

Glencoe Mathematics

TEXAS

Algebra 2

Noteables™
Interactive Study Notebook
with **FOLDABLES™**

Contributing Author

Dinah Zike

FOLDABLES™

Consultant

Douglas Fisher, Ph.D.
Director of Professional Development
San Diego, CA



Glencoe



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Algebra 2 (Texas Student Edition)
Noteables™: Interactive Study Notebook with Foldables™

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Organizing Your Foldables

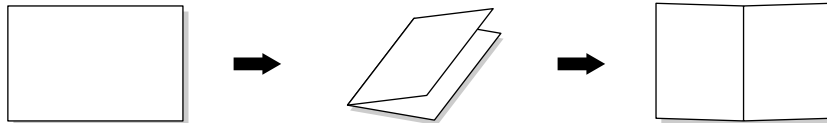


Make this Foldable to help you organize and store your chapter Foldables. Begin with one sheet of 11" × 17" paper.

STEP 1

Fold

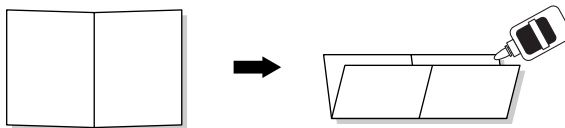
Fold the paper in half lengthwise. Then unfold.



STEP 2

Fold and Glue

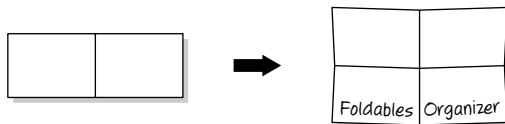
Fold the paper in half widthwise and glue all of the edges.



STEP 3

Glue and Label

Glue the left, right, and bottom edges of the Foldable to the inside back cover of your Noteables notebook.



Reading and Taking Notes As you read and study each chapter, record notes in your chapter Foldable. Then store your chapter Foldables inside this Foldable organizer.

Using Your Noteables™ with FOLDABLES® Interactive Study Notebook


This note-taking guide is designed to help you succeed in *Algebra 2*. Each chapter includes:

CHAPTER 5 Quadratic Functions and Inequalities

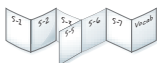
FOLDABLES Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" by 17" paper.

STEP 1 Fold in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.



STEP 2 Staple along the lengthwise fold and staple the uncut section at the top. Label each section with a lesson number and close to form a booklet.



NOTE-TAKING TIP: When you take notes, you may wish to use a highlighting marker to emphasize important concepts.

The Chapter Opener contains instructions and illustrations on how to make a Foldable that will help you to organize your notes.

A Note-Taking Tip provides a helpful hint you can use when taking notes.

The Build Your Vocabulary table allows you to write definitions and examples of important vocabulary terms together in one convenient place.

CHAPTER 5 BUILD YOUR VOCABULARY

This is an alphabetical list of the new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
axis of symmetry			
completing the square			
complex conjugates			
complex number			
constant term			
discriminant dih-s-KRIH-muh-nu			
imaginary unit			
linear term			
maximum value			
minimum value			

(continued on the next page)

Glencoe Algebra 2 121

Chapter 5

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Within each chapter, Build Your Vocabulary boxes will remind you to fill in this table.

5-7 Analyzing Graphs of Quadratic Functions

BUILD YOUR VOCABULARY (pages 121–122)

A function written in the form, $y = (x - h)^2 + k$, where (h, k) is the of the parabola and $x = h$ is its is referred to as the vertex form.

EXAMPLE Graph a Quadratic Function in Vertex Form

1 Analyze $y = (x - 3)^2 + 2$. Then draw its graph.

The vertex is at (h, k) or and the axis of symmetry is $x = \input{type="text"}$. The graph has the same shape as the graph of $y = x^2$, but is translated 3 units right and 2 units up.

Now use this information to draw the graph.

Step 1 Plot the vertex, .

Step 2 Draw the axis of symmetry, .

Step 3 Find and plot two points on one side of the axis of symmetry, such as $(2, 3)$ and $(1, 6)$.

Step 4 Use symmetry to complete the graph.

FOLDABLES

ORGANIZE IT

On the page for Lesson 5-7, sketch a graph of a parabola. Then sketch the graph of the parabola after a vertical translation and a horizontal translation.

Each lesson is correlated to the Texas Essential Knowledge and Skills.

Lessons cover the content of the lessons in your textbook. As your teacher discusses each example, follow along and complete the fill-in boxes. Take notes as appropriate.

Examples parallel the examples in your textbook.

Foldables feature reminds you to take notes in your Foldable.

CHAPTER 5 BRINGING IT ALL TOGETHER

STUDY GUIDE

<p>FOLDABLES</p> <p>Use your Chapter 5 Foldable to help you study for your chapter test.</p>	<p>VOCABULARY PUZZLEMAKER</p> <p>To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to: glencoe.com</p>	<p>BUILD YOUR VOCABULARY</p> <p>You can use your Vocabulary Builder (pages 121–122) to solve the puzzle.</p>
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5-1 Graphing Quadratic Functions

Refer to the graph at the right as you complete the following sentences.

- The curve is called a .
- The line $x = -2$ is called the .
- The point $(-2, 4)$ is called the .

Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.

- $f(x) = -x^2 + 2x + 5$
- $f(x) = 3x^2 - 4x - 2$

5-2 Solving Quadratic Equations by Graphing

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

- $x^2 - 2x = 8$
- $x^2 + 5x - 7 = 0$

5-8

EXAMPLE Solve a Quadratic Inequality

1 Solve $x^2 + x \leq 2$ algebraically.

First, solve the related equation $x^2 + x = 2$.

$$x^2 + x = 2 \quad \text{Related quadratic equation}$$

$$\input{type="text"} = 0 \quad \text{Subtract 2 from each side.}$$

$$\input{type="text"} = 0 \quad \text{Factor.}$$

$$\input{type="text"} = 0 \text{ or } \input{type="text"} = 0 \quad \text{Zero Product Property}$$

$$x = \input{type="text"} \quad x = \input{type="text"} \quad \text{Solve each equation.}$$

Plot the values on a number line. Use closed circles since these solutions are included. Note that the number line is separated into 3 intervals.

Test a value in each interval to see if it satisfies the original inequality.

$x \leq -2$	$-2 \leq x \leq 1$	$x \geq 1$
Test $x = -3$.	Test $x = 0$.	Test $x = 2$.
$x^2 + x \leq 2$	$x^2 + x \leq 2$	$x^2 + x \leq 2$
$(-3)^2 - 3 \leq 2$	$0^2 + 0 \leq 2$	$2^2 + 2 \leq 2$
$6 \leq 2$ X	$0 \leq 2$ ✓	$6 \leq 2$ X

The solution set is .

Check Your Progress Exercises allow you to solve similar exercises on your own.

Bringing It All Together Study Guide reviews the main ideas and key concepts from each lesson.

NOTE-TAKING TIPS

Your notes are a reminder of what you learned in class. Taking good notes can help you succeed in mathematics. The following tips will help you take better classroom notes.

- Before class, ask what your teacher will be discussing in class. Review mentally what you already know about the concept.
- Be an active listener. Focus on what your teacher is saying. Listen for important concepts. Pay attention to words, examples, and/or diagrams your teacher emphasizes.
- Write your notes as clear and concise as possible. The following symbols and abbreviations may be helpful in your note-taking.

Word or Phrase	Symbol or Abbreviation	Word or Phrase	Symbol or Abbreviation
for example	e.g.	not equal	\neq
such as	i.e.	approximately	\approx
with	w/	therefore	\therefore
without	w/o	versus	vs
and	+	angle	\angle

- Use a symbol such as a star (★) or an asterisk (*) to emphasize important concepts. Place a question mark (?) next to anything that you do not understand.
- Ask questions and participate in class discussion.
- Draw and label pictures or diagrams to help clarify a concept.
- When working out an example, write what you are doing to solve the problem next to each step. Be sure to use your own words.
- Review your notes as soon as possible after class. During this time, organize and summarize new concepts and clarify misunderstandings.

Note-Taking Don'ts

- **Don't** write every word. Concentrate on the main ideas and concepts.
- **Don't** use someone else's notes as they may not make sense.
- **Don't** doodle. It distracts you from listening actively.
- **Don't** lose focus or you will become lost in your note-taking.

Solving Equations and Inequalities

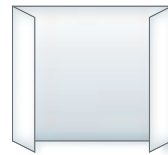


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of notebook paper.

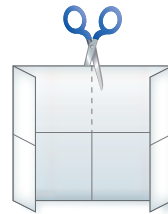
STEP 1

Fold 2" tabs on each of the short sides.



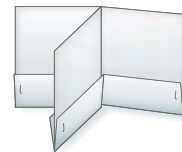
STEP 2

Then **fold** in half in both directions and cut as shown.



STEP 3

Refold along the width. Staple each pocket. Label pockets as *Algebraic Expressions*, *Properties of Real Numbers*, *Solving Equations and Absolute Value Equations*, and *Solve and Graph Inequalities*.



NOTE-TAKING TIP: When you take notes, it is often a good idea to use symbols to emphasize important concepts.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 1. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
absolute value			
algebraic expression			
<u>coefficient</u> KOH-uh-FIH-shuhnt			
compound inequality			
constant			
degree			
empty set			
equation			
formula			
intersection			
irrational numbers			

Vocabulary Term	Found on Page	Definition	Description or Example
like terms			
monomial			
open sentence			
order of operations			
polynomial			
power			
rational numbers			
real numbers			
set-builder notation			
solution			
term			
trinomial			
union			
variable			

Expressions and Formulas



Reinforcement of TEKS A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. **(A)** Use symbols to represent unknowns and variables.

BUILD YOUR VOCABULARY (page 2)

Expressions that contain at least one variable are called **algebraic expressions**.

MAIN IDEAS

- Use the order of operations to evaluate expressions.
- Use formulas.

EXAMPLES Evaluate Algebraic Expressions

1 a. Evaluate $(x - y)^3 + 3$ if $x = 1$ and $y = 4$.

$$\begin{aligned} (x - y)^3 + 3 &= (1 - 4)^3 + 3 && x = \boxed{} \text{ and } y = \boxed{}. \\ &= \boxed{} + 3 && \text{Find } -3^3. \\ &= \boxed{} && \text{Add } -27 \text{ and } 3. \end{aligned}$$

b. Evaluate $s - t(s^2 - t)$ if $s = 2$ and $t = 3.4$.

$$\begin{aligned} s - t(s^2 - t) &= 2 - 3.4(2^2 - 3.4) && \text{Replace } s \text{ with } 2 \text{ and } t \text{ with } 3.4. \\ &= 2 - 3.4(4 - 3.4) && \text{Find } 2^2. \\ &= 2 - 3.4(\boxed{}) && \text{Subtract } 3.4 \text{ from } 4. \\ &= 2 - \boxed{} && \text{Multiply } 3.4 \text{ and } 0.6. \\ &= \boxed{} && \text{Subtract } 2.04 \text{ from } 2. \end{aligned}$$

c. Evaluate $\frac{8xy + z^3}{y^2 + 5}$ if $x = 5$, $y = -2$, and $z = -1$.

$$\begin{aligned} \frac{8xy + z^3}{y^2 + 5} &= \frac{8(5)(-2) + (-1)^3}{(-2)^2 + 5} && x = 5, y = -2, \text{ and } z = -1 \\ &= \frac{\boxed{}(-2) + (-1)}{\boxed{} + 5} && \text{Evaluate the numerator and the denominator separately.} \\ &= \frac{\boxed{} - 1}{4 + 5} && \text{Multiply } 40 \text{ by } -2. \\ &= \frac{-81}{9} \text{ or } \boxed{} && \text{Simplify the numerator and the denominator. Then divide.} \end{aligned}$$

KEY CONCEPT

Order of Operations

1. Evaluate expressions inside grouping symbols, such as parentheses, (), brackets, [], braces, { }, and fraction bars, as in $\frac{5+7}{2}$.
2. Evaluate all powers.
3. Do all multiplications and/or divisions from left to right.
4. Do all additions and/or subtractions from left to right.



Write the order of operations in the Algebraic Expressions pocket of your Foldable.

WRITE IT

Why is it important to follow the order of operations when evaluating expressions?

Check Your Progress

a. Evaluate $x - y^2(x + 5)$ if $x = 2$ and $y = 4$.

b. Evaluate $\frac{3ab + c^2}{b - c}$ if $a = 5$, $b = 2$, and $c = 4$.

EXAMPLE Use a Formula

2 Find the area of a trapezoid with base lengths of 13 meters and 25 meters and a height of 8 meters.

$$A = \frac{1}{2}h(b_1 + b_2)$$

Area of a trapezoid

$$= \frac{1}{2}(8)(13 + 25)$$

Replace h with 8, b_1 with 13, and b_2 with 25.

$$= \boxed{}$$

Add 13 and 25.

$$= \boxed{}$$

Multiply 8 by $\frac{1}{2}$.

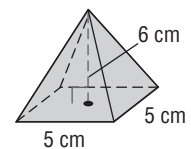
$$= \boxed{}$$

Multiply 4 and 38.

The area of the trapezoid is $\boxed{}$ square meters.

Check Your Progress

The formula for the volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid. Find the volume of the pyramid shown.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE

Classify Numbers

MAIN IDEAS

- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

KEY CONCEPTS

Real Numbers

Rational Numbers A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal.

Irrational Numbers A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.



Reinforcement of TEKS A.4

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(B)** Use the commutative, associative, and distributive properties to simplify algebraic expressions.

- 1 Name the sets of numbers to which each number belongs.

a. $\sqrt{6}$

$\sqrt{6}$ lies between 2 and 3 so it is not a whole number.

b. 5

c. $-\frac{2}{3}$

d. -43

e. -23.3

Check Your Progress

- Name the sets of numbers to which each number belongs.

a. $\frac{3}{5}$

b. $-2.\overline{52}$

c. $\sqrt{5}$

d. $\sqrt{121}$

EXAMPLE Identify Properties of Real Numbers

- 2 Name the property illustrated by the equation $(-8 + 8) + 15 = 0 + 15$.

The Property says that a number plus its opposite is 0.

Check Your Progress Name the property illustrated by each equation.

a. $3 + 0 = 3$

b. $5 \cdot \frac{1}{5} = 1$

KEY CONCEPT**Real Number Properties**

For any real numbers a , b , and c :

Commutative

$$a + b = b + a \text{ and}$$

$$a \cdot b = b \cdot a$$

Associative

$$(a + b) + c = a + (b + c)$$

$$\text{and } (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Identity

$$a + 0 = a = 0 + a \text{ and}$$

$$a \cdot 1 = a = 1 \cdot a$$

Inverse

$$a + (-a) = 0 = (-a) + a$$

$$\text{If } a \neq 0, \text{ then } a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a.$$

Distributive

$$a(b + c) = ab + ac \text{ and}$$

$$(b + c)a = ba + ca$$

EXAMPLE Additive and Multiplicative Inverses

- 3 Identify the additive and multiplicative inverse of -7 .

Since $-7 + 7 = 0$, the additive inverse is . Since

$(-7)\left(-\frac{1}{7}\right) = 1$, the multiplicative inverse is .

Check Your Progress Identify the additive inverse and multiplicative inverse for each number.

a. 5

b. $-\frac{2}{3}$

EXAMPLE Use the Distributive Property to Solve a Problem

- 4 **POSTAGE** Audrey went to the post office and bought eight 39-cent stamps and eight 24-cent postcard stamps. How much did Audrey spend altogether on stamps?

To find the total amount spent on stamps, multiply the price of each type of stamp by 8 and then add.

$$S = 8(0.39) + 8(0.24)$$

$$= \text{} + \text{} \text{ or } \text{}$$

So, Audrey spent stamps.

Check Your Progress

Joel went to the grocery store and bought 3 plain chocolate candy bars for \$0.69 each and 3 chocolate-peanut butter candy bars for \$0.79 each. How much did Joel spend altogether on candy bars?

EXAMPLE**Simplify an Expression****5** Simplify $4(3a - b) + 2(b + 3a)$.

$$\begin{aligned}
 &4(3a - b) + 2(b + 3a) \\
 &= 4(\boxed{}) - 4(\boxed{}) + 2(\boxed{}) + 2(\boxed{}) \quad \text{Distributive Property} \\
 &= 12a - 4b + 2b + 6a \quad \text{Multiply.} \\
 &= 12a + 6a - 4b + 2b \quad \boxed{} \\
 &\quad \text{Property (+)} \\
 &= (\boxed{})a + (\boxed{})b \quad \text{Distributive Property} \\
 &= 18a - 2b \quad \text{Simplify.}
 \end{aligned}$$

Check Your Progress

Simplify $2(3x - y) + 4(2x + 3y)$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Translate verbal expressions into algebraic expressions and equations, and vice versa.
- Solve equations using the properties of equality.



Reinforcement of TEKS A.7

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems. **(B)** Investigate methods for solving linear equations and inequalities using concrete models, the properties of equality, select a method, and solve the equations and inequalities.

EXAMPLE

Verbal to Algebraic Expression

1 Write an algebraic expression to represent

a. 7 less than a number

b. the square of a number decreased by the product of 5 and the number

Check Your Progress

Write an algebraic expression to represent each verbal expression.

a. 2 less than the cube of a number

b. 10 decreased by the product of a number and 2

BUILD YOUR VOCABULARY (page 3)

A mathematical sentence containing one or more

is called an open sentence.

EXAMPLE

Algebraic to Verbal Sentence

2 Write a verbal sentence to represent each equation.

a. $6 = -5 + x$ is equal to plus a number.

b. $7y - 2 = 19$ times a number minus

is

Check Your Progress

Write a verbal sentence to represent each equation.

a. $5 = 2 + x$

b. $3a + 2 = 11$

EXAMPLE Identify Properties of Equality**KEY CONCEPT**

Properties of Equality

Reflexive For any real number a , $a = a$.

Symmetric For all real numbers a and b , if $a = b$, then $b = a$.

Transitive For all real numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.

Substitution If $a = b$, then a may be replaced by b and b may be replaced by a .

3 Name the property illustrated by each statement.

a. $a - 2.03 = a - 2.03$

b. If $9 = x$, then $x = 9$.

Check Your Progress Name the property illustrated by each statement.

a. If $x + 4 = 3$, then $3 = x + 4$.

b. If $3 = x$ and $x = y$, then $3 = y$.

EXAMPLE Solve One-Step Equations

4 Solve each equation.

a. $s - 5.48 = 0.02$

$$s - 5.48 = 0.02 \quad \text{Original equation}$$

$$s - 5.48 + \boxed{} = 0.02 + \boxed{} \quad \text{Add 5.48 to each side.}$$

$$s = \boxed{} \quad \text{Simplify.}$$

b. $18 = \frac{1}{2}t$

$$18 = \frac{1}{2}t \quad \text{Original equation}$$

$$\boxed{} 18 = \frac{1}{2}t \boxed{} \quad \text{Multiply each side by the multiplicative inverse of } \frac{1}{2}.$$

$$\boxed{} = t \quad \text{Simplify.}$$

Check Your Progress Solve each equation.

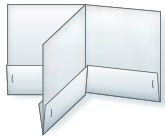
a. $x + 5 = 3$

b. $\frac{2}{3}x = 10$

FOLDABLES™

ORGANIZE IT

Write a multi-step equation in the Equations column of your Foldable. Then solve the equation, justifying each step with one of the properties you have learned.



EXAMPLE Solve a Multi-Step Equation

5 Solve $53 = 3(y - 2) - 2(3y - 1)$.

$$53 = 3(y - 2) - 2(3y - 1) \quad \text{Original equation}$$

$$53 = 3y - 6 - 6y + 2 \quad \text{Distributive and Substitution Properties}$$

$$53 = \boxed{} \quad \text{Commutative, Distributive, and Substitution Properties}$$

$$\boxed{} = \boxed{} \quad \text{Addition and Substitution Properties}$$

$$\boxed{} = y \quad \text{Division and Substitution Properties}$$

Check Your Progress Solve $25 = 3(2x + 2) - 5(2x + 1)$.

EXAMPLE Solve for a Variable

- 6 **GEOMETRY** The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, where A is the area, b_1 is the length of the base, b_2 is the length of the other base, and h is the height of the trapezoid. Solve the formula for h .

$$A = \frac{1}{2}(b_1 + b_2)h \quad \text{Area of a trapezoid}$$

$$\boxed{} A = \boxed{} \cdot \frac{1}{2}(b_1 + b_2)h \quad \text{Multiply each side by 2.}$$

$$2A = (b_1 + b_2)h \quad \text{Simplify.}$$

$$\frac{2A}{\boxed{}} = \frac{(b_1 + b_2)h}{\boxed{}} \quad \text{Divide each side by } (b_1 + b_2).$$

$$\boxed{} = h \quad \text{Simplify.}$$

EXAMPLE Apply Properties of Equality

- 7 **TEST EXAMPLE** If $4g + 5 = \frac{4}{9}$, what is the value of $4g - 2$?

A $-\frac{41}{36}$

B $-\frac{59}{9}$

C $-\frac{41}{9}$

D $-\frac{67}{7}$

You are asked to find the value of the expression .

Your first thought might be to find the value of and then evaluate the expression using this value. Notice that you are *not* required to find the value of g . Instead, you can

use the .

Solve the Test Item.

$$4g + 5 = \frac{4}{9}$$

Original equation

$$4g + 5 - \text{} = \frac{4}{9} - \text{}$$

Subtract from each side.

$$4g - 2 = -\frac{59}{9}$$

$$\frac{4}{9} - 7 = \frac{4}{9} - \frac{63}{9} = \frac{-59}{9}$$

The answer is .

EXAMPLE Write an Equation

- 8 HOME IMPROVEMENT** Carl wants to replace 5 windows in his home. His neighbor Will is a carpenter and he has agreed to help install them for \$250. If Carl has budgeted \$1000 for the total cost, what is the maximum amount he can spend on each window?

Let c represent the cost of each window. Write and solve an equation to find the value of c .

Number of windows	times	cost of each window	plus	installation	equals	total cost.
5	·	c	+	250	=	1000

$$5c + 250 = 1000$$

Original equation

$$5c + 250 - \text{} = 1000 - \text{}$$

Subtract from each side.

$$5c = 750$$

Simplify.

$$c = \text{}$$

Divide each side by 5.

Carl can afford to spend on each window.

Check Your Progress

Kelly wants to repair the siding on her house. Her contractor will charge her \$300 plus \$1.50 per square foot of siding. How much siding can she repair for \$1500?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Evaluate expressions involving absolute values.
- Solve absolute value equations.



Reinforcement of TEKS A.7

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems. **(B)** Investigate methods for solving linear equations and inequalities using concrete models, the properties of equality, select a method, and solve the equations and inequalities.

REVIEW IT

What is the difference between an algebraic expression and an equation?
(Lessons 1-1, 1-3)

BUILD YOUR VOCABULARY (page 2)

The absolute value of a number is its distance from on the number line.

The solution set for an equation that has no solution is the empty set, symbolized by or .

EXAMPLE Evaluate an Expression with Absolute Value

1 Evaluate $2.7 + |6 - 2x|$ if $x = 4$.

$$2.7 + |6 - 2x| = 2.7 + |6 - 2(4)|$$

Replace x with 4.

$$= \text{[]}$$

Simplify $-2(4)$ first.

$$= \text{[]}$$

Subtract 8 from 6.

$$= \text{[]}$$

$$|-2| = 2$$

$$= \text{[]}$$

Add.

Check Your Progress

Evaluate $2.3 - |3y - 10|$

if $y = -2$.

EXAMPLE Solve an Absolute Value Equation

2 Solve $|y + 3| = 8$.

Case 1 $a = b$

$$y + 3 = 8$$

$$y + 3 - \text{[]} = 8 - \text{[]}$$

$$y = \text{[]}$$

Case 2 $a = -b$

$$y + 3 = -8$$

$$y + 3 - \text{[]} = -8 - \text{[]}$$

$$y = \text{[]}$$

The solutions are .

Thus, the solution set is .

EXAMPLE No Solution**REMEMBER IT**

Determine if your solutions are reasonable before checking them. In Example 3, there is no need to consider the two cases because absolute value is never negative.

3 Solve $|6 - 4t| + 5 = 0$.

$$|6 - 4t| + 5 = 0$$

Original equation

$$|6 - 4t| = \boxed{}$$

Subtract $\boxed{}$ from each side.

This sentence is *never* true. So, the solution set is $\boxed{}$.

EXAMPLE One Solution

4 Solve $18 + |y| = 2y - 3$. Check your solutions.

Case 1 $a = b$

$$8 + y = 2y - 3$$

$$\boxed{} = \boxed{}$$

$$\boxed{} = y$$

Case 2 $a = -b$

$$8 + y = -(2y - 3)$$

$$8 + y = -2y + 3$$

$$\boxed{} = 3$$

$$\boxed{} = \boxed{}$$

$$y = \boxed{}$$

There appear to be two solutions.

Check:

$$|8 + y| = 2y - 3$$

$$18 + \boxed{} = 2(\boxed{}) - 3$$

$$\boxed{} \stackrel{?}{=} 19$$

$$19 = \boxed{}$$

$$|8 + y| = 2y - 3$$

$$\left| 8 + \boxed{} \right| \stackrel{?}{=} 2(\boxed{}) - 3$$

$$\left| \frac{19}{13} \right| \stackrel{?}{=} \boxed{} - 3$$

$$\frac{19}{13} = \boxed{}$$

Since $\frac{19}{3} \neq -\frac{19}{3}$, the only solution is $\boxed{}$.

Check Your Progress

Solve each equation. Check your solutions.

a. $|2x + 5| = 15$

$$\boxed{}$$

b. $\left| 5x - \frac{2}{3} \right| + 7 = 0$

$$\boxed{}$$

c. $|3x - 5| = -4x + 37$

$$\boxed{}$$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1-5 Solving Inequalities

BUILD YOUR VOCABULARY (page 3)

MAIN IDEAS

- Solve inequalities.
- Solve real-world problems involving inequalities.



Reinforcement of TEKS A.7

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems. **(B)** Investigate methods for solving linear equations and inequalities using concrete models, the properties of equality, select a method, and solve the equations and inequalities.

The Trichotomy Property says that, for any two real numbers, a and b , exactly of the following statements is true.

$$a < b \qquad a = b \qquad a > b$$

The of an inequality can be expressed by using **set-builder notation**, for example, $\{x \mid x > 9\}$.

EXAMPLE Solve an Inequality Using Addition or Subtraction

- 1** Solve $4y - 3 < 5y + 2$. Graph the solution set on a number line.

$$4y - 3 < 5y + 2 \qquad \text{Original inequality}$$

$$4y - 3 - \text{} < 5y + 2 - \text{} \qquad \text{Subtract } 4y \text{ from each side.}$$

$$\text{} < \text{} \qquad \text{Simplify.}$$

$$\text{} < \text{} \qquad \text{Subtract from each side.}$$

$$\text{} < y \text{ or } y > \text{} \qquad \text{Simplify.}$$

Any real number greater than -5 is a solution of this inequality.



A circle means that this point is not included in the solution set.

Check Your Progress

Solve $6x - 2 < 5x + 7$. Graph the solution on a number line.

KEY CONCEPTS

Properties of Inequality

Addition Property of Inequality For any real numbers a , b , and c :

If $a > b$, then
 $a + c > b + c$.

If $a < b$, then
 $a + c < b + c$.

Subtraction Property of Inequality For any real numbers a , b , and c :

If $a > b$, then
 $a - c > b - c$.

If $a < b$, then
 $a - c < b - c$.

EXAMPLE

Solve an Inequality Using Multiplication or Division

KEY CONCEPTS

Properties of Inequality

Multiplication Property of Inequality For any real numbers a , b , and c , where c is positive:

if $a > b$, then $ac > bc$.

if $a < b$, then $ac < bc$.

c is negative:

if $a > b$, then $ac < bc$.

if $a < b$, then $ac > bc$.

Division Property of Inequality For any real numbers a , b , and c , where c is positive:

if $a > b$, then $\frac{a}{c} > \frac{b}{c}$.

if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

c is negative:

if $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

- 2 Solve $12 \geq -0.3p$. Graph the solution set on a number line.

$$12 \geq -0.3p$$

$$\frac{12}{\boxed{}} \leq \frac{-0.3p}{\boxed{}}$$

$$\boxed{} \leq p$$

$$p \geq \boxed{}$$

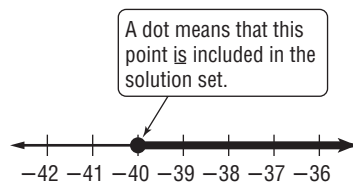
Original inequality

Divide each side by $\boxed{}$, reversing the inequality symbol.

Simplify.

Rewrite with p first.

The solution set is $\{p \mid p \geq -40\}$.



EXAMPLE

Solve a Multi-Step Inequality

- 3 Solve $-x > \frac{x-7}{2}$. Graph the solution set on a number line.

$$-x > \frac{x-7}{2}$$

$$-2x > x-7$$

$$\boxed{} > \boxed{}$$

$$x < \boxed{}$$

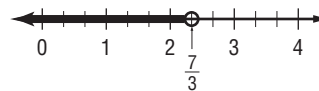
Original inequality

Multiply each side by 2.

Add $-x$ to each side.

Divide each side by -3 , reversing the inequality symbol.

The solution set is $(-\infty, \frac{7}{3})$ and is graphed below.



REMEMBER IT

The symbol ∞ represents *infinity*.



Check Your Progress

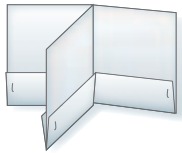
Solve each inequality. Graph each solution on a number line.

a. $-3x \geq 21$

b. $-2x > \frac{x+5}{3}$

FOLDABLES™
ORGANIZE IT

Write the Properties of Inequality in your notes.


EXAMPLE Write an Inequality

- 4 CONSUMER COSTS** Alida has at most \$15.00 to spend at a convenience store. She buys a bag of potato chips and a can of soda for \$1.59. If gasoline at this store costs \$2.89 per gallon, how many gallons of gasoline can Alida buy for her car, to the nearest tenth of a gallon?

Let g = the gallons of gasoline Alida can buy for her car.
Write and solve an inequality.

$$1.59 + 2.89g \leq 15.00$$

Original inequality

$$1.59 + 2.89g + \boxed{} \leq 15.00 - \boxed{}$$

Subtract from each side.

$$\boxed{} \leq \boxed{}$$

Simplify.

$$\boxed{} \leq \boxed{}$$

Divide.

$$g \leq \boxed{}$$

Simplify.

Alida can buy up to $\boxed{}$ gallons of gasoline for her car.

Check Your Progress

Jeb wants to rent a car for his vacation. Value Cars rents cars for \$25 per day plus \$0.25 per mile. How far can he drive for one day if he wants to spend no more than \$200 on car rental?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Solving Compound and Absolute Value Inequalities

BUILD YOUR VOCABULARY (page 2)

MAIN IDEAS

- Solve compound inequalities.
- Solve absolute value inequalities.

A compound inequality consists of two inequalities joined by the word or the word .

The graph of a compound inequality containing *and* is the **intersection** of the solution sets of the two inequalities.

EXAMPLE

Solve an “and” Compound Inequality

1 Solve $10 \leq 3y - 2 < 19$. Graph the solution set.

KEY CONCEPT

“And” Compound Inequalities

A compound inequality containing the word *and* is true if and only if *both* inequalities are true.



Write this concept in your notes.

METHOD 1 Write the compound inequality using the word *and*. Then solve each inequality.

$$10 \leq 3y - 2 \quad \text{and} \quad 3y - 2 < 19$$

$$\boxed{} \leq 3y \qquad 3y < \boxed{}$$

$$\boxed{} \leq y \qquad y < \boxed{}$$

$$\boxed{} \leq y < \boxed{}$$

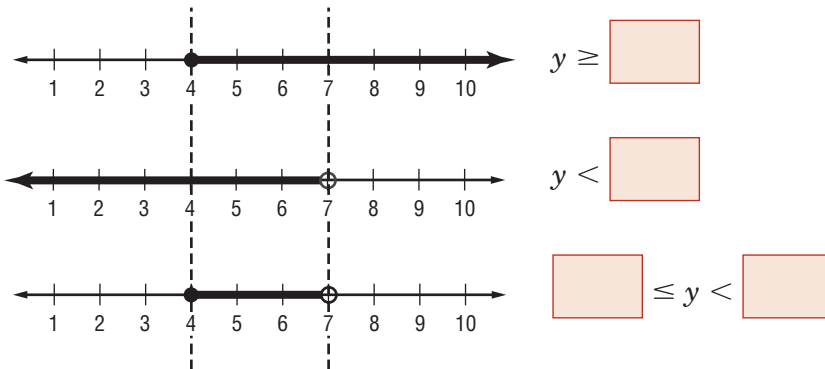
METHOD 2 Solve both parts at the same time by adding 2 to each part. Then divide each part by 3.

$$\boxed{} \leq 3y - 2 < \boxed{}$$

$$\boxed{} \leq 3y < \boxed{}$$

$$\boxed{} \leq y < \boxed{}$$

Graph each solution set. Then find their intersection.



The solution set is $\{y | 4 \leq y < 7\}$.



Reinforcement of TEKS A.7

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems. **(B)** Investigate methods for solving linear equations and inequalities using concrete models, the properties of equality, select a method, and solve the equations and inequalities.

Check Your Progress Solve $11 \leq 2x + 5 < 17$. Graph the solution set.

BUILD YOUR VOCABULARY (page 3)

The graph of a compound inequality containing is the **union** of the solution sets of the two inequalities.

KEY CONCEPT

"Or" Compound Inequalities

A compound inequality containing the word *or* is true if one or more of the inequalities is true.

FOLDABLES Write this concept in your notes.

EXAMPLE Solve an "or" Compound Inequality

2 Solve $x + 3 < 2$ or $-x \leq -4$. Graph the solution set.

Solve each inequality separately.

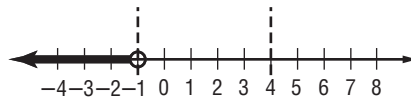
$x + 3 < 2$

or

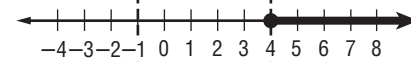
$-x \leq -4$

$x <$

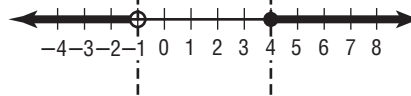
$x \geq$



$x <$



$x \geq$



$x <$ or $x \geq$

The solution set is $\{x \mid x < -1 \text{ or } x \geq 4\}$.

Check Your Progress Solve $x + 5 < 1$ or $-2x \leq -6$. Graph the solution set.

REMEMBER IT



Compound inequalities containing *and* are *conjunctions*. Compound inequalities containing *or* are *disjunctions*.

EXAMPLES Solve an Absolute Value Inequality ($<$)

3 Solve $3 > |d|$. Graph the solution set on a number line.

You can interpret $3 > |d|$ to mean that the distance between d and 0 on a number line is less than units. To make

$3 > |d|$ true, you must substitute numbers for d that are fewer than units from .



Notice that the graph of $3 > |d|$ is the same as the graph of $d > \text{$ and $d < \text{$.

All of the numbers *not* at or between -3 and 3 are less than units from 0 . The solution set is .

Check Your Progress

Solve $|x| < 5$. Graph the solution.

EXAMPLE

Solve a Multi-Step Absolute Value Inequality

4 Solve $|2x - 2| \geq 4$. Graph the solution set.

$|2x - 2| \geq 4$ is equivalent to $2x - 2 \geq 4$ or $2x - 2 \leq -4$. Solve each inequality.

$$2x - 2 \geq 4$$

or

$$2x - 2 \leq -4$$

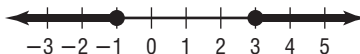
$$\text{} \geq \text{$$

$$\text{} \leq \text{$$

$$x \geq \text{$$

$$x \leq \text{$$

The solution set is .



Check Your Progress

Solve $|3x - 3| > 9$. Graph the solution set.

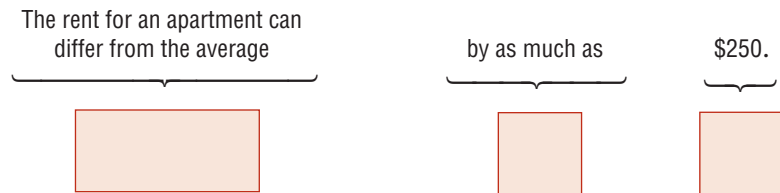
EXAMPLE

Write an Absolute Value Inequality

- 5 HOUSING** According to a recent survey, the average monthly rent for a one-bedroom apartment in one city is \$750. However, the actual rent for any given one-bedroom apartment might vary as much as \$250 from the average.

- a. Write an absolute value inequality to describe this situation.

Let r = the actual monthly rent



- b. Solve the inequality $|750 - r| \leq 250$ to find the range of monthly rent. Rewrite the absolute value inequality as a compound inequality. Then solve for r .

$$-250 \leq 750 - r \leq 250$$

$$-250 \quad \boxed{} \leq -r \leq 250 \quad \boxed{}$$

$$-1000 \leq -r \leq -500$$

$$\boxed{} \geq r \geq \boxed{}$$

The solution set is . The actual rent falls between and , inclusive.

Check Your Progress

The average birth weight of a newborn baby is 7 pounds. However, this weight can vary by as much as 4.5 pounds.

- a. What is an absolute value inequality to describe this situation?

- b. What is the range of birth weights for newborn babies?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BRINGING IT ALL TOGETHER**STUDY GUIDE**

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 1 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 1, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 2–3) to help you solve the puzzle.

1-1

Expressions and Formulas

- Find the value of $30 - 4^2 \div 2 \cdot 4$.
- Evaluate $2x^2 - 3xy$ if $x = -4$ and $y = 5$.
- Why is it important for everyone to use the same order of operations for evaluating expressions?

1-2

Properties of Real Numbers

- Name the sets of numbers to which $-\frac{7}{8}$ belongs.
- Write the Associative Property of Addition in symbols. Then illustrate this property by finding the sum $12 + 18 + 45$.

Complete each sentence.

- The Property of Addition says that adding 0 to any number does not change its value.
- The numbers can be written as ratios of two integers, with the integer in the denominator not being 0.

1-3

Solving Equations

Solve each equation. Check your solution.

8. $3 - 5y = 4y + 6$

9. $\frac{2}{3} - \frac{1}{2}x = \frac{1}{6}$

10. Solve $A = \frac{1}{2}h(a + b)$ for h .

Read the following problem and then write an equation that you could use to solve it. Do not actually solve the equation. In your equation, let m be the number of miles driven.

11. When Louisa rented a moving truck, she agreed to pay \$28 per day plus \$0.42 per mile. If she kept the truck for 3 days and the rental charges (without tax) were \$153.72, how many miles did Louisa drive the truck?

1-4

Solving Absolute Value Equations

12. Evaluate
- $|m - 5n|$
- if
- $m = -3$
- and
- $n = 2$
- .

Solve each equation.

13. $-2|4x - 5| = -46$

14. $|7 + 3x| = x - 5$

15. Explain why the absolute value of a number can never be negative.

1-5

Solving Inequalities

There are several different ways to write or show solution sets of inequalities. Write each of the following in interval notation.

16. $\{x \mid x < -3\}$

17.



Solve each inequality. Graph the solution set.

18. $5y + 9 > 34$

19. $-1 - 5x \leq 4(x + 2)$

1-6

Solving Compound and Absolute Value Inequalities

Complete each sentence.

20. Two inequalities combined by the word *and* or the word *or* form a .

21. The graph of a compound inequality containing the word *and* is the of the graphs of the two separate inequalities.

Solve each inequality. Graph the solution set.

22. $-11 < 3m - 2 < 22$

23. $|x + 3| \geq 1$



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 1.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 1 Practice Test on page 53 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 1 Study Guide and Review on pages 49–52 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 53 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 1 Foldable.
- Then complete the Chapter 1 Study Guide and Review on pages 49–52 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 53 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Linear Relations and Functions

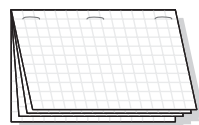


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

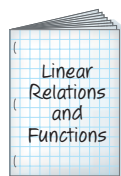
Begin with two sheets of grid paper.

STEP 1

Fold in half along the width and staple along the fold.

**STEP 2**

Turn the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.



NOTE-TAKING TIP: When taking notes, make annotations. Annotations are usually notes taken in the margins of books you own to organize the text for review or study.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 2. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
absolute value function			
boundary			
Cartesian coordinate plane			
constant function			
family of graphs			
function			
greatest integer function			
identity function			
linear function			
line of fit			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
parent graph			
piecewise function PEES-wyz			
point-slope form			
positive correlation			
prediction equation pree-DIHK-shuhn			
relation			
scatter plot			
slope			
slope-intercept form IHN-tuhr-SEHPT			
standard form			
step function			

Relations and Functions



TEKS 2A.1 The student uses properties and attributes of functions and applies functions to problem situations. **(A) Identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.**

BUILD YOUR VOCABULARY (pages 27–28)

MAIN IDEAS

- Analyze and graph relations.
- Find functional values.

A relation is a set of .

A function is a special type of relation in which each element of the domain is paired with element of the range.

A function where each element of the is paired with exactly one element of the is called a **one-to-one function**.

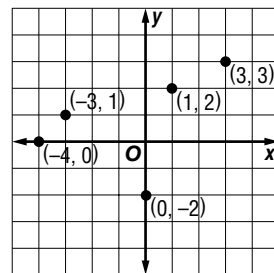
EXAMPLE Domain and Range

1 State the domain and range of the relation shown in the graph. Is the relation a function?

The relation is $\{(1, 2), (3, 3), (0, -2), (-4, 0), (-3, 1)\}$.

The domain is .

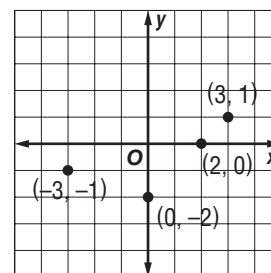
The range is .



Each member of the domain is paired with exactly one member of the range, so this relation is a .

Check Your Progress

State the domain and range of the relation shown in the graph. Is the relation a function?



EXAMPLE Vertical Line Test

- 2 TRANSPORTATION** The table shows the average fuel efficiency in miles per gallon for light trucks for several years. Graph this information and determine whether it represents a function. Is this relation *discrete* or *continuous*?

The year values correspond to the values, and the Fuel Efficiency values correspond to the values.

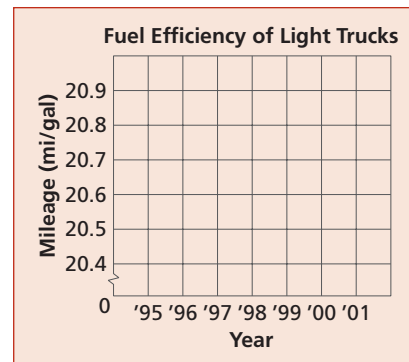
Year	Fuel Efficiency (mi/gal)
1995	20.5
1996	20.8
1997	20.6
1998	20.9
1999	20.5
2000	20.5
2001	20.4

Source: U.S. Environmental Protection Agency

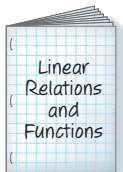
Graph the ordered pairs on a coordinate grid.

Use the vertical line test. Notice that no vertical line can be drawn that contains more than one data point

This relation is a function, and it is .

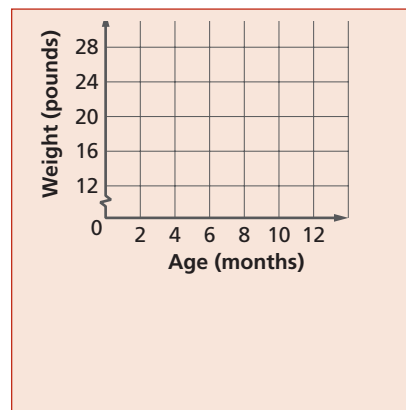
**FOLDABLES™****ORGANIZE IT**

On the page labeled Relations and Functions, define function and relation. Then draw a graph of a function and a relation.

**Check Your Progress**

- HEALTH** The table shows the average weight of a baby for several months during the first year. Graph this information and determine whether it represents a function.

Age (months)	Weight (pounds)
1	12.5
2	16
4	22
6	24
9	25
12	26



EXAMPLE Graph a Relation

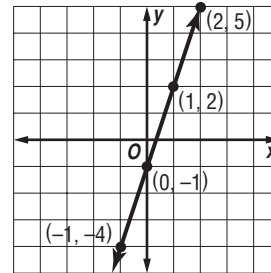
- 3** a. Graph the relation represented by $y = 3x - 1$.

KEY CONCEPT

Vertical Line Test If no vertical line intersects a graph in more than one point, the graph represents a function. If some vertical line intersects a graph in two or more points, the graph does not represent a function.

Make a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y . Then graph the ordered pairs.

x	y
-1	<input type="text"/>
0	<input type="text"/>
1	<input type="text"/>
2	<input type="text"/>



- b. Find the domain and range.

The domain and range are both all numbers.

- c. Determine whether the equation is a function and state whether it is *discrete* or *continuous*.

This graph passes the vertical line test. For each

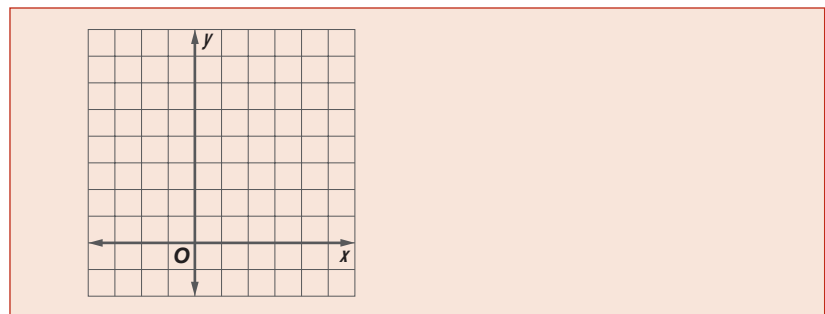
-value, there is exactly one -value.

The equation represents a .

All (x, y) values that satisfy the equation lie on a ,
so the relation is .

Check Your Progress

- Graph $y = 2x + 5$.
- Find the domain and range.
- Determine whether the equation is a function and state whether it is.



EXAMPLE Evaluate a Function

4 Given $f(x) = x^3 - 3$, find each value.

a. $f(-2)$

$$f(x) = x^3 - 3$$

Original function

$$f(-2) = \boxed{}$$

Substitute.

$$= \boxed{} \text{ or } \boxed{}$$

Simplify.

b. $f(2t)$

$$f(x) = x^3 - 3$$

Original function

$$f(2t) = \boxed{}$$

Substitute.

$$= \boxed{}$$

$(ab)^2 = a^2b^2$

REVIEW IT

Explain what it means to evaluate an expression.
(Lesson 1-1)

Check Your Progress Given $f(x) = x^2 + 5$ and $h(x) = 0.5x^2 + 2x + 2.5$, find each value.

a. $f(-1)$

b. $h(1.5)$

c. $f(3a)$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Linear Equations



TEKS 2A.1 The student uses properties and attributes of functions and applies functions to problem situations. **(A) Identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.**

BUILD YOUR VOCABULARY (page 27)

MAIN IDEAS

- Identify linear equations and functions.
- Write linear equations in standard form and graph them.

An equation such as $x + y = 4$ is called a linear equation.

A **linear equation** has no operations other than

, , and

of a variable by a constant.

EXAMPLE

Identify Linear Functions

WRITE IT

Give an example of a linear function and a nonlinear function. Explain how you can tell the difference between the two functions.

- 1 State whether each function is a linear function. Explain.

a. $g(x) = 2x - 5$

This is a function because it is in the form

$$g(x) = mx + b. m = \text{, } b = \text{$$

b. $p(x) = x^3 + 2$

This is not a linear function because x has an exponent other than .

c. $t(x) = 4 + 7x$

This is a function because it is in the form

$$t(x) = mx + b. m = \text{, } b = \text{$$

Check Your Progress

State whether each function is a linear function. Explain.

a. $h(x) = 3x - 2$

b. $g(x, y) = 3xy$

EXAMPLE

Evaluate a Linear Function

2 The linear function $f(C) = 1.8C + 32$ can be used to find the number of degrees Fahrenheit f that are equivalent to a given number of degrees Celsius C .

- a. On the Celsius scale, normal body temperature is 37°C . What is it in degrees Fahrenheit?

$$f(C) = 1.8C + 32 \quad \text{Original function}$$

$$f(\boxed{}) = 1.8(\boxed{}) + 32 \quad \text{Substitute.}$$

$$= \boxed{} \quad \text{Simplify.}$$

- b. There are 100 Celsius degrees between the freezing and boiling points of water and 180 Fahrenheit degrees between these two points. How many Fahrenheit degrees equal 1 Celsius degree?

$$100^\circ\text{C} = \boxed{}^\circ\text{F} \quad \text{Given}$$

$$100^\circ\text{C} = \boxed{}^\circ\text{F} \quad \text{Divide each side by } \boxed{}.$$

$$\boxed{} \quad \boxed{}$$

$$1^\circ\text{C} = \boxed{} \quad \text{Simplify.}$$

Check Your Progress

The linear function $d(s) = \frac{1}{5}s$ can be used to find the distance d in miles from a storm, based on the number of seconds s that it takes to hear thunder after seeing lightning.

- a. If you hear thunder 10 seconds after seeing lightning, how far away is the storm?

- b. If the storm is 3 miles away, how long will it take to hear thunder after seeing lightning?

KEY CONCEPT

Standard Form of a Linear Equation The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero.

EXAMPLE Standard Form

3 Write each equation in standard form. Identify A , B , and C .

a. $y = 3x - 9$

$$y = 3x - 9 \quad \text{Original equation}$$

$$\boxed{} = \boxed{} \quad \text{Subtract } 3x \text{ from each side.}$$

$$\boxed{} = \boxed{} \quad \text{Multiply each side by } -1 \text{ so that } A \geq 0.$$

So, $A = \boxed{}$, $B = \boxed{}$, and $C = \boxed{}$.

b. $-\frac{2}{3}x = 2y - 1$

$$-\frac{2}{3}x = 2y - 1 \quad \text{Original equation}$$

$$\boxed{} = \boxed{} \quad \text{Subtract } 2y \text{ from each side.}$$

$$\boxed{} = \boxed{} \quad \text{Multiply each side by } -3 \text{ so that the coefficients are all integers.}$$

So, $A = \boxed{}$, $B = \boxed{}$, and $C = \boxed{}$.

Check Your Progress Write each equation in standard form. Identify A , B , and C .

a. $y = -2x + 5$

b. $\frac{3}{5}x = -3y + 2$

c. $3x - 9y + 6 = 0$

REMEMBER IT

An equation such as $x = 3$ represents a vertical line, and only has an x -intercept.

An equation such as $y = 5$ represents a horizontal line, and only has a y -intercept.

EXAMPLE Use Intercepts to Graph a Line

- 4** Find the x -intercept and the y -intercept of the graph of $-2x + y - 4 = 0$. Then graph the equation.

The x -intercept is the value of x when $y = 0$.

$$-2x + y - 4 = 0 \quad \text{Original equation}$$

$$-2x + \boxed{} - 4 = 0 \quad \text{Substitute 0 for } y.$$

$$\boxed{} = \boxed{} \quad \text{Add 4 to each side.}$$

$$x = \boxed{-2} \quad \text{Divide each side by } -2.$$

The x -intercept is -2 . The graph crosses the x -axis at $\boxed{-2}$.

Likewise, the y -intercept is the value of y when $x = 0$.

$$-2x + y - 4 = 0 \quad \text{Original equation}$$

$$-2(\boxed{0}) + y - 4 = 0 \quad \text{Substitute 0 for } x.$$

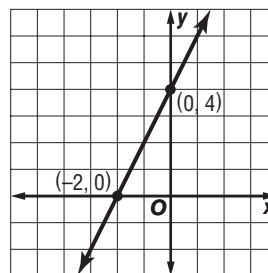
$$y = \boxed{4} \quad \text{Add 4 to each side.}$$

The y -intercept is 4 . The graph crosses the y -axis at $\boxed{4}$.

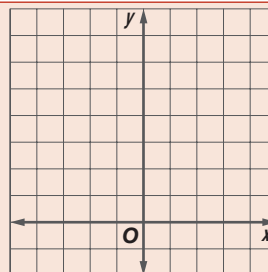
Use the ordered pairs to graph this equation.

The x -intercept is $\boxed{-2}$,

and the y -intercept is $\boxed{4}$.

**Check Your Progress**

Find the x -intercept and the y -intercept of the graph of $3x - y + 6 = 0$. Then graph the equation.

**HOMEWORK ASSIGNMENT**

Page(s): _____

Exercises: _____

EXAMPLE


Find Slope and Use Slope to Graph

MAIN IDEAS

- Find and use the slope of a line.
- Graph parallel and perpendicular lines.

KEY CONCEPT

Slope of a Line The slope of a line is the ratio of the change in y -coordinates to the change in x -coordinates.

 **TEKS 2A.4** The student connects algebraic and geometric representations of functions. **(A) Identify and sketch graphs of parent functions, including linear** ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$), **functions**, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

- 1 Find the slope of the line that passes through $(1, 3)$ and $(-2, -3)$. Then graph the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope formula

$$= \frac{\quad}{\quad}$$

$$(x_1, y_1) = (1, 3), (x_2, y_2) = (-2, -3)$$

$$= \frac{\quad}{\quad} \text{ or } \frac{\quad}{\quad}$$

Simplify.

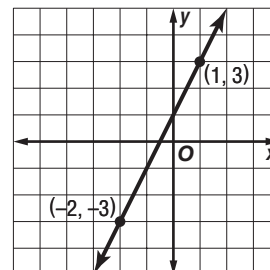
Graph the two ordered pairs and draw the line. Use the slope

to check your graph by selecting any $\frac{\quad}{\quad}$ on the line. Then

go up $\frac{\quad}{\quad}$ units and right $\frac{\quad}{\quad}$

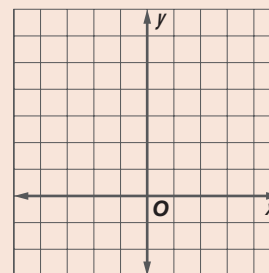
unit or go $\frac{\quad}{\quad}$ 2 units

and left 1 unit. This point should also be on the line.



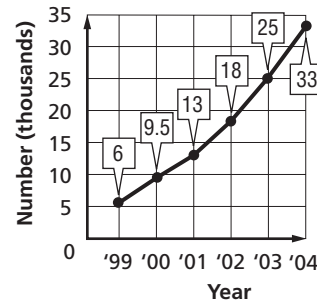
Check Your Progress

Find the slope of the line that passes through $(2, 3)$ and $(-1, 5)$. Then graph the line.



EXAMPLE Find Rate of Change

- 2 BUSINESS** Refer to the graph at the right, which shows data on the fastest-growing restaurant chain in the U.S. during the time period of the graph. Find the rate of change of the number of stores from 1999 to 2004.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope formula

$$= \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}}$$

Substitute.

$$= 5.4$$

Simplify.

Between 1999 and 2004, the number of stores in the U.S. increases at an average rate of or stores per year

Check Your Progress

Refer to the graph in Example 2. Find the rate of change of the number of stores from 2001 to 2004.

KEY CONCEPT

Parallel Lines In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.

EXAMPLE Parallel Lines

- 3** Graph the line through $(1, -2)$ that is parallel to the line with the equation $x - y = -2$.

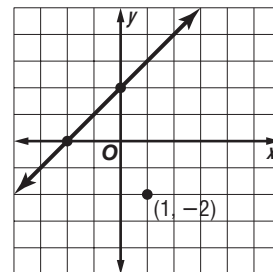
The x -intercept is , and the

y -intercept is .

Use the intercepts to graph $x - y = -2$.

The line rises 1 unit for every 1 unit it moves to the right,

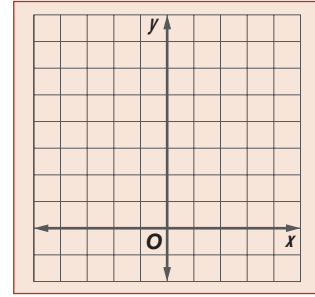
so the slope is .



Now, use the slope and the point at $(1, -2)$ to graph the line parallel to $x - y = -2$.

Check Your Progress

Graph the line through (2, 3) that is parallel to the line with the equation $3x + y = 6$.



EXAMPLE

Perpendicular Lines

4

Graph the line through (2, 1) that is perpendicular to the line with the equation $2x - 3y = 3$.

KEY CONCEPT

Perpendicular Lines In a plane, two oblique lines are perpendicular if and only if the product of their slopes is -1 .

FOLDABLES

On the page opposite the page labeled Slope, graph two lines that are parallel. Then graph two lines that are perpendicular.

The x -intercept is or , and the y -intercept is

. Use the intercepts to graph $2x - 3y = 3$.

The line rises 1 unit for every 1.5 units it moves to the right,

so the slope is or .

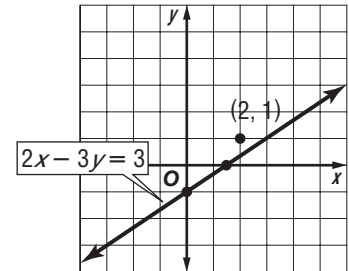
The slope of the line perpendicular

is the opposite reciprocal of

or . Start at (2, 1) and go

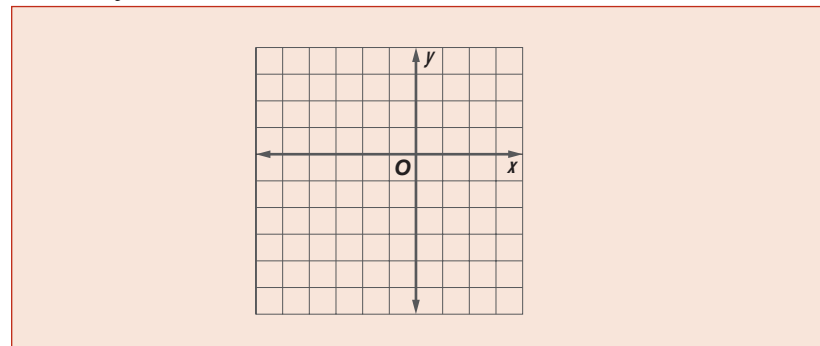
down units and right units.

Use this point and (2, 1) to graph the line.



Check Your Progress

Graph the line through $(-3, 1)$ that is perpendicular to the line with the equation $5x - 10y = -20$.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

The slope is -2 . That eliminates choices B and C.

Now use the point-slope formula to find an equation.

$$y - y_1 = m(x - x_1)$$

Point-slope form

$$y - (-3) = -2(x - 2)$$

$m = -2$; you can use either point for (x_1, y_1) .

$$\boxed{} = \boxed{}$$

Distributive Property

$$y = \boxed{}$$

Subtract 3 from each side.

The answer is $\boxed{}$.

Check Your Progress

What is an equation of the line through $(2, 5)$ and $(-1, 3)$?

A $y = \frac{2}{3}x + \frac{11}{3}$

C $y = \frac{-2}{3}x + \frac{19}{3}$

B $y = \frac{3}{2}x + \frac{9}{2}$

D $y = \frac{-3}{2}x + 8$

EXAMPLE

Write an Equation of a Perpendicular Line

- 3** Write an equation for the line that passes through $(3, -2)$ and is perpendicular to the line whose equation is $y = -5x + 1$.

The slope of the given line is $\boxed{}$. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\boxed{}$.

Use the point-slope form and the ordered pair $\boxed{}$ to write the equation.

$$y - y_1 = m(x - x_1)$$

Point-slope form

$$y - \left(\boxed{}\right) = \boxed{} \left(x - \boxed{}\right)$$

$$(x_1, y_1) = (3, -2), m = \frac{1}{5}$$

$$\boxed{} = \boxed{}$$

Distributive Property

$$y = \boxed{}$$

Subtract 2 from each side.

An equation of the line is $\boxed{}$.

REVIEW IT

Name the property you would use to simplify $8(z + 4)$. (Lesson 1-2)

Check Your Progress

Write an equation for the line that passes through $(3, 5)$ and is perpendicular to the line with equation $y = 3x - 2$.

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____

Statistics: Using Scatter Plots



TEKS 2A.1 The student uses properties and attributes of functions and applies functions to problem situations. **(B) Collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments**

EXAMPLE

Find and use a Prediction Equation

MAIN IDEAS

- Draw scatter plots.
- Find and use prediction equations.

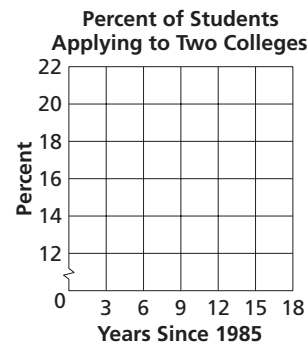
1 EDUCATION The table below shows the approximate percent of students who sent applications to two colleges in various years since 1985.

Years Since 1985	0	3	6	9	12	15
Percent	20	18	15	15	14	13

Source: *U.S. News & World Report*

a. Draw a scatterplot and a line of fit for the data.

Graph the data as ordered pairs, with the number of years since 1985 on the axis and the percentage on the axis.



b. Find a prediction equation. What do the slope and y-intercept indicate?

Find an equation of the line through (3, 18) and (15, 13). Begin by finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope formula

$$= \frac{13 - 18}{15 - 3}$$

Substitute.

$$= \boxed{} \approx \boxed{}$$

Simplify.

(continued on the next page)

WRITE IT

An outlier is a data point that does not appear to belong with the rest of the set. Should you include an outlier when finding a line of fit? Explain.

$$y - y_1 = m(x - x_1)$$

Point-slope form

$$y - \boxed{} = \boxed{}(x - \boxed{})$$

$$m = -0.42, \\ (x^1, y^1) = (3, 18)$$

$$\boxed{} = \boxed{}$$

Distributive Property

$$y = \boxed{}$$

Add 18 to each side.

The slope indicates that the percent of students sending applications to two colleges is falling at about $\boxed{}$ each year. The y -intercept indicates that the percent in 1985 should have been about $\boxed{}$.

c. Predict the percent in 2010.

The year 2010 is 25 years after 1985, so use $x = 25$ to find y .

$$y = -0.42x + 19.26$$

Prediction equation

$$= -0.42(\boxed{}) + 19.26$$

$$x = \boxed{}$$

$$= \boxed{}$$

Simplify.

In 2010 the percent should be about $\boxed{}$.

d. How accurate is the prediction?

The fit is only $\boxed{}$, so the prediction may not be very accurate.

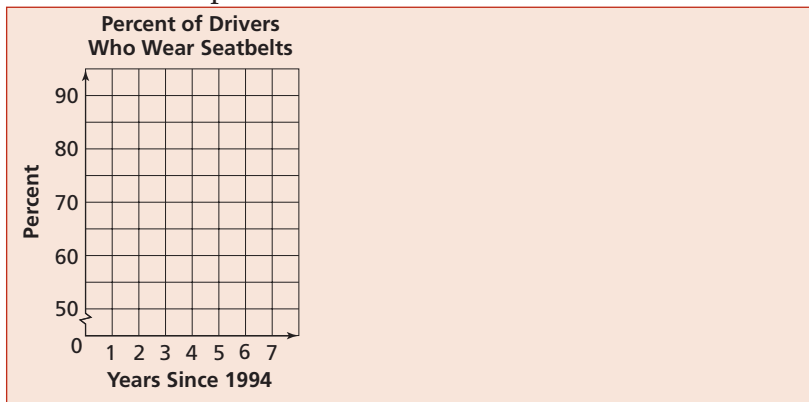
Check Your Progress

The table below shows the approximate percent of drivers who wear seat belts in various years since 1994.

Years Since 1994	0	1	2	3	4	5	6	7
Percent	57	58	61	64	69	68	71	73

Source: National Highway Traffic Safety Administration

a. Make a scatter plot of the data and draw a line of best fit.



- b. Find a prediction equation. What do the slope and y -intercept indicate?

- c. Predict the percent of drivers who will be wearing seat belts in 2005.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____



MAIN IDEAS

- Identify and graph step, constant, and identity functions.
- Identify and graph absolute value and piecewise functions.



TEKS 2A.1 The student uses properties

and attributes of functions and applies functions to problem situations. **(A) Identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.**

2A.4 The student connects algebraic and geometric representations of functions. **(A) Identify and sketch graphs of parent functions, including linear** ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$), functions, **absolute value of x** ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

BUILD YOUR VOCABULARY (pages 27–28)

A function whose graph is a series of line segments is called a **step function**.

The **greatest integer function**, written $f(x) = \llbracket x \rrbracket$, is an example of a step function.

$f(x) = b$ is called a **constant function**.

When a function does not change the value, $f(x) = x$ is called the **identity function**.

Another special function is the **absolute value function**, $f(x) = |x|$.

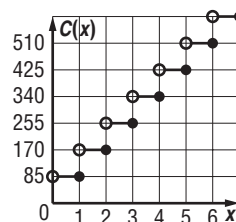
A that is written using two or more expressions is called a **piecewise function**.

EXAMPLE Step Function

1 PSYCHOLOGY One psychologist charges for counseling sessions at the rate of \$85 per hour or any fraction thereof. Draw a graph that represents this situation.

Use the pattern of times and costs to make a table, where x is the number of hours of the session and $C(x)$ is the total cost. Then draw the graph.

x	$C(x)$
$0 < x \leq 1$	<input type="text"/>
$1 < x \leq 2$	<input type="text"/>
$2 < x \leq 3$	<input type="text"/>
$3 < x \leq 4$	<input type="text"/>
$4 < x \leq 5$	<input type="text"/>



EXAMPLE Absolute Value Functions

2 Graph $f(x) = |x - 3|$ and $g(x) = |x + 2|$ on the same coordinate plane. Determine the similarities and differences in the two graphs.

Find several ordered pairs for each function.

x	$ x - 3 $
0	3
1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
4	<input type="text"/>

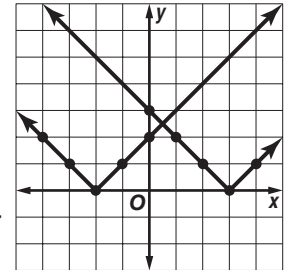
x	$ x + 2 $
-4	2
-3	<input type="text"/>
-2	<input type="text"/>
-1	<input type="text"/>
0	<input type="text"/>

WRITE IT

Consider the general form of an absolute value function, $f(x) = |x|$. Explain why the range of the function only includes nonnegative numbers.

Graph the points and connect them.

- The domain of both graphs is all numbers.
- The range of both graphs is $\{y | y \geq \text{$ }.
- The graphs have the same shape but different .
- The graph of $g(x)$ is the graph of $f(x)$ translated left units.



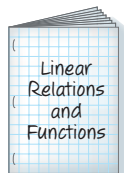
Check Your Progress

a. Graph $f(x) = |x| - 2$ and $g(x) = |x| + 3$ on the same coordinate plane. Determine the similarities and differences in the two graphs.

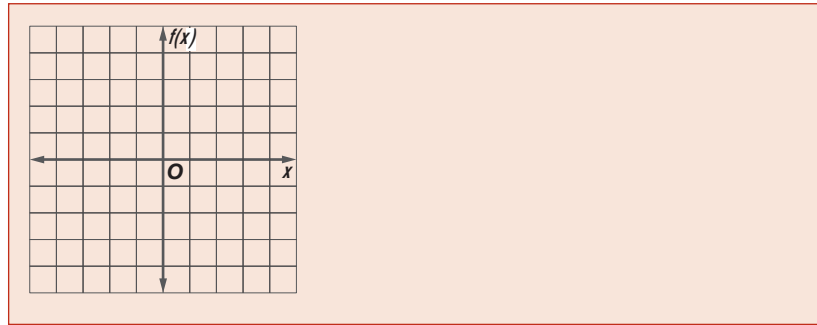
FOLDABLES™

ORGANIZE IT

Under the tab for Graphing Linear Functions, graph an example of each of the following functions: step, constant, absolute value, and piecewise.



- b. Graph the piecewise function $f(x) = \begin{cases} 2x + 1 & \text{if } x > -1 \\ -3 & \text{if } x \leq -1 \end{cases}$.
Identify the domain and range.



EXAMPLE Piecewise Function

- 3 Graph $f(x) = \begin{cases} x - 1 & \text{if } x \leq 3 \\ -1 & \text{if } x > 3 \end{cases}$. Identify the domain and range.

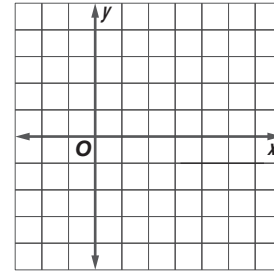
First graph the linear function $f(x) = x - 1$ for $x \leq 3$. Since 3 satisfies this inequality, begin

with a closed circle at .

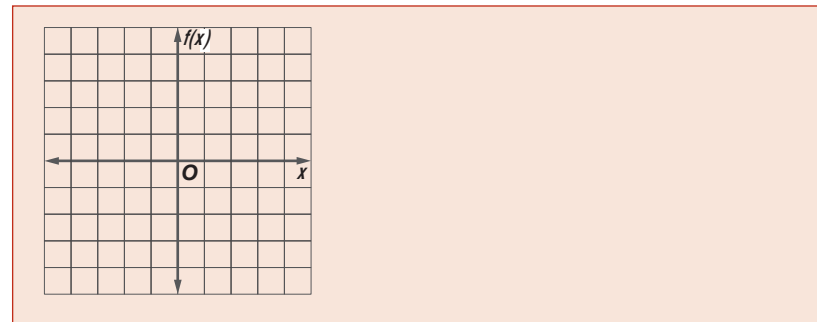
Then graph the function $f(x) = -1$ for $x > 3$. Since 3 does not satisfy this inequality, begin with an

circle at .

The is all real numbers. The is $\{y \mid y \leq 2\}$.

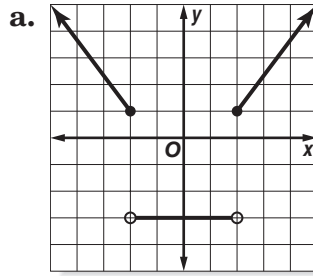


Check Your Progress Graph $f(x) = \begin{cases} 2x + 1 & \text{if } x > -1 \\ -3 & \text{if } x \leq -1 \end{cases}$.
Identify the domain and range.

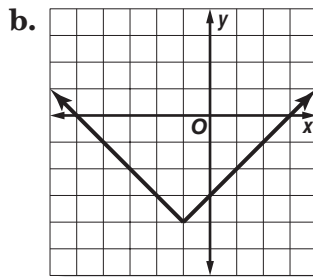


EXAMPLE Identify Functions

4 Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

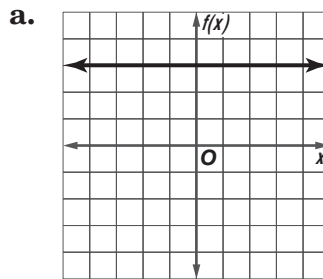


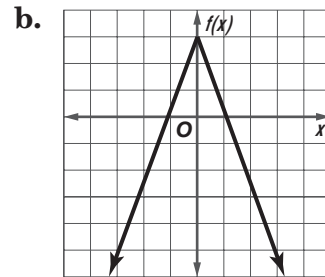
Since this graph consists of different rays and segments, it is a function.



Since this graph is V-shaped, it is an function.

Check Your Progress Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.





HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Graphing Inequalities

MAIN IDEAS

- Graph linear inequalities.
- Graph absolute value inequalities.



Reinforcement of TEKS A.7

The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems. **(B)** Investigate methods for solving linear equations and inequalities using concrete models, the properties of equality, select a method, and solve the equations and inequalities.

BUILD YOUR VOCABULARY (page 27)

When graphing inequalities, the graph of the line is the **boundary** of each region.

EXAMPLE Dashed Boundary

1 Graph $x - 2y < 4$.

The boundary is the graph of $x - 2y = 4$. Since the inequality symbol is $<$, the boundary will be dashed. Use the slope-intercept form, $y = \frac{1}{2}x - 2$. Test $(0, 0)$.

$$x - 2y < 4$$

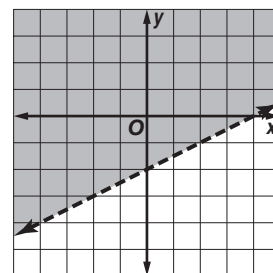
Original inequality

$$\boxed{} < \boxed{}$$

$(x, y) = (0, 0)$

$$0 < 4$$

true



Shade the region that contains $(0, 0)$

EXAMPLE Solid Boundary

2 EDUCATION One tutoring company advertises that it specializes in helping students who have a combined SAT verbal and math score of 900 or less.

- a. Write an inequality to describe the combined scores of students who are prospective tutoring clients. Let x represent the verbal score and y the math score.

$$\underbrace{\text{verbal score}}_x \text{ and } \underbrace{\text{math score}}_y \text{ are at most } \underbrace{\text{nine hundred}}_{900}$$

$$x + y \leq 900$$

The inequality is $\boxed{}$.

- b. Does a student with a verbal score of 480 and a math score of 410 fit the tutoring company's guidelines?

$$\begin{array}{ll} x + y \leq 900 & \text{Original inequality} \\ 480 + 410 \leq 900 & (x, y) = (480, 410) \\ 890 \leq 900 & \text{Simplify} \end{array}$$

So, a student with a verbal score of 480 and a math score of 410 $\boxed{\phantom{\text{fits}}}$ the tutoring company's guidelines.

EXAMPLE Absolute Value Inequality

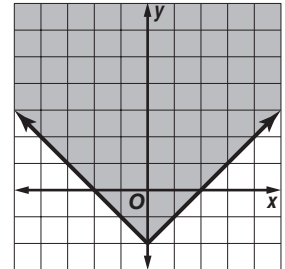
3 Graph $y \geq |x| - 2$.

Since the inequality symbol is \geq , the graph of the related equation $y = |x| - 2$ is solid. Graph the equation. Test $(0, 0)$.

$$y \geq |x| - 2$$

$$0 \stackrel{?}{\geq} |0| - 2$$

$$0 \geq 0 - 2 \text{ or } \boxed{} \geq \boxed{}$$



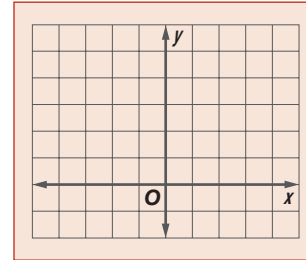
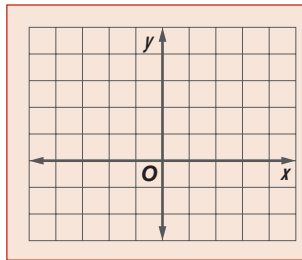
Shade the region that contains $\boxed{}$.

Check Your Progress

Graph each inequality.

a. $2x + 5y > 10$

b. $y < |x| + 3$



- c. **CLASS TRIP** Two social studies classes are going on a field trip. The teachers have asked for parent volunteers to go on the trip as chaperones. However, there is only enough seating for 60 people on the bus. Write an inequality to describe the number of students and chaperones that can ride on the bus. Can 45 students and 10 chaperones go on the trip?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 2 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 2, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 27–28) to help you solve the puzzle.

2-1

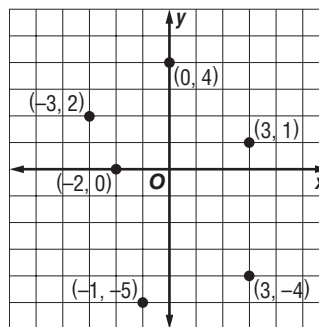
Relations and Functions

For Exercises 1 and 2, refer to the graph shown at the right.

1. Write the domain and range of the relation.

D:

R:



2. Is this relation a function? Explain.

2-2

Linear Equations

3. Write $x - 2 = \frac{1}{6}y$ in standard form. Identify A , B , and C .

Write *yes* or *no* to tell whether each linear equation is in standard form. If it is not, explain why it is not.

4. $-x + 2y = 5$

5. $5x - 7y = 3$

2-3

Slope

6. How are the slopes of two nonvertical parallel lines related?

7. How are the slopes of two oblique perpendicular lines related?

Find the slope of the line that passes through each pair of points.

8. $(3, 7), (8, -1)$

9. $(8, -\frac{1}{4}), (0, -\frac{1}{4})$

2-4

Writing Linear Equations

Write the equation in slope-intercept form for the line that satisfies each set of conditions.

10. slope 4, passes through $(0, 3)$

11. passes through $(5, -6)$ and $(3, 2)$

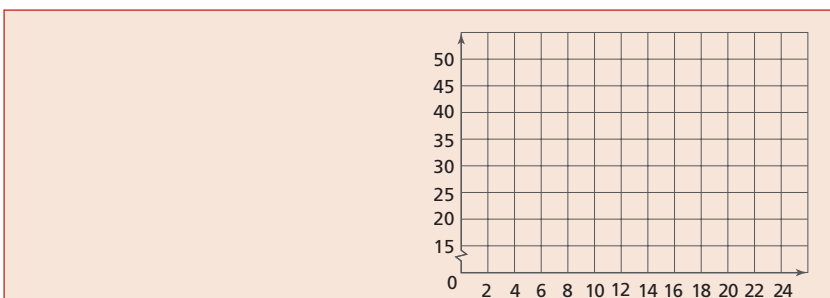
12. Write an equation for the line that passes through $(8, -5)$ and is perpendicular to the line whose equation is $y = \frac{1}{2}x - 8$.

2-5

Statistics: Using Scatter Plots

13. Draw a scatter plot for the data. Then state which of the data points is an outlier.

x	2	6	10	14	20	24
y	15	20	30	16	40	50



2-6

Special Functions

Write the letter of the term that best describes each function.

14. $f(x) = |4x + 3|$

15. $f(x) = \lceil x \rceil + 1$

16. $f(x) = 6$

17. $f(x) = \begin{cases} x + 3 & \text{if } x < 0 \\ 2 - x & \text{if } x \geq 0 \end{cases}$

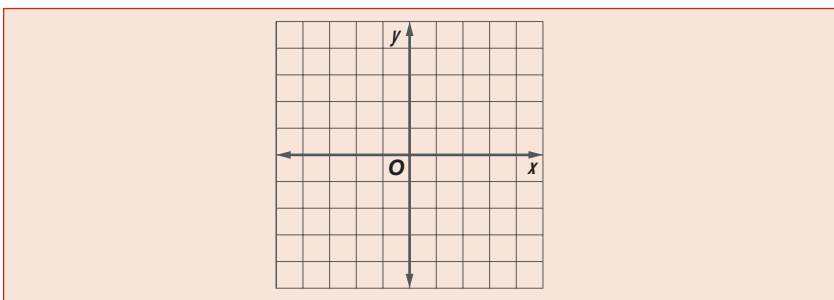
- a. constant function
- b. absolute value function
- c. piecewise function
- d. step function

2-7

Graphing Inequalities

18. When graphing a linear inequality in two variables, how do you know whether to make the boundary a solid line or a dashed line?

18. Graph the inequality $10 - 5y < 2x$.





Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 2.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 2 Practice Test on page 111 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 2 Study Guide and Review on pages 106–110 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 111 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 2 Foldable.
- Then complete the Chapter 2 Study Guide and Review on pages 106–110 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 111 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

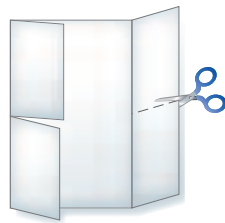
Systems of Equations and Inequalities



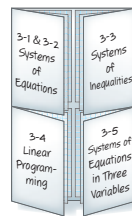
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" × 17" paper and four sheets of grid paper.

STEP 1 **Fold** the short sides of the 11" × 17" paper to meet in the middle. Cut each table in half as shown.



STEP 2 **Cut** 4 sheets of grid paper in half and fold the half-sheets in half. Insert two folded halvesheets under each of the four tabs and staple along the fold. Label each tab as shown.



NOTE-TAKING TIP: When taking notes, summarize the main ideas presented in the lesson. Summaries are useful for condensing data and realizing what is important.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
bounded			
consistent			
<u>constraints</u> kuhn-STRAYNTS			
dependent			
elimination method			
<u>feasible region</u> FEE-zuh-buhl			
<u>inconsistent</u> ihn-kuhn-SIHS-tuhnt			
independent			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
linear programming			
ordered triple			
substitution method			
system of equations			
system of inequalities			
unbounded			
vertex			

MAIN IDEAS

- Solve systems of linear equations by graphing.
- Determine whether a system of linear equations is consistent and independent, consistent and dependent, or inconsistent.

**TEKS 2A.3** The student formulates systems of

equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems. (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities. (C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.**

BUILD YOUR VOCABULARY (pages 57–58)

A system of equations is or more equations with the same variables.

A system of equations is **consistent** if it has at least solution and **inconsistent** if it has solutions.

A consistent system is **independent** if it has exactly solution or **dependent** if it has an number of solutions.

EXAMPLE**Solve the System of Equations by Completing a Table**

- 1** Solve the system of equations by completing a table.

$$x + y = 3$$

$$-2x + y = -6$$

Write each equation in slope-intercept form.

$$x + y = 3 \rightarrow \text{[]}$$

$$-2x + y = -6 \rightarrow \text{[]}$$

Use a table to find the solution that satisfies both equations.

x	$y_1 = -x + 3$	y_1	$y_2 = 2x - 6$	y_2	(x_1, y_1)	(x_2, y_2)
0	$y_1 = (0) + 3$	3	$y_2 = 2(0) - 6$	-6	(0, 3)	(0, -6)
1	$y_1 = -(1) + 3$	2	$y_2 = 2(1) - 6$	-4	(1, 2)	(1, -4)
2	$y_1 = -(2) + 3$	1	$y_2 = 2(2) - 6$	-2	(2, 1)	(2, -2)
3	$y_1 = -(3) + 3$	0	$y_2 = 2(3) - 6$	0	(3, 0)	(3, 0)

The solution of the system of equations is .

EXAMPLE

Solve by Graphing

2 Solve the system of equations by graphing.

$$x - 2y = 0$$

$$x + y = 6$$

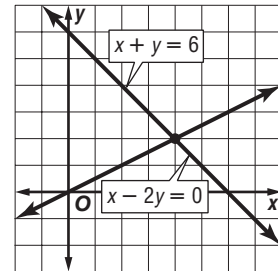
Write each equation in slope-intercept form.

$$x - 2y = 0 \rightarrow \boxed{}$$

$$x + y = 6 \rightarrow \boxed{}$$

The graphs appear to intersect

at $\boxed{}$.



REMEMBER IT



When solving a system of equations by graphing, you should always check the ordered pair in *each* of the original equations.

Check: Substitute the coordinates into each equation.

$$x - 2y = 0 \quad x + y = 6 \quad \text{Original equations.}$$

$$4 - 2(2) \stackrel{?}{=} \boxed{} \quad 4 + 2 \stackrel{?}{=} 6 \quad \text{Replace } x \text{ and } y.$$

$$\boxed{} = 0 \quad \boxed{} = 6 \quad \text{Simplify.}$$

So, the solution of the system is $\boxed{}$.

Check Your Progress

a. Solve the system by completing a table.

$$x + y = 2$$

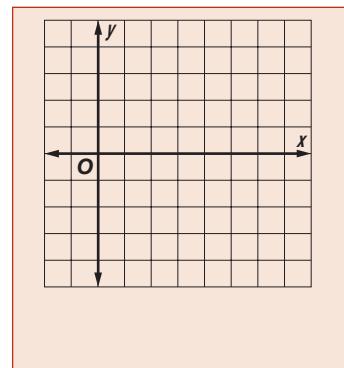
$$x - 3y = -6$$

--	--

b. Solve the system by graphing.

$$x + 3y = 7$$

$$x - y = 3$$



REVIEW IT

Explain the steps you would use to write $2x + 5y = 10$ in slope-intercept form. (Lesson 1-3)

EXAMPLE Break-Even Point Analysis

- 3 SALES** A service club is selling copies of its holiday cookbook to raise funds for a project. The printer's set-up charge is \$200, and each book costs \$2 to print. The cookbooks will sell for \$6 each. How much must the club sell before it makes a profit?

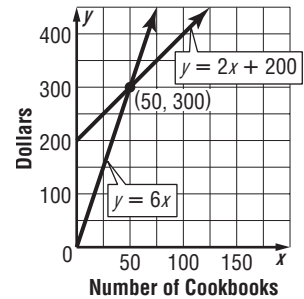
Income from books is price per book times number of books.
 $y = 6x$

The graphs intersect at

This is the break-even point. If the group sells less than 50 books, they

will money. If the group sells more than 50 books, they will

make a .



Check Your Progress

The student government is selling candy bars. It costs \$1 for each candy bar plus a \$60 set-up fee. The group will sell the candy bars for \$2.50 each. How many do they need to sell to break even?

EXAMPLE Same Line

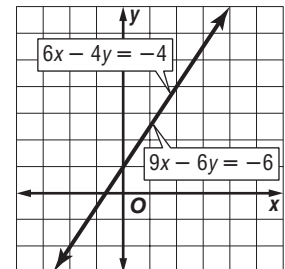
- 4** Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$$9x - 6y = -6$$

$$6x - 4y = -4$$

$$9x - 6y = -6 \rightarrow$$

$$6x - 4y = -4 \rightarrow$$



Since the equations are equivalent, their graphs are the

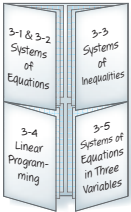
line. There are many solutions.

This system is and .

FOLDABLES™

ORGANIZE IT

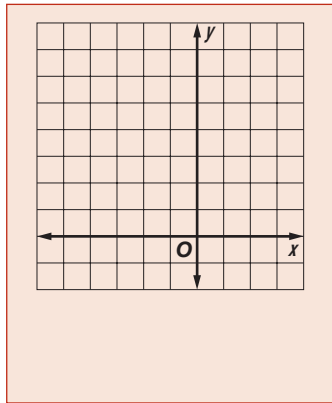
Under the Systems of Equations tab, graph the three possible relationships between a system of equations and the number of solutions. Write the number of solutions below each graph.



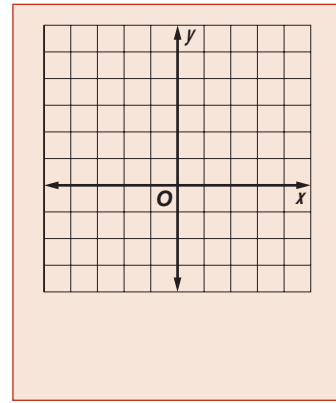
Check Your Progress

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

a. $x + y = 5$
 $2x = y - 11$



b. $x + y = 3$
 $2x = -2y + 6$



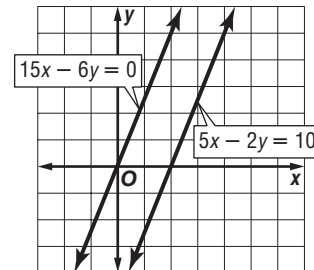
EXAMPLE Parallel Lines

5 Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$15x - 6y = 0$
 $5x - 2y = 10$

$15x - 6y = 0$ →

$5x - 2y = 10$ →

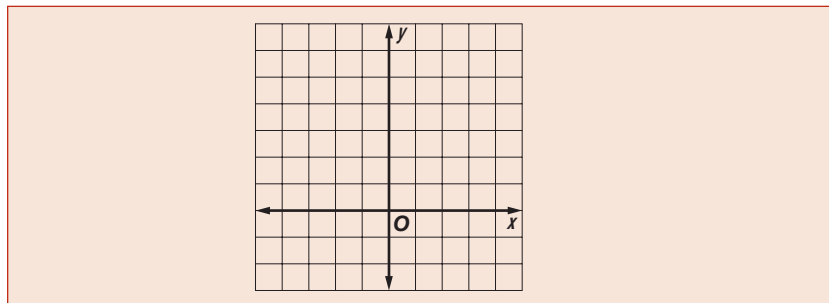


The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system

is .

Check Your Progress

Graph the system of equations $y = 3x + 2$ and $-6x + 2y = 10$ and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.



HOMEWORK ASSIGNMENT

Page(s): _____
 Exercises: _____

MAIN IDEAS

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

REMEMBER IT



In Example 1, you can substitute the value for y in either of the original equations. Choose the equation that is easiest to solve.



TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns** or inequalities in two unknowns **to solve problems.** **(B) Use algebraic methods,** graphs, tables, or matrices, **to solve systems of equations or inequalities.** **(C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.**

BUILD YOUR VOCABULARY (pages 57–58)

Using the **substitution method**, one equation is solved for one in terms of . Then, this expression is substituted for the variable in the other equation. Using the **elimination method**, you eliminate one of the variables by or the equations.

EXAMPLE Solve by Using Substitution

- 1 Use substitution to solve $x + 4y = 26$ and $x - 5y = -10$.

Solve the first equation for x in terms of y .

$$x + 4y = 26$$

First equation

$$x = \text{input type="text"}$$

Subtract $4y$ from each side.

Substitute $26 - 4y$ for x in the second equation and solve for y .

$$x - 5y = -10$$

Second equation

$$\text{input type="text"} - 5y = -10$$

Substitute for x .

$$\text{input type="text"} = \text{input type="text"}$$

Subtract from each side.

$$y = \text{input type="text"}$$

Divide each side.

Now substitute the value for y in either of the original equations and solve for x .

$$x + 4y = 26$$

First equation

$$x + 4(\text{input type="text"}) = 26$$

Replace y with 4.

$$x + \text{input type="text"} = 26$$

Simplify.

$$x = \text{input type="text"}$$

Subtract from each side.

The solution of the system is .

Check Your Progress

Use Substitution to solve the system of equations.

$$x - 3y = 2$$

$$x + 7y = 12$$

EXAMPLE**Solve by Substitution**

- 2 FURNITURE** Lancaster Woodworkers Furniture Store builds two types of wooden outdoor chairs. A rocking chair sells for \$265 and an Adirondack chair with footstool sells for \$320. The books show that last month, the business earned \$13,930 for the 48 outdoor chairs sold. How many chairs were sold?

Read the Test Item

You are asked to find the number of each type of chair sold.

Solve the Test Item.

- Step 1** Define variables and write the system of equations. Let x represent the number of rocking chairs sold and y represent the number of Adirondack chairs.

$$x + y = \boxed{} \quad \text{The total number of chairs sold}$$

$$265x + 320y = \boxed{} \quad \text{The total earned.}$$

- Step 2** Solve one of the equations for one of the variables in terms of the other. Since the coefficient of x is

$\boxed{}$, solve the first equation for x in terms of y .

$$x + y = 48 \quad \text{First equation}$$

$$x = 48 - y \quad \text{Subtract } \boxed{} \text{ from each side.}$$

- Step 3** Substitute $48 - y$ for x in the second equation.

$$265x + 320y = 13,930 \quad \text{Second equation}$$

$$265(\boxed{}) + 320y = 13,930 \quad \text{Substitute } \boxed{} \text{ for } x.$$

$$\boxed{} - \boxed{} + 320y = 13,930 \quad \text{Distributive Property}$$

$$55y = 1210 \quad \text{Simplify.}$$

$$y = \boxed{} \quad \text{Divide each side by } \boxed{}.$$

Step 4 Now find the value of x . Substitute the value for y into either equation.

$$\begin{array}{ll}
 x + y = 48 & \text{First equation} \\
 x + \boxed{} = 48 & \text{Replace } y \text{ with } \boxed{}. \\
 x = 26 & \text{Subtract } \boxed{} \text{ from each side.} \\
 \text{They sold } \boxed{} \text{ rocking chairs and } \boxed{} \text{ Adirondack chairs} &
 \end{array}$$

Check Your Progress

At Amy's Amusement Park, tickets sell for \$24.50 for adults and \$16.50 for children. On one day the amusement park took in \$6405 from selling 330 tickets. How many of each kind of ticket was sold?

EXAMPLE

Multiply, Then Use Elimination

3 Use the elimination method to solve $2x + 3y = 12$ and $5x - 2y = 11$.

Multiply the first equation by 2 and the second equation by 3. Then add the equations to eliminate one of the variables.

$$\begin{array}{rcl}
 2x + 3y = 12 & \xrightarrow{\text{Multiply by 2.}} & \boxed{} + \boxed{} = \boxed{} \\
 5x - 2y = 11 & \xrightarrow{\text{Multiply by 3.}} & (+) \boxed{} - \boxed{} = \boxed{} \\
 \hline
 & & 19x \qquad \qquad = 57 \\
 & & x = \boxed{}
 \end{array}$$

Replace x and solve for y .

$$\begin{array}{ll}
 2x + 3y = 12 & \text{First equation} \\
 2(\boxed{}) + 3y = 12 & \text{Replace } x \text{ with } \boxed{}. \\
 \boxed{} + 3y = 12 & \text{Multiply.} \\
 3y = 6 & \text{Subtract 6 from each side.} \\
 y = 2 & \text{Divide each side by 3.}
 \end{array}$$

The solution is $\boxed{}$.

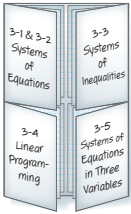
WRITE IT

When solving a system of equations, how do you choose whether to use the substitution method or the elimination method?

FOLDABLES

ORGANIZE IT

Under the Systems of Equations tab, write how you recognize an inconsistent system of equations. Then write how you recognize a consistent system of equations.



EXAMPLE Inconsistent System

- 4 Use the elimination method to solve $-3x + 5y = 12$ and $6x - 10y = -21$.

Use multiplication to eliminate x .

$$\begin{array}{r}
 -3x + 5y = 12 \xrightarrow{\text{Multiply by 2.}} \boxed{} + \boxed{} = \boxed{} \\
 6x - 10y = \boxed{} \quad (+) \quad \boxed{} - \boxed{} = \boxed{} \\
 \hline
 0 = \boxed{}
 \end{array}$$

Since there are no values of x and y that will make the equation $\boxed{} = \boxed{}$ true, there are no solutions for the system of equations.

Check Your Progress Use the elimination method to solve each system of equations.

a. $x + 3y = 7$
 $2x + 5y = 10$

d. $2x + 3y = 11$
 $-4x - 6y = 20$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BUILD YOUR VOCABULARY (page 57)

MAIN IDEAS

- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

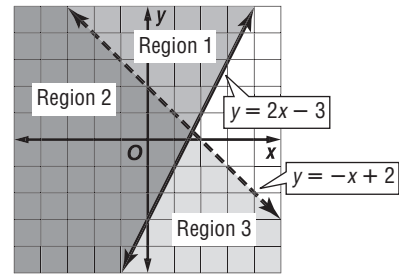
To solve a system of inequalities, find the pairs that satisfy of the inequalities in the system.

EXAMPLE Intersecting Regions

1 Solve each system of inequalities by graphing.

a. $y \geq 2x - 3$
 $y < -x + 2$

Graph each inequality.



solution of $y \geq 2x - 3 \rightarrow$ Regions

solution of $y < -x + 2 \rightarrow$ Regions

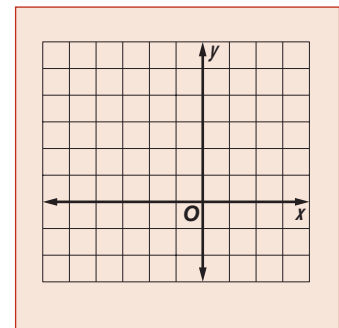
The intersection of these regions is Region , which is the solution of the system of inequalities. Notice that the solution is a region containing an number of ordered pairs.

b. $y \leq -x + 1$
 $|x + 1| < 3$

The inequality $|x + 1| < 3$ can be written as $x + 1 < \text{input type="text" value="3"}$

and $x + 1 > \text{input type="text" value="-3"}$

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.



TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems. (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities. (C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.**

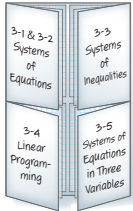
FOLDABLES

ORGANIZE IT

Under the tab for Systems of Inequalities, solve the following by graphing.

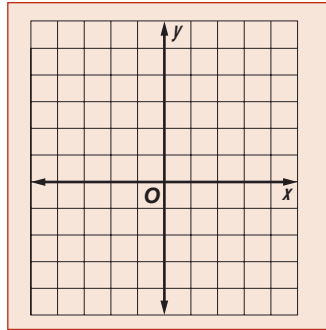
$$y \leq 3x - 6$$

$$y > -4x + 2$$

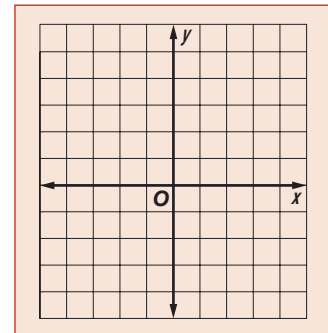


Check Your Progress Solve each system of inequalities by graphing.

a. $y \geq 3x - 3$
 $y > x + 1$



b. $y \geq -2x - 3$
 $|x + 2| < 1$



EXAMPLE Separate Regions

2 Solve the system of inequalities by graphing.

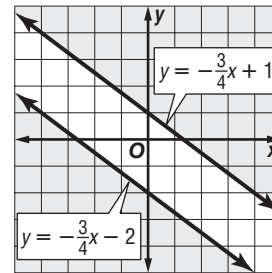
$$y \geq -\frac{3}{4}x + 1$$

$$y \leq -\frac{3}{4}x - 2$$

Graph both inequalities.

The graphs do not overlap, so the solutions have points in common.

The solution set is .



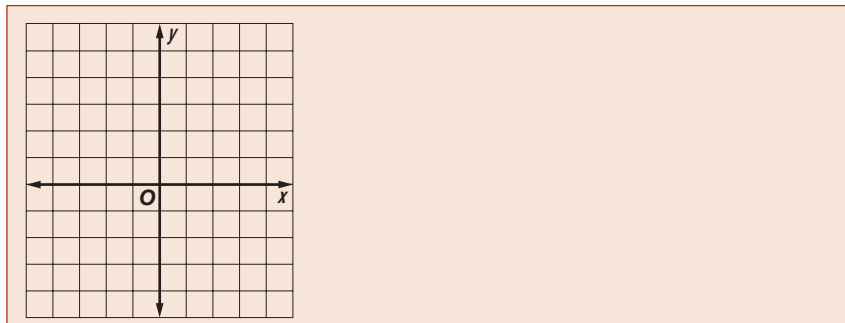
REMEMBER IT



You can indicate that a system of inequalities has no solutions in the following ways: empty set, null set, \emptyset , or $\{\}$.

Check Your Progress Solve the system of inequalities

$$y < \frac{1}{2}x + 2 \text{ and } y > \frac{1}{2}x + 4 \text{ by graphing.}$$



EXAMPLE

Write and Use a System of Inequalities

- 3 MEDICINE** Medical Professionals recommend that patients have a cholesterol level below 200 milligrams per deciliter (mg/dL) of blood and a triglyceride levels below 150 mg/dL. Write and graph a system of inequalities that represents the range of cholesterol level and triglyceride levels for patients.

Source: American Heart Association.

Let c represent the cholesterol levels in mg/dL. It must be less

than mg/dL. Since cholesterol levels cannot be

negative, we can write this as $0 \leq c < \text{}$.

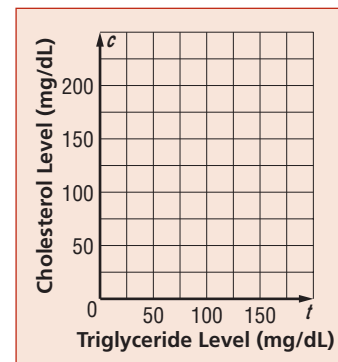
Let t represent the triglyceride levels in mg/dL. It must be less

than mg/dL. Since triglyceride levels also cannot be

negative, we can write this as $0 \leq t < \text{}$.

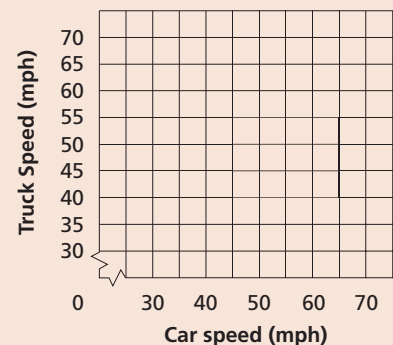
Graph both inequalities. Any ordered pair in the intersection

of the graphs is a of the system.



Check Your Progress

The speed limits while driving on the highway are different for trucks and cars. Cars must drive between 45 and 65 miles per hour, inclusive. Trucks are required to drive between 40 and 55 miles per hour, inclusive. Let c represent the speed range for cars and t represent the speed range for trucks. Write and graph a system of inequalities to represent this situation.



EXAMPLE Find Vertices

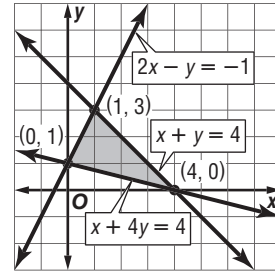
- 4 Find the coordinates of the vertices of the figure formed by $2x - y \geq -1$, $x + y \leq 4$, and $x + 4y \geq 4$.

Graph each inequality.

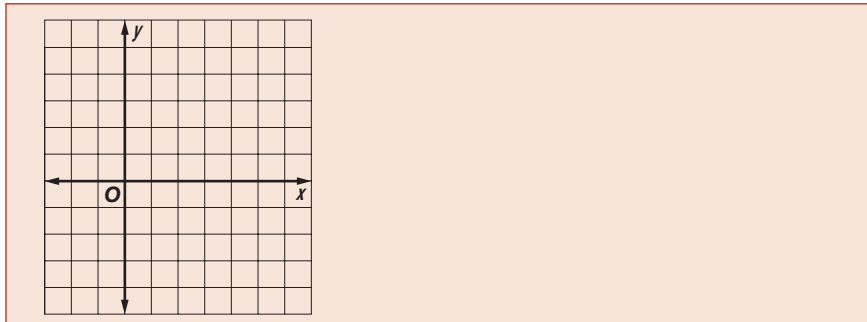
The of the graphs forms a triangular region.

The vertices of the triangle are at

, , and .



Check Your Progress Find the coordinates of the vertices of the figure formed by $x + 2y \geq 1$, $x + y \leq 3$, and $-2x + y \leq 3$.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Find the maximum and minimum values of a function over a region.
- Solve real-world problems using linear programming.



TEKS 2A.3 The student formulates systems of

equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems. (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities. (C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.**

BUILD YOUR VOCABULARY (pages 57–58)

In a graph of a system of inequalities, the are called the **constraints**.

The intersection of the graphs is called the **feasible region**.

When the graph of a system of constraints is a polygonal region, we say that the region is **bounded**.

The maximum or minimum value of a related function

occurs at one of the **vertices** of the

feasible region.

When a system of inequalities forms a region that is

, the region is said to be **unbounded**.

The process of finding or

values of a function for a region defined by inequalities is called **linear programming**.

EXAMPLE Bounded Region

- 1** Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x - 2y$ for this region.

$$x \leq 5$$

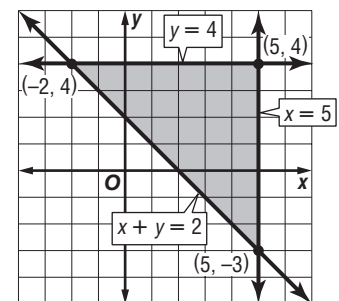
$$y \leq 4$$

$$x + y \geq 2$$

First, find the vertices of the region. Graph the inequalities.

The polygon formed is a triangle with vertices at

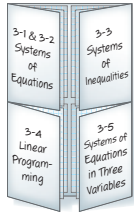
, , and .



FOLDABLES™

ORGANIZE IT

Under the Linear Programming tab, sketch a graph of a system of inequalities like the one shown in Example 1. Then label the constraints, feasible region, and vertices of the graph.



Next, use a table to find the maximum and minimum values of $f(x, y)$. Substitute the coordinates of the vertices into the function.

(x, y)	$3x - 2y$	$f(x, y)$
$(-2, 4)$	<input type="text"/>	<input type="text"/> ← minimum
$(5, -3)$	<input type="text"/>	<input type="text"/> ← maximum
$(5, 4)$	<input type="text"/>	<input type="text"/>

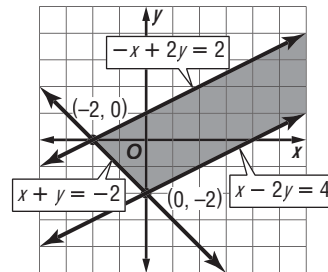
The vertices of the feasible region are , ,
and . The maximum value is at .
The minimum value is at .

EXAMPLE Unbounded Region

2 Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 2x + 3y$ for this region.

$$\begin{aligned}
 -x + 2y &\leq 2 \\
 x - 2y &\leq 4 \\
 x + y &\geq -2
 \end{aligned}$$

Graph the system of inequalities. there are only two points of



intersection,
and .

(x, y)	$2x + 3y$	$f(x, y)$
$(-2, 0)$	<input type="text"/>	<input type="text"/>
$(0, -2)$	<input type="text"/>	<input type="text"/>

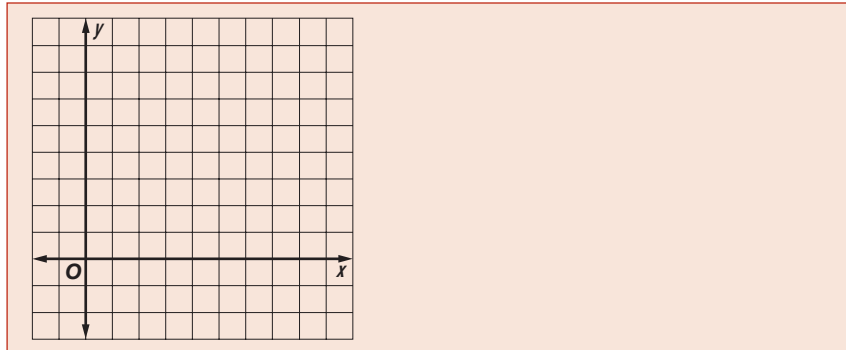
WRITE IT

Explain how you recognize the unbounded region of a system of inequalities.

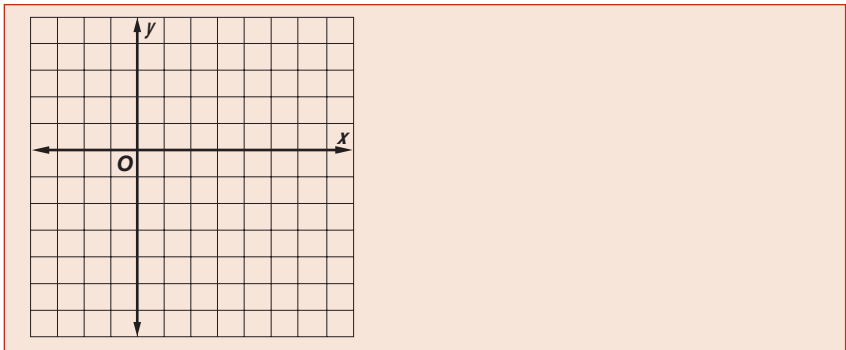
The minimum value is at $(0, -2)$. Although $f(-2, 0)$ is , it is not the maximum value since there are other points that produce greater values. For example, $f(2, 1)$ is and $f(3, 1)$ is . It appears that because the region is unbounded, $f(x, y)$ has no maximum value.

Check Your Progress Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

a. $x \leq 4$
 $y \leq 5$
 $x + y \geq 6$
 $f(x, y) = 4x - 3y$



b. $x + 3y \leq 6$
 $-x - 3y \leq 9$
 $2y - x \geq -6$
 $f(x, y) = x + 2y$



EXAMPLE Linear Programming**KEY CONCEPT**

Linear Programming Procedure

- Step 1** Define the variables.
- Step 2** Write a system of inequalities.
- Step 3** Graph the system of inequalities.
- Step 4** Find the coordinates of the vertices of the feasible region.
- Step 5** Write a function to be maximized or minimized.
- Step 6** Substitute the coordinates of the vertices into the function.
- Step 7** Select the greatest or least result. Answer the problem.

- 3 LANDSCAPING** A landscaping company has crews who mow lawns and prune shrubbery. The company schedules 1 hour for mowing jobs and 3 hours for pruning jobs. Each crew is scheduled for no more than 2 pruning jobs per day. Each crew's schedule is set up for a maximum of 9 hours per day. On the average, the charge for mowing a lawn is \$40 and the charge for pruning shrubbery is \$120. Find a combination of mowing lawns and pruning shrubs that will maximize the income the company receives per day from one of its crews.

- Step 1** Define the variables.

m = the number of mowing jobs

p = the number of pruning jobs

- Step 2** Write a system of inequalities.

Since the number of jobs cannot be negative, m and p must be nonnegative numbers.

$$m \geq \boxed{}, p \geq \boxed{}$$

Mowing jobs take $\boxed{}$ hour. Pruning jobs take $\boxed{}$ hours. There are $\boxed{}$ hours to do the jobs.

$$\boxed{} + \boxed{} \leq 9$$

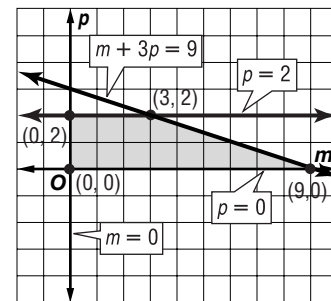
There are no more than 2 pruning jobs a day. $\boxed{} \leq 2$

- Step 3** Graph the system of inequalities.

- Step 4** Find the coordinates of the vertices of the feasible region.

From the graph, the

vertices are at $\boxed{}, \boxed{},$
 $\boxed{}, \boxed{},$ and $\boxed{}.$



- Step 5** Write the function to be maximized.

The function that describes the income is $f(m, p) = 40m + 120p$. We want to find the

$\boxed{}$ value for this function.

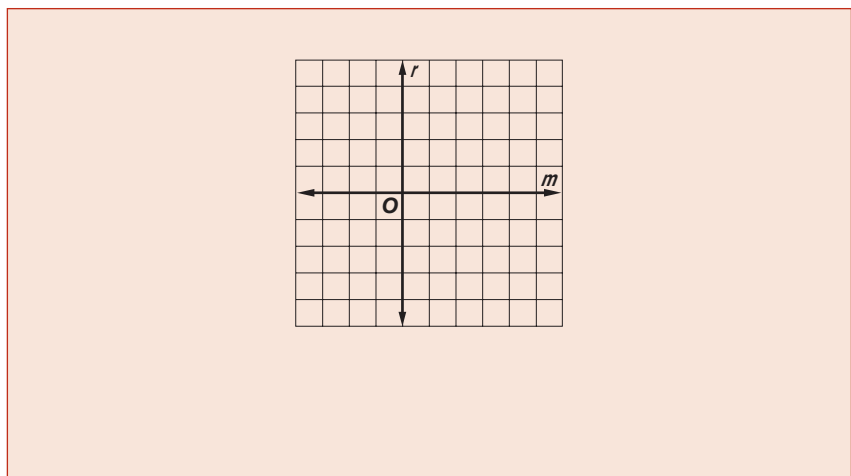
Step 6 Substitute the coordinates of the vertices into the function.

(m, p)	$40m + 120p$	$f(m, p)$
(0, 2)	<input type="text"/>	<input type="text"/>
(3, 2)	<input type="text"/>	<input type="text"/>
(9, 0)	<input type="text"/>	<input type="text"/>
(0, 0)	<input type="text"/>	<input type="text"/>

Step 7 Select the amount. Answer the problem.

The maximum values are at and at . This means that the company receives the most money with mows and prunings or mows and prunings.

Check Your Progress A landscaping company has crews who rake leaves and mulch. The company schedules 2 hours for mulching jobs and 4 hours for raking jobs. Each crew is scheduled for no more than 2 raking jobs per day. Each crew's schedule is set up for a maximum of 8 hours per day. On the average, the charge for raking a lawn is \$50 and the charge for mulching is \$30. Find a combination of raking leaves and mulching that will maximize the income the company receives per day from one of its crews.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Solving Systems of Equations in Three Variables

MAIN IDEAS

- Solve systems of linear equations in three variables.
- Solve real-world problems using systems of linear equations in three variables.

KEY CONCEPT

System of Equations in Three Variables

One solution

- planes intersect in one point

Infinite Solutions

- planes intersect in a line
- planes intersect in the same plane

No solution

- planes have no point in common



TEKS 2A.3 The student formulates systems of

equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems. (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities. (C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.**

BUILD YOUR VOCABULARY (page 57)

The solution of a system of equations in three variables, x , y , and z is called an **ordered triple** and is written as (x, y, z) .

EXAMPLE One Solution

1 Solve the system of equations.

$$5x + 3y + 2z = 2$$

$$2x + y - z = 5$$

$$x + 4y + 2z = 16$$

Use elimination to make a system of two equations in two variables. First, eliminate z in the first and second equations.

$$\begin{array}{r} 5x + 3y + 2z = 2 \\ 2x + y - z = 5 \quad (\times 2) \\ \hline (+) 4x + 2y - 2z = 10 \\ \hline \boxed{} = \boxed{} \end{array}$$

Eliminate z in the first and third equations.

$$\begin{array}{r} 5x + 3y + 2z = 2 \\ (-) x + 4y + 2z = 16 \\ \hline \boxed{} = \boxed{} \end{array}$$

Solve the system of two equations. Eliminate y .

$$\begin{array}{r} 9x + 5y = 12 \\ 4x - y = -14 \quad (\times 5) \\ \hline (+) \boxed{} = \boxed{} \\ \hline \boxed{} = \boxed{} \\ x = \boxed{} \end{array}$$

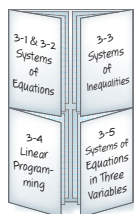
Substitute $\boxed{}$ for x in one of the two equations with two variables and solve for y .

$$\begin{array}{r} 4x - y = -14 \\ 4(\boxed{}) - y = -14 \\ \boxed{} - y = -14 \\ y = \boxed{} \end{array} \quad \begin{array}{l} \text{Equation with two variables} \\ \text{Replace } x. \\ \text{Multiply} \\ \text{Simplify.} \end{array}$$

FOLDABLES™

ORGANIZE IT

Under the Systems of Equations in Three Variables tab, sketch a graph of a system with (a) one solution, (b) infinite solutions, (c) no solutions.



Substitute for x and y in one of the original equations with three variables.

$$2x + y - z = 5$$

Equation with three variables

$$2(\text{input}) + 6 - z = 5$$

Replace x and y .

$$\text{input} + 6 - z = 5$$

Multiply.

$$z = \text{input}$$

Simplify.

The solution is . You can check this solution in the other two original equations.

EXAMPLE Infinite Solutions

2 Solve the system of equations.

$$2x + y - 3z = 5$$

$$x + 2y - 4z = 7$$

$$6x + 3y - 9z = 15$$

Eliminate y in the first and third equations.

$$\begin{array}{r} 2x + y - 3z = 5 \\ 6x + 3y - 9z = 15 \end{array} \quad \begin{array}{l} (\times 3) \\ \rightarrow \end{array} \quad \begin{array}{r} 6x + 3y - 9z = 15 \\ (-)6x + 3y - 9z = 15 \\ \hline \end{array}$$

$$\text{input} = \text{input}$$

The equation $0 = 0$ is always . This indicates that the first and third equations represent the same plane. Check to see if this plane intersects the second plane

$$\begin{array}{r} x + 2y - 4z = 7 \\ 6x + 3y - 9z = 15 \end{array} \quad \begin{array}{l} (\times 6) \\ \rightarrow \end{array} \quad \begin{array}{r} 6x + 12y - 24z = 42 \\ (-)6x + 3y - 9z = 15 \\ \hline \end{array}$$

$$\text{input} = \text{input}$$

$$\text{input} = \text{input}$$

The planes intersect in a . So, there are an of solutions.

EXAMPLE No Solution**3** Solve the system of equations.

$$3x - y - 2z = 4$$

$$6x + 4y + 8z = 11$$

$$9x + 6y + 12z = -3$$

Eliminate x in the second two equations.

$$6x + 4y + 8z = 11 \xrightarrow{(\times 3)} \boxed{} + 12y + \boxed{} = \boxed{}$$

$$9x + 6y + 12z = -3 \xrightarrow{(\times 2)} (-)18x + \boxed{} + 24z = \boxed{}$$

$$\boxed{} = \boxed{}$$

The equation $\boxed{} = \boxed{}$ is never true. So, there is no solution of this system.

Check Your Progress Solve each system of equations.

a. $2x + 3y - 3z = 16$

$x + y + z = -3$

$x - 2y - z = -1$

b. $x + y - 2z = 3$

$-3x - 3y + 6z = -9$

$2x + y - z = 6$

c. $3x + y - z = 5$

$-15x - 5y + 5z = 11$

$x + y + z = 2$

EXAMPLE

Write and Solve a System of Equations

- 4 SPORTS** There are 49,000 seats in a sports stadium. Tickets for the seats in the upper level sell for \$25, the ones in the middle level cost \$30, and the ones in the bottom level are \$35 each. The number of seats in the middle and bottom levels together equals the number of seats in the upper level. When all of the seats are sold for an event, the total revenue is \$1,419,500. How many seats are there in each level?

Explore Read the problem and define the .

u = number of seats in the upper level

m = number of seats in the middle level

b = number of seats in the bottom level

Plan There are seats.

$$u + m + b = 49,000$$

When all the seats are sold, the revenue is .

Seats cost , , and .

$$25u + 30m + 35b = 1,419,500$$

The number of seats in the middle and bottom levels together equal the number of seats in the upper level.

$$m + b = \text{$$

Solve Substitute $m + b$ for u in each of the first two equations.

$$u + m + b = 49,000$$

$$\left(\text{$$

Replace u with .

$$\text{$$

Simplify.

$$m + b = 24,500$$

Divide by 2.

$$25u + 30m + 35b = 1,419,500$$

$$25\left(\text{$$

Replace u with $m + b$.

$$\text{$$

Distributive Property

$$\text{$$

Simplify.

Now, solve the system of two equations in two variables

$$\begin{array}{r}
 m + b = 24,500 \quad (\times 55) \quad \boxed{} + \boxed{} = \boxed{} \\
 55m + 60b = 1,419,500 \quad (-)55m + 60b = 1,419,500 \\
 \hline
 \boxed{} = \boxed{} \\
 b = 14,400
 \end{array}$$

Substitute 14,400 for b in the one of the equations with two variables and solve for m .

$$\begin{array}{l}
 m + b = 24,500 \quad \text{Equation with two variables} \\
 m + \boxed{} = 24,500 \quad b = 14,400 \\
 m = 10,100 \quad \text{Subtract } \boxed{} \text{ from each side.}
 \end{array}$$

Substitute 14,400 for b and 10,100 for m in one of the original equation with three variables.

$$\begin{array}{l}
 m + b = u \quad \text{Equation with three variables} \\
 \boxed{} + \boxed{} = u \quad m = 10,100, b = 14,400 \\
 \boxed{} = u \quad \text{Add.}
 \end{array}$$

There are $\boxed{}$ upper level, $\boxed{}$ middle level, and $\boxed{}$ bottom level seats.

Check Check to see if all the criteria are met.

There are $\boxed{}$ seats in the stadium.

$$24,500 + 10,100 + 14,400 = 49,000 \quad \checkmark$$

The number of seats in the middle and bottom levels equals the number of seats in the upper level.

$$10,100 + 14,400 = 24,500 \quad \checkmark$$

When all of the seats are sold, the revenue is

$$\begin{array}{l}
 \boxed{} \\
 24,500(\$25) + 10,100(\$30) + 14,400(\$35) \\
 = 1,419,500 \quad \checkmark
 \end{array}$$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Check Your Progress

The school store sells pens, pencils, and paper. The pens are \$1.25 each, the pencils are \$0.50 each, and the paper is \$2 per pack. Yesterday the store sold 25 items and earned \$32. The number of pens sold equaled the number of pencils sold plus the number of packs of paper sold minus 5. How many of each item did the store sell?

BRINGING IT ALL TOGETHER**STUDY GUIDE**

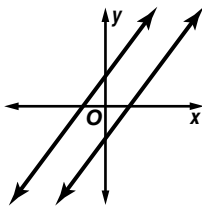
FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 3 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 3, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 57–58) to help you solve the puzzle.

3-1

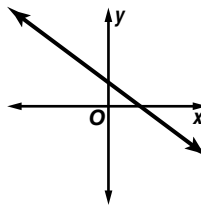
Solving Systems of Equations by Graphing

Under each system graphed below, write all of the following words that apply: *consistent*, *inconsistent*, *dependent*, and *independent*.

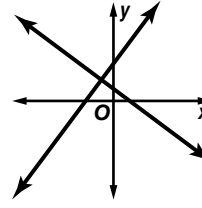
1.



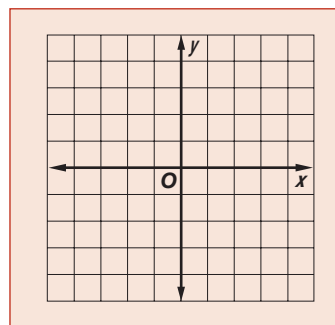
2.



3.



4. Solve the system $x + y = 3$ and $3x - y = 1$ by graphing.



3-2

Solving Systems of Equations Algebraically

5. Solve $x = 4y + 3$ and $x - 2y = 9$ by using substitution.
6. Solve $-2x + 5y = 7$ and $2x + 4y = 11$ by using elimination.

3-3

Solving Systems of Inequalities by Graphing

7. Which system of inequalities matches the graph shown

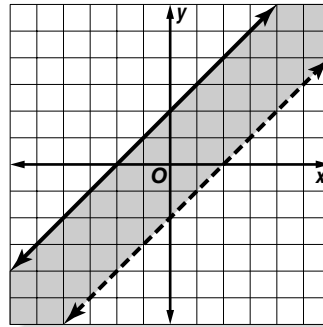
at the right?

a. $x - y \leq -2$
 $x - y > 2$

b. $x - y \geq -2$
 $x - y < 2$

c. $x + y \leq -2$
 $x + y > 2$

d. $x - y > -2$
 $x - y \leq 2$



Find the coordinates of the vertices of the figure formed by each system of inequalities.

8. $x + y \geq 8$
 $y \geq 5$
 $x \geq 0$

9. $x + y \geq 6$
 $x \leq 8$
 $y \leq 5$

3-4

Linear Programming

10. A polygonal feasible region has vertices at (4, -1), (3, 0), (1, -6), and (-2, 2). Find the maximum and minimum of the function $f(x, y) = -2x + y$ over this region.

3-5

Solving Systems of Equations in Three Variables

Solve each system of equations.

11. $5x + y + 4z = 9$
 $-5x + 3y + z = -15$
 $15x + 5y + 7z = 9$

12. $2x + y - 4z = -3$
 $-6x - 3y + 12z = 7$
 $x - 9y + 4z = 1$

13. The sum of three numbers is 22. The sum of the first and second numbers is 19, and the first number is 5 times the third number. Find the numbers.



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 3.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 3 Practice Test on page 157 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 3 Study Guide and Review on pages 153–156 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 157 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 3 Foldable.
- Then complete the Chapter 3 Study Guide and Review on pages 153–156 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 157 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Matrices



Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of notebook paper.

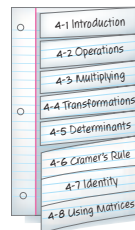
STEP 1

Fold lengthwise to the holes. Cut eight tabs in the top sheet.



STEP 2

Label each tab with a lesson number and title.



NOTE-TAKING TIP: When you take notes, write descriptive paragraphs about your learning experiences.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page numbering in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
<u>Cramer's Rule</u> [KRAY-muhrs]			
determinant			
<u>dilation</u> [dy-LAY-shuhn]			
dimension			
element			
equal matrices			
expansion by minors			
identity matrix			
image			
inverse			
<u>isometry</u> eye-SAH-muh-tree			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
<u>matrix</u> MAY-trihks			
matrix equation			
minor			
preimage			
reflection			
rotation			
<u>scalar multiplication</u> SKAY-luhr			
square matrix			
transformation			
translation			
vertex matrix			
zero matrix			

4-1

Introduction to Matrices



TEKS 2A.1 The student uses properties and attributes of functions and applies functions to problem situations. **(B)** Collect and **organize data**, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgements.

BUILD YOUR VOCABULARY (pages 85–86)

MAIN IDEAS

- Organize data in matrices.
- Solve equations involving matrices.

A **matrix** is a rectangular array of variables or constants in and vertical columns, usually enclosed in brackets.

Each in the matrix is called an **element**.

A matrix that has the same number of rows and is called a **square matrix**.

Another special type of matrix is the **zero matrix**, in which every element is .

EXAMPLE Organize Data into a Matrix

REMEMBER IT

An element of a matrix can be represented by the notation a_{ij} . This refers to the element in row i , column j .



1 COLLEGE Kaitlin wants to attend one of three Iowa universities next year. She has gathered information about tuition (T), room and board (R/B), and enrollment (E) for the universities. Use a matrix to organize the information. Which university's total cost is lowest?

Iowa State University:

T - \$5426 R/B - \$5958 E - 26,380

University of Iowa:

T - \$5612 R/B - \$6560 E - 28,442

University of Northern Iowa:

T - \$5387 R/B - \$5261 E - 12,927

Organize the data into labeled columns and rows.

	T	R/B	E
ISU	<input type="text"/>		
UI			
UNI			

The University of has the lowest total cost.

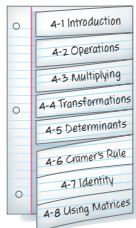
Check Your Progress

Justin is going out for lunch. The information he has gathered from the two fast-food restaurants is listed below. Use a matrix to organize the information. When is each restaurant's total cost less expensive?

Burger Complex		Lunch Express	
Hamburger Meal	\$3.39	Hamburger Meal	\$3.49
Cheeseburger Meal	\$3.59	Cheeseburger Meal	\$3.79
Chicken Sandwich Meal	\$4.99	Chicken Sandwich Meal	\$4.89

FOLDABLES™**ORGANIZE IT**

Under the tab for Lesson 4-1, tell how to find the dimensions of a matrix.

**EXAMPLE****Dimensions of a Matrix**

2 State the dimensions of matrix G if

$$G = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 1 & 5 & -3 & -1 \end{bmatrix}$$

$$G = \left[\begin{array}{cccc} 2 & -1 & 0 & 3 \\ 1 & 5 & -3 & -1 \end{array} \right] \left. \vphantom{\begin{array}{cccc} 2 & -1 & 0 & 3 \\ 1 & 5 & -3 & -1 \end{array}} \right\} \text{rows}$$

columns

Since matrix G has rows and columns, the dimensions of matrix G are .

Check Your Progress

State the dimensions of matrix G

if $G = \begin{bmatrix} 2 & 3 \\ 0 & 4 \\ -1 & 4 \end{bmatrix}$.

EXAMPLE**Solve an Equation Involving Matrices**

3 Solve $\begin{bmatrix} y \\ 3 \end{bmatrix} = \begin{bmatrix} 3x - 2 \\ 2y + x \end{bmatrix}$ for x and y .

Since the matrices are equal, the corresponding elements are equal. When you write the sentences to solve this equation, two linear equations are formed.

$$y = 3x - 2$$

$$3 = 2y + x$$

This system can be solved using substitution.

$$3 = 2y + x \quad \text{Second equation}$$

$$\boxed{} = 2(\boxed{}) + x \quad \text{Substitute } \boxed{} \text{ for } y.$$

$$\boxed{} = \boxed{} + x \quad \text{Distributive Property}$$

$$\boxed{} = \boxed{} \quad \text{Add 4 to each side.}$$

$$\boxed{} = x \quad \text{Divide each side by 7.}$$

To find the value for y , substitute 1 for x in either equation.

$$y = 3x - 2 \quad \text{First equation}$$

$$y = 3(\boxed{}) - 2 \quad \text{Substitute } \boxed{} \text{ for } x.$$

$$y = \boxed{} \quad \text{Simplify.}$$

The solution is $\boxed{}$.

Check Your Progress

Solve $\begin{bmatrix} y \\ 2x \end{bmatrix} = \begin{bmatrix} 3x - 1 \\ y - 2 \end{bmatrix}$ for x and y .

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Operations with Matrices



Preparation for TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(B)** Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

EXAMPLE Add Matrices

1 a. Find $A + B$ if $A = \begin{bmatrix} 6 & 4 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix}$.

$$A + B = \begin{bmatrix} 6 & 4 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix} \quad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \quad \text{Add corresponding elements.}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \quad \text{Simplify.}$$

b. Find $A + B$ if $A = \begin{bmatrix} 4 & -2 & 0 \\ 1 & 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 7 \\ -9 & 3 \end{bmatrix}$.

Since their dimensions are $\boxed{} \times \boxed{}$, these matrices $\boxed{} \times \boxed{}$ be added.

EXAMPLE Subtract Matrices

2 Find $A - B$ if $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$.

$$A - B = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \quad \text{Definition of matrix subtraction}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \quad \text{Subtract corresponding elements.}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \quad \text{Simplify.}$$

Check Your Progress

a. Find $A + B$ if $A = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 5 \\ 3 & 0 \end{bmatrix}$.

MAIN IDEAS

- Add and subtract matrices.
- Multiply by a matrix scalar.

KEY CONCEPTS

Addition of Matrices

If A and B are two $m \times n$ matrices, then $A + B$ is an $m \times n$ matrix in which each element is the sum of the corresponding elements of A and B .

Subtraction of Matrices

If A and B are two $m \times n$ matrices, then $A - B$ is an $m \times n$ matrix in which each element is the difference of the corresponding elements of A and B .

b. Find $A - B$ if $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$.

EXAMPLE Solve Problems

- 3 SCHOOL ATHLETES** The table below shows the total number of student athletes and the number of female athletes in three high schools. Use matrices to find the number of male athletes in each school.

School	Total Number of Athletes	Female Athletes
Jefferson	863	281
South	472	186
Ferguson	1023	346

The data in the table can be organized in matrices.

Find the difference of the matrix that represents total number of and the matrix that represents the number of athletes.

Total	Female	Male	
$\begin{bmatrix} \text{ } \\ \text{ } \\ \text{ } \end{bmatrix}$	$\begin{bmatrix} \text{ } \\ \text{ } \\ \text{ } \end{bmatrix}$	$\begin{bmatrix} \text{ } \\ \text{ } \\ \text{ } \end{bmatrix}$	Subtract corresponding elements
$=$			
$\begin{bmatrix} \text{ } \\ \text{ } \\ \text{ } \end{bmatrix}$			

There are male athletes at Jefferson, male athletes at South, and male athletes at Ferguson.

Check Your Progress

The table shows the percent of students at Clark High School who passed the 9th and 10th grade proficiency tests in 2005 and 2006. Use matrices to find how the percent of passing students changed from 2005 to 2006.

Proficiency Tests Passing Percentages				
Year	2001		2002	
Grade	9	10	9	10
Math	86.2	87.3	88.4	89.6
Reading	83.5	90.1	81.9	91.2
Science	79.6	89.7	85.0	89.9
Citizenship	86.1	87.4	86.4	85.7

BUILD YOUR VOCABULARY (page 85–86)

You can multiply any matrix by a constant called a . This operation is called **scalar multiplication**.

EXAMPLE**Multiply a Matrix by a Scalar****KEY CONCEPT****Scalar Multiplication**

The product of a scalar k and an $m \times n$ matrix is the matrix in which each element equals k times the corresponding elements of the original matrix.

FOLDABLES

Under the tab for Lesson 4-2, write your own example that involves scalar multiplication. Then perform the multiplication.

3 If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$, find $2A$.

$$2A = 2 \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$$

Substitution

$$= \begin{bmatrix} \boxed{} & 1(2) \\ -1(2) & \boxed{} \\ \boxed{} & 5(2) \end{bmatrix}$$

Multiply each element by 2.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Simplify.

EXAMPLE Combination of Matrix Operations

- 5 If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$, find $4A - 3B$.

Perform the scalar multiplication first. Then subtract the matrices.

$$4A - 3B$$

$$= 4 \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 3 \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$$

Substitution

$$= \begin{bmatrix} 4(2) & 4(3) \\ 4(-1) & 4(0) \end{bmatrix} - \begin{bmatrix} 3(-2) & 3(1) \\ 3(0) & 3(-1) \end{bmatrix}$$

Multiply each element in the first matrix by 4 and multiply each element in the second matrix by 3.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} - \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Simplify.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Subtract corresponding elements.

$$= \begin{bmatrix} 14 & 9 \\ -4 & 3 \end{bmatrix}$$

Simplify.

Check Your Progress

- a. If $A = \begin{bmatrix} 2 & 3 & 0 \\ -1 & -5 & 6 \end{bmatrix}$ find $4A$.

- b. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$, find $5A - 2B$.

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____

4-3

Multiplying Matrices



Preparation for TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(B)** Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

EXAMPLE Dimensions of Matrix Products

MAIN IDEAS

- Multiply matrices.
- Use the properties of matrix multiplication.

KEY CONCEPT

Multiplying Matrices

The element a_{ij} of AB is the sum of the products of the corresponding elements in row i of A and column j of B .

FOLDABLES

Under the tab for Lesson 4-3, write an example of multiplying square matrices.

- 1** a. Determine if the product of $A_{3 \times 4}$ and $B_{4 \times 2}$ is defined. If so, state the dimensions of the product.

$$A \cdot B = AB$$

$$3 \times 4 \quad 4 \times 2 \quad \boxed{}$$

The inner dimensions are $\boxed{}$ so the matrix product is $\boxed{}$. The dimensions of the product are $\boxed{}$.

- b. Determine whether the product of $A_{3 \times 2}$ and $B_{4 \times 3}$ is defined. If so, state the dimensions of the product.

$$A \cdot B$$

$$3 \times 2 \quad 4 \times 3$$

The inner dimensions are $\boxed{}$, so the matrix product is $\boxed{}$.

Check Your Progress

Determine if the product of $A_{2 \times 3}$ and $B_{2 \times 3}$ is defined. If so, state the dimensions of the product.

EXAMPLE Multiply Square Matrices

- 2** Find RS if $R = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $S = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$.

$$RS = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{} + \boxed{} & 3(1) + 2(-1) \\ -1(-2) + 0(1) & \boxed{} + \boxed{} \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Check Your Progress

Find RS if $R = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$ and
 $S = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}$.

EXAMPLE**Multiply Matrices with Different Dimensions****WRITE IT**

Is multiplication of matrices commutative? Explain.

- 3** **CHESS** Three teams competed in the final round of the Chess Club's championships. For each win, a team was awarded 3 points and for each draw a team received 1 point. Which team won the tournament?

Team	Wins	Draws
Blue	5	4
Red	6	3
Green	4	5

$$RP = \begin{bmatrix} 5 & 4 \\ 6 & 3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Write an equation.

$$= \begin{bmatrix} \boxed{} + \boxed{} \\ \boxed{} + \boxed{} \\ \boxed{} + \boxed{} \end{bmatrix}$$

Multiply columns by rows.

$$= \begin{bmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{bmatrix}$$

Simplify.

The labels for the product matrix are shown. The red team won the championship with a total of

$\boxed{}$ points.

Total Points

Blue $\begin{bmatrix} \boxed{} \end{bmatrix}$
 Red $\begin{bmatrix} \boxed{} \end{bmatrix}$
 Green $\begin{bmatrix} \boxed{} \end{bmatrix}$

Check Your Progress

Three players made the points listed below. They scored 1 point for the free-throws, 2 points for the 2-point shots, and 3 points for the 3-point shots. How many points did each player score and who scored the most points?

Player	Free-throws	2-point	3-point
Warton	2	3	2
Bryant	5	1	0
Chris	2	4	5

EXAMPLE**Commutative Property**

4 Find each product if $K = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ and $L = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}$.

a. $KL = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}$ Substitution

$$= \begin{bmatrix} -3 + 8 + 0 & \boxed{} \\ \boxed{} & 2 - 6 + 0 \end{bmatrix}$$

Multiply.

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Simplify.

b. $LK = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ Substitution

$$= \begin{bmatrix} -3 + 2 & 2 + 4 & \boxed{} \\ \boxed{} & 8 - 6 & 8 - 0 \\ 0 + 1 & \boxed{} & 0 - 0 \end{bmatrix}$$

Multiply.

$$= \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}$$

Simplify.

Check Your Progress

Find each product if $A = \begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$
and $B = \begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}$.

a. AB

b. BA

EXAMPLE**Distributive Property**

5 Find each product if $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix}$.

$$\begin{aligned} \text{a. } A(B + C) &= \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 2 & -2 \end{bmatrix} && \text{Add.} \\ &= \begin{bmatrix} \boxed{} & -1 - 4 \\ \boxed{} & 0 - 2 \end{bmatrix} \text{ or } \begin{bmatrix} 6 & \boxed{} \\ 2 & \boxed{} \end{bmatrix} \\ &&& \text{Multiply.} \end{aligned}$$

$$\begin{aligned} \text{b. } AB + AC &= \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} && \text{Multiply.} \\ &= \begin{bmatrix} 5 + 1 & \boxed{} \\ 3 - 1 & \boxed{} \end{bmatrix} \text{ or } \begin{bmatrix} 6 & \boxed{} \\ 2 & \boxed{} \end{bmatrix} \\ &&& \text{Add.} \end{aligned}$$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Check Your Progress

Find $AB + AC$ if $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$,

$B = \begin{bmatrix} 2 & 0 \\ 1 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -3 & 3 \end{bmatrix}$.

Transformations with Matrices



Preparation for TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(B)** Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

BUILD YOUR VOCABULARY (pages 85–86)

MAIN IDEAS

- Use matrices to determine the coordinates of a translated or dilated figure.
- Use matrix multiplication to find the coordinates of a reflected or rotated figure.

Transformations are functions that map points of a **preimage** onto its **image**.

A **translation** occurs when a figure is moved from one location to another without changing its , , or orientation.

EXAMPLE Translate a Figure

- Find the coordinates of the vertices of the image of quadrilateral $ABCD$ with $A(-5, -1)$, $B(-2, -1)$, $C(-1, -4)$, and $D(-3, -5)$ if it is moved 3 units to the right and 4 units up.

Write the vertex matrix for quadrilateral $ABCD$.

$$\begin{bmatrix} \boxed{} & \boxed{} & -1 & \boxed{} \\ -1 & -1 & \boxed{} & -5 \end{bmatrix}$$

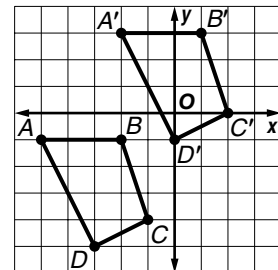
To translate the quadrilateral 3 units to the right, add 3 to each x -coordinate. To translate the figure 4 units up, add 4 to each y -coordinate. This can be done by adding the translation matrix to the vertex matrix.

$$\begin{array}{ccc} \text{Vertex Matrix} & & \text{Translation} \\ \text{of } ABCD & & \text{Matrix} \\ \begin{bmatrix} -5 & -2 & -1 & -3 \\ -1 & -1 & -4 & -5 \end{bmatrix} & + & \begin{bmatrix} & & & \\ & & & \end{bmatrix} \\ & = & \begin{bmatrix} & & & \\ & & & \end{bmatrix} \end{array}$$

The coordinates of $A'B'C'D'$ are

A' , B' ,
 C' , D' .

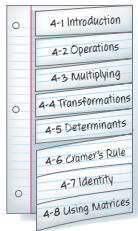
By graphing the preimage and the image, you find that the coordinates of $A'B'C'D'$ are correct.



FOLDABLES

ORGANIZE IT

Under the tab for Lesson 4-4, write each new Vocabulary Builder word. Then give an example of each word.



Check Your Progress

Find the coordinates of the vertices of the image of quadrilateral $HIJK$ with $H(2, 3)$, $I(3, -1)$, $J(-1, -3)$, and $K(-2, 5)$ if it is moved 2 units to the left and 2 units up.

BUILD YOUR VOCABULARY (pages 85–86)

A **reflection** occurs when every point of a figure is mapped to a corresponding image across using a *reflection matrix*.

A **rotation** occurs when a figure is moved around a , usually the .

EXAMPLE**Find a Translation Matrix****2**

TEST EXAMPLE Rectangle $E'F'G'H'$ is the result of a translation of rectangle $EFGH$. A table of the vertices of each rectangle is shown. Find the coordinates of G' .

Rectangle $EFGH$	Rectangle $E'F'G'H'$
$E(-2, 2)$	$E'(-5, 0)$
$F(4, 2)$	$F'(1, 0)$
$G(4, -2)$	$G'(\quad, \quad)$
$H(-2, -2)$	$H'(-5, -4)$

A (1, 0)**C** (7, 0)**B** (1, -4)**D** (7, -4)**Read the Test Item**

You are given the coordinates of the preimage and image of points E , F , and H . Use this information to find the translation matrix. Then you can use the translation matrix to find the coordinates of G .

Solve the Test Item

Step 1 Write a matrix equation. Let (c, d) represent the coordinates of G .

$$\begin{bmatrix} -2 & 4 & 4 & -2 \\ 2 & 2 & -2 & -2 \end{bmatrix} + \begin{bmatrix} x & x & x & x \\ y & y & y & y \end{bmatrix} = \begin{bmatrix} -5 & 1 & c & -5 \\ 0 & 0 & d & -4 \end{bmatrix}$$

$$\left[\begin{array}{cccc} & & & \end{array} \right] = \begin{bmatrix} -5 & 1 & c & -5 \\ 0 & 0 & d & -4 \end{bmatrix}$$

Step 2 The matrices are equal, so corresponding elements are equal.

$$-2 + x = -5 \quad \text{Solve for } x. \quad 2 + y = 0 \quad \text{Solve for } y.$$

$$x = \boxed{} \quad y = \boxed{}$$

Step 3 Use the values for x and y to find the values for $G'(c, d)$.

$$4 + (-3) = c \quad -2 + (-2) = d$$

$$\boxed{} = c \quad \boxed{} = d$$

So, the coordinates for G are $\boxed{}$, and the answer is $\boxed{}$.

EXAMPLE Dilation

3 Dilate $\triangle XYZ$ with $X(1, 2)$, $Y(3, -1)$, and $Z(-1, -2)$ so that its perimeter is twice the original perimeter. Find the coordinates of the vertices of $\triangle X'Y'Z'$.

If the perimeter of a figure is twice the original perimeter, then the lengths of the sides of the figure will be $\boxed{}$ the measure of the original lengths. Multiply the vertex matrix by the scale factor of $\boxed{}$.

$$2 \left[\begin{array}{cc} & \\ & \end{array} \right] = \left[\begin{array}{cc} & \\ & \end{array} \right]$$

The coordinates of the vertices of $\triangle XYZ$ are $X' \boxed{}$, $Y' \boxed{}$, and $Z' \boxed{}$.

Check Your Progress

- a. Rectangle $A'B'C'D'$ is the translation of rectangle $ABCD$ with vertices $A(-4, 5)$, $B(-1, 5)$, $C(-1, 0)$, and $D(-4, 0)$. What are the coordinates of A' ?

A $(-13, 10)$ **B** $(5, 10)$ **C** $(5, 0)$ **D** $(-13, 0)$

- b. $\triangle ABC$ has vertices $A(2, 1)$, $B(-3, -2)$, and $C(1, 4)$. If $\triangle ABC$ is dilated so its perimeter is 4 times the original perimeter, what are the coordinates of the vertices of $\triangle A'B'C'$?

KEY CONCEPT

Reflection Matrices

$$\text{x-axis } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{y-axis } \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{line } y = x \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

EXAMPLE Reflection

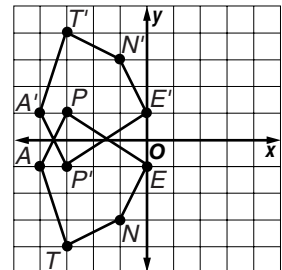
- 3 Find the coordinates of the vertices of the image of pentagon $PENTA$ with $P(-3, 1)$, $E(0, -1)$, $N(-1, -3)$, $T(-3, -4)$, and $A(-4, -1)$ after a reflection across the x -axis.

Write the ordered pairs as a vertex matrix. Then

the vertex matrix by the matrix for the x -axis.

$$\begin{bmatrix} & & & & \\ & & & & \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 & -1 & -3 & -4 \\ 1 & -1 & -3 & -4 & -1 \end{bmatrix} \\ = \begin{bmatrix} -3 & & -1 & & \\ & 1 & & 4 & 1 \end{bmatrix}$$

The coordinates of the vertices of $P'E'N'T'A'$ are $P'(-3, -1)$, $E'(0, 1)$, $N'(-1, 3)$, $T'(-3, 4)$, and $A'(-4, 1)$. The graph of the preimage and image shows that the coordinates of $P'E'N'T'A'$ are correct.



KEY CONCEPT

Reflection Matrices

For a counterclockwise rotation about the origin of:

$$90^\circ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$180^\circ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$270^\circ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Check Your Progress

- Find the coordinates of the vertices of the image of pentagon $PENTA$ with $P(-5, 0)$, $E(-3, 3)$, $N(1, 2)$, $T(1, -1)$, and $A(-4, -2)$ after a reflection across the line $y = x$.

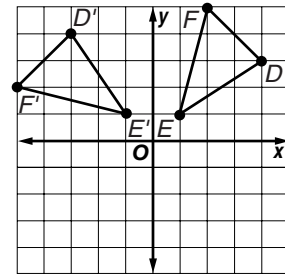
EXAMPLE Rotation

- 5** Find the coordinates of the vertices of the image of $\triangle DEF$ with $D(4, 3)$, $E(1, 1)$, and $F(2, 5)$ after it is rotated 90° counterclockwise about the origin.

Write the ordered pairs in a vertex matrix. Then multiply the vertex matrix by the rotation matrix.

$$\begin{bmatrix} & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} & & -5 \\ 4 & 1 & \end{bmatrix}$$

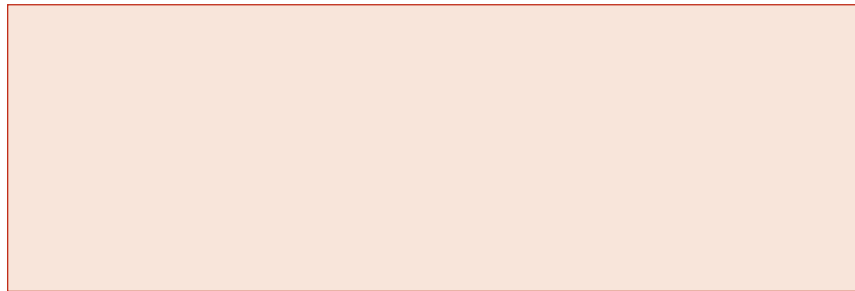
The coordinates of the vertices of triangle $D'E'F'$ are D' , E' , F' .



The graph of the preimage and image show that the coordinates of $D'E'F'$ are correct.

Check Your Progress

Find the coordinates of the vertices of the image of $\triangle TRI$ with $T(-1, 2)$, $R(-3, 0)$ and $I(-2, -2)$ after it is rotated 180° counterclockwise about the origin.

**HOMEWORK ASSIGNMENT**

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Determinants



Preparation for TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(B)** Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

EXAMPLE

Second-Order Determinant

MAIN IDEAS

- Evaluate the determinant of a 2×2 matrix.
- Evaluate the determinant of a 3×3 matrix.

KEY CONCEPT

Second-Order Determinant The value of a second-order determinant is found by calculating the difference of the products of the two diagonals.

1 Find the value of the determinant $\begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix}$.

$$\begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix} = \boxed{} - \boxed{} \quad \text{Definition of determinant}$$

$$= \boxed{} + \boxed{} \quad \text{Multiply.}$$

$$= \boxed{} \quad \text{Simplify.}$$

EXAMPLE

Expansion by Minors

2 Evaluate $\begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 4 & -2 & -3 \end{vmatrix}$ using expansion by minors.

Decide which row of elements to use for the expansion. For this example, let's use the first row.

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 4 & -2 & -3 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -3 & \boxed{} \\ \boxed{} & -3 \end{vmatrix} - 0 \begin{vmatrix} 2 & 3 \\ 4 & -3 \end{vmatrix} + (-1) \begin{vmatrix} \boxed{} & -1 \\ 4 & \boxed{} \end{vmatrix}$$

$$= 1(3 - (\boxed{})) - 0(\boxed{} - 12) - 1(-4 - (\boxed{}))$$

$$= 1(\boxed{}) - \boxed{} - 1(\boxed{})$$

$$= \boxed{}$$

EXAMPLE

- 4 SURVEYING** A survey crew located three points on a map that formed the vertices of a triangular area. A coordinate grid in which one unit equals 10 inches is placed over the map so that the vertices are located at $(0, -1)$, $(-2, -6)$, and $(3, -2)$. Use a determinant to find the area of the triangle.

$$A = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Area Formula

$$= \frac{1}{2} \begin{vmatrix} 0 & -1 & 1 \\ -2 & -6 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

$$(a, b) = \boxed{},$$

$$(c, d) = \boxed{},$$

$$(e, f) = \boxed{}$$

$$= \frac{1}{2} \left[0 \begin{vmatrix} -6 & \boxed{} \\ \boxed{} & 1 \end{vmatrix} - (-1) \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} \boxed{} & -6 \\ 3 & \boxed{} \end{vmatrix} \right]$$

Expansion by minors

$$= \frac{1}{2} \left[0(-6 - (\boxed{})) - (-1)(-2 - \boxed{}) + 1(4 - (\boxed{})) \right]$$

Evaluate 2×2 determinants.

$$= \frac{1}{2} (\boxed{} - \boxed{} + \boxed{})$$
 Multiply.

$$= \frac{1}{2} (\boxed{})$$
 or $\boxed{}$ Simplify.

Remember that 1 unit equals 10 inches, so 1 square unit = 10×10 or 100 square inches. Thus, the area is $\boxed{} \times 100$ or $\boxed{}$ square inches.

HOMEWORK ASSIGNMENT

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BUILD YOUR VOCABULARY (page 85)

Cramer's Rule uses determinants to solve systems of equations.

MAIN IDEAS

- Solve systems of two linear equations by using Cramer's Rule.
- Solve systems of three linear equations by using Cramer's Rule.

EXAMPLE System of Two Equations

1 Use Cramer's Rule to solve the system of equations $5x + 4y = 28$ and $3x - 2y = 8$.

KEY CONCEPT

Cramer's Rule for Two Variables The solution of the system of linear equations $ax + by = e$ and $cx + dy = f$ is (x, y) , where

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.



Under the tab for Lesson 4-6, write this rule.



TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns** or inequalities in two unknowns **to solve problems.** **(B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.**

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

Cramer's Rule

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} \square & \square \\ 8 & \square \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 3 & \square \end{vmatrix}}$$

$a = 5, b = 4,$
 $c = 3, d = -2,$
 $e = 28,$ and
 $f = 8$

$$= \frac{\begin{vmatrix} \square & 28 \\ 3 & \square \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 3 & \square \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} \square & \square \\ \square & \square \end{vmatrix}}{5(-2) - 3(4)}$$

Evaluate.

$$= \frac{5(8) - 3(28)}{5(-2) - 3(4)}$$

$$= \square \text{ or } \square$$

Simplify.

$$= \square \text{ or } \square$$

The solution is \square .

Check Your Progress

Use Cramer's Rule to solve the system of equations $3x + 2y = 1$ and $2x - 5y = -12$.

EXAMPLE

System of Three Equations

KEY CONCEPT

The solution of the system whose equations are

$$ax + by + cz = j$$

$$dx + ey + fz = k$$

$$gx + hy + iz = l$$

is (x, y, z) , where

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}},$$

$$y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}},$$

$$z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \text{ and}$$

$$\text{and } \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0.$$

2 Use Cramer's Rule to solve the system of equations.

$$2x - 3y + z = 5$$

$$x + 2y + z = -1$$

$$x - 3y + 2z = 1$$

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} = \frac{\begin{vmatrix} 5 & -3 & 1 \\ -1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} = \frac{\begin{vmatrix} 2 & 5 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}$$

$$z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} = \frac{\begin{vmatrix} 2 & -3 & 5 \\ 1 & 2 & -1 \\ 1 & -3 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}}$$

Use a calculator to evaluate each determinant.

$$x = \frac{\boxed{}}{\boxed{}} \text{ or } \frac{9}{4} \quad y = \frac{-9}{12} \text{ or } -\frac{\boxed{}}{\boxed{}} \quad z = \frac{\boxed{}}{\boxed{}} \text{ or } -\frac{7}{4}$$

The solution is $\boxed{}$.

Check Your Progress

Use Cramer's Rule to solve the system of equations.

$$2x + y + z = -3$$

$$-3x + 2y - z = 5$$

$$x - y + 3z = 1$$

HOMEWORK ASSIGNMENT

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Identify and Inverse Matrices



Preparation for TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(B)** Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

BUILD YOUR VOCABULARY (page 85)

MAIN IDEAS

- Determine whether two matrices are inverses.
- Find the inverse of a 2×2 matrix.

The **identity matrix** is a square matrix that, when multiplied by another matrix, equals that same matrix.

Two $n \times n$ matrices are **inverses** of each other if their

is the .

EXAMPLE

Verify Inverse Matrices

1 Determine whether each pair of matrices are inverses.

$$X = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Find $X \cdot Y$.

$$X \cdot Y = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{} & 6 - 6 \\ -1 + 1 & \boxed{} \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Find $Y \cdot X$.

$$Y \cdot X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 2 & \boxed{} \\ 3 - 3 & \boxed{} \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

Since $X \cdot Y = Y \cdot X = I$, X and Y are inverses.

Check Your Progress

Determine whether each pair of matrices are inverses.

a. $A = \begin{bmatrix} -2 & 3 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 2 \\ 1 & 1 \end{bmatrix}$

b. $C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

KEY CONCEPTS

Identity Matrix for Multiplication The identity matrix for multiplication I is a square matrix with 1 for every element of the main diagonal, from upper left to lower right, and 0 in all other positions. For any square matrix A of the same dimensions as I , $A \cdot I = I \cdot A = A$.

Inverse of a 2×2 matrix
The inverse of Matrix A

$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} \text{ is } A^{-1} =$$

$$\frac{1}{ad - bc} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix} \text{ where } ad - bc \neq 0.$$

EXAMPLE Find the Inverse of a Matrix**2** Find the inverse of each matrix, if it exists.

a. $S = \begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix}$

Find the value of the determinant.

$$\begin{vmatrix} -1 & 0 \\ 8 & -2 \end{vmatrix} = 2 - \boxed{} = \boxed{}$$

Since the determinant is not equal to 0, S^{-1} exists.

$$\begin{aligned}
 S^{-1} &= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\
 &= \frac{1}{-1(-2) - \boxed{}(0)} \begin{bmatrix} \boxed{} & -1 \\ -8 & \boxed{} \end{bmatrix} \\
 &= \frac{1}{\boxed{}} \begin{bmatrix} -2 & 0 \\ -8 & -1 \end{bmatrix} \text{ or } \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & -\frac{1}{2} \end{bmatrix}
 \end{aligned}$$

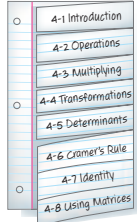
b. $T = \begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix}$

Find the value of the determinant.

$$\begin{vmatrix} -4 & 6 \\ -2 & 3 \end{vmatrix} = \boxed{} + \boxed{} = \boxed{}$$

Since the determinant equals 0, T^{-1} does not exist.**FOLDABLES™****ORGANIZE IT**

Under the tab for Lesson 4-7, write your own 2×2 matrix. Then find the inverse of the matrix, if it exists.

**Check Your Progress** Find the inverse of each matrix if it exists.

a. $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

b. $B = \begin{bmatrix} 6 & -2 \\ 7 & -2 \end{bmatrix}$

EXAMPLE

- 3 a. **CRYPTOGRAPHY** Use the table at the beginning of the lesson to assign a number to each letter in the message ALWAYS_SMILE. Then code the message with the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. Convert the message to numbers using the table.

A	L	W	A	Y	S	_	S	M	I	L	E
1	12	23	1	25	19	0	19	13	9	12	5

Write the message in matrix form. Arrange the numbers in a matrix with 2 columns and as many rows as are needed. Then multiply the message matrix B by the coding matrix A .

$$BA = \begin{bmatrix} 1 & 12 \\ 23 & 1 \\ 25 & 19 \\ 0 & 19 \\ 13 & 9 \\ 12 & 5 \end{bmatrix} \cdot \begin{bmatrix} \\ \end{bmatrix} \quad \text{Write an equation.}$$

$$= \begin{bmatrix} & \\ & \\ & \\ & \\ & \\ & \end{bmatrix} \quad \text{Multiply the matrices.}$$

$$= \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix} \quad \text{Simplify.}$$

The coded message is

b. Use the inverse matrix A^{-1} to decode the message in part a. First find the inverse matrix of $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

$$\begin{aligned}
 A^{-1} &= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} && \text{Definition of inverse} \\
 &= \frac{1}{1(3) - 2(1)} \begin{bmatrix} & \\ & \end{bmatrix} && a = \boxed{}, b = \boxed{}, \\
 &= \boxed{} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} && c = \boxed{}, d = \boxed{}, \\
 &= \begin{bmatrix} & \\ & \end{bmatrix} && \text{Simplify.} \\
 & && \text{Simplify.}
 \end{aligned}$$

Next, decode the message by multiplying the coded matrix C by A^{-1} .

$$CA^{-1} = \begin{bmatrix} 13 & 38 \\ 24 & 49 \\ 44 & 107 \\ 19 & 57 \\ 22 & 53 \\ 17 & 39 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \qquad \text{Write an equation.}$$

$$= \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \qquad \text{Multiply the matrices.}$$

$$= \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix} \qquad \text{Write an equation.}$$

Use the table again to convert the numbers to letters. You can now read the message.

A	L	W	A	Y	S	-	S	M	I	L	E
1	12	23	1	25	19	0	19	13	9	12	5

Check Your Progress

- a. Use the table to assign a number to each letter in the message FUN_MATH. Then code the matrix $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$.

Code		
_ 0	I 9	R 18
A 1	J 10	S 19
B 2	K 11	T 20
C 3	L 12	U 21
D 4	M 13	V 22
E 5	N 14	W 23
F 6	O 15	X 24
G 7	P 16	Y 25
H 8	Q 17	Z 26

- b. Use the inverse matrix of $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ to decode the message 12 | 63 | 28 | 14 | 26 | 16 | 40 | 44.

HOMEWORK ASSIGNMENT

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MAIN IDEAS

- Write matrix equations for systems of equations.
- Solve systems of equations using matrix equations.

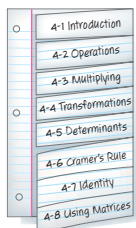
FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 4-8, write a matrix equation for this system of equations.

$$x + 2y = 5$$

$$3x + 5y = 14$$



TEKS 2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems. (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.**

BUILD YOUR VOCABULARY (page 86)

A system of equations can be written with expressing the system of equations as a **matrix equation**.

EXAMPLE Two-Variable Matrix Equation

- 1 Write a matrix equation for the system of equations.

$$x + 3y = 3$$

$$x + 2y = 7$$

Determine the coefficient, variable, and constant matrices.

$$\begin{array}{l} x + 3y = 3 \\ x + 2y = 7 \end{array} \quad \left[\begin{array}{cc} \square & \square \\ \square & \square \end{array} \right] \left[\begin{array}{c} \square \\ \square \end{array} \right] = \left[\begin{array}{c} \square \\ \square \end{array} \right]$$

Write the matrix equation.

$$\begin{array}{c} A \\ \left[\begin{array}{cc} \square & \square \\ \square & \square \end{array} \right] \end{array} \cdot \begin{array}{c} X \\ \left[\begin{array}{c} \square \\ \square \end{array} \right] \end{array} = \begin{array}{c} B \\ \left[\begin{array}{c} \square \\ \square \end{array} \right] \end{array}$$

EXAMPLE Problem Solve with Matrix Equations

- 2 **FABRICS** The table below shows the composition of three types of fabric.

Type	Wool	Silk	Cotton	Cost
R	10%	20%	70%	\$7
S	20%	30%	50%	\$8
T	20%	50%	30%	\$10

- a. Write a system of equations that represents the total cost for each of the three fabric components.

Let w represent the cost of wool.

Let s represent the cost of silk.

Let c represent the cost of cotton.

$$\begin{aligned}0.1w + 0.2s + 0.7c &= 7 \\0.2w + 0.3s + 0.5c &= 8 \\0.2w + 0.5s + 0.3c &= 10\end{aligned}$$

- b. Write a matrix equation for the system of equations. Determine the coefficient, variable, and constant matrices. Then write the matrix equation.

$$\begin{aligned}0.1w + 0.2s + 0.7c &= 7 \\0.2w + 0.3s + 0.5c &= 8 \\0.2w + 0.5s + 0.3c &= 10\end{aligned} \rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} w \\ s \\ c \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

EXAMPLE Solve a System of Equations

- 3 Use a matrix equation to solve the system of equations.

a. $5x + 3y = 13$
 $4x + 7y = -8$

The matrix equation is $\begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$

when $A = \begin{bmatrix} & \\ & \end{bmatrix}$, $X = \begin{bmatrix} \\ \end{bmatrix}$, and $B = \begin{bmatrix} \\ \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{\begin{bmatrix} & \\ & \end{bmatrix}}{\begin{bmatrix} & \\ & \end{bmatrix} - \begin{bmatrix} & \\ & \end{bmatrix}} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \text{ or } \begin{bmatrix} & \\ & \end{bmatrix}$$

Step 2 Multiply each side of the matrix equation by the inverse matrix.

$$\begin{aligned}\frac{1}{23} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{23} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} \cdot \begin{bmatrix} \\ \end{bmatrix} &= \frac{1}{23} \begin{bmatrix} \\ \end{bmatrix} \\ \begin{bmatrix} \\ \end{bmatrix} &= \begin{bmatrix} \\ \end{bmatrix}\end{aligned}$$

The solution is $\begin{bmatrix} \\ \end{bmatrix}$.

b. $10x + 5y = 15$
 $6x + 3y = -6$

The matrix equation is $\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$

when $A = \begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$.

Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{30 - 30} \begin{bmatrix} & \\ & \end{bmatrix} \text{ or } \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -6 & 10 \end{bmatrix}$$

The determinant of the coefficient matrix $\begin{bmatrix} 3 & -5 \\ -6 & 10 \end{bmatrix}$ is

, so A^{-1} does not exist. There is no unique of this system.

Check Your Progress

- a.** Write a matrix equation for the system of equations.
 $x - 2y = 6$ and $3x + 4y = 7$.


- b.** Use a matrix equation to solve the system of equations.
 $3x + 4y = 0$ and $x - 2y = 10$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BRINGING IT ALL TOGETHER**STUDY GUIDE**

	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 4 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 4, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 85–86) to help you solve the puzzle.

4-1

Introduction to Matrices

Match each matrix with its dimensions.

1. $\begin{bmatrix} 3 & 2 & 5 \\ -1 & 0 & 6 \end{bmatrix}$

2. $[30 \quad -84]$

3. $\begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 4 & 0 \\ -1 & 2 \\ 6 & 1 \end{bmatrix}$

a. 3×2

b. 2×3

c. 2×2

d. 1×2

5. Write a system of equations that you could use to solve the following matrix equation for x , y , z .

$$\begin{bmatrix} 3x \\ x + y \\ y - z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 6 \end{bmatrix}$$

4-2

Operations with Matrices

6. Use $M = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -1 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} -2 & 5 & -4 \\ 3 & 1 & 0 \end{bmatrix}$ to find $2M + 3N$.

4-3

Multiplying Matrices

Determine whether each indicated matrix product is defined. If so, state the dimensions of the product. If not, write *undefined*.

7. $M_{3 \times 2}$ and $N_{2 \times 3}$ MN :

8. $M_{1 \times 2}$ and $N_{1 \times 2}$ MN :

9. $M_{4 \times 1}$ and $N_{1 \times 4}$ MN :

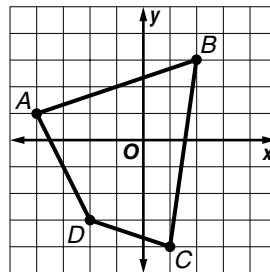
10. Find the product, if possible.

$\begin{bmatrix} 2 & 0 & 3 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 5 & 2 \\ -3 & 0 \end{bmatrix}$

4-4

Transformations with Matrices

Refer to quadrilateral $ABCD$ shown.



11. Write the vertex matrix for the quadrilateral $ABCD$.

12. Write the vertex matrix that represents the position of the quadrilateral $A'B'C'D'$ that results when quadrilateral $ABCD$ is translated 3 units to the right 2 units down.

4-5

Determinants

13. Find the value of $\begin{vmatrix} 8 & 3 \\ 2 & -1 \end{vmatrix}$.

14. Evaluate $\begin{vmatrix} 3 & 12 & -1 \\ 10 & 9 & 0 \\ -5 & 6 & -2 \end{vmatrix}$ using expansion by minors.

4-6

Cramer's Rule

15. The two sides of an angle are contained in the lines whose equations are $3x + y = 5$ and $2x + 3y = 8$. Find the coordinates of the vertex of the angle.

16. Use Cramer's Rule to solve the system of equations.

$$2x + 5y + 3z = 10$$

$$3x - y + 4z = 8$$

$$5x - 2y + 7z = 12$$

4-7

Identity and Inverse Matrices

Indicate whether each of the following statements is *true* or *false*.

17. Every element of an identity matrix is 1.

18. There is a 3×2 identity matrix.

19. If M is a matrix, M^{-1} represents the reciprocal of M .

20. Every square matrix has an inverse.

21. Determine whether $A = \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$ and

$$B = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$$
 are inverses.

4-8

Using Matrices to Solve Systems of Equations

22. Write a matrix equation for the following system equations.

$$3x + 5y = 10$$

$$2x - 4y = -7$$

23. Solve the system of equations $4x - 5y = 4$ and $2x - y = 8$ using inverse matrices.



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 4.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 4 Practice Test on page 229 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 4 Study Guide and Review on pages 224–228 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 229.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 4 Foldable.
- Then complete the Chapter 4 Study Guide and Review on pages 224–228 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 229 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Quadratic Functions and Inequalities

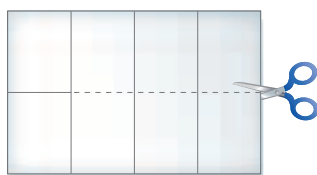


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" by 17" paper.

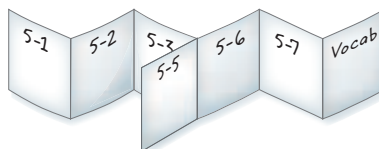
STEP 1

Fold in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.



STEP 2

Staple along the lengthwise fold and staple the uncut section at the top. Label each section with a lesson number and close to form a booklet.



NOTE-TAKING TIP: When you take notes, you may wish to use a highlighting marker to emphasize important concepts.

BUILD YOUR VOCABULARY

This is an alphabetical list of the new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
axis of symmetry			
completing the square			
complex conjugates			
complex number			
constant term			
<u>discriminant</u> dihs-KRIH-muh-nuhnt			
imaginary unit			
linear term			
maximum value			
minimum value			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
parabola puh-RA-buh-luh			
pure imaginary number			
quadratic equation kwah-DRA-tihk			
quadratic function			
quadratic inequality			
quadratic term			
root			
square root			
vertex			
vertex form			
zero			

EXAMPLE

Graph a Quadratic Function

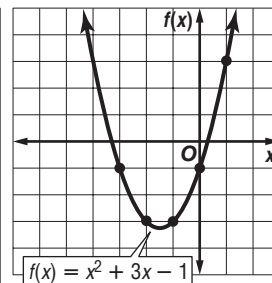
MAIN IDEAS

- Graph quadratic functions.
- Find and interpret the maximum and minimum values of a quadratic function.

1 Graph $f(x) = x^2 + 3x - 1$ by making a table of values.

First, choose integer values for x . Then, evaluate the function for each x value. Graph the function.

x	$x^2 + 3x - 1$	$f(x)$	(x, y)
-3	$(-3)^2 + 3(-3) - 1$	<input type="text"/>	<input type="text"/>
-2	$(-2)^2 + 3(-2) - 1$	<input type="text"/>	<input type="text"/>
-1	$(-1)^2 + 3(-1) - 1$	<input type="text"/>	<input type="text"/>
0	$(0)^2 + 3(0) - 1$	<input type="text"/>	<input type="text"/>
1	$(1)^2 + 3(1) - 1$	<input type="text"/>	<input type="text"/>



TEKS 2A.6

The student understands that quadratic functions can be represented in different ways and translates among their various representations.

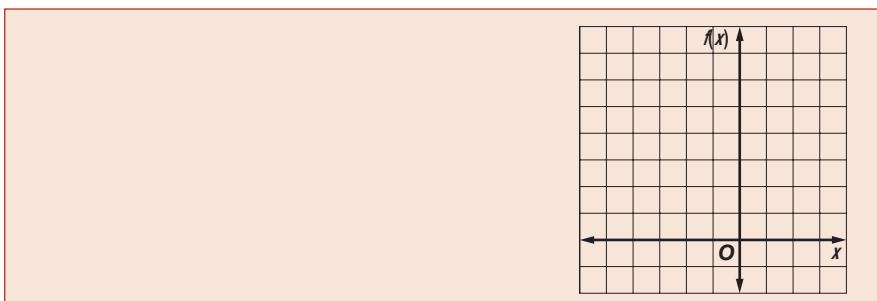
(A) Determine the reasonable domain and range values of quadratic functions,

as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities.

TEKS 2A.8 The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems.** Also addresses TEKS 2A.1(A) and 2A.6(B).

Check Your Progress

Graph $f(x) = 2x^2 + 3x + 2$.



EXAMPLE

Axis of Symmetry, y -intercept, and Vertex2 Consider the quadratic function $f(x) = 2 - 4x + x^2$.

- a. Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex.

Begin by rearranging the terms of the function. Then identify a , b , and c .

$$f(x) = ax^2 + bx + c$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$f(x) = 2 - 4x + x^2 \longrightarrow f(x) = 1x^2 - 4x + 2$$

So, $a = \square$, $b = \square$, and $c = \square$.

KEY CONCEPT

Graph of a Quadratic Function Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The y -intercept is $a(0)^2 + b(0) + c$ or c .
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The x -coordinate of the vertex is $-\frac{b}{2a}$.

The y -intercept is 2. You can find the equation of the axis of symmetry by using a and b .

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

$$x = \boxed{} \quad a = 1, b = -4$$

$$x = \boxed{} \quad \text{Simplify.}$$

The y -intercept is $\boxed{}$. The equation of the axis of symmetry is $x = \boxed{}$. Therefore, the x -coordinate of the vertex is $\boxed{}$.

b. Make a table of values that includes the vertex.

Choose some values for x that are less than 2 and some that are greater than 2.

x	$x^2 - 4x + 2$	$f(x)$	$(x, f(x))$
0	$0^2 - 4(0) + 2$	<input type="text"/>	<input type="text"/>
1	$1^2 - 4(1) + 2$	<input type="text"/>	<input type="text"/>
2	$2^2 - 4(2) + 2$	<input type="text"/>	<input type="text"/>
3	$3^2 - 4(3) + 2$	<input type="text"/>	<input type="text"/>
4	$4^2 - 4(4) + 2$	<input type="text"/>	<input type="text"/>

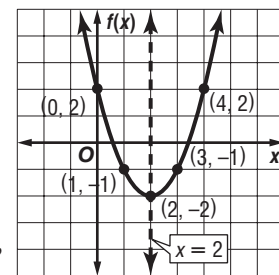
c. Use this information to graph the function.

Graph the vertex and y -intercept.

Then graph the points from your table and the y -intercept, connecting them with a smooth curve.

As a check, draw the axis of symmetry, $x = 2$, as a dashed line.

The graph of the function should be symmetric about this line.



WRITE IT

Why is it helpful to know the axis of symmetry when graphing a quadratic function?

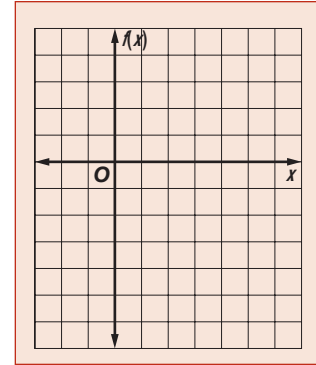
Check Your Progress Consider the quadratic function $f(x) = 3 - 6x + x^2$.

- a. Find the y -intercept, the equation of the axis of symmetry and the x -coordinate of the vertex.

- b. Make a table of values that includes the vertex.

--

- c. Use this information to graph the function.



KEY CONCEPTS

Maximum and Minimum Value The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.

FOLDABLES

On the page for Lesson 5-1, sketch a parabola. Then label the axis of symmetry, vertex, maximum value, and minimum value.

EXAMPLE

Maximum or Minimum Value

- 1 Consider the function $f(x) = -x^2 + 2x + 3$.

- a. Determine whether the function has a maximum or a minimum value.

For this function, $a =$, $b =$, and $c =$.

Since a 0, the graph opens and the function has a .

- b. State the maximum or minimum value of the function.

The maximum value of this function is the y -coordinate of the vertex.

The x -coordinate of the vertex is or .

Find the y -coordinate of the vertex by evaluating the function for $x = 1$.

$$f(x) = -x^2 + 2x + 3 \quad \text{Original function}$$

$$f(1) = \text{} \text{ or } \text{} \quad x = 1$$

The maximum value of the function is .

- c. State the domain and range of the function.

The domain is all real numbers. The range is all reals less than or equal to the value. That is

.

Check Your Progress Consider the function

$$f(x) = x^2 + 4x - 1.$$

- a. Determine whether the function has a maximum or a minimum value.

- b. State the maximum or minimum value of the function.

- c. State the domain and range of the function.

EXAMPLE

- 4 ECONOMICS** A souvenir shop sells about 200 coffee mugs each month for \$6 each. The shop owner estimates that for each \$0.50 increase in the price, he will sell about 10 fewer coffee mugs per month.

- a. How much should the owner charge for each mug in order to maximize the monthly income from their sales?

Words The income is the number of mugs multiplied by the cost per mug.

Variables Let x = the number of \$0.50 price increases.
Then $6 + 0.50x$ = the price per mug and
 $200 - 10x$ = the number of mugs sold.
Let $I(x)$ = income as a function of x .

The income is the number of mugs multiplied by the

per mug.

Equation

$$\begin{aligned} I(x) &= (200 - 10x) \cdot (6 + 0.50x) \\ &= 200(6) + 200(0.50x) - 10x(6) - 10x(0.50x) \\ &= 1200 + 100x - 60x - 5x^2 && \text{Multiply.} \\ &= \text{} + \text{} - \text{} && \text{Simplify.} \\ &= -5x^2 + 40x + 1200 && \text{Rewrite in} \\ & && \text{ax}^2 + \text{bx} + \text{c form.} \end{aligned}$$

$I(x)$ is a quadratic function with $a = -5$, $b = 40$, and $c = 1200$. Since $a < 0$, the function has a maximum value at the of the graph. Use the formula to find the x -coordinate of the vertex.

$$x\text{-coordinate of the vertex} = -\frac{b}{2a}$$

Formula for the x -coordinate of the vertex.

$$= -\frac{40}{2(-5)}$$

$$a = \text{, } b = \text{$$

$$= \text{$$

Simplify.

This means the souvenir shop should make 4 price increases of \$0.50 to maximize their income. Thus, the price of a mug should be or .

b. What is the maximum monthly income the owner can expect to make from the mugs?

To determine maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 4$.

$$I(x) = -5x^2 + 40x + 1200$$

Income function

$$I(\text{$$

$$= \text{$$

Use a calculator.

Thus, the maximum income the souvenir shop can expect is .

Check Your Progress

ECONOMICS A sports team sells about 100 coupon books for \$30 each during their annual fund-raiser. They estimate that for each \$0.50 decrease in the price, they will sell about 10 more coupon books.

a. How much should they charge for each book in order to maximize the income from their sales?

b. What is the maximum monthly income the team can expect to make from these items?

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____

MAIN IDEAS

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.



TEKS 2A.8

The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(C) Compare** and translate between **algebraic and graphical solutions of quadratic equations.** **(D) Solve quadratic equations and inequalities using graphs, tables,** and algebraic methods. Also addresses TEKS 2A.6(B) and 2A.8(A).

BUILD YOUR VOCABULARY (pages 121–122)

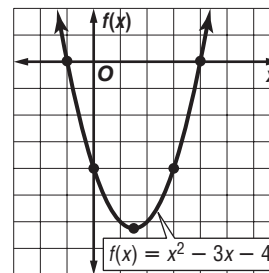
A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

The of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic is to find the **zeros** of the related quadratic .

EXAMPLE Two Real Solutions

1 Solve $x^2 - 3x - 4 = 0$ by graphing.

Graph the related quadratic function $f(x) = x^2 - 3x - 4$. The equation of the axis of symmetry is $x = \frac{-(-3)}{2(1)}$ or $\frac{3}{2}$. Make a table using x -values around $\frac{3}{2}$. Then graph each point.



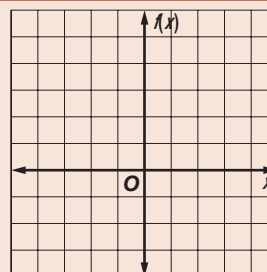
x	-1	0	1	2	3	4
$f(x)$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

From the table and the graph, we can see that the zeros of the function are -1 and 4 . The solutions of the equation are

and .

Check Your Progress

Solve $x^2 + 2x - 3 = 0$ by graphing.



EXAMPLE One Real Solution

KEY CONCEPT

Solutions of a Quadratic Equation A quadratic equation can have one real solution, two real solutions, or no real solution.

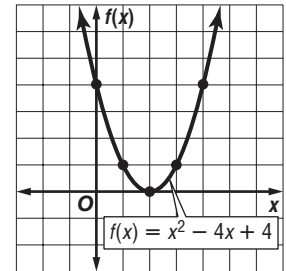
2 Solve $x^2 - 4x = -4$ by graphing.

Write the equation in $ax^2 + bx + c = 0$ form.

$x^2 - 4x = -4 \rightarrow$ $= 0$ Add 4 to each side.

Graph the related quadratic function $f(x) = x^2 - 4x + 4$.

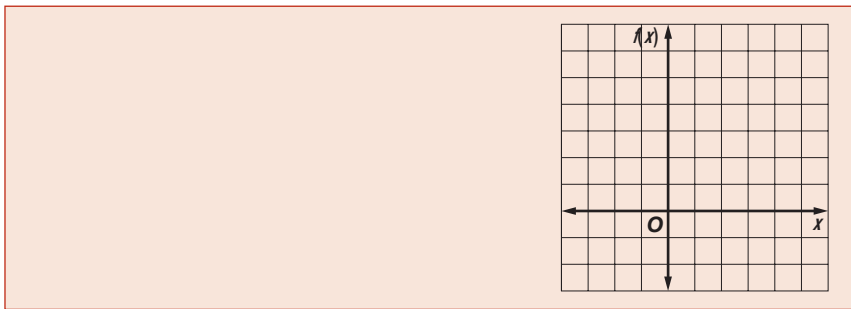
x	0	1	2	3	4
$f(x)$					



Notice that the graph has only one x -intercept, 2.

Check Your Progress

Solve $x^2 - 6x = -9$ by graphing.



EXAMPLE No Real Solution

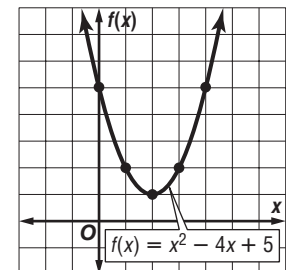
3 **NUMBER THEORY** Find two real numbers whose sum is 4 and whose product is 5 or show that no such numbers exist.

Let $x =$ one of the numbers. Then $4 - x =$ the other number.

Since the product is 5, you know that $x(4 - x) = 5$ or $-x^2 + 4x - 5 = 0$.

You can solve $x^2 - 4x + 5 = 0$ by graphing the related function $f(x) = x^2 - 4x + 5$.

x	0	1	2	3	4
$f(x)$					

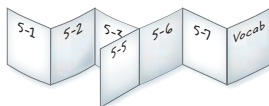


Notice that the graph has no x -intercepts. This means that the original equation has no real solution.

FOLDABLES™

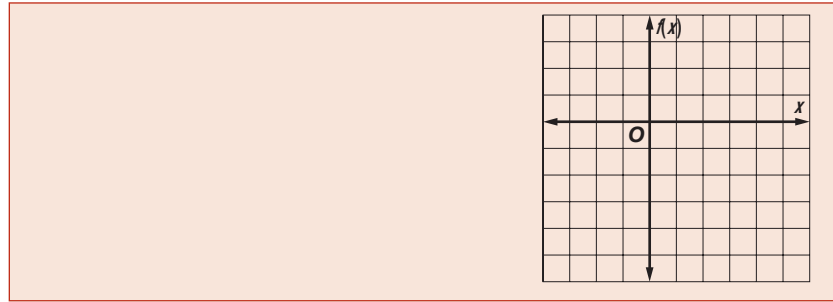
ORGANIZE IT

On the page for Lesson 5-2, sketch a graph of a parabola that has one real solution, two real solutions, and no real solution.



Check Your Progress

Find two real numbers whose sum is 7 and whose product is 14 or show that no such numbers exist.

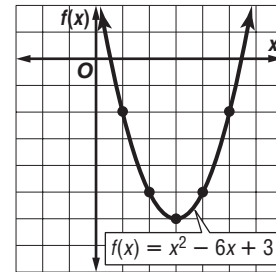


EXAMPLE Estimate Roots

4 Solve $x^2 - 6x + 3 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is

$x =$

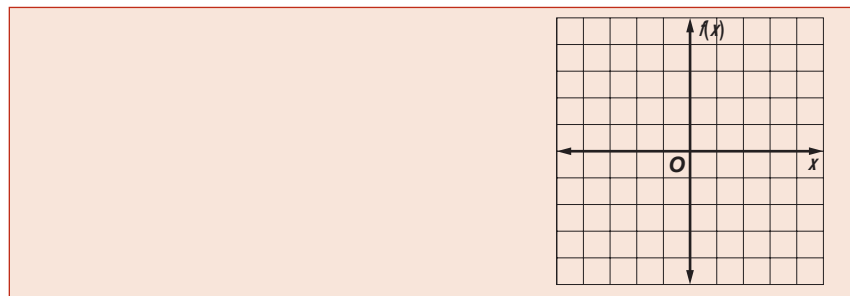


x	0	1	2	3	4	5	6
$f(x)$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

The x -intercepts of the graph are between and between .

Check Your Progress

Solve $x^2 - 4x + 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.



EXAMPLE

- 5 ROYAL GORGE BRIDGE** The highest bridge in the U.S. is the Royal Gorge Bridge in Colorado. The deck is 1053 feet above the river. Suppose a marble is dropped over the railing from a height of 3 feet above the bridge deck. How long will it take the marble to reach the surface of the water, assuming there is no air resistance? Use the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and h_0 is the initial height above the water in feet.

We need to find t when $h_0 = \boxed{}$ and $h(t) = \boxed{}$. Solve $0 = -16t^2 + 1056$.

Graph the related function $y = -16t^2 + 1056$ on a graphing calculator.

Use the ZERO feature, $\boxed{2nd}$ [CALC], to find the positive zero of the function, since time cannot be $\boxed{}$. Use the arrow keys to locate a left bound and press \boxed{ENTER} .

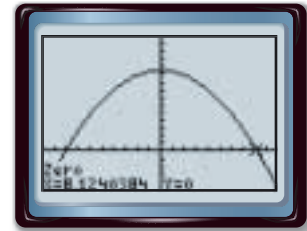
Then, locate a right bound and press \boxed{ENTER} twice.

The positive zero of the function

is approximately $\boxed{}$.

The marble would fall

for about $\boxed{}$ seconds.



$[-10, 10]$ scl: 1 by $[-500, 1500]$ scl: 100

Check Your Progress

One of the larger dams in the United States is the Hoover Dam on the Colorado River, which was built during the Great Depression. The dam is 726.4 feet tall. Suppose a marble is dropped over the railing from a height of 6 feet above the top of the dam. How long will it take the marble to reach the surface of the water, assuming there is no air resistance? Use the formula from Example 5.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Write quadratic equations in intercept form.
- Solve quadratic equations by factoring



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

TEKS 2A.6 The student understands that quadratic functions can be represented in different ways and translates among their various representations. **(C) Determine a quadratic function from its roots or a graph. Also addresses TEKS 2A.6(B), 2A.8(A), 2A.8(C), and 2A.8(D).**

EXAMPLE

Write an Equation Given Roots

- 1 Write a quadratic equation with $-\frac{2}{3}$ and 6 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

$$(x - p)(x - q) = 0 \quad \text{Write the pattern.}$$

$$\left[x - \left(-\frac{2}{3} \right) \right] (x - 6) = 0 \quad \text{Replace } p \text{ with } -\frac{2}{3} \text{ and } q \text{ with } 6.$$

$$\boxed{} = 0 \quad \text{Simplify.}$$

$$\boxed{} = 0 \quad \text{Use FOIL.}$$

$$\boxed{} = 0 \quad \text{Multiply each side by 3 so that } b \text{ is an integer.}$$

Check Your Progress

Write a quadratic equation with $-\frac{3}{4}$ and 5 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

EXAMPLE

Two or Three Terms

- 2 Factor each polynomial.

a. $2y^2 - 3y - 5$

The coefficient of the y terms must be and since

$2(-5) = -10$ and $2 + (-5) = -3$. Rewrite the expression using $-5y$ and $2y$ in place of $-3y$ and factor by grouping.

$$2y^2 - 3y - 5 = 2y^2 - 5y + 2y - 5 \quad \text{Replace } -3y.$$

$$= (2y^2 - 5y) + (2y - 5) \quad \text{Associative Property}$$

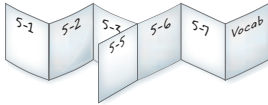
$$= \boxed{}(2y - 5) + \boxed{}(2y - 5) \quad \text{Factor.}$$

$$= \boxed{}(2y - 5) \quad \text{Distributive Property}$$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 5-3, write your own roots to a quadratic equation. Then find a quadratic equation with those roots. Check your result by graphing the related function.



b. $8x^3 - y^3$

$$8x^3 - y^3 = \boxed{}^3 - y^3$$

This is the difference of two cubes.

$$= \boxed{} [(2x)^2 + (2x)y + y^2] \quad \text{Factor.}$$

$$= \boxed{} (4x^2 + 2xy + y^2) \quad \text{Simplify.}$$

Check Your Progress Factor each polynomial.

a. $2x^2 + x - 3$

b. $3x^3 - 12x$

c. $a^3b^3 - 27$

EXAMPLE Two Roots

3 Solve each equation by factoring.

a. $x^2 = -4x$

$$x^2 = -4x$$

Original equation

$$\boxed{} = 0$$

Add 4x to each side.

$$\boxed{} = 0$$

Factor the binomial.

$$\boxed{} = 0 \text{ or } \boxed{} = 0$$

Zero Product Property

$$x = \boxed{}$$

Solve the second equation.

The solution set is $\boxed{}$.

Check Substitute $\boxed{}$ and $\boxed{}$ for x in the original equation.

$$x^2 = -4x$$

$$x^2 = -4x$$

$$\boxed{}^2 \stackrel{?}{=} -4\boxed{}$$

$$\boxed{}^2 = -4\boxed{}$$

$$\boxed{} = \boxed{} \checkmark$$

$$\boxed{} = \boxed{} \checkmark$$

KEY CONCEPT
Zero Product Property

For any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.

b. $3x^2 = 5x + 2$

$3x^2 = 5x + 2$ Original equation

= 0

Subtract 5x and 2 from each side.

= 0

Factor the trinomial.

= 0 or = 0

Zero Product Property

= $x =$

Solve each equation.

$x =$

The solution set is .

Check Your Progress

Solve each equation by

factoring.

a. $x^2 = 3x$

b. $6x^2 + 11x = -4$

EXAMPLE

Double Root

4 Solve $x^2 - 6x = -9$ by factoring.

$x^2 - 6x = -9$

Original equation

= 0

Add 9 to each side.

= 0

Factor.

= 0 or = 0

Zero Product Property

$x =$ $x =$

Solve each equation.

The solution set is .

Check Your Progress

Solve $x^2 + 10x = -25$ by factoring.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

MAIN IDEAS

- Find square roots and perform operations with pure imaginary numbers.
- Perform operations with complex numbers

BUILD YOUR VOCABULARY (pages 121–122)

i is called the imaginary unit ($i = \sqrt{-1}$).

Pure imaginary numbers are square roots of

A complex number, such as the expression $5 + 2i$, is a complex number since it has a real number (5) and a pure imaginary number ($2i$).

Two complex numbers of the form $a + bi$ and are called complex conjugates.

EXAMPLE Properties of Square Roots

1 Simplify.

$$\begin{aligned} \text{a. } \sqrt{18} &= \sqrt{9 \cdot 2} \\ &= \sqrt{9} \cdot \sqrt{2} \\ &= \end{aligned}$$

$$\begin{aligned} \text{b. } \sqrt{\frac{10}{81}} &= \frac{\sqrt{10}}{\sqrt{81}} \\ &= \end{aligned}$$



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. (B) Use complex numbers to describe the solutions of quadratic equations.

Check Your Progress

Simplify

$$\text{a. } \sqrt{75}$$

$$\text{b. } \sqrt{\frac{7}{36}}$$

REMEMBER IT



You can write i to the left or right of the radical symbol. However, i is usually written to the left so it is clear that it is not under the radical.

EXAMPLE

Square Roots of Negative Numbers

2 Simplify $\sqrt{-32y^3}$.

$$\begin{aligned} \sqrt{-32y^3} &= \sqrt{-1 \cdot 4^2 \cdot 2 \cdot y^2 \cdot y} \\ &= \sqrt{-1} \cdot \sqrt{4^2} \cdot \sqrt{2} \cdot \sqrt{y^2} \cdot \sqrt{y} \\ &= \quad \cdot 4|y| \sqrt{2} \cdot \quad \\ &= \end{aligned}$$

Check Your Progress

Simplify.

a. $\sqrt{-32}$

b. $\sqrt{-50x^5}$

EXAMPLE

Multiply Pure Imaginary Numbers

3 Simplify.

a. $-3i \cdot 2i$

$$-3i \cdot 2i = -6i^2$$

$$= -6(\text{ })$$

$$i^2 = \text{ }$$

$$= \text{ }$$

b. $\sqrt{-12} \cdot \sqrt{-2}$

$$\sqrt{-12} \cdot \sqrt{-2} = \text{ } \cdot \text{ }$$

$$= i^2\sqrt{24}$$

$$= \text{ } \text{ or } \text{ }$$

c. i^{35}

$$i^{35} = i \cdot i^{34}$$

$$= i \cdot \text{ }^{17}$$

$$= i \cdot \text{ }^{17}$$

$$= i \cdot \text{ } \text{ or } \text{ }$$

Multiplying powers

Power of a Power

$$i^2 = \text{ }$$

$$(-1)^{17} = \text{ }$$

Check Your Progress

Simplify.

a. $3i \cdot 5i$

b. $\sqrt{-2} \cdot \sqrt{-6}$

c. i^{57}

KEY CONCEPT

Complex Numbers A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.

EXAMPLE Equation with Pure Imaginary Solutions**4** Solve $5y^2 + 20 = 0$.

$$5y^2 + 20 = 0$$

Original equation

$$5y^2 = -\square$$

Subtract \square from each side.

$$y^2 = -4$$

Divide each side by \square .

$$y = \pm\sqrt{-4}$$

Square Root Property

$$y = \square$$

$$\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1}$$

Check Your Progress Solve $2x^2 + 50 = 0$.
EXAMPLE Equate Complex Numbers**5** Find the values of x and y that make the equation $2x + yi = -14 - 3i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$$2x = -14 \quad \text{Real parts}$$

$$x = -7 \quad \text{Divide each side by } \square. \quad y = \square \quad \text{Imaginary parts}$$

$$x = \square, y = \square$$

Check Your Progress Find the value of x and y that make the equation $3x - yi = 15 + 2i$ true.
EXAMPLE Add and Subtract Complex Numbers**6** Simplify.

a. $(3 + 5i) + (2 - 4i)$

$$(3 + 5i) + (2 - 4i)$$

$$= (3 + 2) + (5 - 4)i$$

Commutative and Associative Properties
Simplify.

$$= \square$$

$$\begin{aligned}
 \text{b. } & (4 - 6i) - (3 - 7i) \\
 & (4 - 6i) - (3 - 7i) \\
 & = (4 - 3) + [-6 - (-7)]i && \text{Commutative and Associative} \\
 & = \boxed{} && \text{Properties} \\
 & && \text{Simplify.}
 \end{aligned}$$

Check Your Progress Simplify.

a. $(2 + 6i) + (3 + 4i)$

b. $(3 + 2i) - (-2 + 5i)$

EXAMPLE Multiply Complex Numbers

- 7 ELECTRICITY** In an AC circuit, the voltage E , current I , and impedance Z are related to the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 4j$ amps and impedance $3 - 6j$ ohms.

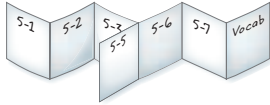
$$\begin{aligned}
 E &= I \cdot Z && \text{Electricity formula} \\
 &= (1 + 4j)(3 - 6j) && I = 1 + 4j, Z = 3 - 6j \\
 &= 1(3) + 1(-6j) + 4j(3) + 4j(-6j) && \text{FOIL} \\
 &= 3 - \boxed{} + 12j - \boxed{} && \text{Multiply} \\
 &= \boxed{} && j^2 = -1 \\
 &= \boxed{} \text{ volts} && \text{Add.}
 \end{aligned}$$

- Check Your Progress** Refer to Example 5. Find the voltage in a circuit with current $1 - 3j$ amps and impedance $3 + 2j$ ohms.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 5-4, write your own example of complex conjugates. Then explain why the product of complex conjugates is always a real number.



EXAMPLE

Divide Complex Numbers

8 Simplify.

a. $\frac{5i}{3 + 2i}$

$$\frac{5i}{3 + 2i} = \frac{5i}{3 + 2i} \cdot \boxed{}$$

$$= \frac{15i + 10i^2}{9 - 4i^2}$$

$$= \frac{15i + 10}{\boxed{}}$$

$$= \boxed{} + \boxed{}$$

$3 - 2i$ and $3 + 2i$ are conjugates.

Multiply

$$i^2 = -1$$

Standard form

b. $\frac{4 - i}{5i}$

$$\frac{4 - i}{5i} = \frac{4 - i}{5i} \cdot \frac{i}{i}$$

$$= \frac{4i - i^2}{-5i^2}$$

$$= \frac{4i + 1}{\boxed{}}$$

$$= \boxed{}$$

Multiply

$$i^2 = -1$$

Standard form

Check Your Progress Simplify.

a. $\frac{3i}{1 + i}$

b. $\frac{3 + 2i}{2i}$

HOMEWORK
ASSIGNMENT

Page(s):

Exercises:

MAIN IDEAS

- Solve quadratic equations by using the Square Root Property.
- Solve quadratic equations by completing the square.

KEY CONCEPT

Square Root Property

For any real number n , if $x^2 = n$, then $x = \pm\sqrt{n}$.

EXAMPLE Equation with Rational Roots

1 Solve $x^2 + 14x + 49 = 64$ by using the Square Root Property.

$$x^2 + 14x + 49 = 64$$

Original equation

$$\boxed{} = 64$$

Factor the trinomial.

$$\boxed{} = \pm \boxed{}$$

Square Root Property

$$\boxed{} = \boxed{}$$

$$\sqrt{64} = 8$$

$$x = -7 \pm 8$$

Subtract 7 from each side.

$$x = -7 + 8 \text{ or } x = -7 - 8$$

Write as two equations.

$$x = \boxed{} \quad x = \boxed{}$$

Solve each equation.

EXAMPLE Equation with Irrational Roots

2 Solve $x^2 - 10x + 25 = 12$ by using the Square Root Property.

$$x^2 - 10x + 25 = 12$$

Original equation

$$(x - 5)^2 = 12$$

Factor the trinomial.

$$x - 5 = \boxed{\phantom{\pm 2\sqrt{3}}}$$

Square Root Property

$$x = \boxed{\phantom{5 \pm 2\sqrt{3}}}$$

Add 5 to each side.

$$\sqrt{12} = 2\sqrt{3}$$

$$x = 5 + \boxed{\phantom{2\sqrt{3}}} \text{ or } x = 5 - \boxed{\phantom{2\sqrt{3}}}$$

Write as two equations.

$$x \approx \boxed{} \quad x \approx \boxed{}$$

Use a calculator.

Check Your Progress

Solve by using the Square Root Property.

a. $x^2 - 16x + 64 = 25$

$$\boxed{}$$

b. $x^2 - 4x + 4 = 8$

$$\boxed{}$$



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(B) Use complex numbers to describe the solutions of quadratic equations.** **TEKS 2A.8** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(D) Solve quadratic equations and inequalities using graphs, tables, and algebraic methods.** Also addresses *TEKS 2A.6(A) and 2A.8(A).*

EXAMPLE

Complete the Square

KEY CONCEPT

Completing the Square

To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b , the coefficient of x .

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

3 Find the value of c that makes $x^2 + 16x + c$ a perfect square. Then write the trinomial as a perfect square.

Step 1 Find one half of 16.

Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + 16x$.

The trinomial can be written as

.

Check Your Progress

Find the value of c that makes $x^2 + 6x + c$ a perfect square. Then write the trinomial as a perfect square.

REMEMBER IT



Be sure to add the same constant to both sides of the equation when solving equations by completing the square.

EXAMPLE

Solve an Equation by Completing the Square

4 Solve $x^2 + 4x - 12 = 0$ by completing the square.

$$x^2 + 4x - 12 = 0$$

Notice that $x^2 + 4x - 12$ is not a perfect square.

$$x^2 + 4x = \text{$$

Rewrite so the left side is of the form $x^2 + bx$.

$$x^2 + 4x + 4 = 12 + 4$$

Add to each side.

$$\text{} = 16$$

Write the left side as a perfect square by factoring.

$$x + 2 = \text{$$

Square Root Property

$$x = \text{$$

Subtract 2 from each side.

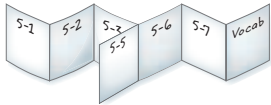
$$x = \text{} \text{ or } x = \text{}$$
 Write as two equations.

$$x = \text{} \qquad x = \text{$$
 Solve each equation.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 5-5, list the steps you would use to solve $w^2 - 8w - 9 = 0$ by completing the square.



Check Your Progress

Solve $x^2 + 5x - 6 = 0$ by completing the square.

EXAMPLE

Equation with $a \neq 1$

5 Solve $3x^2 - 2x - 1 = 0$ by completing the square.

$$3x^2 - 2x - 1 = 0$$

Notice that $3x^2 - 2x - 1$ is not a perfect square.

$$x^2 - \frac{2}{3}x - \frac{1}{3} = 0$$

Divide by the coefficient of the quadratic term, 3.

$$x^2 - \frac{2}{3}x = \frac{1}{3}$$

Add $\frac{1}{3}$ to each side.

$$x^2 - \frac{2}{3}x + \boxed{} = \frac{1}{3} + \boxed{}$$

Since $\left(\frac{-2}{3} \cdot \frac{1}{2}\right)^2 = \boxed{}$,

add $\boxed{}$ to each side.

$$\boxed{} = \boxed{}$$

Write the left side as a perfect square by factoring. Simplify the right side.

$$\boxed{} = \pm \boxed{}$$

Square Root Property

$$x = \boxed{} \pm \boxed{}$$

Add $\boxed{}$ to each side.

$$x = \boxed{} \text{ or } x = \boxed{}$$

Write as two equations.

$$x = \boxed{} \quad x = \boxed{}$$

Solve each equation.

The solution set is $\boxed{}$.

Check Your Progress

Solve $2x^2 + 11x + 15 = 0$ by completing the square.

MAIN IDEAS

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number and types of roots of a quadratic equation.

KEY CONCEPT

Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



TEKS 2A.8

The student formulates

equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(A) Analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems. (B) Analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula. (D) Solve quadratic equations and inequalities using graphs, tables, and algebraic methods. Also addresses TEKS 2A.1(A), 2A.2(B), 2A.6(A), and 2A.6(B).

EXAMPLE Two Rational Roots

1 Solve $x^2 - 8x = 33$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a , b , and c .

$$x^2 - 8x = 33 \rightarrow \begin{array}{c} ax^2 \\ \downarrow \\ \boxed{} \end{array} + \begin{array}{c} x \\ \downarrow \\ \boxed{} \end{array} + \begin{array}{c} x \\ \downarrow \\ \boxed{} \end{array} = 0$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-33)}}{2(1)} \quad \text{Replace } a \text{ with } 1, b \text{ with } -8, \text{ and } c \text{ with } -33.$$

$$x = \frac{8 \pm \sqrt{\boxed{} + \boxed{}}}{2} \quad \text{Simplify.}$$

$$x = \frac{8 \pm \sqrt{\boxed{}}}{2} \quad \text{Simplify.}$$

$$x = \frac{8 \pm \boxed{}}{2} \quad \sqrt{\boxed{}} = \boxed{}$$

$$x = \frac{\boxed{}}{2} \text{ or } x = \frac{\boxed{}}{2} \quad \text{Write as two equations.}$$

$$= \boxed{} = \boxed{} \quad \text{Simplify.}$$

Check Your Progress Solve $x^2 + 13x = 30$ by using the Quadratic Formula.

EXAMPLE One Rational Root**REMEMBER IT**

The Quadratic Formula can be used to solve any quadratic equation.



2 Solve $x^2 - 34x + 289 = 0$ by using the Quadratic Formula.

Identify a , b , and c . Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-(-34) \pm \sqrt{\quad}}{2(1)} \quad \text{Replace } a \text{ with } 1, \text{ } b \text{ with } -34, \text{ and } c \text{ with } 289.$$

$$x = \frac{34 \pm \sqrt{0}}{2} \quad \text{Simplify.}$$

$$x = \frac{34}{2} \text{ or } \quad \sqrt{0} = 0$$

Check Your Progress

Solve $x^2 - 22x + 121 = 0$ by using the Quadratic Formula

EXAMPLE Irrational Roots

3 Solve $x^2 - 6x + 2 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)} \quad \text{Replace } a \text{ with } 1, \text{ } b \text{ with } -6, \text{ and } c \text{ with } 2.$$

$$x = \frac{6 \pm \sqrt{\quad}}{2} \quad \text{Simplify.}$$

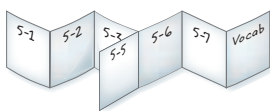
$$x = \frac{\quad}{2} \text{ or } \frac{\quad}{2} \quad \sqrt{28} = \sqrt{4 \cdot 7} \text{ or } 2\sqrt{7}$$

The exact solutions are $\frac{\quad}{2}$ and $\frac{\quad}{2}$.

The approximate solutions are $\frac{\quad}{2}$ and $\frac{\quad}{2}$.

FOLDABLES™**ORGANIZE IT**

On the page for Lesson 5-5, explain why factoring cannot be used to solve the quadratic equation in Example 3.



Check Your Progress

Solve $x^2 - 5x + 3 = 0$ by using the Quadratic Formula.

EXAMPLE**Complex Roots**

4 Solve $x^2 + 13 = 6x$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a , b , and c .

$$x^2 + 13 = 6x \longrightarrow 1x^2 - 6x + 13 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

Replace a with 1, b with -6 , and c with 13.

$$= \frac{\boxed{} \pm \sqrt{\boxed{}}}{\boxed{}}$$

Simplify.

$$= \boxed{}$$

$$\sqrt{-16} = \sqrt{16(-1)} \text{ or } 4i$$

$$= \boxed{}$$

Simplify.

The solutions are the complex numbers $\boxed{}$ and

$$\boxed{}.$$

Check Your Progress

Solve $x^2 + 5 = 4x$ by using the Quadratic Formula.

BUILD YOUR VOCABULARY (pages 121–122)**MAIN IDEAS**

- Analyze quadratic functions of the form $y = a(x - h)^2 + k$.
- Write a quadratic function in the form $y = a(x - h)^2 + k$.

A function written in the form, $y = (x - h)^2 + k$, where (h, k) is the of the parabola and $x = h$ is its , is referred to as the **vertex form**.

EXAMPLE**Graph a Quadratic Function in Vertex Form**

- 1** Analyze $y = (x - 3)^2 + 2$. Then draw its graph.

The vertex is at (h, k) or and the axis of symmetry is $x =$. The graph has the same shape as the graph of $y = x^2$, but is translated 3 units right and 2 units up.

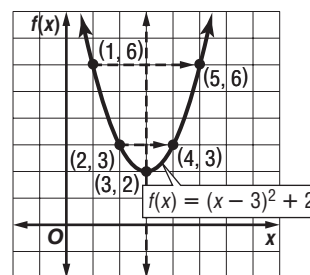
Now use this information to draw the graph.

- Step 1** Plot the vertex, .

- Step 2** Draw the axis of symmetry, .

- Step 3** Find and plot two points on one side of the axis of symmetry, such as $(2, 3)$ and $(1, 6)$.

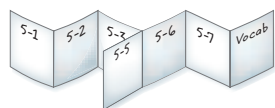
- Step 4** Use symmetry to complete the graph.



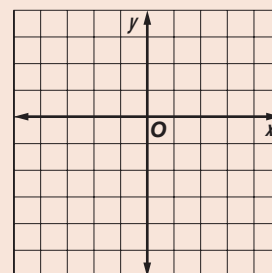
TEKS 2A.4 The student connects algebraic and geometric representations of functions. **(A) Identify and sketch graphs of parent functions, including** linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = ax$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$). **(B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.**

FOLDABLES™**ORGANIZE IT**

On the page for Lesson 5-7, sketch a graph of a parabola. Then sketch the graph of the parabola after a vertical translation and a horizontal translation.

**Check Your Progress**

Analyze $y = (x + 2)^2 - 4$. Then draw its graph.





TEKS 2A.7 The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. **(A) Use characteristics of the quadratic parent function to sketch the related graphs and connect between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions. (B) Use the parent function to investigate, describe, and predict the effects of changes in a , h , and k on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations. Also addresses TEKS 2A.6(C) and 2A.8(A).**

EXAMPLE Vertex Form Parameters

2 TEST EXAMPLE Which function has the widest graph?

- A $y = -12x^2$
- B $y = 12x^2$
- C $y = 1.2x^2$
- D $y = 0.12x^2$

Read the Test Item

You are given four answer choices, each of which is in vertex form.

Solve the Test Item

The value of a determines the width of the graph. Since

$|-12| = \boxed{}$ and $|1.2| > 1$, choices A, B, and C

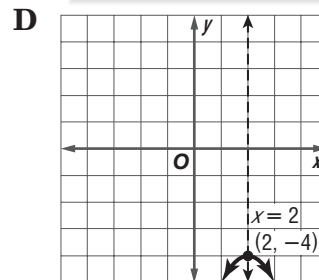
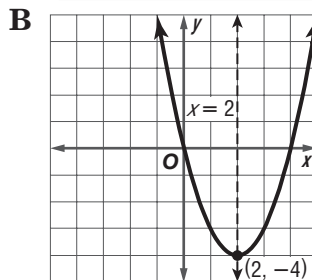
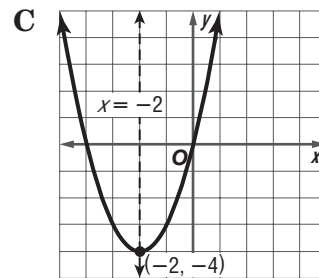
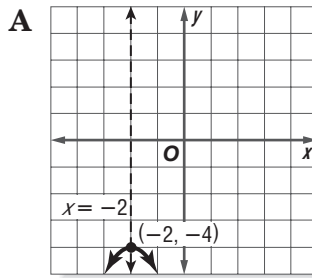
produce graphs that are $\boxed{}$ than $y = x^2$. Since

$|0.12| < 1$, choice D produces a graph that is $\boxed{}$ than

$y = x^2$. The answer is $\boxed{}$.

Check Your Progress

Analyze $y = (x + 2)^2 - 4$. Which graph is the graph of $y = (x + 2)^2 - 4$? $\boxed{}$



WRITE IT

Consider the vertex form of a quadratic function, $y = a(x - h)^2 + k$. Describe what happens to the graph as $|a|$ increases.

EXAMPLE Write Equations in Vertex Form

3 Write each equation in vertex form. Then analyze the function.

a. $y = x^2 + 2x + 4$

$y = x^2 + 2x + 4$

$y = \boxed{}$

$y = \boxed{}$

Notice that $x^2 + 2x + 4$ is not a perfect square.

Complete the square. Add $(\frac{2}{2})^2$ or 1. Balance this addition by subtracting 1. Write $x^2 + 2x + 1$ as a perfect square.

This function can be rewritten as $y = \boxed{} + 3$.

So, $h = \boxed{}$ and $k = \boxed{}$. The vertex is at $\boxed{}$ and the axis of symmetry is $x = \boxed{}$. Since $a = 1$, the graph opens $\boxed{}$ and has the same shape as $y = x^2$ but is translated $\boxed{}$ unit left and $\boxed{}$ units up.

b. $y = -2x^2 - 4x + 2$

$$y = -2x^2 - 4x + 2$$

Original equation

$$y = -2\boxed{} + 2$$

Group $ax^2 - bx$ and factor, dividing by a .

$$y = -2(x + 2x + 1) + 2 - (-2)(1)$$

Complete the square by adding 1 inside the parentheses. Notice that this is an overall addition of $-2(1)$. Balance this addition by subtracting $-2(1)$.

$$y = \boxed{}$$

Write $x^2 + 2x + 1$ as a perfect square.

The vertex is at $(-1, 4)$, and the axis of symmetry is

$x = \boxed{}$. Since $a = -2$, the graph opens $\boxed{}$ and is narrower than the graph of $y = x^2$. It is also translated $\boxed{}$ unit left and $\boxed{}$ units up.

Check Your Progress

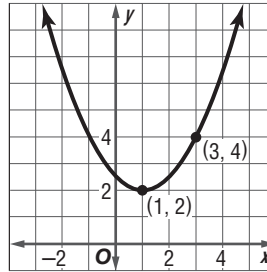
Write each function in vertex form. Then analyze the function.

a. $y = x^2 + 6x + 5$

b. $y = -3x^2 - 6x + 4$

EXAMPLE Write an Equation Given a Graph

Write an equation for the parabola shown in the graph.



The vertex of the parabola is at $(1, 2)$, so $h = \square$ and

$k = \square$. Since $(3, 4)$ is a point on the graph of the parabola,

let $x = \square$ and $y = \square$. Substitute these values into the vertex form of the equation and solve for \square .

$$y = a(x - h)^2 + k \quad \text{Vertex form}$$

$$4 = a(3 - 1)^2 + 2 \quad \text{Substitute } \square \text{ for } y, \square \text{ for } x,$$

$$\square \text{ for } h, \text{ and } \square \text{ for } k.$$

$$4 = \square \quad \text{Simplify.}$$

$$2 = 4a \quad \text{Subtract 2 from each side.}$$

$$\square = a \quad \text{Divide each side by 4.}$$

The equation of the parabola in vertex form is

$$\square.$$

Check Your Progress

Write an equation for the parabola whose vertex is at $(2, 3)$ and passes through $(-2, 1)$.

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____

MAIN IDEAS

- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in one variable.

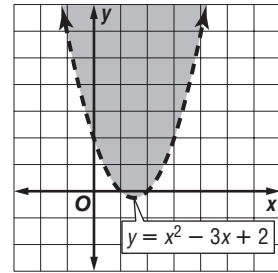
BUILD YOUR VOCABULARY (pages 121–122)

You can graph quadratic inequalities in two variables using the same techniques you used to graph inequalities in two variables.

EXAMPLE Graph a Quadratic Inequality

1 Graph $y > x^2 - 3x + 2$.

Step 1 Graph the related quadratic function, $y = x^2 - 3x + 2$. Since the inequality symbol is $>$, the parabola should be dashed.



Step 2 Test a point inside the parabola, such as $(1, 2)$.

$$y > x^2 - 3x + 2$$

$$2 > \text{[]}$$

$$2 > \text{[]}$$

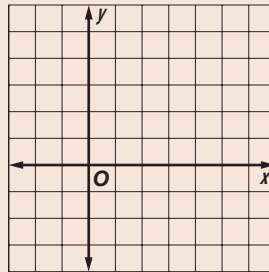
$$2 > \text{[]} \checkmark$$

So, $(1, 2)$ is a solution of the inequality.

Step 3 Shade the region inside the parabola.

Check Your Progress

Graph $y < -x^2 + 4x + 2$.



TEKS 2A.6

The student understands

that quadratic functions can be represented in different ways and translates among their various representations.

(A) Determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities.

TEKS 2A.8 The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. (A) Analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems.

(D) Solve quadratic equations and inequalities using graphs, tables, and algebraic methods. Also addresses TEKS 2A.2(A)

EXAMPLE Solve $ax^2 + bx + c > 0$

2 Solve $x^2 - 4x + 3 > 0$ by graphing.

WRITE IT

Explain how you can check the solution set to a quadratic inequality.

The solution consists of the x values for which the graph of the related quadratic function lies *above* the x -axis. Begin by finding the roots of the related equation.

$$x^2 - 4x + 3 = 0$$

Related equation

$$\boxed{} = 0$$

Factor.

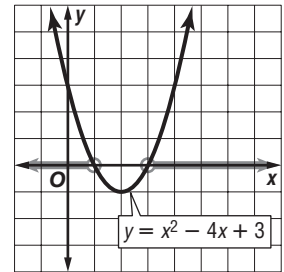
$$\boxed{} = 0 \text{ or } \boxed{} = 0$$

Zero Product Property

$$x = \boxed{} \quad x = \boxed{}$$

Solve each equation.

Sketch the graph of the parabola that has x -intercepts at 3 and 1. The graph lies above the x -axis to the left of $x = 1$ and to the right of $x = 3$.

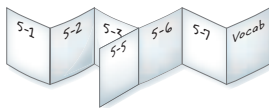


The solution set is $\boxed{}$.

FOLDABLES™

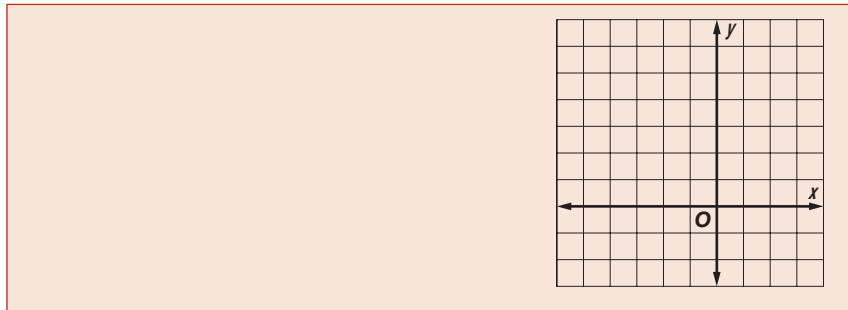
ORGANIZE IT

On the page for Lesson 5-8, use your own words to describe the three steps for graphing quadratic inequalities.

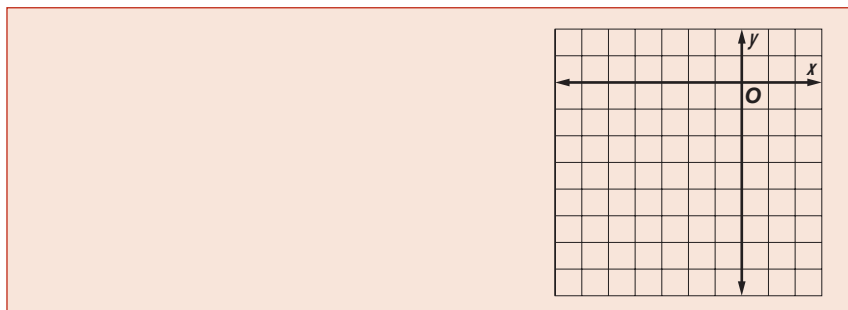


Check Your Progress Solve each inequality by graphing.

a. $x^2 + 5x + 6 > 0$



b. $x^2 + 6x + 2 \leq 0$



EXAMPLE**Solve a Quadratic Inequality****3** Solve $x^2 + x \leq 2$ algebraically.

First, solve the related equation $x^2 + x = 2$.

$$x^2 + x = 2$$

Related quadratic equation

$$\boxed{} = 0$$

Subtract 2 from each side.

$$\boxed{} = 0$$

Factor.

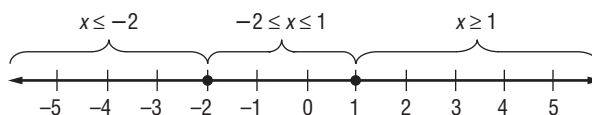
$$\boxed{} = 0 \text{ or } \boxed{} = 0$$

Zero Product Property

$$x = \boxed{} \quad x = \boxed{}$$

Solve each equation.

Plot the values on a number line. Use closed circles since these solutions are included. Note that the number line is separated into 3 intervals.



Test a value in each interval to see if it satisfies the original inequality.

$x \leq -2$	$-2 \leq x \leq 1$	$x \geq 1$
Test $x = -3$.	Test $x = 0$.	Test $x = 2$.
$x^2 + x \leq 2$	$x^2 + x \leq 2$	$x^2 + x \leq 2$
$(-3)^2 - 3 \stackrel{?}{\leq} 2$	$0^2 + 0 \stackrel{?}{\leq} 2$	$2^2 + 2 \stackrel{?}{\leq} 2$
$6 \leq 2 \times$	$0 \leq 2 \checkmark$	$6 \leq 2 \times$

The solution set is $\boxed{}$.



HOMEWORK ASSIGNMENT

Page(s): _____


Exercises: _____

Check Your Progress

Solve $x^2 + 5x < -6$ algebraically.

BRINGING IT ALL TOGETHER

STUDY GUIDE

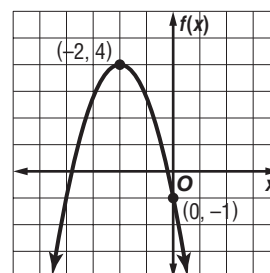
	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 5 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 121–122) to help you solve the puzzle.

5-1

Graphing Quadratic Functions

Refer to the graph at the right as you complete the following sentences.

- The curve is called a .
- The line $x = -2$ is called the .
- The point $(-2, 4)$ is called the .



Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.

4. $f(x) = -x^2 + 2x + 5$

5. $f(x) = 3x^2 - 4x - 2$

5-2

Solving Quadratic Equations by Graphing

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

6. $x^2 - 2x = 8$

7. $x^2 + 5x - 7 = 0$

5-3

Solving Quadratic Equations by Factoring

8. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution.

$$x^2 - 10x = -21$$

Original equation

$$x^2 - 10x + 21 = 0$$

$$(x - 3)(x - 7) = 0$$

$$x - 3 = 0 \text{ or } x - 7 = 0$$

$$x = 3 \quad x = 7$$

The solution set is .

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

9. -4, -2

10. 3, 6

5-4

Complex Numbers

Simplify.

11. $\sqrt{-2} \cdot \sqrt{-10}$

12. $(3 - 8i) - (5 + 2i)$

13. $(4 - i)(5 + 2i)$

14. $\frac{3 - i}{2 + i}$

15. Solve $5x^2 + 60 = 0$.

5-5

Completing the Square

16. Solve $x^2 + 6x + 9 = 49$ by using the Square Root Property.

17. Solve $x^2 - 2x + 10 = 5$ by completing the square.

18. When the dimensions of a cube are reduced by 2 inches on each side, the surface area of the new cube is 486 square inches. What were the dimensions of the original cube?

5-6

The Quadratic Formula and the Discriminant

The value of the discriminant for a quadratic equation with integer coefficients is shown. Give the number and the type of roots for the equation.

	Value of Discriminant	Number of Roots	Type of Roots
19.	64	<input type="text"/>	<input type="text"/>
20.	-8	<input type="text"/>	<input type="text"/>
21.	0	<input type="text"/>	<input type="text"/>
22.	15	<input type="text"/>	<input type="text"/>

23. Solve $x^2 - 8x = 2$ by using the Quadratic Formula. Find exact solutions.

5-7

Analyzing Graphs of Quadratic Functions

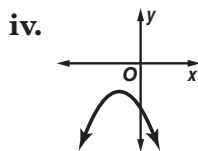
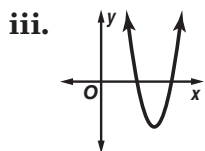
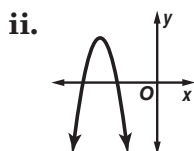
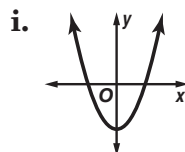
24. Match each graph with the description of the constants in the equation in vertex form.

a. $a > 0, h > 0, k < 0$

b. $a < 0, h < 0, k < 0$

c. $a < 0, h < 0, k > 0$

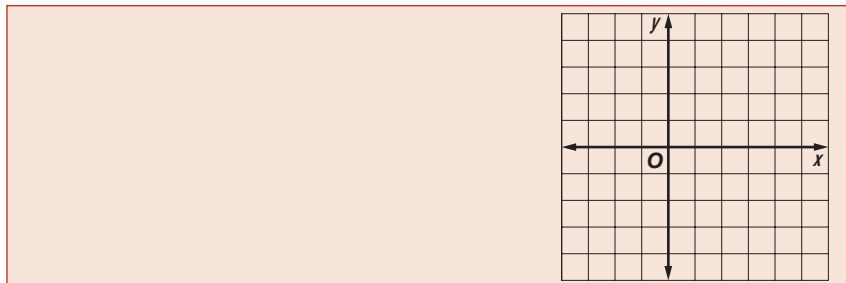
d. $a > 0, h = 0, k < 0$



5-8

Graphing and Solving Quadratic Inequalities

25. Solve $0 < x^2 - 6x + 8$ by graphing.



Solve each inequality algebraically.

26. $x^2 - x > 20$

27. $x^2 - 10x < -16$



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 5.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 5 Practice Test on page 307 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 5 Study Guide and Review on pages 302–306 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 307 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 5 Foldable.
- Then complete the Chapter 5 Study Guide and Review on pages 302–306 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 307 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

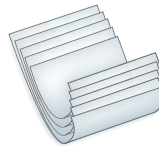
Polynomial Functions



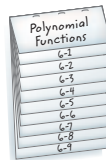
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of notebook paper.

STEP 1 **Stack** sheets of paper with edges $\frac{3}{4}$ -inch apart. Fold up the bottom edges to create equal tabs.



STEP 2 **Staple** along the fold. Label the tabs with lesson numbers.



NOTE-TAKING TIP: When you take notes, think about the order in which the concepts are being presented. Write why you think the concepts were presented in this sequence.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 6. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
degree of a polynomial			
depressed polynomial			
dimensional analysis			
end behavior			
leading coefficient			
Location Principle			
polynomial function			
polynomial in one variable			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
quadratic form			
relative maximum			
relative minimum			
scientific notation			
simplify			
standard notation			
synthetic division			
synthetic substitution			

BUILD YOUR VOCABULARY (pages 161–162)**MAIN IDEAS**

- Use properties of exponents to multiply and divide monomials.
- Use expressions written in scientific notation.

A **monomial** is an expression that is a number, a variable, or the product of a number and one or more variables.

The numerical factor of a monomial is the **coefficient** of the variable(s). The **degree** of a monomial is the sum of the exponents of its variables. A **power** is an expression of the form x^n .

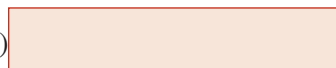
EXAMPLE**Simplify Expressions with Multiplication**

1 Simplify each expression.

a. $(-2a^3b)(-5ab^4)$

$$(-2a^3b)(-5ab^4)$$

$$= (-2 \cdot a \cdot a \cdot a \cdot b)$$



Definition of exponents

$$= -2(-5) \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b$$

C Property

$$= 10a^4b^5$$

Definition of exponents

b. $(3a^5)(c^{-2})(-2a^{-4}b^3)$

$$(3a^5)(c^{-2})(-2a^{-4}b^3)$$

$$= (3a^5)\left(\frac{1}{c^2}\right)\left(\frac{-2}{a^{-4}}\right)(b^3)$$

Definition of negative exponents

$$= (3 \cdot a \cdot a \cdot a \cdot a \cdot a)\left(\frac{1}{c \cdot c}\right)\left(\frac{-2}{a \cdot a \cdot a \cdot a}\right)(b \cdot b \cdot b)$$

Definition of exponents

$$= (3 \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a})\left(\frac{1}{c \cdot c}\right)\left(\frac{-2}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}\right)(b \cdot b \cdot b)$$

Cancel out common factors.

$$= \boxed{}$$

Definition of exponents and fractions

**TEKS 2A.2**

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

KEY CONCEPTS

Negative Exponents

For any real number $a \neq 0$ and any integer n ,
 $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Product of Powers

For any real number a and integers m and n ,
 $a^m \cdot a^n = a^{m+n}$.

Quotient of Powers

For any real number $a \neq 0$, and integers m and n ,
 $\frac{a^m}{a^n} = a^{m-n}$.

Properties of Powers

Suppose a and b are real numbers and m and n are integers. Then the following properties hold.

Power of a Power:

$$(a^m)^n = a^{mn}$$

Power of a Product:

$$(ab)^m = a^m b^m$$

Power of a Quotient:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0, \text{ and}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \text{ or}$$

$$\frac{b^n}{a^n}, a \neq 0, b \neq 0$$

WRITE IT

Write how you read the expression a^3 . Then do the same for z^4 .

EXAMPLE

Simplify Expressions with Division

- 2 Simplify $\frac{s^2}{s^{10}}$. Assume that $s \neq 0$.

$$\frac{s^2}{s^{10}} = \boxed{}$$

Subtract exponents.

$$= \boxed{} \text{ or } \boxed{}$$

A simplified expression cannot contain negative exponents.

EXAMPLE

Simplify Expressions with Powers

- 3 Simplify each expression.

a. $(-3c^2d^5)^3$

$$(-3c^2d^5)^3 = \boxed{}$$

$$= \boxed{}$$

b. $\left(\frac{-2a}{b^2}\right)^5$

$$\left(\frac{-2a}{b^2}\right)^5 = \frac{-2^5 a^5}{(b^2)^5}$$

$$= \boxed{}$$

Check Your Progress

Simplify each expression.

a. $(-3x^2y)(5x^3y^5)$

b. $\frac{x^3}{x^7}$

c. $(x^3)^5$

d. $(-2x^2y^3)^5$

e. $\left(\frac{-3x^2}{y^3}\right)^3$

f. $\left(\frac{y}{2}\right)^{-3}$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 6-1, write each new Vocabulary Builder word and its definition. Give an example of each word.



EXAMPLE

Simplify Expressions Using Several Properties

4 Simplify $\left(\frac{-3a^{5y}}{a^{6y}b^4}\right)^5$.

Simplify the fraction before raising to the fifth power.

$$\begin{aligned}\left(\frac{-3a^{5y}}{a^{6y}b^4}\right)^5 &= \left(\frac{-3a^{5y-6y}}{b^4}\right)^5 \\ &= \left(\frac{-3a^{-y}}{b^4}\right)^5\end{aligned}$$

$$= \boxed{} \text{ or } \boxed{}$$

Check Your Progress

Simplify $\left(\frac{-2a^{3n}}{a^{2n}b^5}\right)^3$.

EXAMPLE

Divide Numbers in Scientific Notation

- 5 **BIOLOGY** There are about 5×10^6 red blood cells in one milliliter of blood. A certain blood sample contains 8.32×10^6 red blood cells. About how many milliliters of blood are in the sample?

Divide the number of red blood cells in the sample by the number of red blood cells in 1 milliliter of blood.

$$\begin{aligned}\frac{\boxed{}}{5 \times 10^6} &\leftarrow \text{number of red blood cells in sample} \\ &\leftarrow \text{number of red blood cells in 1 milliliter} \\ &= \boxed{} \text{ milliliters}\end{aligned}$$

Check Your Progress

A petri dish contains 3.6×10^5 germs. A half hour later, there are 7.2×10^7 . How many times as great is the amount a half hour later?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Add and subtract polynomials.
- Multiply polynomials.



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

BUILD YOUR VOCABULARY (pages 161–162)

A **polynomial** is a monomial or a sum of monomials.

The polynomial $x^2 + 3x + 1$ is a **trinomial** because it has three unlike terms.

A polynomial such as $xy + z^3$ is a **binomial** because it has two unlike terms.

EXAMPLE Degree of a Polynomial

- 1** Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $c^4 - 4\sqrt{c} + 18$

This expression a polynomial because is not a monomial.

b. $-16p^5 + \frac{3}{4}p^2q^7$

The expression a polynomial because each term is a monomial. The degree of the first term is , and the degree of the second term is $2 + 7$ or 9 . The degree of the polynomial is .

EXAMPLE Subtract and Simplify

- 2** Simplify each expression.

a. $(2a^3 + 5a - 7) - (a^3 - 3a + 2)$

$$\begin{aligned} & (2a^3 + 5a - 7) - (a^3 - 3a + 2) \\ &= 2a^3 + 5a - 7 - a^3 + 3a - 2 \\ &= (2a^3 - a^3) + \text{} + \text{} \\ &= \text{} \end{aligned}$$

b. $(4x^2 - 9x + 3) + (-2x^2 - 5x - 6)$

$$\begin{aligned} & (4x^2 - 9x + 3) + (-2x^2 - 5x - 6) \\ &= 4x^2 - 9x + 3 - 2x^2 - 5x - 6 \\ &= (4x^2 - 2x^2) + \text{} + \text{} \\ &= \text{} \end{aligned}$$

REMEMBER IT

The prefix *bi-* means two, so you can remember that a binomial has two unlike terms.

The prefix *tri-* means three, so you can remember that a trinomial has three unlike terms.

Check Your Progress

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{2}a^2b^3 + 3c^5$

b. $\sqrt{c} + 2$

c. Simplify $(3x^2 + 2x - 3) - (4x^2 + x - 5)$.

EXAMPLE**Multiply and Simplify**

3 Find $-y(4y^2 + 2y - 3)$.

$$-y(4y^2 + 2y - 3)$$

$$= -y(4y^2) - y(2y) - y(-3)$$

Distributive Property

$$= \text{[]}$$

Multiply the monomials.

EXAMPLE**Multiply Polynomials**

4 Find $(a^2 + 3a - 4)(a + 2)$.

$$(a^2 + 3a - 4)(a + 2)$$

$$= a^2(a + 2) + 3a(a + 2) - 4(a + 2)$$

Distributive Property

$$= a^2 \cdot a + a^2 \cdot 2 + \text{[]} +$$

$$\text{[]} - \text{[]} - \text{[]}$$

Distributive Property

$$= a^3 + 2a^2 + 3a^2 + 6a - 4a - 8$$

Multiply monomials.

$$= \text{[]}$$

Combine like terms.

KEY CONCEPT**FOIL Method for Multiplying Binomials**

The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.



On the page for Lesson 6-2, write examples of multiplying two binomials.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress

Find each product.

a. $-x(3x^3 - 2x + 5)$

b. $(3p + 2)(5p + 1)$

c. $(x^2 + 3x - 2)(x + 4)$

6-3 Dividing Polynomials

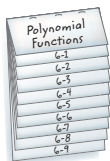
MAIN IDEAS

- Divide polynomials using long division.
- Divide polynomials using synthetic division.

FOLDABLES™

ORGANIZE IT

In your notes, explain how to write the answer to a long division problem that has a quotient and a remainder.



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

EXAMPLE Divide a Polynomial by a Monomial

1 Simplify $\frac{5a^2b - 15ab^3 + 10a^3b^4}{5ab}$.

$$\begin{aligned} & \frac{5a^2b - 15ab^3 + 10a^3b^4}{5ab} \\ &= \frac{5a^2b}{5ab} - \frac{15ab^3}{5ab} + \frac{10a^3b^4}{5ab} && \text{Sum of} \\ & && \text{quotients} \\ &= \frac{5}{5} \cdot a^{2-1}b^{1-1} - \frac{15}{5} \cdot a^{1-1}b^{3-1} + \frac{10}{5} \cdot a^{3-1}b^{4-1} \\ &= \boxed{} && \text{Divide.} \\ & && x^{1-1} = x^0 \text{ or } 1 \end{aligned}$$

Check Your Progress

Simplify $\frac{3x^2y + 6x^5y^2 - 9x^7y^3}{3x^2y}$.

EXAMPLE Division Algorithm

2 Use long division to find $(x^2 - 2x - 15) \div (x - 5)$.

$$\begin{array}{r} x + 3 \\ x - 5 \overline{) x^2 - 2x - 15} \\ \underline{-()} \\ \\ \\ \\ \\ \\ \\ \end{array}$$

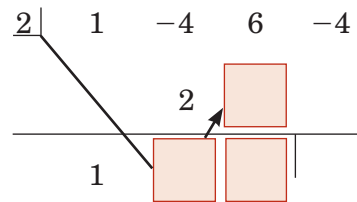
The quotient is $\boxed{}$.

Check Your Progress

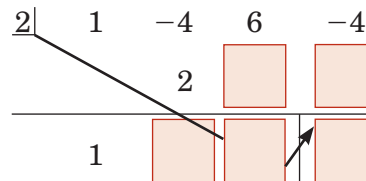
Use long division to find

$(x^2 + 5x + 6) \div (x + 3)$.

Step 4 Multiply this sum by r , 2. Write the product under the next coefficient and add.



Step 5 Multiply this sum by r , 2. Write the product under the next coefficient and add.



The numbers along the bottom are the coefficients of the quotient. So, the quotient is $x^2 - 2x + 2$.

EXAMPLE Divisor with First Coefficient Other than 1

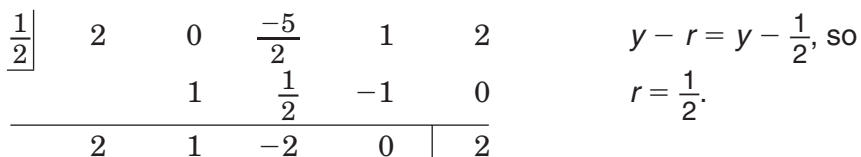
5 Use synthetic division to find $(4y^4 - 5y^2 + 2y + 4) \div (2y - 1)$.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{4y^4 - 5y^2 + 2y + 4}{2y - 1} = \frac{(4y^4 - 5y^2 + 2y + 4) \div 2}{(2y - 1) \div 2}$$

$$= \frac{\boxed{}}{y - \frac{1}{2}}$$

The numerator does not have a y^3 term. So use a coefficient of 0.



The result is $2y^3 + y^2 - 2y + \frac{2}{y - \frac{1}{2}}$. Now simplify the fraction.

$$\frac{2}{y - \frac{1}{2}} = 2 \div \left(y - \frac{1}{2}\right)$$

$$= 2 \div \boxed{\phantom{y - \frac{1}{2}}} = 2 \cdot \boxed{\phantom{y - \frac{1}{2}}} \text{ or } \boxed{\phantom{y - \frac{1}{2}}}$$

The solution is $\boxed{\phantom{2y^3 + y^2 - 2y + \frac{2}{y - \frac{1}{2}}}}$.

HOMEWORK ASSIGNMENT

Page(s): _____
Exercises: _____

Check Your Progress

a. Use long division to find $(x^2 + 5x + 6) \div (x + 3)$.

b. Use synthetic division to find $(16y^4 - 4y^2 + 2y + 8) \div (2y + 1)$.

Polynomial Functions



TEKS 2A.1 The student uses properties and attributes of functions and applies functions to problem situations. **(A) Identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.**

BUILD YOUR VOCABULARY (pages 161–162)

MAIN IDEAS

- Evaluate polynomial functions.
- Identify general shapes of graphs of polynomial functions.

The leading coefficient is the coefficient of the term with the degree.

A common type of function is a **power function**, which has an equation in the form , where a and b are real numbers.

KEY CONCEPT

A Polynomial in One Variable A polynomial of degree n in one variable x is an expression of the form
 $a_0x^n + a_1x^{n-1} + \dots + a_{n-2}x^2 + a_{n-1}x + a_n$,
 where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.



TEKS 2A.6

The student understands that quadratic functions can be represented in different ways and translates among their various representations. **(A) Determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities.**

EXAMPLE

Find Degree and Leading Coefficients

- 1** State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $7z^3 - 4z^2 + z$

This is a polynomial in one variable. The degree is and the leading coefficient is .

b. $6a^3 - 4a^2 + ab^2$

This a polynomial in one variable. It contains two variables, and .

Check Your Progress

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $3x^3 + 2x^2 - 3$

b. $3x^2 + 2xy - 5$

EXAMPLE Function Values of Variables

- 3 Find $b(2x - 1) - 3b(x)$ if $b(m) = 2m^2 + m - 1$.

To evaluate $b(2x - 1)$, replace m in $b(m)$ with .

$$\begin{aligned} b(m) &= 2m^2 + m - 1 && \text{Original function} \\ b(2x - 1) &= 2(2x - 1)^2 + (2x - 1) - 1 && \text{Replace } m \text{ with } 2x - 1. \\ &= \text{} + 2x - 1 - 1 && \text{Evaluate } 2(2x - 1)^2. \\ &= \text{} && \text{Simplify.} \end{aligned}$$

To evaluate $3b(x)$, replace m with x in $b(m)$, then multiply the expression by .

$$\begin{aligned} b(m) &= 2m^2 + m - 1 && \text{Original function} \\ 3b(x) &= 3(2x^2 + x - 1) && \text{Replace } m \text{ with } x. \\ &= \text{} && \text{Distributive Property} \end{aligned}$$

Now evaluate $b(2x - 1) - 3b(x)$.

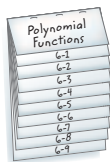
$$\begin{aligned} b(2x - 1) - 3b(x) &= \text{} - \text{} && \text{Replace } b(2x - 1) \text{ and } 3b(x) \\ & && \text{with evaluated expressions.} \\ &= 8x^2 - 6x - 6x^2 - 3x + 3 \\ &= \text{} && \text{Simplify.} \end{aligned}$$

Check Your Progress Find $g(2x + 1) - 2g(x)$ if

$$g(b) = b^2 + 3.$$

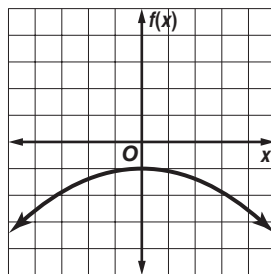
FOLDABLES™**ORGANIZE IT**

On the tab for Lesson 6-4, sketch the graph of a function that has no real zeros, one real zero, and two real zeros.

**EXAMPLE** Graphs of Polynomial Functions

- 4 For each graph, describe the end behavior, determine whether it represents an odd-degree or an even-degree function, and state the number of real zeros.

a.



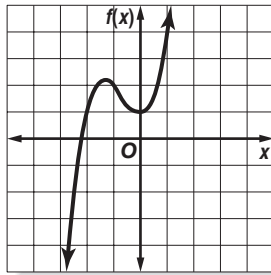
• $f(x) \rightarrow$ as $x \rightarrow +\infty$.

• $f(x) \rightarrow$ as $x \rightarrow -\infty$.

• It is an polynomial function.

• The graph does not intersect the x -axis, so the function has real zeros.

b.

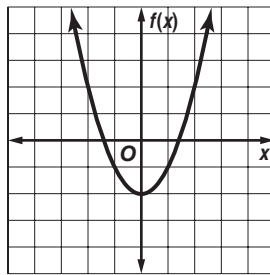


- $f(x) \rightarrow$ as $x \rightarrow +\infty$.
- $f(x) \rightarrow$ as $x \rightarrow -\infty$.
- It is an polynomial function.
- The graph intersects the x -axis at one point, so the function has real zero.

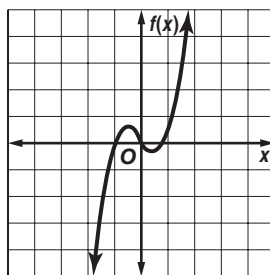
Check Your Progress

For each graph, describe the end behavior, determine whether it represents an odd-degree or an even-degree function, and state the number of real zeros.

a.



b.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Analyzing Graphs of Polynomial Functions



TEKS 2A.1 The student uses properties and attributes of functions and applies functions to problem situations. **(A) Identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.**

EXAMPLE Graph a Polynomial Function

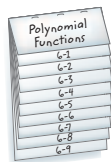
MAIN IDEAS

- Graph polynomial functions and locate their real zeros.
- Find the maxima and minima of polynomial functions.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-5, explain how knowing the end behavior of a graph will assist you in completing the sketch of the graph.



- 1** Graph $f(x) = -x^3 - 4x^2 + 5$ by making a table of values.

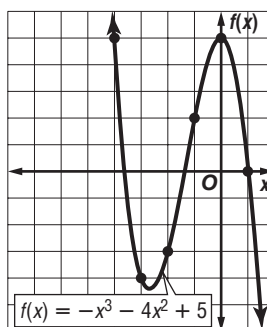
This is an odd degree polynomial with a negative leading coefficient, so

$$f(x) \rightarrow \boxed{} \text{ as } x \rightarrow -\infty \text{ and}$$

$$f(x) \rightarrow \boxed{} \text{ as } x \rightarrow +\infty.$$

The graph intersects the x -axis at $\boxed{}$ points indicating that

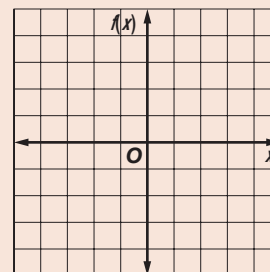
there are $\boxed{}$ real zeros.



x	$f(x)$
-4	<input type="text"/>
-3	<input type="text"/>
-2	<input type="text"/>
-1	<input type="text"/>
0	<input type="text"/>
1	<input type="text"/>
2	<input type="text"/>

Check Your Progress

Graph $f(x) = x^3 + 2x^2 + 1$ by making a table of values.



EXAMPLE Locate Zeros of a Function

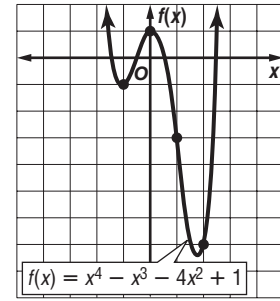
- 2 Determine consecutive values of x between which each real zero of the function $f(x) = x^4 - x^3 - 4x^2 + 1$ is located. Then draw the graph.

KEY CONCEPT**Location Principle**

Suppose $y = f(x)$ represents a polynomial function and a and b are two numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b .

Make a table of values. Since $f(x)$ is a 4th degree polynomial function, it will have between 0 and 4 zeros, inclusive. Look at the value of $f(x)$ to locate the zeros. Then use the points to sketch the graph of the function.

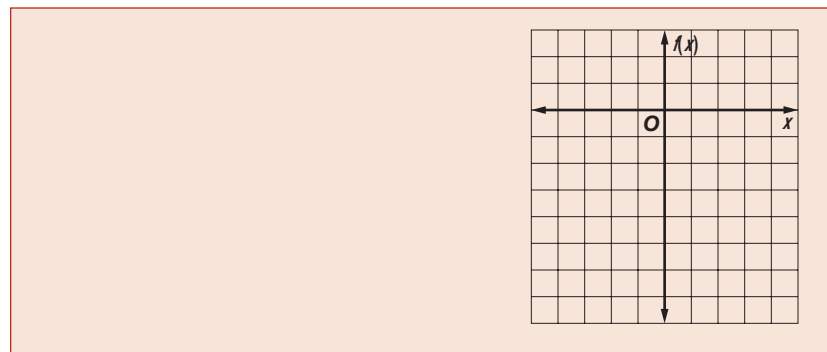
x	$f(x)$	
-2	9	} sign change
-1	-1	
0	1	} sign change
1	-3	
2	-7	} sign change
3	19	



There are zeros between $x = \square$ and \square , $x = \square$ and \square , $x = \square$ and \square , and $x = \square$ and \square .

Check Your Progress

Determine consecutive values of x between which each real zero of the function $f(x) = x^3 - 4x^2 + 2$ is located. Then draw the graph.

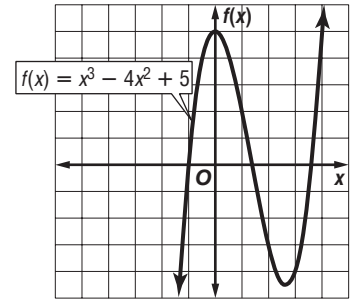
**BUILD YOUR VOCABULARY** (page 162)

A point on a graph is a **relative maximum** of a function if no other nearby points have a \square y-coordinate. Likewise, a point is a **relative minimum** if no other nearby points have a \square y-coordinate.

EXAMPLE Maximum and Minimum Points

- 3** Graph $f(x) = x^3 - 4x^2 + 5$.
 Estimate the x -coordinates at which the relative maximum and relative minimum occur.

Make a table of values and graph the function.



x	$f(x)$
-2	-19
-1	0
0	5
1	2
2	-3
3	-4
4	5
5	30

zero at $x = -1$

← indicates a relative maximum

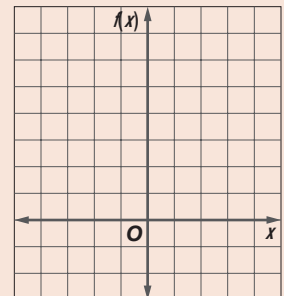
} zero between $x = 1$ and $x = 2$

← indicates a relative minimum
 } and zero between $x = 3$ and $x = 4$

The value of $f(x)$ at $x = \boxed{}$ is greater than the surrounding points, so it is a relative $\boxed{}$. The value of $f(x)$ at about $x = \boxed{}$ is less than the surrounding points, so it is a relative $\boxed{}$.

Check Your Progress

Graph $f(x) = x^3 + 3x^2 + 2$.
 Estimate the x -coordinates at which the relative maximum and relative minimum occur.



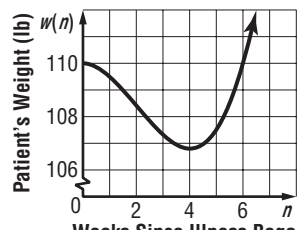
EXAMPLE Graph a Polynomial Model

- 4 HEALTH** The weight w , in pounds, of a patient during a 7-week illness is modeled by the cubic equation $w(n) = 0.1n^3 - 0.6n^2 + 110$, where n is the number of weeks since the patient became ill.

- a. Graph the equation. Describe the turning points of the graph and its end behavior.

Make a table of values for the weeks 1–7. Plot the points and connect with a smooth curve.

w	$w(n)$
1	<input type="text"/>
2	108.4
3	<input type="text"/>
4	106.8
5	<input type="text"/>
6	110



Relative minimum point at week .

End behavior: $w(n)$ increases as increases

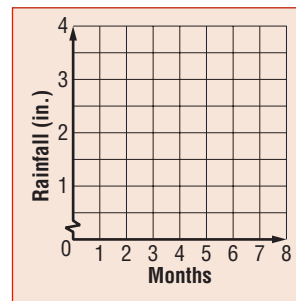
- b. What trends in the patient's weight does the graph suggest? Is it reasonable to assume the trend will continue indefinitely?

The patient lost weight for weeks; then gained weight.

The trend may continue for a few weeks, but it is that the patient's weight will gain so quickly indefinitely.

Check Your Progress The rainfall r , in inches per month, during a 7-month period is modeled by the $r(m) = 0.01m^3 - 0.18m^2 + 0.67m + 3.23$, where m is the number of months after March 1.

- a. Graph the equation. Describe the turning points of the graph and its end behavior.



- b. What trends in the amount of rainfall received by the town does the graph suggest?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

The coefficient of the y terms must be 3 and -5 since $3(-5) = \boxed{}$ and $3 + (-5) = \boxed{}$. Rewrite the expression using $-5y$ and $3y$ in place of $-2y$ and factor by grouping.

$$\begin{aligned} 3y^2 - 2y - 5 &= 3y^2 - 5y + 3y - 5 && \text{Replace } -2y \\ &= (3y^2 - 5y) + (3y - 5) && \text{Associative Property} \\ &= \boxed{}(3y - 5) + \boxed{}(3y - 5) && \text{Factor} \\ &= (3y - 5)(y + 1) && \text{Distributive Property} \\ &= \boxed{} \end{aligned}$$

b. $64x^6 - y^6$

This polynomial could be considered the difference of two $\boxed{}$ or the difference of two $\boxed{}$. The difference of two squares should always be done before the difference of two cubes. This will make the next step of the factorization easier.

$$\begin{aligned} 64x^6 - y^6 &= (8x^3 + y^3)(8x^3 - y^3) && \text{Difference of two squares} \\ &= (2x + y)(4x^2 - 2xy + y^2)(2x - y)(4x^2 + 2xy + y^2) && \text{Sum and difference of two cubes} \end{aligned}$$

Check Your Progress

Factor each polynomial.

a. $2x^2 + x - 3$

b. $a^3b^3 - 27$

KEY CONCEPT

Quadratic Form An expression that is quadratic in form can be written as $au^2 + bu + c$ for any numbers a , b , and c , $a \neq 0$, where u is some expression in x . The expression $au^2 + bu + c$ is called the quadratic form of the original expression.

EXAMPLE

Write an Expression in Quadratic Form

4 Write each expression in quadratic form, if possible.

a. $2x^6 + x^3 + 9$

$$\boxed{} + (x^3) + 9 \quad x^6 = \boxed{}$$

b. $x^4 + 2x^3 + 1$

This cannot be written in quadratic form since $x^4 \neq \boxed{}$.

Check Your Progress Write each expression in quadratic form, if possible.

a. $2x^4 + x^2 + 3$

b. $x^6 + x^4 + 1$

c. $x - 2x^{\frac{1}{2}} + 3$

d. $x^{12} + 5$

EXAMPLE Solve Polynomial Equations

5 a. Solve $x^4 - 29x^2 + 100 = 0$.

$$x^4 - 29x^2 + 100 = 0 \quad \text{Original equation}$$

$$\boxed{} - 29(x^2) + 100 = 0 \quad \text{Write the expression on the left in quadratic form.}$$

$$\boxed{} \boxed{} = 0 \quad \text{Factor the trinomial.}$$

$$\boxed{} = 0 \quad \text{Factor.}$$

Use the Zero Product Property.

$$\boxed{} = 0 \quad \text{or} \quad \boxed{} = 0$$

$$x = \boxed{}$$

$$x = \boxed{}$$

$$\boxed{} = 0 \quad \text{or} \quad \boxed{} = 0$$

$$x = \boxed{}$$

$$x = \boxed{}$$

BUILD YOUR VOCABULARY (page 162)

When synthetic division is used to evaluate a function, it is called **synthetic substitution**.

MAIN IDEAS

- Evaluate functions using synthetic substitution.
- Determine whether a binomial is a factor of a polynomial by using synthetic substitution.

KEY CONCEPTS**Remainder Theorem**

If a polynomial $f(x)$ is divided by $x - a$, the remainder is the constant $f(a)$, and

$$\begin{array}{l} \text{Dividend} \quad \text{equals} \quad \text{quotient} \\ f(x) \quad \quad \quad = \quad \quad q(x) \end{array}$$

times divisor plus remainder.

$$(x - a) + f(a)$$

where $q(x)$ is a polynomial with degree one less than the degree of $f(x)$.

Factor Theorem

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

**TEKS 2A.2**

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring** and properties of exponents **to simplify expressions** and to transform and solve equations.

EXAMPLE Synthetic Substitution

1 If $f(x) = 3x^4 - 2x^3 + x^2 - 2$, find $f(4)$.

METHOD 1 Synthetic Substitution

By the Remainder Theorem, $f(4)$ should be the remainder when you divide the polynomial by $x - 4$.

$$\begin{array}{r|rrrrrr} 4 & & 3 & -2 & 1 & 0 & -2 \\ & & & & & & \\ \hline & & & & & & \end{array}$$

Notice that there is no x term. A zero is placed in this position as a placeholder.

The remainder is . Thus, by using synthetic substitution, $f(4) = \text{input}$.

METHOD 2 Direct Substitution

Replace x with 4.

$$f(x) = 3x^4 - 2x^3 + x^2 - 2$$

Original function

$$f(4) = \text{input}$$

Replace x with 4.

$$f(4) = \text{input} \quad \text{or} \quad \text{input}$$

Simplify.

By using direct substitution, $f(4) = \text{input}$.

Check Your Progress

If $f(x) = 2x^3 - 3x^2 + 7$, find $f(3)$.

BUILD YOUR VOCABULARY (page 162)

When you divide a polynomial by one of its **binomial** factors, the quotient is called a **depressed polynomial**.

EXAMPLE Use the Factor Theorem

2 Show that $x - 3$ is a factor of $x^3 + 4x^2 - 15x - 18$. Then find the remaining factors of the polynomial.

The binomial $x - 3$ is a factor of the polynomial if 3 is a zero of the related polynomial function. Use the factor theorem and synthetic division.

3	1	4	-15	-18

Since the remainder is 0, is a factor of the polynomial. The polynomial $x^3 + 4x^2 - 15x - 18$ can be factored as . The polynomial is the depressed polynomial. Check to see if this polynomial can be factored.

$x^2 + 7x + 6 =$ Factor the trinomial.

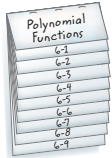
So, $x^3 + 4x^2 - 15x - 18 =$.

Check Your Progress Show that $x + 2$ is a factor of $x^3 + 8x^2 + 17x + 10$. Then find the remaining factors of the polynomial.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-7, use the Factor Theorem to show that $x + 3$ is a factor of $3x^3 - 3x^2 - 36x$.



MAIN IDEAS

- Determine the number and type of roots for a polynomial equation.
- Find the zeros of a polynomial function.

KEY CONCEPTS

Fundamental Theorem of Algebra Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

Corollary A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.

FOLDABLES

On the tab for Lesson 6-8, write these key concepts. Be sure to include examples.



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

EXAMPLE

Determine Number and Type of Roots

- 1 Solve each equation. State the number and type of roots.

a. $x^2 + 2x - 48 = 0$

$$x^2 + 2x - 48 = 0$$

Original equation

$$\boxed{} = 0$$

Factor.

Use the Zero Product Property.

$$\boxed{} = 0 \quad \text{or} \quad \boxed{} = 0$$

$$x = \boxed{}$$

$$x = \boxed{}$$

Solve each equation.

This equation has two real roots, $\boxed{}$ and $\boxed{}$.

b. $y^4 - 16 = 0$

$$y^4 - 16 = 0$$

$$(y^2 + 4)(y^2 - 4) = 0$$

$$(y^2 + 4)(y + 2)(y - 2) = 0$$

$$y^2 + \boxed{} = 0 \quad \text{or} \quad y + \boxed{} = 0 \quad \text{or} \quad y - \boxed{} = 0$$

$$y^2 = \boxed{}$$

$$y = \boxed{}$$

$$y = \boxed{}$$

$$y = \pm\sqrt{-4} \quad \text{or} \quad \pm 2i$$

This equation has two real roots, $\boxed{}$, and twoimaginary roots, $\boxed{}$.

Check Your Progress

Solve each equation. State the number and type of roots.

a. $x^2 - x - 12 = 0$

b. $a^4 - 81 = 0$

EXAMPLE

Find Numbers of Positive and Negative Zeros

KEY CONCEPT

Descartes' Rule of Signs
If $P(x)$ is a polynomial with real coefficients whose terms are arranged in descending powers of the variable,

- the number of positive real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and
- the number of negative real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this number by an even number.

- 2 State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $p(x) = -x^6 + 4x^3 - 2x^2 - x - 1$.

Since $p(x)$ has degree 6, it has 6 zeros. However, some of them may be imaginary. Use Descartes' Rule of Signs to determine the number and type of real zeros. Count the number of changes in sign for the coefficients of $p(x)$.

$$p(x) = -x^6 + 4x^3 - 2x^2 - x - 1$$

Since there are two sign changes, there are 2 or 0 positive real zeros. Find $p(-x)$ and count the number of sign changes for its coefficients.

$$\begin{aligned} p(-x) &= -(-x)^6 + 4(-x)^3 - 2(-x)^2 - (-x) - 1 \\ &= -x^6 - 4x^3 - 2x^2 + x - 1 \end{aligned}$$

Since there are two sign changes, there are 2 or 0 negative real zeros. Make a chart of possible combinations.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total
2	2	<input type="text"/>	6
0	<input type="text"/>	<input type="text"/>	6
2	<input type="text"/>	<input type="text"/>	6
0	<input type="text"/>	<input type="text"/>	6

Check Your Progress

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $p(x) = x^4 - x^3 + x^2 + x + 3$.

EXAMPLE

Use Synthetic Substitution to Find Zeros

3 Find all of the zeros of $f(x) = x^3 - x^2 + 2x + 4$.

Since $f(x)$ has degree 3, the function has three zeros. To determine the possible number and type of real zeros, examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = x^3 - x^2 + 2x + 4$$

$$f(-x) = -x^3 - x^2 - 2x + 4$$

The function has or positive real zeros and exactly 1 negative real zero. Thus the function has either positive real zeros and negative real zero or imaginary zeros and negative real zero.

To find the zeros, list some possibilities and eliminate those that are not zeros. Use a shortened form of synthetic substitution to find $f(a)$ for several values of a .

x	1	-1	2	4
-3	1	-4	14	-38
-2	1	-3	8	-12
-1	1	-2	4	0

From the table, we can see that one zero occurs at $x = \text{$.

Since the depressed polynomial of this zero, $x^2 - 2x + 4$, is quadratic, use the Quadratic Formula to find the roots of the related quadratic equation, $x^2 - 2x + 4 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{(-)\text{} \pm \sqrt{(\text{)^2 - 4(\text{

Replace a with 1, b with -2 , and c with 4.$$

$$= \frac{2 \pm \sqrt{-12}}{2} \quad \text{Simplify.}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2} \text{ or } \text{} \quad \text{Simplify.}$$

Thus, the function has one real zero at $x = \boxed{}$, and two imaginary zeros at $x = \boxed{}$ and $x = \boxed{}$.

Check Your Progress

Find all the zeros of $f(x) = x^3 - 3x^2 - 2x + 4$.

EXAMPLE**Use Zeros to Write a Polynomial Function**

4 Write a polynomial function of least degree with integral coefficients, the zeros of which include 4 and $4 - i$.

Explore If $4 - i$ is a zero, then $\boxed{}$ is also a zero. So,

$x - 4$, $x - (4 - i)$, and $\boxed{}$ are factors of the polynomial function.

Plan Write the polynomial function as a product of its factors: $f(x) = (x - 4)[x - (4 - i)][x - (4 + i)]$

Solve Multiply the factors to find the polynomial function.
 $f(x) = (x - 4)x - (4 - i)[x - (4 + i)]$

$$= (x - 4) \boxed{} [(x - 4) + i]$$

$$= (x - 4) \boxed{}$$

$$= (x - 4) \boxed{}$$

$$= (x - 4) \boxed{}$$

$$= \boxed{}$$

$$= \boxed{}$$

$f(x) = \boxed{}$ is a polynomial function of least degree with integral coefficients and zeros of 4, $4 - i$, and $4 + i$.

Check Your Progress

Write a polynomial function of least degree with integral coefficients whose zeros include 2 and $1 + i$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

MAIN IDEAS

- Identify the possible rational zeros of a polynomial function.
- Find all the rational zeros of a polynomial function.

KEY CONCEPTS

Rational Zero Theorem
Let $f(x) =$

$$a_0x^n + a_1x^{n-1} + \dots +$$

$$a_{n-2}x^2 + a_{n-1}x + a_n$$

represent a polynomial function with integral coefficients. If $\frac{p}{q}$ is a

rational number in simplest form and is a zero of $y = f(x)$, then p is a factor of a_n and q is a factor of a_0 .

Corollary (Integral Zero Theorem) If the coefficients of a polynomial function are integers such that $a_0 = 1$ and $a_n \neq 0$, any rational zeros of the function must be factors of a_n .



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

EXAMPLE

Identify Possible Zeros

- 1 List all of the possible rational zeros of each function.

a. $f(x) = 3x^4 - x^3 + 4$

If $\frac{p}{q}$ is a rational zero, then p is a factor of 4 and q is a

factor of 3. The possible factors of p are , ,

and . The possible factors of q are and .

So, $\frac{p}{q}$.

b. $f(x) = x^4 + 7x^3 - 15$

Since the coefficient of x^4 is the possible rational zeros

must be the factors of the constant term -15 . So, the possible

rational zeros .

Check Your Progress

List all of the possible rational

zeros of each function.

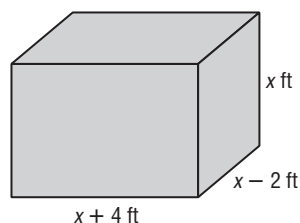
a. $f(x) = 2x^3 + x + 6$

b. $f(x) = x^3 + 3x + 24$

EXAMPLE

Find Rational Zeros

- 2 **GEOMETRY** The volume of a rectangular solid is 1120 cubic feet. The width is 2 feet less than the height, and the length is 4 feet more than the height. Find the dimensions of the solid.



Let x = the height, = the width, and = the length. Write an equation for the volume.

$$\ell wh = V \quad \text{Formula for volume}$$

$$(x - 2)(x + 4)x = 1120 \quad \text{Substitute.}$$

$$\text{} = 1120 \quad \text{Multiply.}$$

$$x^3 + 2x^2 - 8x - 1120 = 0 \quad \text{Subtract 1120.}$$

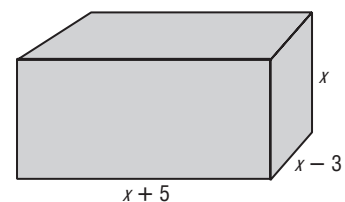
The leading coefficient is 1, so the possible integer zeros are factors of 1120, $\pm 1, \pm 2, \pm 4, \pm 5, \pm 7, \pm 8, \pm 10, \pm 14, \pm 16, \pm 20, \pm 28, \pm 32, \pm 35, \pm 40, \pm 56, \pm 70, \pm 80, \pm 112, \pm 140, \pm 160, \pm 224, \pm 280, \pm 560$, and ± 1120 . Since length can only be positive, we only need to check positive zeros. From Descartes' Rule of Signs, we also know there is only one positive real zero. Make a table and test possible real zeros.

p	1	2	-8	-1120
2	1	3	-5	-1126
4	1	6	16	-1056
7	1	9	55	-735
10	1	12	112	0

Our zero is . Since there is only one positive real zero, we do not have to test the other numbers. The other dimensions are $10 + 4$ or feet and $10 - 2$ or feet.

Check Your Progress

The volume of a rectangular solid is 100 cubic feet. The width is 3 feet less than the height, and the length is 5 feet more than the height. Find the dimensions of the solid.



EXAMPLE Find All Zeros

3 Find all of the zeros of $f(x) = x^4 + x^3 - 19x^2 + 11x + 30$.

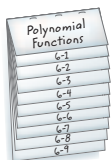
From the corollary to the Fundamental Theorem of Algebra, we know there are exactly 4 complex roots.

According to Descartes' Rule of Signs, there are 2 or 0 positive real roots and 2 or 0 negative real roots.

FOLDABLES™

ORGANIZE IT

On the tab for Lesson 6-9, write how you would find the possible rational zeros of $f(x) = x^3 + 6x^2 + x + 6$.



The possible rational zeros are

Make a table and test some possible rational zeros.

$\frac{p}{q}$	1	1	-19	11	30
0	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
1	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
2	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Since $f(2) = 0$, you know that $x = 2$ is a zero. The depressed polynomial is .

Since $x = 2$ is a positive real zero, and there can only be 2 or 0 positive real zeros, there must be one more positive real zero. Test the next possible rational zeros on the depressed polynomial.

$\frac{p}{q}$	1	3	-13	-15
3	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

There is another zero at $x = 3$. The depressed polynomial is

Factor $x^2 + 6x + 5$.

$$x^2 + 6x + 5 = 0$$

Write the depressed polynomial.

= 0

Factor.

= 0 or = 0

Zero Product Property

$$x = \text{} \quad x = \text{}$$

There are two more real zeros at $x = \text{}$ and $x = \text{}$.

Check Your Progress


Find all of the zeros of $f(x) = x^4 + 4x^3 - 14x^2 - 36x + 45$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

STUDY GUIDE

	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 6 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 6, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 161–162) to help you solve the puzzle.

6-1

Properties of Exponents

Simplify. Assume that no variable equals 0.

1. $(3n^4y^3)(-2ny^{-5})$

2. $\frac{12(x^2y)^3}{4(xy^0)^2}$

6-2

Operations with Polynomials

Determine whether each expression is a polynomial. If the expression is a polynomial, classify it by the number of terms and state the degree of the polynomial.

3. $\sqrt{3x}$

4. $4r^4 - 2r + 1$

5. $2ab + 4ab^2 - 6ab^3$

6. $5x + 4$

Simplify.

7. $(3a - 6) - (2a - 1)$

8. $(2x - 5)(3x + 5)$

6-3

Dividing Polynomials

Simplify.

9. $(c^3 + c^2 - 14c - 24) \div (c - 4)$

10. $\frac{n^2 + 3n - 2}{n + 2}$

6-4

Polynomial Functions

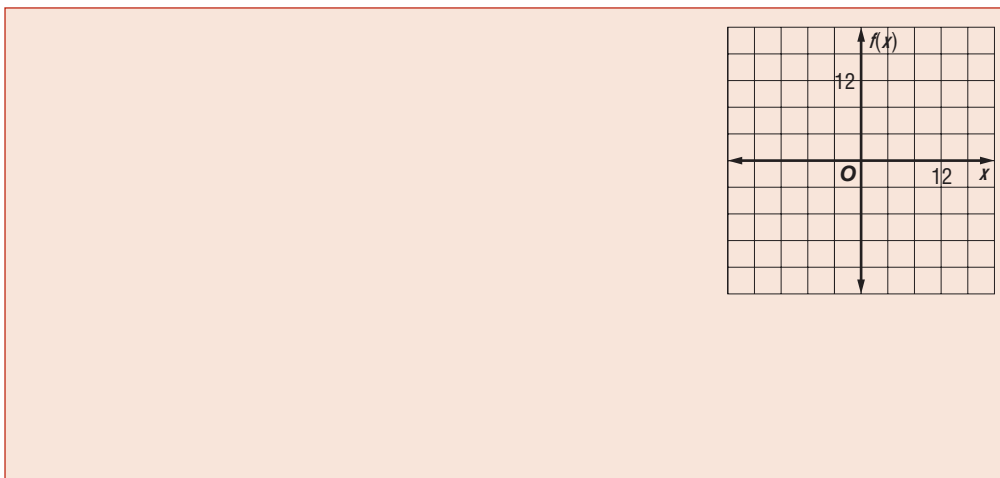
11. Give the degree and leading coefficient of each polynomial.

	degree	leading coefficient
a. $10x^3 + 3x^2 - x + 7$	<input style="width: 40px; height: 25px;" type="text"/>	<input style="width: 60px; height: 25px;" type="text"/>
b. $7y^2 - 2y^5 + y - 4y^3$	<input style="width: 40px; height: 25px;" type="text"/>	<input style="width: 60px; height: 25px;" type="text"/>
c. 100	<input style="width: 40px; height: 25px;" type="text"/>	<input style="width: 60px; height: 25px;" type="text"/>

6-5

Analyzing Graphs of Polynomial Functions

12. Graph $f(x) = x^3 - 6x^2 + 2x + 8$ by making a table of values. Let $x = \{-2, -1, 0, 1, 2, 3\}$. Then determine consecutive values of x between which each real zero is located. Estimate the x -coordinates at which the relative maxima and relative minima occur.



6-6

Solving Polynomial Equations

Factor completely. If the polynomial is not factorable, write *prime*.

13. $3w^2 - 48$

14. $a^3 + 5a - 3a^2 - 15$

15. Simplify $\frac{x^2 + 7x + 10}{x^2 - 4}$. Assume that the denominator is not equal 0.

6-7

The Remainder and Factor Theorems

Find $f(-2)$ for each function.

16. $f(x) = x^3 + 4x^2 - 8x - 6$

17. $f(x) = x^3 + 4x^2 + 4x$

6-8

Roots and Zeros

Let $f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 + 6x - 7$.

18. Write $f(-x)$ in simplified form (with no parentheses).19. What are the possible numbers of positive real zeros of f ?
negative real zeros of f ?

6-9

Rational Zero Theorem

20. List all the possible values of p , all the possible values of q , and all the possible rational zeros $\frac{p}{q}$ for $f(x) = x^3 - 2x^2 - 11x + 12$.



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 6.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 6 Practice Test on page 379 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 6 Study Guide and Review on pages 374–378 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 379 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 6 Foldable.
- Then complete the Chapter 6 Study Guide and Review on pages 374–378 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 379 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

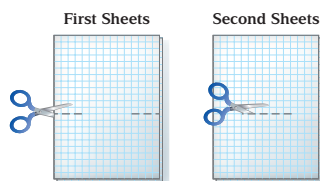
Radical Equations and Inequalities



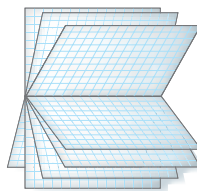
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with four sheets of grid paper.

STEP 1 **Fold** in half along the width. On the first two sheets, cut 5 centimeters along the fold at the ends. On the second two sheets, cut in the center, stopping 5 centimeters from the ends.



STEP 2 **Insert** the first sheets through the second sheets and align the folds. Label the pages with lesson numbers.



NOTE-TAKING TIP: When you take notes, preview the lesson and make generalizations about what you think you will learn. Then compare that with what you actually learned after each lesson.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 7. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
composition of functions			
conjugates			
extraneous solution			
identity function			
inverse function			
inverse relation			
like radical expressions			

Vocabulary Term	Found on Page	Definition	Description or Example
n th root			
one-to-one			
principal root			
radical equation			
radical inequality			
rationalizing the denominator			
square root function			
square root inequality			

Operations on Functions

MAIN IDEAS

- Find the sum, difference, product, and quotient of functions.
- Find the composition of functions.

KEY CONCEPT

Operations with Functions

Sum

$$(f + g)(x) = f(x) + g(x)$$

Difference

$$(f - g)(x) = f(x) - g(x)$$

Product

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Quotient

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$



TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B) Relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions.** **(C) Determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities.**

EXAMPLE

Add and Subtract Functions

1 Given $f(x) = 3x^2 + 7x$ and $g(x) = 2x^2 - x - 1$, find each function.

a. $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= \boxed{} + \boxed{}$$

$$= \boxed{}$$

b. $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= \boxed{} - \boxed{}$$

$$= \boxed{}$$

Check Your Progress

Given $f(x) = 2x^2 + 5x + 2$ and $g(x) = 3x^2 + 3x - 4$, find each function.

a. $(f + g)(x)$

$$\boxed{}$$

b. $(f - g)(x)$

$$\boxed{}$$

EXAMPLE

Multiply and Divide Functions

2 Given $f(x) = 3x^2 - 2x + 1$ and $g(x) = x - 4$, find each function.

a. $(f \cdot g)(x)$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= \left(\boxed{} \right) \left(\boxed{} \right)$$

$$= 3x^2(x - 4) - 2x(x - 4) + 1(x - 4)$$

Distributive Property

$$= \boxed{}$$

Distributive Property

$$= \boxed{}$$

Simplify.

$$\text{b. } \left(\frac{f}{g}\right)(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$=$$

Check Your Progress Given $f(x) = 2x^2 + 3x - 1$ and $g(x) = x + 2$, find each function.

$$\text{a. } (f \cdot g)(x)$$

$$\text{b. } \left(\frac{f}{g}\right)(x)$$

EXAMPLE Evaluate Composition of Relations

3 If $f(x) = \{(2, 6), (9, 4), (7, 7), (0, -1)\}$ and $g(x) = \{(7, 0), (-1, 7), (4, 9), (8, 2)\}$, find $f \circ g$ and $g \circ f$.

To find $f \circ g$, evaluate $g(x)$ first. Then use the range of g as the domain of f and evaluate $f(x)$.

$$f[g(7)] = f(0) \text{ or } \boxed{}$$

$$f[g(4)] = f(9) \text{ or } \boxed{}$$

$$f[g(-1)] = f(7) \text{ or } \boxed{}$$

$$f[g(8)] = f(2) \text{ or } \boxed{}$$

$$f \circ g =$$

To find $g \circ f$, evaluate $f(x)$ first. Then use the range of f as the domain of g and evaluate $g(x)$.

$$g[f(2)] = g(6), \text{ which is } \boxed{}$$

$$g[f(7)] = g(7) \text{ or } \boxed{}$$

$$g[f(9)] = g(4) \text{ or } \boxed{}$$

$$g[f(0)] = g(-1) \text{ or } \boxed{}$$

$$g \circ f =$$

Check Your Progress If $f(x) = \{(1, 2), (0, -3), (6, -5), (2, 1)\}$ and $g(x) = \{(2, 0), (-3, 6), (1, 0), (6, 7)\}$, find $f \circ g$ and $g \circ f$.

EXAMPLE

Simplify Composition of Functions

KEY CONCEPT

Composition of Functions Suppose f and g are functions such that the range of g is a subset of the domain of f . Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

FOLDABLES On the page for Lesson 7-1, write how you would read $[f \circ g](x)$. Then explain which function, f or g , you would evaluate first.

- 4 a. Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = 3x^2 - x + 4$ and $g(x) = 2x - 1$.

$$\begin{aligned}
 [f \circ g](x) &= f[g(x)] \\
 &= f(\boxed{}) \\
 &= \boxed{} - \boxed{} + \boxed{} \\
 &= 3(4x^2 - 4x + 1) - 2x + 1 + 4 \\
 &= \boxed{}
 \end{aligned}$$

$$\begin{aligned}
 [g \circ f](x) &= g(f(x)) \\
 &= g(\boxed{}) \\
 &= \boxed{}(3x^2 - x + 4) - \boxed{} \\
 &= \boxed{}
 \end{aligned}$$

- b. Evaluate $[f \circ g](x)$ and $[g \circ f](x)$ for $x = -2$.

$$\begin{aligned}
 [f \circ g](x) &= 12x^2 - 14x + 8 \\
 [f \circ g](-2) &= \boxed{} \\
 &= \boxed{}
 \end{aligned}$$

$$\begin{aligned}
 [g \circ f](x) &= 6x^2 - 2x + 7 \\
 [g \circ f](-2) &= \boxed{} \\
 &= \boxed{}
 \end{aligned}$$

Check Your Progress

- a. Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = x^2 + 2x + 3$ and $g(x) = x + 5$.

- b. Evaluate $[f \circ g](x)$ and $[g \circ f](x)$ for $x = 1$.

EXAMPLE

Use Composition of Function

- 5 TAXES** Tracey Long has \$100 deducted from every paycheck for retirement. She can have this deduction taken before state taxes are applied, which reduces her taxable income. Her state income tax rate is 4%. If Tracey earns \$1500 every pay period, find the difference in her net income if she has the retirement deduction taken before or after state taxes.

Explore Let x = Tracey's income per paycheck, $r(x)$ = her income after the deduction for retirement, and $t(x)$ = her income after the deduction for state income tax.

Plan Write equations for $r(x)$ and $t(x)$.
\$100 is deducted from every paycheck for retirement:

$$r(x) = x - \boxed{}$$

$$\text{Tracey's tax rate is 4\%: } t(x) = \boxed{}$$

Solve If Tracey has her retirement deducted *before* taxes, then her net income is represented by $[t \circ r](1500)$.

$$[t \circ r](1500) = t(1500 - 100)$$

$$= t \boxed{}$$

$$= 1400 - 0.04(1400)$$

$$= \boxed{}$$

If Tracey has her retirement deducted *after* taxes, then her net income is represented by $[r \circ t](1500)$.

$$[r \circ t](1500) = r[1500 - 0.04(1500)]$$

$$= r \boxed{}$$

$$= \boxed{} - 100$$

$$= \boxed{}$$

Her net pay is $\boxed{}$ more by having her retirement deduction taken before state taxes.

Check Your Progress

Brandi Smith has deducted \$200 from every paycheck for retirement. She can have this deduction taken before state taxes are applied, which reduces her taxable income. Her state income tax is 10%. If Brandi earns \$2200 every pay period, find the difference in her net income if she has the retirement deduction taken before or after state taxes.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Inverse Functions and Relations



TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(B) Relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions.**

EXAMPLE

Find an Inverse Relation

MAIN IDEAS

- Find the inverse of a function or relation.
- Determine whether two functions or relations are inverses.

KEY CONCEPTS

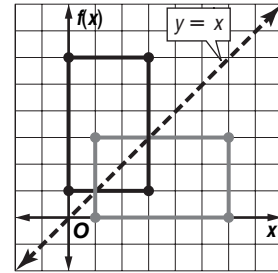
Inverse Relations Two relations are inverse relations if and only if whenever one relation contains the element (a, b) , the other relation contains the element (b, a) .

Property of Inverse Functions Suppose f and f^{-1} are inverse functions. Then, $f(a) = b$ if and only if $f^{-1}(b) = a$.

- 1 GEOMETRY** The ordered pairs of the relation $\{(1, 3), (6, 3), (6, 0), (1, 0)\}$ are the coordinates of the vertices of a rectangle. Find the inverse of this relation and determine whether the resulting ordered pairs are also the coordinates of the vertices of a rectangle.

To find the inverse of this relation, reverse the coordinates of the ordered pairs. The inverse of the relation is

Plotting the points shows that the ordered pairs also describe the vertices of a rectangle. Notice that the graph of the relation and the inverse are reflections over the graph of $y = x$.



EXAMPLE

Find an Inverse Function

- 2 a.** Find the inverse of $f(x) = -\frac{1}{2}x + 1$.

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = -\frac{1}{2}x + 1 \rightarrow$$

Step 2 Interchange x and y

Step 3 Solve for y .

$$x = -\frac{1}{2}y + 1$$

Inverse

$$\text{[]} = \text{[]}$$

Multiply each side by -2 .

$$\text{[]} = \text{[]}$$

Add 2 to each side.

Step 4 Replace y with $f^{-1}(x)$.

$$y = \text{[]} \rightarrow$$

$$f^{-1}(x) =$$

EXAMPLE Verify Two Functions are Inverses

- 3 Determine whether $f(x) = \frac{3}{4}x - 6$ and $g(x) = \frac{4}{3}x + 8$ are inverse functions.

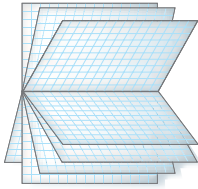
Check to see if the compositions of $f(x)$ and $g(x)$ are identity functions.

$$\begin{aligned}
 [f \circ g](x) &= f\left(\boxed{}\right) & [g \circ f](x) &= g\left(\boxed{}\right) \\
 &= \boxed{}\left(\frac{4}{3}x + 8\right) - \boxed{} & &= \boxed{}\left(\frac{3}{4}x - 6\right) + \boxed{} \\
 &= \boxed{} & &= \boxed{} \\
 &= \boxed{} & &= \boxed{}
 \end{aligned}$$

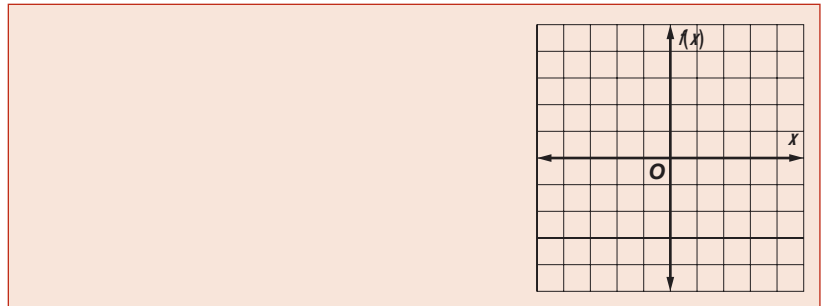
The functions are inverses since both compositions equal $\boxed{}$.

FOLDABLES™**ORGANIZE IT**

On the page for Lesson 7-2, sketch your own graph of two relations that are inverses.

**Check Your Progress**

- a. The ordered pairs of the relation $\{(-3, 4), (-1, 5), (2, 3), (1, 1), (-2, 1)\}$ are the coordinates of the vertices of a pentagon. Find the inverse of this relation and determine whether the resulting ordered pairs are also the coordinates of the vertices of a pentagon.



- b. Find the inverse of $f(x) = \frac{1}{3}x + 6$.

- c. Determine whether $f(x) = 3x + 1$ and $g(x) = \frac{x - 1}{3}$ are inverse functions.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____


Square Root Functions and Inequalities

EXAMPLE

Graph a Square Root Function

MAIN IDEAS

- Graph and analyze square root functions.
- Graph square root inequalities.

 **TEKS 2A.9** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A) Use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges.** **(F)** Analyze situations modeled by square root functions, **formulate equations** or inequalities, **select a method, and solve problems.** **(G) Connect inverses of square root functions with quadratic functions.** Also addresses TEKS 2A.9(B).

- 1 Graph $y = \sqrt{\frac{3}{2}x - 1}$. State the domain, range, and x - and y -intercepts.

Since the radicand cannot be negative, identify the domain

$$\boxed{} \geq 0$$

Write the expression inside the radicand as ≥ 0 .

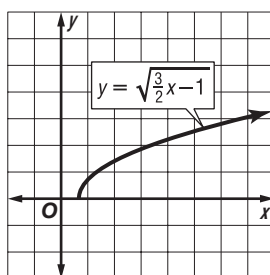
$$x \geq \boxed{}$$

Solve for x .

The x -intercept is $\boxed{}$.

Make a table of values to graph the function. The domain is $x \geq \frac{2}{3}$ and the range is $y \geq 0$.

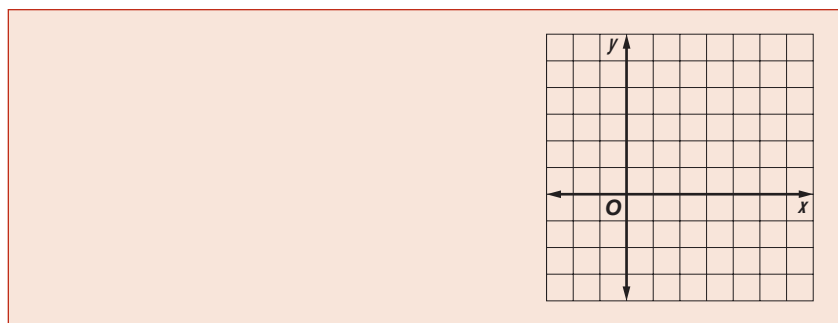
The x -intercept is $\frac{2}{3}$. There is no y -intercept



x	y
$\frac{2}{3}$	0
1	0.71
2	1.41
3	1.87
4	2.24
5	2.55
6	2.83

Check Your Progress

Graph $y = \sqrt{2x - 2}$. State the domain, range, and x - and y -intercepts.



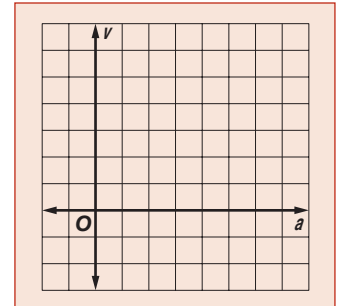
EXAMPLE

- 2 PHYSICS** When an object is spinning in a circular path of radius 2 meters with velocity v , in meters per second, the centripetal acceleration a , in meters per second squared, is directed toward the center of the circle. The velocity v and acceleration a of the object are related by the function $v = \sqrt{2a}$.

- a. Graph the function. State the domain and range.

The domain is , and the

range is .



- b. What would be the centripetal acceleration of an object spinning along the circular path with a velocity of 4 meters per second?

$$v = \sqrt{2a}$$

Original equation

$$\text{[]} = \sqrt{2a}$$

Replace v with .

$$16 = 2a$$

Square each side.

$$\text{[]} = a$$

Divide each side by 2.

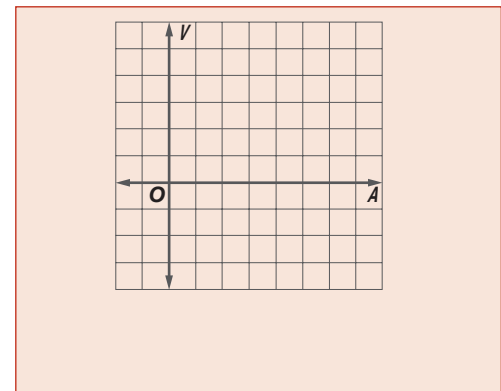
The centripetal acceleration would have to be meters per second squared. Check the reasonableness of this result by comparing it to the graph.

Check Your Progress

The volume V and surface area A of a soap bubble are related by the function

$$V = 0.094\sqrt{A^3}$$

- a. Graph the function. State the domain and range.

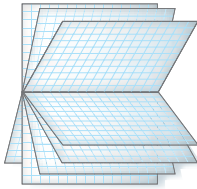


- b. What would the surface area be if the volume were 94 cubic units?

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-3, write how you know if a boundary should be included or not. Then write how you know which region to shade.



EXAMPLE Graph a Square Root Inequality

3 a. Graph $y > \sqrt{3x + 5}$.

Graph the related equation $y = \sqrt{3x + 5}$. Since the boundary is not included, the graph should be dashed.

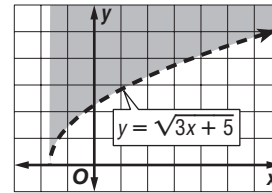
The domain includes values for $x \geq -\frac{5}{3}$. So the graph is to the

right of $x = -\frac{5}{3}$.

Select a point and test its ordered pair. Test $(0, 0)$

$$0 > \boxed{}$$

$$0 > \boxed{} \quad \text{false}$$



Shade the region that does not include $(0, 0)$.

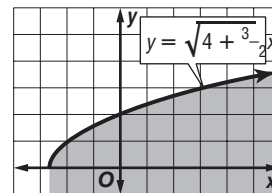
b. Graph $y = \sqrt{4 + \frac{3}{2}x}$.

Graph the related equation $y = \sqrt{4 + \frac{3}{2}x}$.

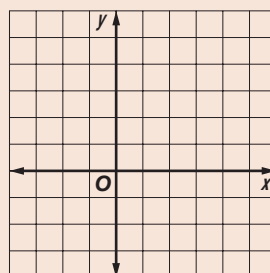
The domain includes values for $x \geq -\frac{8}{3}$. Test $(4, 1)$.

$$1 \leq \boxed{}$$

$$1 \leq \boxed{} \quad \text{true}$$



Shade the region that includes $(4, 1)$.

Check Your Progress Graph $y \leq \sqrt{3 + 3x}$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Simplify radicals.
- Use a calculator to approximate radicals.

BUILD YOUR VOCABULARY (pages 198–199)

The inverse of raising a number to the n th power is finding the n th root of a number. When there is more than one real root, the root is called the principal root.

EXAMPLE Find Roots

1 Simplify

a. $\pm\sqrt{16x^6}$

$$\pm\sqrt{16x^6}$$

$$= \pm\sqrt{\boxed{}^2}$$

$$= \boxed{}$$

b. $-\sqrt{(q^3 + 5)^4}$

$$-\sqrt{(q^3 + 5)^4}$$

$$= -\sqrt{\boxed{}^2}$$

$$= \boxed{}$$

c. $\sqrt[5]{243a^{10}b^{15}}$

$$\sqrt[5]{243a^{10}b^{15}} = \sqrt[5]{\boxed{}^5}$$

$$= \boxed{}$$

d. $\sqrt{-4}$

$$\sqrt{-4} = \boxed{}$$

Since n is even and b is negative, $\sqrt{-4}$ is not a real number.

KEY CONCEPTS

Definition of Square Root

For any real numbers a and b , if $a^2 = b$, then a is a square root of b .

Definition of n th Root

For any real numbers a and b , and any positive integer n , if $a^n = b$, then a is an n th root of b .



TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(C) Determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities. (D) Determine solutions of square root equations using graphs, tables, and algebraic methods.

(C) Determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities. (D) Determine solutions of square root equations using graphs, tables, and algebraic methods.

Check Your Progress

a. $\pm\sqrt{9x^8}$

$$\boxed{}$$

c. $\sqrt[5]{32x^5y^{10}}$

$$\boxed{}$$

Simplify.

b. $-\sqrt{(a^3 + 2)^6}$

$$\boxed{}$$

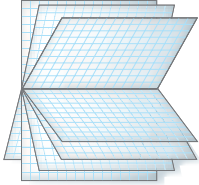
d. $\sqrt{-16}$

$$\boxed{}$$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-4, explain how to simplify using absolute value. Include an example.



EXAMPLE Simplify Using Absolute Value

2 Simplify.

a. $\sqrt[6]{t^6}$

Note that t is a sixth root of t^6 . The index is even, so the principal root is nonnegative. Since t could be negative, you must take the absolute value of t to identify the principal root.

$$\sqrt[6]{t^6} = \boxed{}$$

b. $\sqrt[5]{243(x+2)^{15}}$

$$\sqrt[5]{243(x+2)^{15}} = \sqrt[5]{\boxed{}^5}$$

Since the index is odd, you do not need absolute value.

$$\sqrt[5]{243(x+2)^{15}} = \boxed{}$$

Check Your Progress

Simplify.

a. $\sqrt[4]{x^4}$

$$\boxed{}$$

b. $\sqrt[3]{27(x+2)^9}$

$$\boxed{}$$

EXAMPLE

- 3 **FISH** The relationship between the length and mass of Pacific halibut can be approximated by the equation $L = 0.46\sqrt[3]{M}$, where L is the length of the fish in meters and M is the mass in kilograms. Use a calculator to approximate the length of a 30-kilogram Pacific halibut.

$$L = 0.46\sqrt[3]{M} \qquad \text{Original equation}$$

$$L = 0.46\sqrt[3]{\boxed{}} \text{ or about } \boxed{} \quad M = \boxed{}$$

The length of a 30-kilogram Pacific halibut is about

$$\boxed{}.$$

Check Your Progress

The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt[3]{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. Find the value of T for a 2-foot-long pendulum.

$$\boxed{}$$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Radical Expressions



Preparation for TEKS 2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(D)** Determine solutions of square root equations using graphs, tables, and algebraic methods.

EXAMPLE

Simplify Expressions

MAIN IDEAS

- Simplify radical expressions.
- Add, subtract, multiply, and divide radical expressions.

KEY CONCEPTS

Product Property of Radicals For any real numbers a and b and any integer $n > 1$,

1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, and

2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

Quotient Property of Radicals For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

1 Simplify.

a. $\sqrt{25a^4b^9}$

$$\sqrt{25a^4b^9} = \sqrt{\boxed{}} (b^4)^2 b$$

Factor into squares.

$$= \boxed{} \sqrt{(b^4)^2} \sqrt{b}$$

Product Property

$$= \boxed{} b^4 \sqrt{b}$$

Simplify.

b. $\sqrt{\frac{y^8}{x^7}}$

$$\sqrt{\frac{y^8}{x^7}} = \sqrt{\frac{y^8}{x^7}}$$

Quotient Property

$$= \frac{\sqrt{\boxed{}^2}}{\sqrt{\boxed{}^2} \cdot x}$$

Factor into squares.

$$= \frac{\sqrt{(y^4)^2}}{\sqrt{(x^3)^2} \cdot \sqrt{x}}$$

Product Property

$$= \frac{y^4}{x^3 \sqrt{x}}$$

$$\sqrt{(y^4)^2} = y^4$$

$$= \frac{y^4}{x^3 \sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

Rationalize the denominator.

$$= \boxed{\phantom{\frac{y^4 \sqrt{x}}{x^3 \sqrt{x}}}}$$

$$\sqrt{x} \cdot \sqrt{x} = x$$

c. $\sqrt[3]{\frac{2}{9x}}$

$$\sqrt[3]{\frac{2}{9x}} = \frac{\sqrt[3]{2}}{\sqrt[3]{9x}}$$

Quotient Property

$$= \frac{\sqrt[3]{2}}{\sqrt[3]{9x}} \cdot \boxed{} \quad \text{Rationalize the denominator.}$$

$$= \frac{\sqrt[3]{2 \cdot 3x^2}}{\sqrt[3]{9x \cdot 3x^2}} \quad \text{Product Property}$$

$$= \boxed{} \quad \text{Multiply.}$$

$$= \boxed{} \quad \sqrt[3]{27x^3} = 3x$$

Check Your Progress Simplify each expression.

a. $\sqrt{16x^4y^{11}}$

b. $\sqrt{\frac{x^3}{y^7}}$

c. $\sqrt[3]{\frac{2}{3a}}$

BUILD YOUR VOCABULARY (page 198)

Two radical expressions are called **like radical expressions** if both the indices and radicands are alike.

EXAMPLE Multiply Radicals

2 Simplify $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$.

$$5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a} = 5 \cdot \sqrt[3]{100a^2 \cdot 10a} \quad \text{Product Property of Radicals}$$

$$= 5 \cdot \sqrt{\boxed{}} \quad \text{Factor into cubes where possible.}$$

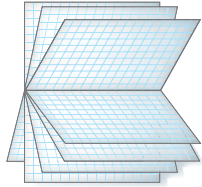
$$= 5 \cdot \boxed{} \cdot \boxed{} \quad \text{Product Property of Radicals}$$

$$= 5 \cdot \boxed{} \cdot \boxed{} \text{ or } \boxed{} \quad \text{Multiply.}$$

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-5, write your own example of two like radical expressions. Then find their sum.



Check Your Progress Simplify $3\sqrt[3]{16a^2} \cdot 2\sqrt[3]{4a}$.

EXAMPLE

Add and Subtract Radicals

3 Simplify $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$.

$$3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$$

$$= 3\sqrt{3^2 \cdot 5} - 5\sqrt{4^2 \cdot 5} + 4\sqrt{2^2 \cdot 5}$$

Factor.

$$= 3\sqrt{3^2} \cdot \sqrt{5} - 5\sqrt{4^2} \cdot \sqrt{5} + 4\sqrt{2^2} \cdot \sqrt{5}$$

Product Property

$$= 3 \cdot \boxed{} \sqrt{5} - 5 \cdot 4\sqrt{5} + 4 \cdot \boxed{} \sqrt{5}$$

$$= \boxed{}$$

Multiply.

$$= \boxed{}$$

Combine like radicals.

Check Your Progress Simplify $3\sqrt{75} - 2\sqrt{48} + \sqrt{3}$.

EXAMPLE

Multiply Radicals

3 Simplify.

a. $(2\sqrt{3} + 3\sqrt{5})(3 - \sqrt{3})$

$$(2\sqrt{3} + 3\sqrt{5})(3 - \sqrt{3})$$

F	O	I	L
$= 2\sqrt{3} \cdot 3 -$	$\boxed{}$	$+ \boxed{}$	$- 3\sqrt{5} \cdot \sqrt{3}$

$$= 6\sqrt{3} - \boxed{} + \boxed{} - 3\sqrt{15}$$

$$= 6\sqrt{3} - \boxed{} - 3\sqrt{15}$$

b. $(4\sqrt{2} + 7)(4\sqrt{2} - 7)$

$$(4\sqrt{2} + 7)(4\sqrt{2} - 7)$$

$$= 4\sqrt{2} \cdot 4\sqrt{2} - \boxed{} + \boxed{} - 7 \cdot 7$$

$$= \boxed{} - 7 \cdot 4\sqrt{2} + 7 \cdot 4\sqrt{2} - \boxed{}$$

$$= 32 - 49$$

$$= \boxed{}$$

Check Your Progress

Simplify each expression.

a. $(2\sqrt{5} + 4\sqrt{6})(5 - \sqrt{7})$

b. $(3\sqrt{5} - 2)(3\sqrt{5} + 2)$

EXAMPLE

Use a Conjugate to Rationalize a Denominator

5 Simplify $\frac{2 + \sqrt{3}}{4 - \sqrt{3}}$.

$$\frac{2 + \sqrt{3}}{4 - \sqrt{3}} = \frac{(2 + \sqrt{3})(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$$

$$= \frac{2 \cdot 4 + 2\sqrt{3} + \boxed{} + \boxed{}}{4^2 - (\sqrt{3})^2}$$

$$= \frac{\boxed{} + 2\sqrt{3} + 4\sqrt{3} + \boxed{}}{16 - 3}$$

$$= \boxed{}$$

 $4 + \sqrt{3}$ is the conjugate of $4 - \sqrt{3}$.FOIL
Difference of squares.

Multiply.

Combine like terms.

Check Your Progress

Simplify $\frac{3 + \sqrt{5}}{2 - \sqrt{5}}$.

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____

Rational Exponents



TEKS 2A.4 The student connects algebraic and geometric representations of functions. **(C)** Describe and analyze the relationship between a function and its inverse.

EXAMPLE

Radical and Exponential Forms

MAIN IDEAS

- Write expressions with rational exponents in radical form, and vice versa.
- Simplify expressions in exponential or radical form.

KEY CONCEPTS

$b^{\frac{1}{n}}$ For any real number b and for any positive integer n , $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.

Rational Exponents

For any nonzero real number b , and any integers m and n , with $n > 1$,

$b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, except when $b < 0$ and n is even.

- 1 a. Write $a^{\frac{1}{6}}$ in radical form.

$$a^{\frac{1}{6}} = \boxed{}$$

- b. Write \sqrt{w} in exponential form.

$$\sqrt{w} = \boxed{}$$

Check Your Progress

- a. Write $x^{\frac{1}{5}}$ in radical form.

- b. Write $\sqrt[3]{y}$ in exponential form.

EXAMPLE

Evaluate Expressions with Rational Exponents

- 2 Evaluate each expression.

a. $49^{-\frac{1}{2}}$

$$49^{-\frac{1}{2}} = \frac{1}{\boxed{}}$$

$$= \frac{1}{\boxed{}}$$

$$= \boxed{}$$

$$b^{-n} = \frac{1}{b^n}$$

$$49^{\frac{1}{2}} = \boxed{}$$

Simplify.

REMEMBER IT

Since $b^{\frac{m}{n}}$ is defined as $(b^{\frac{1}{n}})^m$ or $(b^m)^{\frac{1}{n}}$, either can be used to evaluate an expression. Choose depending on the given expression.

b. $32^{\frac{2}{5}}$

$$32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}}$$

=

=

=

$32 = 2^5$

Power of a Power

Multiply exponents.

$2^2 = 4$

Check Your Progress

Evaluate each expression.

a. $25^{-\frac{1}{2}}$

b. $16^{\frac{3}{4}}$

EXAMPLE

Simplify Expressions with Rational Exponents

3 Simplify each expression.

a. $y^{\frac{1}{7}} \cdot y^{\frac{4}{7}}$

$y^{\frac{1}{7}} \cdot y^{\frac{4}{7}} =$

=

Multiply powers.

Add exponents.

b. $x^{-\frac{2}{3}}$

$x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}}$

= $\frac{1}{x^{\frac{2}{3}}}$ ·

$b^{-n} = \frac{1}{b^n}$

Multiply by .

= or

$x^{\frac{2}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{2}{3} + \frac{1}{3}}$

Check Your Progress Simplify each expression.

a. $x^{\frac{1}{5}} \cdot x^{\frac{2}{5}}$

b. $y^{-\frac{3}{4}}$

EXAMPLE Simplify Radical Expressions**4** Simplify each expression.

a. $\frac{\sqrt[6]{16}}{\sqrt[3]{2}}$

$$\frac{\sqrt[6]{16}}{\sqrt[3]{2}} = \frac{16^{\frac{1}{6}}}{2^{\frac{1}{3}}}$$

Rational exponents

$$= \frac{\boxed{}}{2^{\frac{1}{3}}}$$

$16 = 2^4$

$$= \frac{\boxed{}}{2^{\frac{1}{3}}}$$

Power of a Power

$$= \boxed{}$$

Quotient of Powers

$$= \boxed{} \text{ or } \boxed{}$$

Simplify.

b. $\sqrt[6]{4x^4}$

$$\sqrt[6]{4x^4} = (4x^4)^{\frac{1}{6}}$$

Rational exponents

$$= (2^2 \boxed{})^{\frac{1}{6}}$$

$2^2 = 4$

$$= 2^{2\left(\frac{1}{6}\right)} \cdot \boxed{}$$

Power of a Power

$$= \boxed{}$$

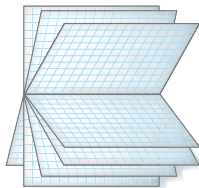
Multiply.

$$= \boxed{}$$

Simplify.

FOLDABLES™**ORGANIZE IT**

On the page for Lesson 7-6, write four conditions that must be met in order for an expression with rational exponents to be in simplest form.



c. $\frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$

$$\frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1} = \frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1} \cdot \frac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} + 1}$$

$$= \frac{y + 2y^{\frac{1}{2}} + 1}{y - 1}$$

$y^{\frac{1}{2}} + 1$ is the conjugate of $y^{\frac{1}{2}} - 1$.

Multiply.

Check Your Progress

Simplify each expression.

a. $\frac{\sqrt[4]{4}}{\sqrt{2}}$

b. $\sqrt[3]{16x^2}$

c. $\frac{x^{\frac{1}{2}} - 1}{x^{\frac{1}{2}} + 1}$

EXAMPLE

5 TEST EXAMPLE If x is a positive number, then $\frac{x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{1}{6}}} = ?$

A $\sqrt{x^3}$

B $\sqrt[3]{x^2}$

C $\sqrt[6]{x^5}$

D $x^{\frac{3}{2}}$

$$\frac{x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{5}{6}}}{x^{\frac{1}{6}}}$$

Add the exponents in the numerator.

$$= x^{\frac{5}{6}} \cdot \text{[]}$$

Quotient of Powers

$$= x^{\frac{2}{3}} \text{ or } \text{[]}$$

The answer is [].

Check Your Progress

If y is a positive number,

then $\frac{y^{\frac{3}{4}} y^{\frac{1}{2}}}{y^{\frac{1}{4}}} = ?$

A y^2

B $y^{\frac{4}{5}}$

C $\sqrt[4]{y^5}$

D $\sqrt[4]{y}$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Solve equations containing radicals.
- Solve inequalities containing radicals.

BUILD YOUR VOCABULARY (pages 198–199)

Equations with radicals that have variables in the radicands are called **radical equations**.

When you solve a radical equation and obtain a number that does not satisfy the original equation, the number is called an **extraneous solution**.

A **radical inequality** is an inequality that has a variable in a radicand.

EXAMPLE Solve Radical Equations

1 a. Solve $\sqrt{y - 2} - 1 = 5$.

$$\sqrt{y - 2} - 1 = 5$$

Original equation

$$\boxed{} = \boxed{}$$

Add to isolate the radical.

$$\boxed{} = \boxed{}$$

Square each side.

$$\boxed{} = \boxed{}$$

Find the squares.

$$y = \boxed{}$$

Add 2 to each side.

b. Solve $\sqrt{x - 12} = 2 - \sqrt{x}$.

$$\sqrt{x - 12} = 2 - \sqrt{x}$$

Original equation

$$(\sqrt{x - 12})^2 = (2 - \sqrt{x})^2$$

Square each side.

$$x - 12 = \boxed{}$$

Find the squares.

$$-16 = -4\sqrt{x}$$

Isolate the radical.

$$\boxed{} = \boxed{}$$

Divide each side by -4 .

$$\boxed{} = \boxed{}$$

Square each side.

$$16 = x$$

Evaluate the squares.

Since $\sqrt{16 - 12} \neq \boxed{}$, the solution does not check

and there is $\boxed{}$



TEKS 2A.9

The student formulates

equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(C) Determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities. (D)

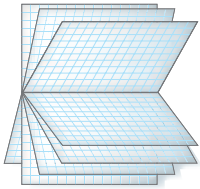
Determine solutions of square root equations using graphs, tables, and algebraic methods. (E) Determine solutions of square root inequalities using graphs and tables. (F)

Analyze situations modeled by square root functions, **formulate equations or inequalities, select a method, and solve problems.** Also addresses TEKS 2A.4(A) and 2A.9(B).

FOLDABLES™

ORGANIZE IT

On the page for Lesson 7-7, write your own problem similar to Example 1. Then explain how you eliminate the radical in order to solve the equation.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress Solve.

a. $\sqrt{x-3} - 2 = 6$

b. $\sqrt{x+5} = -1 - \sqrt{x}$

EXAMPLE Solve a Cube Root Equation

3 Solve $(3y + 1)^{\frac{1}{3}} + 5 = 0$.

In order to remove the $\frac{1}{3}$ power, you must first isolate it and then raise each side of the equation to the third power.

$(3y + 1)^{\frac{1}{3}} + 5 = 0$

Original equation

$(3y + 1)^{\frac{1}{3}} =$

Subtract 5 from each side.

$[(3y + 1)^{\frac{1}{3}}]^3 =$

Cube each side.

$3y + 1 =$

Evaluate the cubes.

$y =$

Simplify.

EXAMPLE Solve a Radical Inequality

3 Solve $\sqrt{3x - 6} + 4 \leq 7$.

Find values of x for which the left side is defined.

$3x - 6 \geq 0$

Radicand must be positive or 0.

$3x \geq 6$

$x \geq 2$

Now solve $\sqrt{3x - 6} + 4 \leq 7$.

$\sqrt{3x - 6} + 4 \leq$

Original inequality

$\sqrt{3x - 6} \leq$

Isolate the radical.

\leq

Eliminate the radical.

$3x \leq$

Add 6 to each side.

$x \leq$

Divide each side by 3.

The solution is .

Check Your Progress Solve.

a. $(2y + 1)^{\frac{1}{3}} - 3 = 0$

b. $\sqrt{2x + 5} - 2 \leq 9$

BRINGING IT ALL TOGETHER**STUDY GUIDE**

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 7 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 7, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 198–199) to help you solve the puzzle.

7-1**Operations on Functions**

1. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for

$$f(x) = x^2 - 5x + 2 \text{ and } g(x) = 3x + 6.$$

2. Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = x^2 + 8x - 7$ and $h(x) = x - 3$.

7-2**Inverse Functions and Relations**

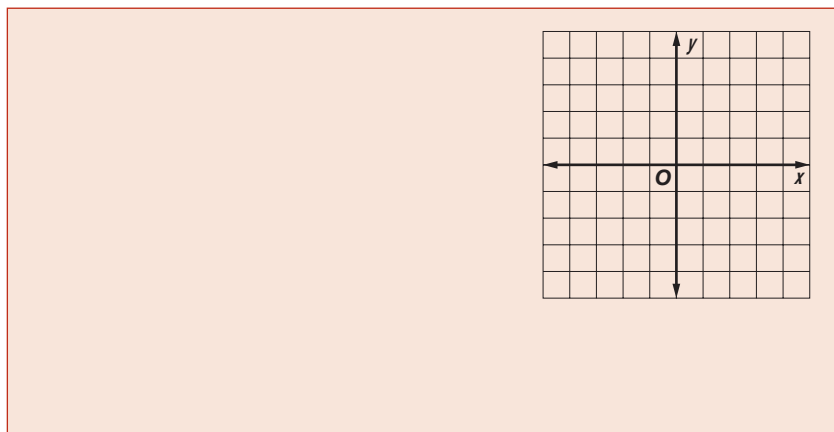
3. Find the inverse of the function $f(x) = -5x + 4$.

4. Determine whether $g(x) = 2x + 4$ and $f(x) = \frac{x}{2} + 2$ are inverse functions.

7-3

Square Root Functions and Inequalities

5. Graph $y < 2 + \sqrt{3x + 4}$. Then state the domain and range of the function.



7-4

*n*th Roots

6. Use a calculator to approximate $\sqrt[3]{-280}$ to three decimal places.

Simplify.

7. $\sqrt[3]{-64}$

8. $\sqrt[8]{x^{16}}$

9. $\sqrt{100a^{12}}$

10. $\sqrt[3]{8y^3}$

11. $\sqrt{49x^6y^8}$

12. $\sqrt[3]{125c^6d^{15}}$

7-5

Operations with Radical Expressions

Simplify.

13. $\sqrt{\frac{5}{6x}}$

14. $2\sqrt{45} - 7\sqrt{8} + \sqrt{80}$

7-6

Rational Exponents

Evaluate.

15. $(-32)^{\frac{1}{5}}$

16. $25^{-\frac{3}{2}}$

17. $\left(\frac{1}{64}\right)^{-\frac{1}{3}}$

Simplify.

18. $x^{\frac{1}{4}} \cdot x^{\frac{5}{4}}$

19. $\left(y^{-\frac{5}{6}}\right)^{-\frac{1}{5}}$

20. $\sqrt[10]{36a^2b^{10}}$

7-7

Solving Radical Equations and Inequalities

Solve each equation or inequality.

21. $\sqrt[3]{5u - 2} = -3$

22. $\sqrt{4z - 3} = \sqrt{9z + 2}$

23. $3 + \sqrt{2x - 1} \leq 6$

24. $\sqrt{5x + 4} + 9 > 13$



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 7.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 7 Practice Test on page 435 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 7 Study Guide and Review on pages 430–434 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 435 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 7 Foldable.
- Then complete the Chapter 7 Study Guide and Review on pages 430–434 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 435 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Rational Expressions and Equations



Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of plain $8\frac{1}{2}$ " by 11" paper.

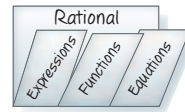
STEP 1

Fold in half lengthwise leaving a $\frac{1}{2}$ inch margin at the top. Fold again in thirds.



STEP 2

Open. Cut along the folds on the short tab to make three tabs. Label as shown.



NOTE-TAKING TIP: Remember to always take notes on your own. Don't use someone else's notes as they may not make sense.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 8. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
<u>asymptote</u> A-suhm(p)-TOHT			
complex fraction			
constant of variation			
<u>continuity</u> KAHN-tuhn-OO-uh-tee			
direct variation			
<u>inverse variation</u> IHN-VUHRS			

Vocabulary Term	Found on Page	Definition	Description or Example
joint variation			
point discontinuity			
rational equation			
rational expression			
rational function			
rational inequality			

BUILD YOUR VOCABULARY (page 226)

MAIN IDEAS

- Simplify rational expressions.
- Simplify complex fractions.

A ratio of two expressions is called a **rational expression**.

EXAMPLE Simplify a Rational Expression

1 a. Simplify $\frac{3y(y+7)}{(y+7)(y^2-9)}$.

$$\frac{3y(y+7)}{(y+7)(y^2-9)} = \frac{3y}{y^2-9} \cdot \frac{\cancel{y+7}}{\cancel{y+7}} \text{ Factor.}$$

$$= \text{} \text{ Simplify.}$$

b. Under what conditions is this expression undefined?

To find out when this expression is undefined, completely factor the denominator.

$$\frac{3y(y+7)}{(y+7)(y^2-9)} = \frac{3y(y+7)}{(y+7) \text{ }}$$


The values that would make the denominator equal to 0

are , , and .

Check Your Progress

a. Simplify $\frac{x(x+5)}{(x+5)(x^2-16)}$.

b. Under what conditions is this expression undefined?

TEKS 2A.2
 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. **(A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

EXAMPLE Use the Process of Elimination

- 2 TEST EXAMPLE** For what value(s) of p is $\frac{p^2 + 2p - 3}{p^2 - 2p - 15}$ undefined?
 A 5 B -3, 5 C 3, -5 D 5, 1, -3

Read the Test Item

You want to determine which values of p make the denominator equal to 0.

Solve the Test Item

Look at the possible answers. Notice that the p term and the constant term are both , so there will be one positive solution and one negative solution. Therefore, you can eliminate choices A and D. Factor the denominator.

$$p^2 - 2p - 15 = \text{} \text{} \quad \text{Factor the denominator.}$$

$$p - 5 = 0 \quad \text{or} \quad p + 3 = 0 \quad \text{Zero Product Property}$$

$$p = \text{} \quad p = \text{} \quad \text{Solve each equation.}$$

Since the denominator equals 0 when $p = \text{}$, the answer is .

Check Your Progress

For what values of p is $\frac{p^2 + 5p + 6}{p^2 + 8p + 15}$ undefined?

- A -5, 3, 2 B -5 C 5 D -5, -3

EXAMPLE Simplify by Factoring Out -1

- 3 Simplify** $\frac{a^4b - 2a^4}{2a^3 - a^3b}$.

$$\frac{a^4b - 2a^4}{2a^3 - a^3b} = \text{} \quad \text{Factor the numerator and the denominator.}$$

$$= \frac{a^4(-1)(2 - b)}{a^3(2 - b)}$$

$$= \text{} \text{ or } -a \quad \text{Simplify.}$$

$$b - 2 = -(-b + 2) \text{ or } -1(2 - b)$$

KEY CONCEPT**Rational Expressions**

- To multiply two rational expressions, multiply the numerators and the denominators.
- To divide two rational expressions, multiply by the reciprocal of the divisor.

EXAMPLE Multiply Rational Expressions

- 4 Simplify each expression.**

a. $\frac{8x}{21y^3} \cdot \frac{7y^2}{16x^3}$

= Simplify.

b. $\frac{2d + 6}{d^2 + d - 2} \div \frac{d + 3}{d^2 + 3d + 2}$
 $\frac{2d + 6}{d^2 + d - 2} \div \frac{d + 3}{d^2 + 4d + 2}$

= $\frac{2d + 6}{d^2 + d - 2} \cdot$ Multiply by the reciprocal of the divisor.

= $\frac{2(\cancel{d+3})(\cancel{d+2})(d+1)}{(\cancel{d+2})(d-1)(\cancel{d+3})}$ Factor.

= Simplify.

EXAMPLE Simplify a Complex Fraction

6 Simplify $\frac{\frac{x^2}{9x^2 - 4y^2}}{\frac{x^3}{2y - 3x}}$.

$\frac{\frac{x^2}{9x^2 - 4y^2}}{\frac{x^3}{2y - 3x}} = \frac{x^2}{9x^2 - 4y^2} \div \frac{x^3}{2y - 3x}$ Express as a division expression.

= $\frac{x^2}{9x^2 - 4y^2} \cdot$ Multiply by the reciprocal of the divisor.

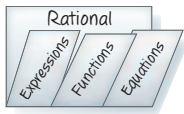
= $\frac{\overset{1}{x} \cdot \overset{1}{x}(\cancel{2y-3x})}{(\cancel{3x-2y})(3x+2y)\overset{1}{x} \cdot \overset{1}{x} \cdot x}$ Factor.

= Simplify.

FOLDABLES

ORGANIZE IT

Under the tab for Rational Expressions, write how simplifying rational expressions is similar to simplifying rational numbers.



HOMEWORK ASSIGNMENT

Page(s): _____
 Exercises: _____

Check Your Progress Simplify each expression.

a. $\frac{x - 3}{x + 2} \cdot \frac{x^2 + 5x + 6}{x^2 - 9}$

b. $\frac{\frac{a^2}{a^2 - 9b^2}}{\frac{a^4}{a + 3b}}$

MAIN IDEAS

- Determine the LCM of polynomials.
- Add and subtract rational expressions.

EXAMPLE LCM of Monomials

1 Find the LCM of $15a^2bc^3$, $16b^5c^2$, and $20a^3c^6$.

$$15a^2bc^3 = \boxed{}$$

Factor the first monomial.

$$16b^5c^2 = \boxed{}$$

Factor the second monomial.

$$20a^3c^6 = \boxed{}$$

Factor the third monomial.

$$\text{LCM} = \boxed{}$$

Use each factor the greatest number of times it appears as a factor and simplify.

$$= \boxed{}$$

EXAMPLE LCM of Polynomials

2 Find the LCM of $x^3 - x^2 - 2x$ and $x^2 - 4x + 4$.

$$x^3 - x^2 - 2x = \boxed{}$$

Factor the first polynomial.

$$x^2 - 4x + 4 = \boxed{}$$

Factor the second polynomial.

$$\text{LCM} = \boxed{}$$

Use each factor the greatest number of times it appears as a factor.



TEKS 2A.2

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. (A) Use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.

Check Your Progress Find the LCM of each set of polynomials.

a. $6x^2zy^3$, $9x^3y^2z^2$, $4x^2z$

b. $x^3 + 2x^2 - 3x$, $x^2 + 6x + 9$

EXAMPLE Monomial Denominators**REMEMBER IT**

When adding and subtracting rational expressions, find the LCD. Then, rewrite the expressions using the common denominator. Finally, add or subtract the fractions and simplify the result.

3 Simplify $\frac{5a^2}{6b} + \frac{9}{14a^2b^2}$.

$$\begin{aligned} & \frac{5a^2}{6b} + \frac{9}{14a^2b^2} \\ &= \frac{5a^2 \cdot \boxed{}}{6b \cdot \boxed{}} + \frac{9 \cdot \boxed{}}{14a^2b^2 \cdot \boxed{}} \\ &= \frac{\boxed{}}{42a^2b^2} + \frac{\boxed{}}{42a^2b^2} \\ &= \frac{35a^4b + 27}{42a^2b^2} \end{aligned}$$

The LCD is $42a^2b^2$. Find equivalent fractions that have this denominator.

Simplify each numerator and denominator.

Add the numerators.

EXAMPLE Polynomial Denominators

4 Simplify $\frac{x+10}{3x-15} - \frac{3x+15}{6x-30}$.

$$\begin{aligned} & \frac{x+10}{3x-15} - \frac{3x+15}{6x-30} \\ &= \frac{x+10}{\boxed{}} - \frac{3x+15}{\boxed{}} \\ &= \frac{2(x+10)}{2 \cdot 3(x-5)} - \frac{3x+15}{6(x-5)} \\ &= \frac{2(x+10) - (3x+15)}{6(x-5)} \\ &= \frac{\boxed{}}{6(x-5)} \\ &= \frac{\boxed{}}{6(x-5)} \\ &= \frac{-1(x-5)}{6(x-5)} \text{ or } \boxed{} \end{aligned}$$

Factor the denominators.

The LCD is $6(x-5)$.

Subtract the numerators.

Distributive Property

Combine like terms.

Simplify.

Check Your Progress

Simplify each expression.

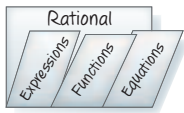
a. $\frac{3x^2}{2y} + \frac{5}{12xy^2}$

b. $\frac{x+5}{2x-4} - \frac{3x+8}{4x-8}$

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Expressions, write your own example similar to Example 5. Then simplify your rational expression. Give the reason for each step.



EXAMPLE

Simplify Complex Fractions

5 Simplify $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{b} - 1}$.

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{b}} - 1 = \frac{\boxed{} + \frac{a}{ab}}{\frac{1}{b} + \boxed{}}$$

The LCD of the numerator is ab .
The LCD of the denominator is b .

$$= \frac{\boxed{}}{\boxed{}}$$

Simplify the numerator and denominator.

$$= \boxed{} \div \boxed{}$$

Write as a division expression.

$$= \frac{b+a}{a\cancel{b}} \cdot \frac{\cancel{b}}{1-b}$$

Multiply by the reciprocal of the divisor.

$$= \frac{b+a}{a(1-b)} \text{ or } \boxed{}$$

Simplify.

Check Your Progress Simplify $\frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{b} + \frac{1}{a}}$.

EXAMPLE Use a Complex Fraction to Solve a Problem

6 COORDINATE GEOMETRY Find the slope of the line that passes through $P\left(\frac{3}{k}, \frac{1}{3}\right)$ and $Q\left(\frac{1}{2}, \frac{2}{k}\right)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Definition of slope

$$= \frac{\frac{2}{k} - \frac{1}{3}}{\frac{1}{2} - \frac{3}{k}}$$

$$y_2 = \frac{2}{k}, y_1 = \frac{1}{3}, x_2 = \frac{1}{2}, x_1 = \frac{3}{k}$$

$$= \frac{6 - k}{k - 6}$$

The LCD of the numerator is $3k$.
The LCD of the denominator is $2k$.

$$= \frac{6 - k}{3k} \div \frac{2k}{k - 6}$$

Write as division expression.

$$= \frac{6 - k}{3k} \cdot \frac{2k}{k - 6}$$

Multiply by the reciprocal of the divisor.

$$= \frac{-(k - 6)}{3k} \cdot \frac{2k}{k - 6}$$

$$6 - k = -(k - 6)$$

$$= -\frac{\overset{1}{\cancel{(k - 6)}}}{3k} \cdot \frac{2\cancel{k}}{\underset{1}{\cancel{k - 6}}} \text{ or } \frac{2}{3}$$

The slope is $\frac{2}{3}$.

Check Your Progress Find the slope of the line that passes through $P\left(\frac{4}{k}, \frac{1}{4}\right)$ and $Q\left(\frac{1}{5}, \frac{5}{k}\right)$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Graphing Rational Functions



TEKS 2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

BUILD YOUR VOCABULARY (page 226)

MAIN IDEAS

- Determine the vertical asymptotes and the point discontinuity for the graphs of rational functions.
- Graph rational functions.

KEY CONCEPTS

Vertical Asymptotes

If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote.

Point Discontinuity

If the original function is undefined for $x = a$ but the rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.



TEKS 2A.10 (A) Use quotients of polynomials

to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior. (C) Determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities. Also addresses TEKS 2A.1(A), 2A.2(A), 2A.4(B), 2A.10(B), and 2A.10(F).

A rational function is an equation of the form

$$f(x) = \frac{p(x)}{q(x)}, \text{ where } p(x) \text{ and } q(x) \text{ are } \boxed{}$$

functions and $q(x) \neq 0$.

The graphs of rational functions may have breaks in **continuity**. This means that not all rational functions are traceable. Breaks in $\boxed{}$ can appear as a **vertical asymptote** or as a **point discontinuity**.

EXAMPLE Vertical Asymptotes and Point Discontinuity

- Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of

$$f(x) = \frac{x^2 - 4}{x^2 + 5x + 6}$$

Factor the numerator and denominator of the rational expression.

$$\frac{x^2 - 4}{x^2 + 5x + 6} = \boxed{}$$

The function is undefined for $x = \boxed{}$ and $\boxed{}$.

Since $\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)} = \boxed{}$, $x = \boxed{}$ is a vertical

asymptote and $x = \boxed{}$ is a hole in the graph.

Check Your Progress

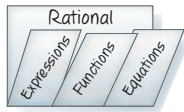
Determine the equations of any vertical asymptotes and the values of x for any holes in the

$$\text{graph of } f(x) = \frac{x^2 - 9}{x^2 + 8x + 15}$$

FOLDABLES™

ORGANIZE IT

Under the tab for Rational Functions, write how you can tell from a rational function where the breaks in continuity will appear in the graph of the function.



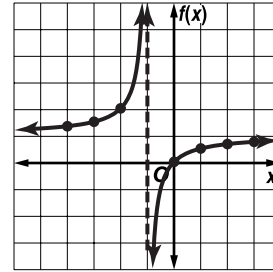
EXAMPLE

Graph with a Vertical Asymptote

2 Graph $f(x) = \frac{x}{x+1}$.

The function is undefined for $x = \square$. Since $\frac{x}{x+1}$ is in its simplest form, $x = \square$ is a vertical asymptote. Make a table of values. Plot the points and draw the graph.

x	$f(x)$
-4	1.33
-3	1.5
-2	2
0	0
1	0.5
2	0.67
3	0.75



EXAMPLE

Graph with Point Discontinuity

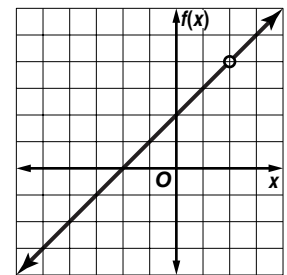
3 Graph $f(x) = \frac{x^2 - 4}{x - 2}$.

Notice that $\frac{x^2 - 4}{x - 2} = \frac{(x+2)(x-2)}{x-2}$

or \square . Therefore, the

graph of $f(x) = \frac{x^2 - 4}{x - 2}$ is the graph

$f(x) = \square$ with a hole at $x = \square$.

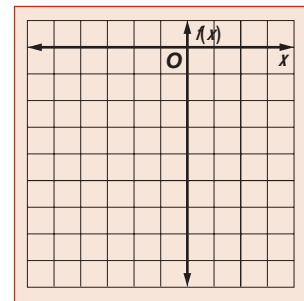
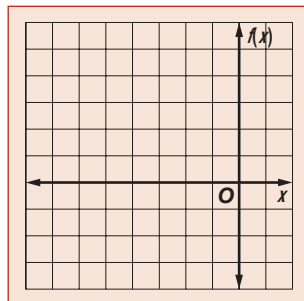


Check Your Progress

Graph each rational function.

a. $f(x) = \frac{x}{x+3}$

b. $f(x) = \frac{x^2 - 16}{x + 4}$



EXAMPLE Use Graphs of Rational Functions

4 AVERAGE SPEED Use the situation and formula given in Example 4 of your textbook.

a. Draw the graph of $r_2 = 15$ miles per hour.

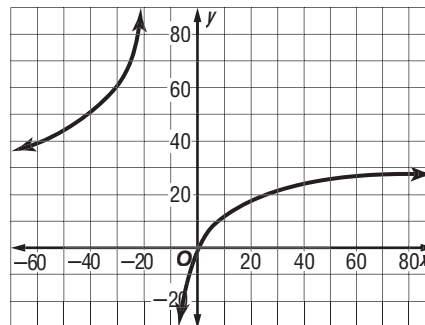
The function is $R = \frac{2r_1(15)}{r_1 + 15}$

or $R = \frac{30r_1}{r_1 + 15}$. The vertical

asymptote is $r_1 = \boxed{}$.

Graph the vertical asymptote and the function. Notice that the horizontal asymptote is

$R = \boxed{}$.



b. What is the R -intercept of the graph?

The R -intercept is $\boxed{}$.

c. What domain and range values are meaningful in the context of the problem?

In the problem context, the speeds are nonnegative values.

Therefore, only values of r_1 greater than or equal to $\boxed{}$

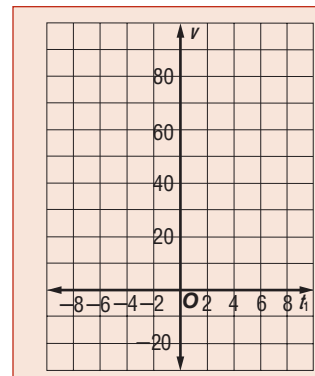
and values of R between $\boxed{}$ are meaningful.

Check Your Progress A train travels at one velocity V_1 for a given amount of time t_1 and then another velocity V_2 for a different amount of time t_2 . The average velocity is given by $V = \frac{V_1t_1 + V_2t_2}{t_1 + t_2}$.

a. Let t_1 be the independent variable and let V be the dependent variable. Draw the graph if $V_1 = 60$ miles per hour, $V_2 = 30$ miles per hour, and $t_2 = 1$ hour.

b. What is the V -intercept of the graph?

c. What values of t_1 and V are meaningful in the context of the problem?



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Recognize and solve direct and joint variation problems.
- Recognize and solve inverse variation problems.

KEY CONCEPTS

Direct Variation y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.

Joint Variation y varies jointly as x and z if there is some number k such that $y = kxz$, where $k \neq 0$, $x \neq 0$, and $z \neq 0$.



TEKS 2A.10

The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(F) Analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem. (G) Use functions to model and make predictions in problem situations involving direct and inverse variation.**

Also addresses TEKS 2A.1(A), 2A.2(A), 2A.4(B), 2A.10(B), and 2A.10(F).

EXAMPLE Direct Variation

- 1 If y varies directly as x and $y = -15$ when $x = 5$, find y when $x = 3$.

Use a proportion that relates the values.

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

Direct proportion.

$$\boxed{} = \boxed{}$$

$$y_1 = -15, x_1 = 5, \text{ and } x_2 = 3$$

$$\boxed{} = \boxed{}$$

Cross multiply.

$$\boxed{} = \boxed{}$$

Simplify.

$$\boxed{} = y_2$$

Divide each side by 5.

When $x = 3$, the value of $y = \boxed{}$.

EXAMPLE Joint Variation

- 2 Suppose y varies jointly as x and z . Find y when $x = 10$ and $z = 5$ if $y = 12$ when $x = 3$ and $z = 8$.

Use a proportion that relates the values.

$$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2}$$

Joint variation.

$$\boxed{} = \boxed{}$$

$$y_1 = 12, x_1 = 3, z_1 = 8, \\ x_2 = 10, \text{ and } z_2 = 5$$

$$\boxed{} = \boxed{}$$

Cross multiply.

$$\boxed{} = \boxed{}$$

Simplify.

$$\boxed{} = y_2$$

Divide each side by 24.

When $x = 10$ and $z = \boxed{}$, $y = \boxed{}$.

EXAMPLE Inverse Variation**KEY CONCEPT**

Inverse Variation y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

- 3** If r varies inversely as t and $r = -6$ when $t = 2$, find r when $t = -7$.

Use a proportion that relates the values.

$$\frac{r_1}{t_2} = \frac{r_2}{t_1}$$

Inverse variation

$$\boxed{} = \boxed{}$$

$$r_1 = -6, t_1 = 2, \text{ and } t_2 = -7$$

$$\boxed{} = \boxed{}$$

Cross multiply.

$$\boxed{} = \boxed{}$$

Simplify.

$$\boxed{} = \boxed{}$$

Divide each side by -7 .

When $t = -7$, r is $\boxed{}$ or $1\frac{5}{7}$.

Check Your Progress

- a.** If y varies directly as x and $y = 12$ when $x = -3$, find y when $x = 7$.

- b.** Suppose y varies jointly as x and z . Find y when $x = 3$ and $z = 2$ if $y = 11$ when $x = 5$ and $z = 22$.

- c.** If a varies inversely as b and $a = 3$ when $b = 8$, find a when $b = 6$.


HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

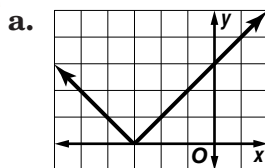
- Identify graphs as different types of functions.
- Identify equations as different types of functions.

 **TEKS 2A.4** The student connects algebraic and geometric representations of functions. (A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$), functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$). (B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

EXAMPLE

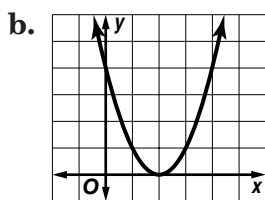
Identify a Function Given the Graph

- 1 Identify the type of function represented by each graph.



The graph is a V shape, so it is

function

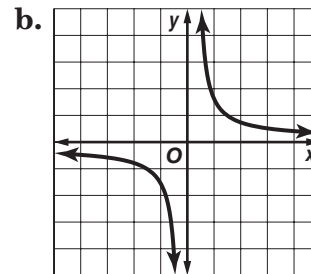
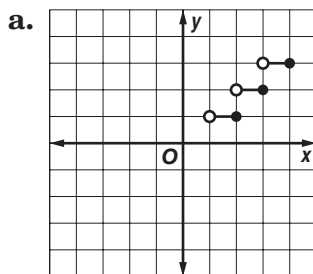


The graph is a parabola, so it is

function.

Check Your Progress

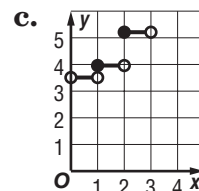
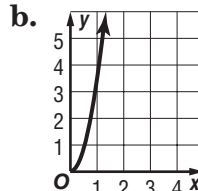
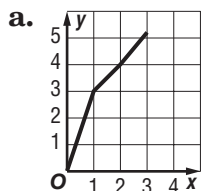
Identify the type of function represented by each graph.



EXAMPLE

Match Equation with Graph

- 2 **SHIPPING CHARGES** A chart gives the shipping rates for an Internet company. They charge \$3.50 to ship less than 1 pound, \$3.95 for 1 pound and over up to 2 pounds, and \$5.20 for 2 pounds and over up to 3 pounds. Which graph depicts these rates?



The shipping rate is constant for x values from 0 to 1. Then it jumps at $x =$ and remains constant until $x =$. The

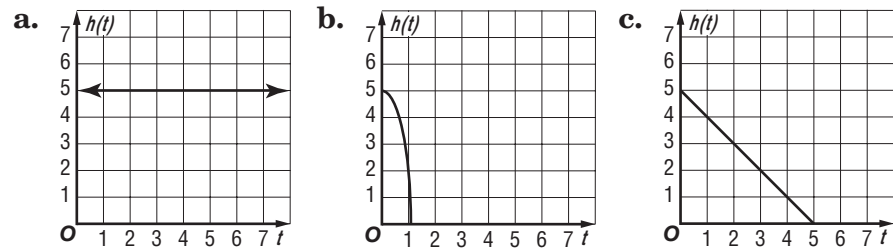
REVIEW IT

In order for a square root to be a real number, what is true about the radicand of the square root function? (Lesson 7-3)

graph jumps again at $x =$ and remains constant until $x =$. The graph of this function looks like steps, so this is , the step or .

Check Your Progress

A ball is thrown into the air. The path of the ball is represented by the equation $h(t) = -16t^2 + 5$. Which graph represents this situation?



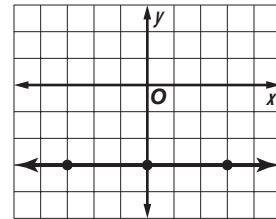
EXAMPLE

Identify a Function Given its Equation

3 Identify the type of function represented by each equation. Then graph the equation.

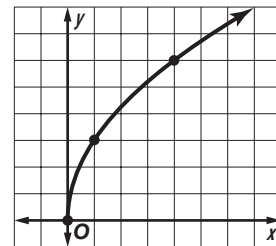
a. $y = -3$

Since the equation has no x -intercept, it is the constant function. Determine some points on the graph and graph it.



b. $y = \sqrt{9x}$

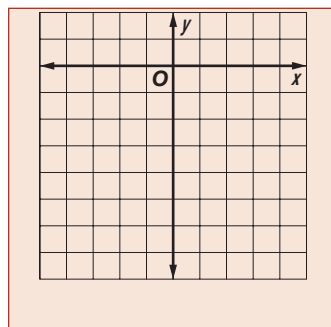
Since the equation includes an expression with a square root, it is a square root function. Plot some points and use what you know about square root graphs to graph it.



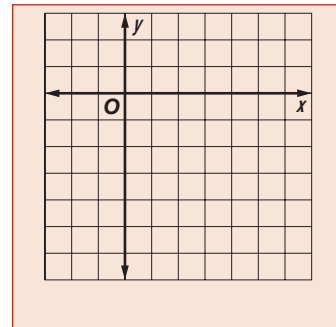
Check Your Progress

Identify the type of function represented by each equation. Then graph the equation.

a. $y = \frac{x^2 - 9}{x + 3}$



b. $y = -2x$



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

MAIN IDEAS

- Solve rational equations.
- Solve rational inequalities.

BUILD YOUR VOCABULARY (page 227)

Any equation that contains one or more rational expressions is called a **rational equation**.

Inequalities that contain one or more rational expressions are called **rational inequalities**.

REMEMBER IT 

Rational equations are often easier to solve when you eliminate the fractions first.

EXAMPLE

Solve a Rational Equation

1 Solve $\frac{5}{24} + \frac{2}{3-x} = \frac{1}{4}$.

The LCD for the three denominators is $24(3-x)$.

$$\frac{5}{24} + \frac{2}{3-x} = \frac{1}{4}$$

Original equation

$$\boxed{} \left(\frac{5}{24} + \frac{2}{3-x} \right) = \frac{1}{4} \boxed{}$$

Multiply each side by $\boxed{}$.

$$24(3-x) \left(\frac{5}{24} \right) + 24(3-x) \left(\frac{2}{3-x} \right) = \frac{1}{4} 24(3-x)$$

Simplify.

$$\boxed{} = \boxed{}$$

Simplify.

$$\boxed{} = \boxed{}$$

Add.

$$x = \boxed{}$$



TEKS 2A.10

The student formulates

equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(C) Determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities. (F) Analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem. Also addresses TEKS 2A.2(A), 2A.10(B), and 2A.10(D).

Check Your Progress

Solve $\frac{5}{2} + \frac{3}{x-1} = \frac{1}{2}$.

EXAMPLE Elimination of a Possible Solution

2 Solve $\frac{p^2 - p + 1}{p + 1} = \frac{p^2 - 7}{p^2 - 1} + p$.

The LCD is $p^2 - 1$.

$$\frac{p^2 - p + 1}{p + 1} = \frac{p^2 - 7}{p^2 - 1} + p \quad \text{Original equation}$$

$$\cancel{(p^2 - 1)}^{\cancel{p-1}} \frac{p^2 - p + 1}{\cancel{p+1}_1} = \frac{p^2 - 7}{\cancel{p^2 - 1}_1} \cancel{(p^2 - 1)}^{\cancel{p^2 - 1}} + p(p^2 - 1)$$

$$(p - 1)(p^2 - p + 1) = p^2 - 7 + (p^2 - 1)p \quad \text{Distributive Property}$$

$$p^3 - p^2 + p - p^2 + p - 1 = p^2 - 7 + p^3 - p \quad \text{Simplify.}$$

$$-2p^2 + 2p - 1 = p^2 - p - 7 \quad \text{Simplify.}$$

$$\boxed{} = \boxed{} \quad \text{Add } (2p^2 - 2p + 1) \text{ to each side.}$$

$$0 = \boxed{} \quad \text{Divide each side by 3.}$$

$$0 = \boxed{} \quad \text{Factor.}$$

$$\boxed{} = 0 \text{ or } \boxed{} = 0 \quad \text{Zero Product Property}$$

$$p = \boxed{} \quad p = \boxed{} \quad \text{Solve each equation.}$$

Since $p = -1$ results in a zero in the denominator, eliminate -1 .

Check Your Progress Solve $\frac{1}{x - 2} = \frac{2x + 1}{x^2 + 2x - 8} + \frac{2}{x + 4}$.

WRITE IT

When solving a rational equation, why must the solutions be checked in the original equation rather than in any of the derived equations?

EXAMPLE

- 3 MOWING LAWNS** Tim and Ashley mow lawns together. Tim working alone could complete a particular job in 4.5 hours, and Ashley could complete it alone in 3.7 hours. How long does it take to complete the job when they work together?

In 1 hour, Tim could complete of the job.

In 1 hour, Ashley could complete of the job.

In t hours, Tim could complete $\frac{1}{4.5} \cdot t$ or of the job.

In t hours Ashley could complete $\frac{1}{3.7} \cdot t$ or of the job.

Part completed by Tim	plus	part completed by Ashley	equals	entire job.
$\frac{t}{4.5}$	+	$\frac{t}{3.7}$	=	1

$$\frac{t}{4.5} + \frac{t}{3.7} = 1 \quad \text{Original equation}$$

$$\left(\text{input}\right) \left(\frac{t}{4.5} + \frac{t}{3.7}\right) = (1) \left(\text{input}\right) \quad \text{Multiply each side by 16.65.}$$

$$8.2t = 16.65 \quad \text{Simplify.}$$

$$t \approx \text{input} \quad \text{Divide each side by 8.2.}$$

It would take about hours working together to complete the job.

Check Your Progress

Libby and Nate clean together. Nate working alone could complete the job in 3 hours, and Libby could complete it alone in 5 hours. How long does it take to complete the job when they work together?

EXAMPLE

4 SWIMMING Janine swims for 5 hours in a stream that has a current of 1 mile per hour. She leaves her dock and swims upstream for 2 miles and then swims back to her dock. What is her swimming speed in still water?

Words The formula that relates distance, time, and rate is $d = rt$ or $\frac{d}{r} = t$.

Variables Let r = speed in still water. Then her speed with the current is , and her speed against the current is .

Equation

Time going with the current	plus	Time going against the current	equals	total time.
<input style="width: 60px; height: 30px;" type="text"/>	+	<input style="width: 60px; height: 30px;" type="text"/>	=	<input style="width: 40px; height: 30px;" type="text"/>

Solve the equation.

$$\frac{2}{r-1} + \frac{2}{r+1} = 5$$

Original equation

$$(r^2 - 1)\left(\frac{2}{r-1} + \frac{2}{r+1}\right) = (r^2 - 1)5$$

Multiply each side by $r^2 - 1$.

$$\cancel{(r^2 - 1)} \frac{2}{\cancel{r-1}} + \cancel{(r^2 - 1)} \frac{2}{\cancel{r+1}} = 5(r^2 - 1)$$

Distributive Property

$$\text{[]} = 5r^2 - 5$$

Simplify.

$$\text{[]} = 5r^2 - 5$$

Simplify.

$$0 = \text{[]}$$

Subtract $4r$ from each side.

Use the Quadratic Formula to solve for r .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-5)}}{2(5)}$$

$x = r$, $a = 5$, $b = -4$, and $c = -5$

$$r = \frac{4 \pm \sqrt{\text{[]}}}{10}$$

Simplify.

$$r = \text{[]} \text{ or } \text{[]}$$

Use a calculator.

Since speed must be positive, it is miles per hour.

Check Your Progress

Lynne swims for 1 hour in a stream that has a current of 2 miles per hour. She leaves her dock and swims upstream for 3 miles and then back to her dock. What is her swimming speed in still water?

EXAMPLE**Solve a Rational Inequality**

5 Solve $\frac{1}{3s} + \frac{2}{9s} < \frac{2}{3}$.

Step 1 Values that make the denominator equal to 0 are excluded from the denominator. For this inequality the excluded value is 0.

Step 2 Solve the related equation.

$$\frac{1}{3s} + \frac{2}{9s} = \frac{2}{3}$$

Related equation.

$$\boxed{} \left(\frac{1}{3s} + \frac{2}{9s} \right) = \frac{2}{3} \boxed{}$$

Multiply each side

by $\boxed{}$.

$$\boxed{} = \boxed{}$$

Simplify.

$$\boxed{} = \boxed{}$$

Add.

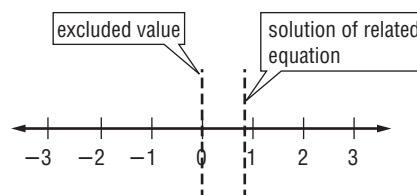
$$\boxed{} = \boxed{}$$

Divide each side

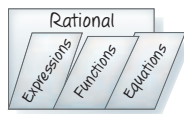
by $\boxed{}$.

Step 3 Draw vertical lines at the excluded value and at the solution to separate the number line into regions.

Now test a sample value in each region to determine if the values in the region satisfy the inequality.

**FOLDABLES™****ORGANIZE IT**

Under the tab for Rational Expressions, write how simplifying rational expressions is similar to simplifying rational numbers.



Test $s = -1$.

$$\frac{1}{3(-1)} + \frac{2}{9(-1)} \stackrel{?}{<} \frac{2}{3}$$

$$\boxed{} - \boxed{} \stackrel{?}{<} \frac{2}{3}$$

$$\boxed{} < \frac{2}{3}$$

So, $s < 0$ is a solution.

Test $s = \frac{1}{3}$.

$$\frac{1}{3\left(\frac{1}{3}\right)} + \frac{2}{9\left(\frac{1}{3}\right)} \stackrel{?}{<} \frac{2}{3}$$

$$\boxed{} + \boxed{} \stackrel{?}{<} \frac{2}{3}$$

$$\boxed{} \not< \frac{2}{3}$$

So, $0 < s < \frac{5}{6}$ is not a solution.

Test $s = 1$.

$$\frac{1}{3(1)} + \frac{2}{9(1)} \stackrel{?}{<} \frac{2}{3}$$

$$\boxed{} + \boxed{} \stackrel{?}{<} \frac{2}{3}$$

$$\boxed{} < \frac{2}{3}$$

So $s > \frac{5}{6}$ is not a solution.

Check Your Progress


Solve $\frac{1}{x} + \frac{3}{5x} < \frac{2}{5}$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

STUDY GUIDE

	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 8 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 8, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 226–227) to help you solve the puzzle.

8-1

Multiplying and Dividing Rational Expressions

1. Which expressions are complex fractions?
-

i. $\frac{7}{12}$	ii. $\frac{\frac{3}{8}}{\frac{5}{16}}$	iii. $\frac{r+5}{r-5}$	vi. $\frac{\frac{z+1}{z}}{z}$	v. $\frac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$
-------------------	--	------------------------	-------------------------------	---

Simplify each expression.

2. $\frac{6r^2s^3}{t^3} \cdot \frac{3s^2t^2}{12rst}$

3. $\frac{3y^2 + 3y - 6}{4y - 8} \div \frac{y^2 - 4}{2y^2 - 6y + 4}$

8-2

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

4. $4y, 9xy, 6y^2$

5. $x^2 - 5x + 6, x^3 - 4x^2 + 4x$

Simplify each expression.

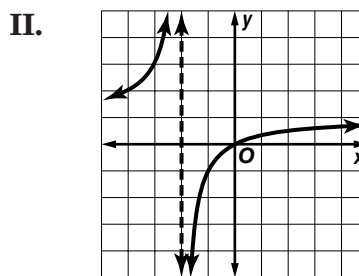
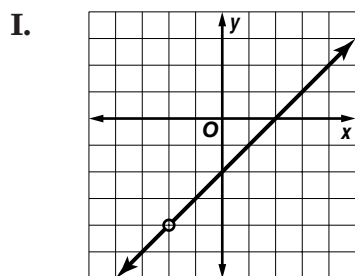
6. $\frac{6}{4ab} - \frac{3}{ab^2}$

7. $\frac{5q}{2p} + 8$

8-3

Graphing Rational Functions

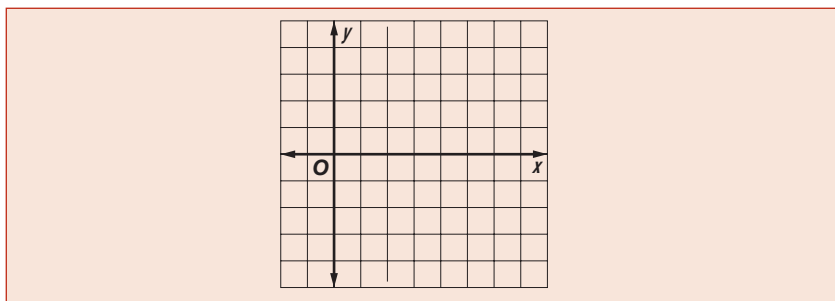
For Exercises 8 and 9, refer to the two rational functions shown.



8. Graph I has a at $x =$.

9. Graph II has a at $x =$.

10. Graph $f(x) = \frac{3}{x - 2}$.

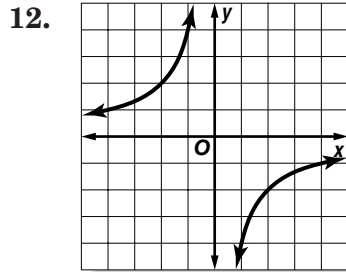


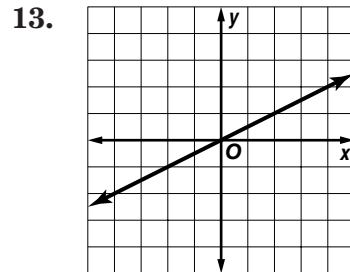
8-4

Direct, Joint, and Inverse Variation

11. Suppose y varies inversely as x . Find y when $x = 4$, $y = 8$ when $x = 3$.

Which type of variation, direct or inverse, is represented by each graph?



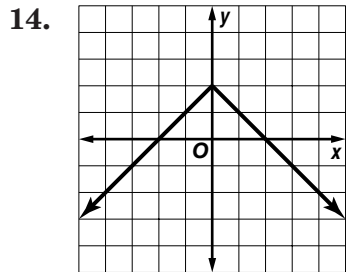


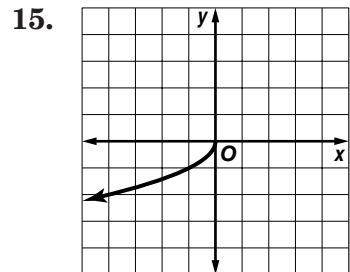
8-5

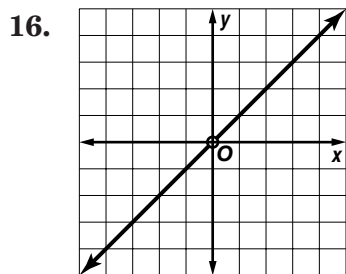
Classes of Functions

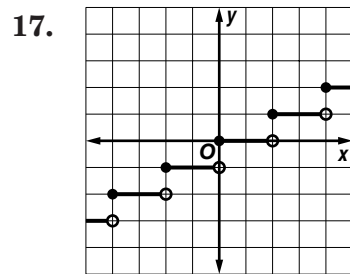
Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

- | | |
|----------------|---------------------|
| a. square root | b. absolute value |
| c. rational | d. greatest integer |
| e. constant | f. identity |









8-6

Solving Rational Equations and Inequalities

Solve each equation or inequality.

18. $\frac{1}{2x} + \frac{x+1}{x} = 4$

19. $\frac{y+1}{y-1} - \frac{2}{y-1} = \frac{y}{3}$

18. $\frac{3}{z+2} - \frac{6}{z} > 0$



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 8.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want take the Chapter 8 Practice Test on page 493 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 8 Study Guide and Review on pages 489–492 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 493.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 8 Foldable.
- Then complete the Chapter 8 Study Guide and Review on pages 489–492 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 493.

Student Signature

Parent/Guardian Signature

Teacher Signature

Exponential and Logarithmic Relations

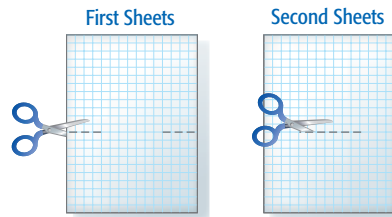


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of notebook paper.

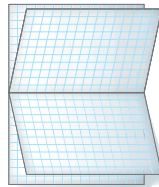
STEP 1

Fold in half along the width. On the first sheet, cut 5 cm along the fold at the ends. On the second sheet, cut in the center, stopping 5 cm from the ends.



STEP 2

Insert the first sheet through the second sheet and align the folds. Label the pages with lesson numbers.



NOTE-TAKING TIP: When you take notes, listen or read for main ideas. Then record those ideas in a simplified form for future reference.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. As you complete the study notes for the chapter, complete you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
Change of Base Formula			
common <u>logarithm</u> [LAW-guh-rih-thuhm]			
<u>exponential decay</u> [EHK-spuh-NEHN-chuhl]			
exponential equation			
exponential function			
exponential growth			
exponential inequality			
logarithm			

Vocabulary Term	Found on Page	Definition	Description or Example
logarithmic equation LAW-guh-RIHTH-mihk			
logarithmic function LAW-guh-RIHTH-mihk			
logarithmic inequality LAW-guh-RIHTH-mihk			
natural base, e			
natural base exponential function			
natural logarithm			
natural logarithmic function			
rate of decay			
rate of growth			

EXAMPLE

Graph an Exponential Function

MAIN IDEAS

- Graph exponential functions.
- Solve exponential equations and inequalities.

REMEMBER IT 

Polynomial functions like $y = x^2$ have a variable for the base and a constant for the exponent. Exponential functions like $y = 2^x$ have a constant for the base and a variable for the exponent.



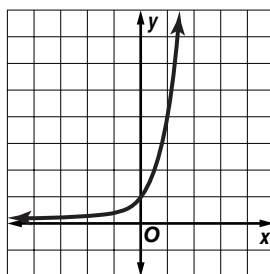
TEKS 2A.11 The student formulates equations and inequalities based

on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(B) Use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior. (F) Analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem. Also addresses TEKS 2A.4(A), 2A.11(C), 2A.11(D), and 2A.11(E).

- 1 Sketch the graph of $y = 4^x$. Then state the function's domain and range.

Make a table of values. Connect the points to sketch a smooth curve.



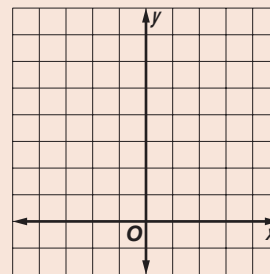
x	$y = 4^x$
-2	<input type="text"/>
-1	<input type="text"/>
0	<input type="text"/>
1	<input type="text"/>
2	<input type="text"/>

The domain is , and the range is

.

Check Your Progress

Sketch the graph of $y = 3^x$. Then state the function's domain and range.



BUILD YOUR VOCABULARY (page 254)

In general, an equation of the form , where $a \neq 0$, $b > 0$, and $b \neq 1$, is called an exponential function with base b .

EXAMPLE Identify Exponential Growth and Decay**KEY CONCEPT****Exponential Growth and Decay**

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

- 2** Determine whether each function represents exponential *growth* or *decay*.

a. $y = 10\left(\frac{4}{3}\right)^x$ The function represents exponential , since the base, , is .

b. $y = (0.7)^x$ The function represents exponential , since the base, , is .

Check Your Progress Determine whether each function represents exponential *growth* or *decay*.

a. $y = 10\left(\frac{2}{5}\right)^x$

b. $y = (0.5)^x$

BUILD YOUR VOCABULARY (page 254)

Exponential equations are equations in which

occur as .

Exponential inequalities are inequalities involving

functions.

EXAMPLE Write an Exponential Function

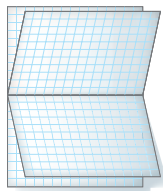
- 3 PHONES** In 1995, there were an estimated 28,154,000 cellular telephone subscribers in the United States. By 2005, this estimated number had risen to 194,633,000.

- a. Write an exponential function of the form $y = ab^x$ that could be used to model the number of cellular telephone subscribers in the U.S. Write the function in terms of x , the number of years since 1995.

For 1995, the time x equals , and the initial number of cellular telephone subscribers y is 28,154,000. Thus the y -intercept, and the value of a , is .

FOLDABLES™**ORGANIZE IT**

Use the page for Lesson 9-1. Write your own example of an exponential growth and an exponential decay function. Then sketch a graph of each function.



For 2005, the time x equals $2005 - 1995$ or .

Substitute these values and the value of a into an exponential function to approximate the value of b .

$$y = ab^x$$

$$194,633,000 = 28,154,000b^{10}$$

$$\text{[]} \approx b^{10}$$

$$\text{[]} \approx b$$

Exponential function

Replace x with 10, y with 194,633,000, and a with 28,154,000.

Divide each side by 28,154,000.

Take the 10th root of each side.

To find the 10th root of , use selection 5: $\sqrt[10]{}$ under the MATH menu on the TI-83 Plus.

Keystrokes: 10 5 6.91 1.21324302926

Answer: An equation that models the number of cellular telephone subscribers in the U.S. from 1995 to 2005 is .

- b. Suppose the number of cellular subscribers continues to increase at the same rate. Estimate the number of U.S. subscribers in 2015.**

For 2015, the time x equals $2015 - 1995$ or .

$$y = 28,154,000(1.213)^x$$

Modeling equation

$$y = 28,154,000(1.213)^{\text{[]}}$$

Replace x with .

$$y = \text{[]}$$

Use a calculator.

The number of cellular phone subscribers will be about in 2015.

Check Your Progress

In 1991, 4.9% of Americans had diabetes. By 2000, this percent had risen to 7.3%.

- a.** Write an exponential function of the form $y = ab^x$ that could be used to model the percentage of Americans with diabetes. Write the function in terms of x , the number of years since 1991.

- b.** Suppose the percent of Americans with diabetes continues to increase at the same rate. Estimate the percent of Americans with diabetes in 2010. .

EXAMPLE Solve Exponential Equations

KEY CONCEPTS

Property of Equality for Exponential Functions

If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Property of Inequality for Exponential Functions

If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

4 Solve each equation.

a. $4^{9n - 2} = 256$.

$$4^{9n - 2} = 256$$

$$4^{9n - 2} = 4^4$$

$$\boxed{} = \boxed{}$$

$$\boxed{}$$

$$n = \boxed{}$$

Original equation

Rewrite 256 as 4^4 so each side has the same base.

Property of Equality for Exponential Functions

Add 2 to each side.

Divide each side by 9.

b. $3^{5x} = 9^{2x - 1}$

$$3^{5x} = 9^{2x - 1}$$

$$3^{5x} = 3^{2(2x - 1)}$$

$$5x = \boxed{}(2x - 1)$$

$$5x = \boxed{}$$

$$x = \boxed{}$$

Original equation

Rewrite 9 as 3^2 so each side has the same base.

Property of Equality for Exponential Functions

Distributive Property

Subtract $4x$ from each side.

EXAMPLE Solve Exponential Inequalities

5 Solve $5^{3 - 2k} > \frac{1}{625}$.

$$5^{3 - 2k} > \frac{1}{625}$$

$$5^{3 - 2k} > 5^{-4}$$

$$\boxed{} > \boxed{}$$

$$\boxed{} > \boxed{}$$

$$k < \boxed{}$$

Original inequality

Rewrite $\frac{1}{625}$ as $\frac{1}{5^4}$ or 5^{-4} .

Property of Inequality for Exponential Functions

Subtract 3 from each side.

Divide each side by -2 .

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress Solve.

a. $2^{3x + 1} = 32$

b. $5^{2x} = 25^{2x - 1}$

c. $3^{3 - 2k} > \frac{1}{27}$

Logarithms and Logarithmic Functions

MAIN IDEAS

- Evaluate logarithmic expressions.
- Solve logarithmic equations and inequalities.

KEY CONCEPT

Logarithm with Base b
Let b and x be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.



TEKS 2A.11 The student formulates equations and

inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(A)** Develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses. **(D)** Determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods. Also addresses TEKS 2A.4(A) and 2A.11(C).

EXAMPLE

Logarithmic to Exponential Form

1 Write each equation in exponential form.

a. $\log_3 9 = 2$

$$\log_3 9 = 2 \rightarrow \boxed{}$$

b. $\log_{10} \frac{1}{100} = -2$

$$\log_{10} \frac{1}{100} = -2 \rightarrow \boxed{}$$

EXAMPLE

Exponential to Logarithmic Form

2 Write each equation in logarithmic form.

a. $5^3 = 125$

$$5^3 = 125 \rightarrow \boxed{}$$

b. $27^{\frac{1}{3}} = 3$

$$27^{\frac{1}{3}} = 3 \rightarrow \boxed{}$$

Check Your Progress

Write each equation in exponential form.

a. $\log_2 8 = 3$

b. $\log_3 \frac{1}{9} = -2$

Write each equation in logarithmic form.

c. $3^4 = 81$

d. $81^{\frac{1}{2}} = 9$

EXAMPLE Evaluate Logarithmic Expressions**3** Evaluate $\log_3 243$.

$$\log_3 243 = y$$

Let the logarithm equal y .

$$\square = \square$$

Definition of logarithm

$$\square = \square$$

$$243 = 3^5$$

$$\square = y$$

Property of Equality for Exponential Functions

Check Your ProgressEvaluate $\log_{10} 100$.**EXAMPLE** Solve a Logarithmic Equation**4** Solve $\log_8 n = \frac{4}{3}$.

$$\log_8 n = \frac{4}{3}$$

Original equation

$$n = \square$$

Definition of logarithm

$$n = \square$$

$$8 = 2^3$$

$$n = \square$$

Power of a Power

$$n = \square$$

Simplify.

Check Your ProgressSolve $\log_{27} n = \frac{2}{3}$.**KEY CONCEPTS****Logarithmic to Exponential Inequality**If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.**Property of Equality for Logarithmic Functions**If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.**Property of Inequality for Logarithmic Functions**If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.**EXAMPLE** Solve a Logarithmic Inequality**5** Solve $\log_6 x > 3$.

$$\log_6 x > 3$$

Original inequality

$$\square$$

Logarithmic to exponential inequality

$$\square$$

Simplify.

Check Your Progress Solve $\log_3 x < 2$.

EXAMPLE Solve Equations with Logarithms on Each Side

6 Solve $\log_4 x^2 = \log_4 (4x - 3)$.

$\log_4 x^2 = \log_4 (4x - 3)$ Original equation

$x^2 =$ Property of Equality for Logarithmic Functions

$x^2 - 4x + 3 = 0$ Subtract $4x$ and add 3 to each side.

= 0 Factor.

= 0 or = 0 Zero Product Property

$x =$ $x =$ Solve each equation.

EXAMPLE Solve Inequalities with Logarithms on Each Side

7 Solve $\log_7 (2x + 8) > \log_7 (x + 5)$.

$\log_7 (2x + 8) > \log_7 (x + 5)$ Original inequality

$>$ Property of Inequality for Logarithmic Functions

$x >$ Addition and Subtraction Properties of Inequalities

We must exclude all values of x such that $2x + 8 \leq 0$ or $x + 5 \leq 0$. Thus the solution set is , , and . This compound inequality simplifies to .

Check Your Progress Solve.

a. $\log_5 x^2 = \log_5 (x + 6)$

b. $\log_5 (2x + 6) > \log_3 (x + 2)$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Properties of Logarithms



TEKS 2A.11 The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(D) Determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods.**

EXAMPLE Use the Product Property

1 Use $\log_5 2 \approx 0.4307$ to approximate the value of $\log_5 250$.

$$\log_5 250 = \log_5 (5^3 \cdot 2)$$

Replace 250 with $5^3 \cdot 2$.

$$= \boxed{} + \boxed{}$$

Product Property

$$= \boxed{} + \boxed{}$$

Inverse Property of Exponents and Logarithms

$$\approx \boxed{}$$

Replace $\log_5 2$.

MAIN IDEAS

- Simplify and evaluate expressions using the properties of logarithms.
- Solve logarithmic equations using the properties of logarithms.

KEY CONCEPTS

Product Property of Logarithms The logarithm of a product is the sum of the logarithms of its factors.

Quotient Property of Logarithms The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.

EXAMPLE Use the Quotient Property

2 Use $\log_6 8 \approx 1.1606$ and $\log_6 32 \approx 1.9343$ to approximate the value of $\log_6 4$.

$$\log_6 4 = \log_6 \left(\frac{32}{8} \right)$$

Replace 4 with $\frac{32}{8}$.

$$= \boxed{} - \boxed{}$$

Quotient Property

$$\approx \boxed{} - \boxed{}$$

or $\boxed{}$

Check Your Progress

a. Use $\log_2 3 \approx 1.5850$ to approximate the value of $\log_2 96$.

b. Use $\log_5 4 \approx 0.8614$ and $\log_5 32 \approx 2.1534$ to approximate the value of $\log_5 8$.

KEY CONCEPT

Power Property of Logarithms The logarithm of a power is the product of the logarithm and the exponent.

FOLDABLES

Use the page for Lesson 9-3. Write your own examples that show the Product, Quotient, and Power Properties of Logarithms.

EXAMPLE Power Property of Logarithms

- 3 Given that $\log_5 6 \approx 1.1133$, approximate the value of $\log_5 216$.

$$\begin{aligned} \log_5 216 &= \boxed{} && \text{Replace 216 with } 6^3. \\ &= \boxed{} && \text{Power Property} \\ &\approx \boxed{} && \text{Replace } \log_5 6 \text{ with } 1.1133. \end{aligned}$$

Check Your Progress

Given that $\log_4 6 \approx 1.2925$, approximate the value of $\log_4 1296$.

EXAMPLE Solve Equations Using Properties of Logarithms

- 4 Solve each equation.

a. $4 \log_2 x - \log_2 5 = \log_2 125$

$$4 \log_2 x - \log_2 5 = \log_2 125 \quad \text{Original equation}$$

$$\boxed{} - \log_2 5 = \log_2 125 \quad \text{Power Property}$$

$$\boxed{} = \log_2 125 \quad \text{Quotient Property}$$

$$\boxed{} = 125 \quad \text{Property of Equality for Logarithmic Functions}$$

$$\boxed{} = \boxed{} \quad \text{Multiply each side by 5.}$$

$$x = \boxed{} \quad \text{Take the 4th root of each side.}$$

$$\mathbf{b. \log_8 x + \log_8 (x - 12) = 2}$$

$$\log_8 x + \log_8 (x - 12) = 2$$

Original equation

$$\log_8 [x(x - 12)] = 2$$

Product Property

$$x(x - 12) = \boxed{}$$

Definition of logarithm

$$\boxed{} = 0$$

Subtract 64 from each side.

$$\boxed{} \boxed{} = 0$$

Factor.

$$\boxed{} = 0 \quad \text{or} \quad \boxed{} = 0$$

Zero Product Property

$$x = \boxed{}$$

$$x = \boxed{}$$

Solve each equation.

Check Substitute each value into the original equation.

$$\log_8 (-4) + \log_8 [(-4) - 12] \stackrel{?}{=} 2 \quad \text{Replace } x \text{ with } -4.$$

$$\log_8 (-4) + \log_8 (\boxed{}) \stackrel{?}{=} 2$$

Since $\log_8 (-4)$ and $\log_8 (-16)$ are $\boxed{}$, -4 is an extraneous solution and must be eliminated.

$$\log_8 16 + \log_8 (16 - 12) \stackrel{?}{=} 2 \quad \text{Replace } x \text{ with } 16.$$

$$\log_8 16 + \log_8 4 \stackrel{?}{=} 2 \quad 16 - 12 = 4$$

$$\log_8 (16 \cdot 4) \stackrel{?}{=} 2 \quad \text{Product Property}$$

$$\log_8 \boxed{} \stackrel{?}{=} 2 \quad 16 \cdot 4 = 64$$

$$2 = 2 \checkmark \quad \text{Definition of logarithm}$$

The only solution is $\boxed{}$.

Check Your Progress Solve each equation.

a. $2 \log_3 x - 2 \log_3 6 = \log_3 4$

b. $\log_2 x + \log_2 (x - 6) = 4$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Solve exponential equations and inequalities using common logarithms.
- Evaluate logarithmic expressions using the Change of Base Formula.



TEKS 2A.11 The student formulates equations and

inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(C)** Determine the reasonable domain and range values of exponential and logarithmic functions, as well as **interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities.** **(D)** Determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods.

EXAMPLE Find Common Logarithms

- 1 Use a calculator to evaluate each expression to four decimal places.

a. $\log 6$ Keystrokes: 6 .7781512504

b. $\log 0.35$ Keystrokes: 0.35

-.4559319556

Check Your Progress

Use a calculator to evaluate each expression to four decimal places.

a. $\log 5$

b. $\log 0.62$

EXAMPLE Solve Logarithmic Equations

- 2 **EARTHQUAKES** Refer to Example 2 in your textbook. The San Fernando Valley earthquake of 1994 measured 6.6 on the Richter scale. How much energy did this earthquake release?

$$\log E = 11.8 + 1.5M$$

Write the formula.

$$\log E = 11.8 + 1.5(\text{input})$$

Replace M with .

$$\log E = \text{input}$$

Simplify.

$$10 \log E = 10^{21.7}$$

Write each side using 10 as a base.

$$E = \text{input}$$

Inverse Property of Exponents and Logarithms

$$E \approx \text{input}$$

Use a calculator.

The amount of energy released was about

ergs.

REMEMBER IT

When solving an exponential equation using logarithms, the first step is often referred to as *taking the logarithm of each side*.

Check Your Progress

The amount of energy E , in ergs, that an earthquake releases is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. In 1999 an earthquake in Turkey measured 7.4 on the Richter scale. How much energy did this earthquake release?

EXAMPLE**Solve Exponential Equations Using Logarithms**

3 Solve $5^x = 62$.

$$5^x = 62$$

Original equation

$$\boxed{} = \boxed{}$$

Property of Equality for Logarithmic Functions

$$\boxed{} = \boxed{}$$

Power Property of Logarithms

$$x = \boxed{}$$

Divide each side by $\log 5$.

$$x \approx \boxed{}$$

Use a calculator.

Check Your Progress

Solve $3^x = 17$.

EXAMPLE**Solve Exponential Inequalities Using Logarithms**

4 Solve $2^{7x} > 3^{5x-3}$.

$$2^{7x} > 3^{5x-3}$$

$$\log 2^{7x} > \log 3^{5x-3}$$

$$\boxed{} > \boxed{}$$

$$7x \log 2 > \boxed{} - \boxed{}$$

$$7x \log 2 - \boxed{} > \boxed{}$$

KEY CONCEPT

Change of Base Formula
For all positive numbers a , b , and n , where $a \neq 1$ and $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\begin{aligned} & \boxed{} > -3 \log 3 \\ x & < \frac{-3 \log 3}{7 \log 2 - 5 \log 3} \\ x & < \frac{-3(0.4771)}{7(0.3010) - 5(0.4771)} \\ x & < \boxed{} \end{aligned}$$

Check Your ProgressSolve $5^{3x} < 10^{x-2}$.
EXAMPLE**Change of Base Formula**

- 5** Express $\log_3 18$ in terms of common logarithms. Then approximate its value to four decimal places.

$$\begin{aligned} \log_3 18 &= \boxed{} && \text{Change of Base Formula} \\ &\approx \boxed{} && \text{Use a calculator.} \end{aligned}$$

Check Your Progress

Express $\log_5 16$ in terms of common logarithms. Then approximate its value to four decimal places.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

MAIN IDEAS

- Evaluate expressions involving the natural base and natural logarithms.
- Solve exponential equations and inequalities using natural logarithms.



TEKS 2A.11

The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(D) Determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods.** **(F) Analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem. Also addresses TEKS 2A.11(A).**

BUILD YOUR VOCABULARY (page 255)

The number 2.71828 ... is referred to as the **natural base, e** .

An function with base is called a **natural base exponential function**.

The with base is called the **natural logarithm**.

The **natural logarithmic function, $y = \ln x$** , is the of the natural base exponential function, $y = e^x$.

EXAMPLE

Evaluate Natural Base Expressions

- 1 Use a calculator to evaluate each expression to four decimal places.

a. $e^{0.5}$

Keystrokes: 2nd [e^x] 0.5 ENTER

1.648721271

b. e^{-8}

Keystrokes: 2nd [e^x] -8 ENTER

0.0003354626

Check Your Progress

Use a calculator to evaluate each expression to four decimal places.

a. $e^{0.3}$

b. e^{-2}

EXAMPLE**Evaluate Natural Logarithmic Expressions****WRITE IT**

Write and evaluate three logarithms, one with base 2, one with base e , and one with base 10.

- 2** Use a calculator to evaluate each expression to four decimal places.

a. $\ln 3$

Keystrokes: 3

1.098612289

b. $\ln \frac{1}{4}$

Keystrokes: 1 4

-1.386294361

Check Your Progress

Use a calculator to evaluate each expression to four decimal places.

a. $\ln 2$

b. $\ln \frac{1}{2}$

EXAMPLE**Write Equivalent Expressions**

- 3** Write an equivalent exponential or logarithmic equation.

a. $e^x = 23$

$$e^x = 23 \rightarrow \begin{array}{l} \boxed{} = \boxed{} \\ \boxed{} = \boxed{} \end{array}$$

b. $\ln x \approx 1.2528$

$$\ln x \approx 1.2528 \rightarrow \begin{array}{l} \boxed{} \approx \boxed{} \\ \boxed{} \approx \boxed{} \end{array}$$

Check Your Progress

Write an equivalent exponential or logarithmic equation.

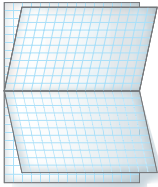
a. $e^x = 6$

b. $\ln x = 2.25$

FOLDABLES™

ORGANIZE IT

Use the page for Lesson 9-5. On the same grid, sketch the graph for $y = e^x$ and $y = \ln x$. Then write how you can tell that the two functions are inverses.

EXAMPLE Solve Base e Equations

4 Solve $3e^{-2x} + 4 = 10$.

$3e^{-2x} + 4 = 10$

Original equation

$\square = \square$

Subtract 4 from each side.

$\square = \square$

Divide each side by 3.

$\ln e^{-2x} = \ln 2$

Property of Equality for Logarithms

$\square = \square$

Inverse Property of Exponents and Logarithms

$x = \square$

Divide each side by -2 .

$x \approx \square$

Use a calculator.

Check Your Progress

Solve $2e^{-2x} + 5 = 15$.

EXAMPLE Solve Base e Inequalities

5 SAVINGS Suppose you deposit \$700 into an account paying 3% annual interest, compounded continuously.

a. What is the balance after 8 years?

$A = Pe^{rt}$

Continuous compounding formula

$= \square e^{(0.03)(8)}$

Replace P with \square , r with 0.03, and t with 8.

$= 700e^{0.24}$

Simplify.

$\approx \square$

Use a calculator.

The balance after 8 years would be \square .

b. How long will it take for the balance in your account to reach at least \$1200?

$A \geq 1200$

The balance is at least \$1200.

$700e^{(0.03)t} \geq 1200$

Replace A with $700e^{(0.03)t}$.

$e^{(0.03)t} \geq \square$

Divide each side by 700.

$\ln e^{(0.03)t} \geq \ln \frac{12}{7}$

Property of Inequality for Logarithms

$\square \geq \ln \frac{12}{7}$

Inverse Property of Exponents and Logarithms

$t \geq \frac{\ln \frac{12}{7}}{0.03}$

Divide each side by 0.03.

$t \geq \square$

Use a calculator.

It will take at least \square years for the balance to reach \$1200.

Check Your Progress

Suppose you deposit \$700 into an account paying 6% annual interest, compounded continuously.

- a. What is the balance after 7 years?

- b. How long will it take for the balance in your account to reach at least \$2500?

EXAMPLE**Solve Natural Log Equations and Inequalities**

- 6 Solve each equation or inequality.

- a. $\ln 3x = 0.5$

$$\ln 3x = 0.5$$

$$e^{\ln 3x} = e^{0.5}$$

$$\boxed{} = \boxed{}$$

$$x = \frac{e^{0.5}}{3}$$

$$x \approx \boxed{}$$

Original equation

Write each side using exponents and base e .

Inverse Property of Exponents and Logarithms

Divide each side by 3.

Use a calculator.

- b. $\ln (2x - 3) < 2.5$

$$\ln (2x - 3) < 2.5$$

$$e^{\ln(2x - 3)} < e^{2.5}$$

$$\boxed{} < \boxed{}$$

$$\boxed{} < \boxed{}$$

$$x < \boxed{}$$

$$x < \boxed{}$$

Original inequality

Write each side using exponents and base e .

Inverse Property of Exponents and Logarithms

Add 3 to each side.

Divide each side by 2.

Use a calculator.

Check Your Progress

Solve each equation or inequality.

- a. $\ln 2x = 0.7$

- b. $\ln (x - 3) > 3$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BUILD YOUR VOCABULARY (page 255)**MAIN IDEAS**

- Use logarithms to solve problems involving exponential decay.
- Use logarithms to solve problems involving exponential growth.



TEKS 2A.11 The student formulates equations and

inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. **(D)**

Determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods.

(F) Analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

The percent of decrease r is also referred to as the **rate of decay** in the equation for exponential decay of the form $y = a(1 - r)^t$.

EXAMPLE**Exponential Decay of the Form $y = a(1 - r)^t$**

- 1 CAFFEINE** A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for 90% of this caffeine to be eliminated from a person's body?

$$y = a(1 - r)^t$$

Exponential decay formula

$$13 = 130(1 - 0.11)^t$$

Replace y with 13, a with 130, and r with 0.11.

$$\boxed{} = \boxed{}$$

Divide each side by 130.

$$\log \boxed{} = \log \boxed{}$$

Property of Equality for Logarithms

$$\log 0.10 = \boxed{}$$

Power Property for Logarithms

$$\boxed{} = t$$

Divide each side by $\log 0.89$.

$$\boxed{} \approx t$$

Use a calculator.

It will take approximately $\boxed{}$ hours for 90% of the caffeine to be eliminated from a person's body.

Check Your Progress

Refer to Example 1. How long will it take for 80% of this caffeine to be eliminated from a person's body?

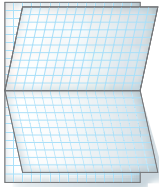
EXAMPLE

Exponential Decay of the Form $y = ae^{-kt}$

FOLDABLES™

ORGANIZE IT

Use the page for Lesson 9-6. If an exponential decay problem involves finding a period of time, which model do you use? Why?



2 GEOLOGY The half-life of Sodium-22 is 2.6 years.

- a. What is the value of k and the equation of decay for Sodium-22?

$$y = ae^{-kt}$$

Exponential decay formula

$$0.5a = ae^{-k(2.6)}$$

Replace y and t .

$$0.5 = e^{-2.6k}$$

Divide each side by a .

$$\ln \boxed{} = \ln \boxed{}$$

Property of Equality for Logarithmic Functions

$$\ln 0.5 = \boxed{}$$

Inverse Property of Exponents and Logarithms

$$\boxed{} = k$$

Divide each side by -2.6 .

$$\boxed{} = k$$

Use a calculator.

The constant k for Sodium-22 is $\boxed{}$. The equation

for the decay of Sodium-22 is $\boxed{}$, where t is years.

- b. A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth?

$$y = ae^{-0.2666t}$$

Decay formula

$$0.1a = ae^{-0.2666t}$$

Replace y with $0.1a$.

$$0.1 = e^{-0.2666t}$$

Divide each side by a .

$$\ln \boxed{} = \ln \boxed{}$$

Property of Equality for Logarithms

$$\ln \boxed{} = \boxed{}$$

Inverse Property

$$\boxed{} = t$$

Divide each side by -0.2666 .

$$\boxed{} \approx t$$

Use a calculator.

It was formed about $\boxed{}$ years ago.

Check Your Progress The half-life of radioactive iodine used in medical studies is 8 hours.

- a. What is the value of k for radioactive iodine?

- b. A doctor wants to know when the amount of radioactive iodine in a patient's body is 20% of the original amount. When will this occur?

BUILD YOUR VOCABULARY (page 255)

The percent of increase r is also referred to as the **rate of growth** in the equation for exponential growth of the form $y = a(1 + r)^t$.

WRITE IT

How can you tell whether an exponential equation involves growth or decay?

EXAMPLE Exponential Growth of the Form $y = a(1 + r)^t$

- 3 TEST EXAMPLE** The population of a city of one million is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled?

A 4 years B 5 years C 20 years D 23 years

$$y = a(1 + r)^t$$

Growth formula

$$2,000,000 = 1,000,000(1 + 0.03)^t$$

Replace y , a , and r .

$$\boxed{} = \boxed{}$$

Divide each side by 1,000,000.

$$\ln \boxed{} = \ln \boxed{}$$

Property of Equality for Logarithms

$$\ln 2 = \boxed{}$$

Power Property of Logarithms

$$\frac{\ln 2}{\ln 1.03} = t$$

Divide.

$$t \approx 23.45$$

Use a calculator.

The population will have doubled in years.

Check Your Progress

The population of a city of 10,000 is increasing at a rate of 5% per year. If the population continues to grow at this rate, in about how many years will the population have doubled?

- A 10 years B 12 years C 14 years D 18 years

EXAMPLE**Exponential Growth of the Form $y = ae^{kt}$**

- 4 POPULATION** As of 2005, Nigeria had an estimated population of 129 million people, and the United States had an estimated population of 296 million people. Assume that the populations of Nigeria and the United States can be modeled by $N(t) = 129e^{0.024t}$ and $U(t) = 296e^{0.009t}$, respectively. According to these models, when will Nigeria's population be more than the population of the United States?

You want to find t such that $N(t) > U(t)$.

$$N(t) > U(t)$$

$$129e^{0.024t} > 296e^{0.009t}$$

Replace $N(t)$ with $129e^{0.024t}$ and $U(t)$ with $296e^{0.009t}$.

$$\ln 129e^{0.024t} > \ln 296e^{0.009t}$$

Property of Inequality for Logarithms

$$\ln 129 + \ln e^{0.024t} > \boxed{} + \boxed{}$$

Product Property of Logarithms

$$\ln 129 + 0.024t > \boxed{} + \boxed{}$$

Inverse Property of Exponents and Logarithms

$$\ln 129 \boxed{} > \boxed{} t$$

Subtract 296 and $0.024t$ from each side.

$$\boxed{} < t$$

Divide each side by
-0.015.

$$\boxed{} < t$$

Use a calculator.

After $\boxed{}$ years or in $\boxed{}$, Nigeria's population will be greater than the population of the U.S.

Check Your Progress

As of 2000, Saudi Arabia had an estimated population of 20.7 million people and the United States had an estimated population of 278 million people. The growth of the populations of Saudi Arabia and the United States can be modeled by $S(t) = 20.7e^{0.0327t}$ and $U(t) = 278e^{0.009t}$, respectively. According to these models, when will Saudi Arabia's population be more than the population of the United States?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BRINGING IT ALL TOGETHER**STUDY GUIDE**

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 9 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 9, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 254–255) to help you solve the puzzle.

9-1

Exponential Functions

Determine whether each function represents exponential growth or decay.

1. $y = 0.2(3)^x$

2. $y = 3\left(\frac{2}{5}\right)^x$

3. $y = 0.4(1.01)^x$

4. Simplify $4^x \cdot 4^{2x}$.

5. Solve $25^{y+3} = \left(\frac{1}{5}\right)^y$.

9-2

Logarithms and Logarithmic Functions

6. What is the inverse of the function $y = 5^x$?

7. What is the inverse of the function $y = \log_{10} x$?

8. Evaluate $\log_{27} 9$.

9. Solve $\log_8 x = -\frac{1}{3}$.

9-3

Properties of Logarithms

State whether each of the following equations is *true* or *false*. If the statement is true, name the property of logarithms that is illustrated.

10. $\log_3 10 = \log_3 30 - \log_3 3$

11. $\log_4 12 = \log_4 4 + \log_4 8$

12. $\log_2 81 = 2\log_2 9$

Solve each equation.

13. $\log_5 14 - \log_5 (2x) = \log_5 21$ 14. $\log_2 x + \log_2 (x + 2) = 3$

9-4

Common Logarithms

Match each expression from the first column with an expression from the second column that has the same value.

15. $\log_2 2$

16. $\log 12$

17. $\log_3 1$

18. $\log_5 \frac{1}{5}$

19. $\log 1000$

a. $\log_4 1$

b. $\log_2 8$

c. $\log_{10} 12$

d. $\log_5 5$

e. $\log 0.1$

20. Solve $6^{2x-1} = 2^{5x}$. Round to four decimal places.

21. Express $\log_8 5$ in terms of common logarithms. Then approximate its value to four decimal places.

9-5

Base e and Natural Logarithms

Match each expression from the first column with its value in the second column. Some choices may be used more than once or not at all.

22. $e^{\ln 5}$	<input type="text"/>	I. 1 II. 10 III. -1 IV. 5 V. 0 VI. e
23. $\ln 1$	<input type="text"/>	
24. $e^{\ln e}$	<input type="text"/>	
25. $\ln e^5$	<input type="text"/>	
26. $\ln e$	<input type="text"/>	
27. $\ln\left(\frac{1}{e}\right)$	<input type="text"/>	

28. Solve $\ln(x - 8) = 5$. Round to four decimal places.

29. Evaluate $e^{\ln 5.2}$.

9-6

Exponential Growth and Decay

State whether each equation represents exponential growth or decay.

30. $y = 5e^{0.15t}$

31. $y = 1000(1 - 0.05)^t$

32. $y = 0.3e^{-1200t}$

33. $y = 2(1 + 0.0001)^t$

34. Leroy bought a lawn mower for \$1,200. It is expected to depreciate at a rate of 20% per year. What will be the value of the lawn mower in 5 years?

35. The population of a school has increased at a steady rate each year from 375 students to 580 students in 8 years. Find the annual rate of growth.



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 9.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 9 Practice Test on page 557 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 9 Study Guide and Review on pages 552–556 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 557 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 9 Foldable.
- Then complete the Chapter 9 Study Guide and Review on pages 552–556 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 557 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Conic Sections

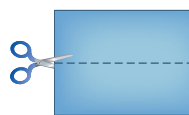
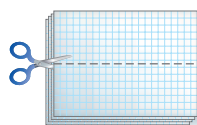


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

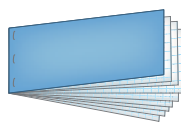
Begin with five sheets of notebook paper.

STEP 1

Cut each sheet of grid paper in half lengthwise. Cut the sheet of construction paper in half lengthwise to form a front and back cover for the booklet of grid paper.

**STEP 2**

Staple all the sheets together to form a long, thin notepad of grid paper.



NOTE-TAKING TIP: When you take notes, include personal experiences that relate to the lesson and ways in which what you have learned will be used in your daily life.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 10. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
<u>asymptote</u> A-suhm(p)-toht			
center			
circle			
conic section			
<u>conjugate axis</u> KAHN-jih-guht			
<u>directrix</u> duh-REHK-trihks			
<u>ellipse</u> ih-LIHPS			
foci			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
focus FOH-kuhs			
hyperbola hy-PUHR-buh-luh			
latus rectum LA-tuhs REHK-tuhm			
major axis			
minor axis			
parabola puh-RA-buh-luh			
transverse axis			
vertex			

EXAMPLE

3 TEST EXAMPLE A coordinate grid is placed over a scale drawing of Jenny's patio. A grill is located at $(2, -3)$. A flowerpot is located at $(-6, -1)$. A picnic table is at the midpoint between the grill and the flowerpot. In coordinate units, about how far is it from the grill to the picnic table?

- A 0.2 B 2.24 C 4.1 D 5

Read the Test Item

The question asks us to find the distance between the grill and the midpoint. Find the midpoint and then use the Distance Formula.

Solve the Test Item

Use the Midpoint Formula to find the coordinates of the picnic table.

$$\begin{aligned} \text{midpoint} &= \left(\frac{2 + (-6)}{2}, \frac{(-3) + (-1)}{2} \right) && \text{Midpoint Formula} \\ &= \boxed{} && \text{Simplify.} \end{aligned}$$

Use the Distance Formula to find the distance between the grill

$\boxed{}$ and the picnic table $\boxed{}$.

$$\begin{aligned} \text{distance} &= \sqrt{[(-2) - 2]^2 + [-2 - (-3)]^2} && \text{Distance Formula} \\ &= \sqrt{(\boxed{})^2 + (\boxed{})^2} && \text{Subtract.} \\ &= \boxed{} \text{ or about } \boxed{} && \text{Simplify.} \end{aligned}$$

The answer is $\boxed{}$.

Check Your Progress

A coordinate grid is placed over a floor plan of the school. The gymnasium is located at $(4, 6)$. The science lab is located at $(-2, -4)$. The office is located at the midpoint between the gymnasium and the science lab. In coordinate units, about how far is the office from the science lab?

- A 1.4 B 2.8 C 9.1 D 10.3

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

10-2 Parabolas

BUILD YOUR VOCABULARY (pages 283–284)

MAIN IDEAS

- Write equations of parabolas in standard form.
- Graph parabolas.



TEKS 2A.5

The student knows the relationship between the geometric and algebraic descriptions of conic sections. (A) Describe a conic section as the intersection of a plane and a cone. (B) Sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph. (C) Identify symmetries from graphs of conic sections. Also addresses TEKS 2A.5(E), 2A.7(A), and 2A.7(B).

KEY CONCEPT

Equation of a Parabola

The standard form of the equation of a parabola with vertex (h, k) and axis of symmetry $x = h$ is $y = a(x - h)^2 + k$.

- If $a > 0$, k is the minimum value of the related function and the parabola opens upward.
- If $a < 0$, k is the maximum value of the related function and the parabola opens downward.

The graph of an equation of the form, $a \neq 0$,

is a **parabola**.

Any figure that can be obtained by slicing a

is called a **conic section**.

A parabola can also be defined as the set of all points in a plane that are the same from a given point called the **focus** and a given line called the **directrix**.

EXAMPLE Analyze the Equation of a Parabola

- 1 Write $y = -x^2 - 2x + 3$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

$$y = -x^2 - 2x + 3$$

Original equation

$$y = -1(x^2 + 2x) + 3$$

Factor.

$$y = -1(x^2 + 2x + \square) + 3 - (-1)(\square)$$

Complete the square.

$$y = -1\left(x^2 + 2x + \square\right) + 3 + 1\left(\square\right)$$

Multiply by -1 .

$$y = \square$$

$$y = \square$$

$(h, k) = (-1, 4)$

The vertex of this parabola is located at , and the equation of the axis of symmetry is $x = \square$. The parabola opens downward.

Check Your Progress

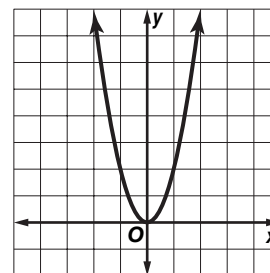
Write $y = 2x^2 + 4x + 5$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

EXAMPLE**Graph Parabolas****2 a. Graph $y = 2x^2$.**

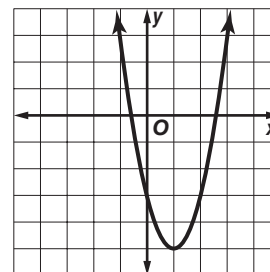
For this equation, $h = 0$ and $k = 0$. The vertex is at the origin. Substitute positive integers for x and find the corresponding y -values.

x	y
1	2
2	8
3	18

Since the graph is symmetric about the y -axis, the points at , , and are also on the parabola. Use all of these points to draw the graph.

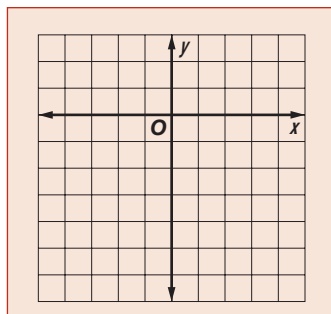
**b. Graph $y = 2(x - 1)^2 - 5$.**

The equation is of the form $y = a(x - h)^2 + k$, where $h = 1$ and $k = -5$. The graph of this equation is the graph of $y = 2x^2$ in part **a** translated unit right and units down. The vertex is now at .

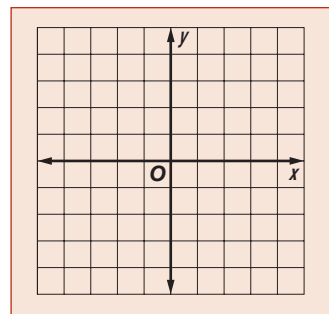
**Check Your Progress**

Graph each equation.

a. $y = -3x^2$



b. $y = -3(x + 1)^2 + 3$



BUILD YOUR VOCABULARY (page 284)

The line segment through the focus of a parabola and perpendicular to the axis of is called the **latus rectum**.

EXAMPLE Graph an Equation Not in Standard Form**3** Graph $x + y^2 = 4y - 1$.

First write the equation in the form $x = a(y - k)^2 + h$.

$$x + y^2 = 4y - 1$$

$$x = -y^2 + 4y - 1$$

$$x = -1(\text{input}) - 1$$

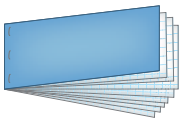
$$x = -1(y^2 - 4y + \text{input}) - 1 - (-1)(\text{input})$$

$$x = -1(y^2 - 4y + \text{input}) - 1 + 1(\text{input})$$

$$x = \text{input}$$

FOLDABLES™**ORGANIZE IT**

Use the page for Vocabulary. Make a sketch of each vocabulary term in this lesson.



Then use the following information to draw the graph.

vertex:

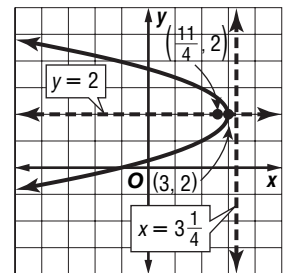
axis of symmetry: $y =$

focus: $(3 + \frac{1}{4(-1)}, 2)$ or $(\frac{11}{4}, 2)$

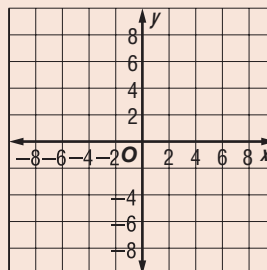
directrix: $x =$ or $3\frac{1}{4}$

direction of opening: left, since $a < 0$

length of the latus rectum: or unit

**Check Your Progress**

Graph $x - y^2 = 6y + 2$.



EXAMPLE

Write and Graph an Equation for a Parabola

4 BRIDGES The 52-meter-long Hulme Arch Bridge in Manchester, England, is supported by cables suspended from a parabolic steel arch. The highest point of the arch is 25 meters above the bridge, and the focus of the arch is about 18 meters above the bridge.

- a. Let the bridge be the x -axis, and let the y -axis pass through the arch's vertex. Write an equation that models the arch.

The vertex is at $(0, 25)$, so $h = \square$ and $k = \square$.

The focus is at $(0, 12)$. Use the focus to find a .

$$18 = 25 + \frac{1}{4a} \quad k = 25; \text{ The focus } y\text{-coordinate is } 18.$$

$$\square = \frac{1}{4a} \quad \text{Subtract 25 from each side.}$$

$$\square = 1 \quad \text{Multiply each side by } 4a.$$

$$a = \square \quad \text{Divide each side by } -28.$$

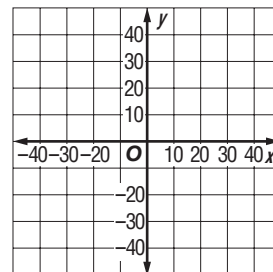
An equation of the parabola is \square .

- b. Graph the equation.

The length of the latus rectum is $\frac{1}{a}$ or

\square units, so the graph must pass through $(-14, 18)$ and $(14, 18)$.

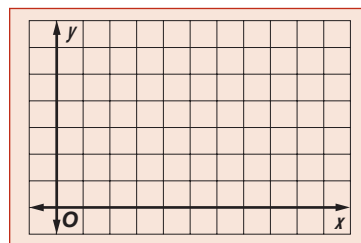
According to the length of the bridge, the graph must pass through the points $(-26, 0)$ and $(26, 0)$. Use these points and the information from part a to draw the graph.



Check Your Progress

The water stream follows a parabolic path. The highest point of the water stream is $6\frac{1}{4}$ feet above the ground and the water hits the ground 10 feet from the jet. The focus of the fountain is $5\frac{1}{2}$ feet above the ground.

- a. Write an equation that models the path of the water fountain.
- b. Graph the equation.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

10-3 Circles

MAIN IDEAS

- Write equations of circles.
- Graph circles.

BUILD YOUR VOCABULARY (pages 283–284)

A circle is the set of all points in a plane that are from a given in the plane, called the **center**.

A line that the circle in exactly point is said to be **tangent** to the circle.

EXAMPLE Write an Equation Given the Center and Radius

- 1 LANDSCAPING** The plan for a park puts the center of a circular pond, of radius 0.6 miles, 2.5 miles east and 3.8 miles south of the park headquarters. Write an equation to represent the border of the pond, using the headquarters as the origin.

Since the headquarters is at , the center of the pond is at with radius 0.6 mile.

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$\left(x - \text{input}\right)^2 + \left(y + \text{input}\right)^2 = \text{input}^2 \quad (h, k) = (2.5, -3.8), r = 0.6$$

$$(x - 2.5)^2 + (y + 3.8)^2 = \text{input} \quad \text{Simplify.}$$

The equation is

Check Your Progress The plan for a park puts the center of a circular pond, of radius 0.5 mile, 3.5 miles west and 2.6 miles north of the park headquarters. Write an equation to represent the border of the pond, using the headquarters as the origin.

KEY CONCEPT

Equation of a Circle
The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$

FOLDABLES On the page for Circles, write this equation. Be sure to explain h , k , and r .

TEKS 2A.5
The student knows the relationship between the geometric and algebraic descriptions of conic sections. **(B)** Sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph. **(C)** Identify symmetries from graphs of conic sections.

EXAMPLE Graph an Equation in Standard Form

WRITE IT

What must you do to the equation of a circle if you want to graph the circle on a calculator?

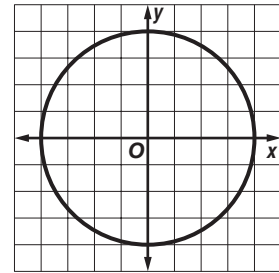
3 Find the center and radius of the circle with equation $x^2 + y^2 = 16$. Then graph the circle.

The center is at , and the radius is .

The table lists some values for x and y that satisfy the equation.

x	y
0	4
1	3.9
2	3.5
3	2.6
4	0

Since the circle is centered at the origin, it is symmetric about the y -axis. Use these points and the concept of symmetry to graph $x^2 + y^2 = 16$.



EXAMPLE Graph an Equation Not in Standard Form

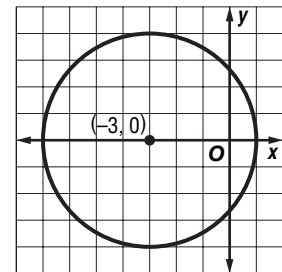
4 Find the center and radius of the circle with equation $x^2 + y^2 + 6x - 7 = 0$. Then graph the circle.

Complete the square.

$$x^2 + 6x + \square + y^2 = 7$$

$$x^2 + 6x + \square + y^2 = 7 + \square$$

$$\square + y^2 = \square$$

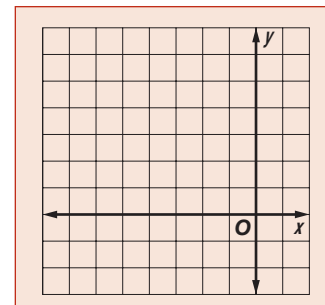
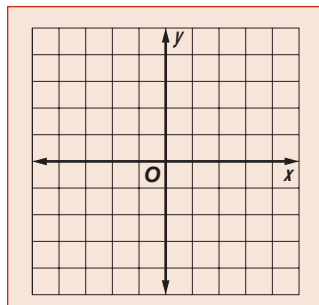


The center is at , and the radius is .

Check Your Progress Graph each equation.

a. $x^2 + y^2 = 9$

b. $x^2 + y^2 + 8x - 4y + 11 = 0$



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Ellipses



TEKS 2A.5 The student knows the relationship between the geometric and algebraic descriptions of conic sections. **(B) Sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph. (C) Identify symmetries from graphs of conic sections.** Also addresses TEKS 2A.5(E).

BUILD YOUR VOCABULARY (pages 283–284)

MAIN IDEAS

- Write equations of ellipses.
- Graph ellipses.

An ellipse is the set of all points in a plane such that the sum of the from two fixed is constant. The two fixed points are called the **foci** of the ellipse.

The points at which the ellipse intersects its axes of symmetry determine two with endpoints on the ellipse called the **major axis** and the **minor axis**. The axes intersect at the **center** of the ellipse.

EXAMPLE

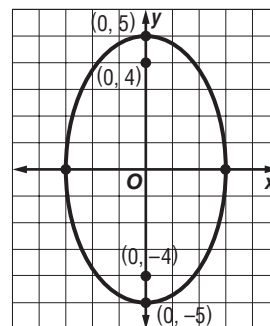
Write an Equation for a Graph

1 Write an equation for the ellipse.

Find the values of a and b for the ellipse. The length of the major axis of any ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between $(0, 5)$ and $(0, -5)$. 10 units.

$$2a = \text{input} \quad \text{Length of major axis} = 10$$

$$a = \text{input} \quad \text{Divide each side by 2.}$$



The foci are located at $(0, 4)$ and $(0, -4)$, so $c = 4$.

Use the relationship between a , b , and c to find b .

$$c^2 = a^2 - b^2$$

Equation relating a , b , and c

$$\text{input} = \text{input}$$

$$c = 4 \text{ and } a = 5$$

$$b^2 = \text{input}$$

Solve for b^2 .

Since the major axis is vertical, substitute 25 for a^2 and 9 for b^2 in the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$. An equation of the ellipse is

$$\text{input}$$

WRITE IT

Write the name for the endpoints of each axis of an ellipse.

$$c^2 = a^2 - b^2$$

$$c = \boxed{}$$

$$\boxed{c} = \boxed{} \sqrt{a^2 - b^2}$$

$$2c = 2\sqrt{\left(\boxed{}\right)^2 - \left(\boxed{}\right)^2}$$

$$2c \approx \boxed{}$$

Equation relating a , b , and c .

Take the square root of each side.

Multiply each side by 2.

Substitute $a = 160$ and $b = 75$.

Use a calculator.

The points where two people should stand to hear each other whisper are about $\boxed{}$ feet apart.

Check Your Progress

A listener is standing in an elliptical room 60 feet wide and 120 feet long. When a speaker stands at one focus and whispers, the best place for the listener to stand is at the other focus.

- a. Write an equation to model this ellipse, assuming the major axis is horizontal and the center is at the origin.

- b. How far apart should the speaker and the listener be in this room?

EXAMPLE

Graph an Equation in Standard Form

- 3** Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{9} = 1$. Graph the ellipse.

The center of this ellipse is at $(0, 0)$.

Since $a^2 = 36$, $a = \boxed{}$ and since $b^2 = 9$, $b = \boxed{}$.

The length of the major axis is $2(6)$ or $\boxed{}$ units, and the

length of the minor axis is $2(3)$ or $\boxed{}$. Since the x^2 term has

the greatest denominator, the major axis is $\boxed{}$.

$$c^2 = a^2 - b^2$$

$$c^2 = \boxed{} - \boxed{}$$

$$c = \boxed{} \text{ or } \boxed{}$$

Equation relating a , b , and c .

$$a = 6 \text{ and } b = 3$$

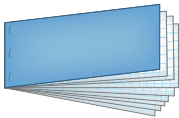
Take a square root of each side.

The foci are at $\boxed{}$ and $\boxed{}$.

FOLDABLES™

ORGANIZE IT

Use the page for Ellipses. Make a sketch of each vocabulary term in this lesson.



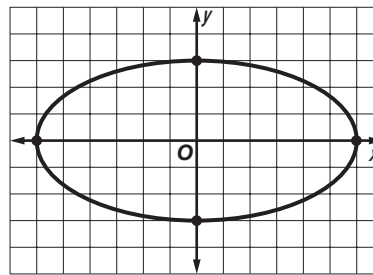
You can use a calculator to find some approximate nonnegative values for x and y that satisfy the equation.

Since the ellipse is centered at the origin, it is symmetric about the y -axis. So the points at and lie on the graph.

The ellipse is also symmetric about the x -axis, so the points at and also lie on the graph.

Graph the intercepts , , , and , and draw the ellipse that passes through them and the other points. center: $(0, 0)$; foci: $(3\sqrt{3}, 0)$, $(-3\sqrt{3}, 0)$; major axis: 12; minor axis: 6

x	y
0	3
1	2.96
2	2.83
3	2.60
4	2.24
5	1.66
6	0



Check Your Progress

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Then graph the equation.

EXAMPLE

Graph an Equation Not in Standard Form

- 3 Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $x^2 + 4y^2 - 6x - 16y - 11 = 0$. Graph the ellipse.

Complete the square to write in standard form.

$$x^2 + 4y^2 - 6x - 16y - 11 = 0$$

$$x^2 - 6x + \square + 4(y^2 - 4y + \square) = 11 + \square + 4(\square)$$

$$(x^2 - 6x + \square) + 4(y^2 - 4y + \square) = 11 + 9 + 4(\square)$$

$$\square$$

$$\square = 1$$

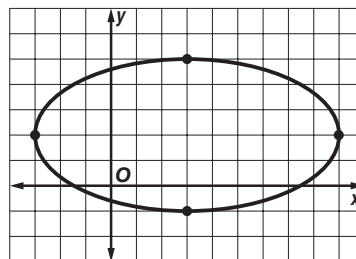
The center is \square , and the foci

are located at \square

and \square . The

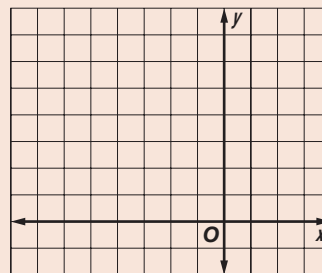
length of the major axis is

\square units, and the length of the minor axis is \square .



Check Your Progress

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $4x^2 + 25y^2 + 16x - 150y + 141 = 0$. Graph the ellipse.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Hyperbolas



TEKS 2A.5 The student knows the relationship between the geometric and algebraic descriptions of conic sections. **(B) Sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph. (C) Identify symmetries from graphs of conic sections. Also addresses TEKS 2A.5(E).**

EXAMPLE

Write an Equation for a Graph

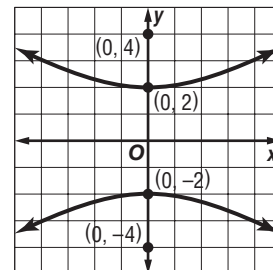
MAIN IDEAS

- Write equations of hyperbolas.
- Graph hyperbolas.

1 Write an equation for the hyperbola shown.

The center is .

The value of a is the distance from the center to a vertex, or units. The value of c is the distance from the center to a focus, or units.



$c^2 = a^2 + b^2$ Equation relating a , b , and c

= $c = 4, a = 2$

= Evaluate the squares.

$12 = b^2$ Solve for b^2 .

Since the transverse axis is vertical, the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$. Substitute the values for a^2 and b^2 .

An equation of the hyperbola is .

KEY CONCEPT

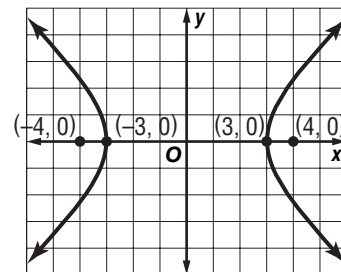
Equations of Hyperbolas with Centers at the Origin

Horizontal Transverse Axis $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertical Transverse Axis $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Check Your Progress

Write an equation for the hyperbola.



EXAMPLE

Graph an Equation in Standard Form

2 Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $x^2 - y^2 = 1$. Graph the hyperbola.

The center of the hyperbola is at the origin. According to the equation, $a^2 = 1$ and $b^2 = 1$, $a = 1$ and $b = 1$. The coordinates of the vertices are and .

$$c^2 = a^2 + b^2 \quad \text{Equation relating } a, b, \text{ and } c \text{ for a hyperbola.}$$

$$c^2 = \boxed{}^2 + \boxed{}^2 \quad a = 1, b = 1$$

$$c^2 = 2 \text{ or } c = \boxed{} \quad \text{Simplify and take the square root.}$$

The equations of the asymptotes are $y = \pm \frac{b}{ax}$ or $y = \pm x$.

Use a calculator to find some approximate nonnegative values for x and y that satisfy the equation. Since the hyperbola is centered at the origin, it is symmetric about the y -axis. Therefore, the points at $(-5, 4.9)$,

$\boxed{}$, $(-3, 2.8)$, $(-2, 1.7)$, and

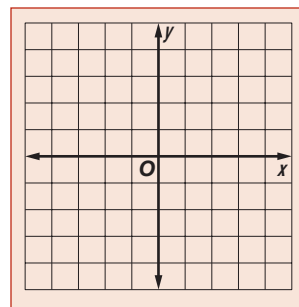
$\boxed{}$ lie on the graph.

x	y
1	0
2	1.7
3	2.8
4	3.9
5	4.9

The hyperbola is also symmetric about the x -axis, so the points at $(-5, -4.9)$, $(-4, 3.9)$, $\boxed{}$, $(-2, -1.7)$, $(3, -2.8)$, $\boxed{}$, and $(5, -4.9)$ also lie on the graph.

Draw a 2-unit by 2-unit square. The asymptotes contain the diagonals of the square. Graph the vertices, which, in this case, are the x -intercepts. Use the asymptotes as a guide to draw the hyperbola that passes through the vertices and the other points. The

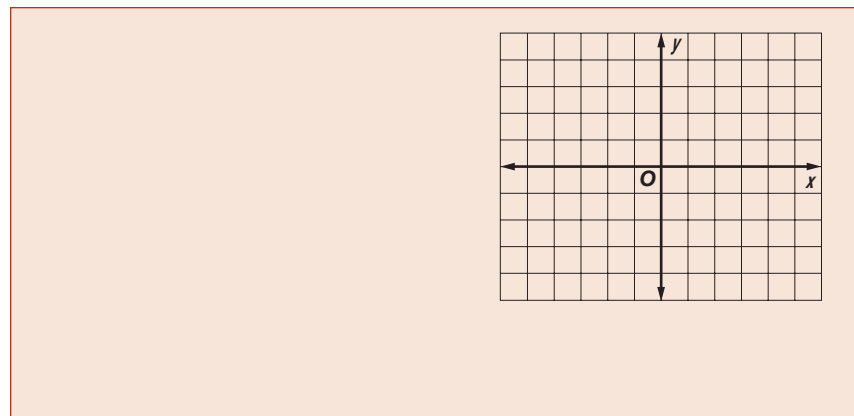
graph $\boxed{}$ intersect the asymptotes.



Check Your Progress

Find the vertices and foci and the equations of the asymptotes for the hyperbola with equation

$$\frac{x^2}{4 - y^2} = 1. \text{ Then graph the hyperbola.}$$



EXAMPLE Graph an Equation Not in Standard Form

- 3 Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $x^2 - y^2 + 6x + 10y - 17 = 0$. Then graph the hyperbola.

Complete the square for each variable.

$$x^2 - y^2 + 6x + 10y - 17 = 0$$

$$x^2 - 6x + \square - 1(y^2 - 10y + \square) = 17 + \square - 1(\square)$$

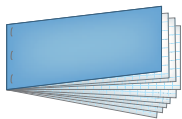
$$x^2 + 6x + \square - 1(y^2 - 10y + \square) = 17 + \square - 1(25)$$

$$\square - \square = \square$$

FOLDABLES™

ORGANIZE IT

Under the page for Hyperbolas, describe two similarities between hyperbolas and ellipses.



The vertices are \square and

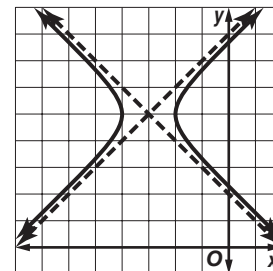
\square , and the foci are

$(\sqrt{2} - 3, 5)$ and $(-\sqrt{2} - 3, 5)$.

The equations of the asymptotes are

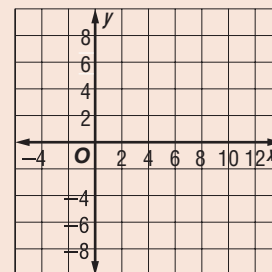
\square or $y = \square$

and $y = -x + 2$.



Check Your Progress

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $9x^2 - 16y^2 - 72x - 64y + 224 = 0$. Then graph the hyperbola.



HOMEWORK ASSIGNMENT

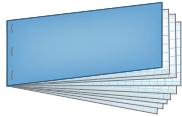
Page(s): _____

Exercises: _____

FOLDABLES

ORGANIZE IT

Under the page for Conic Sections, sketch and label each of the conic sections. Then write the standard form on the conic section below each label.



b. $2x^2 + 2y^2 + 16x - 20y = -32$

$A =$ and $C =$

Since , the graph is a .

c. $y^2 - 2x - 4y + 10 = 0$

$A =$ and $C =$

Since , this graph is a .

Check Your Progress

Without writing the equation in standard form, state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

a. $2y^2 - x^2 + 16 = 0$

b. $3x^2 + y^2 + 15x - 21y = -11$

c. $y^2 - 3x + 2y - 10 = 0$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____


EXAMPLE Linear-Quadratic System

MAIN IDEAS

- Solve systems of quadratic equations algebraically and graphically.
- Solve systems of quadratic inequalities graphically.

REVIEW IT

By the Square Root Property, for any real number n , if $x^2 = n$, then $x = \underline{\hspace{2cm}}$?
(Lesson 5-4)

 **TEKS 2A.3** The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. **(B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.**

2A.5 The student knows the relationship between the geometric and algebraic descriptions of conic sections. **(B) Sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph.**

1 Solve the system of equations.

$$4x^2 - 16y^2 = 25$$

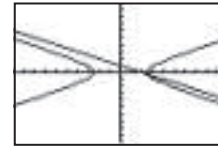
$$2y + x = 2$$

You can use a graphing calculator to help visualize the relationships of the graphs of the equations and predict the number of solutions.

Solve each equation for y to obtain

$$y = \pm \frac{\sqrt{4x^2 - 25}}{4} \text{ and}$$

$$y = \boxed{\hspace{2cm}}.$$



Enter the functions on the Y= screen. The graph indicates that the hyperbola and the line intersect in one point. So, the system has one solution.

Use substitution to solve the system.

First, rewrite $2y + x = 2$ as $x = 2 - 2y$.

$$4x^2 - 16y^2 = 25$$

First equation in the system

$$4(\boxed{\hspace{2cm}})^2 - 16y^2 = 25$$

Substitute $\boxed{\hspace{2cm}}$ for x .

$$\boxed{\hspace{2cm}} + 16 = 25$$

Simplify.

$$\boxed{\hspace{2cm}} = \boxed{\hspace{2cm}}$$

Subtract $\boxed{\hspace{2cm}}$ from each side.

$$y = \boxed{\hspace{2cm}}$$

Divide each side by $\boxed{\hspace{2cm}}$.

Now solve for x .

$$x = 2 - 2y$$

Equation for x in terms of y

$$x = 2 - 2(\boxed{\hspace{2cm}})$$

Substitute the y -value.

$$x = \boxed{\hspace{2cm}}$$

Simplify.

$$\text{The solution is } (\boxed{\hspace{2cm}}).$$

Check Your Progress

Solve $x^2 - y^2 = 4$ and $2y + x = 2$.

EXAMPLE

Quadratic-Quadratic System

2 Solve the system of equations.

$$x^2 + y^2 = 16$$

$$4x^2 + y^2 = 23$$

REMEMBER IT



When graphing conic sections, press ZOOM 5. This window gives the graphs a more realistic look.

A graphing calculator indicates that the circle and ellipse intersect in four points. So, this system has four solutions.



Use the elimination method to solve.

$$-x^2 - y^2 = -16$$

Rewrite the first original equation.

$$(+)\ 4x^2 + y^2 = 23$$

Second original equation

$$\boxed{} = 7$$

Add.

$$\boxed{} = \boxed{}$$

Divide each side by 3.

$$\boxed{} = \pm \boxed{}$$

Take the square root of each side.

Substitute $\sqrt{\frac{7}{3}}$ and $-\sqrt{\frac{7}{3}}$ in either of the original equations and solve for y .

$$x^2 + y^2 = 16$$

$$x^2 + y^2 = 16$$

Original equation

$$\left(\sqrt{\frac{7}{3}}\right)^2 + y^2 = 16$$

$$\left(-\sqrt{\frac{7}{3}}\right)^2 + y^2 = 16$$

Substitute for x .

$$y^2 = \boxed{}$$

$$y^2 = \boxed{}$$

Subtract from each side

$$y = \boxed{}$$

$$y = \boxed{}$$

Take the square root of each side.

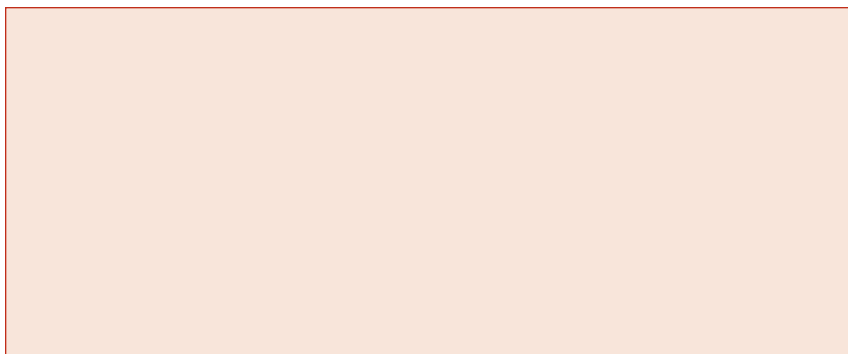
The solutions are

 $\boxed{}, \boxed{},$
 $\boxed{}, \text{ and } \boxed{}.$

Check Your Progress

Solve $x^2 + y^2 = 10$ and

$3x^2 + y^2 = 28$.



EXAMPLE

System of Quadratic Inequalities

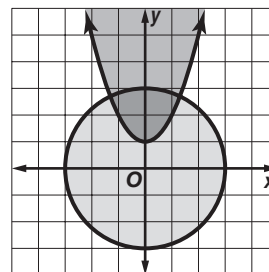
3 Solve the system of inequalities by graphing.

$y \geq x^2 + 1$
 $x^2 + y^2 \leq 9$

The graph of $y \geq x^2 + 1$ is the

$y = x^2 + 1$ and the

region inside and above it. Shade the region dark gray.



The graph of $x^2 + y^2 \leq 9$ is the

interior of $x^2 + y^2 = 9$.

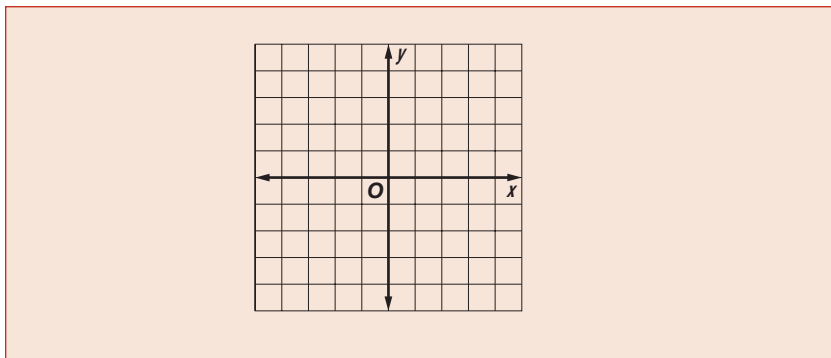
Shade the region medium gray.

The intersection of these regions, shaded darker gray, represents the solution of the system of inequalities.

Check Your Progress

Solve $y < -x^2 + 1$ and $x^2 + y^2 \leq 4$

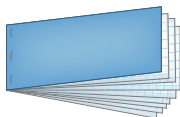
by graphing.



FOLDABLES™

ORGANIZE IT

On the page for Quadratic Systems, sketch five graphs of systems of quadratic equations. Write the number of solutions below each graph.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 10 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 10, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 283–284) to help you solve the puzzle.

10-1**Midpoint and Distance Formulas**

Consider the segment connecting the points $(-3, 5)$ and $(9, 11)$.

- Find the midpoint of this segment.
- Find the length of the segment. Write your answer in simplified radical form.

- Circle P has a diameter \overline{CD} . If C is at $(4, -3)$ and D is at $(-3, 5)$, find the center of the circle and the length of the diameter.

10-2**Parabolas**

Write each equation in standard form.

4. $y = 2x^2 - 8x + 1$

5. $y = -2x^2 + 6x + 1$

6. $y = \frac{1}{2}x^2 - 5x + 12$

10-3

Circles

7. Write the equation of the circle with center $(4, -3)$ and radius 5.

8. The circle with equation $(x + 8)^2 + y^2 = 121$ has center

 and radius .

9. a. In order to find the center and radius of the circle with equation

$$x^2 + y^2 + 4x - 6y - 3 = 0, \text{ it is necessary to } \text{[]}.$$

Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

$$x^2 + y^2 + 4x - 6y = \text{[]}$$

$$x^2 + 4x + \text{[]} + y^2 - 6y + \text{[]} = \text{[]} + \text{[]} + \text{[]}$$

$$\left(x + \text{[]}\right)^2 + \left(y - \text{[]}\right)^2 = \text{[]}$$

- b. This circle has radius 4 and center at .

10-4

Ellipses

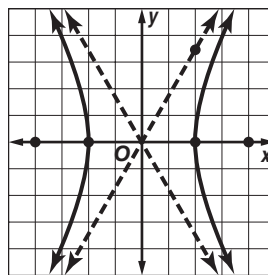
10. Complete the table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis	<input type="text"/>	<input type="text"/>
Direction of Minor Axis	<input type="text"/>	<input type="text"/>
Foci	<input type="text"/>	<input type="text"/>
Length of Major Axis	<input type="text"/>	<input type="text"/>
Length of Minor Axis	<input type="text"/>	<input type="text"/>

10-5

Hyperbolas

Study the hyperbola graphed at the right.



11. The center is .
12. The value of a is .
13. The value of c is .
14. To find b^2 , solve = + .
15. The equation in standard form for this hyperbola is .

10-6

Conic Sections

Name the conic section that is the graph of each equation. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.

- | | |
|--|--|
| <p>16. $\frac{(x - 3)^2}{36} + \frac{(y + 5)^2}{15} = 1$</p> <input type="text"/> | <p>17. $x = -2(y + 1)^2 + 7$</p> <input type="text"/> |
| <p>18. $(x - 5)^2 - (y + 5)^2 = 1$</p> <input type="text"/> | <p>19. $(x + 6)^2 + (y - 2)^2 = 1$</p> <input type="text"/> |

10-7

Solving Quadratic Systems

Draw a sketch to illustrate each of the following possibilities.

- | | | |
|--|--|--|
| <p>20. a parabola and a line that intersect in 2 points</p> <input type="text"/> | <p>21. an ellipse and a circle that intersect in 4 points</p> <input type="text"/> | <p>22. a hyperbola and a line that intersect in 1 point</p> <input type="text"/> |
|--|--|--|



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 10.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 10 Practice Test on page 615 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 10 Study Guide and Review on pages 609–614 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 615 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 10 Foldable.
- Then complete the Chapter 10 Study Guide and Review on pages 609–614 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 615 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Sequences and Series

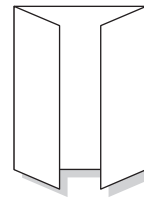


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" × 17" paper and four sheets of notebook paper.

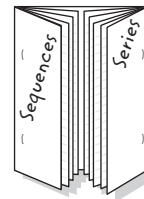
STEP 1

Fold the short sides of the 11" × 17" paper to meet in the middle.



STEP 2

Fold the notebook paper in half lengthwise. Insert two sheets of notebook paper in each tab and staple the edges. Label with lesson numbers. Take notes under the appropriate tabs.



NOTE-TAKING TIP: When you take notes, write questions you have about the lessons in the margin of your notes. Then include the answers to these questions as you work through the lesson.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 11. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
arithmetic means <u>AR-ihth-MEH-tihk</u>			
arithmetic sequence			
arithmetic series			
common difference			
common ratio			
factorial			
Fibonacci sequence <u>fih-buh-NAH-chee</u>			
geometric means			
geometric sequence			
geometric series			

Vocabulary Term	Found on Page	Definition	Description or Example
index of summation			
induction hypothesis			
infinite geometric series			
iteration IH-tuh-RAY-shuhn			
mathematical induction			
partial sum			
Pascal's triangle pas-KALZ			
recursive formula rih-KUHR-sihv			
sequence			
series			
sigma notation SIHG-muh			
term			

Arithmetic Sequence



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(A)** Represent patterns using arithmetic and geometric sequences and series. **(B)** Use arithmetic, geometric, and other sequences and series to solve real-life problems.

BUILD YOUR VOCABULARY (pages 312–313)

MAIN IDEAS

- Use arithmetic sequences.
- Find arithmetic means.

An arithmetic sequence is a sequence in which each after the first is found by a constant, called the **common difference** d , to the term.

EXAMPLE Find the Next Terms

KEY CONCEPT

n th Term of an Arithmetic Sequence The n th term a_n of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is any positive integer.

FOLDABLES On the tab for Lesson 11-1, write your own arithmetic sequence.

- 1** Find the next four terms of the arithmetic sequence $-8, -6, -4, \dots$

Find the common difference d by subtracting 2 consecutive terms.

$$-6 - (-8) = \boxed{} \text{ and } -4 - (-6) = \boxed{} \text{ So, } d = \boxed{}.$$

Now add 2 to the third term of the sequence and then continue adding 2 until the next four terms are found.

$$\begin{array}{cccc}
 -4 & \boxed{} & \boxed{} & \boxed{} & \boxed{} \\
 \swarrow & \nearrow & \swarrow & \nearrow & \swarrow & \nearrow \\
 & + 2 & + 2 & + 2 & + 2 &
 \end{array}$$

Check Your Progress

Find the next four terms of the arithmetic sequence $5, 3, 1, \dots$

EXAMPLE Find a Particular Term

- 2 CONSTRUCTION** The table below shows typical costs for a construction company to rent a crane for one, two, three, or four months. If the sequence continues, how much would it cost to rent the crane for 24 months?

Months	Costs(\$)
1	75,000
2	90,000
3	105,000
4	120,000

Explore Since the difference between any two successive costs is \$15,000, the costs form an arithmetic sequence with common difference .

Plan You can use the formula for the n th term of an arithmetic sequence with $a_1 = 75,000$ and $d = 15,000$ to find a_{24} the cost for 24 months.

Solve $a_n =$ Formula for the n th term

$$a_{24} = 75,000 + (24 - 1)(15,000) \quad n = 24, \\ a_1 = 75,000, \\ d = 15,000$$

$a_{24} =$ Simplify.

It would cost to rent the crane for 24 months.

Check Your Progress Refer to Example 2. How much would it cost to rent the crane for 8 months?

REVIEW IT

When finding the value of an expression, you must follow the order of operations. Briefly list the order of operations. (Lesson 1-1)

EXAMPLE Write an Equation for the n th Term

3 Write an equation for the n th term of the arithmetic sequence $-8, -6, -4, \dots$

In this sequence, $a_1 = -8$ and $d = 2$. Use the n th formula to write an equation.

$a_n = a_1 + (n - 1)d$ Formula for the n th term

$a_n =$ $a_1 = -8, d = 2$

$a_n =$ Distributive Property

$a_n =$ Simplify.

Check Your Progress

Write an equation for the n th term of the arithmetic sequence 5, 3, 1,

BUILD YOUR VOCABULARY (pages 312–313)

The terms between any two nonsuccessive terms of an sequence are called **arithmetic means**.

EXAMPLE**Find Arithmetic Means****4 Find the three arithmetic means between 21 and 45.**

You can use the n th term formula to find the common difference. In the sequence 21, , , , 45, . . . , $a_1 =$ and $a_5 =$.

$$a_n = a_1 + (n - 1)d \quad \text{Formula for the } n\text{th term}$$

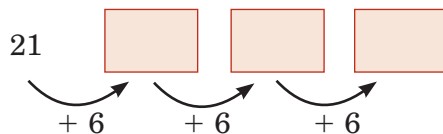
$$a_5 = \text{} \quad n = 5, a_1 = 21$$

$$\text{} = \text{} \quad a_5 = 45$$

$$\text{} = \text{} \quad \text{Subtract 21 from each side.}$$

$$\text{} = d \quad \text{Divide each side by 4.}$$

Now use the value of d to find the three arithmetic means.

**Check Your Progress**

Find the three arithmetic means between 13 and 25.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Arithmetic Series



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(A)** Represent patterns using arithmetic and geometric sequences and series. **(B)** Use arithmetic, geometric, and other sequences and series to solve real-life problems.

BUILD YOUR VOCABULARY (page 313)

MAIN IDEAS

- Find sums of arithmetic series.
- Use sigma notation.

A series is an indicated sum of the of a sequence. Since 18, 22, 26, 30 is an arithmetic sequence, $18 + 22 + 26 + 30$ is an arithmetic series.

EXAMPLE

Find the Sum of an Arithmetic Series

- 1** Find the sum of the first 20 even numbers, beginning with 2.

The series is $2 + 4 + 6 + \dots + 40$. Since $a_1 = \text{$, $a_{20} = \text{$, and $d = \text{$, you can use either sum formula for this series.

Method 1

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Sum formula

$$S_{20} = \frac{20}{2}(2 + 40)$$

$$n = 20, a_1 = 2, a_{20} = 40$$

$$S_{20} = \text{$$

Simplify.

$$S_{20} = \text{$$

Multiply.

Method 2

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

Sum formula

$$S_2 = \frac{20}{2}[2(2) + (20 - 1)2]$$

$$n = 20, a_1 = 2, d = 2$$

$$S_2 = \text{$$

Simplify.

$$S_2 = \text{$$

Multiply.

The sum of the first 20 even numbers is .

Check Your Progress

Find the sum of the first 15 counting numbers, beginning with 1.

KEY CONCEPT

Sum of an Arithmetic Series The sum S_n of the first n terms of an arithmetic series is given by

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d] \text{ or}$$

$$S_n = \frac{n}{2}(a_1 + a_n).$$

EXAMPLE Find the First Term

- 2 RADIO** A radio station is giving away money every day in the month of September for a total of \$124,000. They plan to increase the amount of money given away by \$100 each day. How much should they give away on the first day of September, rounded to the nearest cent?

You know the values of n , S_n , and d . Use the sum formula that contains d .

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d] \quad \text{Sum formula}$$

$$S_{30} = \frac{30}{2}[2a_1 + (30 - 1)100] \quad n = 30, d = 100$$

$$\boxed{} = 15(2a_1 + 2900) \quad S_{30} = 124,000$$

$$124,000 = \boxed{} + \boxed{} \quad \text{Distributive Property}$$

$$80,500 = 30a_1 \quad \text{Subtract 43,500 from each side.}$$

$$\boxed{} = a_1 \quad \text{Divide each side by 30.}$$

They should give away $\boxed{}$ the first day.

Check Your Progress

A television game show gives contestants a chance to win a total of \$1,000,000 by answering 16 consecutive questions correctly. If the value from question to question increases by \$5,000, how much is the first question worth?

EXAMPLE Find the First Three Terms

- 3** Find the first four terms of an arithmetic series in which $a_1 = 14$, $a_n = 29$, and $S_n = 129$.

Step 1 Since you know a_1 , a_n , and S_n , use

$$S_n = \frac{n}{2}(a_1 + a_n) \text{ to find } n.$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

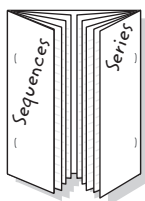
$$\boxed{} = \frac{n}{2} \boxed{}$$

$$\boxed{} = \boxed{}$$

$$\boxed{} = n$$

FOLDABLES**ORGANIZE IT**

Use the tab for Lesson 11-2. Write an example of an arithmetic sequence and an arithmetic series. Then explain the difference between the two.



Step 2 Find d .

$$a_n = a_1 + (n - 1)d$$

$$\boxed{} = \boxed{} + \boxed{}d$$

$$\boxed{} = \boxed{}$$

$$\boxed{} = d$$

Step 3 Use d to determine a_2 , a_3 , and a_4 .

$$a_2 = \boxed{} + \boxed{} \text{ or } \boxed{}$$

$$a_3 = \boxed{} + \boxed{} \text{ or } \boxed{}$$

$$a_4 = \boxed{} + \boxed{} \text{ or } \boxed{}$$

The first four terms are $\boxed{}$, $\boxed{}$, $\boxed{}$, and $\boxed{}$.

Check Your Progress

Find the first three terms of an arithmetic series in which $a_1 = 11$, $a_n = 31$ and $S_n = 105$.

BUILD YOUR VOCABULARY (page 313)

A concise notation for writing out a series is called **sigma notation**. The series $3 + 6 + 9 + 12 + \dots + 30$ can be expressed as $\sum_{n=1}^{10} 3n$. The variable, in this case n , is called the **index of summation**.

$$\begin{array}{c} \text{last value of } n \rightarrow 10 \\ \sum_{n=1}^{10} 3n \\ \uparrow \\ \text{first value of } n \end{array}$$

formula for the terms of the series

EXAMPLE

Evaluate a Sum in Sigma Notation

4 Evaluate $\sum_{k=3}^{10} (2k + 1)$.

Method 1 Find the terms by replacing k with 3, 4, ..., 10. Then add.

$$\begin{aligned} & \sum_{k=3}^{10} (2k + 1) \\ &= [2(3) + 1] + [2(4) + 1] + [2(5) + 1] + [2(6) + 1] + \\ & \quad \boxed{} + \boxed{} + \\ & \quad \boxed{} + \boxed{} \\ &= \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \\ & \quad \boxed{} + \boxed{} + \boxed{} \\ &= \boxed{} \end{aligned}$$

Method 2 Since the sum is an arithmetic series, use the formula $S_n = \frac{n}{2}(a_1 + a_n)$. There are $\boxed{}$ terms, $a_1 = 2(\boxed{}) + 1$ or $\boxed{}$, and $a_8 = 2(\boxed{}) + 1$ or $\boxed{}$.

$$S_n = \boxed{} = \boxed{} \text{ or } \boxed{}$$

Check Your Progress Evaluate $\sum_{i=5}^{10} (2i + 3)$.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Geometric Sequences



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(A)** Represent patterns using arithmetic and geometric sequences and series. **(B)** Use arithmetic, geometric, and other sequences and series to solve real-life problems.

MAIN IDEAS

- Use geometric sequences.
- Find geometric means.

BUILD YOUR VOCABULARY (pages 312–313)

A **geometric sequence** is a sequence in which each term after the first is found by multiplying the previous term by a constant r called the **common ratio**.

The missing term(s) between two nonsuccessive terms of a geometric sequence are called **geometric means**.

EXAMPLE Find the Next Term

1 TEST EXAMPLE What is the missing term in the geometric sequence 324, 108, 36, 12, ___?

- A 972 B 4 C 0 D -12

Read the Test Item

Since $\frac{108}{324} = \frac{1}{3}$, $\frac{36}{108} = \frac{1}{3}$, and $\frac{12}{36} = \frac{1}{3}$, the sequence has the common ratio of .

Solve the Test Item

Find the missing term, multiply the last given term by $\frac{1}{3}$:

$$12\left(\frac{1}{3}\right) = \text{}. \text{ The answer is } \text{}.$$

Check Your Progress

Find the missing term in the geometric sequence 100, 50, 25, ___.

- A 200 B 0 C 12.5 D -12.5

KEY CONCEPT

n th Term of a Geometric Sequence The n th term a_n of a geometric sequence with first term a_1 and common ratio r is given by $a_n = a_1 \cdot r^{n-1}$, where n is any positive integer.

EXAMPLE Find a Term Given the First Term and the Ratio

2 Find the sixth term of a geometric sequence for which $a_1 = -3$ and $r = -2$.

$$a_n = a_1 \cdot r^{n-1}$$

$$a_6 = \text{} \cdot \text{}$$

$$a_6 = \text{} \cdot \text{}$$

$$a_6 = \text{}$$

Formula for the n th term

$$n = 6, a_1 = -3, r = -2$$

$$(-2^5) = -32$$

Multiply.

Check Your Progress

Find the fifth term of a geometric sequence for which $a_1 = 6$ and $r = 2$.

EXAMPLE**Write an Equation for the n th Term**

- 3** Write an equation for the n th term of the geometric sequence 5, 10, 20, 40,

$$a_n = a_1 r^{n-1}$$

Formula for the n th term

$$a_n = \boxed{} \left(\boxed{} \right)^{n-1}$$

$$a_1 = \boxed{}, r = \boxed{}$$

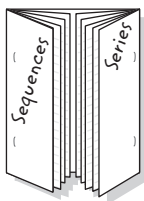
An equation is .

Check Your Progress

Write an equation for the n th term of the geometric sequence 2, 6, 18, 54,

FOLDABLES™**ORGANIZE IT**

Use the tab for Lesson 11-3. Write your own example of an arithmetic sequence and a geometric sequence. Then explain the difference between the two.

**EXAMPLE****Find a Term Given One Term and the Ratio**

- 4** Find the seventh term of a geometric sequence for which $a_3 = 96$ and $r = 2$.

First find the value of a_1 .

$$a_n = a_1 r^{n-1}$$

$$\boxed{} = \boxed{}$$

$$\boxed{} = a_1 (2)^2$$

$$\boxed{} = a_1$$

Now find a_7 .

$$a_n = a_1 r^{n-1}$$

$$a_7 = \boxed{}$$

$$a_7 = \boxed{}$$

The seventh term is .

Check Your Progress

Find the sixth term of a geometric sequence for which $a_4 = 27$ and $r = 3$.

EXAMPLE Find Geometric Means**5** Find three geometric means between 3.12 and 49.92.

In the sequence a_1 is 3.12 and a_5 is 49.92.

$$a_n = a_1 r^{n-1} \quad \text{Formula for the } n\text{th term}$$

$$a_5 = \boxed{} \quad n = 5, a_1 = 3.12$$

$$\boxed{} = \boxed{} \quad a_5 = 49.92$$

$$\boxed{} = r^4 \quad \text{Divide by 3.12.}$$

$$\boxed{} = r \quad \text{Take the fourth root of each side.}$$

Use each value of r to find three geometric means.

$$r = 2$$

$$r = -2$$

$$a_2 = 3.12(2) \text{ or } \boxed{}$$

$$a_2 = 3.12(-2) \text{ or } \boxed{}$$

$$a_3 = 6.24(2) \text{ or } \boxed{}$$

$$a_3 = -6.24(-2) \text{ or } \boxed{}$$

$$a_4 = 12.48(2) \text{ or } \boxed{}$$

$$a_4 = 12.48(-2) \text{ or } \boxed{}$$

The geometric means are $\boxed{}$, $\boxed{}$, and $\boxed{}$
or $\boxed{}$, $\boxed{}$, and $\boxed{}$.

Check Your Progress

Find three geometric means between 12 and 0.75.

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____

Geometric Series



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(A)** Represent patterns using arithmetic and geometric sequences and series. **(B)** Use arithmetic, geometric, and other sequences and series to solve real-life problems.

EXAMPLE

Find the Sum of the First n Terms

MAIN IDEAS

- Find sums of geometric series.
- Find specific terms of geometric series.

KEY CONCEPT

Sum of a Geometric Series The sum S_n of the first n terms of a geometric series is given by $S_n = \frac{a_1 - a_1 r^n}{1 - r}$ or $S_n = \frac{a_1(1 - r^n)}{1 - r}$, where $r \neq 1$.

- 1 HEALTH** Contagious disease can spread very quickly. Suppose five people are ill during the first week of an epidemic, and each person who is ill spreads the disease to four people by the end of the week. By the end of the sixth week of the epidemic, how many people have been affected by the illness?

This is a geometric series with $a_1 = 5$, $r = 4$, and $n = 6$.

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad \text{Sum formula}$$

$$= \boxed{}$$

$$n = 6, a_1 = 5, r = 4$$

$$= \boxed{}$$

Use a calculator.

After 6 weeks, $\boxed{}$ people have been affected.

EXAMPLE

Evaluate a Sum Written in Sigma Notation

- 2** Evaluate $\sum_{n=1}^{12} 3 \cdot 2^{n-1}$.

The sum is a geometric series.

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad \text{Sum formula}$$

$$S_{12} = \boxed{}$$

$$n = 12, a_1 = 3, r = 2$$

$$S_{12} = \frac{3(4095)}{1}$$

$$2^{12} = 4096$$

$$S_{12} = \boxed{}$$

Simplify.

REMEMBER IT



The sum in Example 2 can also be found by using

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

Check Your Progress

- a. How many direct ancestors would a person have after 7 generations?

b. Evaluate $\sum_{n=1}^4 5 \cdot 3^{n-1}$.

EXAMPLE Use the Alternate Formula for a Sum

- 3 Find the sum of a geometric series for which $a_1 = 7776$, $a_n = 6$, and $r = -\frac{1}{6}$.

Since you do not know the value of n , use $S_n = \frac{a_1 - a_n r}{1 - r}$.

$S_n = \frac{a_1 - a_n r}{1 - r}$ Alternate sum formula

$= \frac{\text{[]}}{\text{[]}}$

$a_1 = 7776$, $a_n = 6$, and $r = -\frac{1}{6}$

$1 - \text{[]}$

$= \frac{7777}{7} \text{ or } \text{[]}$

Simplify.

Check Your Progress Find the sum of a geometric series for which $a_1 = 64$, $a_n = 729$, and $r = -1.5$.

EXAMPLE Find the First Term of a Series

- 4 Find a_1 in a geometric series for which $S_8 = 765$ and $r = 2$.

$S_n = \frac{a_1(1 - r^n)}{1 - r}$ Sum formula

$\text{[]} = \text{[]}$

$S_8 = 765$, $r = 2$, and $n = 8$

$765 = 255a_1$

Simplify.

$\text{[]} = a_1$

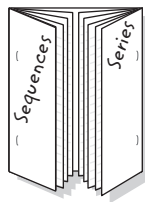
Divide each side by 255.

Check Your Progress Find a_1 in a geometric series for which $S_6 = 364$ and $r = 3$.

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-4. Write in words the sigma notation that is used in Example 2.



HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Infinite Geometric Series



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(C)** Describe limits of sequences and apply their properties to investigate convergent and divergent series.

BUILD YOUR VOCABULARY (page 312)

MAIN IDEAS

- Find the sum of an infinite geometric series.
- Write repeating decimals as fractions.

If a geometric series has no last term, it is called an **infinite geometric series**. For infinite series, S_n is called a **partial sum** of the series.

EXAMPLE

Sum of an Infinite Geometric Series

KEY CONCEPT

Sum of an Infinite Geometric Series The sum S of an infinite geometric series with $-1 < r < 1$ is given by $S = \frac{a_1}{1-r}$.

- 1 Find the sum of each infinite geometric series, if it exists.

a. $-\frac{4}{3} + 4 - 12 + 36 - 108 + \dots$

Find the value of r to determine if the sum exists.

$a_1 = -\frac{4}{3}$ and $a_2 = 4$, so $r =$ or .

Since $|-3| \geq 1$, the sum exist.

b. $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \dots$

$a_1 =$ and $a_2 =$, so $r =$ or $-\frac{1}{2}$.

Since , the sum .

Use the formula for the sum of an infinite geometric series.

$S = \frac{a_1}{1-r}$ Sum formula

$= \frac{3}{1 - \left(-\frac{1}{2}\right)}$ $a_1 = 3, r = -\frac{1}{2}$

$= \frac{\text{}}{\text{}}$ or Simplify.

Check Your Progress

Find the sum of each infinite geometric series, if it exists.

a. $2 + 4 + 8 + 16 + \dots$

b. $1 + \frac{1}{2} + \frac{1}{4} + \dots$

EXAMPLE Infinite Series in Sigma Notation

2 Evaluate $\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$.

In this infinite geometric series, $a_1 = 5$ and $r = \frac{1}{2}$.

$$S = \frac{a_1}{1-r} \quad \text{Sum formula}$$

$$= \frac{5}{1-\frac{1}{2}} \quad a_1 = 5, r = \frac{1}{2}$$

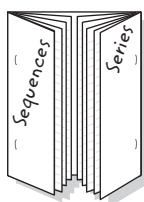
= or Simplify.

Check Your Progress Evaluate $\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$.

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 11-5, write $0.\overline{54}$ as a fraction. Use either of the methods shown in Example 3 of your textbook.



EXAMPLE Write a Repeating Decimal a Fraction

3 Write $0.\overline{25}$ as a fraction.

$S = 0.\overline{25}$ Label the given decimal.

$S = 0.25252525\dots$ Repeating decimal

$100S =$ Multiply each side by 100.

$99S =$ Subtract the second equation from the third.

$S =$ Divide each side by 99.

Check Your Progress Write $0.\overline{37}$ a fraction.

HOMEWORK ASSIGNMENT

Page(s): _____

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Recursion and Special Sequences



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(B)** Use arithmetic, geometric, and other sequences and series to solve real-life problems.

BUILD YOUR VOCABULARY (pages 312–313)

MAIN IDEAS

- Recognize and use special sequences.
- Iterate functions.

The sequence 1, 1, 2, 3, 5, 8, 13, ... , where each term in the sequence after the second is the sum of the two previous terms, is called the **Fibonacci sequence**.

The formula $a_n = a_{n-2} + a_{n-1}$ is an example of a **recursive formula**.

EXAMPLE Use a Recursive Formula

- 1** Find the first five terms of the sequence in which $a_1 = 5$ and $a_{n+1} = 2a_n + 7$, $n \geq 1$.

$$a_{n+1} = 2a_n + 7 \quad \text{Recursive formula}$$

$$a_{1+1} = 2a_1 + 7 \quad n = 1$$

$$a_2 = 2 \left(\boxed{} \right) + 7 \text{ or } \boxed{} \quad a_1 = 5$$

$$a_{2+1} = 2a_2 + 7 \quad n = 2$$

$$a_3 = 2 \left(\boxed{} \right) + 7 \text{ or } \boxed{} \quad a_2 = 17$$

$$a_{3+1} = 2a_3 + 7 \quad n = 3$$

$$a_4 = 2 \left(\boxed{} \right) + 7 \text{ or } \boxed{} \quad a_3 = 41$$

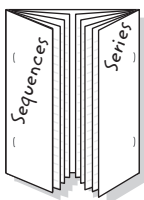
$$a_{4+1} = 2a_4 + 7 \quad n = 4$$

$$a_5 = 2 \left(\boxed{} \right) + 7 \text{ or } \boxed{} \quad a_4 = 89$$

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-6. Describe the pattern in the sequence 1, 2, 6, 24, 120, Then find the next three terms of the sequence.



Check Your Progress Find the first five terms of the sequence in which $a_1 = 2$ and $a_{n+1} = 3a_n + 2$, $n \geq 1$.

BUILD YOUR VOCABULARY (page 313)

Iteration is the process of composing a with itself repeatedly.

EXAMPLE Iterate a Function

- 3** Find the first three iterates x_1 , x_2 , and x_3 of the function $f(x) = 3x - 1$ for an initial value of $x_0 = 5$.

To find the first iterate x_1 , find the value of the function for $x_0 = 5$.

$$\begin{aligned} x_1 &= f(x_0) && \text{Iterate the function.} \\ &= f(\text{ }) && x_0 = 5 \\ &= \text{ } \text{ or } \text{ } && \text{Simplify.} \end{aligned}$$

To find the second iterate x_2 , substitute x_1 for x .

$$\begin{aligned} x_2 &= f(x_1) && \text{Iterate the function.} \\ &= f(\text{ }) && x_1 = 14 \\ &= \text{ } \text{ or } \text{ } && \text{Simplify.} \end{aligned}$$

Substitute x_2 for x to find the third iterate.

$$\begin{aligned} x_3 &= f(x_2) && \text{Iterate the function.} \\ &= f(\text{ }) && x_2 = 41 \\ &= \text{ } \text{ or } \text{ } && \text{Simplify.} \end{aligned}$$

Check Your Progress

Find the first three iterates x_1 , x_2 , and x_3 of the function $f(x) = 2x + 1$ for an initial value of $x_0 = 2$.

REVIEW IT

If $f(x) = 4x + 10$ and $g(x) = 2x^2 - 5$, find $f(g(x))$. (Lesson 7-7)

HOMEWORK ASSIGNMENT

Page(s):

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The Binomial Theorem



Preparation for TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(D)** Apply sequences and series to solve problems including sums and binomial expansion.

BUILD YOUR VOCABULARY (pages 312–313)

MAIN IDEAS

- Use Pascal's triangle to expand powers of binomials.
- Use the Binomial Theorem to expand powers of binomials.

The in powers of form a pattern that is often displayed in a triangular formation known as **Pascal's triangle**.

The factors in the coefficients of binomial involve special products called **factorials**.

EXAMPLE Use Pascal's Triangle

1 Expand $(p + q)^5$.

Write the row of Pascal's triangle corresponding to $n = 5$.

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Use the patterns of a binomial expansion and the coefficients to write the expansion of $(p + q)^5$.

$$(p + q)^5$$

$$= \text{$$

$$= \text{$$

EXAMPLE Use the Binomial Theorem

2 Expand $(t - s)^8$.

The expression will have terms. Use the sequence $1, \frac{8}{1}, \frac{8 \cdot 7}{1 \cdot 2}, \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}, \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}$ to find the coefficients for the first five terms. Use symmetry to find the remaining coefficients.

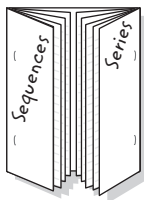
$$(t - s)^8 = 1t^8(-s)^0 + \frac{8}{1}t^7(-s) + \frac{8 \cdot 7}{1 \cdot 2}t^6(-s)^2 + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}t^5(-s)^3 + \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}t^4(-s)^4 + \dots + 1t^0(-s)^8$$

$$= \text{$$

FOLDABLES™

ORGANIZE IT

Use the tab for Lesson 11-7. Refer to Example 1, and describe what happens to the exponents for p as the exponents for q increase.



KEY CONCEPT

Binomial Theorem If n is a nonnegative integer, then

$$(a + b)^n = 1a^n b^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \dots + 1a^0b^n.$$

Check Your Progress

a. Expand $(x + y)^6$.

b. Expand $(x - y)^4$.

EXAMPLE Factorials

3 Evaluate $\frac{6!}{2!4!}$.

$$\frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

Note that $6! = 6 \cdot 5 \cdot 4!$, so

$$\frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot 4!}{2!4!} \text{ or } \frac{6 \cdot 5}{2 \cdot 1}$$

$$= \boxed{} \text{ or } \boxed{}$$

Check Your Progress

Evaluate $\frac{7!}{2!3!}$.

KEY CONCEPT

Binomial Theorem, Factored Form

$$\begin{aligned} (a + b)^n &= \frac{n!}{n!0!} a^n b^0 + \\ &\frac{n!}{(n-1)!1!} a^{n-1} b^1 + \\ &\frac{n!}{(n-2)!2!} a^{n-2} b^2 + \\ &\dots + \frac{n!}{n!0!} a^0 b^n + \\ &\sum_{k=0}^n \frac{n!}{(n-k)!k!} a^{n-k} b^k \end{aligned}$$

EXAMPLE Use a Factorial Form for the Binomial Theorem

4 Expand $(3x - y)^4$.

$$(3x - y)^4 = \boxed{}$$

$$= \frac{4!}{4!0!} (3x)^4 (-y)^0 + \frac{4!}{3!1!} (3x)^3 (-y)^1 + \frac{4!}{2!2!} (3x)^2 (-y)^2 +$$

$$\boxed{} + \boxed{}$$

$$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} (3x)^4 + \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} (3x)^3 (-y) +$$

$$\frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} (3x)^2 y^2 + \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1} (3x)^1 (-y^3) +$$

$$\boxed{}$$

$$= \boxed{}$$

Check Your ProgressExpand $(2x + y)^4$.
EXAMPLE**Find a Particular Term****5 Find the fourth term in the expansion of $(a + 3b)^4$.**

First, use the Binomial Theorem to write the expression in sigma notation.

$$(a + 3b)^4 = \sum_{k=0}^4 \frac{4!}{(4-k)!k!} a^{4-k} (3b)^k$$

In the fourth term, $k =$.

$$\frac{4!}{(4-k)!k!} a^{4-k} (3b)^k = \frac{4!}{(4-3)!3!} a^{4-3} (3b)^3 \quad k = 3$$

$$=$$

$$=$$

$$\frac{4!}{1!3!} =$$

Simplify.

Check Your Progress

Find the fifth term in the expansion of $(x + 2y)^6$.

**HOMEWORK
ASSIGNMENT**

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Exercises: _____

Proof and Mathematical Induction



Reinforcement of TEKS G.3 The student applies logical reasoning to justify and prove mathematical statements. **(D)** Use inductive reasoning to formulate a conjecture.

BUILD YOUR VOCABULARY (page 313)

MAIN IDEAS

- Prove statements using mathematical induction.
- Disprove statements by finding a counterexample.

Mathematical induction is used to statements about integers.

EXAMPLE Summation Formula

1 Prove that $1 + 3 + 5 + \dots + (2n - 1) = n^2$.

Step 1 When $n = 1$, the left side of the given equation is - or . The right side is or . Thus, the equation is true for $n = 1$.

Step 2 Assume $1 + 3 + 5 + \dots + (2k - 1) = k^2$ for a positive integer k .

Step 3 Show that the given equation is true for

$$n = \text{}.$$

$$1 + 3 + 5 + \dots + (2k - 1) + (2(k + 1) - 1)$$

$$= \text{} \quad \text{Add } (2(k + 1) - 1) \text{ to each side.}$$

$$= \text{} \quad \text{Add.}$$

$$= \text{} \quad \text{Simplify.}$$

$$= \text{} \quad \text{Factor.}$$

The last expression is the right side of the equation to be proved, where n has been replaced by $k + 1$. Thus, the equation is true for $n = k + 1$.

This proves that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ is true for all positive integers n .

KEY CONCEPT

Mathematical Induction

Step 1 Show that the statement is true for some integer n .

Step 2 Assume that the statement is true for some positive integer k , where $k \geq n$. This assumption is called the **inductive hypothesis**.

Step 3 Show that the statement is true for the next integer $k + 1$.

EXAMPLE Divisibility

2 Prove that $6n - 1$ is divisible by 5 for all positive integers n .

Step 1 When $n = 1$, $6^n - 1 = \boxed{}$ or $\boxed{}$. Since $\boxed{}$ is divisible by 5, the statement is true for $n = 1$.

Step 2 Assume that $6^k - 1$ is divisible by 5 for some positive integer k . This means that there is a whole number r such that $\boxed{}$.

Step 3 Show that the statement is true for $n = k + 1$.

$$\boxed{} = \boxed{} \quad \text{Inductive hypothesis}$$

$$6^k = \boxed{} \quad \text{Add 1 to each side.}$$

$$6 \cdot 6^k = 6 \cdot \boxed{} \quad \text{Multiply each side by 6.}$$

$$6^{k+1} = \boxed{} \quad \text{Simplify.}$$

$$6^{k+1} - 1 = \boxed{} \quad \text{Subtract 1 from each side.}$$

$$6^{k+1} - 1 = \boxed{} \quad \text{Factor.}$$

Since r is a whole number, $6r + 1$ is a whole number.

Therefore, $\boxed{}$.

Thus, the statement is true for $\boxed{}$. This proves that $6^n - 1$ is divisible by 5.

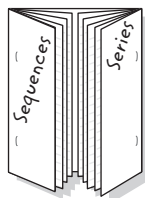
Check Your Progress

a. Prove that $2 + 4 + 6 + 8 + \dots + 2n = n(n + 1)$.

b. Prove that $10^n - 1$ is divisible by 9 for all positive integers n .

EXAMPLE Counterexample**FOLDABLES™****ORGANIZE IT**

On the tab for Lesson 11-8, write a real-world statement. Then find a counterexample to your statement.



- 3 Find a counterexample for the statement that $n^2 + n + 5$ is always a prime number for any positive integer n .

Check the first few positive integers.

n	Formula	Prime?
1	<input type="text"/>	<input type="text"/>
2	<input type="text"/>	<input type="text"/>
3	<input type="text"/>	<input type="text"/>
4	<input type="text"/>	<input type="text"/>

The value $n =$ is a counterexample for the formula.

Check Your Progress

Find a counterexample for the formula that $2n - 1$ is always a prime number for any positive integer n .

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

BRINGING IT ALL TOGETHER**STUDY GUIDE**

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 11 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 11, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 312–313) to help you solve the puzzle.

11-1

Arithmetic Sequences

1. Find the next four terms of the arithmetic sequence 3, 6, 9, 12,

2. Find the first five terms of the arithmetic sequence in which $a_1 = 2$ and $d = 9$.

3. Write an equation for the n th term of the arithmetic sequence 10, 6, 2, -2 ,

11-2

Arithmetic Series

4. Find S_n for the arithmetic series in which $a_1 = -6$, $n = 18$, and $d = 2$.

5. Find the sum of the arithmetic series $30 + 25 + \dots + (-10)$.

Find the sum of each arithmetic series.

6. $\sum_{j=3}^9 (6 - j)$

7. $\sum_{k=10}^{25} (2k + 1)$

11-3

Geometric Sequences

8. In the sequence 5, 8, 11, 14, 17, 20, the numbers 8, 11, 14, and 17 are between 5 and 20.
9. In the sequence 12, 4, $\frac{4}{3}$, $\frac{4}{9}$, $\frac{4}{27}$, the numbers 4, $\frac{4}{3}$, and $\frac{4}{9}$ are between 12 and $\frac{4}{27}$.
10. Find three geometric means between 4 and 324.

11-4

Geometric Series

11. Consider the formula $S_n = \frac{a_1(1 - r^n)}{1 - r}$. Suppose that you want to use the formula to evaluate the sum $\sum_{n=1}^6 8(-2)^{n-1}$. Indicate the values you would substitute into the formula in order to find S_n .

$$n = \text{} \quad a_1 = \text{} \quad r = \text{} \quad r^n = \text{}$$

12. Find the sum of a geometric series for which $a_1 = 5$, $n = 9$, and $r = 3$.

11-5

Infinite Geometric Series

13. Consider the formula $S = \frac{a_1}{1 - r}$. For what values of r does an infinite geometric sequence have a sum?
14. For the geometric series $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$, give the values of a_1 and r . Then state whether the sum of the series exists.

11-6

Recursion and Special Sequences

15. Find the first five terms of the sequence in which $a_1 = -3$ and $a_{n+1} = 2a_n + 5$.

16. Find the first three iterates of $f(x) = 3x - 2$ for an initial value of $x_0 = 4$.

11-7

The Binomial Theorem

Consider the expansion of $(w + z)^5$.

17. How many terms does this expansion have?
18. In the second term of the expansion, what is the exponent of w ?
19. In the fourth term of the expansion, what is the exponent of w ?
20. What is the last term of this expansion?

11-8

Proof and Mathematical Induction

Suppose that you wanted to prove that the following statement is true for all positive integers.

$$3 + 6 + 9 + \dots + 3n = \frac{3n(n + 1)}{2}$$

21. Which statement shows that the statement is true for $n = 1$?

i. $3 = \frac{3 \cdot 2 + 1}{2}$	ii. $3 = \frac{3 \cdot 1 \cdot 2}{2}$	iii. $\frac{3 + 1 + 2}{2}$
----------------------------------	---------------------------------------	----------------------------



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 11.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 11 Practice Test on page 679 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 11 Study Guide and Review on pages 674–678 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 679 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 11 Foldable.
- Then complete the Chapter 11 Study Guide and Review on pages 674–678 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 679 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Probability and Statistics

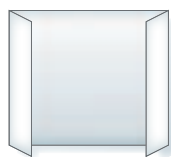


Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" × 17" paper.

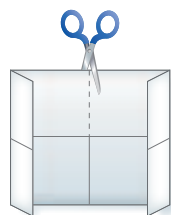
STEP 1

Fold 2" tabs on each of the short sides.



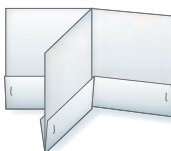
STEP 2

Fold in half in both directions. Open and cut as shown.



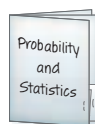
STEP 3

Refold along the width. Staple each pocket.



STEP 4

Label pockets as *The Counting Principle*, *Permutations and Combinations*, *Probability*, and *Statistics*.



NOTE-TAKING TIP: When you take notes, look for written real-world examples in your everyday life. Comment on how writers use statistics to prove or disprove points of view and discuss the ethical responsibilities writers have when using statistics.

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 12. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
binomial experiment			
combination			
compound event			
dependent and independent events			
event			
<u>inclusive events</u> ihn-KLOO-sihv			
measure of variation			
<u>mutually exclusive events</u> MYOO-chuh-lee			
normal distribution			
outcome			
<u>permutation</u> PUHR-myoo-TAY-shuhn			

(continued on the next page)

Vocabulary Term	Found on Page	Definition	Description or Example
probability			
probability distribution			
random			
random variable			
relative-frequency histogram			
sample space			
simple event			
standard deviation			
unbiased sample			
uniform distribution			
univariate data			
variance VEHR-ee-uhn(t)s			

The Counting Principle



Reinforcement of TEKS 7.10 The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events. **(A)** Construct sample spaces for simple or composite experiments.

BUILD YOUR VOCABULARY (pages 341–342)

MAIN IDEAS

- Solve problems involving independent events.
- Solve problems involving dependent events.

The set of all possible is called the **sample space**.

An **event** consists of one or more outcomes of a trial. Events that do not affect each other are called **independent events**.

EXAMPLE

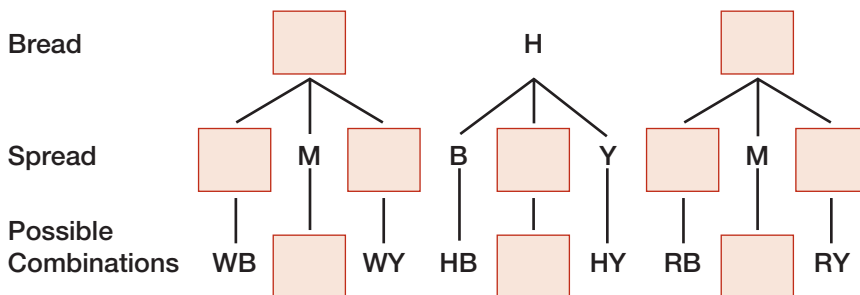
Independent Events

- 1 FOOD** A sandwich menu offers customers a choice of white, wheat, or rye bread with one spread chosen from butter, mustard, or mayonnaise. How many different combinations of bread and spread are possible?

First note that the choice of the type of bread does not affect the choice of the type of spread, so these events are independent.

Method 1 Tree Diagram

W represents white, H, wheat, R, rye, B, butter, M, mustard, Y, mayonnaise.



Method 2 Make a Table

Make a table in which each row represents a type of bread and each column represents a type of spread.

	Butter	Mustard	Mayonnaise
White	<input type="text"/>	WM	<input type="text"/>
Wheat	HB	<input type="text"/>	HY
Rye	<input type="text"/>	RM	<input type="text"/>

There are possible outcomes.

KEY CONCEPT

Fundamental Counting Principle If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Check Your Progress

A pizza place offers customers a choice of American, mozzarella, Swiss, feta, or provolone cheese with one topping chosen from pepperoni, mushrooms, or sausage. How many different combinations of cheese and toppings are there?

EXAMPLE**Fundamental Counting Principle**

- 2 TEST EXAMPLE** The Murray family is choosing from a trip to the beach or a trip to the mountains. The family can select transportation from a car, plane, or train. How many different ways can they select a destination followed by a means of transportation?

A 2 B 5 C 6 D 9

There are · or ways to choose a trip.

The answer is .

Check Your Progress

For their vacation, the Esper family is going on a trip. They can select their transportation from a car, plane, or train. They can also select from 4 different hotels. How many different ways can they select a means of transportation followed by a hotel?

A 8 B 12 C 16 D 7

EXAMPLE**More than Two Independent Events**

- 3 COMMUNICATION** How many answering machine codes are possible if the code is just 2-digits?

The choice of any digit does not affect the other digit, so the choices of digits are independent events.

There are possible choices for the first digit and

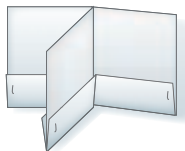
possible choices for the second digit.

So, there are · or possible different codes.

FOLDABLES™

ORGANIZE IT

Write and solve your own examples of independent and dependent events. Place your work in The Counting Principle pocket.



Check Your Progress Many automated teller machines (ATM) require a 4-digit code to access an account. How many codes are possible?

BUILD YOUR VOCABULARY (page 341)

With dependent events, the outcome of one event

affect the outcome of another event.

EXAMPLE Dependent Events

4 SCHOOL Refer to the table in Example 4 of your textbook. How many different schedules could a student who is planning to take only four different classes have?

The choices of which class to schedule each period are dependent events.

There are 4 classes that can be taken during the first period. That leaves 3 classes for the second period, 2 classes for the third period, and so on.

Period	1st	2nd	3rd	4th
Number of Choices	<div style="border: 1px solid black; width: 40px; height: 30px;"></div>	<div style="border: 1px solid black; width: 40px; height: 30px;"></div>	<div style="border: 1px solid black; width: 40px; height: 30px;"></div>	<div style="border: 1px solid black; width: 40px; height: 30px;"></div>

There are $\square \cdot \square \cdot \square \cdot \square$ or \square different schedules for a student who is taking 4 classes.

Check Your Progress How many different schedules could a student have who is planning to take 5 different classes?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Check Your Progress

- a. Ten people are competing in a swim race where 4 ribbons will be given. How many ways can blue, red, green, and yellow ribbons be awarded?

- b. How many different ways can the letters of the word ALGEBRA be arranged?

KEY CONCEPTS

Combinations The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$.

FOLDABLES Write a real-world example of a permutation and a combination. Place your work in the Permutations and Combinations pocket.

BUILD YOUR VOCABULARY (page 341)

An arrangement or selection of objects in which order is *not* important is called a **combination**.

EXAMPLE Combination

- 3** Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people who will go?

Since the order they choose the three people is not important, you must find the number of combinations of 5 cousins taken 3 at a time.

$$C(n, r) = \frac{n!}{(n-r)!r!} \quad \text{Combination formula}$$

$$C(5, 3) = \boxed{} \quad n = 5 \text{ and } r = 3$$

$$= \boxed{} \text{ or } \boxed{} \quad \text{Simplify.}$$

There are $\boxed{}$ ways to choose the three cousins.

EXAMPLE Multiple Events

- 4 Six cards are drawn from a standard deck of cards. How many hands consist of two hearts and four spades?

Multiply the number of ways to select two hearts and the number of ways to select four spades. Only the cards in the hand matter, not the order in which they were drawn, so use

$$\boxed{} \cdot $$

$$C(13, 2) \cdot C(13, 4)$$

$$= \frac{13!}{(13-2)!2!} \cdot \boxed{} \quad \text{Combination formula}$$

$$= \boxed{} \cdot \boxed{} \quad \text{Subtract.}$$

$$= \boxed{} \cdot \boxed{} \text{ or } \boxed{} \quad \text{Simplify.}$$

There are $\boxed{}$ hands consisting of 2 hearts and 4 spades.

Check Your Progress

- a. Six friends at a party decide that three of them will go to pick up a movie. How many ways can they choose three people to go?

- b. Thirteen cards are drawn from a standard deck of cards. How many hands consist of six hearts and seven diamonds?

**HOMEWORK
ASSIGNMENT**

Page(s): _____

Exercises: _____



Reinforcement of TEKS 8.11 The student applies concepts of theoretical and experimental probability to make predictions. **(B)** Use theoretical probabilities and experimental results to make predictions and decisions.

BUILD YOUR VOCABULARY (page 342)

MAIN IDEAS

- Use combinations and permutations to find probability.
- Create and use graphs of probability distributions.

The **probability** of an event is a that measures the chances of the event occurring.

EXAMPLE Probability

- 1** Roman has a collection of 26 books—16 are fiction and 10 are nonfiction. He randomly chooses 8 books to take with him on vacation. What is the probability that he happens to choose 4 fiction and 4 nonfiction books?

Step 1 Determine how many 8-book selections meet the conditions. Notice that order doesn't matter.

$C(16, 4)$ Select 4 fiction books.

$C(10, 4)$ Select 4 nonfiction books.

Step 2 Use the Fundamental Counting Principle to find s , the number of successes.

$$C(16, 4) \cdot C(10, 4) = \text{[]} \cdot \text{[]} \text{ or } \text{[]}$$

Step 3 Find the total number, $s + f$, of possible 8-book selections.

$$C(26, 8) = \text{[]} \text{ or } \text{[]} \quad s + f = \text{[]}$$

Step 4 Determine the probability.

$P(4 \text{ fiction and } 4 \text{ nonfiction})$

$$= \frac{s}{s + f} \quad \text{Probability formula}$$

$$= \frac{\text{[]}}{\text{[]}} \text{ or about } \text{[]}$$

The probability of selecting 4 fiction and 4 nonfiction is about

$$\text{[]} \text{ or } \text{[]}.$$

Check Your Progress

When three coins are tossed, what is the probability that exactly two are heads?

KEY CONCEPT

Probability of Success and Failure If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows.

$$\bullet P(S) = \frac{s}{s + f}$$

$$\bullet P(F) = \frac{f}{s + f}$$

EXAMPLE Odds

- 2** For next semester, Alisa has signed up for English, precalculus, Spanish, geography, and chemistry classes. If class schedules are assigned randomly and each class is equally likely to be at any time of the day, what is the probability that Alisa's first two classes in the morning will be precalculus and chemistry, in either order?

KEY CONCEPT

Odds The odds that an event will occur can be expressed as the ratio of the number of ways it can succeed to the number of ways it can fail. If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows.

- Odds of success = $s:f$
- Odds of failure = $f:s$

Step 1 Determine how many schedule arrangements meet the conditions. Notice that order matters.

$P(2, 2)$ Place the two earliest classes.

$P(3, 3)$ Place the other 3 classes.

Step 2 Use the Fundamental Counting Principle to find the number of successes.

$$P(2, 2) \cdot P(3, 3) = \boxed{} \text{ or } \boxed{}$$

Step 3 Find the total number, $s + f$, of possible 5-class arrangements.

$$P(5, 5) = \boxed{} \text{ or } \boxed{} \quad s + f = \boxed{}$$

Step 4 Determine the probability.

$P(\text{Precalculus, chemistry, followed by the other classes})$

$$= \frac{s}{s + f} \quad \text{Probability formula}$$

$$= \boxed{} \text{ or about } \boxed{} \quad \text{Substitute.}$$

The probability that Alisa's first two classes are precalculus and chemistry is about $\boxed{}$ or $\boxed{}$.

Check Your Progress The chances of a male born in 1980 to live to be at least 65 years of age are about 7 in 10. For females, the chances are about 21 in 25.

- a. What are the odds that a male born in 1980 will live to age 65?

- b. What are the odds that a female born in 1980 will live to age 65?

EXAMPLE Probability With Permutations

3 Use the table and graph in Example 3 of your textbook.

a. Use the graph to determine which outcomes are least likely. What are their probabilities?

The least likely outcomes are and , with a probability of for each.

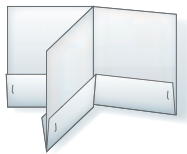
b. Use the table to find $P(S = 11)$. What other sum has the same probability?

According to the table, the probability of a sum of 11 is . The other outcome with a probability of is .

FOLDABLES™

ORGANIZE IT

Write your own probability example. Then write your own odds example. Explain the difference between the two. Place your work in the Probability pocket.

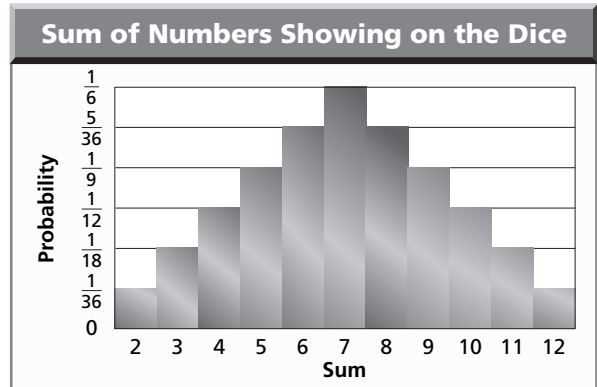


Check Your Progress

Suppose two dice are rolled. The table and the relative frequency histogram show the distribution of the sum of the numbers rolled.

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

a. Use the graph to determine which outcomes are the second most likely. What are their probabilities?



b. Use the table to find $P(S = 4)$. What other sum has the same probability?

c. What are the odds of rolling a sum of 3?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Multiplying Probabilities



Reinforcement of TEKS 8.11 The student applies concepts of theoretical and experimental probability to make predictions. **(A)** Find the probabilities of dependent and independent events. **(B)** Use theoretical probabilities and experimental results to make predictions and decisions.

EXAMPLE Two Independent Events

- 1** Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both of the coins he selects are dimes?

$P(\text{both dimes})$

$$= P(\text{dime}) \cdot P(\text{dime})$$

Probability of independent events

$$= \boxed{} \cdot \boxed{} \text{ or } \boxed{}$$

Substitute and multiply.

The probability is $\boxed{}$ or about $\boxed{}\%$.

EXAMPLE Three Independent Events

- 2** When three dice are rolled, what is the probability that two dice show a 5 and the third die shows an even number?

Let A be the event that the first die shows a 5. $\longrightarrow P(A) = \boxed{}$

Let B be the event that the second die shows a 5. $\longrightarrow P(B) = \boxed{}$

Let C be the event that the third die shows an even number. $\longrightarrow P(C) = \boxed{}$

$$P(A, B, \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$

Probability of Independent events

$$= \boxed{} \cdot \boxed{} \cdot \boxed{} \text{ or } \boxed{}$$

Substitute and multiply.

The probability that the first and second dice show a 5 and the third die shows an even number is $\boxed{}$.

Check Your Progress

- a. Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both of the coins he selects are pennies?

MAIN IDEAS

- Find the probability of two independent events.
- Find the probability of two dependent events.

KEY CONCEPT

Probability of Two Independent Events If two events, A and B , are independent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

- b. When three dice are rolled, what is the probability that one die is a multiple of 3, one die shows an even number, and one die shows a 5?

EXAMPLE Two Dependent Events

- 3 The host of a game show draws chips from a bag to determine the prizes for which contestants will play. Of the 20 chips, 11 show *computer*, 8 show *trip*, and 1 shows *truck*. If the host draws the chips at random and does not replace them, find the probability of drawing a computer, then a truck.

$$P(C \text{ then } T)$$

$$= P(C) \cdot P(T \text{ following } C) \quad \text{Dependent events}$$

$$= \boxed{} \cdot \boxed{} \text{ or } \boxed{} \quad \text{After the first chip is drawn, there are 19 left.}$$

The probability is $\boxed{}$ or about $\boxed{}$.

KEY CONCEPT

Probability of Two Dependent Events If two events, A and B , are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

Check Your Progress Refer to Example 3. Find the probability of drawing each group of prizes.

- a. a truck, then a trip b. two computers

EXAMPLE Three Dependent Events

- 4 Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart, another heart, and a spade in that order.

Since the cards are not replaced, the events are dependent. Let H represent a heart and S represent a spade.

$$P(H, H, S) = P(H) \cdot P(H \text{ following } H) \cdot P(S \text{ following } H \text{ and } H)$$

$$= \boxed{} \cdot \boxed{} \cdot \boxed{} \text{ or } \boxed{}$$

The probability is $\boxed{}$ or about $\boxed{}$.

Check Your Progress Refer to Example 4. Find the probability of drawing a diamond, another diamond, and another diamond in that order.

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____



12-5 Adding Probabilities

BUILD YOUR VOCABULARY (page 341)

MAIN IDEAS

- Find the probability of mutually exclusive events.
- Find the probability of inclusive events.

An event that consists of two or more events is called a **compound event**.

If two events cannot occur at the same , they are called **mutually exclusive events**.

EXAMPLE Two Mutually Exclusive Events

KEY CONCEPT

Probability of Mutually Exclusive Events If two events, A and B , are mutually exclusive, then the probability that A or B occurs is the sum of their probabilities.

- 1 Sylvia has a stack of playing cards consisting of 10 hearts, 8 spades, and 7 clubs. If she selects a card at random from this stack, what is the probability that it is a heart or club?

These are mutually exclusive events since the card cannot be both a heart and a club.

$P(\text{heart or club})$

$$= P(H) + P(C) \quad \text{Mutually exclusive events}$$

$$= \text{} + \text{} \quad \text{Substitute.}$$

$$= \text{} \quad \text{Add.}$$

The probability that Sylvia selects a heart or a club is .

Check Your Progress

Sylvia has a stack of playing cards consisting of 10 hearts, 8 spades, and 7 clubs. If she selects a card at random from this stack, what is the probability that it is a spade or a club?

EXAMPLE Three Mutually Exclusive Events

- 2** The Film Club makes a list of 9 comedies and 5 adventure movies they want to see. They plan to select 4 titles at random to show this semester. What is the probability that at least two of the films they select are comedies?

At least two comedies mean that the club may have , , or comedies. The events are mutually exclusive.

Add the probabilities of each type.

$P(\text{at least 2 comedies})$

$$\begin{aligned}
 &= P(2 \text{ comedies}) + \text{} + \text{} \\
 &= \frac{C(9, 2) \cdot C(5, 2)}{C(14, 4)} + \text{} + \text{} \\
 &= \text{} + \text{} + \text{} \\
 &= \text{}
 \end{aligned}$$

The probability of at least 2 comedies being shown this semester is or about .

Check Your Progress

The Book Club makes a list of 9 mysteries and 3 romance books they want to read. They plan to select 3 titles at random to read this semester. What is the probability that at least two of the books they select are romances?

BUILD YOUR VOCABULARY (page 341)

If two events are not , they are called **inclusive events**.

KEY CONCEPT

Probability of Inclusive Events If two events, A and B , are inclusive, then the probability that A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

EXAMPLE Inclusive Events

- 3 There are 2400 subscribers to an Internet service provider. Of these, 1200 own Brand A computers, 500 own Brand B, and 100 own both A and B. What is the probability that a subscriber selected at random owns either Brand A or Brand B?

Since some subscribers own both A and B , the events are inclusive.

$$P(A) = \boxed{} \quad P(B) = \boxed{} \quad P(\text{both}) = \boxed{}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \boxed{} + \boxed{} - \boxed{}$$

$$= \boxed{} \quad \text{Substitute and simplify.}$$

The probability that a subscriber owns either A or B

is $\boxed{}$.

Check Your Progress

There are 200 students taking Calculus, 500 taking Spanish, and 100 taking both. There are 1000 students in the school. What is the probability that a student selected at random is taking Calculus or Spanish?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Statistical Measures



Reinforcement of TEKS 8.12 The student uses statistical procedures to describe data. (A) Select the appropriate measure of central tendency or range to describe a set of data and justify the choice for a particular situation.

MAIN IDEAS

- Use measures of central tendency to represent a set of data.
- Find measures of variation for a set of data.

BUILD YOUR VOCABULARY (pages 341–342)

A number that describes a set of data is called a **measure of central tendency** because it represents the or middle of the data.

Measures of variation or dispersion measure how

or a set of data is.

The **standard deviation** σ is the of the **variance**.

EXAMPLE

Choose a Measure of Central Tendency

- 1 SALARIES** A new Internet company has 3 employees who are paid \$300,000, ten who are paid \$100,000, and sixty who are paid \$50,000. Which measure of central tendency best represents the pay at this company?

Since most of the employees are paid \$50,000, the higher values are outliers.

Thus, the or best represents the pay at this company.

Check Your Progress

In a cereal contest, there is 1 Grand Prize of \$1,000,000, 10 first prizes of \$100, and 50 second prizes of \$10.

- a. Which measure of central tendency best represents the prizes?

- b. Which measure of central tendency would advertisers be most likely to use?

REVIEW IT

In your own words, what is an outlier? (Lesson 2-5)

EXAMPLE Standard Deviation

- 2 RIVERS** This table shows the length in thousands of miles of some of the longest rivers in the world. Find the standard deviation for these data.

River	Length (thousands of miles)
Nile	4.16
Amazon	4.08
Missouri	2.35
Rio Grande	1.90
Danube	1.78

KEY CONCEPT

Standard Deviation If a set of data consists of the n values x_1, x_2, \dots, x_n and has mean \bar{x} , then the standard deviation σ is given by the following formula.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}}$$

FOLDABLES

Suppose you have a small standard of deviation for your test scores. Does this mean that you have been consistent or inconsistent? Place your explanation in the Statistics pocket.

Find the mean. Add the data and divide by the number of items.

$$\bar{x} = \frac{4.16 + 4.08 + 2.35 + 1.90 + 1.78}{5}$$

$$= \boxed{} \text{ thousand miles}$$

Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}$$

Variance formula

$$\approx \frac{(4.16 - 2.85)^2 + (4.08 - 2.85)^2 + \dots + (1.78 - 2.85)^2}{5}$$

$$\approx \boxed{}^{**}$$

Simplify.

$$\approx \boxed{} \text{ thousand miles}$$

Find the standard deviation.

$$\sigma^2 = \boxed{}$$

Take the square root of each side.

$$\sigma \approx \boxed{} \text{ thousand miles}$$

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Check Your Progress

A teacher has the following test scores: 100, 4, 76, 85, and 92. Find the standard deviation for these data.

BUILD YOUR VOCABULARY (page 342)

MAIN IDEAS

- Determine whether a set of data appears to be normally distributed or skewed.
- Solve problems involving normally distributed data.

A curve that is , often called a *bell curve*, is called a **normal distribution**.

A curve or histogram that is not symmetric represents a **skewed distribution**.

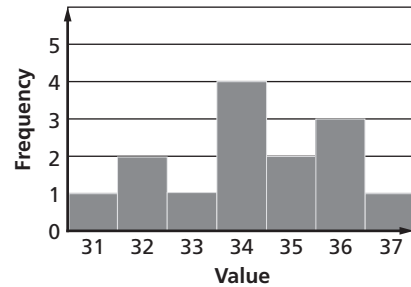
EXAMPLE Classify a Data Distribution

- Determine whether the data {31, 37, 35, 36, 34, 36, 32, 36, 33, 32, 34, 34, 35, 34} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Make a frequency table for the data. Then use the table to make a histogram.

Value	31	32	33	34	35	36	37
Frequency	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Since the data are somewhat symmetric, this is a



REMEMBER IT



To help remember how skewed distributions are labeled, think about the long tail being in the direction of the skew. For example, a positively skewed distribution has a long tail in the positive direction.

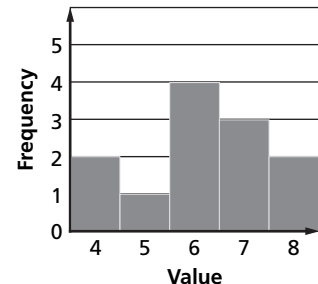


Reinforcement of TEKS 8.12

The student uses statistical procedures to describe data. (C) Select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology.

Check Your Progress

Determine whether the data {7, 5, 6, 7, 8, 4, 6, 8, 7, 6, 6, 4} shown in the histogram appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.



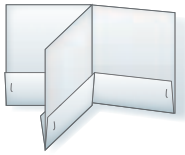
EXAMPLE Normal Distribution

2 Students counted the number of candies in 100 small packages. They found that the number of candies per package was normally distributed with a mean of 23 candies per package and a standard deviation of 1 piece of candy.

FOLDABLES™

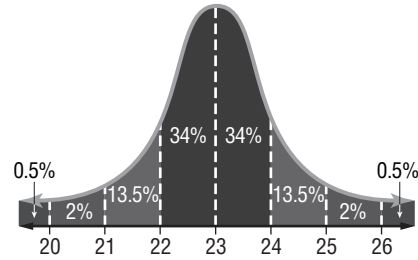
ORGANIZE IT

Describe real-world situations where you would expect the data to be positively skewed, negatively skewed, and normally distributed. Place your explanations in the Statistics pocket.



a. About how many packages had between 24 and 22 candies?

Draw a normal curve. Label the mean and positive and negative multiples of the standard deviation.



The values of 22 and 24 are standard deviation *below* and *above* the mean, respectively. Therefore, of the data are located here.

$100 \cdot \text{} = \text{}$ packages Multiply 100 by 0.68.

About packages contained between 22 and 24 pieces.

b. What is the probability that a package selected at random had more than 25 candies.

The value 25 is standard deviations above the mean. You know that about $100\% - 95\%$ or of the data are more than one standard deviation away from the mean. By the symmetry of the normal curve, half of 5% or , of the data are more than two standard deviations above the mean.

The probability that a package selected at random has more than 25 candies is about or .

Check Your Progress Refer to Example 2.

a. About how many packages had between 25 and 21 candies?

b. What is the probability that a package selected at random had more than 24 candies?

HOMEWORK ASSIGNMENT

Page(s): _____
Exercises: _____

12-8

Binomial Experiments



Reinforcement of TEKS P.4 The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. **(D)** Apply sequences and series to solve problems including sums and binomial expansion.

EXAMPLE Binomial Theorem

MAIN IDEAS

- Use binomial expansions to find probabilities.
- Find probabilities for binomial experiments.

1 If a family has 4 children, what is the probability that they have 2 girls and 2 boys?

There are two possible outcomes for the gender of each of their children: boy or girl. The probability of a boy b is , and the probability of a girl g is .

$$(b + g)^4 = \text{} + \text{} + \text{} + \text{} + \text{}$$

The term $6b^2g^2$ represents 2 girls and 2 boys.

$P(2 \text{ girls and } 2 \text{ boys})$

$$= 6b^2g^2$$

$$6 \left(\text{} \right)^2 \left(\text{} \right)^2$$

$$b \text{ , } g \text{$$

$$\text{$$

Simplify.

The probability of 2 boys and 2 girls is or .

Check Your Progress

If a family has 4 children, what is the probability that they have 4 boys?

REVIEW IT

Use the Binomial Theorem to expand $(x - 3)^5$. (Lesson 11-7)

EXAMPLE

Binomial Experiment

KEY CONCEPT

Binomial Experiments
A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The probabilities for each trial are the same.

- 2 SALES** A report said that approximately 1 out of 6 cars sold in a certain year was green. Suppose a salesperson sells 7 cars per week.

- a. What is the probability that this salesperson will sell exactly 3 green cars in a week?

The probability that a sold car is green is .

The probability that a sold car is not green is .

There are $C(7, 3)$ ways to choose the three green cars that sell.

$P(3 \text{ green cars})$

$$= C(7, 3) \left(\text{input} \right)^3 \left(\text{input} \right)^4$$

If he sells three green cars, he sells four that are not green.

$$= \text{input} \left(\text{input} \right)^3 \left(\text{input} \right)^4$$

$$C(7, 3) = \frac{7!}{4!3!}$$

$$= \text{input}$$

Simplify.

The probability that he will sell exactly 3 green cars is

or about .

- b. What is the probability that this salesperson will sell at least 3 green cars in a week?

Instead of adding the probabilities of selling exactly 3, 4, 5,

6, and 7 green cars, it is easier to the

probabilities of selling exactly , , or green

cars from .

$$\begin{aligned}
 &P(\text{at least 3 green cars}) \\
 &= 1 - P(0 \text{ green cars}) - P(1 \text{ green car}) - P(2 \text{ green cars}) \\
 &= 1 - C(7, 0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^7 - \boxed{} \\
 &\quad - \boxed{} \\
 &= 1 - \frac{\boxed{}}{279,936} - \frac{\boxed{}}{279,936} - \frac{\boxed{}}{279,936} \\
 &= \boxed{}
 \end{aligned}$$

The probability that this salesperson will sell at least three green cars in a week is $\boxed{}$ or about $\boxed{}$.

Check Your Progress

A report said that approximately 1 out of 6 cars sold in a certain year was green. Suppose a salesperson sells 7 cars per week.

- a. What is the probability that this salesperson will sell exactly 4 green cars in a week?

- b. What is the probability that this salesperson will sell at least 2 green cars in a week?

HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

Sampling and Error



Reinforcement of TEKS 8.13 The student evaluates predictions and conclusions based on statistical data. **(A)** Evaluate methods of sampling to determine validity of an inference made from a set of data.

EXAMPLE

Biased and Unbiased Samples

MAIN IDEAS

- Determine whether a sample is unbiased.
- Find margins of sampling error.

KEY CONCEPT

Margin of Sampling Error

If the percent of people in a sample responding in a certain way is p and the size of the sample is n , then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$

1 State whether each method would produce a random sample. Explain.

a. surveying people going into an action movie to find out the most popular kind of movie

; they will most likely think that action movies are the most popular kind of movie.

b. calling every 10th person on the list of subscribers to a newspaper to ask about the quality of the delivery service

; no obvious bias exists in calling every 10th subscriber.

Check Your Progress

State whether each method would produce a random sample. Explain.

a. surveying people going into a football game to find out the most popular sport

b. surveying every fifth person going into a mall to find out the most popular kind of movie

EXAMPLE

Find a Margin of Error

2 In a survey of 100 randomly selected adults, 37% answered “yes” to a particular question. What is the margin of error?

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$

$$= \text{$$

$$\approx \text{ or 10\%$$

Formula for margin of sampling error

$$p = 37\% \text{ or } 0.37, \\ n = 100$$

Use a calculator.

This means that there is a 95% chance that the percent of people in the whole population who would answer “yes”

is between $37 - 10$ or % and $37 + 10$ or %.

Check Your Progress

In a survey of 100 randomly selected adults, 50% answered “no” to a particular question. What is the margin of error?

EXAMPLE

Analyze a Margin of Error

3 HEALTH In an earlier survey, 30% of the people surveyed said they had smoked cigarettes in the past week. The margin of error was 2%. How many people were surveyed?

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$

Formula for margin of sampling error

$$0.02 = 2\sqrt{\frac{0.3(1-0.3)}{n}}$$

$ME = 0.02, p = 0.3$

$$\text{[]} = \text{[]}$$

Divide by 2.

$$\text{[]} = \text{[]}$$

Square each side.

$$n = \text{[]}$$

Multiply by n and divide by 0.0001.

$$n = \text{[]}$$

Use a calculator.

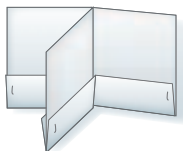
Check Your Progress

In an earlier survey, 25% of the people surveyed said they had exercised in the past week. The margin of error was 2%. How many people were surveyed?

FOLDABLES™

ORGANIZE IT

Write your own example of a biased and an unbiased survey. Place your work in the Statistics pocket.




HOMEWORK ASSIGNMENT

Page(s): _____

Exercises: _____

STUDY GUIDE

	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your Chapter 12 Foldable to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 12, go to: glencoe.com	You can use your completed Vocabulary Builder (pages 335–336) to help you solve the puzzle.

12-1

The Counting Principle

A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.

- A marble is drawn out of the jar and is not replaced. A second marble is drawn.
- A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn.
- A man owns two suits, ten ties, and eight shirts. How many different outfits can he wear if each is made up of a suit, a tie, and a shirt?

12-2

Permutations and Combinations

- Indicate whether arranging five pictures in a row on a wall involves a *permutation* or a *combination*.

Evaluate each expression.

5. $P(5, 3)$

6. $C(7, 2)$

12-3

Probability

A weather forecast says that the chance of rain tomorrow is 40%.

7. Write the probability that it will rain tomorrow as a fraction in lowest terms.
8. What are the odds in favor of rain?
9. Balls are numbered 1 through 15. Find the probability that a ball drawn at random will show a number less than 4. Then find the odds that a number less than 4 is drawn.

12-4

Multiplying Probabilities

A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. A ball is drawn from the bag and is not replaced. A second ball is drawn.

10. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black.
- a. $P(Y \text{ and } B) = \frac{P(Y)}{P(Y) + P(B)}$ b. $P(Y \text{ and } B) = P(Y) \cdot P(B)$
 c. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$
11. Which equation shows the correct calculation of this probability?
- a. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$ b. $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$
 c. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ d. $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$
12. A pair of dice is thrown. What is the probability that both dice show a number greater than 5?

12-5

Adding Probabilities

Marla took a quiz on this lesson that contained the following problem. Her solution is shown.

Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?

$$P(\text{odd}) = \frac{13}{25} \quad P(\text{multiple of } 5) = \frac{5}{25} \text{ or } \frac{1}{5}$$

$$\begin{aligned} P(\text{odd or multiple of } 5) &= P(\text{odd}) + P(\text{multiple of } 5) \\ &= \frac{13}{25} + \frac{5}{25} = \frac{18}{25} \end{aligned}$$

13. Why is Marla's work incorrect?

14. Show the corrected work.

15. A card is drawn from a standard deck of 52 playing cards. What is the probability that an ace or a black card is drawn?

12-6

Statistical Measures

Consider the data set {25, 31, 49, 52, 68, 79, 105}.

16. Find the variance to the nearest tenth.

17. Find the standard deviation to the nearest tenth.

12-7

The Normal Distribution

Indicate whether each of the following statements is *true* or *false*.

18. In a continuous probability distribution, there is a finite number of possible outcomes.

19. Every normal distribution can be represented by a bell curve.

12-8

Binomial Experiments

Indicate whether each of the following is a *binomial experiment* or *not a binomial experiment*. If the experiment is not a binomial experiment, explain why.

20. A fair coin is tossed 10 times and “heads” or “tails” is recorded each time.

21. A pair of dice is thrown 5 times and the sum of the numbers that come up is recorded each time.

Find each probability if a coin is tossed four times.

22. $P(\text{exactly three heads})$

23. $P(\text{exactly four heads})$

12-9

Sampling and Error

25. In a survey of 200 people, 36% voted in the last presidential election. Find the margin of sampling error.



Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 12.

ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 12 Practice Test on page 745 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

- You should complete the Chapter 12 Study Guide and Review on pages 740–744 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 745.

I asked for help from someone else to complete the review of all or most lessons.

- You should review the examples and concepts in your Study Notebook and Chapter 12 Foldable.
- Then complete the Chapter 12 Study Guide and Review on pages 740–744 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 745.

Student Signature

Parent/Guardian Signature

Teacher Signature