12 = 0$ by completing the square.**

$x^2 + 4x - 12 = 0$ Notice that $x^2 + 4x - 12$ is not a perfect square.

$x^2 + 4x =$ Rewrite so the left side is of the form $x^2 + bx$.

$x^2 + 4x + 4 = 12 + 4$ Add to each side.

$= 16$ Write the left side as a perfect square by factoring.

$x + 2 =$ Square Root Property

$x =$ Subtract 2 from each side.

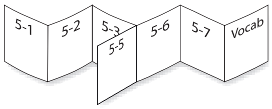
$x =$ or $x =$ Write as two equations.

$x =$ $x =$ Solve each equation.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 5-5, list the steps you would use to solve $w^2 - 8w - 9 = 0$ by completing the square.



Check Your Progress Solve $x^2 + 5x - 6 = 0$ by completing the square.

EXAMPLE **Equation with $a \neq 1$**

5 **Solve $3x^2 - 2x - 1 = 0$ by completing the square.**

$3x^2 - 2x - 1 = 0$	Notice that $3x^2 - 2x - 1$ is not a perfect square.
$x^2 - \frac{2}{3}x - \frac{1}{3} = 0$	Divide by the coefficient of the quadratic term, 3.
$x^2 - \frac{2}{3}x = \frac{1}{3}$	Add $\frac{1}{3}$ to each side.
$x^2 - \frac{2}{3}x + \boxed{} = \frac{1}{3} + \boxed{}$	Since $\left(\frac{-2}{3} \cdot \frac{1}{2}\right)^2 = \boxed{}$, add $\boxed{}$ to each side.
$\boxed{} = \boxed{}$	Write the left side as a perfect square by factoring. Simplify the right side.
$\boxed{} = \pm\boxed{}$	Square Root Property
$x = \boxed{} \pm \boxed{}$	Add $\boxed{}$ to each side.
$x = \boxed{}$ or $x = \boxed{}$	Write as two equations.
$x = \boxed{}$ $\quad x = \boxed{}$	Solve each equation.

The solution set is $\boxed{}$.

Check Your Progress Solve $2x^2 + 11x + 15 = 0$ by completing the square.

EXAMPLE Equation with Complex Solutions

⑥ **Solve $x^2 + 2x + 3 = 0$ by completing the square.**

$x^2 + 2x + 3 = 0$ Notice that $x^2 + 2x + 3 = 0$ is not a perfect square.

$= -3$ Rewrite so the left side is of the form $x^2 + bx$.

$x^2 + 2x + 1 = -3 + 1$ Since $\left(\frac{2}{2}\right)^2$, add 1 to each side.

$^2 = -2$ Write the left side as a perfect square by factoring.

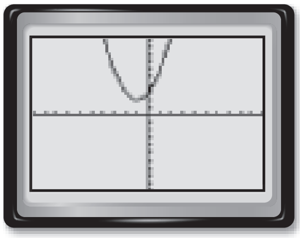
$= \pm\sqrt{-2}$ Square Root Property

$= \pm i\sqrt{2}$ $\sqrt{-1} = i$

$x =$ Subtract 1 from each side.

The solution set is . Notice that these are imaginary solutions.

Check A graph of the related function shows that the equation has no real solutions since the graph had no .



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Check Your Progress Solve $x^2 + 4x + 5 = 0$ by completing the square.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–6 The Quadratic Formula and the Discriminant

MAIN IDEAS

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number and types of roots of a quadratic equation.

KEY CONCEPT

Quadratic Formula
The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE Two Rational Roots

① **Solve $x^2 - 8x = 33$ by using the Quadratic Formula.**

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$\begin{array}{cccccccc} & & ax^2 & + & x & + & x & = 0 \\ & & \downarrow & & \downarrow & & \downarrow & \\ x^2 - 8x = 33 & \longrightarrow & \boxed{} & - & \boxed{} & - & \boxed{} & = 0 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)}$$ Replace a with 1, b with -8, and c with -33.

$$x = \frac{8 \pm \sqrt{\boxed{} + \boxed{}}}{2}$$ Simplify.

$$x = \frac{8 \pm \sqrt{\boxed{}}}{2}$$ Simplify.

$$x = \frac{8 \pm \boxed{}}{2} \qquad \sqrt{\boxed{}} = \boxed{}$$

$$x = \frac{\boxed{}}{2} \text{ or } x = \frac{\boxed{}}{2}$$ Write as two equations.

$$= \boxed{} \qquad = \boxed{}$$ Simplify.

Check Your Progress Solve $x^2 + 13x = 30$ by using the Quadratic Formula.

EXAMPLE One Rational Root

REMEMBER IT The Quadratic Formula can be used to solve any quadratic equation.

② **Solve $x^2 - 34x + 289 = 0$ by using the Quadratic Formula.**

Identify a, b, and c. Then, substitute these values into the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$x = \dfrac{-(-34) \pm \sqrt{\boxed{}}}{2(1)}$ Replace a with 1, b with −34, and c with 289.

$x = \dfrac{34 \pm \sqrt{0}}{2}$ Simplify.

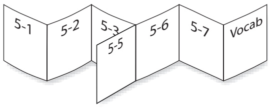
$x = \dfrac{34}{2}$ or $\boxed{}$ $\sqrt{0} = 0$

Check Your Progress Solve $x^2 - 22x + 121 = 0$ by using the Quadratic Formula

$\boxed{}$

EXAMPLE Irrational Roots

FOLDABLES™ ORGANIZE IT On the page for Lesson 5-5, explain why factoring cannot be used to solve the quadratic equation in Example 3.



③ **Solve $x^2 - 6x + 2 = 0$ by using the Quadratic Formula.**

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$ Replace a with 1, b with −6, and c with 2.

$x = \dfrac{6 \pm \sqrt{\boxed{}}}{2}$ Simplify.

$x = \dfrac{\boxed{}}{2}$ or $\dfrac{\boxed{}}{2}$ $\sqrt{28} = \sqrt{4 \cdot 7}$ or $2\sqrt{7}$

The exact solutions are $\boxed{}$ and $\boxed{}$.

The approximate solutions are $\boxed{}$ and $\boxed{}$.

Check Your Progress Solve $x^2 - 5x + 3 = 0$ by using the Quadratic Formula.

EXAMPLE Complex Roots

4 Solve $x^2 + 13 = 6x$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$x^2 + 13 = 6x \longrightarrow 1x^2 - 6x + 13 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$ Replace a with 1, b with -6, and c with 13.

$$= \frac{\boxed{} \pm \sqrt{\boxed{}}}{\boxed{}}$$ Simplify.

$$= \boxed{}$$ $\sqrt{-16} = \sqrt{16(-1)}$ or $4i$

$$= \boxed{}$$ Simplify.

The solutions are the complex numbers $\boxed{}$ and $\boxed{}$.

Check Your Progress Solve $x^2 + 5 = 4x$ by using the Quadratic Formula.

KEY CONCEPT

Discriminant

Consider $ax^2 + bx + c = 0$.

- If $b^2 - 4ac > 0$ and $b^2 - 4ac$ is a perfect square, then there are two real, rational roots.
- If $b^2 - 4ac > 0$ and $b^2 - 4ac$ is *not* a perfect square, then there are two real, irrational roots.
- If $b^2 - 4ac = 0$, then there is one real, rational root.
- If $b^2 - 4ac < 0$, then there are two complex roots.

EXAMPLE Describe Roots

⑤ **Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.**

a. $x^2 + 3x + 5 = 0$

$a =$, $b =$, $c =$

$b^2 - 4ac =$

$=$

$=$

The discriminant is a number, so there

.

b. $x^2 - 11x + 10 = 0$

$a =$, $b =$, $c =$

$b^2 - 4ac =$

$=$

$=$

The discriminant is a number, so there

.

Check Your Progress **Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.**

a. $x^2 + 8x + 16 = 0$

b. $x^2 + 2x + 7 = 0$

c. $x^2 + 3x + 1 = 0$

d. $x^2 + 4x - 12 = 0$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–7 Analyzing Graphs of Quadratic Functions

MAIN IDEAS

- Analyze quadratic functions of the form $y = a(x - h)^2 + k$.
- Write a quadratic function in the form $y = a(x - h)^2 + k$.

BUILD YOUR VOCABULARY (pages 121–122)

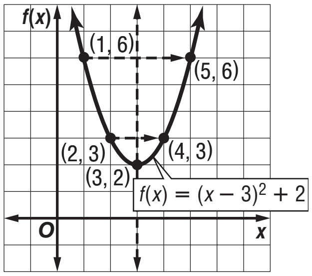
A function written in the form, $y = (x - h)^2 + k$, where (h, k) is the of the parabola and $x = h$ is its , is referred to as the **vertex form**.

EXAMPLE Graph a Quadratic Function in Vertex Form

① **Analyze $y = (x - 3)^2 + 2$. Then draw its graph.**

The vertex is at (h, k) or and the axis of symmetry is $x =$. The graph has the same shape as the graph of $y = x^2$, but is translated 3 units right and 2 units up.

Now use this information to draw the graph.



Step 1 Plot the vertex, .

Step 2 Draw the axis of symmetry, .

Step 3 Find and plot two points on one side of the axis of symmetry, such as (2, 3) and (1, 6).

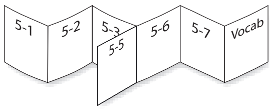
Step 4 Use symmetry to complete the graph.

Check Your Progress Analyze $y = (x + 2)^2 - 4$. Then draw its graph.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 5-7, sketch a graph of a parabola. Then sketch the graph of the parabola after a vertical translation and a horizontal translation.



EXAMPLE Vertex Form Parameters

2 TEST EXAMPLE Which function has the widest graph?

A $y = -12x^2$ **C** $y = 1.2x^2$

B $y = 12x^2$ **D** $y = 0.12x^2$

Read the Item

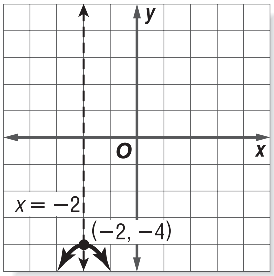
You are given four answer choices, each of which is in vertex form.

Solve the Item

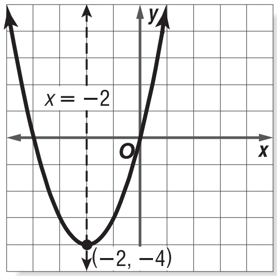
The value of a determines the width of the graph. Since $|-12| =$ and $|1.2| > 1$, choices A, B, and C produce graphs that are than $y = x^2$. Since $|0.12| < 1$, choice D produces a graph that is than $y = x^2$. The answer is .

Check Your Progress Analyze $y = (x + 2)^2 - 4$. Which graph is the graph of $y = (x + 2)^2 - 4$?

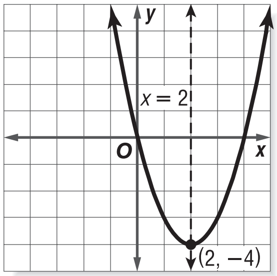
A



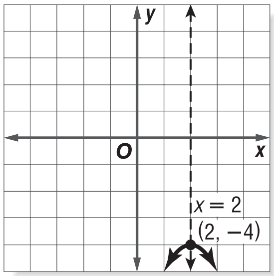
C



B



D



WRITE IT

Consider the vertex form of a quadratic function, $y = a(x - h)^2 + k$. Describe what happens to the graph as $|a|$ increases.

EXAMPLE Write Equations in Vertex Form

3 Write each equation in vertex form. Then analyze the function.

a. $y = x^2 + 2x + 4$

$y = x^2 + 2x + 4$

Notice that $x^2 + 2x + 4$ is not a perfect square.

$y =$

Complete the square. Add $\left(\frac{2}{2}\right)^2$ or 1. Balance this addition by subtracting 1.

$y =$

Write $x^2 + 2x + 1$ as a perfect square.

This function can be rewritten as $y =$ $+ 3$.

So, $h =$ and $k =$. The vertex is at

and the axis of symmetry is $x =$. Since $a = 1$, the

graph opens and has the same shape as $y = x^2$ but

is translated unit left and units up.

b. $\mathbf{y = -2x^2 - 4x + 2}$

$y = -2x^2 - 4x + 2$	Original equation
$y = -2$ $+ 2$	Group $ax^2 - bx$ and factor, dividing by a.
$y = -2(x + 2x + 1) + 2 - (-2)(1)$	Complete the square by adding 1 inside the parentheses. Notice that this is an overall addition of $-2(1)$. Balance this addition by subtracting $-2(1)$.
$y =$	Write $x^2 + 2x + 1$ as a perfect square.

The vertex is at $(-1, 4)$, and the axis of symmetry is

$x =$. Since $a = -2$, the graph opens

and is narrower than the graph of $y = x^2$. It is also

translated unit left and units up.

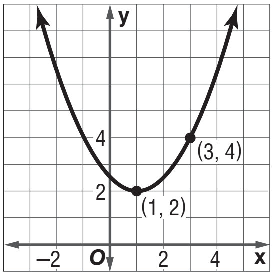
Check Your Progress **Write each function in vertex form. Then analyze the function.**

a. $y = x^2 + 6x + 5$

b. $y = -3x^2 - 6x + 4$

EXAMPLE Write an Equation Given a Graph

④ **Write an equation for the parabola shown in the graph.**



The vertex of the parabola is at (1, 2), so $h =$ and $k =$. Since (3, 4) is a point on the graph of the parabola, let $x =$ and $y =$. Substitute these values into the vertex form of the equation and solve for .

$y = a(x - h)^2 + k$ Vertex form

$4 = a(3 - 1)^2 + 2$ Substitute for y, for x, for h, and for k.

$4 =$ Simplify.

$2 = 4a$ Subtract 2 from each side.

$= a$ Divide each side by 4.

The equation of the parabola in vertex form is

.

Check Your Progress Write an equation for the parabola whose vertex is at (2, 3) and passes through (−2, 1).

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–8 Graphing and Solving Quadratic Inequalities

MAIN IDEAS

- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in one variable.

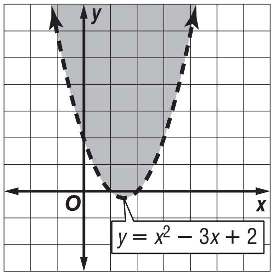
BUILD YOUR VOCABULARY (pages 121–122)

You can graph **quadratic inequalities** in two variables using the same techniques you used to graph inequalities in two variables.

EXAMPLE Graph a Quadratic Inequality

① **Graph $y > x^2 - 3x + 2$.**

Step 1 Graph the related quadratic function, $y = x^2 - 3x + 2$. Since the inequality symbol is $>$, the parabola should be dashed.



Step 2 Test a point inside the parabola, such as (1, 2).

$y > x^2 - 3x + 2$

$2 >$

$2 >$

$2 >$ ✔

So, (1, 2) is a solution of the inequality.

Step 3 Shade the region inside the parabola.

Check Your Progress Graph $y < -x^2 + 4x + 2$.

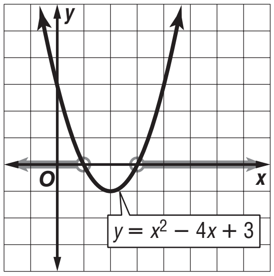
EXAMPLE Solve $ax^2 + bx + c > 0$

1 Solve $x^2 - 4x + 3 > 0$ by graphing.

The solution consists of the x values for which the graph of the related quadratic function lies *above* the x-axis. Begin by finding the roots of the related equation.

$x^2 - 4x + 3 = 0$	Related equation
$= 0$	Factor.
$= 0$ or $= 0$	Zero Product Property
$x =$ $x =$	Solve each equation.

Sketch the graph of the parabola that has x-intercepts at 3 and 1. The graph lies above the x-axis to the left of $x = 1$ and to the right of $x = 3$.



The solution set is .

WRITE IT

Explain how you can check the solution set to a quadratic inequality.

Check Your Progress **Solve each inequality by graphing.**

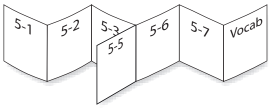
a. $x^2 + 5x + 6 > 0$

b. $x^2 + 6x + 2 \leq 0$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 5-8, use your own words to describe the three steps for graphing quadratic inequalities.



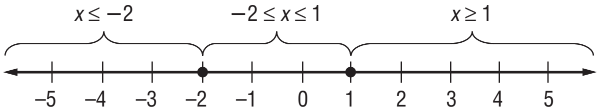
EXAMPLE Solve a Quadratic Inequality

3 Solve $x^2 + x \le 2$ algebraically.

First, solve the related equation $x^2 + x = 2$.

$x^2 + x = 2$	Related quadratic equation
$\boxed{\quad} = 0$	Subtract 2 from each side.
$\boxed{\quad} = 0$	Factor.
$\boxed{\quad} = 0$ or $\boxed{\quad} = 0$	Zero Product Property
$x = \boxed{\quad}$ $x = \boxed{\quad}$	Solve each equation.

Plot the values on a number line. Use closed circles since these solutions are included. Note that the number line is separated into 3 intervals.



Test a value in each interval to see if it satisfies the original inequality.

$x \le -2$	$-2 \le x \le 1$	$x \ge 1$
Test $x = -3$.	Test $x = 0$.	Test $x = 2$.
$x^2 + x \le 2$	$x^2 + x \le 2$	$x^2 + x \le 2$
$(-3)^2 - 3 \overset{?}{\le} 2$	$0^2 + 0 \overset{?}{\le} 2$	$2^2 + 2 \overset{?}{\le} 2$
$6 \le 2$ ✗	$0 \le 2$ ✓	$6 \le 2$ ✗

The solution set is $\boxed{\quad}$.

Check Your Progress Solve $x^2 + 5x < -6$ algebraically.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 5

BRINGING IT ALL TOGETHER

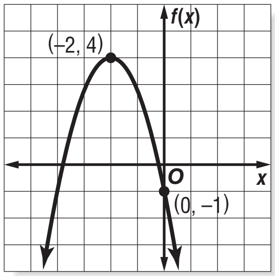
STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 5 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 121–122)* to help you solve the puzzle.

5-1

Graphing Quadratic Functions

Refer to the graph at the right as you complete the following sentences.



1. The curve is called a .

2. The line $x = -2$ called the .

3. The point $(-2, 4)$ is called the .

Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.

4. $f(x) = -x^2 + 2x + 5$

5. $f(x) = 3x^2 - 4x - 2$

5-2

Solving Quadratic Equations by Graphing

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

6. $x^2 - 2x = 8$

7. $x^2 + 5x - 7 = 0$

5-3

Solving Quadratic Equations by Factoring

8. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution.

$x^2 - 10x = -21$ Original equation

$x^2 - 10x + 21 = 0$

$(x - 3)(x - 7) = 0$

$x - 3 = 0$ or $x - 7 = 0$

$x = 3 \qquad x = 7$

The solution set is .

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

9. $-4, -2$

10. $3, 6$

5-4

Complex Numbers

Simplify.

11. $\sqrt{-2} \cdot \sqrt{-10}$

12. $(3 - 8\boldsymbol{i}) - (5 + 2\boldsymbol{i})$

13. $(4 - \boldsymbol{i})(5 + 2\boldsymbol{i})$

14. $\frac{3 - \boldsymbol{i}}{2 + \boldsymbol{i}}$

15. Solve $5x^2 + 60 = 0$.

5-5

Completing the Square

16. Solve $x^2 + 6x + 9 = 49$ by using the Square Root Property.

17. Solve $x^2 - 2x + 10 = 5$ by completing the square.

18. When the dimensions of a cube are reduced by 2 inches on each side, the surface area of the new cube is 486 square inches. What were the dimensions of the original cube?

5-6

The Quadratic Formula and the Discriminant

The value of the discriminant for a quadratic equation with integer coefficients is shown. Give the number and the type of roots for the equation.

	Value of Discriminant	Number of Roots	Type of Roots
19.	64		
20.	−8		
21.	0		
22.	15		

23. Solve $x^2 - 8x = 2$ by using the Quadratic Formula. Find exact solutions.

5-7

Analyzing Graphs of Quadratic Functions

24. Match each graph with the description of the constants in the equation in vertex form.

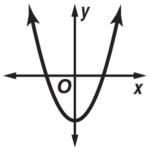
a. $a > 0, h > 0, k < 0$

b. $a < 0, h < 0, k < 0$

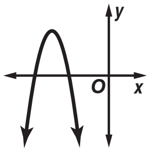
c. $a < 0, h < 0, k > 0$

d. $a > 0, h = 0, k < 0$

i.



ii.



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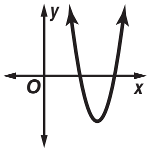
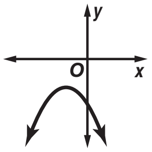


iv.



5-8

Graphing and Solving Quadratic Inequalities

25. Solve $0 < x^2 - 6x + 8$ by graphing.

Solve each inequality algebraically.

26. $x^2 - x > 20$

27. $x^2 - 10x < -16$

CHAPTER

5

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 5.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 5 Practice Test on page 307 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 5 Study Guide and Review on pages 302–306 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 307 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 5 Foldable.
- Then complete the Chapter 5 Study Guide and Review on pages 302–306 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 307 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

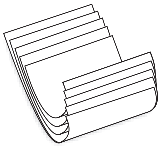
CHAPTER 6

Polynomial Functions

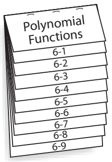
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of notebook paper.

STEP 1 **Stack** sheets of paper with edges $\frac{3}{4}$-inch apart. Fold up the bottom edges to create equal tabs.



STEP 2 **Staple** along the fold. Label the tabs with lesson numbers.



NOTE-TAKING TIP: When you take notes, think about the order in which the concepts are being presented. Write why you think the concepts were presented in this sequence.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 6. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
degree of a polynomial			
depressed polynomial			
dimensional analysis			
end behavior			
leading coefficient			
Location Principle			
polynomial function			
polynomial in one variable			

(*continued on the next page*)

Vocabulary Term	Found on Page	Definition	Description or Example
quadratic form			
relative maximum			
relative minimum			
scientific notation			
simplify			
standard notation			
synthetic division			
synthetic substitution			

6–1 Properties of Exponents

MAIN IDEAS

- Use properties of exponents to multiply and divide monomials.
- Use expressions written in scientific notation.

BUILD YOUR VOCABULARY (pages 161–162)

A **monomial** is an expression that is a number, a variable, or the product of a number and one or more variables.

The numerical factor of a monomial is the **coefficient** of the variable(s). The **degree** of a monomial is the sum of the exponents of its variables. A **power** is an expression of the form x^n.

EXAMPLE Simplify Expressions with Multiplication

1 Simplify each expression.

a. $(-2a^3b)(-5ab^4)$

$(-2a^3b)(-5ab^4)$

$= (-2 \cdot a \cdot a \cdot a \cdot b)$ Definition of exponents

$= -2(-5) \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b$ Commutative Property

$= 10a^4b^5$ Definition of exponents

b. $(3a^5)(c^{-2})(-2a^{-4}b^3)$

$(3a^5)(c^{-2})(-2a^{-4}b^3)$

$= (3a^5)\left(\frac{1}{c^2}\right)\left(\frac{-2}{a^{-4}}\right)(b^3)$ Definition of negative exponents

$= (3 \cdot a \cdot a \cdot a \cdot a \cdot a)\left(\frac{1}{c \cdot c}\right)\left(\frac{-2}{a \cdot a \cdot a \cdot a}\right)(b \cdot b \cdot b)$ Definition of exponents

$= (3 \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a)\left(\frac{1}{c \cdot c}\right)\left(\frac{-2}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}\right)(b \cdot b \cdot b)$ Cancel out common factors.

$=$ Definition of exponents and fractions

KEY CONCEPTS

Negative Exponents For any real number $a \neq 0$ and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Product of Powers For any real number a and integers m and n, $a^m \cdot a^n = a^{m+n}$.

Quotient of Powers For any real number $a \neq 0$, and integers m and $\frac{a^m}{a^n} = a^{m-n}$.

Properties of Powers Suppose a and b are real numbers and m and n are integers. Then the following properties hold.

Power of a Power: $(a^m)^n = a^{mn}$

Power of a Product: $(ab)^m = a^m b^m$

Power of a Quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$, and $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0$, $b \neq 0$

EXAMPLE Simplify Expressions with Division

② **Simplify $\frac{s^2}{s^{10}}$. Assume that $s \neq 0$.**

$\frac{s^2}{s^{10}} =$ Subtract exponents.

$=$ or A simplified expression cannot contain negative exponents.

EXAMPLE Simplify Expressions with Powers

③ **Simplify each expression.**

a. $(-3c^2d^5)^3$

$(-3c^2d^5)^3 =$

$=$

b. $\left(\frac{-2a}{b^2}\right)^5$

$\left(\frac{-2a}{b^2}\right)^5 = \frac{-2^5a^5}{(b^2)^5}$

$=$

Check Your Progress **Simplify each expression.**

a. $(-3x^2y)(5x^3y^5)$

b. $\frac{x^3}{x^7}$

c. $(x^3)^5$

d. $(-2x^2y^3)^5$

e. $\left(\frac{-3x^2}{y^3}\right)^3$

f. $\left(\frac{y}{2}\right)^{-3}$

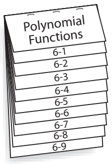
WRITE IT

Write how you read the expression a^3. Then do the same for z^4.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 6-1, write each new Vocabulary Builder word and its definition. Give an example of each word.



EXAMPLE **Simplify Expressions Using Several Properties**

④ **Simplify $\left(\frac{-3a^{5y}}{a^{6y}b^4}\right)^5$.**

Simplify the fraction before raising to the fifth power.

$$\left(\frac{-3a^{5y}}{a^{6y}b^4}\right)^5 = \left(\frac{-3a^{5y-6y}}{b^4}\right)^5$$

$$= \left(\frac{-3a^{-y}}{b^4}\right)^5$$

$$= \qquad \text{or} \qquad$$

Check Your Progress Simplify $\left(\frac{-2a^{3n}}{a^{2n}b^5}\right)^3$.

EXAMPLE **Divide Numbers in Scientific Notation**

⑤ **BIOLOGY** **There are about 5×10^6 red blood cells in one milliliter of blood. A certain blood sample contains 8.32×10^6 red blood cells. About how many milliliters of blood are in the sample?**

Divide the number of red blood cells in the sample by the number of red blood cells in 1 milliliter of blood.

$\overline{\qquad\qquad}$ ← number of red blood cells in sample

5×10^6 ← number of red blood cells in 1 milliliter

$=$ milliliters

Check Your Progress A petri dish contains 3.6×10^5 germs. A half hour later, there are 7.2×10^7. How many times as great is the amount a half hour later?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–2 Operations with Polynomials

MAIN IDEAS

- Add and subtract polynomials.
- Multiply polynomials.

BUILD YOUR VOCABULARY (pages 161–162)

A **polynomial** is a monomial or a sum of monomials.

The polynomial $x^2 + 3x + 1$ is a **trinomial** because it has three unlike terms.

A polynomial such as $xy + z^3$ is a **binomial** because it has two unlike terms.

EXAMPLE Degree of a Polynomial

1 Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $c^4 - 4\sqrt{c} + 18$

This expression ____ a polynomial because ____ is not a monomial.

b. $-16p^5 + \frac{3}{4}p^2q^7$

The expression ____ a polynomial because each term is a monomial. The degree of the first term is ____, and the degree of the second term is 2 + 7 or 9. The degree of the polynomial is ____.

EXAMPLE Subtract and Simplify

2 Simplify each expression.

a. $(2a^3 + 5a - 7) - (a^3 - 3a + 2)$.

$(2a^3 + 5a - 7) - (a^3 - 3a + 2)$

$= 2a^3 + 5a - 7 - a^3 + 3a - 2$

$= (2a^3 - a^3) +$ ____ $+$ ____

$=$ ____

b. $(4x^2 - 9x + 3) + (-2x^2 - 5x - 6)$

$(4x^2 - 9x + 3) + (-2x^2 - 5x - 6)$

$= 4x^2 - 9x + 3 - 2x^2 - 5x - 6$

$= (4x^2 - 2x^2) +$ ____ $+$ ____

$=$ ____

REMEMBER IT

The prefix *bi-* means two, so you can remember that a binomial has two unlike terms.

The prefix *tri-* means three, so you can remember that a trinomial has three unlike terms.

Check Your Progress **Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.**

a. $\frac{1}{2}a^2b^3 + 3c^5$

b. $\sqrt{c} + 2$

c. Simplify $(3x^2 + 2x - 3) - (4x^2 + x - 5)$.

EXAMPLE Multiply and Simplify

③ Find $-y(4y^2 + 2y - 3)$.

$-y(4y^2 + 2y - 3)$

$= -y(4y^2) - y(2y) - y(-3)$ Distributive Property

$= \boxed{}$ Multiply the monomials.

KEY CONCEPT

FOIL Method for Multiplying Binomials The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

FOLDABLES On the page for Lesson 6-2, write examples of multiplying two binomials.

EXAMPLE Multiply Polynomials

④ Find $(a^2 + 3a - 4)(a + 2)$.

$(a^2 + 3a - 4)(a + 2)$

$= a^2(a + 2) + 3a(a + 2) - 4(a + 2)$ Distributive Property

$= a^2 \cdot a + a^2 \cdot 2 + \boxed{} + \boxed{} - \boxed{} - \boxed{}$ Distributive Property

$= a^3 + 2a^2 + 3a^2 + 6a - 4a - 8$ Multiply monomials.

$= \boxed{}$ Combine like terms.

Check Your Progress **Find each product.**

a. $-x(3x^3 - 2x + 5)$

b. $(3p + 2)(5p + 1)$

c. $(x^2 + 3x - 2)(x + 4)$

HOMEWORK ASSIGNMENT

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6–3 Dividing Polynomials

MAIN IDEAS

- Divide polynomials using long division.
- Divide polynomials using synthetic division.

EXAMPLE Divide a Polynomial by a Monomial

① Simplify $\dfrac{5a^2b - 15ab^3 + 10a^3b^4}{5ab}$.

$$\frac{5a^2b - 15ab^3 + 10a^3b^4}{5ab}$$

$$= \frac{5a^2b}{5ab} - \frac{15ab^3}{5ab} + \frac{10a^3b^4}{5ab}$$ Sum of quotients

$$= \frac{5}{5} \cdot a^{2-1}b^{1-1} - \frac{15}{5} \cdot a^{1-1}b^{3-1} + \frac{10}{5} \cdot a^{3-1}b^{4-1}$$ Divide.

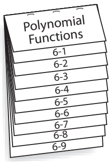
$$=$$ $x^{1-1} = x^0$ or 1

Check Your Progress Simplify $\dfrac{3x^2y + 6x^5y^2 - 9x^7y^3}{3x^2y}$.

FOLDABLES™

ORGANIZE IT

In your notes, explain how to write the answer to a long division problem that has a quotient and a remainder.



EXAMPLE Division Algorithm

② Use long division to find $(x^2 - 2x - 15) \div (x - 5)$.

$$\begin{array}{r} x + 3 \\ x - 5\,\overline{)\,x^2 - 2x - 15} \end{array}$$

(−) $x(x - 5) = x^2 - 5x$

$-2x - (-5x) = 3x$

(−) $3(x - 5) = 3x - 15$

The quotient is .

Check Your Progress Use long division to find $(x^2 + 5x + 6) \div (x + 3)$.

EXAMPLE Quotient with Remainder

3 **TEST EXAMPLE** **Which expression is equal to $(a^2 - 5a + 3)(2 - a)^{-1}$?**

A $a + 3$

B $-a + 3 + \frac{3}{2 - a}$

C $-a - 3 + \frac{3}{2 - a}$

D $-a + 3 - \frac{3}{2 - a}$

Read the Item

Since the second factor has an exponent of ☐, this is a ☐ problem.

$$(a^2 - 5a + 3)(2 - a)^{-1} = \frac{a^2 - 5a + 3}{2 - a}$$

Solve the Item

$$\begin{array}{r l}
 & \quad\;\; -a + 3 \\
-a + 2 \,\big) & \overline{a^2 - 5a + 3} \\
 & \underline{(-)a^2 - 2a} \\
 & \quad\;\; -3a + 3 \\
 & \quad \underline{(-)-3a + 6} \\
 & \qquad\qquad -3
\end{array}$$

The quotient is ☐, and the remainder is ☐.

Therefore, $(a^2 - 5a + 3)(2 - a)^{-1} = -a + 3 - \frac{3}{2 - a}$. The answer is ☐.

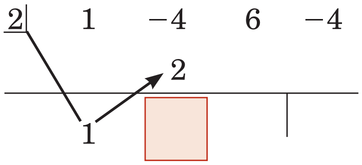
EXAMPLE Synthetic Division

4 **Use synthetic division to find $(x^3 - 4x^2 + 6x - 4) \div (x - 2)$.**

Step 1 With the terms of the dividend in descending order by degree, write just the coefficients.

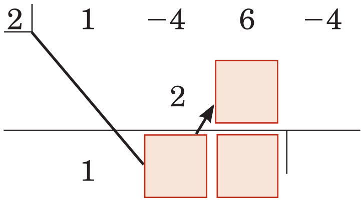
$$\begin{array}{cccc}
x^3 & -4x^2 & +6x & -4 \\
\downarrow & \downarrow & \downarrow & \downarrow \\
1 & -4 & 6 & -4
\end{array}$$

Steps 2 & 3 Write the constant of the divisor $x - r$, 2, to the left. Bring down the first coefficient, 1.

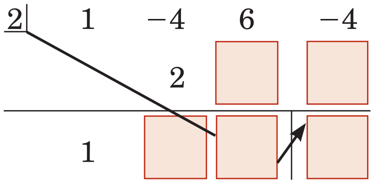


Multiply the first coefficient by r, $1 \cdot 2 = 2$. Write the product under the next coefficient and add.

Step 4 Multiply this sum by r, 2. Write the product under the next coefficient and add.



Step 5 Multiply this sum by r, 2. Write the product under the next coefficient and add.



The numbers along the bottom are the coefficients of the quotient. So, the quotient is $x^2 - 2x + 2$.

EXAMPLE Divisor with First Coefficient Other than 1

5 **Use synthetic division to find $(4y^4 - 5y^2 + 2y + 4) \div (2y - 1)$.**

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{4y^4 - 5y^2 + 2y + 4}{2y - 1} = \frac{(4y^4 - 5y^2 + 2y + 4) \div 2}{(2y - 1) \div 2}$$

$$= \frac{\qquad\qquad}{y - \frac{1}{2}}$$

The numerator does not have a y^3 term. So use a coefficient of 0.

$$\begin{array}{c|ccccc} \frac{1}{2} & 2 & 0 & \frac{-5}{2} & 1 & 2 \\ & & 1 & \frac{1}{2} & -1 & 0 \\ \hline & 2 & 1 & -2 & 0 & 2 \end{array}$$

$y - r = y - \frac{1}{2}$, so $r = \frac{1}{2}$.

The result is $2y^3 + y^2 - 2y + \frac{2}{y - \frac{1}{2}}$. Now simplify the fraction.

$$\frac{2}{y - \frac{1}{2}} = 2 \div \left(y - \frac{1}{2}\right)$$

$$= 2 \div \qquad = 2 \cdot \qquad \text{or}$$

The solution is .

Check Your Progress

a. Use long division to find $(x^2 + 5x + 6) \div (x + 3)$.

b. Use synthetic division to find $(16y^4 - 4y^2 + 2y + 8) \div (2y + 1)$.

HOMEWORK ASSIGNMENT

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6–4 Polynomial Functions

MAIN IDEAS

- Evaluate polynomial functions.
- Identify general shapes of graphs of polynomial functions.

BUILD YOUR VOCABULARY (pages 161–162)

The **leading coefficient** is the coefficient of the term with the degree.

A common type of function is a **power function**, which has an equation in the form , where *a* and *b* are real numbers.

KEY CONCEPT

A Polynomial in One Variable A polynomial of degree *n* in one variable *x* is an expression of the form $a_0x^n + a_1x^{n-1} + \cdots + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, \ldots, a_n$, represent real numbers, a_0 is not zero, and *n* represents a nonnegative integer.

EXAMPLE Find Degree and Leading Coefficients

① **State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.**

a. $7z^3 - 4z^2 + z$

This is a polynomial in one variable. The degree is and the leading coefficient is .

b. $6a^3 - 4a^2 + ab^2$

This a polynomial in one variable. It contains two variables, and .

Check Your Progress **State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.**

a. $3x^3 + 2x^2 - 3$

b. $3x^2 + 2xy - 5$

KEY CONCEPT

Definition of a Polynomial Function A polynomial function of degree n can be described by an equation of the form $P(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, \ldots, a_n$, represent real numbers, a_0 is not zero, and n represents a nonnegative integer.

EXAMPLE Evaluate a Polynomial Function

2 **NATURE** **Refer to the application at the beginning of Lesson 6-4 in your textbook. A sketch of the arrangement of hexagons shows a fourth ring of 18 hexagons, a fifth ring of 24 hexagons, and a sixth ring of 30 hexagons.**

a. Show that the polynomial function $f(r) = 3r^2 - 3r + 1$ gives the total number of hexagons when $r = 4$, 5, and 6.

$f(4) = 3(4)^2 - 3(4) + 1$, or ;

$f(5) = 3(5)^2 - 3(5) + 1$, or ;

$f(6) = 3(6)^2 - 3(6) + 1$, or ;

The total number of hexagons for four rings is 19 + 18, or , five rings is , or 61, and six rings is 61 + 30, or . These the functional values for $r = 4$, 5, and 6, respectively.

b. Find the total number of hexagons in a honeycomb with 25 rings.

$f(r) = 3r^2 - 3r + 1$ Original function

$f(25) =$ Replace r with 25.

$=$ or Simplify.

Check Your Progress **Refer to Example 2.**

a. What are the total number of hexagons when $r = 7$, 8, and 9?

b. Find the total number of hexagons in a honeycomb with 30 rings.

EXAMPLE **Function Values of Variables**

3 **Find $b(2x - 1) - 3b(x)$ if $b(m) = 2m^2 + m - 1$.**

To evaluate $b(2x - 1)$, replace m in $b(m)$ with .

$b(m) = 2m^2 + m - 1$ Original function

$b(2x - 1) = 2(2x - 1)^2 + (2x - 1) - 1$ Replace m with $2x - 1$.

$= \quad + 2x - 1 - 1$ Evaluate $2(2x - 1)^2$.

$=$ Simplify.

To evaluate $3b(x)$, replace m with x in $b(m)$, then multiply the expression by .

$b(m) = 2m^2 + m - 1$ Original function

$3b(x) = 3(2x^2 + x - 1)$ Replace m with x.

$=$ Distributive Property

Now evaluate $b(2x - 1) - 3b(x)$.

$b(2x - 1) - 3b(x) = \quad -$

Replace $b(2x - 1)$ and $3b(x)$ with evaluated expressions.

$= 8x^2 - 6x - 6x^2 - 3x + 3$

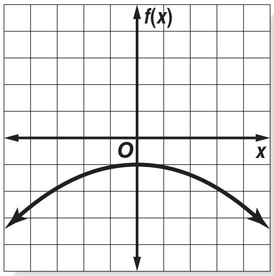
$=$ Simplify.

Check Your Progress Find $g(2x + 1) - 2g(x)$ if $g(b) = b^2 + 3$.

EXAMPLE **Graphs of Polynomial Functions**

4 **For each graph, describe the end behavior, determine whether it represents an odd-degree or an even-degree function, and state the number of real zeros.**

a.

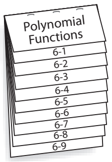


- $f(x) \to$ as $x \to +\infty$.
- $f(x) \to$ as $x \to -\infty$.
- It is an polynomial function.
- The graph does not intersect the x-axis, so the function has real zeros.

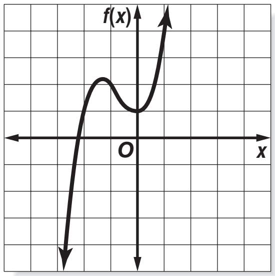
FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-4, sketch the graph of a function that has no real zeros, one real zero, and two real zeros.



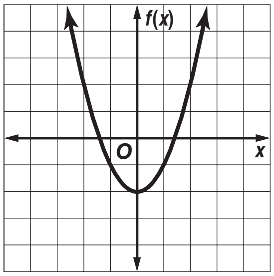
b.



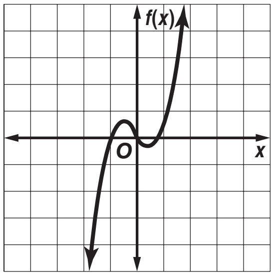
- $f(x) \rightarrow$ ☐ as $x \rightarrow +\infty$.
- $f(x) \rightarrow$ ☐ as $x \rightarrow -\infty$.
- It is an ☐ polynomial function.
- The graph intersects the x-axis at one point, so the function has ☐ real zero.

Check Your Progress **For each graph, describe the end behavior, determine whether it represents an odd-degree or an even-degree function, and state the number of real zeros.**

a.



b.



HOMEWORK ASSIGNMENT

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6–5 Analyzing Graphs of Polynomial Functions

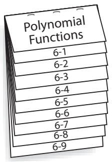
MAIN IDEAS

- Graph polynomial functions and locate their real zeros.
- Find the maxima and minima of polynomial functions.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-5, explain how knowing the end behavior of a graph will assist you in completing the sketch of the graph.



EXAMPLE Graph a Polynomial Function

① **Graph $f(x) = -x^3 - 4x^2 + 5$ by making a table of values.**

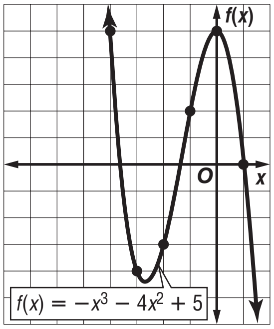
This is an odd degree polynomial with a negative leading coefficient, so

$f(x) \rightarrow$ ______ as $x \rightarrow -\infty$ and

$f(x) \rightarrow$ ______ as $x \rightarrow +\infty$.

The graph intersects the x-axis at ______ points indicating that there are ______ real zeros.

x	$f(x)$
−4	
−3	
−2	
−1	
0	
1	
2	



Check Your Progress Graph $f(x) = x^3 + 2x^2 + 1$ by making a table of values.

EXAMPLE Locate Zeros of a Function

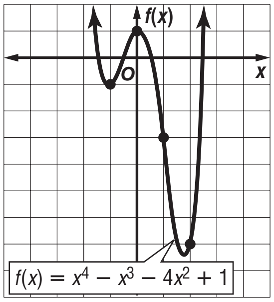
1 Determine consecutive values of x between which each real zero of the function $f(x) = x^4 - x^3 - 4x^2 + 1$ is located. Then draw the graph.

Make a table of values. Since $f(x)$ is a 4th degree polynomial function, it will have between 0 and 4 zeros, inclusive. Look at the value of $f(x)$ to locate the zeros. Then use the points to sketch the graph of the function.

KEY CONCEPT

Location Principle Suppose $y = f(x)$ represents a polynomial function and a and b are two numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b.

x	$f(x)$	
-2	9	} sign change
-1	-1	} sign change
0	1	} sign change
1	-3	
2	-7	} sign change
3	19	



There are zeros between $x =$ ____ and ____, $x =$ ____ and ____, $x =$ ____ and ____, and $x =$ ____ and ____.

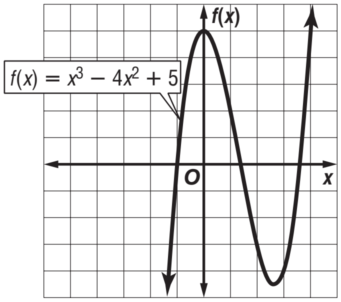
Check Your Progress Determine consecutive values of x between which each real zero of the function $f(x) = x^3 - 4x^2 + 2$ is located. Then draw the graph.

BUILD YOUR VOCABULARY (page 162)

A point on a graph is a **relative maximum** of a function if no other nearby points have a ____ y-coordinate. Likewise, a point is a **relative minimum** if no other nearby points have a ____ y-coordinate.

EXAMPLE Maximum and Minimum Points

3 Graph $f(x) = x^3 - 4x^2 + 5$. Estimate the x-coordinates at which the relative maximum and relative minimum occur.



Make a table of values and graph the function.

x	$f(x)$
−2	−19
−1	0
0	5
1	2
2	−3
3	−4
4	5
5	30

zero at $x = -1$

← indicates a relative maximum

} zero between $x = 1$ and $x = 2$

← indicates a relative minimum

} and zero between $x = 3$ and $x = 4$

The value of $f(x)$ at $x =$ is greater than the surrounding points, so it is a relative . The value of $f(x)$ at about $x =$ is less than the surrounding points, so it is a relative .

Check Your Progress

Graph $f(x) = x^3 + 3x^2 + 2$. Estimate the x-coordinates at which the relative maximum and relative minimum occur.

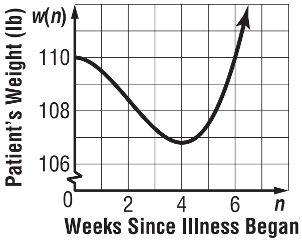
EXAMPLE Graph a Polynomial Model

④ **HEALTH The weight w, in pounds, of a patient during a 7-week illness is modeled by the cubic equation $w(n) = 0.1n^3 - 0.6n^2 + 110$, where n is the number of weeks since the patient became ill.**

a. Graph the equation. Describe the turning points of the graph and its end behavior.

Make a table of values for the weeks 1–7. Plot the points and connect with a smooth curve.

w	*w*(*n*)
1	
2	108.4
3	
4	106.8
5	
6	110



Relative minimum point at week ____.

End behavior: $w(n)$ increases as ____ increases

b. What trends in the patient's weight does the graph suggest? Is it reasonable to assume the trend will continue indefinitely?

The patient lost weight for ____ weeks; then gained weight.

The trend may continue for a few weeks, but it is ____ that the patient's weight will gain so quickly indefinitely.

Check Your Progress **The rainfall r, in inches per month, during a 7-month period is modeled by the $r(m) = 0.01m^3 - 0.18m^2 + 0.67m + 3.23$, where m is the number of months after March 1.**

a. Graph the equation. Describe the turning points of the graph and its end behavior.

b. What trends in the amount of rainfall received by the town does the graph suggest?

HOMEWORK ASSIGNMENT

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6–6 Solving Polynomial Equations

MAIN IDEAS

- Factor polynomials.
- Simplify polynomial quotients by factoring.

EXAMPLE GCF

1 **Factor $10a^3b^2 + 15a^2b - 5ab^3$.**

$10a^3b^2 + 15a^2b - 5ab^3$

$= (2 \cdot 5 \cdot a \cdot a \cdot a \cdot b \cdot b) + (3 \cdot 5 \cdot a \cdot a \cdot b) - (5 \cdot a \cdot b \cdot b \cdot b)$

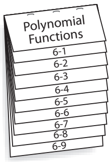
$= \left(5ab \cdot \boxed{}\right) + \left(5ab \cdot \boxed{}\right) - \left(5ab \cdot \boxed{}\right)$ The GCF is $5ab$.

$= \boxed{}$ Distributive Property

FOLDABLES™

ORGANIZE IT

On the page for Lesson 6-6, write an ordered list describing what you look for as you factor a polynomial.



EXAMPLE Grouping

2 **Factor $x^3 + 5x^2 - 2x - 10$.**

$x^3 + 5x^2 - 2x - 10$

$= \boxed{} + \boxed{}$ Group to find the GCF.

$= x^2\boxed{} + (-2)\boxed{}$ Factor the GCF of each binomial.

$= \boxed{}$ Distributive Property

Check Your Progress **Factor each polynomial.**

a. $6x^4y^2 + 9x^2y^2 - 3xy^2$ $\boxed{}$

b. $x^3 - 3x^2 + 4x - 12$ $\boxed{}$

EXAMPLE Factor Polynomials

3 **Factor each polynomial.**

a. $\mathbf{12y^3 - 8y^2 - 20y}$

This trinomial does not fit any of the factoring patterns.

First, factor out the $\boxed{}$.

$12y^3 - 8y^2 - 20y = \boxed{}(3y^2 - 2y - 5)$ Factor out the GCF.

The coefficient of the y terms must be 3 and -5 since

$3(-5) = \boxed{\quad}$ and $3 + (-5) = \boxed{\quad}$. Rewrite the

expression using $-5y$ and $3y$ in place of $-2y$ and factor by grouping.

$3y^2 - 2y - 5 = 3y^2 - 5y + 3y - 5$	Replace $-2y$
$= (3y^2 - 5y) + (3y - 5)$	Associative Property
$= \boxed{\quad}(3y - 5) + \boxed{\quad}(3y - 5)$	Factor
$= (3y - 5)(y + 1)$	Distributive Property
$= \boxed{\qquad\qquad}$	

b. $64x^6 - y^6$

This polynomial could be considered the difference of two $\boxed{\qquad}$ or the difference of two $\boxed{\qquad}$. The difference of two squares should always be done before the difference of two cubes. This will make the next step of the factorization easier.

$64x^6 - y^6$

$= (8x^3 + y^3)(8x^3 - y^3)$	Difference of two squares
$= (2x + y)(4x^2 - 2xy + y^2)(2x - y)(4x^2 + 2xy + y^2)$	Sum and difference of two cubes

Check Your Progress **Factor each polynomial.**

a. $2x^2 + x - 3$

$\boxed{\qquad\qquad}$

b. $a^3b^3 - 27$

$\boxed{\qquad\qquad}$

KEY CONCEPT

Quadratic Form An expression that is quadratic in form can be written as $au^2 + bu + c$ for any numbers a, b, and c, $a \neq 0$, where u is some expression in x. The expression $au^2 + bu + c$ is called the quadratic form of the original expression.

EXAMPLE Write an Expression in Quadratic Form

4 **Write each expression in quadratic form, if possible.**

a. $2x^6 - x^3 + 9$ $\boxed{\qquad} - (x^3) + 9$ $x^6 = \boxed{\qquad}$

b. $x^4 - 2x^3 - 1$ This cannot be written in quadratic form since $x^4 \neq \boxed{\qquad}$.

Check Your Progress **Write each expression in quadratic form, if possible.**

a. $2x^4 + x^2 + 3$

$\boxed{}$

b. $x^6 + x^4 + 1$

$\boxed{}$

c. $x - 2x^{\frac{1}{2}} + 3$

$\boxed{}$

d. $x^{12} + 5$

$\boxed{}$

EXAMPLE Solve Polynomial Equations

⑤ **a. Solve $x^3 + 216 = 0$.**

$x^3 + 216 = 0$ Original equation

$x^3 + \boxed{} = 0$ Sum of two cubes

$(x + 6)\boxed{} = 0$ Factor.

$\boxed{} = 0$ or $\boxed{} = 0$ Zero Product Property

The solution of the first equation is $\boxed{}$. The second equation can be solved by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(36)}}{2(1)}$$ Replace *a* with 1, *b* with −6, and *c* with 36.

$$x = \frac{6 \pm \boxed{}}{2}$$ Simplify.

$x = \dfrac{6 \pm \boxed{\quad\quad}}{2}$ or $\dfrac{6 \pm \boxed{\quad\quad}}{2}$ Multiply $\sqrt{108}$ and $\sqrt{-1}$.

$x = \boxed{\quad\quad\quad}$ Simplify.

b. Solve $x^4 - 29x^2 + 100 = 0$.

$x^4 - 29x^2 + 100 = 0$ Original equation

$\boxed{\quad\quad} - 29(x^2) + 100 = 0$ Write the expression on the left in quadratic form.

$\boxed{\quad\quad}\boxed{\quad\quad} = 0$ Factor the trinomial.

$\boxed{\quad\quad\quad\quad\quad\quad} = 0$ Factor.

Use the Zero Product Property.

$\boxed{\quad\quad} = 0$ or $\boxed{\quad\quad} = 0$

$x = \boxed{\quad\quad}$ $x = \boxed{\quad\quad}$

$\boxed{\quad\quad} = 0$ or $\boxed{\quad\quad} = 0$

$x = \boxed{\quad\quad}$ $x = \boxed{\quad\quad}$

Check Your Progress **Solve each equation.**

a. $x^4 - 10x^2 + 9 = 0$

b. $x^3 + 8 = 0$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–7 The Remainder and Factor Theorems

MAIN IDEAS

- Evaluate functions using synthetic substitution.
- Determine whether a binomial is a factor of a polynomial by using synthetic substitution.

KEY CONCEPTS

Remainder Theorem
If a polynomial $f(x)$ is divided by $x - a$, the remainder is the constant $f(a)$, and

Dividend	equals	quotient
$f(x)$	$=$	$q(x)$

times	divisor	plus	remainder.
$\cdot$	$(x - a)$	$+$	$f(a)$.

where $q(x)$ is a polynomial with degree one less than the degree of $f(x)$.

Factor Theorem
The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

BUILD YOUR VOCABULARY (page 162)

When synthetic division is used to evaluate a function, it is called **synthetic substitution**.

EXAMPLE Synthetic Substitution

① **If $f(x) = 3x^4 - 2x^3 + x^2 - 2$, find $f(4)$.**

METHOD 1 Synthetic Substitution

By the Remainder Theorem, $f(4)$ should be the remainder when you divide the polynomial by $x - 4$.

$$\underline{4|} \quad 3 \quad -2 \quad 1 \quad 0 \quad -2$$

Notice that there is no x term. A zero is placed in this position as a placeholder.

The remainder is ____. Thus, by using synthetic substitution, $f(4) =$ ____.

METHOD 2 Direct Substitution

Replace x with 4.

$f(x) = 3x^4 - 2x^3 + x^2 - 2$ Original function

$f(4) =$ ____ Replace x with 4.

$f(4) =$ ____ or ____ Simplify.

By using direct substitution, $f(4) =$ ____.

Check Your Progress If $f(x) = 2x^3 - 3x^2 + 7$, find $f(3)$.

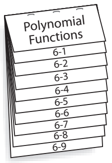
BUILD YOUR VOCABULARY (page 161)

When you divide a polynomial by one of its **binomial** factors, the quotient is called a **depressed polynomial**.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-7, use the Factor Theorem to show that $x + 3$ is a factor of $3x^3 - 3x^2 - 36x$.



EXAMPLE Use the Factor Theorem

1 **Show that $x - 3$ is a factor of $x^3 + 4x^2 - 15x - 18$. Then find the remaining factors of the polynomial.**

The binomial $x - 3$ is a factor of the polynomial if 3 is a zero of the related polynomial function. Use the factor theorem and synthetic division.

$\underline{3}$	1	4	−15	−18

Since the remainder is 0, is a factor of the polynomial. The polynomial $x^3 + 4x^2 - 15x - 18$ can be factored as . The polynomial is the depressed polynomial. Check to see if this polynomial can be factored.

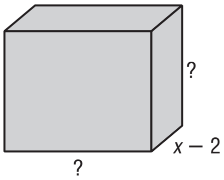
$x^2 + 7x + 6 =$ Factor the trinomial.

So, $x^3 + 4x^2 - 15x - 18 =$.

Check Your Progress Show that $x + 2$ is a factor of $x^3 + 8x^2 + 17x + 10$. Then find the remaining factors of the polynomial.

EXAMPLE Find All Factors

③ **GEOMETRY The volume of the rectangular prism is given by $V(x) = x^3 + 7x^2 + 2x - 40$. Find the missing measures.**



The volume of a rectangular prism is $\ell \times w \times h$.

You know that one measure is , so $x - 2$ is a factor
of $V(x)$.

2	1	7	2	−40

The quotient is $x^2 + 9x + 20$. Use this to factor $V(x)$.

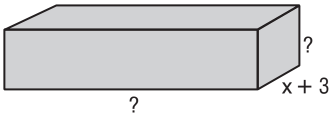
$V(x) = x^3 + 7x^2 + 2x - 40$ Volume function

$= \quad (x^2 + 9x + 20)$ Factor.

$=$ Factor the trinomial $x^2 + 9x + 20$.

So the missing measures of the prism are and .

Check Your Progress The volume of a rectangular prism is given by $V(x) = x^3 + 6x^2 - x - 30$. Find the missing measures.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–8 Roots and Zeros

MAIN IDEAS

- Determine the number and type of roots for a polynomial equation.
- Find the zeros of a polynomial function.

KEY CONCEPTS

Fundamental Theorem of Algebra Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

Corollary A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.

FOLDABLES™ On the tab for Lesson 6-8, write these key concepts. Be sure to include examples.

EXAMPLE Determine Number and Type of Roots

① **Solve each equation. State the number and type of roots.**

a. $x^2 + 2x - 48 = 0$

$x^2 + 2x - 48 = 0$ Original equation

$= 0$ Factor.

Use the Zero Product Property.

$= 0$ or $= 0$

$x =$ $x =$ Solve each equation.

This equation has two real roots, and .

b. $y^4 - 256 = 0$

$$y^4 - 256 = 0$$
$$(y^2 + 16)(y^2 - 16) = 0$$
$$(y^2 + 16)(y + 4)(y - 4) = 0$$

$y^2 +$ $= 0$ or $y +$ $= 0$ or $y -$ $= 0$

$y^2 =$ $y =$ $y =$

$y = \pm\sqrt{-16}$ or $\pm 4i$

This equation has two real roots, , and two imaginary roots, .

Check Your Progress **Solve each equation. State the number and type of roots.**

a. $x^2 - x - 12 = 0$

b. $a^4 - 81 = 0$

EXAMPLE Find Numbers of Positive and Negative Zeros

KEY CONCEPT

Descartes' Rule of Signs If $P(x)$ is a polynomial with real coefficients whose terms are arranged in descending powers of the variable,

- the number of positive real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and
- the number of negative real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this number by an even number.

① **State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $p(x) = -x^6 + 4x^3 - 2x^2 - x - 1$.**

Since $p(x)$ has degree 6, it has 6 zeros. However, some of them may be imaginary. Use Descartes' Rule of Signs to determine the number and type of real zeros. Count the number of changes in sign for the coefficients of $p(x)$.

$$p(x) = -x^6 \quad + \quad 4x^3 \quad - \quad 2x^2 \quad - \quad x \quad - \quad 1$$

Since there are two sign changes, there are 2 or 0 positive real zeros. Find $p(-x)$ and count the number of sign changes for its coefficients.

$$\begin{aligned} p(-x) &= -(-x)^6 \quad + \quad 4(-x)^3 \quad - \quad 2(-x)^2 \quad - \quad (-x) \quad - \quad 1 \\ &= -x^6 \quad - \quad 4x^3 \quad - \quad 2x^2 \quad + \quad x \quad - \quad 1 \end{aligned}$$

Since there are two sign changes, there are 2 or 0 negative real zeros. Make a chart of possible combinations.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total
2	2		6
0			6
2			6
0			6

Check Your Progress State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $p(x) = x^4 - x^3 + x^2 + x + 3$.

EXAMPLE Use Synthetic Substitution to Find Zeros

3 Find all of the zeros of $f(x) = x^3 - x^2 + 2x + 4$.

Since $f(x)$ has degree 3, the function has three zeros. To determine the possible number and type of real zeros, examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = x^3 \quad - \quad x^2 \quad + \quad 2x \quad + \quad 4$$

$$\boxed{} \quad \boxed{} \quad \boxed{}$$

$$f(-x) = -x^3 \quad - \quad x^2 \quad - \quad 2x \quad + \quad 4$$

$$\boxed{} \quad \boxed{} \quad \boxed{}$$

The function has $\boxed{}$ or $\boxed{}$ positive real zeros and exactly

1 negative real zero. Thus the function has either $\boxed{}$

positive real zeros and $\boxed{}$ negative real zero or $\boxed{}$

imaginary zeros and $\boxed{}$ negative real zero.

To find the zeros, list some possibilities and eliminate those that are not zeros. Use a shortened form of synthetic substitution to find $f(a)$ for several values of a.

x	1	−1	2	4
−3	1	−4	14	−38
−2	1	−3	8	−12
−1	1	−2	4	0

From the table, we can see that one zero occurs at $x = \boxed{}$.

Since the depressed polynomial of this zero, $x^2 - 2x + 4$, is quadratic, use the Quadratic Formula to find the roots of the related quadratic equation, $x^2 - 2x + 4 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{(-)\boxed{} \pm \sqrt{\left(\boxed{}\right)^2 - 4\left(\boxed{}\right)\left(\boxed{}\right)}}{2\left(\boxed{}\right)}$$ Replace *a* with 1, *b* with −2, and *c* with 4.

$$= \frac{2 \pm \sqrt{-12}}{2}$$ Simplify.

Thus, the function has one real zero at $x =$, and two imaginary zeros at $x =$ and $x =$.

Check Your Progress Find all the zeros of $f(x) = x^3 - 3x^2 - 2x + 4$.

EXAMPLE Use Zeros to Write a Polynomial Function

4 Write a polynomial function of least degree with integral coefficients, the zeros of which include 4 and 4 − $\boldsymbol{i}$.

Explore If $4 - \boldsymbol{i}$ is a zero, then is also a zero. So, $x - 4$, $x - (4 - \boldsymbol{i})$, and are factors of the polynomial function.

Plan Write the polynomial function as a product of its factors: $f(x) = (x - 4)[x - (4 - \boldsymbol{i})][x - (4 + \boldsymbol{i})]$

Solve Multiply the factors to find the polynomial function.

$f(x) = (x - 4)x - (4 - \boldsymbol{i})[x - (4 + \boldsymbol{i})]$

$= (x - 4)$ $[(x - 4) + \boldsymbol{i}]$

$= (x - 4)$

$= (x - 4)$

$= (x - 4)$

$=$

$=$

$f(x) =$ is a polynomial function of least degree with integral coefficients and zeros of 4, $4 - \boldsymbol{i}$, and $4 + \boldsymbol{i}$.

Check Your Progress Write a polynomial function of least degree with integral coefficients whose zeros include 2 and $1 + \boldsymbol{i}$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–9 Rational Zero Theorem

MAIN IDEAS

- Identify the possible rational zeros of a polynomial function.
- Find all the rational zeros of a polynomial function.

KEY CONCEPTS

Rational Zero Theorem Let $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-2}x^2 + a_{n-1}x + a_n$ represent a polynomial function with integral coefficients. If $\frac{p}{q}$ is a rational number in simplest form and is a zero of $y = f(x)$, then p is a factor of a_n and q is a factor of a_0.

Corollary (Integral Zero Theorem) If the coefficients of a polynomial function are integers such that $a_0 = 1$ and $a_n \neq 0$, any rational zeros of the function must be factors of a_n.

EXAMPLE Identify Possible Zeros

1 List all of the possible rational zeros of each function.

a. $f(x) = 3x^4 - x^3 + 4$

If $\frac{p}{q}$ is a rational zero, then p is a factor of 4 and q is a factor of 3. The possible factors of p are ______, ______, and ______. The possible factors of q are ______ and ______.

So, $\frac{p}{q}$ ______.

b. $f(x) = x^4 + 7x^3 - 15$

Since the coefficient of x^4 is ______ the possible rational zeros must be the factors of the constant term -15. So, the possible rational zeros ______.

Check Your Progress **List all of the possible rational zeros of each function.**

a. $f(x) = 2x^3 + x + 6$

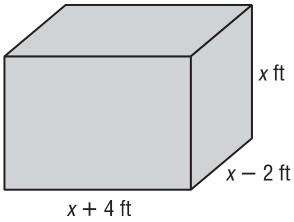
______

b. $f(x) = x^3 + 3x + 24$

______

EXAMPLE Find Rational Zeros

2 GEOMETRY The volume of a rectangular solid is 1120 cubic feet. The width is 2 feet less than the height, and the length is 4 feet more than the height. Find the dimensions of the solid.



Let x = the height, $\qquad$ = the width, and $\qquad$ = the length. Write an equation for the volume.

$\ell wh = V$	Formula for volume
$(x - 2)(x + 4)x = 1120$	Substitute.
$\qquad = 1120$	Multiply.
$x^3 + 2x^2 - 8x - 1120 = 0$	Subtract 1120.

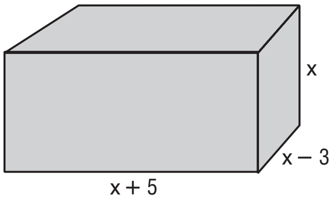
The leading coefficient is 1, so the possible integer zeros are factors of 1120, ±1, ±2, ±4, ±5, ±7, ±8, ±10, ±14, ±16, ±20, ±28, ±32, ±35, ±40, ±56, ±70, ±80, ±112, ±140, ±160, ±224, ±280, ±560, and ±1120. Since length can only be positive, we only need to check positive zeros. From Descartes' Rule of Signs, we also know there is only one positive real zero. Make a table and test possible real zeros.

p	1	2	−8	−1120
2	1	3	−5	−1126
4	1	6	16	−1056
7	1	9	55	−735
10	1	12	112	0

Our zero is $\qquad$. Since there is only one positive real zero, we do not have to test the other numbers. The other dimensions are 10 + 4 or $\qquad$ feet and 10 − 2 or $\qquad$ feet.

Check Your Progress

The volume of a rectangular solid is 100 cubic feet. The width is 3 feet less than the height, and the length is 5 feet more than the height. Find the dimensions of the solid.



EXAMPLE

Find All Zeros

③ **Find all of the zeros of $f(x) = x^4 + x^3 - 19x^2 + 11x + 30$.**

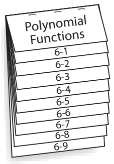
From the corollary to the Fundamental Theorem of Algebra, we know there are exactly 4 complex roots.

According to Descartes' Rule of Signs, there are 2 or 0 positive real roots and 2 or 0 negative real roots.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-9, write how you would find the possible rational zeros of $f(x) = x^3 + 6x^2 + x + 6$.



The possible rational zeros are

.

Make a table and test some possible rational zeros.

$\frac{p}{q}$	1	1	−19	11	30
0					
1					
2					

Since $f(2) = 0$, you know that $x = 2$ is a zero. The depressed polynomial is .

Since $x = 2$ is a positive real zero, and there can only be 2 or 0 positive real zeros, there must be one more positive real zero. Test the next possible rational zeros on the depressed polynomial.

$\frac{p}{q}$	1	3	−13	−15
3				

There is another zero at $x = 3$. The depressed polynomial is

.

Factor $x^2 + 6x + 5$.

$x^2 + 6x + 5 = 0$ Write the depressed polynomial.

$= 0$ Factor.

$= 0$ or $= 0$ Zero Product Property

$x =$ $x =$

There are two more real zeros at $x =$ and $x =$.

Check Your Progress Find all of the zeros of $f(x) = x^4 + 4x^3 - 14x^2 - 36x + 45$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 6

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 6 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 6, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 161–162)* to help you solve the puzzle.

6-1

Properties of Exponents

Simplify. Assume that no variable equals 0.

1. $(3n^4y^3)(-2ny^{-5})$

2. $\dfrac{12(x^2y)^3}{4(xy^0)^2}$

6-2

Operations with Polynomials

Determine whether each expression is a polynomial. If the expression is a polynomial, classify it by the number of terms and state the degree of the polynomial.

3. $\sqrt{3x}$

4. $4r^4 - 2r + 1$

5. $2ab + 4ab^2 - 6ab^3$

6. $5x + 4$

Simplify.

7. $(3a - 6) - (2a - 1)$

8. $(2x - 5)(3x + 5)$

6-3

Dividing Polynomials

Simplify.

9. $(c^3 + c^2 - 14c - 24) \div (c - 4)$

10. $\frac{n^2 + 3n - 2}{n + 2}$

6-4

Polynomial Functions

11. Give the degree and leading coefficient of each polynomial.

	degree	**leading coefficient**
a. $10x^3 + 3x^2 - x + 7$		
b. $7y^2 - 2y^5 + y - 4y^3$		
c. 100		

6-5

Analyzing Graphs of Polynomial Functions

12. Graph $f(x) = x^3 - 6x^2 + 2x + 8$ by making a table of values. Let $x = \{-2, -1, 0, 1, 2, 3\}$. Then determine consecutive values of x between which each real zero is located. Estimate the x-coordinates at which the relative maxima and relative minima occur.

6-6

Solving Polynomial Equations

Factor completely. If the polynomial is not factorable, write *prime*.

13. $3w^2 - 48$

14. $a^3 + 5a - 3a^2 - 15$

15. Simplify $\frac{x^2 + 7x + 10}{x^2 - 4}$. Assume that the denominator is not equal 0.

6-7

The Remainder and Factor Theorems

Find $f(-2)$ for each function.

16. $f(x) = x^3 + 4x^2 - 8x - 6$

17. $f(x) = x^3 + 4x^2 + 4x$

6-8

Roots and Zeros

Let $f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 + 6x - 7$.

18. Write $f(-x)$ in simplified form (with no parentheses).

19. What are the possible numbers of positive real zeros of f? negative real zeros of f?

6-9

Rational Zero Theorem

20. List all the possible values of p, all the possible values of q, and all the possible rational zeros $\frac{p}{q}$ for $f(x) = x^3 - 2x^2 - 11x + 12$.

CHAPTER 6

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 6.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 6 Practice Test on page 379 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 6 Study Guide and Review on pages 374–378 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 379 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 6 Foldable.
- Then complete the Chapter 6 Study Guide and Review on pages 374–378 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 379 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

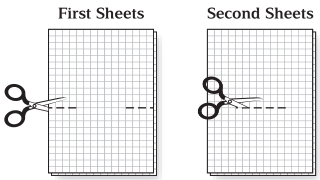
CHAPTER 7

Radical Equations and Inequalities

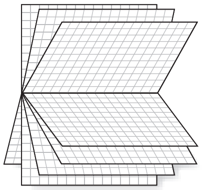
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with four sheets of grid paper..

STEP 1 **Fold** in half along the width. On the first two sheets, cut 5 centimeters along the fold at the ends. On the second two sheets, cut in the center, stopping 5 centimeters from the ends.



STEP 2 **Insert** the first sheets through the second sheets and align the folds. Label the pages with lesson numbers.



NOTE-TAKING TIP: When you take notes, preview the lesson and make generalizations about what you think you will learn. Then compare that with what you actually learned after each lesson.

CHAPTER

7

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 7. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
composition of functions			
conjugates			
extraneous solution			
identity function			
inverse function			
inverse relation			
like radical expressions			

Vocabulary Term	Found on Page	Definition	Description or Example
nth root			
one-to-one			
principal root			
radical equation			
radical inequality			
rationalizing the denominator			
square root function			
square root inequality			

7–1 Operations on Functions

MAIN IDEAS

- Find the sum, difference, product, and quotient of functions.
- Find the composition of functions.

KEY CONCEPT

Operations with Functions

Sum
$(f+g)(x)=f(x)+g(x)$

Difference
$(f-g)(x)=f(x)-g(x)$

Product
$(f\cdot g)(x)=f(x)\cdot g(x)$

Quotient
$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0$

EXAMPLE Add and Subtract Functions

1 Given $f(x)=3x^2+7x$ and $g(x)=2x^2-x-1$, find each function.

a. $(f+g)(x)$

$(f+g)(x)=f(x)+g(x)$

$=\boxed{}+\boxed{}$

$=\boxed{}$

b. $(f-g)(x)$

$(f-g)(x)=f(x)-g(x)$

$=\boxed{}-\boxed{}$

$=\boxed{}$

Check Your Progress **Given $f(x)=2x^2+5x+2$ and $g(x)=3x^2+3x-4$, find each function.**

a. $(f+g)(x)$ $\boxed{}$

b. $(f-g)(x)$ $\boxed{}$

EXAMPLE Multiply and Divide Functions

2 Given $f(x)=3x^2-2x+1$ and $g(x)=x-4$, find each function.

a. $(f\cdot g)(x)$

$(f\cdot g)(x)=f(x)\cdot g(x)$

$=\left(\boxed{}\right)\left(\boxed{}\right)$

$=3x^2(x-4)-2x(x-4)+1(x-4)$ Distributive Property

$=\boxed{}$ Distributive Property

$=\boxed{}$ Simplify.

b. $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(f)}$$

$$=$$

Check Your Progress **Given $f(x) = 2x^2 + 3x - 1$ and $g(x) = x + 2$, find each function.**

a. $(f \cdot g)(x)$

b. $\left(\frac{f}{g}\right)(x)$

EXAMPLE Evaluate Compositon of Relations

3 **If $f(x) = \{(2, 6), (9, 4), (7, 7), (0, -1)\}$ and $g(x) = \{(7, 0), (-1, 7), (4, 9), (8, 2)\}$, find $f \circ g$ and $g \circ f$.**

To find $f \circ g$, evaluate $g(x)$ first. Then use the range of g as the domain of f and evaluate $f(x)$.

$f[g(7)] = f(0)$ or

$f[g(4)] = f(9)$ or

$f[g(-1)] = f(7)$ or

$f[g(8)] = f(2)$ or

$f \circ g =$

To find $g \circ f$, evaluate $f(x)$ first. Then use the range of f as the domain of g and evaluate $g(x)$.

$g[f(2)] = g(6)$, which is

$g[f(7)] = g(7)$ or

$g[f(9)] = g(4)$ or

$g[f(0)] = g(-1)$ or

$g \circ f =$

Check Your Progress If $f(x) = \{(1, 2), (0, -3), (6, -5), (2, 1)\}$ and $g(x) = \{(2, 0), (-3, 6), (1, 0), (6, 7)\}$, find $f \circ g$ and $g \circ f$.

KEY CONCEPT

Composition of Functions Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

FOLDABLES On the page for Lesson 7-1, write how you would read $[f \circ g](x)$. Then explain which function, f or g, you would evaluate first.

EXAMPLE Simplify Composition of Functions

④ **a. Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = 3x^2 - x + 4$ and $g(x) = 2x - 1$.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\boxed{}\right) \\ &= \boxed{} - \boxed{} + \boxed{} \\ &= 3(4x^2 - 4x + 1) - 2x + 1 + 4 \\ &= \boxed{}\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g(f(x)] \\ &= g\left(\boxed{}\right) \\ &= \boxed{}(3x^2 - x + 4) - \boxed{} \\ &= \boxed{}\end{aligned}$$

b. Evaluate $[f \circ g](x)$ and $[g \circ f](x)$ for $x = -2$.

$$\begin{aligned}[f \circ g](x) &= 12x^2 - 14x + 8 \\ [f \circ g](-2) &= \boxed{} \\ &= \boxed{}\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= 6x^2 - 2x + 7 \\ [g \circ f](-2) &= \boxed{} \\ &= \boxed{}\end{aligned}$$

Check Your Progress

a. Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = x^2 + 2x + 3$ and $g(x) = x + 5$.

b. Evaluate $[f \circ g](x)$ and $[g \circ f](x)$ for $x = 1$.

EXAMPLE Use Composition of Function

5 **TAXES Tracey Long has \$100 deducted from every paycheck for retirement. She can have this deduction taken before state taxes are applied, which reduces her taxable income. Her state income tax rate is 4%. If Tracey earns \$1500 every pay period, find the difference in her net income if she has the retirement deduction taken before or after state taxes.**

Explore Let x = Tracey's income per paycheck, $r(x)$ = her income after the deduction for retirement, and $t(x)$ = her income after the deduction for state income tax.

Plan Write equations for $r(x)$ and $t(x)$.
\$100 is deducted from every paycheck for retirement:

$r(x) = x -$ ☐

Tracey's tax rate is 4%: $t(x) =$ ☐

Solve If Tracey has her retirement deducted before taxes, then her net income is represented by $[t \circ r](1500)$.

$[t \circ r](1500) = t(1500 - 100)$

$= t$ ☐

$= 1400 - 0.04(1400)$

$=$ ☐

If Tracey has her retirement deducted *after* taxes, then her net income is represented by $[r \circ t](1500)$.

$[r \circ t](1500) = r[1500 - 0.04(1500)]$

$= r$ ☐

$=$ ☐ $- 100$

$=$ ☐

Her net pay is ☐ more by having her retirement deduction taken before state taxes.

Check Your Progress Brandi Smith has deducted \$200 from every paycheck for retirement. She can have this deduction taken before state taxes are applied, which reduces her taxable income. Her state income tax is 10%. If Brandi earns \$2200 every pay period, find the difference in her net income if she has the retirement deduction taken before or after state taxes.

☐

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–2 Inverse Functions and Relations

MAIN IDEAS

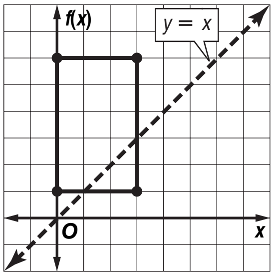
- Find the inverse of a function or relation.
- Determine whether two functions or relations are inverses.

EXAMPLE Find an Inverse Relation

① **GEOMETRY The ordered pairs of the relation {(1, 3), (6, 3), (6, 0), (1, 0)} are the coordinates of the vertices of a rectangle. Find the inverse of this relation and determine whether the resulting ordered pairs are also the coordinates of the vertices of a rectangle.**

To find the inverse of this relation, reverse the coordinates of the ordered pairs. The inverse of the relation is

.



Plotting the points shows that the ordered pairs also describe the vertices of a rectangle. Notice that the graph of the relation and the inverse are reflections over the graph of $y = x$.

KEY CONCEPTS

Inverse Relations Two relations are inverse relations if and only if whenever one relation contains the element (*a*, *b*), the other relation contains the element (*b*, *a*).

Property of Inverse Functions Suppose f and f^{-1} are inverse functions. Then, $f(a) = b$ if and only if $f^{-1}(b) = a$.

EXAMPLE Find an Inverse Function

② **a. Find the inverse of $f(x) = -\frac{1}{2}x + 1$.**

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = -\frac{1}{2}x + 1 \longrightarrow$$

Step 2 Interchange x and y

Step 3 Solve for y.

$x = -\frac{1}{2}y + 1$ Inverse

= Multiply each side by −2.

= Add 2 to each side.

Step 4 Replace y with $f^{-1}(x)$.

$$y = \qquad \longrightarrow \quad f^{-1}(x) =$$

EXAMPLE Verify Two Functions are Inverses

③ Determine whether $f(x) = \frac{3}{4}x - 6$ and $g(x)\ \frac{4}{3}x + 8$ are inverse functions.

Check to see if the compositions of $f(x)$ and $g(x)$ are identity functions.

$[f \circ g](x)$

$= f\left(\boxed{}\right)$

$= \boxed{}\left(\frac{4}{3}x + 8\right) - \boxed{}$

$= \boxed{}$

$= \boxed{}$

$[g \circ f](x)$

$= g\left(\boxed{}\right)$

$= \boxed{}\left(\frac{3}{4}x - 6\right) + \boxed{}$

$= \boxed{}$

$= \boxed{}$

The functions are inverses since both compositions equal $\boxed{}$.

Check Your Progress

a. The ordered pairs of the relation {(−3, 4), (−1, 5), (2, 3), (1, 1), (−2, 1)} are the coordinates of the vertices of a pentagon. Find the inverse of this relation and determine whether the resulting ordered pairs are also the coordinates of the vertices of a pentagon.

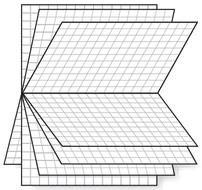
b. Find the inverse of $f(x) = \frac{1}{3}x + 6$.

c. Determine whether $f(x) = 3x + 1$ and $g(x) = \frac{x - 1}{3}$ are inverse functions.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-2, sketch your own graph of two relations that are inverses.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–3

Square Root Functions and Inequalities

MAIN IDEAS

- Graph and analyze square root functions.
- Graph square root inequalities.

EXAMPLE Graph a Square Root Function

① **Graph $y = \sqrt{\frac{3}{2}x - 1}$. State the domain, range, and x- and y-intercepts.**

Since the radicand cannot be negative, identify the domain

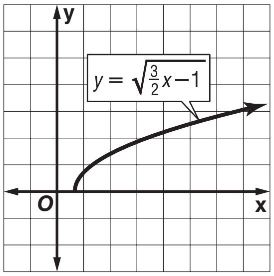
≥ 0 Write the expression inside the radicand as ≥ 0.

$x \geq$ Solve for x.

The x-intercept is .

Make a table of values to graph the function. The domain is $x \geq \frac{2}{3}$ and the range is $y \geq 0$.

The x-intercept is $\frac{2}{3}$. There is no y-intercept.



x	y
$\frac{2}{3}$	0
1	0.71
2	1.41
3	1.87
4	2.24
5	2.55
6	2.83

Check Your Progress Graph $y = \sqrt{2x - 2}$. State the domain, range, and x- and y-intercepts.

EXAMPLE

1 PHYSICS When an object is spinning in a circular path of radius 2 meters with velocity *v*, in meters per second, the centripetal acceleration *a*, in meters per second squared, is directed toward the center of the circle. The velocity *v* and acceleration *a* of the object are related by the function $v = \sqrt{2a}$.

a. Graph the function. State the domain and range.

The domain is , and the range is .

b. What would be the centripetal acceleration of an object spinning along the circular path with a velocity of 4 meters per second?

$v = \sqrt{2a}$	Original equation
$= \sqrt{2a}$	Replace *v* with .
$16 = 2a$	Square each side.
$= a$	Divide each side by 2.

The centripetal acceleration would have to be meters per second squared. Check the reasonableness of this result by comparing it to the graph.

Check Your Progress **The volume *V* and surface area *A* of a soap bubble are related by the function $V = 0.094\sqrt{A^3}$.**

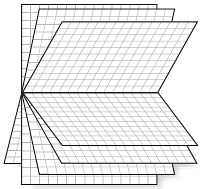
a. Graph the function. State the domain and range.

b. What would the surface area be if the volume were 94 cubic units?

FOLDABLES™

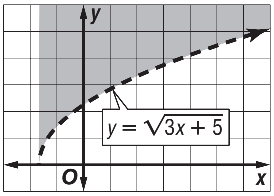
ORGANIZE IT

On the page for Lesson 7-3, write how you know if a boundary should be included or not. Then write how you know which region to shade.



EXAMPLE Graph a Square Root Inequality

③ **Graph $y > \sqrt{3x + 5}$.**



Graph the related equation $y = \sqrt{3x + 5}$. Since the boundary is not included, the graph should be dashed.

The domain includes values for $x \geq -\frac{5}{3}$. So the graph is to the right of $x = -\frac{5}{3}$.

Select a point and test its ordered pair. Test (0, 0)

$0 >$

$0 >$ false

Shade the region that does not include (0, 0).

Check Your Progress Graph $y \leq \sqrt{3 + 3x}$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–4

*n*th Roots

MAIN IDEAS

- Simplify radicals.
- Use a calculator to approximate radicals.

BUILD YOUR VOCABULARY (pages 198–199)

The inverse of raising a number to the *n*th power is finding the ***n*th root** of a number. When there is more than one real root, the ______ root is called the **principal root**.

KEY CONCEPTS

Definition of Square Root For any real numbers *a* and *b*, if $a^2 = b$, then *a* is a square root of *b*.

Definition of *n*th Root For any real numbers *a* and *b*, and any positive integer *n*, if $a^n = b$, then *a* is an *n*th root of *b*.

EXAMPLE Find Roots

1 **Simplify**

a. $\pm\sqrt{16x^8}$

$\pm\sqrt{16x^8}$

$= \pm\sqrt{\boxed{}^{\,2}}$

$= \boxed{}$

b. $-\sqrt{(q^3 + 5)^4}$

$-\sqrt{(q^3 + 5)^4}$

$= -\sqrt{\left(\boxed{}\right)^2}$

$= \boxed{}$

c. $\sqrt[5]{243a^{10}b^{15}}$

$\sqrt[5]{243a^{10}b^{15}} = \sqrt[5]{\left(\boxed{}\right)^5}$

$= \boxed{}$

d. $\sqrt{-4}$

$\sqrt{-4} = \boxed{}$

Since n is even and b is negative, $\sqrt{-4}$ is not a real number.

Check Your Progress

Simplify.

a. $\pm\sqrt{9x^8}$

b. $-\sqrt{(a^3 + 2)^6}$

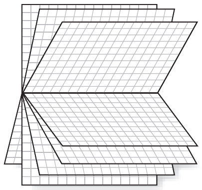
c. $\sqrt[5]{32x^5y^{10}}$

d. $\sqrt{-16}$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-4, explain how to simplify using absolute value. Include an example.



EXAMPLE Simplify Using Absolute Value

② **Simplify.**

a. $\sqrt[6]{t^6}$

Note that t is a sixth root of t^6. The index is even, so the principal root is nonnegative. Since t could be negative, you must take the absolute value of t to identify the principal root.

$$\sqrt[6]{t^6} = \boxed{}$$

b. $\sqrt[5]{243(x+2)^{15}}$

$$\sqrt[5]{243(x+2)^{15}} = \sqrt[5]{\left[\boxed{}\right]^5}$$

Since the index is odd, you do not need absolute value.

$$\sqrt[5]{243(x+2)^{15}} = \boxed{}$$

Check Your Progress **Simplify.**

a. $\sqrt[4]{x^4}$ $\boxed{}$

b. $\sqrt[3]{27(x+2)^9}$ $\boxed{}$

EXAMPLE

③ FISH **The relationship between the length and mass of Pacific halibut can be approximated by the equation $L = 0.46\sqrt[3]{M}$, where L is the length of the fish in meters and M is the mass in kilograms. Use a calculator to approximate the length of a 30-kilogram Pacific halibut.**

$L = 0.46\sqrt[3]{M}$ Original equation

$L = 0.46\sqrt[3]{\boxed{}}$ or about $\boxed{}$ $M = \boxed{}$

The length of a 30-kilogram Pacific halibut is about $\boxed{}$.

Check Your Progress The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt[3]{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. Find the value of T for a 2-foot-long pendulum.

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–5 Radical Expressions

MAIN IDEAS

- Simplify radical expressions.
- Add, subtract, multiply, and divide radical expressions.

KEY CONCEPTS

Product Property of Radicals For any real numbers a and b and any integer $n > 1$,

1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, and
2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

Quotient Property of Radicals For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} \cdot \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

EXAMPLE Simplify Expressions

① **Simplify.**

a. $\sqrt{25a^4b^9}$

$$\sqrt{25a^4b^9} = \sqrt{\boxed{}\,(b^4)^2b}$$ Factor into squares.

$$= \boxed{}\,\sqrt{(b^4)^2}\sqrt{b}$$ Product Property

$$= \boxed{}\,b^4\sqrt{b}$$ Simplify.

b. $\sqrt{\frac{y^8}{x^7}}$

$$\sqrt{\frac{y^8}{x^7}} = \sqrt{\frac{y^8}{x^7}}$$ Quotient Property

$$= \frac{\sqrt{\left(\boxed{}\right)^2}}{\sqrt{\left(\boxed{}\right)^2 \cdot x}}$$ Factor into squares.

$$= \frac{\sqrt{(y^4)^2}}{\sqrt{(x^3)^2} \cdot \sqrt{x}}$$ Product Property

$$= \frac{y^4}{x^3\sqrt{x}}$$ $\sqrt{(y^4)^2} = y^4$

$$= \frac{y^4}{x^3\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$ Rationalize the denominator.

$$= \boxed{}$$ $\sqrt{x} \cdot \sqrt{x} = x$

c. $\sqrt[3]{\frac{2}{9x}}$

$$\sqrt[3]{\frac{2}{9x}} = \frac{\sqrt[3]{2}}{\sqrt[3]{9x}}$$ Quotient Property

$$= \frac{\sqrt[3]{2}}{\sqrt[3]{9x}} \cdot \boxed{}$$ Rationalize the denominator.

$$= \frac{\sqrt[3]{2 \cdot 3x^2}}{\sqrt[3]{9x \cdot 3x^2}}$$ Product Property

$$= \boxed{}$$ Multiply.

$$= \boxed{}$$ $\sqrt[3]{27x^3} = 3x$

Check Your Progress **Simplify each expression.**

a. $\sqrt{16x^4y^{11}}$

b. $\sqrt{\frac{x^3}{y^7}}$

c. $\sqrt[3]{\frac{2}{3a}}$

BUILD YOUR VOCABULARY (page 198)

Two radical expressions are called **like radical expressions** if both the indices and radicands are alike.

EXAMPLE Multiply Radicals

① **Simplify $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$.**

$5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a} = 5 \cdot \sqrt[3]{100a^2 \cdot 10a}$ Product Property of Radicals

$= 5 \cdot \sqrt{\boxed{}}$ Factor into cubes where possible.

$= 5 \cdot \boxed{} \cdot \boxed{}$ Product Property of Radicals

$= 5 \cdot \boxed{} \cdot \boxed{}$ or $\boxed{}$ Multiply.

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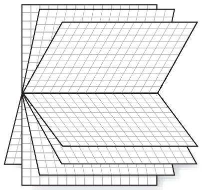
ORGANIZE IT

On the page for Lesson 7-5, write your own example of two like radical expressions. Then find their sum.



Check Your Progress Simplify $3\sqrt[3]{16a^2} \cdot 2\sqrt[3]{4a}$.

$\boxed{\quad}$

EXAMPLE

Add and Subtract Radicals

3 **Simplify $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$.**

$3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$

$= 3\sqrt{3^2 \cdot 5} - 5\sqrt{4^2 \cdot 5} + 4\sqrt{2^2 \cdot 5}$ Factor.

$= 3\sqrt{3^2} \cdot \sqrt{5} - 5\sqrt{4^2} \cdot \sqrt{5} + 4\sqrt{2^2} \cdot \sqrt{5}$ Product Property

$= 3 \cdot \boxed{\quad} \sqrt{5} - 5 \cdot 4\sqrt{5} + 4 \cdot \boxed{\quad} \sqrt{5}$

$= \boxed{\quad}$ Multiply.

$= \boxed{\quad}$ Combine like radicals.

Check Your Progress Simplify $3\sqrt{75} - 2\sqrt{48} + \sqrt{3}$.

$\boxed{\quad}$

EXAMPLE

Multiply Radicals

4 **Simplify.**

a. $(2\sqrt{3} + 3\sqrt{5})(3 - \sqrt{3})$

$(2\sqrt{3} + 3\sqrt{5})(3 - \sqrt{3})$

F O I L

$= 2\sqrt{3} \cdot 3 - \boxed{\quad} + \boxed{\quad} - 3\sqrt{5} \cdot \sqrt{3}$

$= 6\sqrt{3} - \boxed{\quad} + \boxed{\quad} - 3\sqrt{15}$

$= 6\sqrt{3} - \boxed{\quad} - 3\sqrt{15}$

b. $(4\sqrt{2} + 7)(4\sqrt{2} - 7)$

$$\begin{aligned}
&(4\sqrt{2} + 7)(4\sqrt{2} - 7)\\
&= 4\sqrt{2} \cdot 4\sqrt{2} - \boxed{} + \boxed{} - 7 \cdot 7\\
&= \boxed{} - 7 \cdot 4\sqrt{2} + 7 \cdot 4\sqrt{2} - \boxed{}\\
&= 32 - 49\\
&= \boxed{}
\end{aligned}$$

Check Your Progress **Simplify each expression.**

a. $(2\sqrt{5} + 4\sqrt{6})(5 - \sqrt{7})$

$\boxed{}$

b. $(3\sqrt{5} - 2)(3\sqrt{5} + 2)$

$\boxed{}$

EXAMPLE Use a Conjugate to Rationalize a Denominator

5 Simplify $\frac{2 + \sqrt{3}}{4 - \sqrt{3}}$.

$$\frac{2 + \sqrt{3}}{4 - \sqrt{3}} = \frac{(2 + \sqrt{3})(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$$

$4 + \sqrt{3}$ is the conjugate of $4 - \sqrt{3}$.

$$= \frac{2 \cdot 4 + 2\sqrt{3} + \boxed{} + \boxed{}}{4^2 - (\sqrt{3})^2}$$

FOIL
Difference of squares.

$$= \frac{\boxed{} + 2\sqrt{3} + 4\sqrt{3} + \boxed{}}{16 - 3}$$

Multiply.

$$= \boxed{}$$

Combine like terms.

Check Your Progress Simplify $\frac{3 + \sqrt{5}}{2 - \sqrt{5}}$.

$\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):
Exercises:

7–6 Rational Exponents

MAIN IDEAS

- Write expressions with rational exponents in radical form, and vice versa.
- Simplify expressions in exponential or radical form.

EXAMPLE 1 Radical and Exponential Forms

a. Write $a^{\frac{1}{6}}$ in radical form.

$$a^{\frac{1}{6}} = \boxed{}$$

b. Write $\sqrt{w}$ in exponential form.

$$\sqrt{w} = \boxed{}$$

Check Your Progress

a. Write $x^{\frac{1}{5}}$ in radical form.

$\boxed{}$

b. Write $\sqrt[3]{y}$ in exponential form.

$\boxed{}$

KEY CONCEPTS

$b^{\frac{1}{n}}$ For any real number b and for any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.

Rational Exponents For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, except when $b < 0$ and n is even.

EXAMPLE 2 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. $49^{-\frac{1}{2}}$

$$49^{-\frac{1}{2}} = \frac{1}{\boxed{}} \qquad b^{-n} = \frac{1}{b^n}$$

$$= \frac{1}{\boxed{}} \qquad 49^{\frac{1}{2}} = \boxed{}$$

$$= \boxed{} \qquad \text{Simplify.}$$

REMEMBER IT

Since $b^{\frac{m}{n}}$ is defined as $\left(b^{\frac{1}{n}}\right)^m$ or $(b^m)^{\frac{1}{n}}$, either can be used to evaluate an expression. Choose depending on the given expression.

b. $32^{\frac{2}{5}}$

$32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}}$	$32 = 2^5$
$=$	Power of a Power
$=$	Multiply exponents.
$=$	$2^2 = 4$

Check Your Progress **Evaluate each expression.**

a. $25^{-\frac{1}{2}}$

b. $16^{\frac{3}{4}}$

EXAMPLE Simplify Expressions with Rational Exponents

3 **Simplify each expression.**

a. $y^{\frac{1}{7}} \cdot y^{\frac{4}{7}}$

$y^{\frac{1}{7}} \cdot y^{\frac{4}{7}} =$	Multiply powers.
$=$	Add exponents.

b. $x^{-\frac{2}{3}}$

$x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}}$	$b^{-n} = \frac{1}{b^n}$
$= \frac{1}{x^{\frac{2}{3}}} \cdot$	Multiply by .
$=$ or	$x^{\frac{2}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{2}{3} + \frac{1}{3}}$

Check Your Progress **Simplify each expression.**

a. $x^{\frac{1}{5}} \cdot x^{\frac{2}{5}}$ $\boxed{}$

b. $y^{-\frac{3}{4}}$ $\boxed{}$

EXAMPLE Simplify Radical Expressions

4 **Simplify each expression.**

a. $\dfrac{\sqrt[6]{16}}{\sqrt[3]{2}}$

$$\frac{\sqrt[6]{16}}{\sqrt[3]{2}} = \frac{16^{\frac{1}{6}}}{2^{\frac{1}{3}}}$$ Rational exponents

$$= \frac{\boxed{}}{2^{\frac{1}{3}}}$$ $16 = 2^4$

$$= \frac{\boxed{}}{2^{\frac{1}{3}}}$$ Power of a Power

$$= \boxed{}$$ Quotient of Powers

$$= \boxed{} \text{ or } \boxed{}$$ Simplify.

b. $\sqrt[6]{4x^4}$

$$\sqrt[6]{4x^4} = (4x^4)^{\frac{1}{6}}$$ Rational exponents

$$= (2^2 \boxed{})^{\frac{1}{6}}$$ $2^2 = 4$

$$= 2^{2\left(\frac{1}{6}\right)} \cdot \boxed{}$$ Power of a Power

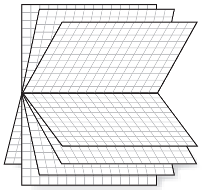
$$= \boxed{}$$ Multiply.

$$= \boxed{}$$ Simplify.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-6, write four conditions that must be met in order for an expression with rational exponents to be in simplest form.



c. $\dfrac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$

$$\frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1} = \frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1} \cdot \frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} + 1}$$

$y^{\frac{1}{2}} + 1$ is the conjugate of $y^{\frac{1}{2}} - 1$.

$$= \frac{y + 2y^{\frac{1}{2}} + 1}{y - 1}$$

Multiply.

Check Your Progress **Simplify each expression.**

a. $\dfrac{\sqrt[4]{4}}{\sqrt{2}}$

b. $\sqrt[3]{16x^2}$

c. $\dfrac{x^{\frac{1}{2}} - 1}{x^{\frac{1}{2}} + 1}$

EXAMPLE

5 **TEST EXAMPLE** **If x is a positive number, then $\dfrac{x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{1}{6}}} = ?$**

A $\sqrt[2]{x^3}$ **B** $\sqrt[3]{x^2}$ **C** $\sqrt[6]{x^5}$ **D** $x^{\frac{3}{2}}$

$$\frac{x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{5}{6}}}{x^{\frac{1}{6}}}$$

Add the exponents in the numerator.

$$= x^{\frac{5}{6}} \cdot \boxed{\quad}$$

Quotient of Powers

$$= x^{\frac{2}{3}} \text{ or } \boxed{\quad}$$

The answer is $\boxed{\quad}$.

Check Your Progress If y is a positive number, then $\dfrac{y^{\frac{3}{4}} y^{\frac{1}{2}}}{y^{\frac{1}{4}}} = ?$

A y^2 **B** $y^{\frac{4}{5}}$ **C** $\sqrt[4]{y^5}$ **D** $\sqrt[4]{y}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–7 Solving Radical Equations and Inequalities

MAIN IDEAS

- Solve equations containing radicals.
- Solve inequalities containing radicals.

BUILD YOUR VOCABULARY (pages 198–199)

Equations with radicals that have variables in the radicands are called **radical equations**.

When you solve a radical equation and obtain a number that does not satisfy the original equation, the number is called an **extraneous solution**.

A **radical inequality** is an inequality that has a variable in a radicand.

EXAMPLE Solve Radical Equations

① **a. Solve $\sqrt{y-2}-1=5$.**

$\sqrt{y-2}-1=5$ Original equation

$\boxed{} = \boxed{}$ Add to isolate the radical.

$\boxed{} = \boxed{}$ Square each side.

$\boxed{} = \boxed{}$ Find the squares.

$y = \boxed{}$ Add 2 to each side.

b. Solve $\sqrt{x-12}=2-\sqrt{x}$.

$\sqrt{x-12}=2-\sqrt{x}$ Original equation

$(\sqrt{x-12})^2=(2-\sqrt{x})^2$ Square each side.

$x-12=\boxed{}$ Find the squares.

$-16=-4\sqrt{x}$ Isolate the radical.

$\boxed{} = \boxed{}$ Divide each side by −4.

$\boxed{} = \boxed{}$ Square each side.

$16=x$ Evaluate the squares.

Since $\sqrt{16-12}\neq\boxed{}$, the solution does not check

and there is $\boxed{}$

Check Your Progress **Solve.**

a. $\sqrt{x-3} - 2 = 6$

b. $\sqrt{x+5} = -1 - \sqrt{x}$

EXAMPLE Solve a Cube Root Equation

2 **Solve $(3y + 1)^{\frac{1}{3}} + 5 = 0$.**

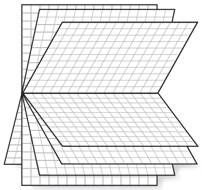
In order to remove the $\frac{1}{3}$ power, you must first isolate it and then raise each side of the equation to the third power.

$(3y + 1)^{\frac{1}{3}} + 5 = 0$	Original equation
$(3y + 1)^{\frac{1}{3}} =$	Subtract 5 from each side.
$[(3y + 1)^{\frac{1}{3}}]^3 =$	Cube each side.
$3y + 1 =$	Evaluate the cubes.
$y =$	Simplify.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-7, write your own problem similar to Example 1. Then explain how you eliminate the radical in order to solve the equation.



EXAMPLE Solve a Radical Inequality

3 **Solve $\sqrt{3x - 6} + 4 \leq 7$.**

Find values of x for which the left side is defined.

$3x - 6 \geq 0$ Radicand must be positive or 0.

$3x \geq 6$

$x \geq 2$

Now solve $\sqrt{3x - 6} + 4 \leq 7$.

$\sqrt{3x - 6} + 4 \leq$	Original inequality
$\sqrt{3x - 6} \leq$	Isolate the radical.
$\leq$	Eliminate the radical.
$3x \leq$	Add 6 to each side.
$x \leq$	Divide each side by 3.

The solution is .

Check Your Progress **Solve.**

a. $(2y + 1)^{\frac{1}{3}} - 3 = 0$

b. $\sqrt{2x + 5} - 2 \leq 9$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 7

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 7 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 7, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 198–199)* to help you solve the puzzle.

7-1

Operations on Functions

1. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 - 5x + 2$ and $g(x) = 3x + 6$.

2. Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = x^2 + 8x - 7$ and $h(x) = x - 3$.

7-2

Inverse Functions and Relations

3. Find the inverse of the function $f(x) = -5x + 4$.

4. Determine whether $g(x) = 2x + 4$ and $f(x)\ \frac{x}{2} + 2$ are inverse functions.

7-3

Square Root Functions and Inequalities

5. Graph $y < 2 + \sqrt{3x + 4}$. Then state the domain and range of the function.

7-4

*n*th Roots

6. Use a calculator to approximate $\sqrt[3]{-280}$ to three decimal places.

Simplify.

7. $\sqrt[3]{-64}$

8. $\sqrt[8]{x^{16}}$

9. $\sqrt{100a^{12}}$

10. $\sqrt[3]{8y^3}$

11. $\sqrt{49x^6y^8}$

12. $\sqrt[3]{125c^6d^{15}}$

7-5

Operations with Radical Expressions

Simplify.

13. $\sqrt{\frac{5}{6x}}$

14. $2\sqrt{45} - 7\sqrt{8} + \sqrt{80}$

7-6

Rational Exponents

Evaluate.

15. $(-32)^{\frac{1}{5}}$

16. $25^{-\frac{3}{2}}$

17. $\left(\frac{1}{64}\right)^{-\frac{1}{3}}$

Simplify.

18. $x^{\frac{1}{4}} \cdot x^{\frac{5}{4}}$

19. $\left(y^{-\frac{5}{6}}\right)^{-\frac{1}{5}}$

20. $\sqrt[10]{36a^2b^{10}}$

7-7

Solving Radical Equations and Inequalities

Solve each equation or inequality.

21. $\sqrt[3]{5u - 2} = -3$

22. $\sqrt{4z - 3} = \sqrt{9z + 2}$

23. $3 + \sqrt{2x - 1} \leq 6$

24. $\sqrt{5x + 4} + 9 > 13$

CHAPTER 7

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 7.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 7 Practice Test on page 435 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 7 Study Guide and Review on pages 430–434 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 435 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 7 Foldable.
- Then complete the Chapter 7 Study Guide and Review on pages 430–434 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 435 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

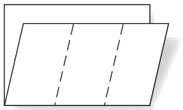
CHAPTER 8

Rational Expressions and Equations

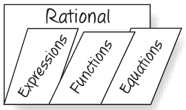
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

STEP 1 **Fold** in half lengthwise leaving a $\frac{1}{2}$ inch margin at the top. Fold again in thirds.



STEP 2 **Open.** Cut along the folds on the short tab to make three tabs. Label as shown.



NOTE-TAKING TIP: Remember to always take notes on your own. Don't use someone else's notes as they may not make sense.

CHAPTER

8

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 8. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
asymptote A-suhm(p)-TOHT			
complex fraction			
constant of variation			
continuity KAHN-tuhn-OO-uh-tee			
direct variation			
inverse variation IHN-VUHRS			

Vocabulary Term	Found on Page	Definition	Description or Example
joint variation			
point discontinuity			
rational equation			
rational expression			
rational function			
rational inequality			

8–1 Multiplying and Dividing Rational Expressions

MAIN IDEAS

- Simplify rational expressions.
- Simplify complex fractions.

BUILD YOUR VOCABULARY (page 227)

A ratio of two expressions is called a **rational expression**.

EXAMPLE Simplify a Rational Expression

① **a. Simplify $\frac{3y(y+7)}{(y+7)(y^2-9)}$.**

$$\frac{3y(y+7)}{(y+7)(y^2-9)} = \frac{3y}{y^2-9} \cdot \frac{\overset{1}{\cancel{y+7}}}{\underset{1}{\cancel{y+7}}} \quad \text{Factor.}$$

$$= \qquad \qquad \text{Simplify.}$$

b. Under what conditions is this expression undefined?

To find out when this expression is undefined, completely factor the denominator.

$$\frac{3y(y+7)}{(y+7)(y^2-9)} = \frac{3y(y+7)}{(y+7)\qquad}$$

The values that would make the denominator equal to 0

are , , and .

Check Your Progress

a. Simplify $\frac{x(x+5)}{(x+5)(x^2-16)}$.

b. Under what conditions is this expression undefined?

EXAMPLE Use the Process of Elimination

2 **TEST EXAMPLE** **For what value(s) of p is $\dfrac{p^2 + 2p - 3}{p^2 - 2p - 15}$ undefined?**

A 5 **B** −3, 5 **C** 3, −5 **D** 5, 1, −3

Read the Item

You want to determine which values of p make the denominator equal to 0.

Solve the Item

Look at the possible answers. Notice that the p term and the constant term are both $\boxed{\qquad\qquad}$, so there will be one positive solution and one negative solution. Therefore, you can eliminate choices A and D. Factor the denominator.

$p^2 - 2p - 15 = \boxed{\qquad}\boxed{\qquad}$ Factor the denominator.

$p - 5 = 0$ or $p + 3 = 0$ Zero Product Property

$p = \boxed{\quad}$ $p = \boxed{\quad}$ Solve each equation.

Since the denominator equals 0 when $p = \boxed{\quad}$, the answer is $\boxed{\quad}$.

Check Your Progress For what values of p is $\dfrac{p^2 + 5p + 6}{p^2 + 8p + 15}$ undefined? $\boxed{\quad}$

A −5, 3, 2 **B** −5 **C** 5 **D** −5, −3

EXAMPLE Simplify by Factoring Out −1

3 **Simplify $\dfrac{a^4b - 2a^4}{2a^3 - a^3b}$.**

$\dfrac{a^4b - 2a^4}{2a^3 - a^3b} = \boxed{\qquad\qquad}$ Factor the numerator and the denominator.

$= \dfrac{\overset{a}{\cancel{a^4}}(-1)\overset{1}{\cancel{(2-b)}}}{\underset{1}{\cancel{a^3}}\underset{1}{\cancel{(2-b)}}}$ $b - 2 = -(-b + 2)$ or $-1(2 - b)$

$= \boxed{\quad}$ or $-a$ Simplify.

KEY CONCEPT

Rational Expressions

- To multiply two rational expressions, multiply the numerators and the denominators.
- To divide two rational expressions, multiply by the reciprocal of the divisor.

EXAMPLE Multiply Rational Expressions

4 **Simplify each expression.**

a. $\dfrac{8x}{21y^3} \cdot \dfrac{7y^2}{16x^3}$

$$\frac{8x}{21y^3} \cdot \frac{7y^2}{16x^3} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}}}{3 \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}} \cdot y \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{x}} \cdot x \cdot x}$$ Factor.

$$= \qquad\qquad \text{or} \qquad\qquad$$ Simplify.

b. $\mathbf{\dfrac{10ps^2}{3c^2d} \div \dfrac{5ps}{6c^2d^2}}$

$$\frac{10ps^2}{3c^2d} \div \frac{5ps}{6c^2d^2}$$

$$= \frac{10ps^2}{3c^2d} \cdot \frac{6c^2d^2}{5ps}$$ Multiply by the reciprocal of the divisor.

$$=$$ Factor.

$$=$$ Simplify.

$$=$$ Simplify.

Check Your Progress **Simplify each expression.**

a. $\dfrac{x^4y - 3x^4}{3x^3 - x^3y}$

b. $\dfrac{3x^2y}{20ab} \div \dfrac{6xy}{5a^2b^3}$

EXAMPLE Polynomials in the Numerator and Denominator

5 Simplify each expression.

a. $\mathbf{\dfrac{k-3}{k+1} \cdot \dfrac{1-k^2}{k^2-4k+3}}$

$$\frac{k-3}{k+1} \cdot \frac{1-k^2}{k^2-4k+3}$$

$$= \frac{k-3}{k+1} \cdot$$ Multiply by the reciprocal of the divisor.

$$= \frac{\overset{1}{\cancel{(k-3)}}\overset{-1}{\cancel{(1-k)}}\overset{1}{\cancel{(1+k)}}}{\underset{1}{\cancel{(k+1)}}\underset{1}{\cancel{(k-1)}}\underset{1}{\cancel{(k-3)}}}$$

$1 + k = k + 1$,
$1 - k = -1(k - 1)$

$=$ Simplify.

b. $\mathbf{\dfrac{2d+6}{d^2+d-2} \div \dfrac{d+3}{d^2+3d+2}}$

$\dfrac{2d+6}{d^2+d-2} \div \dfrac{d+3}{d^2+4d+2}$

$= \dfrac{2d+6}{d^2+d-2} \cdot$ Multiply by the reciprocal of the divisor.

$= \dfrac{2\overset{1}{\cancel{(d+3)}}\overset{1}{\cancel{(d+2)}}(d+1)}{\underset{1}{\cancel{(d+2)}}(d-1)\underset{1}{\cancel{(d+3)}}}$ Factor.

$=$ Simplify.

EXAMPLE Simplify a Complex Fraction

⑥ **Simplify** $\mathbf{\dfrac{\frac{x^2}{9x^2-4y^2}}{\frac{x^3}{2y-3x}}}$.

$\dfrac{\frac{x^2}{9x^2-4y^2}}{\frac{x^3}{2y-3x}} = \dfrac{x^2}{9x^2-4y^2} \div \dfrac{x^3}{2y-3x}$ Express as a division expression.

$= \dfrac{x^2}{9x^2-4y^2} \cdot$ Multiply by the reciprocal of the divisor.

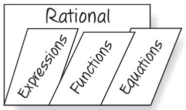
$= \dfrac{\overset{1}{\cancel{x}}\cdot\overset{1}{\cancel{x}}\overset{-1}{\cancel{(2y-3x)}}}{\underset{1}{\cancel{(3x-2y)}}(3x+2y)\underset{1}{\cancel{x}}\cdot\underset{1}{\cancel{x}}\cdot x}$ Factor.

$=$ Simplify.

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Expressions, write how simplifying rational expressions is similar to simplifying rational numbers.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress **Simplify each expression.**

a. $\dfrac{x-3}{x+2} \cdot \dfrac{x^2+5x+6}{x^2-9}$

b. $\dfrac{\frac{a^2}{a^2-9b^2}}{\frac{a^4}{a+3b}}$

8–2 Adding and Subtracting Rational Expressions

MAIN IDEAS

- Determine the LCM of polynomials.
- Add and subtract rational expressions.

EXAMPLE LCM of Monomials

① **Find the LCM of $15a^2bc^3$, $16b^5c^2$, and $20a^3c^6$.**

$15a^2bc^3 =$ Factor the first monomial.

$16b^5c^2 =$ Factor the second monomial.

$20a^3c^6 =$ Factor the third monomial.

$\text{LCM} =$ Use each factor the greatest number of times it appears as a factor and simplify.

$=$

EXAMPLE LCM of Polynomials

② **Find the LCM of $x^3 - x^2 - 2x$ and $x^2 - 4x + 4$.**

$x^3 - x^2 - 2x =$ Factor the first polynomial.

$x^2 - 4x + 4 =$ Factor the second polynomial.

$\text{LCM} =$ Use each factor the greatest number of times it appears as a factor.

Check Your Progress **Find the LCM of each set of polynomials.**

a. $6x^2zy^3$, $9x^3y^2z^2$, $4x^2z$

b. $x^3 + 2x^2 - 3x$, $x^2 + 6x + 9$

REMEMBER IT When adding and subtracting rational expressions, find the LCD. Then, rewrite the expressions using the common denominator. Finally, add or subtract the fractions and simplify the result.

EXAMPLE Monomial Denominators

③ Simplify $\frac{5a^2}{6b} + \frac{9}{14a^2b^2}$.

$$\frac{5a^2}{6b} + \frac{9}{14a^2b^2}$$

$$= \frac{5a^2 \cdot \boxed{}}{6b \cdot \boxed{}} + \frac{9 \cdot \boxed{}}{14a^2b^2 \cdot \boxed{}}$$

The LCD is $42a^2b^2$. Find equivalent fractions that have this denominator.

$$= \frac{\boxed{}}{42a^2b^2} + \frac{\boxed{}}{42a^2b^2}$$

Simplify each numerator and denominator.

$$= \frac{35a^4b + 27}{42a^2b^2}$$

Add the numerators.

EXAMPLE Polynomial Denominators

④ Simplify $\frac{x + 10}{3x - 15} - \frac{3x + 15}{6x - 30}$.

$$\frac{x + 10}{3x - 15} - \frac{3x + 15}{6x - 30}$$

$$= \frac{x + 10}{\boxed{}} - \frac{3x + 15}{\boxed{}}$$

Factor the denominators.

$$= \frac{2(x + 10)}{2 \cdot 3(x - 5)} - \frac{3x + 15}{6(x - 5)}$$

The LCD is $6(x - 5)$.

$$= \frac{2(x + 10) - (3x + 15)}{6(x - 5)}$$

Subtract the numerators.

$$= \frac{\boxed{}}{6(x - 5)}$$

Distributive Property

$$= \frac{\boxed{}}{6(x - 5)}$$

Combine like terms.

$$= \frac{-1\overset{1}{\cancel{(x - 5)}}}{6\underset{1}{\cancel{(x - 5)}}} \text{ or } \boxed{}$$

Simplify.

Check Your Progress **Simplify each expression.**

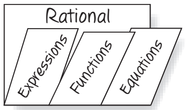
a. $\frac{3x^2}{2y} + \frac{5}{12xy^2}$

b. $\frac{x+5}{2x-4} - \frac{3x+8}{4x-8}$

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Expressions, write your own example similar to Example 5. Then simplify your rational expression. Give the reason for each step.



EXAMPLE Simplify Complex Fractions

5 **Simplify** $\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{b}-1}$.

$$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{b}} - 1 = \frac{\boxed{\quad}+\frac{a}{ab}}{\frac{1}{b}+\boxed{\quad}}$$

The LCD of the numerator is *ab*. The LCD of the denominator is *b*.

$$= \frac{\frac{\boxed{\quad}}{ab}}{\frac{\boxed{\quad}}{b}}$$

Simplify the numerator and denominator.

$$= \boxed{\quad} \div \boxed{\quad}$$

Write as a division expression.

$$= \frac{b+a}{a\underset{1}{\cancel{b}}} \cdot \frac{\overset{1}{\cancel{b}}}{1-b}$$

Multiply by the reciprocal of the divisor.

$$= \frac{b+a}{a(1-b)} \text{ or } \boxed{\quad}$$

Simplify.

Check Your Progress Simplify $\dfrac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{b} + \frac{1}{a}}$.

EXAMPLE Use a Complex Fraction to Solve a Problem

6 **COORDINATE GEOMETRY** **Find the slope of the line that passes through $P\left(\frac{3}{k}, \frac{1}{3}\right)$ and $Q\left(\frac{1}{2}, \frac{2}{k}\right)$.**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$= \dfrac{\frac{2}{k} - \frac{1}{3}}{\frac{1}{2} - \frac{3}{k}}$ $y_2 = \frac{2}{k}, y_1 = \frac{1}{3}, x_2 = \frac{1}{2}, x_1 = \frac{3}{k}$

$= \dfrac{\frac{6 - k}{\boxed{\quad}}}{\frac{k - 6}{\boxed{\quad}}}$ The LCD of the numerator is $3k$.
The LCD of the denominator is $2k$.

$= \dfrac{6 - k}{3k} \div \boxed{\quad}$ Write as division expression.

$= \dfrac{6 - k}{3k} \cdot \dfrac{2k}{k - 6}$ Multiply by the reciprocal of the divisor.

$= \dfrac{-(k - 6)}{3k} \cdot \dfrac{2k}{k - 6}$ $6 - k = -(k - 6)$

$= -\dfrac{\overset{1}{\cancel{(k - 6)}}}{3\cancel{k}} \cdot \dfrac{2\cancel{k}}{\underset{1}{\cancel{k - 6}}}$ or $\boxed{\quad}$

The slope is $\boxed{\quad}$.

Check Your Progress Find the slope of the line that passes through $P\left(\frac{4}{k}, \frac{1}{4}\right)$ and $Q\left(\frac{1}{5}, \frac{5}{k}\right)$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–3 Graphing Rational Functions

MAIN IDEAS

- Determine the vertical asymptotes and the point discontinuity for the graphs of rational functions.
- Graph rational functions.

KEY CONCEPTS

Vertical Asymptotes If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote.

Point Discontinuity If the original function is undefined for $x = a$ but the rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.

BUILD YOUR VOCABULARY (pages 226–227)

A **rational function** is an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are functions and $q(x) \neq 0$.

The graphs of rational functions may have breaks in **continuity**. This means that not all rational functions are traceable. Breaks in can appear as a vertical **asymptote** or as a **point discontinuity**.

EXAMPLE Vertical Asymptotes and Point Discontinuity

① **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2 - 4}{x^2 + 5x + 6}$.**

Factor the numerator and denominator of the rational expression.

$$\frac{x^2 - 4}{x^2 + 5x + 6} =$$

The function is undefined for $x =$ and .

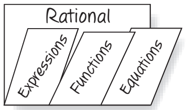
Since $\frac{(x - 2)\overset{1}{\cancel{(x + 2)}}}{\underset{1}{\cancel{(x + 2)}}(x + 3)} =$, $x =$ is a vertical asymptote and $x =$ is a hole in the graph.

Check Your Progress Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2 - 9}{x^2 + 8x + 15}$.

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Functions, write how you can tell from a rational function where the breaks in continuity will appear in the graph of the function.

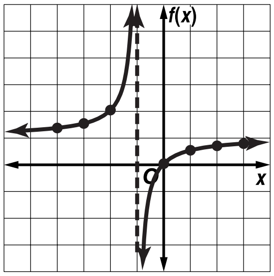


EXAMPLE Graph with a Vertical Asymptote

② **Graph** $f(x) = \dfrac{x}{x+1}$.

The function is undefined for $x =$. Since $\dfrac{x}{x+1}$ is in its simplest form, $x =$ is a vertical asymptote. Make a table of values. Plot the points and draw the graph.

x	$f(x)$
−4	1.33
−3	1.5
−2	2
0	0
1	0.5
2	0.67
3	0.75



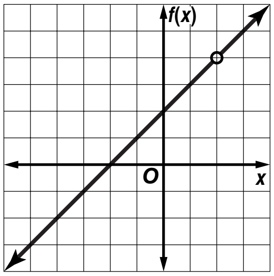
EXAMPLE Graph with Point Discontinuity

③ **Graph** $f(x) = \dfrac{x^2 - 4}{x - 2}$.

Notice that $\dfrac{x^2 - 4}{x - 2} = \dfrac{(x + 2)(x - 2)}{x - 2}$

or . Therefore, the graph of $f(x) = \dfrac{x^2 - 4}{x - 2}$ is the graph

$f(x) =$ with a hole at $x =$.



Check Your Progress **Graph each rational function.**

a. $f(x) = \dfrac{x}{x + 3}$

b. $f(x) = \dfrac{x^2 - 16}{x + 4}$

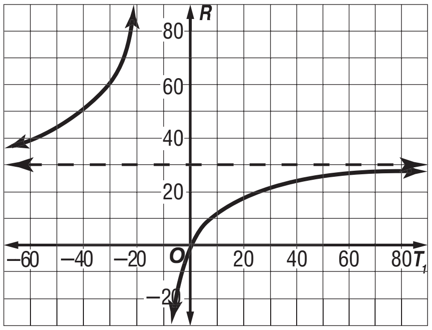
EXAMPLE Use Graphs of Rational Functions

④ **AVERAGE SPEED** **Use the situation and formula given in Example 4 of your textbook.**

a. Draw the graph if r_2 = 15 miles per hour.

The function is $R = \frac{2r_1(15)}{r_1 + 15}$ or $R = \frac{30r_1}{r_1 + 15}$. The vertical asymptote is $r_1 =$.

Graph the vertical asymptote and the function. Notice that the horizontal asymptote is $R =$.



b. What is the R-intercept of the graph?

The R-intercept is .

c. What domain and range values are meaningful in the context of the problem?

In the problem context, the speeds are nonnegative values. Therefore, only values of r_1 greater than or equal to and values of R between are meaningful.

Check Your Progress **A train travels at one velocity V_1 for a given amount of time t_1 and then another velocity V_2 for a different amount of time t_2. The average velocity is given by $V = \frac{V_1t_1 + V_2t_2}{t_1 + t_2}$.**

a. Let t_1 be the independent variable and let V be the dependent variable. Draw the graph if V_1 = 60 miles per hour, V_2 = 30 miles per hour, and t_2 = 1 hour.

b. What is the V-intercept of the graph?

c. What values of t_1 and V are meaningful in the context of the problem?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–4 Direct, Joint, and Inverse Variation

MAIN IDEAS

- Recognize and solve direct and joint variation problems.
- Recognize and solve inverse variation problems.

KEY CONCEPTS

Direct Variation y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.

Joint Variation y varies jointly as x and z if there is some number k such that $y = kxz$, where $k \neq 0$, $x \neq 0$, and $z \neq 0$.

EXAMPLE Direct Variation

① **If y varies directly as x and $y = -15$ when $x = 5$, find y when $x = 3$.**

Use a proportion that relates the values.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$	Direct proportion.
$\boxed{} = \boxed{}$	$y_1 = -15$, $x_1 = 5$, and $x_2 = 3$
$\boxed{} = \boxed{}$	Cross multiply.
$\boxed{} = \boxed{}$	Simplify.
$\boxed{} = y_2$	Divide each side by 5.

When $x = 3$, the value of $y = \boxed{}$.

EXAMPLE Joint Variation

② **Suppose y varies jointly as x and z. Find y when $x = 10$ and $z = 5$ if $y = 12$ when $x = 3$ and $z = 8$.**

Use a proportion that relates the values.

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2}$	Joint variation.
$\boxed{} = \boxed{}$	$y_1 = 12$, $x_1 = 3$, $z_1 = 8$, $x_2 = 10$, and $z_2 = 5$
$\boxed{} = \boxed{}$	Cross multiply.
$\boxed{} = \boxed{}$	Simplify.
$\boxed{} = y_2$	Divide each side by 24.

When $x = 10$ and $z = \boxed{}$, $y = \boxed{}$.

KEY CONCEPT

Inverse Variation y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

EXAMPLE Inverse Variation

3 **If r varies inversely as t and $r = -6$ when $t = 2$, find r when $t = -7$.**

Use a proportion that relates the values.

$\frac{r_1}{t_2} = \frac{r_2}{t_1}$ Inverse variation

= $r_1 = -6$, $t_1 = 2$, and $t_2 = -7$

= Cross multiply.

= Simplify.

= Divide each side by -7.

When $t = -7$, r is or $1\frac{5}{7}$.

Check Your Progress

a. If y varies directly as x and $y = 12$ when $x = -3$, find y when $x = 7$.

b. Suppose y varies jointly as x and z. Find y when $x = 3$ and $z = 2$ if $y = 11$ when $x = 5$ and $z = 22$.

c. If a varies inversely as b and $a = 3$ when $b = 8$, find a when $b = 6$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–5 Classes of Function

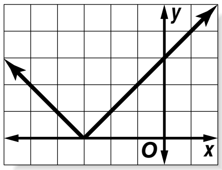
MAIN IDEAS

- Identify graphs as different types of functions.
- Identify equations as different types of functions.

EXAMPLE Identify a Function Given the Graph

① **Identify the type of function represented by each graph.**

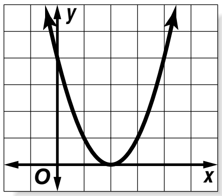
a.



The graph is a V shape, so it is

function

b.

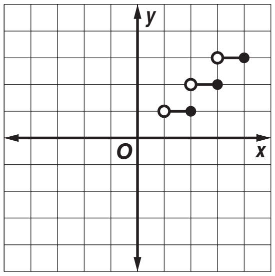


The graph is a parabola, so it is

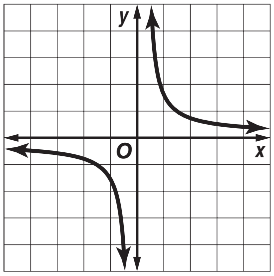
function.

Check Your Progress **Identify the type of function represented by each graph.**

a.



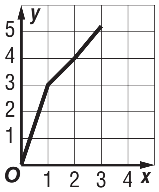
b.



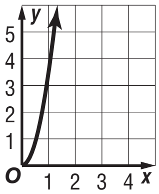
EXAMPLE Match Equation with Graph

② SHIPPING CHARGES **A chart gives the shipping rates for an Internet company. They charge $3.50 to ship less than 1 pound, $3.95 for 1 pound and over up to 2 pounds, and $5.20 for 2 pounds and over up to 3 pounds. Which graph depicts these rates?**

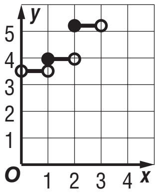
a.



b.



c.



The shipping rate is constant for x values from 0 to 1. Then it jumps at $x =$ and remains constant until $x =$.

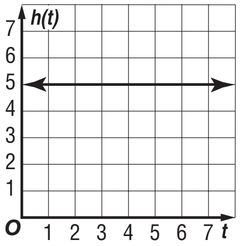
The graph jumps again at $x =$ ____ and remains constant until $x =$ ____. The graph of this function looks like steps, so this is ____, the step or ____.

REVIEW IT

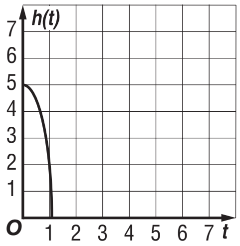
In order for a square root to be a real number, what is true about the radicand of the square root function? (*Lesson 7-3*)

Check Your Progress A ball is thrown into the air. The path of the ball is represented by the equation $h(t) = -16t^2 + 5$. Which graph represents this situation?

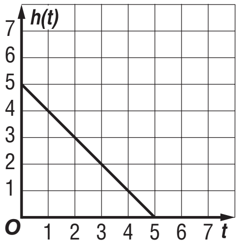
a.



b.



c.

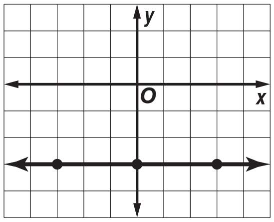


EXAMPLE Identify a Function Given its Equation

3 **Identify the type of function represented by each equation. Then graph the equation.**

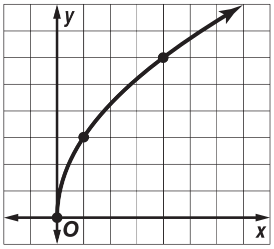
a. $y = -3$

Since the equation has no x-intercept, it is the constant function. Determine some points on the graph and graph it.



b. $y = \sqrt{9x}$

Since the equation includes an expression with a square root, it is a square root function. Plot some points and use what you know about square root graphs to graph it.



Check Your Progress **Identify the type of function represented by each equation. Then graph the equation.**

a. $y = \dfrac{x^2 - 9}{x + 3}$

b. $y = -2x$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–6 Solving Rational Equations and Inequalities

MAIN IDEAS

- Solve rational equations.
- Solve rational inequalities.

BUILD YOUR VOCABULARY (page 227)

Any equation that contains one or more rational expressions is called a **rational equation**.

Inequalities that contain one or more rational expressions are called **rational inequalities**.

REMEMBER IT

Rational equations are often easier to solve when you eliminate the fractions first.

EXAMPLE Solve a Rational Equation

① **Solve $\frac{5}{24} + \frac{2}{3-x} = \frac{1}{4}$.**

The LCD for the three denominators is $24(3 - x)$.

$\frac{5}{24} + \frac{2}{3-x} = \frac{1}{4}$	Original equation
$\boxed{\qquad}\left(\frac{5}{24} + \frac{2}{3-x}\right) = \frac{1}{4}\boxed{\qquad}$	Multiply each side by $\boxed{\qquad}$.
$\overset{1}{\cancel{24}}(3-x)\left(\frac{5}{\underset{1}{\cancel{24}}}\right) + 24\overset{1}{\cancel{(3-x)}}\left(\frac{2}{\underset{1}{\cancel{3-x}}}\right) = \frac{1}{\underset{1}{\cancel{4}}}\overset{6}{\cancel{24}}(3-x)$	Simplify.
$\boxed{\qquad} = \boxed{\qquad}$	Simplify.
$\boxed{\qquad} = \boxed{\qquad}$	Add.
$x = \boxed{\qquad}$	

Check Your Progress Solve $\frac{5}{2} + \frac{3}{x-1} = \frac{1}{2}$.

WRITE IT

When solving a rational equation, why must the solutions be checked in the original equation rather than in any of the derived equations?

____

____

____

____

____

____

EXAMPLE Elimination of a Possible Solution

2 Solve $\frac{p^2 - p + 1}{p + 1} = \frac{p^2 - 7}{p^2 - 1} + p$.

The LCD is $p^2 - 1$.

$$\frac{p^2 - p + 1}{p + 1} = \frac{p^2 - 7}{p^2 - 1} + p$$

Original equation

$$\overset{p-1}{\cancel{(p^2 - 1)}}\frac{p^2 - p + 1}{\underset{1}{\cancel{p + 1}}} = \frac{p^2 - 7}{\underset{1}{\cancel{p^2 - 1}}}\overset{1}{\cancel{(p^2 - 1)}} + p(p^2 - 1)$$

$$(p - 1)(p^2 - p + 1) = p^2 - 7 + (p^2 - 1)p$$

Distributive Property

$$p^3 - p^2 + p - p^2 + p - 1 = p^2 - 7 + p^3 - p$$

Simplify.

$$-2p^2 + 2p - 1 = p^2 - p - 7$$

Simplify.

$$\boxed{} = \boxed{}$$

Add $(2p^2 - 2p + 1)$ to each side.

$$0 = \boxed{}$$

Divide each side by 3.

$$0 = \boxed{}$$

Factor.

$$\boxed{} = 0 \text{ or } \boxed{} = 0$$

Zero Product Property

$$p = \boxed{} \qquad p = \boxed{}$$

Solve each equation.

Since $p = -1$ results in a zero in the denominator, eliminate -1.

Check Your Progress Solve $\frac{1}{x - 2} = \frac{2x + 1}{x^2 + 2x - 8} + \frac{2}{x + 4}$.

EXAMPLE

③ **MOWING LAWNS** **Tim and Ashley mow lawns together. Tim working alone could complete a particular job in 4.5 hours, and Ashley could complete it alone in 3.7 hours. How long does it take to complete the job when they work together?**

In 1 hour, Tim could complete $\boxed{}$ of the job.

In 1 hour, Ashley could complete $\boxed{}$ of the job.

In t hours, Tim could complete $\frac{1}{4.5} \cdot t$ or $\boxed{}$ of the job.

In t hours Ashley could complete $\frac{1}{3.7} \cdot t$ or $\boxed{}$ of the job.

$$\underbrace{\text{Part completed by Tim}}_{\frac{t}{4.5}} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{part completed by Ashley}}_{\frac{t}{3.7}} \quad \underbrace{\text{equals}}_{=} \quad \underbrace{\text{entire job.}}_{1}$$

$\frac{t}{4.5} + \frac{t}{3.7} = 1$ Original equation

$\left(\boxed{}\right)\left(\frac{t}{4.5} + \frac{t}{3.7}\right) = (1)\left(\boxed{}\right)$ Multiply each side by 16.65.

$8.2t = 16.65$ Simplify.

$t \approx \boxed{}$ Divide each side by 8.2.

It would take about $\boxed{}$ hours working together to complete the job.

Check Your Progress Libby and Nate clean together. Nate working alone could complete the job in 3 hours, and Libby could complete it alone in 5 hours. How long does it take to complete the job when they work together?

EXAMPLE

④ **SWIMMING** **Janine swims for 5 hours in a stream that has a current of 1 mile per hour. She leaves her dock and swims upstream for 2 miles and then swims back to her dock. What is her swimming speed in still water?**

Words The formula that relates distance, time, and rate is $d = rt$ or $\frac{d}{r} = t$.

Variables Let r = speed in still water. Then her speed with the current is $\boxed{}$, and her speed against the current is $\boxed{}$.

Equation

$$\underbrace{\text{Time going with the current}} \quad \underbrace{\text{plus}} \quad \underbrace{\text{Time going against the current}} \quad \underbrace{\text{equals}} \quad \underbrace{\text{total time.}}$$

$$\boxed{} \quad + \quad \boxed{} \quad = \quad \boxed{}$$

Solve the equation.

$$\frac{2}{r-1} + \frac{2}{r+1} = 5$$

Original equation

$$(r^2-1)\left(\frac{2}{r-1} + \frac{2}{r+1}\right) = (r^2-1)5$$

Multiply each side by $r^2 - 1$.

$$\overset{r+1}{\cancel{(r^2-1)}}\frac{2}{\underset{1}{\cancel{r-1}}} + \overset{r-1}{\cancel{(r^2-1)}}\frac{2}{\underset{1}{\cancel{r+1}}} = 5(r^2-1)$$

Distributive Property

$$\boxed{} = 5r^2 - 5$$

Simplify.

$$\boxed{} = 5r^2 - 5$$

Simplify.

$$0 = \boxed{}$$

Subtract $4r$ from each side.

Use the Quadratic Formula to solve for r.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-5)}}{2(5)}$$

$x = r$, $a = 5$, $b = -4$, and $c = -5$

$$r = \frac{4 \pm \sqrt{\boxed{}}}{10}$$

Simplify.

$$r = \boxed{} \text{ or } \boxed{}$$

Use a calculator.

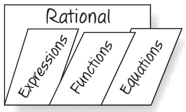
Since speed must be positive, it is $\boxed{}$ miles per hour.

Check Your Progress Lynne swims for 1 hour in a stream that has a current of 2 miles per hour. She leaves her dock and swims upstream for 3 miles and then back to her dock. What is her swimming speed in still water?

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Expressions, write how simplifying rational expressions is similar to simplifying rational numbers.



EXAMPLE

Solve a Rational Inequality

⑤ **Solve $\frac{1}{3s} + \frac{2}{9s} < \frac{2}{3}$.**

Step 1 Values that make the denominator equal to 0 are excluded from the denominator. For this inequality the excluded value is 0.

Step 2 Solve the related equation.

$$\frac{1}{3s} + \frac{2}{9s} = \frac{2}{3}$$ Related equation.

$$\boxed{}\left(\frac{1}{3s} + \frac{2}{9s}\right) = \frac{2}{3}\boxed{}$$ Multiply each side by $\boxed{}$.

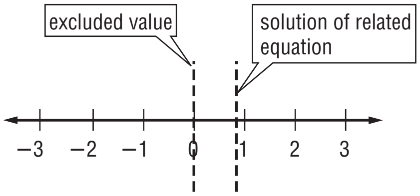
$$\boxed{} = \boxed{}$$ Simplify.

$$\boxed{} = \boxed{}$$ Add.

$$\boxed{} = \boxed{}$$ Divide each side by $\boxed{}$.

Step 3 Draw vertical lines at the excluded value and at the solution to separate the number line into regions.

Now test a sample value in each region to determine if the values in the region satisfy the inequality.



Test $s = -1$.

$$\frac{1}{3(-1)} + \frac{2}{9(-1)} \overset{?}{<} \frac{2}{3}$$

$$\boxed{} - \boxed{} \overset{?}{<} \frac{2}{3}$$

$$\boxed{} < \frac{2}{3}$$

So, $s < 0$ is a solution.

Test $s = \frac{1}{3}$.

$$\frac{1}{3\left(\frac{1}{3}\right)} + \frac{2}{9\left(\frac{1}{3}\right)} \overset{?}{<} \frac{2}{3}$$

$$\boxed{} + \boxed{} \overset{?}{<} \frac{2}{3}$$

$$\boxed{} \not< \frac{2}{3}$$

So, $0 < s < \frac{5}{6}$ is not a solution.

Test $s = 1$.

$$\frac{1}{3(1)} + \frac{2}{9(1)} \overset{?}{<} \frac{2}{3}$$

$$\boxed{} + \boxed{} \overset{?}{<} \frac{2}{3}$$

$$\boxed{} < \frac{2}{3}$$

So $s > \frac{5}{6}$ is not a solution.

Check Your Progress Solve $\frac{1}{x} + \frac{3}{5x} < \frac{2}{5}$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 8

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 8 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 8, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 226–227)* to help you solve the puzzle.

8-1

Multiplying and Dividing Rational Expressions

1. Which expressions are complex fractions?

i. $\frac{7}{12}$ **ii.** $\frac{\frac{3}{8}}{\frac{5}{16}}$ **iii.** $\frac{r+5}{r-5}$ **vi.** $\frac{\frac{z+1}{z}}{z}$ **v.** $\frac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$

Simplify each expression.

2. $\frac{6r^2s^3}{t^3} \cdot \frac{3s^2t^2}{12rst}$

3. $\frac{3y^2+3y-6}{4y-8} \div \frac{y^2-4}{2y^2-6y+4}$

8-2

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

4. $4y, 9xy, 6y^2$

5. x^2-5x+6, x^3-4x^2+4x

Simplify each expression.

6. $\frac{6}{4ab} - \frac{3}{ab^2}$

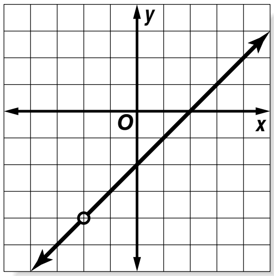
7. $\frac{5q}{2p} + 8$

8-3

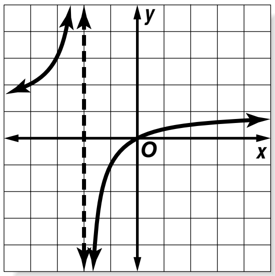
Graphing Rational Functions

For Exercises 8 and 9, refer to the two rational functions shown.

I.



II.



8. Graph I has a at $x =$.

9. Graph II has a at $x =$.

10. Graph $f(x) = \frac{3}{x - 2}$.

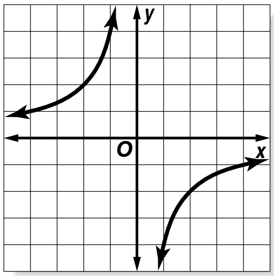
8-4

Direct, Joint, and Inverse Variation

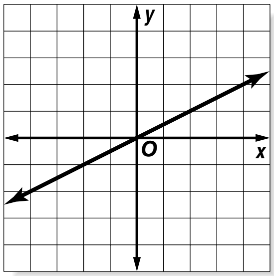
11. Suppose y varies inversely as x. Find y when $x = 4$, $y = 8$ when $x = 3$.

Which type of variation, direct or inverse, is represented by each graph?

12.



13.



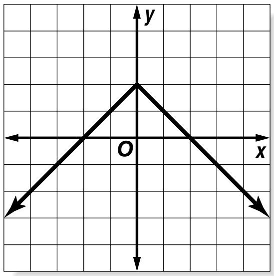
8-5

Classes of Functions

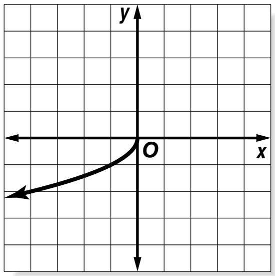
Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

a. square root
b. absolute value
c. rational
d. greatest integer
e. constant
f. identity

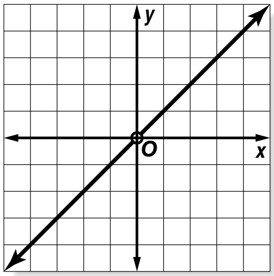
14.



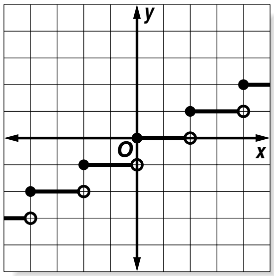
15.



16.



17.



8-6

Solving Rational Equations and Inequalities

Solve each equation or inequality.

18. $\frac{1}{2x} + \frac{x + 1}{x} = 4$

19. $\frac{y + 1}{y - 1} - \frac{2}{y - 1} = \frac{y}{3}$

18. $\frac{3}{z + 2} - \frac{6}{z} > 0$

CHAPTER
8
Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 8.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want take the Chapter 8 Practice Test on page 493 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 8 Study Guide and Review on pages 489–492 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 493.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 8 Foldable.
- Then complete the Chapter 8 Study Guide and Review on pages 489–492 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 493.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 9

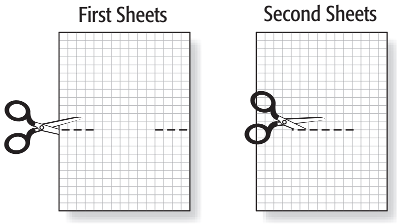
Exponential and Logarithmic Relations

FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of notebook paper.

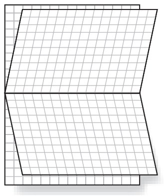
STEP 1

Fold in half along the width. On the first sheet, cut 5 cm along the fold at the ends. On the second sheet, cut in the center, stopping 5 cm from the ends.



STEP 2

Insert the first sheet through the second sheet and align the folds. Label the pages with lesson numbers.



NOTE-TAKING TIP: When you take notes, listen or read for main ideas. Then record those ideas in a simplified form for future reference.

CHAPTER
9

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. As you complete the study notes for the chapter, complete you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
Change of Base Formula			
common logarithm [LAW-guh-rih-thuhm]			
exponential decay [EHK-spuh-NEHN-chuhl]			
exponential equation			
exponential function			
exponential growth			
exponential inequality			
logarithm			

Vocabulary Term	Found on Page	Definition	Description or Example
logarithmic equation LAW-guh-RIHTH-mihk			
logarithmic function LAW-guh-RIHTH-mihk			
logarithmic inequality LAW-guh-RIHTH-mihk			
natural base, *e*			
natural base exponential function			
natural logarithm			
natural logarithmic function			
rate of decay			
rate of growth			

9–1 Exponential Functions

MAIN IDEAS

- Graph exponential functions.
- Solve exponential equations and inequalities.

REMEMBER IT

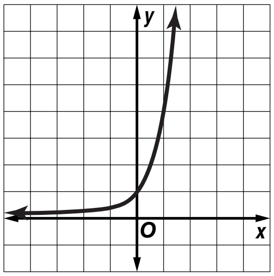
Polynomial functions like $y = x^2$ have a variable for the base and a constant for the exponent. Exponential functions like $y = 2^x$ have a constant for the base and a variable for the exponent.

EXAMPLE Graph an Exponential Function

① **Sketch the graph of $y = 4^x$. Then state the function's domain and range.**

Make a table of values. Connect the points to sketch a smooth curve.

x	$y = 4^x$
−2	
−1	
0	
1	
2	



The domain is ______, and the range is ______.

Check Your Progress Sketch the graph of $y = 3^x$. Then state the function's domain and range.

BUILD YOUR VOCABULARY (page 254)

In general, an equation of the form ______, where $a \neq 0$, $b > 0$, and $b \neq 1$, is called an **exponential function** with base b.

KEY CONCEPT

Exponential Growth and Decay

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

EXAMPLE Identify Exponential Growth and Decay

2 **Determine whether each function represents exponential *growth* or *decay*.**

a. $y = 10\left(\frac{4}{3}\right)^x$ The function represents exponential , since the base, .

is

b. $y = (0.7)^x$ The function represents exponential , since the base, ,

is .

Check Your Progress **Determine whether each function represents exponential *growth* or *decay*.**

a. $y = 10\left(\frac{2}{5}\right)^x$

b. $y = (0.5)^x$

BUILD YOUR VOCABULARY (page 254)

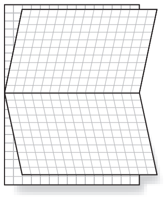
Exponential equations are equations in which occur as .

Exponential inequalities are inequalities involving functions.

FOLDABLES™

ORGANIZE IT

Use the page for Lesson 9-1. Write your own example of an exponential growth and an exponential decay function. Then sketch a graph of each function.



EXAMPLE Write an Exponential Function

3 **PHONES In 1995, there were an estimated 28,154,000 cellular telephone subscribers in the United States. By 2005, this estimated number had risen to 194,633,000.**

a. Write an exponential function of the form $y = ab^x$ that could be used to model the number of cellular telephone subscribers in the U.S. Write the function in terms of x, the number of years since 1995.

For 1995, the time x equals , and the initial number of cellular telephone subscribers y is 28,154,000. Thus the y-intercept, and the value of a, is .

For 2005, the time x equals $2005 - 1995$ or .

Substitute these values and the value of a into an exponential function to approximate the value of b.

$y = ab^x$	Exponential function
$194{,}633{,}000 = 28{,}154{,}000b^{10}$	Replace x with 10, y with 194,633,000, and a with 28,154,000.
$\approx b^{10}$	Divide each side by 28,154,000.
$\approx b$	Take the 10th root of each side.

To find the 10th root of , use selection 5: $\sqrt[x]{\ }$ under the MATH menu on the TI-83 Plus.

Keystrokes: 10 $\boxed{\text{MATH}}$ 5 6.91 $\boxed{\text{ENTER}}$ 1.21324302926

Answer: An equation that models the number of cellular telephone subscribers in the U.S. from 1995 to 2005 is .

b. Suppose the number of cellular subscribers continues to increase at the same rate. Estimate the number of U.S. subscribers in 2015.

For 2015, the time x equals $2015 - 1995$ or .

$y = 28{,}154{,}000(1.213)^x$	Modeling equation
$y = 28{,}154{,}000(1.213)$	Replace x with .
$y =$	Use a calculator.

The number of cellular phone subscribers will be about in 2015.

Check Your Progress **In 1991, 4.9% of Americans had diabetes. By 2000, this percent had risen to 7.3%.**

a. Write an exponential function of the form $y = ab^x$ that could be used to model the percentage of Americans with diabetes. Write the function in terms of x, the number of years since 1991.

b. Suppose the percent of Americans with diabetes continues to increase at the same rate. Estimate the percent of Americans with diabetes in 2010.

KEY CONCEPTS

Property of Equality for Exponential Functions If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Property of Inequality for Exponential Functions If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

EXAMPLE Solve Exponential Equations

4 **Solve each equation.**

a. $4^{9n-2} = 256$.

$4^{9n-2} = 256$	Original equation
$4^{9n-2} = 4^4$	Rewrite 256 as 4^4 so each side has the same base.
$\boxed{} = \boxed{}$	Property of Equality for Exponential Functions
$\boxed{}$	Add 2 to each side.
$n = \boxed{}$	Divide each side by 9.

b. $3^{5x} = 9^{2x-1}$

$3^{5x} = 9^{2x-1}$	Original equation
$3^{5x} = 3^{2(2x-1)}$	Rewrite 9 as 3^2 so each side has the same base.
$5x = \boxed{}(2x-1)$	Property of Equality for Exponential Functions
$5x = \boxed{}$	Distributive Property
$x = \boxed{}$	Subtract $4x$ from each side.

EXAMPLE Solve Exponential Inequalities

5 **Solve $5^{3-2k} > \frac{1}{625}$.**

$5^{3-2k} > \frac{1}{625}$	Original inequality
$5^{3-2k} > 5^{-4}$	Rewrite $\frac{1}{625}$ as $\frac{1}{5^4}$ or 5^{-4}.
$\boxed{} > \boxed{}$	Property of Inequality for Exponential Functions
$\boxed{} > \boxed{}$	Subtract 3 from each side.
$k < \boxed{}$	Divide each side by -2.

Check Your Progress **Solve.**

a. $2^{3x+1} = 32$

b. $5^{2x} = 25^{2x-1}$

c. $3^{3-2k} > \frac{1}{27}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–2 Logarithms and Logarithmic Functions

MAIN IDEAS

- Evaluate logarithmic expressions.
- Solve logarithmic equations and inequalities.

KEY CONCEPT

Logarithm with Base *b*
Let *b* and *x* be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent *y* that makes the equation $b^y = x$ true.

EXAMPLE Logarithmic to Exponential Form

1 Write each equation in exponential form.

a. $\log_3 9 = 2$

$\log_3 9 = 2 \longrightarrow$

b. $\log_{10} \frac{1}{100} = -2$

$\log_{10} \frac{1}{100} = -2 \longrightarrow$

EXAMPLE Exponential to Logarithmic Form

2 Write each equation in logarithmic form.

a. $5^3 = 12^5$

$5^3 = 125 \longrightarrow$

b. $27^{\frac{1}{3}} = 3$

$27^{\frac{1}{3}} = 3 \longrightarrow$

Check Your Progress

Write each equation in exponential form.

a. $\log_2 8 = 3$

b. $\log_3 \frac{1}{9} = -2$

Write each equation in logarithmic form.

c. $3^4 = 81$

d. $81^{\frac{1}{2}} = 9$

EXAMPLE Evaluate Logarithmic Expressions

3 Evaluate $\log_3 243$.

$\log_3 243 = y$ Let the logarithm equal *y*.

= Definition of logarithm

= $243 = 3^5$

$= y$ Property of Equality for Exponential Functions

Check Your Progress Evaluate $\log_{10} 100$.

EXAMPLE Solve a Logarithmic Equation

4 Solve $\log_8 n = \frac{4}{3}$.

$\log_8 n = \frac{4}{3}$ Original equation

$n =$ Definition of logarithm

$n =$ $8 = 2^3$

$n =$ Power of a Power

$n =$ Simplify.

Check Your Progress Solve $\log_{27} n = \frac{2}{3}$.

KEY CONCEPTS

Logarithmic to Exponential Inequality If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.

If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.

Property of Equality for Logarithmic Functions If *b* is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.

Property of Inequality for Logarithmic Functions If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

EXAMPLE Solve a Logarithmic Inequality

5 Solve $\log_6 x > 3$.

$\log_6 x > 3$ Original inequality

Logarithmic to exponential inequality

Simplify.

Check Your Progress Solve $\log_3 x < 2$.

EXAMPLE Solve Equations with Logarithms on Each Side

⑥ **Solve $\log_4 x^2 = \log_4 (4x - 3)$.**

$\log_4 x^2 = \log_4 (4x - 3)$ Original equation

$x^2 =$ Property of Equality for Logarithmic Functions

$x^2 - 4x + 3 = 0$ Subtract $4x$ and add 3 to each side.

$= 0$ Factor.

$= 0$ or $= 0$ Zero Product Property

$x =$ $x =$ Solve each equation.

EXAMPLE Solve Inequalities with Logarithms on Each Side

⑦ **Solve $\log_7 (2x + 8) > \log_7 (x + 5)$.**

$\log_7 (2x + 8) > \log_7 (x + 5)$ Original inequality

$>$ Property of Inequality for Logarithmic Functions

$x >$ Addition and Subtraction Properties of Inequalities

We must exclude all values of x such that $2x + 8 \leq 0$ or $x + 5 \leq 0$. Thus the solution set is , , and . This compound inequality simplifies to .

Check Your Progress **Solve.**

a. $\log_5 x^2 = \log_5 (x + 6)$

b. $\log_5 (2x + 6) > \log_3 (x + 2)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–3 Properties of Logarithms

MAIN IDEAS

- Simplify and evaluate expressions using the properties of logarithms.
- Solve logarithmic equations using the properties of logarithms.

KEY CONCEPTS

Product Property of Logarithms The logarithm of a product is the sum of the logarithms of its factors.

Quotient Property of Logarithms The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.

EXAMPLE Use the Product Property

① **Use $\log_5 2 \approx 0.4307$ to approximate the value of $\log_5 250$.**

$\log_5 250 = \log_5 (5^3 \cdot 2)$	Replace 250 with $5^3 \cdot 2$.
$= \boxed{} + \boxed{}$	Product Property
$= \boxed{} + \boxed{}$	Inverse Property of Exponents and Logarithms
$\approx \boxed{}$	Replace $\log_5 2$.

EXAMPLE Use the Quotient Property

② **Use $\log_6 8 \approx 1.1606$ and $\log_6 32 \approx 1.9343$ to approximate the value of $\log_6 4$.**

$\log_6 4 = \log_6 \left(\frac{32}{8}\right)$	Replace 4 with $\frac{32}{8}$.
$= \boxed{} - \boxed{}$	Quotient Property
$\approx \boxed{} - \boxed{}$	
or $\boxed{}$	

Check Your Progress

a. Use $\log_2 3 \approx 1.5850$ to approximate the value of $\log_2 96$.

b. Use $\log_5 4 \approx 0.8614$ and $\log_5 32 \approx 2.1534$ to approximate the value of $\log_5 8$.

KEY CONCEPT

Power Property of Logarithms The logarithm of a power is the product of the logarithm and the exponent.

FOLDABLES™ Use the page for Lesson 9-3. Write your own examples that show the Product, Quotient, and Power Properties of Logarithms.

EXAMPLE Power Property of Logarithms

③ Given that $\log_5 6 \approx 1.1133$, approximate the value of $\log_5 216$.

$\log_5 216 =$ Replace 216 with 6^3.

$=$ Power Property

$\approx$ Replace $\log_5 6$ with 1.1133.

Check Your Progress Given that $\log_4 6 \approx 1.2925$, approximate the value of $\log_4 1296$.

EXAMPLE Solve Equations Using Properties of Logarithms

④ Solve each equation.

a. $4 \log_2 x - \log_2 5 = \log_2 125$

$4 \log_2 x - \log_2 5 = \log_2 125$ Original equation

$- \log_2 5 = \log_2 125$ Power Property

$= \log_2 125$ Quotient Property

$= 125$ Property of Equality for Logarithmic Functions

$=$ Multiply each side by 5.

$x =$ Take the 4th root of each side.

b. $\log_8 x + \log_8 (x - 12) = 2$

$\log_8 x + \log_8 (x - 12) = 2$	Original equation
$\log_8 [x(x - 12)] = 2$	Product Property
$x(x - 12) = \boxed{\quad}$	Definition of logarithm
$\boxed{\qquad\qquad} = 0$	Subtract 64 from each side.
$\boxed{\qquad}\boxed{\qquad} = 0$	Factor.
$\boxed{\qquad} = 0$ or $\boxed{\qquad} = 0$	Zero Product Property
$x = \boxed{\quad}$ $x = \boxed{\quad}$	Solve each equation.

Check Substitute each value into the original equation.

$\log_8 (-4) + \log_8 [(-4) - 12] \stackrel{?}{=} 2$	Replace *x* with −4.

$\log_8 (-4) + \log_8 \left(\boxed{\qquad}\right) \stackrel{?}{=} 2$

Since $\log_8 (-4)$ and $\log_8 (-16)$ are $\boxed{\qquad\qquad}$, −4 is an extraneous solution and must be eliminated.

$\log_8 16 + \log_8 (16 - 12) \stackrel{?}{=} 2$	Replace *x* with 16.
$\log_8 16 + \log_8 4 \stackrel{?}{=} 2$	$16 - 12 = 4$
$\log_8 (16 \cdot 4) \stackrel{?}{=} 2$	Product Property
$\log_8 \boxed{\quad} \stackrel{?}{=} 2$	$16 \cdot 4 = 64$
$2 = 2$ ✓	Definition of logarithm

The only solution is $\boxed{\qquad}$.

Check Your Progress **Solve each equation.**

a. $2 \log_3 x - 2 \log_3 6 = \log_3 4$

b. $\log_2 x + \log_2 (x - 6) = 4$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–4 Common Logarithms

MAIN IDEAS

- Solve exponential equations and inequalities using common logarithms.
- Evaluate logarithmic expressions using the Change of Base Formula.

EXAMPLE Find Common Logarithms

① **Use a calculator to evaluate each expression to four decimal places.**

a. log 6 **Keystrokes:** LOG 6 ENTER .7781512504

b. log 0.35 **Keystrokes:** LOG 0.35 ENTER

−.4559319556

Check Your Progress **Use a calculator to evaluate each expression to four decimal places.**

a. log 5

b. log 0.62

EXAMPLE Solve Logarithmic Equations

② **EARTHQUAKES** **Refer to Example 2 in your textbook. The San Fernando Valley earthquake of 1994 measured 6.6 on the Richter scale. How much energy did this earthquake release?**

$\log E = 11.8 + 1.5M$ Write the formula.

$\log E = 11.8 + 1.5(\quad)$ Replace *M* with .

$\log E =$ Simplify.

$10 \log E = 10^{21.7}$ Write each side using 10 as a base.

$E =$ Inverse Property of Exponents and Logarithms

$E \approx$ Use a calculator.

The amount of energy released was about

ergs.

REMEMBER IT When solving an exponential equation using logarithms, the first step is often referred to as *taking the logarithm of each side*.

Check Your Progress The amount of energy E, in ergs, that an earthquake releases is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. In 1999 an earthquake in Turkey measured 7.4 on the Richter scale. How much energy did this earthquake release?

EXAMPLE Solve Exponential Equations Using Logarithms

3 **Solve $5^x = 62$.**

$5^x = 62$ Original equation

$=$ Property of Equality for Logarithmic Functions

$=$ Power Property of Logarithms

$x =$ Divide each side by log 5.

$x \approx$ Use a calculator.

Check Your Progress Solve $3^x = 17$.

EXAMPLE Solve Exponential Inequalities Using Logarithms

4 **Solve $2^{7x} > 3^{5x-3}$.**

$$2^{7x} > 3^{5x-3}$$

$$\log 2^{7x} > \log 3^{5x-3}$$

$$>$$

$$7x \log 2 > \quad - $$

$$7x \log 2 - \quad > $$

$$\boxed{} > -3 \log 3$$

$$x < \frac{-3 \log 3}{7 \log 2 - 5 \log 3}$$

$$x < \frac{-3(0.4771)}{7(0.3010) - 5(0.4771)}$$

$$x < \boxed{}$$

KEY CONCEPT

Change of Base Formula For all positive numbers a, b, and n, where $a \neq 1$ and $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}.$$

Check Your Progress Solve $5^{3x} < 10^{x-2}$.

EXAMPLE Change of Base Formula

5 **Express $\log_3 18$ in terms of common logarithms. Then approximate its value to four decimal places.**

$\log_3 18 = \boxed{}$ Change of Base Formula

$\approx \boxed{}$ Use a calculator.

Check Your Progress Express $\log_5 16$ in terms of common logarithms. Then approximate its value to four decimal places.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–5 Base *e* and Natural Logarithms

MAIN IDEAS

- Evaluate expressions involving the natural base and natural logarithms.
- Solve exponential equations and inequalities using natural logarithms.

BUILD YOUR VOCABULARY (page 255)

The number 2.71828 … is referred to as the **natural base, *e*.**

An function with base is called a **natural base exponential function.**

The with base is called the **natural logarithm.**

The **natural logarithmic function**, $y = \ln x$, is the of the natural base exponential function, $y = e^x$.

EXAMPLE Evaluate Natural Base Expressions

1 Use a calculator to evaluate each expression to four decimal places.

a. $e^{0.5}$

Keystrokes: 2nd $[e^x]$ 0.5 ENTER

1.648721271

b. e^{-8}

Keystrokes: 2nd $[e^x]$ -8 ENTER

0.0003354626

Check Your Progress **Use a calculator to evaluate each expression to four decimal places.**

a. $e^{0.3}$

b. e^{-2}

EXAMPLE Evaluate Natural Logarithmic Expressions

WRITE IT

Write and evaluate three logarithms, one with base 2, one with base e, and one with base 10.

② **Use a calculator to evaluate each expression to four decimal places.**

a. ln 3

Keystrokes: LN 3 ENTER

1.098612289 ___

b. $\mathbf{ln\ \frac{1}{8}}$

Keystrokes: LN 1 ÷ 8 ENTER

−2.079441542 ___

Check Your Progress **Use a calculator to evaluate each expression to four decimal places.**

a. ln 2 ___

b. $\ln \frac{1}{2}$ ___

EXAMPLE Write Equivalent Expressions

③ **Write an equivalent exponential or logarithmic equation.**

a. $\mathbf{e^x = 23}$

$e^x = 23 \rightarrow$ ___ = ___

___ = ___

b. $\mathbf{\ln x \approx 1.2528}$

$\ln x \approx 1.2528 \rightarrow$ ___ ≈ ___

___ ≈ ___

Check Your Progress **Write an equivalent exponential or logarithmic equation.**

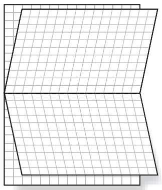
a. $e^x = 6$ ___

b. $\ln x = 2.25$ ___

FOLDABLES™

ORGANIZE IT

Use the page for Lesson 9-5. On the same grid, sketch the graph for $y = e^x$ and $y = \ln x$. Then write how you can tell that the two functions are inverses.



EXAMPLE Solve Base *e* Equations

④ **Solve $3e^{-2x} + 4 = 10$.**

$3e^{-2x} + 4 = 10$	Original equation
$=$	Subtract 4 from each side.
$=$	Divide each side by 3.
$\ln e^{-2x} = \ln 2$	Property of Equality for Logarithms
$=$	Inverse Property of Exponents and Logarithms
$x =$	Divide each side by -2.
$x \approx$	Use a calculator.

Check Your Progress Solve $2e^{-2x} + 5 = 15$.

EXAMPLE Solve Base *e* Inequalities

⑤ **SAVINGS Suppose you deposit \$700 into an account paying 3% annual interest, compounded continuously.**

a. What is the balance after 8 years?

$A = Pe^{rt}$	Continuous compounding formula
$= \quad e^{(0.03)(8)}$	Replace P with , r with 0.03, and t with 8.
$= 700e^{0.24}$	Simplify.
$\approx$	Use a calculator.

The balance after 8 years would be .

b. How long will it take for the balance in your account to reach at least \$1200?

$A \geq 1200$	The balance is at least \$1200.
$700e^{(0.03)t} \geq 1200$	Replace A with $700e^{(0.03)t}$.
$e^{(0.03)t} \geq$	Divide each side by 700.
$\ln e^{(0.03)t} \geq \ln \frac{12}{7}$	Property of Inequality for Logarithms
$\geq \ln \frac{12}{7}$	Inverse Property of Exponents and Logarithms
$t \geq \frac{\ln \frac{12}{7}}{0.03}$	Divide each side by 0.03.
$t \geq$	Use a calculator.

It will take about years for the balance to reach \$1200.

Check Your Progress **Suppose you deposit $700 into an account paying 6% annual interest, compounded continously.**

a. What is the balance after 7 years?

b. How long will it take for the balance in you account to reach at least $2500?

EXAMPLE Solve Natural Log Equations and Inequalities

6 **Solve each equation or inequality.**

a. ln $3x = 0.5$

$\ln 3x = 0.5$ Original equation

$e^{\ln 3x} = e^{0.5}$ Write each side using exponents and base *e*.

$\boxed{\quad} = \boxed{\quad}$ Inverse Property of Exponents and Logarithms

$x = \frac{e^{0.5}}{3}$ Divide each side by 3.

$x \approx \boxed{\quad}$ Use a calculator.

b. ln $(2x - 3) < 2.5$

$\ln (2x - 3) < 2.5$ Original inequality

$e^{\ln(2x - 3)} < e^{2.5}$ Write each side using exponents and base *e*.

$\boxed{\quad} < \boxed{\quad}$ Inverse Property of Exponents and Logarithms

$\boxed{\quad} < \boxed{\quad}$ Add 3 to each side.

$x < \boxed{\quad}$ Divide each side by 2.

$x < \boxed{\quad}$ Use a calculator.

Check Your Progress **Solve each equation or inequality.**

a. $\ln 2x = 0.7$

b. $\ln (x - 3) > 3$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–6 Exponential Growth and Decay

MAIN IDEAS

- Use logarithms to solve problems involving exponential decay.
- Use logarithms to solve problems involving exponential growth.

BUILD YOUR VOCABULARY (page 255)

The percent of decrease r is also referred to as the **rate of decay** in the equation for exponential decay of the form $y = a(1 - r)^t$.

EXAMPLE Exponential Decay of the Form $y = a(1 - r)^t$

① CAFFEINE **A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for 90% of this caffeine to be eliminated from a person's body?**

$y = a(1 - r)^t$	Exponential decay formula
$13 = 130(1 - 0.11)^t$	Replace y with 13, a with 130, and r with 0.11.
$\boxed{\quad} = \boxed{\quad}$	Divide each side by 130.
$\log \boxed{\quad} = \log \boxed{\quad}$	Property of Equality for Logarithms
$\log 0.10 = \boxed{\quad}$	Power Property for Logarithms
$\boxed{\quad} = t$	Divide each side by log 0.89.
$\boxed{\quad} \approx t$	Use a calculator.

It will take approximately $\boxed{\quad}$ hours for 90% of the caffeine to be eliminated from a person's body.

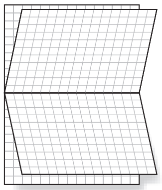
Check Your Progress Refer to Example 1. How long will it take for 80% of this caffeine to be eliminated from a person's body?

$\boxed{\quad}$

FOLDABLES™

ORGANIZE IT

Use the page for Lesson 9-6. If an exponential decay problem involves finding a period of time, which model do you use? Why?



EXAMPLE Exponential Decay of the Form $y = ae^{-kt}$

① **GEOLOGY** **The half-life of Sodium-22 is 2.6 years.**

a. What is the value of *k* and the equation of decay for Sodium-22?

$y = ae^{-kt}$	Exponential decay formula
$0.5a = ae^{-k(2.6)}$	Replace *y* and *t*.
$0.5 = e^{-2.6k}$	Divide each side by *a*.
$\ln \boxed{\quad} = \ln \boxed{\quad}$	Property of Equality for Logarithmic Functions
$\ln 0.5 = \boxed{\quad}$	Inverse Property of Exponents and Logarithms
$\boxed{\quad} = k$	Divide each side by −2.6.
$\boxed{\quad} = k$	Use a calculator.

The constant *k* for Sodium-22 is $\boxed{\quad}$. The equation for the decay of Sodium-22 is $\boxed{\quad}$, where *t* is years.

b. A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth?

$y = ae^{-0.2666t}$	Decay formula
$0.1a = ae^{-0.2666t}$	Replace *y* with 0.1*a*.
$0.1 = e^{-0.2666t}$	Divide each side by *a*.
$\ln \boxed{\quad} = \ln \boxed{\quad}$	Property of Equality for Logarithms
$\ln \boxed{\quad} = \boxed{\quad}$	Inverse Property
$\boxed{\quad} = t$	Divide each side by −0.2666.
$\boxed{\quad} \approx t$	Use a calculator.

It was formed about $\boxed{\quad}$ years ago.

Check Your Progress **The half-life of radioactive iodine used in medical studies is 8 hours.**

a. What is the value of k for radioactive iodine?

b. A doctor wants to know when the amount of radioactive iodine in a patient's body is 20% of the original amount. When will this occur?

BUILD YOUR VOCABULARY (page 255)

The percent of increase r is also referred to as the **rate of growth** in the equation for exponential growth of the form $y = a(1 + r)^t$.

EXAMPLE Exponential Growth of the Form $y = a(1 + r)^t$

③ **TEST EXAMPLE** **The population of a city of one million is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled?**

A 4 years **B** 5 years **C** 20 years **D** 23 years

$y = a(1 + r)^t$	Growth formula
$2{,}000{,}000 = 1{,}000{,}000(1 + 0.03)^t$	Replace y, a, and r.
$\boxed{} = \boxed{}$	Divide each side by 1,000,000.
$\ln \boxed{} = \ln \boxed{}$	Property of Equality for Logarithms
$\ln 2 = \boxed{}$	Power Property of Logarithms
$\frac{\ln 2}{\ln 1.03} = t$	Divide.
$t \approx 23.45$	Use a calculator.

The population will have doubled in $\boxed{}$ years.

WRITE IT

How can you tell whether an exponential equation involves growth or decay?

____

____

____

____

Check Your Progress The population of a city of 10,000 is increasing at a rate of 5% per year. If the population continues to grow at this rate, in about how many years will the population have doubled?

A 10 years **B** 12 years **C** 14 years **D** 18 years

EXAMPLE Exponential Growth of the Form $y = ae^{kt}$

4 POPULATION As of 2005, Nigeria had an estimated population of 129 million people, and the United States had an estimated population of 296 million people. Assume that the populations of Nigeria and the United States can be modeled by $N(t) = 129e^{0.024t}$ and $U(t) = 296e^{0.009t}$, respectively, where *t* represents the number of years since 2005. According to these models, when will Nigeria's population be more than the population of the United States?

You want to find *t* such that $N(t) > U(t)$.

$N(t) > U(t)$

$129e^{0.024t} > 296e^{0.009t}$ Replace $N(t)$ with $129e^{0.024t}$ and $U(t)$ with $296e^{0.009t}$.

$\ln 129e^{0.024t} > \ln 296e^{0.009t}$ Property of Inequality for Logarithms

$\ln 129 + \ln e^{0.024t} > \boxed{\quad} + \boxed{\quad}$ Product Property of Logarithms

$\ln 129 + 0.024t > \boxed{\quad} + \boxed{\quad}$ Inverse Property of Exponents and Logarithms

$\ln 129 \boxed{\quad} > \boxed{\quad} t$ Subtract 296 and 0.024*t* from each side.

$\boxed{} < t$ Divide each side by −0.015.

$\boxed{} < t$ Use a calculator.

After ☐ years or in ☐, Nigeria’s population will be greater than the population of the U.S.

Check Your Progress As of 2000, Saudi Arabia had an estimated population of 20.7 million people and the United States had an estimated population of 278 million people. The growth of the populations of Saudi Arabia and the United States can be modeled by $S(t) = 20.7e^{0.0327t}$ and $U(t) = 278e^{0.009t}$, respectively, where t represents the number of years since 2000. According to these models, when will Saudi Arabia’s population be more than the population of the United States?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 9

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 9 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 9, go to: glencoe.com	You can use your completed **Vocabulary Builder** (*pages 254–255*) to help you solve the puzzle.

9-1

Exponential Functions

Determine whether each function represents exponential *growth* or *decay*.

1. $y = 0.2(3)^x$

2. $y = 3\left(\frac{2}{5}\right)^x$

3. $y = 0.4(1.01)$

4. Simplify $4^x \cdot 4^{2x}$.

5. Solve $25^{y+3} = \left(\frac{1}{5}\right)^y$.

9-2

Logarithms and Logarithmic Functions

6. What is the inverse of the function $y = 5^x$?

7. What is the inverse of the function $y = \log_{10} x$?

8. Evaluate $\log_{27} 9$.

9. Solve $\log_8 x = -\frac{1}{3}$.

9-3

Properties of Logarithms

State whether each of the following equations is *true* or *false*. If the statement is true, name the property of logarithms that is illustrated.

10. $\log_3 10 = \log_3 30 - \log_3 3$

11. $\log_4 12 = \log_4 4 + \log_4 8$

12. $\log_2 81 = 2\log_2 9$

Solve each equation.

13. $\log_5 14 - \log_5 (2x) = \log_5 21$

14. $\log_2 x + \log_2 (x + 2) = 3$

9-4

Common Logarithms

Match each expression from the first column with an expression from the second column that has the same value.

15. $\log_2 2$

16. $\log 12$

17. $\log_3 1$

18. $\log_5 \frac{1}{5}$

19. $\log 1000$

a. $\log_4 1$

b. $\log_2 8$

c. $\log_{10} 12$

d. $\log_5 5$

e. $\log 0.1$

20. Solve $6^{2x-1} = 2^{5x}$. Round to four decimal places.

21. Express $\log_8 5$ in terms of common logarithms. Then approximate its value to four decimal places.

9-5

Base *e* and Natural Logarithms

Match each expression from the first column with its value in the second column. Some choices may be used more than once or not at all.

22. $e^{\ln 5}$

23. $\ln 1$

24. $e^{\ln e}$

25. $\ln e^5$

26. $\ln e$

27. $\ln \left(\frac{1}{e}\right)$

I. 1
II. 10
III. −1
IV. 5
V. 0
VI. *e*

28. Solve $\ln (x - 8) = 5$. Round to four decimal places.

29. Evaluate $e^{\ln 5.2}$.

9-6

Exponential Growth and Decay

State whether each equation represents exponential *growth* or *decay*.

30. $y = 5e^{0.15t}$

31. $y = 1000(1 - 0.05)^t$

32. $y = 0.3e^{-1200t}$

33. $y = 2(1 + 0.0001)^t$

34. Leroy bought a lawn mower for $1,200. It is expected to depreciate at a rate of 20% per year. What will be the value of the lawn mower in 5 years?

35. The population of a school has increased at a steady rate each year from 375 students to 580 students in 8 years. Find the annual rate of growth.

CHAPTER 9

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 9.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 9 Practice Test on page 557 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 9 Study Guide and Review on pages 552–556 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 557 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 9 Foldable.
- Then complete the Chapter 9 Study Guide and Review on pages 552–556 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 557 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER

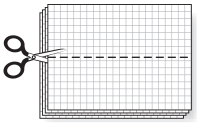
10

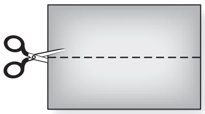
Conic Sections

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

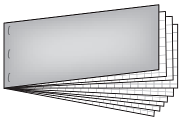
Begin with five sheets of notebook paper.

STEP 1 **Cut** each sheet of grid paper in half lengthwise. Cut the sheet of construction paper in half lengthwise to form a front and back cover for the booklet of grid paper.





STEP 2 **Staple** all the sheets together to form a long, thin notepad of grid paper.



NOTE-TAKING TIP: When you take notes, include personal experiences that relate to the lesson and ways in which what you have learned will be used in your daily life.

CHAPTER

10

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 10. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
center			
circle			
conic section			
conjugate axis KAHN-jih-guht			
directrix duh-REHK-trihks			
ellipse ih-LIHPS			
foci			

(continued on the next page)

Chapter 10 BUILD YOUR VOCABULARY

Vocabulary Term	Found on Page	Definition	Description or Example
focus FOH-kuhs			
hyperbola hy-PUHR-buh-luh			
latus rectum LA-tuhs REHK-tuhm			
major axis			
minor axis			
parabola puh-RA-buh-luh			
transverse axis			
vertex			

10–1 Midpoint and Distance Formulas

MAIN IDEAS

- Find the midpoint of a segment on the coordinate plane.
- Find the distance between two points on the coordinate plane.

KEY CONCEPTS

Midpoint Formula If a line segment has endpoints at (x_1, y_1) and (x_2, y_2), then the midpoint of the segment has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Distance Formula The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE Find a Midpoint

1 COMPUTERS **A graphing program draws a line segment on a computer screen so that its endpoints are at (5, 2) and (7, 8). What are the coordinates of its midpoint?**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\boxed{}}{2}, \frac{\boxed{}}{2}\right)$$

$$= \left(\frac{\boxed{}}{2}, \frac{\boxed{}}{2}\right) \text{ or } \boxed{}$$

The coordinates of the midpoint are $\boxed{}$

EXAMPLE Find the Distance Between Two Points

2 **Find the distance between $P(-1, 4)$ and $Q(2, -3)$.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{\boxed{}}$	Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (2, -3)$.
$= \sqrt{\boxed{}}$	Subtract.
$= \sqrt{\boxed{}}$ or $\sqrt{\boxed{}}$	Simplify.

The distance between the two points is $\boxed{}$ units.

Check Your Progress

a. Find the midpoint of the segment with endpoints at (3, 6) and (−1, −8).

b. What is the distance between $P(2, 3)$ and $Q(-3, 1)$?

EXAMPLE

3 **TEST EXAMPLE** **A coordinate grid is placed over a scale drawing of Jenny's patio. A grill is located at (2, −3). A flowerpot is located at (−6, −1). A picnic table is at the midpoint between the grill and the flowerpot. In coordinate units, about how far is it from the grill to the picnic table?**

A 0.2 **B** 2.24 **C** 4.1 **D** 5

Read the Item
The question asks us to find the distance between the grill and the midpoint. Find the midpoint and then use the Distance Formula.

Solve the Item
Use the Midpoint Formula to find the coordinates of the picnic table.

$$\text{midpoint} = \left(\frac{2 + (-6)}{2}, \frac{(-3) + (-1)}{2}\right) \quad \text{Midpoint Formula}$$

$$= \boxed{} \quad \text{Simplify.}$$

Use the Distance Formula to find the distance between the grill $\boxed{}$ and the picnic table $\boxed{}$.

$$\text{distance} = \sqrt{[(-2) - 2]^2 + [-2 - (-3)]^2} \quad \text{Distance Formula}$$

$$= \sqrt{\left(\boxed{}\right)^2 + \left(\boxed{}\right)^2} \quad \text{Subtract.}$$

$$= \boxed{} \text{ or about } \boxed{} \quad \text{Simplify.}$$

The answer is $\boxed{}$.

Check Your Progress A coordinate grid is placed over a floor plan of the school. The gymnasium is located at (4, 6). The science lab is located at (−2, −4). The office is located at the midpoint between the gymnasium and the science lab. In coordinate units, about how far is the office from the science lab?

A 1.4 **B** 2.8 **C** 9.1 **D** 10.3

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–2 Parabolas

MAIN IDEAS

- Write equations of parabolas in standard form.
- Graph parabolas.

BUILD YOUR VOCABULARY (pages 283–284)

The graph of an equation of the form, $a \neq 0$,

☐ is a **parabola.**

Any figure that can be obtained by slicing a

☐ is called a **conic section.**

A parabola can also be defined as the set of all points in a plane that are the same ☐ from a given point called the **focus** and a given line called the **directrix.**

EXAMPLE Analyze the Equation of a Parabola

① **Write $y = -x^2 - 2x + 3$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.**

$y = -x^2 - 2x + 3$ Original equation

$y = -1(x^2 + 2x) + 3$ Factor.

$y = -1(x^2 + 2x + \square) + 3 - (-1)(\square)$ Complete the square.

$y = -1\left(x^2 + 2x + \boxed{}\right) + 3 + 1\left(\boxed{}\right)$ Multiply ☐ by −1.

$y = \boxed{}$

$y = \boxed{}$ $(h, k) = (-1, 4)$

The vertex of this parabola is located at ☐, and the equation of the axis of symmetry is $x =$ ☐. The parabola opens downward.

KEY CONCEPT

Equation of a Parabola The standard form of the equation of a parabola with vertex (h, k) and axis of symmetry $x = h$ is $y = a(x - h)^2 + k$.

- If $a > 0$, k is the minimum value of the related function and the parabola opens upward.
- If $a < 0$, k is the maximum value of the related function and the parabola opens downward.

Check Your Progress Write $y = 2x^2 + 4x + 5$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

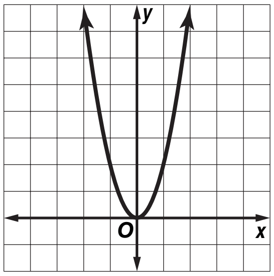
EXAMPLE Graph Parabolas

2 **a. Graph $y = 2x^2$.**

For this equation, $h = 0$ and $k = 0$. The vertex is at the origin. Substitute positive integers for x and find the corresponding y-values.

x	y
1	2
2	8
3	18

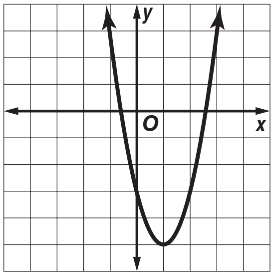
Since the graph is symmetric about the y-axis, the points at , , and are also on the parabola. Use all of these points to draw the graph.



b. Graph $y = 2(x - 1)^2 - 5$.

The equation is of the form $y = a(x - h)^2 + k$, where $h = 1$ and $k = -5$. The graph of this equation is the graph of $y = 2x^2$ in part **a** translated unit right and units down.

The vertex is now at .



Check Your Progress **Graph each equation.**

a. $y = -3x^2$

b. $y = -3(x + 1)^2 + 3$

BUILD YOUR VOCABULARY (page 284)

The line segment through the focus of a parabola and perpendicular to the axis of $\boxed{}$ is called the **latus rectum.**

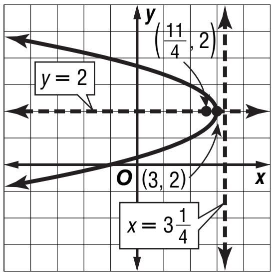
EXAMPLE Graph an Equation Not in Standard Form

1 **Graph $x + y^2 = 4y - 1$.**

First write the equation in the form $x = a(y - k)^2 + h$.

$$x + y^2 = 4y - 1$$
$$x = -y^2 + 4y - 1$$
$$x = -1\left(\boxed{}\right) - 1$$
$$x = -1(y^2 - 4y + \square) - 1 - (-1)(\square)$$
$$x = -1\left(y^2 - 4y + \boxed{}\right) - 1 + 1\left(\boxed{}\right)$$
$$x = \boxed{}$$

Then use the following information to draw the graph.



vertex: $\boxed{}$

axis of symmetry: $y = \boxed{}$

focus: $\left(3 + \frac{1}{4(-1)}, 2\right)$ or $\left(\frac{11}{4}, 2\right)$

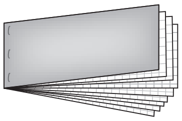
directrix: $x = \boxed{}$ or $3\frac{1}{4}$

direction of opening: left, since $a < 0$

length of the latus rectum: $\boxed{}$ or $\boxed{}$ unit

FOLDABLES ORGANIZE IT

Use the page for Vocabulary. Make a sketch of each vocabulary term in this lesson.



Check Your Progress Graph $x - y^2 = 6y + 2$.

EXAMPLE Write and Graph an Equation for a Parabola

④ **BRIDGES** **The 52-meter-long Hulme Arch Bridge in Manchester, England, is supported by cables suspended from a parabolic steel arch. The highest point of the arch is 25 meters above the bridge, and the focus of the arch is about 18 meters above the bridge.**

a. Let the bridge be the *x*-axis, and let the *y*-axis pass through the arch's vertex. Write an equation that models the arch.

The vertex is at (0, 25), so $h = \boxed{}$ and $k = \boxed{}$.

The focus is at (0, 12). Use the focus to find a.

$18 = 25 + \frac{1}{4a}$ $\quad$ $k = 25$; The focus y-coordinate is 18.

$\boxed{} = \frac{1}{4a}$ $\quad$ Subtract 25 from each side.

$\boxed{} = 1$ $\quad$ Multiply each side by $4a$.

$a = \boxed{}$ $\quad$ Divide each side by -28.

An equation of the parabola is $\boxed{}$.

b. Graph the equation.

The length of the latus rectum is $\frac{1}{a}$ or $\boxed{}$ units, so the graph must pass through $(-14, 18)$ and $(14, 18)$. According to the length of the bridge, the graph must pass through the points $(-26, 0)$ and $(26, 0)$. Use these points and the information from part a to draw the graph.

Check Your Progress **The water stream follows a parabolic path. The highest point of the water stream is $6\frac{1}{4}$ feet above the ground and the water hits the ground 10 feet from the jet. The focus of the fountain is $5\frac{1}{2}$ feet above the ground.**

a. Write an equation that models the path of the water fountain.

b. Graph the equation.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–3 Circles

MAIN IDEAS

- Write equations of circles.
- Graph circles.

BUILD YOUR VOCABULARY (pages 283–284)

A circle is the set of all points in a plane that are $\boxed{}$ from a given $\boxed{}$ in the plane, called the **center.**

A line that $\boxed{}$ the circle in exactly $\boxed{}$ point is said to be **tangent** to the circle.

KEY CONCEPT

Equation of a Circle
The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$

FOLDABLES™ On the page for Circles, write this equation. Be sure to explain h, k, and r.

EXAMPLE Write an Equation Given the Center and Radius

① **LANDSCAPING The plan for a park puts the center of a circular pond, of radius 0.6 miles, 2.5 miles east and 3.8 miles south of the park headquarters. Write an equation to represent the border of the pond, using the headquarters as the origin.**

Since the headquarters is at $\boxed{}$, the center of the pond is at $\boxed{}$ with radius 0.6 mile.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$\left(x - \boxed{}\right)^2 + \left(y + \boxed{}\right)^2 = \boxed{}^{\,2}$ $(h, k) = (2.5, -3.8)$, $r = 0.6$

$(x - 2.5)^2 + (y + 3.8)^2 = \boxed{}$ Simplify.

The equation is $\boxed{}$

Check Your Progress The plan for a park puts the center of a circular pond, of radius 0.5 mile, 3.5 miles west and 2.6 miles north of the park headquarters. Write an equation to represent the border of the pond, using the headquarters as the origin.

EXAMPLE Write an Equation Given a Diameter

② **Write an equation for a circle if the endpoints of the diameter are at (2, 8) and (2, −2).**

First, find the center of the circle.

$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ Midpoint Formula

$$= \left(\frac{\boxed{}}{2}, \frac{\boxed{}}{2}\right)$$ $(x_1, y_1) = (2, 8)$, $(x_2, y_2) = (2, -2)$

$$= \left(\boxed{}, \boxed{}\right)$$ Add.

$$= \left(\boxed{}\right)$$ Simplify.

Now find the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula

$$= \sqrt{\boxed{}}$$ $(x_1, y_1) = (2, 8)$, $(x_2, y_2) = (2, 3)$

$$= \sqrt{\boxed{}}$$ Subtract.

$$= \boxed{} \text{ or } \boxed{}$$ Simplify.

The radius of the circle is $\boxed{}$ units, so $r^2 = \boxed{}$.

Substitute h, k, and r^2 into the standard form of the equation of a circle. An equation of the circle is

$\boxed{}$

Check Your Progress Write an equation for a circle if the endpoints of the diameter are at (3, 5) and (3, −7).

WRITE IT

What must you do to the equation of a circle if you want to graph the circle on a calculator?

______

______

______

______

EXAMPLE Graph an Equation in Standard Form

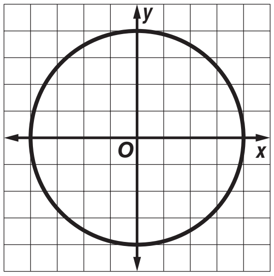
3 Find the center and radius of the circle with equation $x^2 + y^2 = 16$. Then graph the circle.

The center is at ______, and the radius is ______.

x	y
0	4
1	3.9
2	3.5
3	2.6
4	0

The table lists some values for x and y that satisfy the equation.

Since the circle is centered at the origin, it is symmetric about the y-axis. Use these points and the concept of symmetry to graph $x^2 + y^2 = 16$.



EXAMPLE Graph an Equation Not in Standard Form

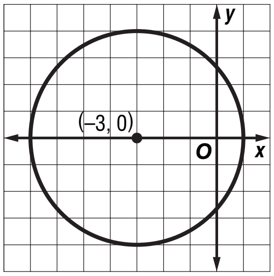
4 Find the center and radius of the circle with equation $x^2 + y^2 + 6x - 7 = 0$. Then graph the circle.

Complete the square.

$x^2 + 6x + \square + y^2 = 7$

$x^2 + 6x +$ ______ $+ y^2 = 7 +$ ______

______ $+ y^2 =$ ______



The center is at ______, and the radius is ______.

Check Your Progress **Graph each equation.**

a. $x^2 + y^2 = 9$

b. $x^2 + y^2 + 8x - 4y + 11 = 0$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–4 Ellipses

MAIN IDEAS

- Write equations of ellipses.
- Graph ellipses.

BUILD YOUR VOCABULARY (pages 283–284)

An **ellipse** is the set of all points in a plane such that the sum of the from two fixed is constant. The two fixed points are called the **foci** of the ellipse.

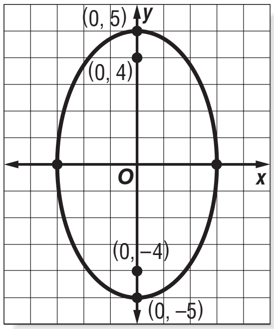
The points at which the ellipse intersects its axes of symmetry determine two with endpoints on the ellipse called the **major axis** and the **minor axis.** The axes intersect at the **center** of the ellipse.

WRITE IT

Write the name for the endpoints of each axis of an ellipse.

EXAMPLE Write an Equation for a Graph

① **Write an equation for the ellipse.**



Find the values of a and b for the ellipse. The length of the major axis of any ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between (0, 5) and (0, −5). 10 units.

$2a =$ Length of major axis = 10

$a =$ Divide each side by 2.

The foci are located at (0, 4) and (0, −4), so $c = 4$.

Use the relationship between a, b, and c to find b.

$c^2 = a^2 - b^2$ Equation relating a, b, and c

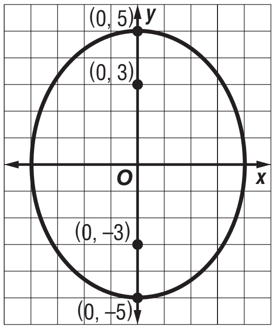
$=$ $c = 4$ and $a = 5$

$b^2 =$ Solve for b^2.

Since the major axis is vertical, substitute 25 for a^2 and 9 for b^2 in the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$. An equation of the ellipse is

.

Check Your Progress Write an equation for the ellipse shown.



EXAMPLE Write an Equation Given the Lengths of the Axes

2 SOUND A listener is standing in an elliptical room 150 feet wide and 320 feet long. When a speaker stands at one focus and whispers, the best place for the listener to stand is at the other focus.

a. Write an equation to model this ellipse, assuming the major axis is horizontal and the center is at the origin.

The length of the major axis is 320 feet.

$2a = 320$ Length of major axis $= 320$

$a =$ Divide each side by 2.

The length of the minor axis is 150 feet.

$2b = 150$ Length of minor axis $= 150$

$b =$ Divide each side by 2.

Substitute $a =$ and $b =$ into the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

An equation for the ellipses is .

b. How far apart should the speaker and the listener be in this room?

The two people should stand at the foci of the ellipse. The distance between the two foci is units.

$c^2 = a^2 - b^2$	Equation relating *a*, *b*, and *c*.
$c = \boxed{}$	Take the square root of each side.
$\boxed{}\,c = \boxed{}\sqrt{a^2 - b^2}$	Multiply each side by 2.
$2c = 2\sqrt{\left(\boxed{}\right)^2 - \left(\boxed{}\right)^2}$	Substitute *a* = 160 and *b* = 75.
$2c \approx \boxed{}$	Use a calculator.

The points where two people should stand to hear each other whisper are about $\boxed{}$ feet apart.

Check Your Progress **A listener is standing in an elliptical room 60 feet wide and 120 feet long. When a speaker stands at one focus and whispers, the best place for the listener to stand is at the other focus.**

a. Write an equation to model this ellipse, assuming the major axis is horizontal and the center is at the origin.

b. How far apart should the speaker and the listener be in this room?

EXAMPLE Graph an Equation in Standard Form

3 **Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{9} = 1$. Graph the ellipse.**

The center of this ellipse is at (0, 0).

Since $a^2 = 36$, $a = \boxed{}$ and since $b^2 = 9$, $b = \boxed{}$.

The length of the major axis is 2(6) or $\boxed{}$ units, and the length of the minor axis is 2(3) or $\boxed{}$. Since the x^2 term has the greatest denominator, the major axis is $\boxed{}$.

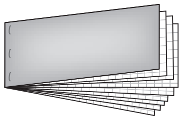
$c^2 = a^2 - b^2$	Equation relating *a*, *b*, and *c*.
$c^2 = \boxed{} - \boxed{}$	*a* = 6 and *b* = 3
$c = \boxed{}$ or $\boxed{}$	Take a square root of each side.

The foci are at $\boxed{}$ and $\boxed{}$.

FOLDABLES™

ORGANIZE IT

Use the page for Ellipses. Make a sketch of each vocabulary term in this lesson.



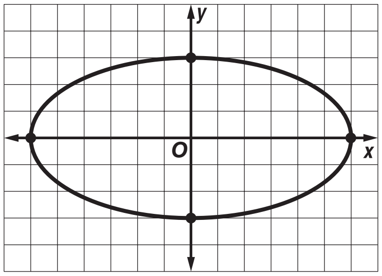
You can use a calculator to find some approximate nonnegative values for x and y that satisfy the equation.

x	y
0	3
1	2.96
2	2.83
3	2.60
4	2.24
5	1.66
6	0

Since the ellipse is centered at the origin, it is symmetric about the y-axis. So the points at and lie on the graph.

The ellipse is also symmetric about the x-axis, so the points at and also lie on the graph.

Graph the intercepts , , , and , and draw the ellipse that passes through them and the other points. center: (0, 0); foci:$(3\sqrt{3}, 0)$, $(-3\sqrt{3}, 0)$; major axis: 12; minor axis: 6



Check Your Progress Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Then graph the equation.

EXAMPLE Graph an Equation Not in Standard Form

④ **Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $x^2 + 4y^2 - 6x - 16y - 11 = 0$. Graph the ellipse.**

Complete the square to write in standard form.

$$x^2 + 4y^2 - 6x - 16y - 11 = 0$$

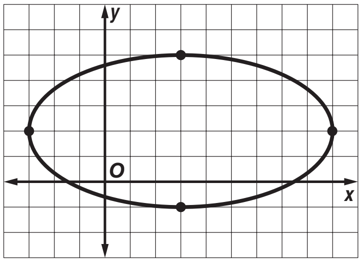
$$x^2 - 6x + \square + 4(y^2 - 4y + \square) = 11 + \square + 4(\square)$$

$$\left(x^2 - 6x + \boxed{}\right) + 4\left(y^2 - 4y + \boxed{}\right) = 11 + 9 + 4\left(\boxed{}\right)$$

$$\boxed{}$$

$$\boxed{} = 1$$

The center is ☐, and the foci are located at ☐ and ☐. The length of the major axis is ☐ units, and the length of the minor axis is ☐.



Check Your Progress Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $4x^2 + 25y^2 + 16x - 150y + 141 = 0$. Graph the ellipse.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

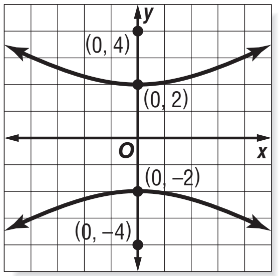
10–5 Hyperbolas

MAIN IDEAS

- Write equations of hyperbolas.
- Graph hyperbolas.

EXAMPLE Write an Equation for a Graph

1 Write an equation for the hyperbola shown.



The center is .

The value of a is the distance from the center to a vertex, or units. The value of c is the distance from the center to a focus, or units.

$c^2 = a^2 + b^2$	Equation relating a, b, and c
=	$c = 4$, $a = 2$
=	Evaluate the squares.
$12 = b^2$	Solve for b^2.

Since the transverse axis is vertical, the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$. Substitute the values for a^2 and b^2.

An equation of the hyperbola is .

KEY CONCEPT

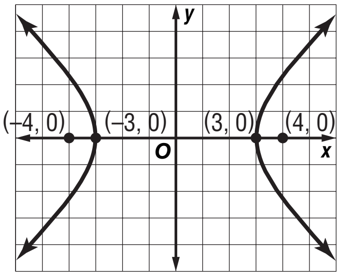
Equations of Hyperbolas with Centers at the Origin

Horizontal Transverse Axis $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertical Transverse Axis $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Check Your Progress

Write an equation for the hyperbola.



EXAMPLE Graph an Equation in Standard Form

2 Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $x^2 - y^2 = 1$. Graph the hyperbola.

The center of the hyperbola is at the origin. According to the equation, $a^2 = 1$ and $b^2 = 1$, $a = 1$ and $b = 1$. The coordinates of the vertices are and .

$c^2 = a^2 + b^2$ Equation relating *a*, *b*, and *c* for a hyperbola.

$c^2 = \boxed{}^2 + \boxed{}^2$ *a* = 1, *b* = 1

$c^2 = 2$ or $c = \boxed{}$ Simplify and take the square root.

The equations of the asymptotes are $y = \pm\frac{b}{ax}$ or $y = \pm x$.

Use a calculator to find some approximate nonnegative values for x and y that satisfy the equation. Since the hyperbola is centered at the origin, it is symmetric about the y-axis. Therefore, the points at $(-5, 4.9)$, $\boxed{}$, $(-3, 2.8)$, $(-2, 1.7)$, and $\boxed{}$ lie on the graph.

x	*y*
1	0
2	1.7
3	2.8
4	3.9
5	4.9

The hyperbola is also symmetric about the x-axis, so the points at $(-5, -4.9)$, $(-4, 3.9)$, $\boxed{}$, $(-2, -1.7)$, $(3, -2.8)$, $\boxed{}$, and $(5, -4.9)$ also lie on the graph.

Draw a 2-unit by 2-unit square. The asymptotes contain the diagonals of the square. Graph the vertices, which, in this case, are the x-intercepts. Use the asymptotes as a guide to draw the hyperbola that passes through the vertices and the other points. The graph $\boxed{}$ intersect the asymptotes.

Check Your Progress Find the vertices and foci and the equations of the asymptotes for the hyperbola with equation $\frac{x^2}{4 - y^2} = 1$. Then graph the hyperbola.

EXAMPLE Graph an Equation Not in Standard Form

③ **Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $x^2 - y^2 + 6x + 10y - 17 = 0$. Then graph the hyperbola.**

Complete the square for each variable.

$$x^2 - y^2 + 6x + 10y - 17 = 0$$

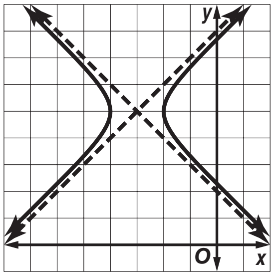
$$x^2 - 6x + \square - 1(y^2 - 10y + \square) = 17 + \square - 1(\square)$$

$$x^2 + 6x + \boxed{} - 1\left(y^2 - 10y + \boxed{}\right) = 17 + \boxed{} - 1(25)$$

$$\boxed{} - \boxed{} = \boxed{}$$

The vertices are $\boxed{}$ and $\boxed{}$, and the foci are $(\sqrt{2} - 3, 5)$ and $(-\sqrt{2} - 3, 5)$.

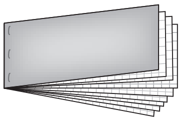
The equations of the asymptotes are $\boxed{}$ or $y = \boxed{}$ and $y = -x + 2$.



FOLDABLES™

ORGANIZE IT

Under the page for Hyperbolas, describe two similarities between hyperbolas and ellipses.



Check Your Progress Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $9x^2 - 16y^2 - 72x - 64y + 224 = 0$. Then graph the hyperbola.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–6 Conic Sections

EXAMPLE Rewrite an Equation of a Conic Section

MAIN IDEAS

- Write equations of conic sections in standard form
- Identify conic sections from their equations.

REMEMBER IT

If $a = b$ in the equation for an ellipse, the graph of the equation is a circle.

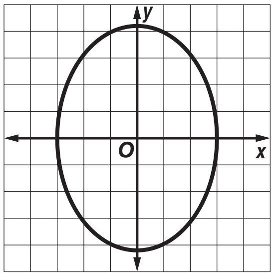
① **Write the equation $y^2 = 18 - 2x^2$ in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola.* Graph the equation.**

Write the equation in standard form.

$y^2 = 18 - 2x^2$

= Isolate terms

= Divide each side by .



The graph is an ellipse with center at .

Check Your Progress Write $x^2 + y^2 - 6x - 7 = 0$ in standard form. State whether the graph of the equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.

EXAMPLE Analyze an Equation of a Conic Section

② **Without writing the equation in standard form, state whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola.***

a. $3y^2 - x^2 - 9 = 0$

$A =$ and $C =$

Since ,

the graph is a .

b. $2x^2 + 2y^2 + 16x - 20y = -32$

$A =$ and $C =$

Since , the graph is a .

c. $y^2 - 2x - 4y + 10 = 0$

$A =$ and $C =$

Since , this graph is a .

Check Your Progress **Without writing the equation in standard form, state whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*.**

a. $2y^2 - x^2 + 16 = 0$

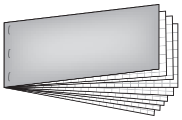
b. $3x^2 + y^2 + 15x - 21y = -11$

c. $y^2 - 3x + 2y - 10 = 0$

FOLDABLES™

ORGANIZE IT

Under the page for Conic Sections, sketch and label each of the conic sections. Then write the standard form on the conic section below each label.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–7 Solving Quadratic Systems

MAIN IDEAS

- Solve systems of quadratic equations algebraically and graphically.
- Solve systems of quadratic inequalities graphically.

REVIEW IT

By the Square Root Property, for any real number *n*, if $x^2 = n$, then $x =$ ________ ? *(Lesson 5-4)*

EXAMPLE Linear-Quadratic System

① **Solve the system of equations.**

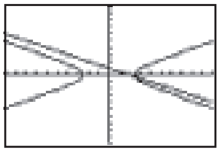
$$4x^2 - 16y^2 = 25$$
$$2y + x = 2$$

You can use a graphing calculator to help visualize the relationships of the graphs of the equations and predict the number of solutions.

Solve each equation for y to obtain

$$y = \pm\frac{\sqrt{4x^2 - 25}}{4} \text{ and}$$

$$y = \boxed{\quad}.$$



Enter the functions on the Y= screen. The graph indicates that the hyperbola and the line intersect in one point. So, the system has one solution.

Use substitution to solve the system.

First, rewrite $2y + x = 2$ as $x = 2 - 2y$.

$4x^2 - 16y^2 = 25$ First equation in the system

$4\left(\boxed{\quad}\right)^2 - 16y^2 = 25$ Substitute $\boxed{\quad}$ for *x*.

$\boxed{\quad} + 16 = 25$ Simplify.

$\boxed{\quad} = \boxed{\quad}$ Subtract $\boxed{\quad}$ from each side.

$y = \boxed{\quad}$ Divide each side by $\boxed{\quad}$.

Now solve for x.

$x = 2 - 2y$ Equation for *x* in terms of *y*

$x = 2 - 2\left(\boxed{\quad}\right)$ Substitute the *y*-value.

$x = \boxed{\quad}$ Simplify.

The solution is $\left(\boxed{\quad}\right)$.

Check Your Progress Solve $x^2 - y^2 = 4$ and $2y + x = 2$.

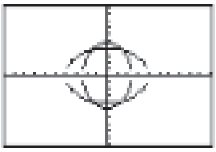
EXAMPLE Quadratic-Quadratic System

② **Solve the system of equations.**

$$\mathbf{x^2 + y^2 = 16}$$
$$\mathbf{4x^2 + y^2 = 23}$$

REMEMBER IT When graphing conic sections, press ZOOM 5. This window gives the graphs a more realistic look.

A graphing calculator indicates that the circle and ellipse intersect in four points. So, this system has four solutions.



Use the elimination method to solve.

$-x^2 - y^2 = -16$	Rewrite the first original equation.
$(+)\ 4x^2 + y^2 = 23$	Second original equation
$\boxed{} = 7$	Add.
$\boxed{} = \boxed{}$	Divide each side by 3.
$\boxed{} = \pm\boxed{}$	Take the square root of each side.

Substitute $\sqrt{\frac{7}{3}}$ and $-\sqrt{\frac{7}{3}}$ in either of the original equations and solve for y.

$x^2 + y^2 = 16$	$x^2 + y^2 = 16$	Original equation
$\left(\sqrt{\frac{7}{3}}\right)^2 + y^2 = 16$	$\left(-\sqrt{\frac{7}{3}}\right)^2 + y^2 = 16$	Substitute for x.
$y^2 = \boxed{}$	$y^2 = \boxed{}$	Subtract from each side.
$y = \boxed{}$	$y = \boxed{}$	Take the square root of each side.

After rationalizing the denominators the solutions are

$\boxed{}$, $\boxed{}$,

$\boxed{}$, and $\boxed{}$.

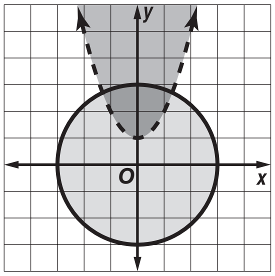
Check Your Progress Solve $x^2 + y^2 = 10$ and $3x^2 + y^2 = 28$.

EXAMPLE System of Quadratic Inequalities

3 **Solve the system of inequalities by graphing.**

$$y > x^2 + 1$$
$$x^2 + y^2 \leq 9$$

The graph of $y > x^2 + 1$ is the ______ $y = x^2 + 1$ and the region inside and above it. Shade the region dark gray.



The graph of $x^2 + y^2 \leq 9$ is the interior of ______ $x^2 + y^2 = 9$. Shade the region medium gray.

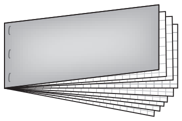
The intersection of these regions, shaded darker gray, represents the solution of the system of inequalities.

Check Your Progress Solve $y < -x^2 + 1$ and $x^2 + y^2 \leq 4$ by graphing.

FOLDABLES™

ORGANIZE IT

On the page for Quadratic Systems, sketch five graphs of systems of quadratic equations. Write the number of solutions below each graph.



HOMEWORK ASSIGNMENT

Page(s):

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CHAPTER 10

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 10 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 10, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 283–284)* to help you solve the puzzle.

10-1

Midpoint and Distance Formulas

Consider the segment connecting the points (−3, 5) and (9, 11).

1. Find the midpoint of this segment.

2. Find the length of the segment. Write your answer in simplified radical form.

3. Circle P has a diameter $\overline{CD}$. If C is at $(4, -3)$ and D is at $(-3, 5)$, find the center of the circle and the length of the diameter.

10-2

Parabolas

Write each equation in standard form.

4. $y = 2x^2 - 8x + 1$

5. $y = -2x^2 + 6x + 1$

6. $y = \frac{1}{2}x^2 - 5x + 12$

10-3

Circles

7. Write the equation of the circle with center (4, −3) and radius 5.

8. The circle with equation $(x + 8)^2 + y^2 = 121$ has center $\boxed{}$ and radius $\boxed{}$.

9. a. In order to find the center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$, it is necessary to $\boxed{}$.

Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$
$$x^2 + y^2 + 4x - 6y = \boxed{}$$
$$x^2 + 4x + \boxed{} + y^2 - 6y + \boxed{} = \boxed{} + \boxed{} + \boxed{}$$
$$\left(x + \boxed{}\right)^2 + \left(y - \boxed{}\right)^2 = \boxed{}$$

b. This circle has radius 4 and center at $\boxed{}$.

10-4

Ellipses

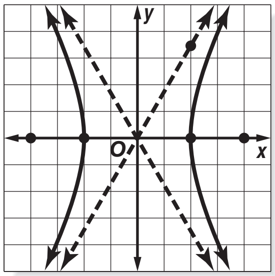
10. Complete the table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis		
Direction of Minor Axis		
Foci		
Length of Major Axis		
Length of Minor Axis		

10-5

Hyperbolas

Study the hyperbola graphed at the right.



11. The center is ____.

12. The value of a is ____.

13. The value of c is ____.

14. To find b^2, solve ____ = ____ + ____.

15. The equation in standard form for this hyperbola is ____.

10-6

Conic Sections

Name the conic section that is the graph of each equation. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.

16. $\frac{(x-3)^2}{36} + \frac{(y+5)^2}{15} = 1$

17. $x = -2(y+1)^2 + 7$

18. $(x-5)^2 - (y+5)^2 = 1$

19. $(x+6)^2 + (y-2)^2 = 1$

10-7

Solving Quadratic Systems

Draw a sketch to illustrate each of the following possibilities.

20. a parabola and a line that intersect in 2 points

21. an ellipse and a circle that intersect in 4 points

22. a hyperbola and a line that intersect in 1 point

CHAPTER 10

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 10.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 10 Practice Test on page 615 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 10 Study Guide and Review on pages 609–614 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 615 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 10 Foldable.
- Then complete the Chapter 10 Study Guide and Review on pages 609–614 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 615 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

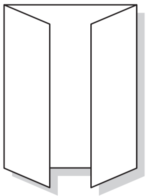
CHAPTER 11

Sequences and Series

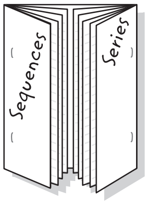
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11” × 17” paper and four sheets of notebook paper.

STEP 1 **Fold** the short sides of the 11″ × 17″ paper to meet in the middle.



STEP 2 **Fold** the notebook paper in half lengthwise. Insert two sheets of notebook paper in each tab and staple the edges. Label with lesson numbers. Take notes under the appropriate tabs.



NOTE-TAKING TIP: When you take notes, write questions you have about the lessons in the margin of your notes. Then include the answers to these questions as you work through the lesson.

CHAPTER 11

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 11. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
arithmetic means AR-ihth-MEH-tihk			
arithmetic sequence			
arithmetic series			
common difference			
common ratio			
factorial			
Fibonacci sequence fih-buh-NAH-chee			
geometric means			
geometric sequence			
geometric series			

Vocabulary Term	Found on Page	Definition	Description or Example
index of summation			
induction hypothesis			
infinite geometric series			
iteration IH-tuh-RAY-shuhn			
mathematical induction			
partial sum			
Pascal's triangle pas-KALZ			
recursive formula rih-KUHR-sihv			
sequence			
series			
sigma notation SIHG-muh			
term			

11–1 Arithmetic Sequence

MAIN IDEAS

- Use arithmetic sequences.
- Find arithmetic means.

BUILD YOUR VOCABULARY (pages 312–313)

An **arithmetic sequence** is a sequence in which each after the first is found by a constant, called the **common difference** *d*, to the term.

KEY CONCEPT

***n*th Term of an Arithmetic Sequence** The *n*th term a_n of an arithmetic sequence with first term a_1 and common difference *d* is given by $a_n = a_1 + (n - 1)d$, where *n* is any positive integer.

FOLDABLES On the tab for Lesson 11-1, write your own arithmetic sequence.

EXAMPLE Find the Next Terms

① **Find the next four terms of the arithmetic sequence $-8, -6, -4, \ldots$.**

Find the common difference *d* by subtracting 2 consecutive terms.

$-6 - (-8) =$ and $-4 - (-6) =$ So, $d =$.

Now add 2 to the third term of the sequence and then continue adding 2 until the next four terms are found.

-4

$+ 2$ $+ 2$ $+ 2$ $+ 2$

Check Your Progress Find the next four terms of the arithmetic sequence 5, 3, 1, ...

EXAMPLE Find a Particular Term

② **CONSTRUCTION** **The table below shows typical costs for a construction company to rent a crane for one, two, three, or four months. If the sequence continues, how much would it cost to rent the crane for 24 months?**

Months	Costs($)
1	75,000
2	90,000
3	105,000
4	120,000

Explore Since the difference between any two successive costs is \$15,000, the costs form an arithmetic sequence with common difference .

Plan You can use the formula for the nth term of an arithmetic sequence with $a_1 = 75{,}000$ and $d = 15{,}000$ to find a_{24} the cost for 24 months.

Solve $a_n =$ Formula for the *n*th term

$a_{24} = 75{,}000 + (24 - 1)(15{,}000)$ $n = 24$, $a_1 = 75{,}000$, $d = 15{,}000$

$a_{24} =$ Simplify.

It would cost to rent the crane for 24 months.

Check Your Progress Refer to Example 2. How much would it cost to rent the crane for 8 months?

EXAMPLE Write an Equation for the *n*th Term

3 Write an equation for the nth term of the arithmetic sequence $-8, -6, -4, \ldots$.

In this sequence, $a_1 = -8$ and $d = 2$. Use the nth formula to write an equation.

$a_n = a_1 + (n - 1)d$ Formula for the *n*th term

$a_n =$ $a_1 = -8$, $d = 2$

$a_n =$ Distributive Property

$a_n =$ Simplify.

REVIEW IT

When finding the value of an expression, you must follow the order of operations. Briefly list the order of operations. *(Lesson 1-1)*

Check Your Progress Write an equation for the *n*th term of the arithmetic sequence 5, 3, 1, ...

BUILD YOUR VOCABULARY (pages 312–313)

The terms between any two nonsuccessive terms of an sequence are called **arithmetic means**.

EXAMPLE Find Arithmetic Means

④ **Find the three arithmetic means between 21 and 45.**

You can use the *n*th term formula to find the common difference. In the sequence 21, __, __, __, 45, . . . , $a_1 =$ and $a_5 =$.

$a_n = a_1 + (n - 1)d$ Formula for the *n*th term

$a_5 =$ $n = 5$, $a_1 = 21$

$=$ $a_5 = 45$

$=$ Subtract 21 from each side.

$= d$ Divide each side by 4.

Now use the value of *d* to find the three arithmetic means.

21

+ 6 + 6 + 6

Check Your Progress Find the three arithmetic means between 13 and 25.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–2 Arithmetic Series

MAIN IDEAS

- Find sums of arithmetic series.
- Use sigma notation.

BUILD YOUR VOCABULARY (page 313)

A series is an indicated **sum** of the of a sequence. Since 18, 22, 26, 30 is an **arithmetic** sequence, 18 + 22 + 26 + 30 is an **arithmetic series.**

KEY CONCEPT

Sum of an Arithmetic Series The sum S_n of the first n terms of an arithmetic series is given by

$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$ or

$S_n = \frac{n}{2}(a_1 + a_n)$.

EXAMPLE Find the Sum of an Arithmetic Series

1 Find the sum of the first 20 even numbers, beginning with 2.

The series is 2 + 4 + 6 + . . . + 40. Since $a_1 =$, $a_{20} =$, and $d =$, you can use either sum formula for this series.

Method 1

$S_n = \frac{n}{2}(a_1 + a_n)$	Sum formula
$S_{20} = \frac{20}{2}(2 + 40)$	$n = 20$, $a_1 = 2$, $a_{20} = 40$
$S_{20} =$	Simplify.
$S_{20} =$	Multiply.

Method 2

$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$	Sum formula
$S_2 = \frac{20}{2}[2(2) + (20 - 1)2]$	$n = 20$, $a_1 = 2$, $d = 2$
$S_2 =$	Simplify.
$S_2 =$	Multiply.

The sum of the first 20 even numbers is .

Check Your Progress Find the sum of the first 15 counting numbers, beginning with 1.

EXAMPLE **Find the First Term**

② **RADIO** **A radio station is giving away money every day in the month of September for a total of \$124,000. They plan to increase the amount of money given away by \$100 each day. How much should they give away on the first day of September, rounded to the nearest cent?**

You know the values of n, S_n, and d. Use the sum formula that contains d.

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d] \qquad \text{Sum formula}$$

$$S_{30} = \frac{30}{2}[2a_1 + (30 - 1)100] \qquad n = 30,\ d = 100$$

$$\boxed{} = 15(2a_1 + 2900) \qquad S_{30} = 124{,}000$$

$$124{,}000 = \boxed{} + \boxed{} \qquad \text{Distributive Property}$$

$$80{,}500 = 30a_1 \qquad \text{Subtract 43,500 from each side.}$$

$$\boxed{} = a_1 \qquad \text{Divide each side by 30.}$$

They should give away $\boxed{}$ the first day.

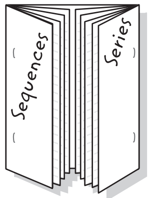
Check Your Progress A television game show gives contestants a chance to win a total of \$1,000,000 by answering 16 consecutive questions correctly. If the value from question to question increases by \$5,000, how much is the first question worth?

$\boxed{}$

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-2. Write an example of an arithmetic sequence and an arithmetic series. Then explain the difference between the two.



EXAMPLE **Find the First Three Terms**

③ **Find the first four terms of an arithmetic series in which $a_1 = 14$, $a_n = 29$, and $S_n = 129$.**

Step 1 Since you know a_1, a_n, and S_n, use $S_n = \frac{n}{2}(a_1 + a_n)$ to find n.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\boxed{} = \frac{n}{2}\boxed{}$$

$$\boxed{} = \boxed{}$$

$$\boxed{} = n$$

Step 2 Find d.

$$a_n = a_1 + (n - 1)d$$

$$\boxed{} = \boxed{} + \boxed{}\ d$$

$$\boxed{} = \boxed{}$$

$$\boxed{} = d$$

Step 3 Use d to determine a_2, a_3, and a_4.

$$a_2 = \boxed{} + \boxed{} \text{ or } \boxed{}$$

$$a_3 = \boxed{} + \boxed{} \text{ or } \boxed{}$$

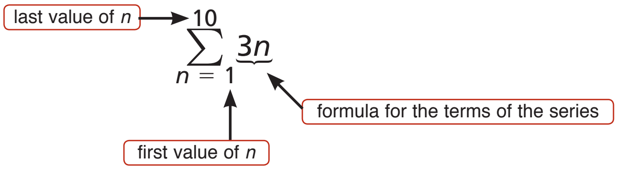
$$a_4 = \boxed{} + \boxed{} \text{ or } \boxed{}$$

The first four terms are $\boxed{}$, $\boxed{}$, $\boxed{}$, and $\boxed{}$.

Check Your Progress Find the first three terms of an arithmetic series in which $a_1 = 11$, $a_n = 31$ and $S_n = 105$.

BUILD YOUR VOCABULARY (page 313)

A concise notation for writing out a series is called **sigma notation**. The series 3 + 6 + 9 + 12 + . . . + 30 can be expressed as $\sum_{n=1}^{10} 3n$. The variable, in this case n, is called the **index of summation**.



EXAMPLE **Evaluate a Sum in Sigma Notation**

④ **Evaluate $\sum_{k=3}^{10}(2k + 1)$.**

Method 1 Find the terms by replacing k with 3, 4, …, 10. Then add.

$$\sum_{k=3}^{10}(2k + 1)$$

$$= [2(3) + 1] + [2(4) + 1] + [2(5) + 1] + [2(6) + 1] + \boxed{\qquad} + \boxed{\qquad} + \boxed{\qquad} + \boxed{\qquad}$$

$$= \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad}$$

$$= \boxed{\quad}$$

Method 2 Since the sum is an arithmetic series, use the formula $S_n = \frac{n}{2}(a_1 + a_n)$. There are $\boxed{\quad}$ terms, $a_1 = 2\left(\boxed{\quad}\right) + 1$ or $\boxed{\quad}$, and $a_8 = 2\left(\boxed{\quad}\right) + 1$ or $\boxed{\quad}$.

$$S_n = \boxed{\qquad} = \boxed{\quad} \text{ or } \boxed{\quad}$$

Check Your Progress Evaluate $\sum_{i=5}^{10}(2i + 3)$.

$\boxed{\qquad}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–3 Geometric Sequences

MAIN IDEAS

- Use geometric sequences.
- Find geometric means.

BUILD YOUR VOCABULARY (pages 312–313)

A **geometric sequence** is a sequence in which each term after the first is found by multiplying the previous term by a constant r called the **common ratio**.

The missing term(s) between two nonsuccessive terms of a geometric sequence are called **geometric means**.

EXAMPLE Find the Next Term

① **TEST EXAMPLE What is the missing term in the geometric sequence 324, 108, 36, 12, ____?**

A 972 **B** 4 **C** 0 **D** −12

Read the Item

Since $\frac{108}{324} = \frac{1}{3}$, $\frac{36}{108} = \frac{1}{3}$, and $\frac{12}{36} = \frac{1}{3}$, the sequence has the common ratio of $\boxed{\quad}$.

Solve the Item

Find the missing term, multiply the last given term by $\frac{1}{3}$:

$12\left(\frac{1}{3}\right) = \boxed{\quad}$. The answer is $\boxed{\quad}$.

Check Your Progress Find the missing term in the geometric sequence 100, 50, 25, ____.

A 200 **B** 0 **C** 12.5 **D** −12.5

$\boxed{\qquad\qquad\qquad\qquad}$

KEY CONCEPT

***n*th Term of a Geometric Sequence** The *n*th term a_n of a geometric sequence with first term a_1 and common ratio r is given by $a_n = a_1 \cdot r^{n-1}$, where n is any positive integer.

EXAMPLE Find a Term Given the First Term and the Ratio

② **Find the sixth term of a geometric sequence for which $a_1 = -3$ and $r = -2$.**

$a_n = a_1 \cdot r^{n-1}$ Formula for the *n*th term

$a_6 = \boxed{\quad} \cdot \boxed{\quad}$ $n = 6$, $a_1 = -3$, $r = -2$

$a_6 = \boxed{\quad} \cdot \boxed{\quad}$ $(-2^5) = -32$

$a_6 = \boxed{\quad}$ Multiply.

Check Your Progress Find the fifth term of a geometric sequence for which $a_1 = 6$ and $r = 2$.

EXAMPLE Write an Equation for the *n*th Term

3 **Write an equation for the *n*th term of the geometric sequence 5, 10, 20, 40, … .**

$a_n = a_1 r^{n-1}$ Formula for the *n*th term

$a_n = \boxed{}\left(\boxed{}\right)^{n-1}$ $a_1 = \boxed{}$, $r = \boxed{}$

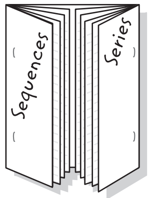
An equation is $\boxed{}$.

Check Your Progress Write an equation for the *n*th term of the geometric sequence 2, 6, 18, 54 . . . .

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-3. Write your own example of an arithmetic sequence and a geometric sequence. Then explain the difference between the two.



EXAMPLE Find a Term Given One Term and the Ratio

4 **Find the seventh term of a geometric sequence for which $a_3 = 96$ and $r = 2$.**

First find the value of a_1.

$a_n = a_1 r^{n-1}$

$\boxed{} = \boxed{}$

$\boxed{} = a_1(2)^2$

$\boxed{} = a_1$

Now find a_7.

$a_n = a_1 r^{n-1}$

$a_7 = \boxed{}$

$a_7 = \boxed{}$

The seventh term is $\boxed{}$.

Check Your Progress Find the sixth term of a geometric sequence for which $a_4 = 27$ and $r = 3$.

EXAMPLE Find Geometric Means

5 **Find three geometric means between 3.12 and 49.92.**

In the sequence a_1 is 3.12 and a_5 is 49.92.

$a_n = a_1 r^{n-1}$ Formula for the *n*th term

$a_5 =$ $n = 5$, $a_1 = 3.12$

$=$ $a_5 = 49.92$

$= r^4$ Divide by 3.12.

$= r$ Take the fourth root of each side.

Use each value of r to find three geometric means.

$r = 2$

$a_2 = 3.12(2)$ or

$a_3 = 6.24(2)$ or

$a_4 = 12.48(2)$ or

$r = -2$

$a_2 = 3.12(-2)$ or

$a_3 = -6.24(-2)$ or

$a_4 = 12.48(-2)$ or

The geometric means are , , and

or , , and .

Check Your Progress Find three geometric means between 12 and 0.75.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–4 Geometric Series

MAIN IDEAS

- Find sums of geometric series.
- Find specific terms of geometric series.

KEY CONCEPT

Sum of a Geometric Series The sum S_n of the first n terms of a geometric series is given by $S_n = \frac{a_1 - a_1r^n}{1 - r}$ or $S_n = \frac{a_1(1 - r^n)}{1 - r}$, where $r \neq 1$.

EXAMPLE Find the Sum of the First *n* Terms

① **HEALTH** **Contagious disease can spread very quickly. Suppose five people are ill during the first week of an epidemic, and each person who is ill spreads the disease to four people by the end of the week. By the end of the sixth week of the epidemic, how many people have been affected by the illness?**

This is a geometric series with $a_1 = 5$, $r = 4$, and $n = 6$.

$S_n = \frac{a_1(1 - r^n)}{1 - r}$ Sum formula

$=$ $n = 6$, $a_1 = 5$, $r = 4$

$=$ Use a calculator.

After 6 weeks, people have been affected.

EXAMPLE Evaluate a Sum Written in Sigma Notation

② **Evaluate $\sum_{n=1}^{12} 3 \cdot 2^{n-1}$.**

The sum is a geometric series.

$S_n = \frac{a_1(1 - r^n)}{1 - r}$ Sum formula

$S_{12} =$ $n = 12$, $a_1 = 3$, $r = 2$

$S_{12} = \frac{3(4095)}{1}$ $2^{12} = 4096$

$S_{12} =$ Simplify.

REMEMBER IT

The sum in Example 2 can also be found by using $S_n = \frac{a_1 - a_1r^n}{1 - r}$.

Check Your Progress

a. How many direct ancestors would a person have after 7 generations?

b. Evaluate $\sum_{n=1}^{4} 5 \cdot 3^{n-1}$.

EXAMPLE Use the Alternate Formula for a Sum

3 **Find the sum of a geometric series for which $a_1 = 7776$, $a_n = 6$, and $r = -\frac{1}{6}$.**

Since you do not know the value of n, use $S_n = \frac{a_1 - a_n r}{1 - r}$.

$S_n = \frac{a_1 - a_n r}{1 - r}$ Alternate sum formula

$= \frac{\boxed{\quad}}{1 - \boxed{\quad}}$ $a_1 = 7776$, $a_n = 6$, and $r = -\frac{1}{6}$

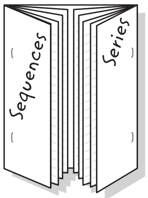
$= \frac{7777}{\frac{7}{6}}$ or $\boxed{\quad}$ Simplify.

Check Your Progress Find the sum of a geometric series for which $a_1 = 64$, $a_n = 729$, and $r = -1.5$.

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-4. Write in words the sigma notation that is used in Example 2.



EXAMPLE Find the First Term of a Series

4 **Find a_1 in a geometric series for which $S_8 = 765$ and $r = 2$.**

$S_n = \frac{a_1(1 - r^n)}{1 - r}$ Sum formula

$\boxed{\quad} = \boxed{\quad}$ $S_8 = 765$, $r = 2$, and $n = 8$

$765 = 255a_1$ Simplify.

$\boxed{\quad} = a_1$ Divide each side by 255.

Check Your Progress Find a_1 in a geometric series for which $S_6 = 364$ and $r = 3$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–5 Infinite Geometric Series

MAIN IDEAS

- Find the sum of an infinite geometric series.
- Write repeating decimals as fractions.

KEY CONCEPT

Sum of an Infinite Geometric Series The sum S of an infinite geometric series with $-1 < r < 1$ is given by $S = \frac{a_1}{1 - r}$.

BUILD YOUR VOCABULARY (page 312)

If a geometric series has no last term, it is called an **infinite geometric series**. For infinite series, S_n is called a **partial sum** of the series.

EXAMPLE Sum of an Infinite Geometric Series

① **Find the sum of each infinite geometric series, if it exists.**

a. $-\frac{4}{3} + 4 - 12 + 36 - 108 + \ldots$

Find the value of r to determine if the sum exists.

$a_1 = -\frac{4}{3}$ and $a_2 = 4$, so $r = \boxed{}$ or $\boxed{}$.

Since $|-3| \geq 1$, the sum $\boxed{}$ exist.

b. $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \ldots$

$a_1 = \boxed{}$ and $a_2 = \boxed{}$, so $r = \boxed{}$ or $-\frac{1}{2}$.

Since $\boxed{}$, the sum $\boxed{}$.

Use the formula for the sum of an infinite geometric series.

$$S = \frac{a_1}{1 - r} \qquad \text{Sum formula}$$

$$= \frac{3}{1 - \left(-\frac{1}{2}\right)} \qquad a_1 = 3,\ r = -\frac{1}{2}$$

$$= \frac{\boxed{}}{\boxed{}} \text{ or } \boxed{} \qquad \text{Simplify.}$$

Check Your Progress **Find the sum of each infinite geometric series, if it exists.**

a. $2 + 4 + 8 + 16 + \ldots$ $\boxed{}$

b. $1 + \frac{1}{2} + \frac{1}{4} + \ldots$

EXAMPLE Infinite Series in Sigma Notation

2 **Evaluate $\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$.**

In this infinite geometric series, $a_1 = 5$ and $r = \frac{1}{2}$.

$S = \frac{a_1}{1 - r}$ Sum formula

$= \frac{5}{1 - \frac{1}{2}}$ $a_1 = 5$, $r = \frac{1}{2}$

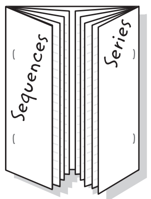
$=$ or Simplify.

Check Your Progress Evaluate $\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$.

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 11-5, write $0.\overline{54}$ as a fraction. Use either of the methods shown in Example 3 of your textbook.



EXAMPLE Write a Repeating Decimal a Fraction

3 **Write $0.\overline{25}$ as a fraction.**

$S = 0.\overline{25}$ Label the given decimal.

$S = 0.25252525\ldots$ Repeating decimal

$100S =$ Multiply each side by 100.

$99S =$ Subtract the second equation from the third.

$S =$ Divide each side by 99.

Check Your Progress Write $0.\overline{37}$ a fraction.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–6 Recursion and Special Sequences

MAIN IDEAS

- Recognize and use special sequences.
- Iterate functions.

BUILD YOUR VOCABULARY (pages 312–313)

The sequence 1, 1, 2, 3, 5, 8, 13, … , where each term in the sequence after the second is the sum of the two previous terms, is called the **Fibonacci sequence**.

The formula $a_n = a_{n-2} + a_{n-1}$ is an example of a **recursive formula**.

EXAMPLE Use a Recursive Formula

① **Find the first five terms of the sequence in which $a_1 = 5$ and $a_{n+1} = 2a_n + 7, n \geq 1$.**

$a_{n+1} = 2a_n + 7$ Recursive formula

$a_{1+1} = 2a_1 + 7$ $n = 1$

$a_2 = 2\left(\boxed{}\right) + 7$ or $\boxed{}$ $a_1 = 5$

$a_{2+1} = 2a_2 + 7$ $n = 2$

$a_3 = 2\left(\boxed{}\right) + 7$ or $\boxed{}$ $a_2 = 17$

$a_{3+1} = 2a_3 + 7$ $n = 3$

$a_4 = 2\left(\boxed{}\right) + 7$ or $\boxed{}$ $a_3 = 41$

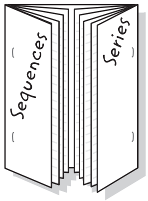
$a_{4+1} = 2a_4 + 7$ $n = 4$

$a_5 = 2\left(\boxed{}\right) + 7$ or $\boxed{}$ $a_4 = 89$

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-6. Describe the pattern in the sequence 1, 2, 6, 24, 120, ... Then find the next three terms of the sequence.



Check Your Progress Find the first five terms of the sequence in which $a_1 = 2$ and $a_{n+1} = 3a_n + 2, n \geq 1$.

BUILD YOUR VOCABULARY (page 313)

Iteration is the process of composing a with itself repeatedly.

EXAMPLE Iterate a Function

1 **Find the first three iterates x_1, x_2, and x_3 of the function $f(x) = 3x - 1$ for an initial value of $x_0 = 5$.**

To find the first iterate x_1, find the value of the function for $x_0 = 5$.

$x_1 = f(x_0)$	Iterate the function.
$= f$	$x_0 = 5$
$=$ or	Simplify.

To find the second iterate x_2, substitute x_1 for x.

$x_2 = f(x_1)$	Iterate the function.
$= f$	$x_1 = 14$
$=$ or	Simplify.

Substitute x_2 for x to find the third iterate.

$x_3 = f(x_2)$	Iterate the function.
$= f$	$x_2 = 41$
$=$ or	Simplify.

Check Your Progress Find the first three iterates x_1, x_2, and x_3 of the function $f(x) = 2x + 1$ for an initial value of $x_0 = 2$.

REVIEW IT

If $f(x) = 4x + 10$ and $g(x) = 2x^2 - 5$, find $f(g(x))$. *(Lesson 7-7)*

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–7 The Binomial Theorem

MAIN IDEAS

- Use Pascal's triangle to expand powers of binomials.
- Use the Binomial Theorem to expand powers of binomials.

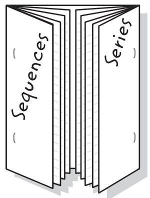
BUILD YOUR VOCABULARY (pages 312–313)

The ____ in powers of ____ form a pattern that is often displayed in a triangular formation known as **Pascal's triangle**.

The factors in the coefficients of binomial ____ involve special products called **factorials**.

FOLDABLES™ ORGANIZE IT

Use the tab for Lesson 11-7. Refer to Example 1, and describe what happens to the exponents for *p* as the exponents for *q* increase.



EXAMPLE Use Pascal's Triangle

1 Expand $(p + q)^5$.

Write the row of Pascal's triangle corresponding to $n = 5$.

Use the patterns of a binomial expansion and the coefficients to write the expansion of $(p + q)^5$.

$(p + q)^5$

= ____

= ____

EXAMPLE Use the Binomial Theorem

2 Expand $(t - s)^8$.

The expression will have ____ terms. Use the sequence $1, \frac{8}{1}, \frac{8 \cdot 7}{1 \cdot 2}, \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}, \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}$ to find the coefficients for the first five terms. Use symmetry to find the remaining coefficients.

$$(t - s)^8 = 1t^8(-s)^0 + \frac{8}{1}t^7(-s) + \frac{8 \cdot 7}{1 \cdot 2}t^6(-s)^2 + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}t^5(-s)^3 + \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}t^4(-s)^4 + \ldots + 1t^0(-s)^8$$

= ____

KEY CONCEPT

Binomial Theorem If n is a nonnegative integer, then

$(a + b)^n$

$= 1a^nb^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \ldots + 1a^0b^n.$

Check Your Progress

a. Expand $(x + y)^6$.

b. Expand $(x - y)^4$.

EXAMPLE Factorials

3 Evaluate $\frac{6!}{2!4!}$.

$$\frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{2 \cdot 1 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}$$

$$= \qquad \text{or} \qquad$$

Note that $6! = 6 \cdot 5 \cdot 4!$, so $\frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{2!\cancel{4!}}$ or $\frac{6 \cdot 5}{2 \cdot 1}$.

KEY CONCEPT

Binomial Theorem, Factored Form

$$(a + b)^n$$
$$= \frac{n!}{n!0!}a^n b^0 + \frac{n!}{(n-1)!1!}a^{n-1}b^1 + \frac{n!}{(n-2)!2!}a^{n-2}b^2 + \ldots + \frac{n!}{n!0!}a^0 b^n + \sum_{k=0}^{n}\frac{n!}{(n-k)!k!}a^{n-k}b^k$$

Check Your Progress Evaluate $\frac{7!}{2!3!}$.

EXAMPLE Use a Factorial Form for the Binomial Theorem

4 Expand $(3x - y)^4$.

$$(3x - y)^4 =$$

$$= \frac{4!}{4!0!}(3x)^4(-y)^0 + \frac{4!}{3!1!}(3x)^3(-y)^1 + \frac{4!}{2!2!}(3x)^2(-y)^2 + \qquad + \qquad$$

$$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1}(3x)^4 + \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1}(3x)^3(-y) + \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}(3x)^2y^2 + \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1}(3x)^1(-y^3) +$$

$$=$$

Check Your Progress Expand $(2x + y)^4$.

EXAMPLE Find a Particular Term

5 Find the fourth term in the expansion of $(a + 3b)^4$.

First, use the Binomial Theorem to write the expression in sigma notation.

$$(a + 3b)^4 = \sum_{k=0}^{4} \frac{4!}{(4-k)!k!} a^{4-k}(3b)^k$$

In the fourth term, $k = \boxed{}$.

$$\frac{4!}{(4-k)!k!} a^{4-k}(3b)^k = \frac{4!}{(4-3)!3!} a^{4-3}(3b)^3 \qquad k = 3$$

$$= \boxed{} \qquad \frac{4!}{1!3!} = \boxed{}$$

$$= \boxed{} \qquad \text{Simplify.}$$

Check Your Progress Find the fifth term in the expansion of $(x + 2y)^6$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–8 Proof and Mathematical Induction

MAIN IDEAS

- Prove statements using mathematical induction.
- Disprove statements by finding a counterexample.

KEY CONCEPT

Mathematical Induction

Step 1 Show that the statement is true for some integer n.

Step 2 Assume that the statement is true for some positive integer k, where $k \geq n$. This assumption is called the **inductive hypothesis**.

Step 3 Show that the statement is true for the next integer $k + 1$.

BUILD YOUR VOCABULARY (page 313)

Mathematical induction is used to ______ statements about ______ integers.

EXAMPLE Summation Formula

① **Prove that $1 + 4 + 7 + \ldots + (3n - 2) = \dfrac{n(3n - 1)}{2}$.**

Step 1 When $n = 1$, the left side of the given equation is ______ – ______ or ______. The right side is ______ or ______. Thus, the equation is true for $n = 1$.

Step 2 Assume $1 + 4 + 7 + \ldots + (3k - 2) = \dfrac{k(3k - 1)}{2}$ for a positive integer k.

Step 3 Show that the given equation is true for $n =$ ______.

$1 + 4 + 7 + \ldots + (3k - 2) + (3(k + 1) - 2)$

$\dfrac{k(3k - 1)}{2} + 3(k + 1) - 2$ Add $3(k + 1) - 2$ to each side.

$\dfrac{3k^2 + 5k + 2}{2}$ Add.

$\dfrac{(3k + 2)(k + 1)}{2}$ Simplify.

$\dfrac{(k + 1)(3(k + 1) - 1)}{2}$ Factor.

The last expression is the right side of the equation to be proved, where n has been replaced by $k + 1$. Thus, the equation is true for $n = k + 1$.

This proves that $1 + 4 + 7 + \ldots + (3n - 2) = \dfrac{n(3n - 1)}{2}$ is true for all positive integers n.

EXAMPLE Divisibility

① **Prove that $6n - 1$ is divisible by 5 for all positive integers n.**

Step 1 When $n = 1$, $6^n - 1 =$ or . Since is divisible by 5, the statement is true for $n = 1$.

Step 2 Assume that $6^k - 1$ is divisible by 5 for some positive integer k. This means that there is a whole number r such that .

Step 3 Show that the statement is true for $n = k + 1$.

$=$ Inductive hypothesis

$6^k =$ Add 1 to each side.

$6 \cdot 6^k = 6$ Multiply each side by 6.

$6^{k+1} =$ Simplify.

$6^{k+1} - 1 =$ Subtract 1 from each side.

$6^{k+1} - 1 =$ Factor.

Since r is a whole number, $6r + 1$ is a whole number.

Therefore, .

Thus, the statement is true for . This proves that $6^n - 1$ is divisible by 5.

Check Your Progress

a. Prove that $2 + 4 + 6 + 8 + \ldots + 2n = n(n + 1)$.

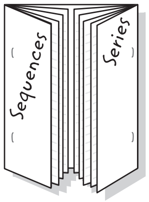
b. Prove that $10^n - 1$ is divisible by 9 for all positive integers n.

EXAMPLE Counterexample

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 11-8, write a real-world statement. Then find a counterexample to your statement.



3 **Find a counterexample for the statement that $n^2 + n + 5$ is always a prime number for any positive integer n.**

Check the first few positive integers.

n	Formula	Prime?
1		
2		
3		
4		

The value $n =$ ☐ is a counterexample for the formula.

Check Your Progress Find a counterexample for the formula that $2n - 1$ is always a prime number for any positive integer n.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER
11

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 11 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 11, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 312–313)* to help you solve the puzzle.

11-1
Arithmetic Sequences

1. Find the next four terms of the arithmetic sequence 3, 6, 9, 12, ...

2. Find the first five terms of the arithmetic sequence in which $a_1 = 2$ and $d = 9$.

3. Write an equation for the nth term of the arithmetic sequence 10, 6, 2, −2, ...

11-2
Arithmetic Series

4. Find S_n for the arithmetic series in which $a_1 = -6$, $n = 18$, and $d = 2$.

5. Find the sum of the arithmetic series 30 + 25 + . . . + (−10).

Find the sum of each arithmetic series.

6. $\sum_{j=3}^{9}(6 - j)$

7. $\sum_{k=10}^{25}(2k + 1)$

11-3

Geometric Sequences

8. In the sequence 5, 8, 11, 14, 17, 20, the numbers 8, 11, 14, and 17 are ______ between 5 and 20.

9. In the sequence 12, 4, $\frac{4}{3}$, $\frac{4}{9}$, $\frac{4}{27}$, the numbers 4, $\frac{4}{3}$, and $\frac{4}{9}$ are ______ between 12 and $\frac{4}{27}$.

10. Find three geometric means between 4 and 324.

11-4

Geometric Series

11. Consider the formula $S_n = \frac{a_1(1 - r^n)}{1 - r}$. Suppose that you want to use the formula to evaluate the sum $\sum_{n=1}^{6} 8(-2)^{n-1}$. Indicate the values you would substitute into the formula in order to find S_n.

$n =$ ______ $a_1 =$ ______ $r =$ ______ $r^n =$ ______

12. Find the sum of a geometric series for which $a_1 = 5$, $n = 9$, and $r = 3$.

11-5

Infinite Geometric Series

13. Consider the formula $S = \frac{a_1}{1 - r}$. For what values of r does an infinite geometric sequence have a sum? ______

14. For the geometric series $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$, give the values of a_1 and r. Then state whether the sum of the series exists.

11-6
Recursion and Special Sequences

15. Find the first five terms of the sequence in which $a_1 = -3$ and $a_{n+1} = 2a_n + 5$.

16. Find the first three iterates of $f(x) = 3x - 2$ for an initial value of $x_0 = 4$.

11-7
The Binomial Theorem

Consider the expansion of $(w + z)^5$.

17. How many terms does this expansion have?

18. In the second term of the expansion, what is the exponent of w?

19. In the fourth term of the expansion, what is the exponent of w?

20. What is the last term of this expansion?

11-8
Proof and Mathematical Induction

Suppose that you wanted to prove that the following statement is true for all positive integers.

$$\mathbf{3 + 6 + 9 + \ldots + 3n = \frac{3n(n + 1)}{2}}$$

21. Which statement shows that the statement is true for $n = 1$?

i. $3 = \frac{3 \cdot 2 + 1}{2}$ **ii.** $3 = \frac{3 \cdot 1 \cdot 2}{2}$ **iii.** $\frac{3 + 1 + 2}{2}$

CHAPTER 11

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 11.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 11 Practice Test on page 679 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 11 Study Guide and Review on pages 674–678 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 679 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 11 Foldable.
- Then complete the Chapter 11 Study Guide and Review on pages 674–678 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 679 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER

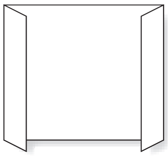
12

Probability and Statistics

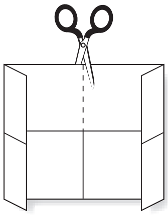
FOLDABLES™ Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11” × 17” paper.

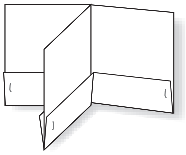
STEP 1 **Fold** 2” tabs on each of the short sides.



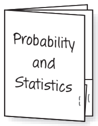
STEP 2 **Fold** in half in both directions. Open and cut as shown.



STEP 3 **Refold** along the width. Staple each pocket.



STEP 4 **Label** pockets as *The Counting Principle, Permutations and Combinations, Probability*, and *Statistics*.



NOTE-TAKING TIP: When you take notes, look for written real-world examples in your everyday life. Comment on how writers use statistics to prove or disprove points of view and discuss the ethical responsibilities writers have when using statistics.

CHAPTER

12

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 12. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
binomial experiment			
combination			
compound event			
dependent and independent events			
event			
inclusive events ihn-KLOO-sihv			
measure of variation			
mutually exclusive events MYOO-chuh-lee			
normal distribution			
outcome			
permutation PUHR-myoo-TAY-shuhn			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
probability			
probability distribution			
random			
random variable			
relative-frequency histogram			
sample space			
simple event			
standard deviation			
unbiased sample			
uniform distribution			
univariate data			
$\underbrace{\text{variance}}$ VEHR-ee-uhn(t)s			

12–1 The Counting Principle

MAIN IDEAS

- Solve problems involving independent events.
- Solve problems involving dependent events.

BUILD YOUR VOCABULARY (pages 341–342)

The set of all possible is called the **sample space.**

An **event** consists of one or more outcomes of a trial. Events that do not affect each other are called **independent events.**

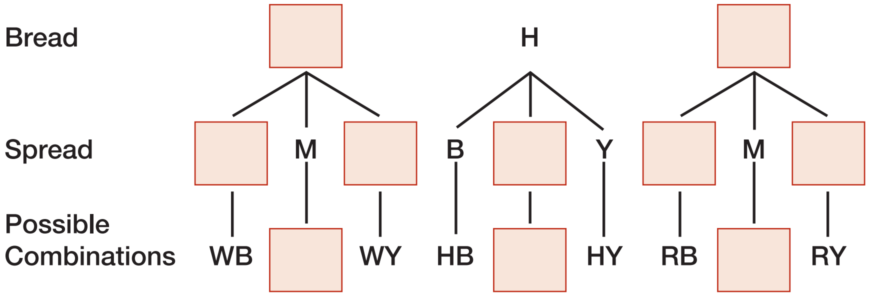
EXAMPLE Independent Events

① FOOD **A sandwich menu offers customers a choice of white, wheat, or rye bread with one spread chosen from butter, mustard, or mayonnaise. How many different combinations of bread and spread are possible?**

First note that the choice of the type of bread does not affect the choice of the type of spread, so these events are independent.

Method 1 Tree Diagram

W represents white, H, wheat, R, rye, B, butter, M, mustard, Y, mayonnaise.



Method 2 Make a Table

Make a table in which each row represents a type of bread and each column represents a type of spread.

	Butter	Mustard	Mayonnaise
White		WM	
Wheat	HB		HY
Rye		RM	

There are possible outcomes.

Check Your Progress A pizza place offers customers a choice of American, mozzarella, Swiss, feta, or provolone cheese with one topping chosen from pepperoni, mushrooms, or sausage. How many different combinations of cheese and toppings are there?

KEY CONCEPT

Fundamental Counting Principle If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then event *M* followed by event *N* can occur in $m \cdot n$ ways.

EXAMPLE Fundamental Counting Principle

② **TEST EXAMPLE** **The Murray family is choosing from a trip to the beach or a trip to the mountains. The family can select transportation from a car, plane, or train. How many different ways can they select a destination followed by a means of transportation?**

A 2 **B** 5 **C** 6 **D** 9

There are · or ways to choose a trip.

The answer is .

Check Your Progress For their vacation, the Esper family is going on a trip. They can select their transportation from a car, plane, or train. They can also select from 4 different hotels. How many different ways can they select a means of transportation followed by a hotel?

A 8 **B** 12 **C** 16 **D** 7

EXAMPLE More than Two Independent Events

③ **COMMUNICATION** **How many answering machine codes are possible if the code is just 2-digits?**

The choice of any digit does not affect the other digit, so the choices of digits are independent events.

There are possible choices for the first digit and

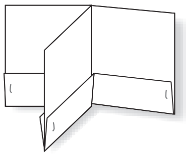
possible choices for the second digit.

So, there are · or possible different codes.

FOLDABLES™

ORGANIZE IT

Write and solve your own examples of independent and dependent events. Place your work in The Counting Principle pocket.



Check Your Progress Many automated teller machines (ATM) require a 4-digit code to access an account. How many codes are possible?

BUILD YOUR VOCABULARY (page 341)

With **dependent events**, the outcome of one event ____ affect the outcome of another event.

EXAMPLE Dependent Events

④ **SCHOOL Refer to the table in Example 4 of your textbook. How many different schedules could a student who is planning to take only four different classes have?**

The choices of which class to schedule each period are dependent events.

There are 4 classes that can be taken during the first period. That leaves 3 classes for the second period, 2 classes for the third period, and so on.

Period	1st	2nd	3rd	4th
Number of Choices				

There are ____ · ____ · ____ · ____ or ____ different schedules for a student who is taking 4 classes.

Check Your Progress How many different schedules could a student have who is planning to take 5 different classes?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–2 Permutations and Combinations

MAIN IDEAS

- Solve problems involving permutations.
- Solve problems involving combinations.

KEY CONCEPTS

Permutations The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.

Permutations with Repetitions The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

BUILD YOUR VOCABULARY (page 341)

When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

EXAMPLE Permutation

1 COOKING **Eight people enter the Best Pie contest. How many ways can blue, red, and yellow ribbons be awarded?**

Since each winner will receive a different ribbon, order is important. You must find the number of permutations of 8 things taken 3 at a time.

$P(n, r) = \frac{n!}{(n-r)!}$ Permutation formula

$= \boxed{}$ $n = 8$, $r = 3$

$= \boxed{}$ Simplify.

$= \frac{8 \cdot 7 \cdot 6 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{4}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{1}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{1}}}$ Divide.

$= \boxed{}$

EXAMPLE Permutation with Repetition

2 **How many different ways can the letters of the word BANANA be arranged?**

The second, fourth, and sixth letters are each A.

The third and fifth letters are each N.

Find the number of permutations of $\boxed{}$ letters of which $\boxed{}$ of one letter and $\boxed{}$ of another letter are the same.

$\frac{6!}{3!2!} = \boxed{}$ or $\boxed{}$

Check Your Progress

a. Ten people are competing in a swim race where 4 ribbons will be given. How many ways can blue, red, green, and yellow ribbons be awarded?

b. How many different ways can the letters of the word ALGEBRA be arranged?

KEY CONCEPTS

Combinations The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$.

FOLDABLES Write a real-world example of a permutation and a combination. Place your work in the Permutations and Combinations pocket.

BUILD YOUR VOCABULARY (page 341)

An arrangement or selection of objects in which order is *not* important is called a **combination**.

EXAMPLE Combination

3 **Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people who will go?**

Since the order they choose the three people is not important, you must find the number of combinations of 5 cousins taken 3 at a time.

$C(n, r) = \frac{n!}{(n-r)!r!}$ Combination formula

$C(5, 3) =$ ☐ $n = 5$ and $r = 3$

$=$ ☐ or ☐ Simplify.

There are ☐ ways to choose the three cousins.

EXAMPLE Multiple Events

④ **Six cards are drawn from a standard deck of cards. How many hands consist of two hearts and four spades?**

Multiply the number of ways to select two hearts and the number of ways to select four spades. Only the cards in the hand matter, not the order in which they were drawn, so use ☐.

$C(13, 2) \cdot C(13, 4)$

$= \frac{13!}{(13 - 2)!2!} \cdot$ ☐ Combination formula

$=$ ☐ $\cdot$ ☐ Subtract.

$=$ ☐ $\cdot$ ☐ or ☐ Simplify.

There are ☐ hands consisting of 2 hearts and 4 spades.

Check Your Progress

a. Six friends at a party decide that three of them will go to pick up a movie. How many ways can they choose three people to go?

b. Thirteen cards are drawn from a standard deck of cards. How many hands consist of six hearts and seven diamonds?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–3 Probability

MAIN IDEAS

- Use combinations and permutations to find probability.
- Create and use graphs of probability distributions.

BUILD YOUR VOCABULARY (page 342)

The **probability** of an event is a that measures the chances of the event occurring.

KEY CONCEPT

Probability of Success and Failure If an event can succeed in *s* ways and fail in *f* ways, then the probabilities of success, *P*(*S*), and of failure, *P*(*F*), are as follows.

- $P(S) = \frac{s}{s+f}$
- $P(F) = \frac{f}{s+f}$

EXAMPLE Probability

① **Roman has a collection of 26 books—16 are fiction and 10 are nonfiction. He randomly chooses 8 books to take with him on vacation. What is the probability that he happens to choose 4 fiction and 4 nonfiction books?**

Step 1 Determine how many 8-book selections meet the conditions. Notice that order doesn't matter.

$C(16, 4)$ Select 4 fiction books.
$C(10, 4)$ Select 4 nonfiction books.

Step 2 Use the Fundamental Counting Principle to find s, the number of successes.

$C(16, 4) \cdot C(10, 4) =$ $\cdot$ or

Step 3 Find the total number, $s + f$, of possible 8-book selections.

$C(26, 8) =$ or $s + f =$

Step 4 Determine the probability.

P(4 fiction and 4 nonfiction)

$= \frac{s}{s+f}$ Probability formula

$= \frac{}{}$ or about

The probability of selecting 4 fiction and 4 nonfiction is about or .

Check Your Progress When three coins are tossed, what is the probability that exactly two are heads?

KEY CONCEPT

Odds The odds that an event will occur can be expressed as the ratio of the number of ways it can succeed to the number of ways it can fail. If an event can succeed in *s* ways and fail in *f* ways, then the odds of success and of failure are as follows.

- Odds of success = s:f
- Odds of failure = f:s

EXAMPLE Odds

1 **For next semester, Alisa has signed up for English, precalculus, Spanish, geography, and chemistry classes. If class schedules are assigned randomly and each class is equally likely to be at any time of the day, what is the probability that Alisa's first two classes in the morning will be precalculus and chemistry, in either order?**

Step 1 Determine how many schedule arrangements meet the conditions. Notice that order matters.

$P(2, 2)$ Place the two earliest classes.
$P(3, 3)$ Place the other 3 classes.

Step 2 Use the Fundamental Counting Principle to find the number of successes.

$P(2, 2) \cdot P(3, 3) =$ ______ or ______

Step 3 Find the total number, $s + f$, of possible 5-class arrangements.

$P(5, 5) =$ ______ or ______ $s + f =$ ______

Step 4 Determine the probability.

P(Precalculus, chemistry, followed by the other classes)

$= \frac{s}{s + f}$ Probability formula

$=$ ______ or about ______ Substitute.

The probability that Alisa's first two classes are precalculus and chemistry is about ______ or ______.

Check Your Progress **The chances of a male born in 1980 to live to be at least 65 years of age are about 7 in 10. For females, the chances are about 21 in 25.**

a. What are the odds that a male born in 1980 will live to age 65?

b. What are the odds that a female born in 1980 will live to age 65?

EXAMPLE Probability With Permutations

3 Use the table and graph in Example 3 of your textbook.

a. Use the graph to determine which outcomes are least likely. What are their probabilities?

The least likely outcomes are ____ and ____, with a probability of ____ for each.

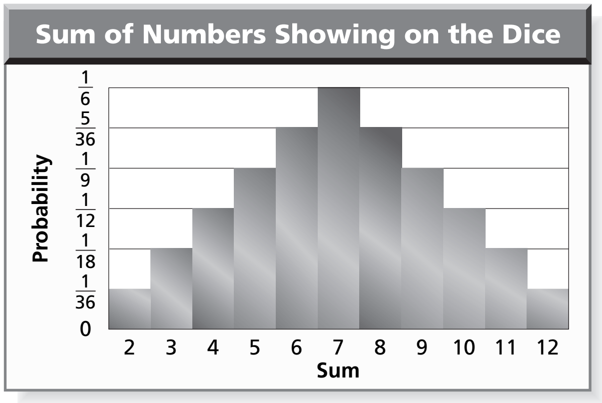
b. Use the table to find $P(S = 11)$. What other sum has the same probability?

According to the table, the probability of a sum of 11 is ____. The other outcome with a probability of ____ is ____.

Check Your Progress **Suppose two dice are rolled. The table and the relative frequency histogram show the distribution of the sum of the numbers rolled.**

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

a. Use the graph to determine which outcomes are the second most likely. What are their probabilities?


Sum of Numbers Showing on the Dice

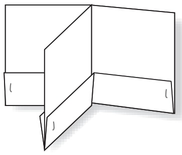
b. Use the table to find $P(S = 4)$. What other sum has the same probability?

c. What are the odds of rolling a sum of 3?

FOLDABLES™

ORGANIZE IT

Write your own probability example. Then write your own odds example. Explain the difference between the two. Place your work in the Probability pocket.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–4 Multiplying Probabilities

MAIN IDEAS

- Find the probability of two independent events.
- Find the probability of two dependent events.

KEY CONCEPT

Probability of Two Independent Events If two events, *A* and *B*, are independent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE Two Independent Events

① Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both of the coins he selects are dimes?

$P(\text{both dimes})$

$= P(\text{dime}) \cdot P(\text{dime})$ Probability of independent events

$= \boxed{} \cdot \boxed{}$ or $\boxed{}$ Substitute and multiply.

The probability is $\boxed{}$ or about $\boxed{}$%.

EXAMPLE Three Independent Events

② **When three dice are rolled, what is the probability that the first two show a 5 and the third shows an even number?**

Let *A* be the event that the first die shows a 5. ⟶ $P(A) = \boxed{}$

Let *B* be the event that the second die shows a 5. ⟶ $P(B) = \boxed{}$

Let C be the event that the third die shows an even number. ⟶ $P(C) = \boxed{}$

$P(A, B, \text{ and } C)$

$= P(A) \cdot P(B) \cdot P(C)$ Probability of Independent events

$= \boxed{} \cdot \boxed{} \cdot \boxed{}$ or $\boxed{}$ Substitute and multiply.

The probability that the first and second dice show a 5 and the third die shows an even number is $\boxed{}$.

Check Your Progress

a. Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both of the coins he selects are pennies?

$\boxed{}$

b. When three dice are rolled, what is the probability that one die is a multiple of 3, one die shows an even number, and one die shows a 5?

$\boxed{}$

EXAMPLE Two Dependent Events

3 The host of a game show draws chips from a bag to determine the prizes for which contestants will play. Of the 20 chips, 11 show *computer*, 8 show *trip*, and 1 shows *truck*. If the host draws the chips at random and does not replace them, find the probability of drawing a computer, then a truck.

$P(C \text{ then } T)$

$= P(C) \cdot P(T \text{ following } C)$ Dependent events

$= \boxed{} \cdot \boxed{}$ or $\boxed{}$ After the first chip is drawn, there are 19 left.

The probability is $\boxed{}$ or about $\boxed{}$.

KEY CONCEPT

Probability of Two Dependent Events If two events, *A* and *B*, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

Check Your Progress **Refer to Example 3. Find the probability of drawing each group of prizes.**

a. a truck, then a trip

$\boxed{}$

b. two computers

$\boxed{}$

EXAMPLE Three Dependent Events

4 Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart, another heart, and a spade in that order.

Since the cards are not replaced, the events are dependent. Let *H* represent a heart and *S* represent a spade.

$P(H, H, S) = P(H) \cdot P(H \text{ following } H) \cdot P(S \text{ following } H \text{ and } H)$

$= \boxed{} \cdot \boxed{} \cdot \boxed{}$ or $\boxed{}$

The probability is $\boxed{}$ or about $\boxed{}$.

Check Your Progress Refer to Example 4. Find the probability of drawing a diamond, another diamond, and another diamond in that order. $\boxed{}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–5 Adding Probabilities

MAIN IDEAS

- Find the probability of mutually exclusive events.
- Find the probability of inclusive events.

BUILD YOUR VOCABULARY (page 341)

An event that consists of two or more ☐ events is called a **compound event**.

If two events cannot occur at the same ☐, they are called **mutually exclusive events**.

KEY CONCEPT

Probability of Mutually Exclusive Events If two events, *A* and *B*, are mutually exclusive, then the probability that *A* or *B* occurs is the sum of their probabilities.

EXAMPLE Two Mutually Exclusive Events

① **Sylvia has a stack of playing cards consisting of 10 hearts, 8 spades, and 7 clubs. If she selects a card at random from this stack, what is the probability that it is a heart or club?**

These are mutually exclusive events since the card cannot be both a heart and a club.

P(heart or club)

$= P(H) + P(C)$ Mutually exclusive events

$= \boxed{} + \boxed{}$ Substitute.

$= \boxed{}$ Add.

The probability that Sylvia selects a heart or a club is ☐.

Check Your Progress Sylvia has a stack of playing cards consisting of 10 hearts, 8 spades, and 7 clubs. If she selects a card at random from this stack, what is the probability that it is a spade or a club?

EXAMPLE Three Mutually Exclusive Events

② **The Film Club makes a list of 9 comedies and 5 adventure movies they want to see. They plan to select 4 titles at random to show this semester. What is the probability that at least two of the films they select are comedies?**

At least two comedies mean that the club may have $\boxed{}$, $\boxed{}$, or $\boxed{}$ comedies. The events are mutually exclusive.

Add the probabilities of each type.

P(at least 2 comdies)

$$= P(2 \text{ comedies}) + \boxed{} + \boxed{}$$

$$= \frac{C(9, 2) \cdot C(5, 2)}{C(14, 4)} + \boxed{} + \boxed{}$$

$$= \boxed{} + \boxed{} + \boxed{}$$

$$= \boxed{}$$

The probability of at least 2 comedies being shown this semester is $\boxed{}$ or about $\boxed{}$.

Check Your Progress The Book Club makes a list of 9 mysteries and 3 romance books they want to read. They plan to select 3 titles at random to read this semester. What is the probability that at least two of the books they select are romances?

BUILD YOUR VOCABULARY (page 341)

If two events are not $\boxed{}$, they are called **inclusive events.**

KEY CONCEPT

Probability of Inclusive Events If two events, *A* and *B*, are inclusive, then the probability that *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occurring.

EXAMPLE Inclusive Events

1 **There are 2400 subscribers to an Internet service provider. Of these, 1200 own Brand A computers, 500 own Brand B, and 100 own both A and B. What is the probability that a subscriber selected at random owns either Brand A or Brand B?**

Since some subscribers own both *A* and *B*, the events are inclusive.

$P(\mathrm{A}) = \boxed{}$ $\quad P(\mathrm{B}) = \boxed{}$ $\quad P(\mathrm{both}) = \boxed{}$

$$P(\mathrm{A\ or\ B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A\ and\ B})$$

$$= \boxed{} + \boxed{} - \boxed{}$$

$$= \boxed{}$$ Substitute and simplify.

The probability that a subscriber owns either A or B is $\boxed{}$.

Check Your Progress There are 200 students taking Calculus, 500 taking Spanish, and 100 taking both. There are 1000 students in the school. What is the probability that a student selected at random is taking Calculus or Spanish?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–6 Statistical Measures

MAIN IDEAS

- Use measures of central tendency to represent a set of data.
- Find measures of variation for a set of data.

BUILD YOUR VOCABULARY (pages 341–342)

A number that describes a set of data is called a **measure of central tendency** because it represents the ______ or middle of the data.

Measures of variation or dispersion measure how ______ or ______ a set of data is.

The **standard deviation** σ is the ______ of the **variance**.

EXAMPLE Choose a Measure of Central Tendency

① **SALARIES** **A new Internet company has 3 employees who are paid $300,000, ten who are paid $100,000, and sixty who are paid $50,000. Which measure of central tendency best represents the pay at this company?**

Since most of the employees are paid $50,000, the higher values are outliers.

Thus, the ______ or ______ best represents the pay at this company.

Check Your Progress **In a cereal contest, there is 1 Grand Prize of $1,000,000, 10 first prizes of $100, and 50 second prizes of $10.**

a. Which measure of central tendency best represents the prizes?

______

b. Which measure of central tendency would advertisers be most likely to use?

______

REVIEW IT

In your own words, what is an outlier? (*Lesson 2-5*)

______

______

______

______

KEY CONCEPT

Standard Deviation If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean x, then the standard deviation σ is given by the following formula.

$$\sigma = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n}}$$

FOLDABLES Suppose you have a small standard of deviation for your test scores. Does this mean that you have been consistent or inconsistent? Place your explanation in the Statistics pocket.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Standard Deviation

① **RIVERS This table shows the lengths in thousands of miles of some of the longest rivers in the world. Find the standard deviation for these data.**

River	Length (thousands of miles)
Nile	4.16
Amazon	4.08
Missouri	2.35
Rio Grande	1.90
Danube	1.78

Find the mean. Add the data and divide by the number of items.

$$\overline{x} = \frac{4.16 + 4.08 + 2.35 + 1.90 + 1.78}{5}$$

$$= \boxed{} \text{ thousand miles}$$

Find the variance.

$$\sigma^2 = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n} \qquad \text{Variance formula}$$

$$\approx \frac{(4.16 - 2.85)^2 + (4.08 - 2.85)^2 + \cdots + (1.78 - 2.85)^2}{5}$$

$$\approx \boxed{} \qquad \text{Simplify.}$$

$$\approx \boxed{} \text{ thousand miles}$$

Find the standard deviation.

$$\sigma^2 = \boxed{} \qquad \text{Take the square root of each side.}$$

$$\sigma \approx \boxed{} \text{ thousand miles}$$

Check Your Progress A teacher has the following test scores: 100, 4, 76, 85, and 92. Find the standard deviation for these data.

12–7 The Normal Distribution

MAIN IDEAS

- Determine whether a set of data appears to be normally distributed or skewed.
- Solve problems involving normally distributed data.

BUILD YOUR VOCABULARY (page 342)

A curve that is ______, often called a *bell curve*, is called a **normal distribution**.

A curve or histogram that is not symmetric represents a **skewed distribution**.

EXAMPLE Classify a Data Distribution

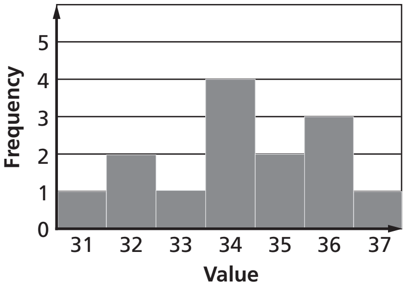
① **Determine whether the data {31, 37, 35, 36, 34, 36, 32, 36, 33, 32, 34, 34, 35, 34} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**

Make a frequency table for the data. Then use the table to make a histogram.

Value	31	32	33	34	35	36	37
Frequency							

Since the data are somewhat symmetric, this is a

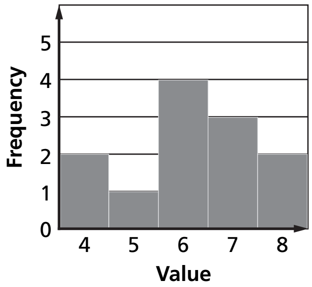
______.



REMEMBER IT

To help remember how skewed distributions are labeled, think about the long tail being in the direction of the skew. For example, a positively skewed distribution has a long tail in the positive direction.

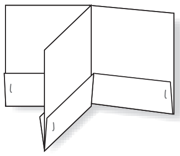
Check Your Progress Determine whether the data {7, 5, 6, 7, 8, 4, 6, 8, 7, 6, 6, 4} shown in the histogram appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.



FOLDABLES™

ORGANIZE IT

Describe real-world situations where you would expect the data to be positively skewed, negatively skewed, and normally distributed. Place your explanations in the Statistics pocket.

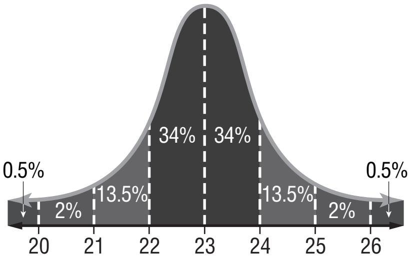


EXAMPLE Normal Distribution

2 **Students counted the number of candies in 100 small packages. They found that the number of candies per package was normally distributed with a mean of 23 candies per package and a standard deviation of 1 piece of candy.**

a. About how many packages had between 24 and 22 candies?

Draw a normal curve. Label the mean and positive and negative multiples of the standard deviation.



The values of 22 and 24 are standard deviation *below and above* the mean, respectively. Therefore, of the data are located here.

100 · = packages   Multiply 100 by 0.68.

About packages contained between 22 and 24 pieces.

b. What is the probability that a package selected at random had more than 25 candies.

The value 25 is standard deviations above the mean. You know that about 100% − 95% or of the data are more than one standard deviation away from the mean. By the symmetry of the normal curve, half of 5% or , of the data are more than two standard deviations above the mean.

The probability that a package selected at random has more than 25 candies is about or .

Check Your Progress **Refer to Example 2.**

a. About how many packages had between 25 and 21 candies?

b. What is the probability that a package selected at random had more than 24 candies?

HOMEWORK ASSIGNMENT

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12–8 Exponential and Binomial Distribution

EXAMPLE Exponential Distribution

① **The average amount of time a North High School student spends studying for math tests is 45 minutes. What is the probability that a randomly chosen student studies for longer than 120 minutes?**

First, find the m, the inverse of the mean. Because the mean is ____, the multiplicative inverse is ____.

$f(x) = e^{-mx}$	Exponential Distribution Function
$f(120) =$ ____	Replace x with 120 and m with $\frac{1}{45}$.
$=$ ____	Simplify.
$\approx$ ____ or ____	Use a calculator.

There is a 7% chance that a randomly selected North High School student studies for math tests for more than 120 minutes. This appears to be a reasonable solution because few students spend either no or an unusually long amount of time studying.

Check Your Progress If each branch of The Portrait Studio hires an average of 10 seasonal employees, what is the probability that a randomly selected branch hired more than 15 seasonal employees?

EXAMPLE Exponential Distribution

② **If blank CDRs last an average of 5 years, what is the probability that a randomly selected CDR will last less than 1 year?**

The question asks for the probability that a CDR lasts *less* than 1 year, so use the exponential function $1 - e^{-mx}$. The mean is , so the multiplicative inverse m is .

$f(x) = 1 - e^{-mx}$	Exponential Distribution Function
$f(1) = 1 -$	Replace x with 1 and m with $\frac{1}{5}$.
$= 1 -$	Simplify.
$\approx$ or	Use a calculator.

There is an 18.13% chance that a randomly selected CDR will last less than 1 year.

Check Your Progress If each library employee shelves on average 75 books an hour, what is the probability that a randomly selected library employee shelves less than 50 books an hour?

EXAMPLE Binomial Distribution

3 **A candy company produces bags of strawberry and vanilla flavored candies, 65% of which are strawberry on average.**

a. What is the probability that a bag of 10 candies has at least 4 strawberry candies?

In this situation it is easier to calculate the probability of the bag having exactly 0, 1, 2, or 3 strawberry candies and then subtracting that sum from 1.

$P(\geq 4 \text{ strawberry candies})$

$= 1 - P(< 4)$

$= 1 - P(0) - P(1) - P(2) - P(3)$

$= 1 - C(10, 0)(0.65)^0(0.35)^{10} - C(10, 1)\,(0.65)^1(0.35)^9 -$
$C(10, 2)(0.65)^2(0.35)^8 - C(10, 3)\,(0.65)^3(0.35)^7$

$=$ ☐ or ☐

The probability of at least 4 strawberry candies being in a 10-piece bag is 97.47%.

b. What is the expected number of strawberry candies in a 20-piece bag?

$E(X) = np$	Expectation for a binomial distribution
$=$ ☐	$n = 20$ and $p = 0.65$
$=$ ☐	Multiply.

The expected number of strawberry candies in a 20-piece bag is 13.

Check Your Progress A paperclip company produces assorted colors of paperclips, 20% of which are yellow on average.

a. What is the probability that a box of 50 paperclips has at least 5 yellow paperclips?

b. What is the expected number of yellow paperclips in a box of 50?

HOMEWORK ASSIGNMENT

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Exercises:

12–9 Binomial Experiments

MAIN IDEAS

- Use binomial expansions to find probabilities.
- Find probabilities for binomial experiments.

REVIEW IT

Use the Binomial Theorem to expand $(x - 3)^5$. (*Lesson 11-7*)

____

____

____

____

____

EXAMPLE Binomial Theorem

① **If a family has 4 children, what is the probability that they have 2 girls and 2 boys?**

There are two possible outcomes for the gender of each of their children: boy or girl. The probability of a boy b is $\boxed{\quad}$, and the probability of a girl g is $\boxed{\quad}$.

$$(b + g)^4 = \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \boxed{\quad}$$

The term $6b^2g^2$ represents 2 girls and 2 boys.

P(2 girls and 2 boys)

$$= 6b^2g^2$$

$$= 6\left(\boxed{\quad}\right)^2\left(\boxed{\quad}\right)^2 \qquad b = \boxed{\quad}, g = \boxed{\quad}$$

$$= \boxed{\quad} \qquad \text{Simplify.}$$

The probability of 2 boys and 2 girls is $\boxed{\quad}$ or $\boxed{\quad}$.

Check Your Progress If a family has 4 children, what is the probability that they have 4 boys?

KEY CONCEPT

Binomial Experiments A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The probabilities for each trial are the same.

EXAMPLE Binomial Experiment

① **SALES** **A report said that approximately 1 out of 6 cars sold in a certain year was green. Suppose a salesperson sells 7 cars per week.**

a. What is the probability that this salesperson will sell exactly 3 green cars in a week?

The probability that a sold car is green is ☐.

The probability that a sold car is not green is ☐.

There are $C(7, 3)$ ways to choose the three green cars that sell.

$P(3 \text{ green cars})$

$= C(7, 3)\left(\square\right)^3\left(\square\right)^4$ If he sells three green cars, he sells four that are not green.

$= \square\left(\square\right)^3\left(\square\right)^4$ $C(7, 3) = \frac{7!}{4!3!}$

$= \square$ Simplify.

The probability that he will sell exactly 3 green cars is ☐ or about ☐.

b. What is the probability that this salesperson will sell at least 3 green cars in a week?

Instead of adding the probabilities of selling exactly 3, 4, 5, 6, and 7 green cars, it is easier to ☐ the probabilities of selling exactly ☐, ☐, or ☐ green cars from ☐.

P(at least 3 green cars)

$$= 1 - P(0 \text{ green cars}) - P(1 \text{ green car}) - P(2 \text{ green cars})$$

$$= 1 - C(7, 0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^7 - \boxed{}$$

$$- \boxed{}$$

$$= 1 - \frac{\boxed{}}{279{,}936} - \frac{\boxed{}}{279{,}936} - \frac{\boxed{}}{279{,}936}$$

$$= \boxed{}$$

The probability that this salesperson will sell at least three green cars in a week is $\boxed{}$ or about $\boxed{}$.

Check Your Progress **A report said that approximately 1 out of 6 cars sold in a certain year was green. Suppose a salesperson sells 7 cars per week.**

a. What is the probability that this salesperson will sell exactly 4 green cars in a week?

b. What is the probability that this salesperson will sell at least 2 green cars in a week?

HOMEWORK ASSIGNMENT

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12–10 Sampling and Error

MAIN IDEAS

- Determine whether a sample is unbiased.
- Find margins of sampling error.

KEY CONCEPT

Margin of Sampling Error If the percent of people in a sample responding in a certain way is p and the size of the sample is n, then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where $ME = 2\sqrt{\frac{p(1-p)}{n}}$.

EXAMPLE Biased and Unbiased Samples

① **State whether each method would produce a random sample. Explain.**

a. surveying people going into an action movie to find out the best kind of movie

; they will most likely think that action movies are the best kind of movie.

b. calling every 10th person on the list of subscribers to a newspaper to ask about the quality of the delivery service

; no obvious bias exists in calling every 10th subscriber.

Check Your Progress **State whether each method would produce a random sample. Explain.**

a. surveying people going into a football game to find out the most popular sport

b. surveying every fifth person going into a mall to find out the most popular kind of movie

EXAMPLE Find a Margin of Error

② **In a survey of 100 randomly selected adults, 37% answered "yes" to a particular question. What is the margin of error?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$=$ p = 37% or 0.37, n = 100

$\approx$ or 10% Use a calculator.

This means that there is a 95% chance that the percent of people in the whole population who would answer "yes"

is between 37 − 10 or ☐ % and 37 + 10 or ☐ %.

Check Your Progress In a survey of 100 randomly selected adults, 50% answered "no" to a particular question. What is the margin of error?

EXAMPLE Analyze a Margin of Error

3 **HEALTH In an earlier survey, 30% of the people surveyed said they had smoked cigarettes in the past week. The margin of error was 2%. How many people were surveyed?**

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$ Formula for margin of sampling error

$$0.02 = 2\sqrt{\frac{0.3(1-0.3)}{n}}$$ $ME = 0.02$, $p = 0.3$

☐ = ☐ Divide by 2.

☐ = ☐ Square each side.

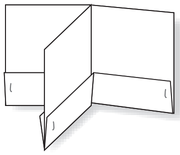
n = ☐ Multiply by n and divide by 0.0001.

n = ☐ Use a calculator.

FOLDABLES™

ORGANIZE IT

Write your own example of a biased and an unbiased survey. Place your work in the Statistics pocket.



Check Your Progress In an earlier survey, 25% of the people surveyed said they had exercised in the past week. The margin of error was 2%. How many people were surveyed?

HOMEWORK ASSIGNMENT

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CHAPTER

12

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 12 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 12, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 341–342)* to help you solve the puzzle.

12-1

The Counting Principle

A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.

1. A marble is drawn out of the jar and is not replaced. A second marble is drawn.

2. A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn.

3. A man owns two suits, ten ties, and eight shirts. How many different outfits can he wear if each is made up of a suit, a tie, and a shirt?

12-2

Permutations and Combinations

4. Indicate whether arranging five pictures in a row on a wall involves a *permutation* or a *combination*.

Evaluate each expression.

5. $P(5, 3)$

6. $C(7, 2)$

12-3

Probability

A weather forecast says that the chance of rain tomorrow is 40%.

7. Write the probability that it will rain tomorrow as a fraction in lowest terms.

8. What are the odds in favor of rain?

9. Balls are numbered 1 through 15. Find the probability that a ball drawn at random will show a number less than 4. Then find the odds that a number less than 4 is drawn.

12-4

Multiplying Probabilities

A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. A ball is drawn from the bag and is not replaced. A second ball is drawn.

10. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black.

a. $P(Y \text{ and } B) = \dfrac{P(Y)}{P(Y) + P(B)}$ **b.** $P(Y \text{ and } B) = P(Y) \cdot P(B)$

c. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$

11. Which equation shows the correct calculation of this probability?

a. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$ **b.** $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$

c. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ **d.** $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

12. A pair of dice is thrown. What is the probability that both dice show a number greater than 5?

12-5

Adding Probabilities

Marla took a quiz on this lesson that contained the following problem. Her solution is shown.

Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?

$P(\text{odd}) = \frac{13}{25} \quad P(\text{multiple of } 5) = \frac{5}{25} \text{ or } \frac{1}{5}$

$P(\text{odd or multiple of } 5) = P(\text{odd}) + P(\text{multiple of } 5)$

$$= \frac{13}{25} + \frac{5}{25} = \frac{18}{25}$$

13. Why is Marla's work incorrect?

14. Show the corrected work.

15. A card is drawn from a standard deck of 52 playing cards. What is the probability that an ace or a black card is drawn?

12-6

Statistical Measures

Consider the data set {25, 31, 49, 52, 68, 79, 105}.

16. Find the variance to the nearest tenth.

17. Find the standard deviation to the nearest tenth.

12-7 The Normal Distribution

Indicate whether each of the following statements is *true* or *false*.

18. In a continuous probability distribution, there is a finite number of possible outcomes.

19. Every normal distribution can be represented by a bell curve.

12-8 Exponential and Binomial Distribution

20. If the average dishwasher lasts 10 years, what is the probability that a randomly selected dishwasher will last 12 years?

21. A sticker company produces stickers in assorted shapes, on average 25% of all stickers produced are stars.

a. What is the probability that a sheet of 20 stickers has at least 2 stars?

b. What is the expected number of stars on a sheet of 20 stickers?

12-9 Binomial Experiments

Indicate whether each of the following is a *binomial experiment* or *not a binomial experiment*. If the experiment is not a binomial experiment, explain why.

22. A fair coin is tossed 10 times and "heads" or "tails" is recorded each time.

23. A pair of dice is thrown 5 times and the sum of the numbers that come up is recorded each time.

Find each probability if a coin is tossed four times.

24. P(exactly three heads)

25. P(exactly four heads)

12-10

Sampling and Error

26. In a survey of 200 people, 36% voted in the last presidential election. Find the margin of sampling error.

CHAPTER
12
Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 12.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 12 Practice Test on page 745 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 12 Study Guide and Review on pages 740–744 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 745.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 12 Foldable.
- Then complete the Chapter 12 Study Guide and Review on pages 740–744 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 745.

Student Signature

Parent/Guardian Signature

Teacher Signature

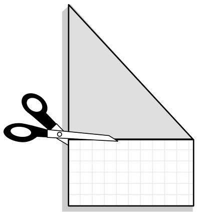
CHAPTER 13

Trigonometric Functions

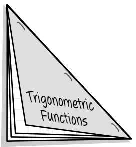
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of construction paper and two pieces of grid paper.

STEP 1 **Fold and Cut**
Stack and fold on the diagonal. Cut to form a triangular stack.



STEP 2 **Staple and Label**
Staple edge to form a booklet.



NOTE-TAKING TIP: When you take notes, include visuals. Clearly label the visuals and write captions when needed.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 13. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
angle of depression or elevation			
Arccosine function [AHRK-KOH-SYN]			
Arcsine function [AHRK-SYN]			
Arctangent function [AHRK-TAN-juhnt]			
cosecant [KOH-SEE-KANT]			
cosine			
coterminal angles			
cotangent			
Law of Cosines			

Vocabulary Term	Found on Page	Definition	Description or Example
Law of Sines			
period			
principal values			
quadrantal angles [kwah-DRAN-tuhl]			
radian [RAY-dee-uhn]			
reference angle			
secant			
sine			
standard position			
tangent			
trigonometry [TRIH-guh-NAH-muh-tree]			

13–1 Right Triangle Trigonometry

MAIN IDEAS

- Find values of trigonometric functions for acute angles.
- Solve problems involving right angles.

KEY CONCEPT

Trigonometric Functions

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\cot\theta = \frac{\text{adj}}{\text{opp}}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

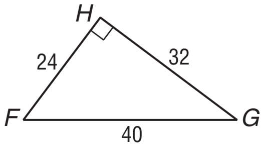
$\cot\theta = \frac{1}{\tan\theta}$

BUILD YOUR VOCABULARY (pages 376–377)

Trigonometry is the study of the relationships among the and of a right triangle.

EXAMPLES Find Trigonometric Values

1 Find the value of the six trigonometric functions for angle G.



For this triangle, the leg opposite $\angle G$ is and the leg adjacent to $\angle G$ is . The hypotenuse is .

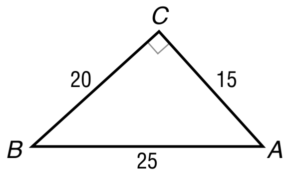
Use opp = , adj = , and hyp = to write each trigonometric ratio.

$\sin G = \frac{\text{opp}}{\text{hyp}} =$ $\cos G = \frac{\text{adj}}{\text{hyp}} =$

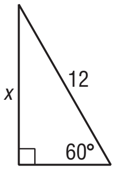
$\tan G = \frac{\text{opp}}{\text{adj}} =$ $\csc G = \frac{\text{hyp}}{\text{opp}} =$

$\sec G = \frac{\text{hyp}}{\text{adj}} =$ $\cot G = \frac{\text{adj}}{\text{opp}} =$

Check Your Progress Find the value of the six trigonometric functions for angle A.



EXAMPLE Find a Missing Side Length of a Right Triangle



1 Write an equation involving sin, cos, or tan that can be used to find the value of *x*. Then solve the equation. Round to the nearest tenth.

The measure of the hypotenuse is 12.

The side with the missing length is *opposite* the angle measuring 60°. The trigonometric function relating the opposite side of a right triangle and the hypotenuse is the sine function.

$\sin \theta = \frac{\text{opp}}{\text{hyp}}$ Sine ratio

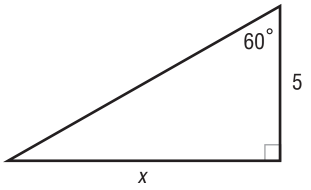
sin ____ = ____ Replace θ with 60°, *opp* with *x*, and *hyp* with 12.

____ = ____ $\sin 60° = \frac{\sqrt{3}}{2}$

____ $= x$ Multiply each side by 12.

The value of x is ____ or about ____.

Check Your Progress



Write an equation involving sin, cos, or tan that can be used to find the value of x. Then solve the equation. Round to the nearest tenth.

____

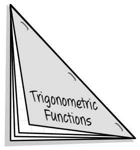
BUILD YOUR VOCABULARY (pages 376)

The angle between a ____ line and the line of sight from the observer to an object at a ____ level is called the **angle of elevation.** The angle between a ____ line and the line of sight from the observer to an object at a ____ level is called the **angle of depression.**

FOLDABLES™

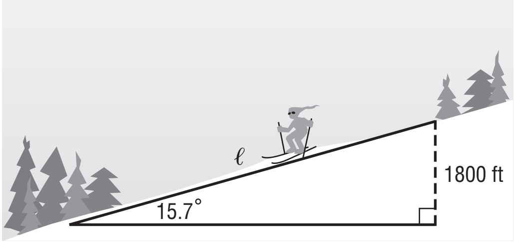
ORGANIZE IT

On the first page of your Trigonometric Functions booklet, write each of the six trigonometric ratios introduced in this lesson.



EXAMPLE Use an Angle of Elevation

① **SKIING** **A run has an angle of elevation of 15.7° and a vertical drop of 1800 feet. Estimate the length of this run.**



Let ℓ represent the length of the run. Write an equation using a trigonometric function that involves the ratio of ℓ and 1800.

$\sin 15.7^\circ =$ $\qquad$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$\ell =$ $\qquad$ Solve for ℓ.

$\ell \approx$ $\qquad$ Use a calculator.

The length of the run is about $\qquad$ feet.

Check Your Progress A run has an angle of elevation of 23° and a vertical drop of 1000 feet. Estimate the length of this run.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–2 Angles and Angle Measure

MAIN IDEAS

- Change radian measure to degree measure and vice versa.
- Identify coterminal angles.

BUILD YOUR VOCABULARY (pages 377)

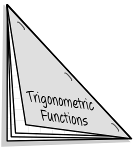
An angle positioned so that its is at the origin and its initial side is along the positive *x*-axis is said to be in **standard position**.

One **radian** is the measure of an angle θ in position whose rays intercept an arc of length 1 unit on the unit circle.

When two angles in standard position have the same sides, they are called **coterminal angles**.

FOLDABLES™ ORGANIZE IT

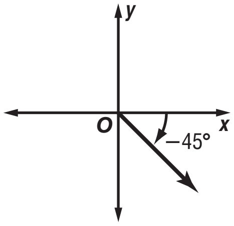
Use the second page of your Trigonometric Functions booklet. Define and give an example of each new Vocabulary Builder term from the lesson.



EXAMPLES Draw an Angle in Standard Position

① **Draw the angle −45° in standard position.**

The angle is negative. Draw the terminal side clockwise from the *x*-axis.



Check Your Progress Draw the angle 225° in standard position.

KEY CONCEPT

Radian and Degree Measure

- To rewrite the radian measure of an angle in degrees, multiply the number of radians by $\frac{180^\circ}{\pi \text{ radians}}$
- To rewrite the degree measure of an angle in radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180^\circ}$

EXAMPLE Convert Between Degree and Radian Measure

2 **Rewrite the degree measure in radians and the radian measure in degrees.**

a. 30°

$$30^\circ = 30\cancel{^\circ}\left(\frac{\pi \text{ radians}}{180\cancel{^\circ}}\right)$$

$=$ radians or

b. $-\frac{5\pi}{3}$

$$-\frac{5\pi}{3} = \left(-\frac{5\cancel{\pi}}{3}\ \cancel{\text{radians}}\right)\left(\frac{180^\circ}{\cancel{\pi}\ \cancel{\text{radians}}}\right)$$

$=$ or

Check Your Progress **Rewrite the degree measure in radians and the radian measure in degrees.**

a. 45°

b. $\frac{\pi}{6}$

EXAMPLES Find Coterminal Angles

3 **Find one angle with positive measure and one angle with negative measure coterminal with**

a. 210°

A positive angle is 210° + 360° or .

A negative angle is 210° − 360° or .

b. $\frac{7\pi}{3}$

A positive angle is $\frac{7\pi}{3} + 2\pi$ or .

A negative angle is $\frac{7\pi}{3} - 2(2\pi)$ or .

Check Your Progress **Find one angle with positive measure and one angle with negative measure coterminal with each angle.**

a. 150°

b. $\frac{\pi}{6}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–3 Trigonometric Functions of General Angles

MAIN IDEAS

- Find values of trigonometric functions for general angles.
- Use reference angles to find values of trigonometric functions.

KEY CONCEPTS

Trigonometric Functions, θ in Standard Position

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. Using the Pythagorean Theorem, the distance r from the origin to P is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

$\sin \theta = \frac{y}{r}$

$\cos \theta = \frac{x}{r}$

$\tan \theta = \frac{y}{x}, x \neq 0$

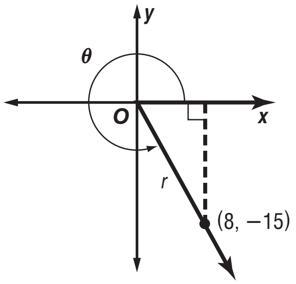
$\csc \theta = \frac{r}{y}, y \neq 0$

$\sec \theta = \frac{r}{x}, x \neq 0$

$\cot \theta = \frac{x}{r}, y \neq 0$

EXAMPLES Evaluate Trigonometric Functions for a Given Point

① **Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point (8, −15).**



From the coordinates given, you know that $x = 8$ and $y = -15$. Use the Pythagorean Theorem to find r.

$r = \sqrt{x^2 + y^2}$ Pythagorean Theorem

$=$ Replace x and y.

$=$ or Simplify.

Now use the values of x, y, and r to write the ratios.

$\sin \theta = \frac{y}{r}$

$=$ or

$\cos \theta = \frac{x}{r}$

$=$

$\tan \theta = \frac{y}{x}$

$=$ or

$\csc \theta = \frac{r}{y}$

$=$ or

$\sec \theta = \frac{r}{x}$

$=$

$\cot \theta = \frac{x}{r}$

$=$ or

Check Your Progress Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point (−3, 4).

WRITE IT

Give an example of an angle in each quadrant. Be sure to label each angle in degrees and radians.

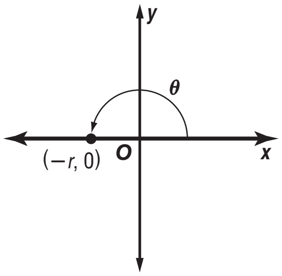
BUILD YOUR VOCABULARY (pages 377)

If the side of angle θ lies on one of the axes, θ is called a **quadrantal angle**.

If θ is a nonquadrantal angle in standard position, its **reference angle**, θ', is defined as the angle formed by the terminal side of θ and the *x*-axis.

EXAMPLE Quadrantal Angles

1 **Find the values of the six trigonometric functions for an angle in standard position that measures 180°.**



When $\theta = 180$, $x = -r$, and $y = 0$.

$\sin\theta = \frac{y}{r}$

$=$

$\cos\theta = \frac{x}{r}$

$=$

$\tan\theta = \frac{y}{x}$

$=$

$\csc\theta = \frac{r}{y}$

$=$

$\sec\theta = \frac{r}{x}$

$=$

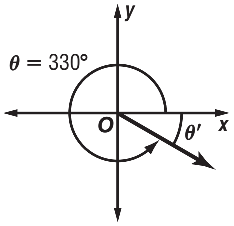
$\cot\theta = \frac{x}{y}$

$=$

Check Your Progress Find the exact values of the six trigonometric functions for an angle in standard position that measures 90°.

EXAMPLE Find the Reference Angle for a Given Angle

③ **Sketch the angle 330°. Then find its reference angle.**



Because the terminal side of 330° lies in quadrant ____, the reference angle is ____ – ____ or ____.

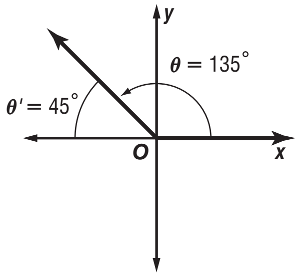
Check Your Progress **Sketch each angle. Then find its reference angle.**

a. 315°

b. $\frac{-3\pi}{4}$

EXAMPLE Use a Reference Angle to Find a Trigonometric Value

④ **Find the exact value of sin 135°.**



Because the terminal side of 135° lies in Quadrant ____, the reference angle θ' is ____ – ____ or ____.

The sine function is ____ in Quadrant ____, so sin 135° = sin 45° or ____.

Check Your Progress **Find the exact value of each trigonometric function.**

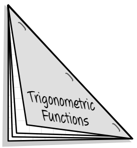
a. sin 120° ____

b. cot $\frac{11\pi}{6}$ ____

FOLDABLES™

ORGANIZE IT

On the third page of your Trigonometric Functions booklet, make a sketch of a quadrantal angle and a reference angle.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–4 Law of Sines

MAIN IDEAS

- Solve problems by using the Law of Sines.
- Determine whether a triangle has one, two, or no solutions.

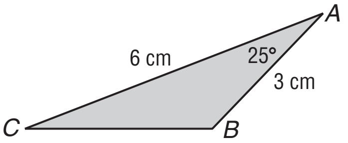
KEY CONCEPTS

Area of a Triangle The area of a triangle is one half the product of the lengths of two sides and the sine of their included angle.

Law of Sines Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measurements A, B, and C, respectively. Then, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

EXAMPLES Find the Area of a Triangle

① **Find the area of $\triangle ABC$ to the nearest tenth.**



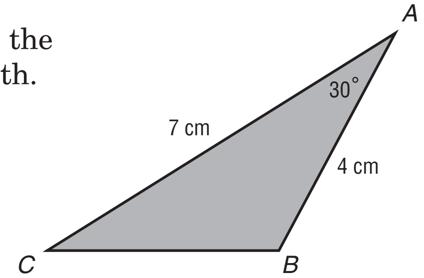
In this triangle, $b = 6$, $c = 3$, and $A = 25°$. Use the formula $A = \frac{1}{2}bc \sin A$.

Area $= \frac{1}{2}bc \sin A$ Area formula

$= \frac{1}{2}\boxed{} \sin \boxed{}$ Replace b, c, and A.

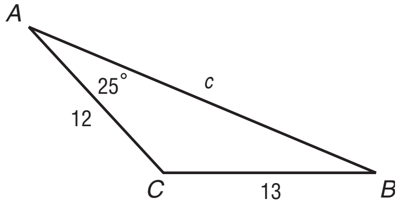
$\approx \boxed{}$ square centimeters Use a calculator.

Check Your Progress Find the area of $\triangle ABC$ to the nearest tenth.



EXAMPLE One Solution

② **In $\triangle ABC$, $A = 25°$, $a = 13$, and $b = 12$. Determine whether $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve $\triangle ABC$.**



Angle A is acute and $a > b$, so one solution exists. Make a sketch and then use the Law of Sines to find B.

$\frac{\sin B}{\boxed{}} = \frac{\sin \boxed{}}{\boxed{}}$ Law of Sines

$\sin B = \boxed{}$ Multiply by 12.

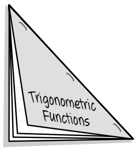
$\sin B \approx \boxed{}$ Use a calculator.

$B \approx \boxed{}$ Use the $\sin^{-1}$ function.

FOLDABLES™

ORGANIZE IT

On the fourth page of your Trigonometric Functions booklet, write the Law of Sines. Then explain what triangle measures you must know in order to apply the Law of Sines.



The measure of angle C is approximately

$180 - \boxed{\quad}$ or $\boxed{\quad}$.

Use the Law of Sines again to find c.

$$\frac{\sin \boxed{\quad}}{c} \approx \frac{\sin \boxed{\quad}}{\boxed{\quad}} \qquad \text{Law of Sines}$$

$$c \approx \boxed{\quad} \text{ or about } \boxed{\quad}$$

So, $B \approx \boxed{\quad}$, $C \approx \boxed{\quad}$, and $c \approx \boxed{\quad}$.

Check Your Progress In $\triangle ABC$, $A = 33°$, $a = 15$, and $b = 10$. Determine whether $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve $\triangle ABC$.

$\boxed{\quad}$

EXAMPLE **Two Solutions**

3 **In $\triangle ABC$, $A = 25°$, $a = 5$, and $b = 10$. Determine whether $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve $\triangle ABC$.**

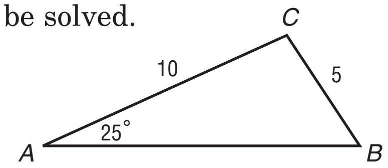
Since angle A is acute, find $b \sin A$ and compare it with a.

$$b \sin A = \boxed{\quad} \qquad \text{Replace } b \text{ and } A.$$

$$\approx \boxed{\quad} \qquad \text{Use a calculator.}$$

Since $10 > 5 > 4.23$, there are two possible solutions. Thus, there are two possible triangles to be solved.

CASE 1 Acute Angle B



First use the Law of Sines to find B.

$$\frac{\sin B}{10} = \frac{\sin 25°}{5}$$

$$\sin B = \boxed{\quad}$$

$\sin B \approx \boxed{\quad}$. So B $\approx \boxed{\quad}$.

The measure of angle C is approximately

$180 - (25 + 58)$ or .

Use the Law of Sines again to find c.

$$\frac{\sin 97^\circ}{c} \approx \frac{\sin 25^\circ}{5}$$

$c \approx$

$c \approx$

Therefore, $B \approx$, $C \approx$, and

$c \approx$.

CASE 2 Obtuse Angle B

To find B, you need to find an obtuse angle whose sin is also 0.8452. To do this, subtract the angle given by your calculator, 58°, from 180°. So B is

approximately $180 -$ or .

The measure of angle C is approximately

$180 -$ or .

Use the Law of Sines to find c.

$$\frac{\sin 33^\circ}{c} \approx \frac{\sin 25^\circ}{5}$$

$c \approx$

$c \approx$

Therefore, $B \approx$, $C \approx$, and

$c \approx$.

Check Your Progress In $\triangle ABC$, $A = 27^\circ$, $a = 12$, and $b = 20$. Determine whether $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve $\triangle ABC$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–5 Law of Cosines

MAIN IDEAS

- Solve problems by using the Law of Cosines.
- Determine whether a triangle can be solved by first using the Law of Sines or the Law of Cosines.

KEY CONCEPTS

Law of Cosines Let △*ABC* be any triangle with *a*, *b*, and *c* representing the measures of sides, and opposite angles with measures *A*, *B*, and *C*, respectively. Then the following equations are true.

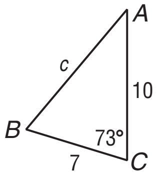
$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

EXAMPLES Solve a Triangle Given Two Sides and Included Angle

① **Solve △ABC.**



Use the Law of Cosines to find c.

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = \boxed{} + \boxed{} - \boxed{}$

$c^2 \approx \boxed{}$ Simplify using a calculator.

$c \approx \boxed{}$ Take the square root of each side.

Next, use the Law of Sines to find the measure of angle A.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ Law of Sines

$\frac{\sin A}{\boxed{}} \approx \frac{\sin 73°}{\boxed{}}$ $a = 7$, $C = 73°$, and $c \approx 10.4$

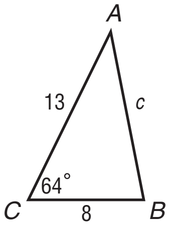
$\sin A \approx \boxed{}$ Multiply each side by 7.

$\sin A \approx \boxed{}$ Use a calculator.

$A \approx \boxed{}$ Use the $\sin^{-1}$ function.

The measure of angle B is approximately $180 - \boxed{}$ or $\boxed{}$. So, $c \approx \boxed{}$, $A \approx \boxed{}$, and $B \approx \boxed{}$.

Check Your Progress Solve △*ABC*.

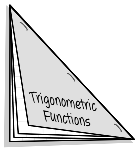


$\boxed{}$

FOLDABLES™

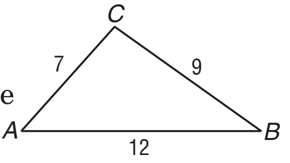
ORGANIZE IT

On the fifth page of your Trigonometric Functions booklet, write the Law of Cosines. Then explain what triangle measures you must know in order to apply the Law of Cosines.



EXAMPLE Solve a Triangle Given Three Sides

1 **Solve △*ABC*.**



You are given the measures of three sides. Use the Law of Cosines to find the measure of the largest angle first, angle *C*.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\boxed{\quad} = \boxed{\quad} + \boxed{\quad} - \boxed{\qquad\qquad}$$

$$\boxed{\quad} - 9^2 - \boxed{\quad} = \boxed{\qquad\qquad}$$

$$\frac{144 - 81 - 49}{\boxed{\quad}} = \boxed{\quad}$$

$$-0.1111 \approx \cos C$$

$$C \approx \boxed{\quad}$$

Use the Law of Sines to find the measure of angle *B*.

$\frac{\sin B}{b} = \frac{\sin C}{c}$	Law of Sines
$\frac{\sin B}{\boxed{\quad}} \approx \frac{\sin \boxed{\quad}}{\boxed{\quad}}$	$b = 7$, $C \approx 96°$, and $c = 12$
$\sin B \approx \boxed{\qquad}$	Multiply each side by 7.
$\sin B \approx 0.5801$	Use a calculator.
$B \approx \boxed{\quad}$	Use the $\sin^{-1}$ function.

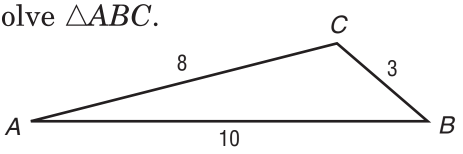
The angle of measure *A* is approximately

$180 - \boxed{\qquad}$ or $\boxed{\quad}$. So, $A \approx \boxed{\quad}$, $B \approx \boxed{\quad}$,

and $C \approx \boxed{\quad}$.

Check Your Progress Solve △*ABC*.

$\boxed{\qquad\qquad}$



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–6 Circular Functions

MAIN IDEAS

- Define and use the trigonometric functions based on the unit circle.
- Find the exact values of trigonometric functions of angles.

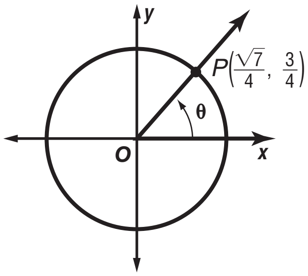
KEY CONCEPTS

Definition of Sine and Cosine If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$. Therefore, the coordinates of P can be written as $P(\cos \theta, \sin \theta)$

Periodic Function A function is called periodic if there is a number a such that $f(x) = f(x + a)$ for all x in the domain of the function. The least positive value of a for which $f(x) = f(x + a)$ is called the period of the function.

EXAMPLES Find Sine and Cosine Given Point on Unit Circle

1 **Given an angle θ in standard position, if $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$ lies on the terminal side of θ and on the unit circle, find $\sin \theta$ and $\cos \theta$.**



$P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right) =$

$\sin \theta =$ and $\cos \theta =$

Check Your Progress Given an angle θ in standard position, if $P\left(\frac{\sqrt{15}}{4}, -\frac{1}{4}\right)$ lies on the terminal side of θ and on the unit circle, find $\sin \theta$ and $\cos \theta$.

BUILD YOUR VOCABULARY (pages 377)

Every 360° or 2π radians, the sine and cosine functions their values. Therefore, these functions are **periodic**, each having a **period** of or radians.

EXAMPLE Find the Value of a Trigonometric Function

1 **Find the exact value of each function.**

a. cos 690°

$$\cos 690° = \cos\left(\boxed{} + \boxed{}\right)$$

$$= \cos \boxed{}$$

$$= \boxed{}$$

b. sin $\left(-\frac{3\pi}{4}\right)$

$$\sin\left(-\frac{3\pi}{4}\right) = \sin\left(-\frac{3\pi}{4}\right) + \boxed{}$$

$$= \sin\left(\boxed{}\right)$$

$$= \boxed{}$$

Check Your Progress **Find the exact value of each function.**

a. cos 405°

b. sin $\left(-\frac{\pi}{2}\right)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–7 Inverse Trigonometric Function

MAIN IDEAS

- Solve equations by using inverse trigonometric functions.
- Find values of expressions involving trigonometric functions.

BUILD YOUR VOCABULARY (pages 376–377)

The domains of trigonometric functions must be restricted so that their are functions. The values in these restricted domains are called **principal values**.

The inverse of the Sine function is called the **Arcsine function** and is symbolized by or .

The definitions of the **Arccosine** and **Arctangent functions** are similar to the definition of the Arcsine function.

KEY CONCEPT

Principal Values of Sine, Cosine, and Tangent

- $y = \text{Sin } x$ if and only if $y = \sin x$ and $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- $y = \text{Cos } x$ if and only if $y = \cos x$ and $0 \leq x \leq \pi$
- $y = \text{Tan } x$ if and only if $y = \tan x$ and $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

EXAMPLES Solve an Equation

① **Solve $\text{Sin } x = \frac{\sqrt{2}}{2}$ by finding the value of x to the nearest degree.**

If $\text{Sin } x = \frac{\sqrt{2}}{2}$, the x is the least value whose sine is .

So, $x =$.

Use a calculator to find x.

Keystrokes:

2nd [SIN^{-1}] 2nd [√] 2) ÷ 2) ENTER

The value of x is .

Check Your Progress Solve $\text{Cos } x = \frac{\sqrt{3}}{2}$ by finding the value of x to the nearest degree.

EXAMPLE Find a Trigonometric Value

① **Find each value. Write the angle measure in radians. Round to the nearest hundredth.**

a. **Arcsin** $\frac{\sqrt{2}}{2}$

Keystrokes:

2nd [SIN^{-1}] 2nd [$\sqrt{\ }$] 2) ÷ 2) ENTER

Arcsin $\frac{\sqrt{2}}{2} \approx$ radian

b. **tan** $\left(\text{Cos}^{-1}\frac{4}{5}\right)$

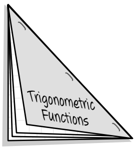
Keystrokes:

2nd 2nd [COS^{-1}] 4 ÷ 5) ENTER

$\tan\left(\cos^{-1}\frac{4}{5}\right) =$ radian

FOLDABLES™ ORGANIZE IT

On the sixth page of your Trigonometric Functions booklet, write the inverse of sine, cosine, and tangent. Then explain when you would use these inverses.



Check Your Progress **Find each value. Write the angle measures in radians. Round to the nearest hundredth.**

a. Arcsin $\left(\frac{\sqrt{3}}{2}\right)$

b. tan $\left(\text{Cos}^{-1}\ \frac{2}{3}\right)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

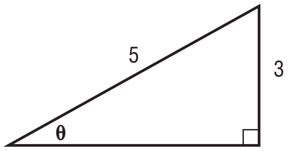
13 BRINGING IT ALL TOGETHER

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 13 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 13, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 376-377)* to help you solve the puzzle.

13-1

Right Triangle Trigonometry

1. Find the values of the six trigonometric functions for angle θ.



13-2

Angles and Angle Measure

Match each degree measure with the corresponding radian measure on the right.

2. 30°	**a.** $\frac{2\pi}{3}$
3. 90°	**b.** $\frac{\pi}{2}$
4. 120°	**c.** π
5. 135°	**d.** $\frac{\pi}{6}$
6. 180°	**e.** $\frac{3\pi}{4}$

13-3

Trigonometric Functions of General Angles

7. Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(1, -2)$.

13-4

Law of Sines

Determine whether $\triangle ABC$ has *no solution*, *one solution*, or *two solutions*. Do not try to solve the triangle.

8. $a = 20, A = 30°, B = 70°$

9. $c = 12, A = 100°, a = 30$

10. $C = 27°, b = 23.5, c = 17.5$

13-5

Law of Cosines

Suppose that you are asked to solve $\triangle ABC$ given the following information about the sides and angles of the triangle. In each case, indicate whether you would begin by using the *Law of Sines* or the *Law of Cosines*.

11. $a = 8, b = 7, c = 6$

12. $b = 9.5, A = 72°, B = 39°$

13. $C = 123°, b = 22.95, a = 34.35$

14. Solve $\triangle ABC$ for $a = 4$, $b = 5$, and $c = 8$.

13-6

Circular Functions

Tell whether each function is periodic. Write *yes* or *no*.

15. $y = 2x$

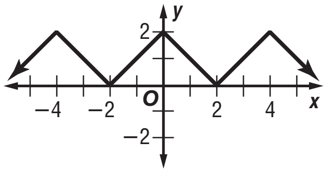
16. $y = x^2$

17. $y = \cos x$

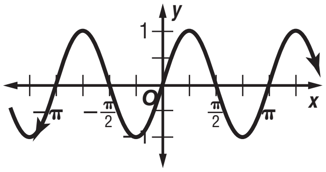
18. $y = |x|$

Find the period of each function by examining its graph.

19.



20.



Find the exact value of each function.

21. $\sin 765°$

22. $\cos\left(-\frac{9\pi}{2}\right)$

13-7

Inverse Trigonometric Functions

Indicate whether each statement is *true* or *false*.

23. The domain of the function $y = \sin x$ is the set of all real numbers.

24. The domain of the function $y = \text{Cos } x$ is $0 \leq x \leq 2\pi$.

25. The range of the function $y = \text{Tan } x$ is $-1 \leq y \leq 1$.

Find each value. Round to the nearest hundredth, if necessary.

26. $\text{Cos}^{-1}\frac{1}{2}$

27. $\sin\left(\text{Tan}^{-1}\frac{3}{2}\right)$

CHAPTER 13

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 13.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 13 Practice Test on page 817 of your textbook as a final check.

☐ **I used my Foldables or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 13 Study Guide and Review on pages 812–816 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 13 Practice Test on page 817.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 13 Foldables.
- Then complete the Chapter 13 Study Guide and Review on pages 812–816 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 13 Practice Test on page 817.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 14

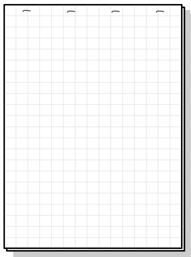
Trigonometric Graphs and Identities

FOLDABLES™

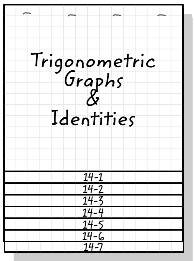
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with eight sheets of grid paper.

STEP 1 **Staple**
Staple the stack of grid paper along the top to form a booklet.



STEP 2 **Cut and Label**
Cut seven lines from the bottom of the top sheet, six lines from the second sheet, and so on. Label with lesson numbers as shown.



NOTE-TAKING TIP: When you take notes, write instructions on how to do something presented in each lesson. Then follow your own instructions to check them for accuracy.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 14. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
amplitude [AM-pluh-TOOD]			
double-angle formula			
half-angle formula			
midline			
phase shift [FAYZ]			
trigonometric equation			
trigonometric identity			
vertical shift			

14–1 Graphing Trigonometric Functions

MAIN IDEAS

- Graph trigonometric functions.
- Find the amplitude and period of variation of the sine, cosine, and tangent functions.

KEY CONCEPT

Amplitudes and Periods
For functions of the form $y = a \sin b\theta$ and $y = a \cos b\theta$, the amplitude is $|a|$, and the period is $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$.
For functions of the form $y = a \tan b\theta$, the amplitude is not defined, and the period is $\frac{180°}{|b|}$ or $\frac{\pi}{b}$.

BUILD YOUR VOCABULARY (page 400)

The **amplitude** of the graph of a periodic function is the value of half the difference between its value and its value.

Graph Trigonometric Functions

Find the amplitude and period of each function. Then graph the function.

a. $y = \sin \frac{1}{3}\theta$

First, find the amplitude.

$|a| =$

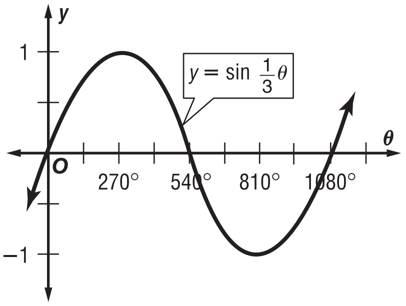
The coefficient of $\sin \frac{1}{3}\theta$ is 1.

Next, find the period.

$\frac{360°}{|b|} =$ $\quad b = \frac{1}{3}$

$= 1080°$ or 6π

Use the amplitude and period to graph the function.



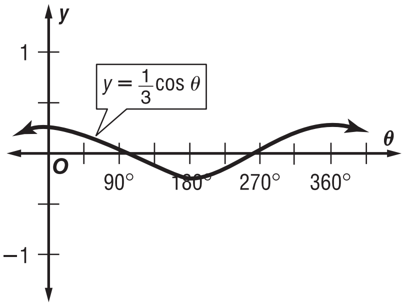
b. $y = \frac{1}{3} \cos \theta$

Amplitude: $|a| =$

$=$

Period: $\frac{360°}{|b|} =$

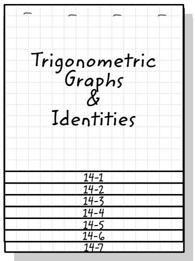
$=$



FOLDABLES™

ORGANIZE IT

On the page for Lesson 14-1, describe three real-world situations that fluctuate in a regular, periodic pattern.



Check Your Progress **Find the amplitude and period for each function. Then graph the function.**

a. $y = \sin \frac{1}{2}\theta$

b. $y = \frac{1}{2}\cos \theta$

c. $y = 3 \sin \frac{1}{2}\theta$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

14–2 Translations of Trigonometric Graphs

MAIN IDEAS

- Graph horizontal translations of trigonometric graphs and find phase shifts.
- Graph vertical translations of trigonometric graphs.

BUILD YOUR VOCABULARY (page 400)

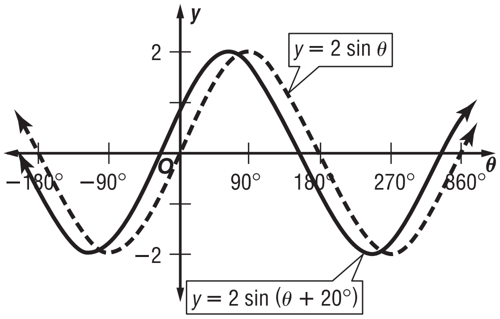
A horizontal translation of a trigonometric function is called a **phase shift.**

EXAMPLE Graph Horizontal Translations

① **State the amplitude, period, and phase shift of $y = 2 \sin (\theta + 20°)$. Then graph the function.**

Since $a =$ and $b =$, the amplitude and period of the function are the same as $y = 2 \cos \theta$, 360°. However, $h =$, so the phase shift is . Because $h < 0$ the parent graph is shifted to the .

To graph $y = 2 \sin (\theta + 20°)$, consider the graph of $y = 2 \sin \theta$. Graph this function and then shift the graph to the .



KEY CONCEPT

Phase Shift The phase shift of the functions $y = a \sin b(\theta - h) + k$, $y = a \cos b(\theta - h) + k$, and $y = a \tan b(\theta - h) + k$ is h, where $b > 0$.

- If $h > 0$, the shift is to the right.
- If $h < 0$, the shift is to the left.

Check Your Progress State the amplitude, period, and phase shift of $y = \frac{1}{4} \cos \left(\theta - \frac{\pi}{4}\right)$. Then graph the function.

BUILD YOUR VOCABULARY (page 400)

Graphs of trigonometric functions can be translated through a **vertical shift**.

A new axis called the **midline** becomes the reference line about which the graph .

KEY CONCEPT

Vertical Shift
The vertical shift of the functions
$y = a \sin b(\theta - h) + k$,
$y = a \cos b(\theta - h) + k$,
and
$y = a \tan b(\theta - h) + k$
is k.

- If $k > 0$, the shift is up.
- If $k < 0$, the shift is down.
- The midline is $y = k$.

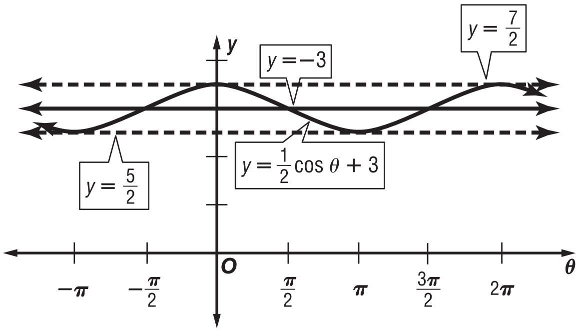
EXAMPLE Graph Vertical Translations

① **State the vertical shift, equation of the midline, amplitude, and period of $y = \frac{1}{2} \cos \theta + 3$. Then graph the function.**

Vertical shift:

$k =$, so the midline is the graph of

$y =$.



Amplitude: $|a| =$

Period: $\frac{2\pi}{|b|} =$

Since the amplitude of the function is , draw dashed lines parallel to the midline that are unit above and below the midline. Then draw the cosine curve.

REMEMBER IT

You may find it easier to graph the parent graph in one color, and then each transformation in a different color.

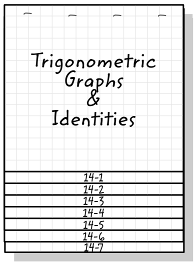
Check Your Progress State the vertical shift, equation of the midline, amplitude, and period of $y = 3 \sin \theta - 2$. Then graph the function.

EXAMPLE Graph Transformations

FOLDABLES™

ORGANIZE IT

Use the page for Lesson 14-2. Sketch a phase shift and a vertical shift for the sine, cosine, and tangent functions.



③ **State the vertical shift, amplitude, period, and phase shift of $y = 3 \sin \left[2\left(\theta - \frac{\pi}{2}\right)\right] + 4$. Then graph the function.**

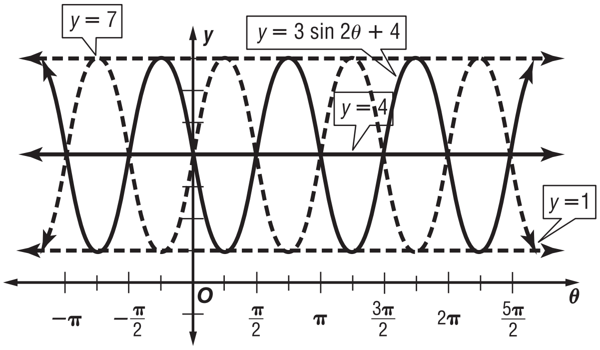
The function is written in the form $y = a \cos[b(\theta - h) + k]$. Identify the values of k, a, b, and h.

Step 1 The vertical shift is ____. Graph the midline $y =$ ____.

Step 2 The amplitude is ____. Draw dashed lines ____ units above and below the midline at $y =$ ____ and $y =$ ____.

Step 3 The period is ____, so the graph is ____. Graph $y = 3 \sin 2\theta + 4$ using the midline as a reference.

Step 4 Shift the graph ____ to the ____.



Check Your Progress State the vertical shift, amplitude, period, and phase shift of $y = 2 \cos\left[3\left(\theta - \frac{\pi}{4}\right)\right] - 2$. Then graph the function.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

14–3 Trigonometric Identities

MAIN IDEAS

- Use identities to find trigonometric values.
- Use trigonometric identities to simplify expressions.

KEY CONCEPT

Basic Trigonometric Identities

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

BUILD YOUR VOCABULARY (page 400)

A **trigonometric identity** is an equation involving trigonometric   that is true for   values for which every expression in the equation is defined.

EXAMPLE Find a Value of a Trigonometric Function

① **Find tan θ if sec $\theta = -2$ and $180° < \theta < 270°$.**

$\tan^2\theta + 1 = \sec^2\theta$	Trigonometric identity
$\tan^2\theta =$	Subtract 1 from each side.
$\tan^2\theta =$	Substitute -2 for sec θ.
$\tan^2\theta =$	Square -2.
$\tan^2\theta =$	Subtract.
$\tan\theta =$	Take the square root of each side.

Since θ is in the third quadrant, tan θ is   .

Thus, $\tan\theta =$.

Check Your Progress **Find the value of each expression.**

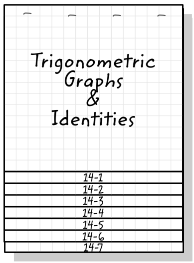
a. $\cos\theta$ if $\sin\theta = \frac{1}{3}$ and $0° < \theta < 90°$

b. $\sec \theta$ if $\tan \theta = -2$, and $\frac{3\pi}{2} < \theta < 2\pi$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 14-3, write the Quotient, Reciprocal, and Pythagorean Identities.



EXAMPLE Simplify an Expression

1 **Simplify $\sin \theta(\csc \theta - \sin \theta)$.**

$\sin \theta(\csc \theta - \sin \theta)$

$= \sin \theta$ $\qquad$ $\csc \theta = \left(\frac{1}{\sin \theta}\right)$

$= \sin \theta$ $\qquad - \sin \theta$

Distributive Property

$=$ $\qquad$ Simplify.

$=$ $\qquad$ $1 - \sin^2 \theta = \cos^2 \theta$

Check Your Progress Simplify $\tan \theta \cot \theta$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

14–4 Verifying Trigonometric Identities

MAIN IDEAS

- Verify trigonometric identities by transforming one side of an equation into the form of the other side.
- Verify trigonometric identities by transforming each side of the equation into the same form.

EXAMPLE Transform One Side of an Equation

① **Verify that csc θ cos θ tan θ = 1 is an identity.**

Transform the left side.

$\csc \theta \cos \theta \tan \theta \overset{?}{=} 1$	Original equation
$\square \cdot \square \cdot \square \overset{?}{=} 1$	$\csc \theta = \frac{1}{\sin \theta}$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$
$\square = 1$	Simplify.

EXAMPLE Verify by Transforming Both Sides

② **Verify that csc θ + sec θ = $\frac{1 + \cot \theta}{\cos \theta}$ is an identity.**

$\csc \theta + \sec \theta \overset{?}{=} \frac{1 + \cot \theta}{\cos \theta}$	Original equation
$\square + \square \overset{?}{=} \square$	Express all terms using sine and cosine.
$\square \overset{?}{=} \square$	Rewrite using the LCD, sin θ cos θ.
$\square = \square$	Simplify the right side.

WRITE IT

Why do you include a question mark above the equality sign when verifying an identity?

____

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Check Your Progress

a. Verify that $\csc^2 \theta \tan^2 \theta = \sec^2 \theta$ is an identity.

b. Verify that $1 + \frac{1}{\cos \theta} = \frac{\tan^2 \theta}{\sec \theta - 1}$ is an identity.

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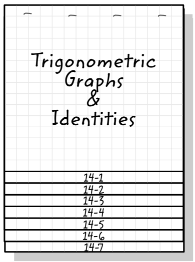
ORGANIZE IT

On the page for Lesson 14-4, write your own checklist for verifying trigonometric identities.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

14–5 Sum and Difference of Angles Formulas

MAIN IDEAS

- Find values of sine and cosine involving sum and difference formulas.
- Verify identities by using sum and difference formulas.

KEY CONCEPT

Sum and Difference of Angles Formulas The following identities hold true for all values of α and β.

$\cos(\alpha \pm \beta) =$
$\cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin(\alpha \pm \beta) =$
$\sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

REMEMBER IT

The symbols α and β are lowercase Greek letters that stand for *alpha* and *beta*.

EXAMPLE Use Sum and Difference of Angles Formulas

① **Find the exact value of each expression.**

a. sin 75°

Use the formula $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

$$\begin{aligned}\sin 75^\circ &= \sin(30^\circ + 45^\circ) \qquad \alpha = 30^\circ;\ \beta = 45^\circ\\ &= \sin \boxed{} \cos \boxed{} + \cos \boxed{} \sin \boxed{}\\ &= \boxed{} \cdot \boxed{} + \boxed{} \cdot \boxed{}\\ &= \boxed{} + \boxed{}\\ &= \boxed{}\end{aligned}$$

b. cos (−75°)

Use the formula $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

$$\begin{aligned}\cos(-75^\circ) &= \cos(60^\circ - 135^\circ) \qquad \alpha = 60^\circ;\ \beta = 135^\circ\\ &= \cos 60^\circ \cos 135^\circ + \sin 60^\circ \sin 135^\circ\\ &= \boxed{} \cdot \left(-\frac{\sqrt{2}}{2}\right) + \boxed{} \cdot \frac{\sqrt{2}}{2}\\ &= \boxed{} + \boxed{}\\ &= \boxed{}\end{aligned}$$

Check Your Progress **Find the exact value of each expression.**

a. sin 105°

$\boxed{}$

b. cos (–120°)

$\boxed{}$

EXAMPLE Verify Identities

① **Verify that each of the following is an identity.**

a. $\mathbf{\cos (90° - \theta) = \sin \theta}$

$\cos (90° - \theta) \overset{?}{=} \sin \theta$

$\overset{?}{=} \sin \theta$

$\overset{?}{=} \sin \theta$

$= \sin \theta$

b. $\mathbf{\cos (180° - \theta) = -\cos \theta}$

$\overset{?}{=} -\cos \theta$

$\overset{?}{=} -\cos \theta$

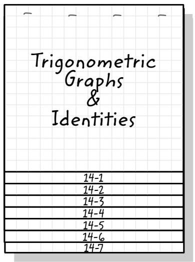
$\overset{?}{=} -\cos \theta$

$= -\cos \theta$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 14-5, write the Sum and Difference of Angles Formulas. Then explain the difference between the ∓ sign used in the cosine formula and the ± sign used in the sine formula.



Check Your Progress **Verify that each of the following is an identity.**

a. $\cos (90° + \theta) = -\sin \theta$

b. $\sin (180° + \theta) = -\sin \theta$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

14–6 Double-Angle and Half-Angle Formulas

MAIN IDEAS

- Find values of sine and cosine involving double-angle formulas.
- Find values of sine and cosine involving half-angle formulas.

KEY CONCEPT

Double-Angles Formulas
The following identities hold true for all values of θ.

$\sin 2\theta = 2 \sin \theta \cos \theta$
$\cos 2\theta = \cos^2 \theta \; 2 \sin^2 \theta$
$\cos 2\theta = 1 - 2 \sin^2 \theta$
$\cos 2\theta = 2 \cos^2 \theta - 1$

EXAMPLE Double-Angle Formulas

① **Find the exact value of sin 2θ if sin $\theta = \frac{3}{4}$ and θ is between 0° and 90°.**

Use the identity $\sin 2\theta = 2 \sin \theta \cos \theta$.
First find the value of $\cos \theta$.

$\cos^2 \theta =$ $\cos^2 \theta + \sin^2 \theta = 1$

$\cos^2 \theta =$ $\sin \theta = \frac{3}{4}$

$\cos^2 \theta =$ Subtract.

$\cos \theta =$ Find the square root of each side.

Since θ is in the first quadrant, cosine is .

Thus, $\cos \theta =$.

Now find $\sin 2\theta$.

$\sin 2\theta =$ Double-angle formula

$=$ $\sin \theta = \frac{3}{4}$, $\cos \theta = \frac{\sqrt{7}}{4}$

$=$ Simplify.

Check Your Progress **Find the value of each expression if sin $\theta = \frac{1}{4}$ and θ is between 0° and 90°.**

a. $\sin 2\theta$

b. $\cos 2\theta$

KEY CONCEPT

Half-Angle Formulas
The following identities hold true for all values of α.

$$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$

EXAMPLE Half-Angle Formulas

1 Find $\cos \frac{\alpha}{2}$ if $\sin \alpha = \frac{4}{5}$ and α is in the second quadrant.

Since $\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$, we must find $\cos \alpha$ first.

$\cos^2 \alpha =$ $\qquad$ $\sin^2 \alpha + \cos^2 \alpha = 1$

$\cos^2 \alpha =$ $\qquad$ $\sin \alpha = \frac{4}{5}$

$\cos^2 \alpha =$ $\qquad$ Simplify.

$\cos \alpha =$ $\qquad$ Take the square root of each side.

Since α is in the second quadrant, $\cos \alpha =$.

$\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$ $\qquad$ Half-angle formula

$= \pm\sqrt{\frac{1 + \left(-\frac{3}{5}\right)}{2}}$ $\qquad$ $\cos \alpha = -\frac{3}{5}$

$=$ $\qquad$ Simplify the radicand.

$=$ $\qquad$ Rationalize.

$=$ $\qquad$ Multiply.

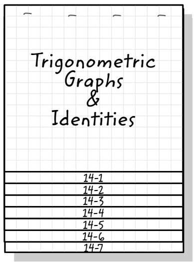
Since α is between 90° and 180°, $\frac{\alpha}{2}$ is between and . Thus, $\cos \frac{\alpha}{2}$ is and equals .

Check Your Progress Find $\cos \frac{\alpha}{2}$ if $\sin \alpha = -\frac{9}{41}$ and α is in the fourth quadrant.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 14-6, write the Double-Angle and Half-Angle Formulas. Explain how you can use the terminal side of an angle to help determine the sign of the function.



EXAMPLE Evaluate Using Half-Angle Formulas

1 **Find the exact value of $\cos \frac{9\pi}{8}$ by using the half-angle formulas.**

$\cos \frac{9\pi}{8} =$

$=$ $\qquad$ $\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos \alpha}{2}}$

$=$ $\qquad$ $\cos \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$

$=$ $\qquad$ Simplify the radicand.

$=$ $\qquad$ Simplify the denominator.

Since $\frac{9\pi}{8}$ is in the third quadrant, $\cos \frac{9\pi}{8}$ is .

Thus, $\cos \frac{9\pi}{8} =$.

Check Your Progress **Find the exact value of each expression by using the half-angle formulas.**

a. sin 67.5°

b. $\cos \frac{3\pi}{8}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

14–7 Solving Trigonometric Equations

EXAMPLE Solve Equations for a Given Interval

MAIN IDEAS

- Solve trigonometric equations.
- Use trigonometric equations to solve real-world problems.

① **Find all solutions of $2\cos^2\theta - 1 = \sin\theta$ for the interval $0° < \theta \leq 360°$.**

$2\cos^2\theta - 1 = \sin\theta$ Original equation

$2(1 - \sin^2\theta) - 1 - \sin\theta = 0$ Solve for 0.

$\boxed{\qquad\qquad} = 0$ Distributive Property

$\boxed{\qquad\qquad} = 0$ Simplify.

$2\sin^2\theta + \boxed{\quad} - \boxed{\quad} = 0$ Divide each side by -1.

$\boxed{\qquad\qquad} = 0$ Factor.

Now use the Zero Product Property.

$2\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$

$2\sin\theta = \boxed{\quad}$ $\sin\theta = \boxed{\quad}$

$\sin\theta = \boxed{\quad}$ $\sin\theta = \boxed{\quad}$

$\theta = \boxed{\qquad}$ $\theta = \boxed{\quad}$

Check Your Progress **Find all solutions of each equation for the given interval.**

a. $\sin^2\theta + \cos 2\theta - \cos\theta = 0;\ 0° \leq \theta < 360°$

$\boxed{\qquad\qquad\qquad\qquad}$

b. $\cos\theta + 1 = 0;\ 0 < \theta \leq 2\pi$

$\boxed{\qquad\qquad\qquad\qquad}$

REVIEW IT

Solve $x^2 - x - 6 = 0$ by factoring. *(Lesson 6-3)*

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EXAMPLE Solve Trigonometric Equations

② **Solve $2 \sin \theta \cos \theta = \cos \theta$ for all values of θ if θ is measured in radians.**

$2 \sin \theta \cos \theta = \cos \theta$ Original equation

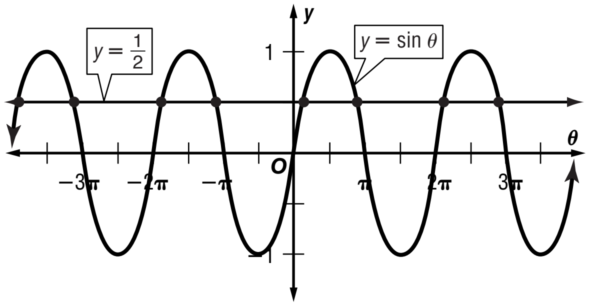
____ $= 0$ Subtract cos θ.

____ $= 0$ Factor.

____ $= 0$ or ____ $=$ 0 Zero Product Property

____ $=$ ____ Solve.

Look at the graph of $y = \sin \theta$ to find solutions of $\sin \theta = \frac{1}{2}$.



The solutions are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$, and so on, and $-\frac{7\pi}{6}, -\frac{11\pi}{6}, -\frac{19\pi}{6}, -\frac{23\pi}{6}$, and so on. The only solutions in the interval 0 to 2π are ____ and ____. The period of the sine function is ____ radians. So, the solutions can be written as ____ and ____, where k is any integer. Similarly, the solutions for $\cos \theta = 0$ are ____ and ____.

Check Your Progress

a. Solve $\cos \theta + 1 = 0$ for all values of θ if θ is measured in radians.

b. Solve $\sin^2 \theta - 1 = \cos^2 \theta$ for all values of θ if θ is measured in degrees.

EXAMPLE Solve Trigonometric Equations Using Identities

3 **Solve $\sin \theta \cot \theta = \cos^2 \theta$.**

$\sin \theta \cot \theta = \cos^2 \theta$ Original equation

$= 0$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$

$= 0$ Multiply.

$= 0$ Factor.

$= 0$ or $= 0$

$\theta =$ $=$

$\theta =$

Check your solutions. θ equals 0° is extraneous.

The solution is .

Check Your Progress Solve $\sin \theta = \cos \theta$.

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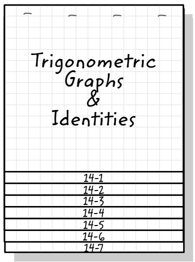
ORGANIZE IT

On the page for Lesson 14-7, describe the similarities and differences in solving the equations $3x + 5 = 0$ and $3 \tan \theta + 5 = 0$.



EXAMPLE Determine Whether a Solution Exists

4 **Solve $\sin^2 \theta = \frac{1}{4} + \cos \theta$.**

$\sin^2 \theta = \frac{1}{4} + \cos \theta$ Original equation

$1 - \cos^2 = \frac{1}{4} + \cos \theta$ $\sin^2 \theta = 1 - \cos^2 \theta$

$0 =$ Subtract. Then add.

$0 =$ Multiply.

$0 =$ Factor.

$= 0$ or $= 0$

$\cos \theta =$ $\cos \theta =$

No solutions exist. $\theta =$

The solutions are and .

Check Your Progress Solve $\sin 2\theta - \sin \theta = 0$.

HOMEWORK ASSIGNMENT

Page(s):

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14 BRINGING IT ALL TOGETHER

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 14 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 14, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 400)* to help you solve the puzzle.

14-1

Graphing Trigonometric Functions

1. Find the amplitude, if it exists, and period of $y = 2 \sin \frac{1}{4}x$. Then graph the function.

Determine whether each statement is *true* or *false*.

2. The period of a function is the distance between the maximum and minimum points.

3. The amplitude of the function $y = \sin \theta$ is 2π.

14-2

Translations of Trigonometric Graphs

Determine whether the graph of each function has an *amplitude change, period change, phase shift,* or *vertical shift* compared to the graph of the parent function. (More than one of these may apply to each function. Do not actually graph the functions.)

4. $y = 3 \sin\left(\frac{\theta + 8\pi}{6}\right)$

5. $y = \cos(2\theta + 70°)$

6. $y = -4 \cos 3\theta$

7. $y = \sec \frac{1}{2}\theta - 3$

14-3

Trigonometric Identities

8. Find the value of sin θ, if $\cos \theta = \frac{1}{2}$ and $270° < \theta < 360°$.

Match each expression from the list on the left with an expression from the list on the right that is equal to it for all values for which each expression is defined.

9. $\sec^2 \theta - \tan^2 \theta$

10. $\frac{\sin \theta}{\cos \theta}$

11. $\csc \theta$

12. $\frac{\cos \theta}{\sin \theta}$

a. $\frac{1}{\sin \theta}$

b. $\tan \theta$

c. 1

d. $\cot \theta$

14-4

Verifying Trigonometric Identities

Determine whether each equation is an *identity* or *not an identity*.

13. $\frac{1}{\sin^2 \theta} - \frac{1}{\tan^2 \theta} = 1$

14. $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \cos \theta \sin \theta$

15. $\frac{\sin^2 \theta}{\cos^2 \theta} + 1 = \sec^2 \theta$

16. $\tan^2 \theta \cos^2 \theta = \frac{1}{\csc^2 \theta}$

17. Verify that $\csc \theta(\cos \theta + \sin \theta) = \cot \theta + 1$.

14-5

Sum and Difference of Angles Formulas

Find the exact value of each expression.

18. sin 165°

19. cos (–210°)

14-6

Double-Angle and Half-Angle Formulas

Find the exact value for each.

20. $\cos \frac{\theta}{2}$, if $\cos \theta = \frac{2}{3}$; $270° < \theta < 360°$

21. $\sin \theta$, if $\cos 2\theta = \frac{4}{5}$; $90° < \theta < 180°$

Match each expression from the list on the left with *all* expressions from the list on the right that are equal to it for all values of β.

22. $\sin \frac{\beta}{2}$

23. $\cos 2\beta$

24. $\cos \frac{\beta}{2}$

25. $\sin 2\beta$

a. $2 \sin \beta \cos \beta$

b. $1 - 2 \sin^2 \beta$

c. $\cos^2 \beta - \sin^2 \beta$

d. $\pm\sqrt{\frac{1 + \cos \beta}{2}}$

e. $\pm\sqrt{\frac{1 - \cos \beta}{2}}$

14-7

Solving Trigonometric Equations

26. Find all solutions for $2 \cot \theta = -2$; $0° \leq \theta < 360°$.

27. Identify which equations have no solution.

a. $\sin \theta = 1$

b. $\tan \theta = 0.001$

c. $\sec \theta = \frac{1}{2}$

d. $\csc \theta = -3$

e. $\cos \theta = 1.01$

f. $\cos \theta + 2 = -1$

CHAPTER

14

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 14.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 14 Practice Test on page 871 of your textbook as a final check.

☐ **I used my Foldables or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 14 Study Guide and Review on pages 867–870 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 14 Practice Test on page 871 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 14 Foldables.
- Then complete the Chapter 14 Study Guide and Review on pages 867–870 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 14 Practice Test on page 871 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature