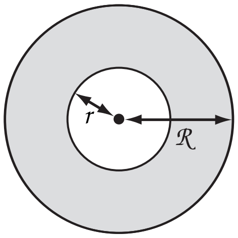
1-1 Word Problem Practice

Expressions and Formulas

1. ARRANGEMENTS The chairs in an auditorium are arranged into two rectangles. Both rectangles are 10 rows deep. One rectangle has 6 chairs per row and the other has 12 chairs per row. Write an expression for the total number of chairs in the auditorium.

2. GEOMETRY The formula for the area of a ring-shaped object is given by $A = \pi(R^2 - r^2)$, where R is the radius of the outer circle and r is the radius of the inner circle. If $R = 10$ inches and $r = 5$ inches, what is the area rounded to the nearest square inch?



3. GUESS AND CHECK Amanda received a worksheet from her teacher. Unfortunately, one of the operations in an equation was covered by a blot. What operation is hidden by the blot?

$$10 + 3(4 \quad 6) = 4$$

4. GAS MILEAGE Rick has d dollars. The formula for the number of gallons of gasoline that Rick can buy with d dollars is given by $g = \frac{d}{3}$. The formula for the number of miles that Rick can drive on g gallons of gasoline is given by $m = 21g$. How many miles can Rick drive on $8 worth of gasoline?

COOKING For Exercises 5 and 6, use the following information.

A steak has thickness w inches. Let T be the time it takes to broil the steak. It takes 12 minutes to broil a one inch thick steak. For every additional inch of thickness, the steak should be broiled for 5 more minutes.

5. Write a formula for T in terms of w.

6. Use your formula to compute the number of minutes it would take to broil a 2 inch thick steak.

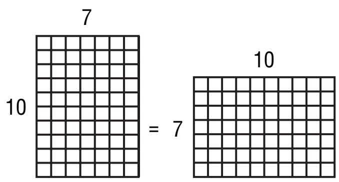
NAME ____ DATE______________ PERIOD _____

1-2 Word Problem Practice

Properties of Real Numbers

1. MENTAL MATH When teaching elementary students to multiply and learn place value, books often show that $54 \times 8 = (50 + 4) \times 8 = (50 \times 8) + (4 \times 8)$. What property is used?

2. MODELS What property of real numbers is illustrated by the figure below?

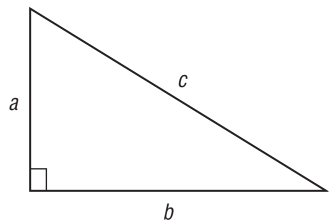


3. VENN DIAGRAMS Make a Venn diagram that shows the relationship between natural numbers, integers, rational numbers, irrational numbers, and real numbers.

4. NUMBER THEORY Consider the following two statements.
I. The product of any two rational numbers is always another rational number.
II. The product of two irrational numbers is always irrational.
Determine if these statements are always, sometimes, or never true. Explain.

RIGHT TRIANGLES For Exercises 5–7, use the following information.

The lengths of the sides of the right triangle shown are related by the formula $c^2 = a^2 + b^2$.



For each set of values for a and b, determine the value of c. State whether c is a natural number.

5. $a = 5, b = 12$

6. $a = 7, b = 14$

7. $a = 7, b = 24$

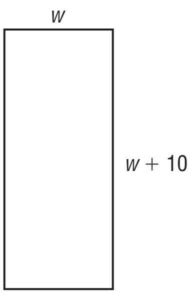
1-3 Word Problem Practice

Solving Equations

1. AGES Robert's father is 5 years older than 3 times Robert's age. Let Robert's age be denoted by R and let Robert's father's age be denoted by F. Write an equation that relates Robert's age and his father's age.

2. AIRPLANES The number of passengers p and the number of suitcases s that an airplane can carry are related by the equation $180p + 60s = 3{,}000$. If 10 people board the aircraft, how many suitcases can the airplane carry?

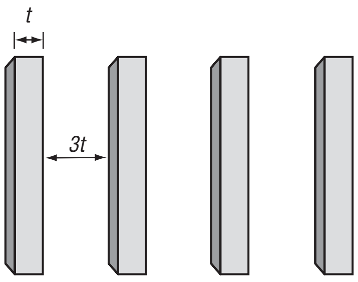
3. GEOMETRY The length of a rectangle is 10 units longer than its width. If the total perimeter of the rectangle is 44 units, what is the width?



4. SAVINGS Jason started with d dollars in his piggy bank. One week later, Jason doubled the amount in his piggy bank. Another week later, Jason was able to add \$20 to his piggy bank. At this point, the piggy bank had \$50 in it. What is d?

DOMINOES For Exercises 5 and 6, use the information below.

Nancy is setting up a train of dominos from the front entrance straight down the hall to the kitchen entrance. The thickness of each domino is t. Nancy places the dominoes so that that the space separating consecutive dominoes is $3t$. The total distance that N dominoes takes up is given by $d = t(4N + 1)$.



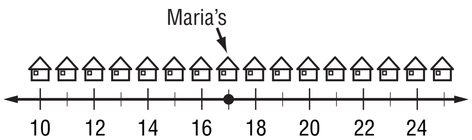
5. Nancy measures her dominoes and finds that $t = 1$ centimeter. She measures the distance of her hallway and finds that $d = 321$ centimeters. Rewrite the equation that relates d, t, and N with the given values substituted for t and d.

6. How many dominoes did Nancy have in her hallway?

1-4 Word Problem Practice

Solving Absolute Value Equations

1. **LOCATIONS** Identical vacation cottages, equally spaced along a street, are numbered consecutively beginning with 10. Maria lives in cottage #17. Joshua lives 4 cottages away from Maria. If n represents Joshua's cottage number, then $|n - 17| = 4$. What are the possible numbers of Joshua's cottage?



2. **HEIGHT** Sarah and Jessica are sisters. Sarah's height is s inches and Jessica's height is j inches. Their father wants to know how many inches separate the two. Write an equation for this difference in such a way that the result will always be positive no matter which sister is taller.

3. **AGES** Rhonda conducts a survey of the ages of students in eleventh grade at her school. On November 1, she finds the average age is 200 months. She also finds that two-thirds of the students are within 3 months of the average age. Write and solve an equation to determine the age limits for this group of students.

4. **TOLERANCE** Martin makes exercise weights. For his 10 pound dumbbells, he guarantees that the actual weight of his dumbbells is within 0.1 pounds of 10 pounds. Write and solve an equation that describes the minimum and maximum weight of his 10 pound dumbbells.

WALKING For Exercises 5–7, use the following information.

Jim is walking along a straight line. An observer watches him. If Jim walks forward, the observer records the distance as a positive number, but if he walks backward, the observer records the distance as a negative number. The observer has recorded that Jim has walked a, then b, then c feet.

5. Write a formula for the total distance that Jim walked.

6. The equation you wrote in part A should not be $T = |a + b + c|$. What does $|a + b + c|$ represent?

7. When would the formula you wrote in part A give the same value as the formula shown in part B?

NAME ____ DATE______________ PERIOD _____

1-5 Word Problem Practice

Solving Inequalities

1. **PANDAS** An adult panda bear will eat at least 20 pounds of bamboo every day. Write an inequality that expresses this situation.

2. **PARTY FAVORS** Janelle would like to give a party bag to every person who is coming to her party. The cost of the party bag is \$7 per person. Write an inequality that describes the number of people P that she can invite if Janelle has D dollars to spend on the party bags.

3. **INCOME** Manuel takes a job translating English instruction manuals to Spanish. He will receive \$15 per page plus \$100 per month. Manuel would like to work for 3 months during the summer and make at least \$1,500. Write and solve an inequality to find the minimum number of pages Manuel must translate in order to reach his goal.

4. **FINDING THE ERROR** The sample below shows how Brandon solved $5 < -2x - 7$. Study his solution and determine if it is correct. Explain your reasoning.

$$5 < -2x - 7$$
$$12 < -2x$$
$$-6 < x$$

CARNIVALS For Exercises 5–7, use the following information.

On a Ferris wheel at a carnival, only two people per car are allowed. The two people together cannot weigh more than 300 pounds. Let x and y be the weights of the people.

5. Write an inequality that describes the weight limitation in terms of x and y.

6. Write an inequality that describes the limit on the average weight a of the two riders.

7. Ron and his father want to go on the ride together. Ron's father weighs 175 pounds. What is the maximum weight Ron can be for the two to be allowed on the ride?

1-6 Word Problem Practice

Solving Compound and Absolute Value Inequalities

1. **AQUARIUM** The depth d of an aquarium tank for dolphins satisfies $|d - 50| < 5$. Rewrite this as a compound inequality that does not involve the absolute value function.

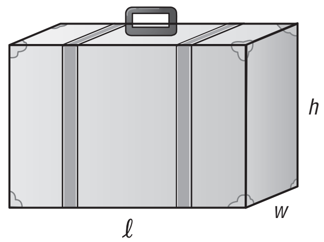
2. **HIKING** For a hiking trip, everybody must bring at least one backpack. However, because of space limitations, nobody is allowed to bring more than two backpacks. Let n be the number of people going on the hiking trip and b be the number of backpacks allowed. Write a compound inequality that describes how b and n are related.

3. **CONCERT** Jacinta is organizing a large fund-raiser concert in a space with a maximum capacity of 10,000 people. Her goal is to raise at least $100,000. Tickets cost $20 per person. Jacinta spends $50,000 to put the event together. Write and solve a compound inequality that describes N, the number of attendees needed to achieve Jacinta's goal.

4. **NUMBERS** Amy is thinking of two numbers a and b. The sum of the two numbers must be within 10 units of zero. If a is between -100 and 100, write a compound inequality that describes the possible values of b.

AIRLINE BAGGAGE For Exercises 5–7, use the following information.

An airline company has a size limitation for carry-on luggage. The limitation states that the sum of the length, width, and height of the suitcase must not exceed 45 inches.



5. Write an inequality that describes the airline's carry-on size limitation.

6. A passenger needs to bring a soil sample on the plane that is at least 1 cubic foot. The passenger is bringing it in a suitcase that is in the shape of a cube with side length s inches. Write an inequality that gives the minimum length for s.

7. Write a compound inequality for s using parts A and B. Find the maximum and minimum values for s.

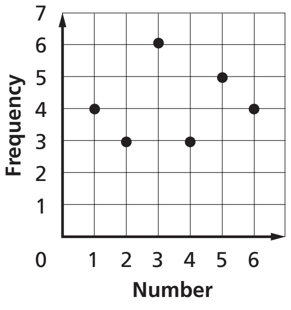
2-1 Word Problem Practice

Relations and Functions

1. PLANETS The table below gives the mean distance from the Sun and orbital period of the nine major planets in our Solar System. Think of the mean distance as the domain and the orbital period as the range of a relation. Is this relation a function? Explain.

Planet	Mean Distance from Sun (AU)	Orbital Period (years)
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.0	1.0
Mars	1.524	1.881
Jupiter	5.204	11.75
Saturn	9.582	29.5
Uranus	19.201	84
Neptune	30.047	165
Pluto	39.236	248

2. PROBABILITY Martha rolls a number cube several times and makes the frequency graph shown. Write a relation to represent this data.



3. SCHOOL The number of students N in Vassia's school is given by $N = 120 + 30G$, where G is the grade level. Is 285 in the range of this function?

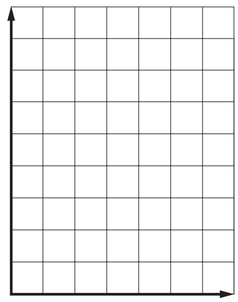
4. FLOWERS Anthony decides to decorate a ballroom with $r = 3n + 20$ roses, where n is the number of dancers. It occurs to Anthony that the dancers always come in pairs. That is, $n = 2p$, where p is the number of pairs. What is r as a function of p?

SALES For Exercises 5–7, use the following information.

Cool Athletics introduced the new Power Sneaker in one of their stores. The table shows the sales for the first 6 weeks.

Week	1	2	3	4	5	6
Pairs Sold	8	10	15	22	31	44

5. Graph the data.



6. Identify the domain and range.

7. Is the relation a function? Explain.

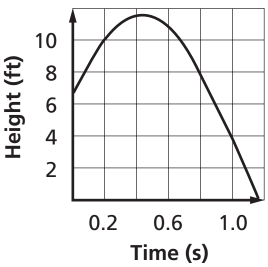
NAME ____ DATE______________ PERIOD _____

2-2 Word Problem Practice

Linear Equations

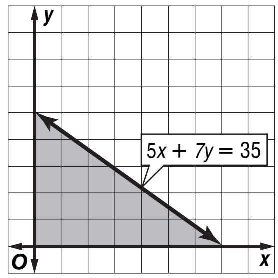
1. WORK RATE The linear equation $n = 10t$ describes n, the number of origami boxes that Holly can fold in t hours. How many boxes can Holly fold in 3 hours?

2. BASKETBALL Tony tossed a basketball. Below is a graph showing the height of the basketball as a function of time. Is this the graph of a linear function? Explain.



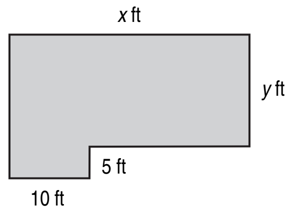
3. PROFIT Paul charges people $25 to test the air quality in their homes. The device he uses to test air quality cost him $500. Write an equation that describes Paul's net profit as a function of the number of clients he gets. How many clients does he need to break even?

4. RAMP A ramp is described by the equation $5x + 7y = 35$. What is the area of the shaded region?



SWIMMING POOL For Exercises 5–7, use the following information.

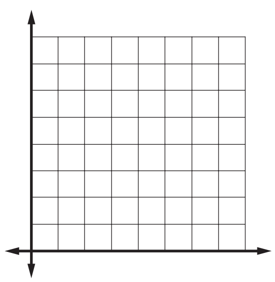
A swimming pool is shaped as shown below. The total perimeter is 110 feet.



5. Write an equation that relates x and y.

6. Write the linear equation from Exercise 5 in standard form.

7. Graph the equation.

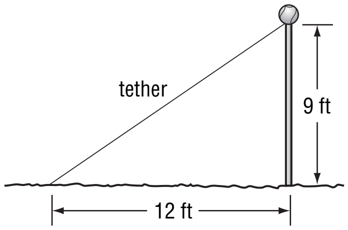


Lesson 2-2

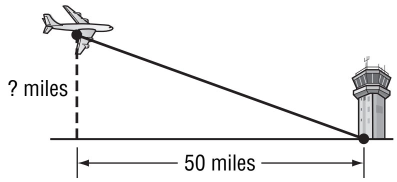
2-3 Word Problem Practice

Slope

1. **TETHER** A tether is tied tautly to the top of a pole as shown. What is the slope of the tether?



2. **AVIATION** An airplane descends along a straight-line path with a slope of -0.1 to land at an airport. Use the information in the diagram to determine the initial height of the airplane.



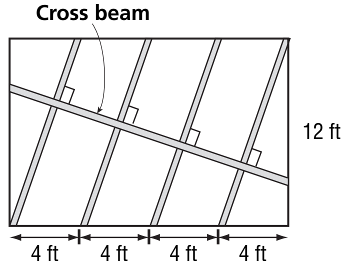
3. **ROCK CLIMBING** The table below shows Gail's altitude above ground during a rock climb up a cliff.

Time	Altitude (m)
10:00	0
10:20	22
10:40	30
11:00	33

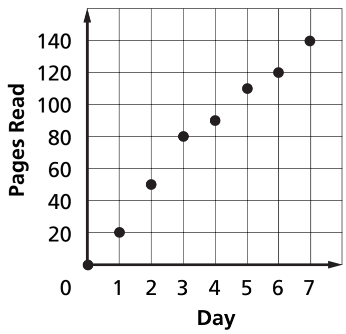
Complete the following table of Gail's average rate of ascent.

Time Period	Average rate of ascent (m/h)
10:00-10:20	
10:20-10:40	
10:40-11:00	

4. **DESIGN** An architect is designing a window with slanted interior bars. The crossbeam is perpendicular to the other four bars. What is the slope of the crossbeam?



READING For Exercises 5–7, use the graph that shows how many pages of her book Bridget read each day.



5. Find the average number of pages Bridget read per day.

6. On which days did Bridget read more pages than her daily average?

7. If Bridget had been able to keep up the pace she had on day 3, how many days would it have taken her to finish the book?

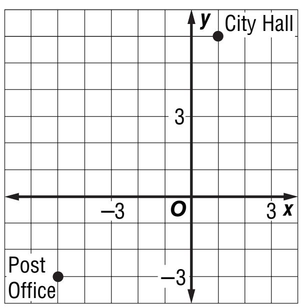
2-4 Word Problem Practice

Writing Linear Equations

1. **HIKING** Tim began a hike at the base of the mountain that is 129 feet above sea level. He is hiking at a steady rate of 5 feet per minute. Let A be his altitude above sea level in feet and let t be the number of minutes he has been hiking. Write an equation in slope-intercept form that represents how many feet above sea level Tim has hiked.

2. **CHARITY** By midnight, a charity had collected 83 shirts. Every hour after that, it collected 20 more shirts. Let h be the number of hours since midnight and s be the number of shirts. Write a linear equation in slope-intercept form that relates the number of shirts collected and the number of hours since midnight.

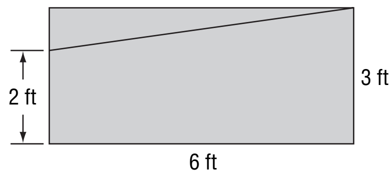
3. **MAPS** The post office and city hall are marked on a coordinate plane. Write the equation of the line in slope-intercept form that passes through these two points.



4. **RIGHT TRIANGLES** The line containing the base of a right triangle has the equation $y = 3x + 4$. The leg perpendicular to the base has an endpoint at (6, 1). What is the slope-intercept form of the equation of the line containing the leg?

DECORATING For Exercises 5–7, use the following information.

A group of students is decorating a bulletin board that measures 3 feet by 6 feet. They want to put a line that stretches from the upper right corner to a point 2 feet up along the left edge as shown in the figure.



5. Using the lower left corner of the bulletin board as the origin, what is the equation of the line in slope-intercept form?

6. The students change their mind and decide that the line should be lowered by 1 foot. What is the equation of the lowered line in slope-intercept form?

7. What are the coordinates of the center of the bulletin board? Does the lowered line pass through the center? Explain.

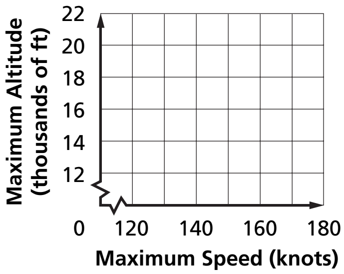
2-5 Word Problem Practice

Modeling Real-World Data: Using Scatter Plots

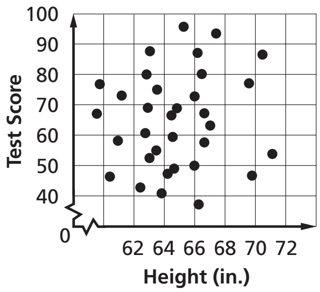
1. AIRCRAFT The table shows the maximum speed and altitude of different aircraft. Draw a scatter plot of this data.

Max. Speed (knots)	121	123	137	173	153
Max. Altitude (1000 feet)	14.2	17.0	15.3	20.7	16.0

Source: www.risingup.com

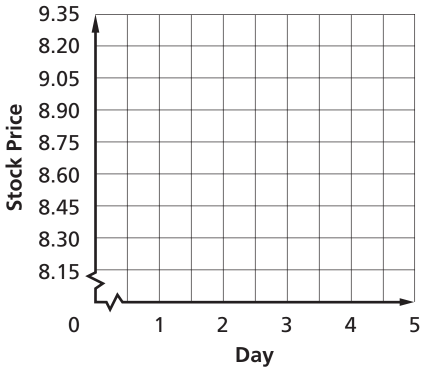


2. TESTING The scatter plot shows the height and test scores of students in a math class. Describe the correlation between heights and test scores.

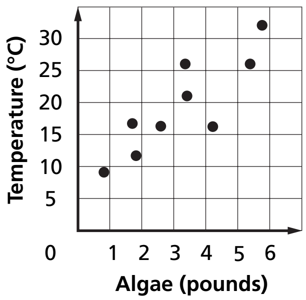


3. STOCKS The prices of a technology stock over 5 days are shown in the table. Draw a scatter plot of the data and a line of fit.

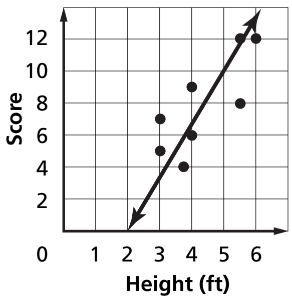
Day	1	2	3	4	5
BTEK	8.30	8.60	8.55	8.90	9.30



4. ALGAE The scatter plot shows data recording the amount of algae and the temperature of the water in various aquarium tanks. Draw a line of fit for this data and write a prediction equation.



SPORTS For Exercises 5 and 6, use the scatter plot showing the height and score of different contestants shooting darts.



5. What is the equation of the line of fit?

6. What do you predict someone 5 feet tall would score?

NAME ____ DATE______________ PERIOD _____

2-6 Word Problem Practice

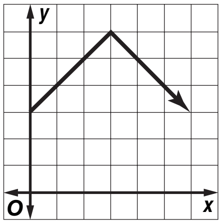
Special Functions

1. **SAVINGS** Nathan puts \$200 into a checking account as soon as he gets his paycheck. The value of his checking account is modeled by the formula $200[\![m]\!]$, where m is the number of months that Nathan has been working. After 105 days, how much money is in the account?

2. **FINANCE** A financial advisor handles the transactions in a bank account. For every transaction, the advisor gets a 5% commission, regardless of whether the transaction is a deposit or withdrawal. Write a formula using the absolute value function for the advisor's commission. Let D represent the value of one transaction.

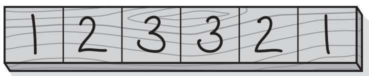
3. **ROUNDING** A science teacher instructs students to round their measurements as follows: If a number is less than 0.5 of a millimeter, students are instructed to round down. If a number is exactly 0.5 or greater, students are told to round up to the next millimeter. Write a formula that takes a measurement x and yields the rounded off number.

4. **ARCHITECTURE** The cross-section of a roof is shown in the figure. Write an absolute value function that models the shape of the roof.



GAMES For Exercises 5 and 6, use the following information.

Some young people are playing a game where a wooden plank is used as a target. It is marked off into 6 equal parts. A value is written in each section to represent the score earned if the dart lands in that section. Let x denote the horizontal position of a dart on the board, where the center of the board is the origin. Negative values correspond to the left half of the dart board, and positive values correspond to the right half. A player's score depends on the distance of the dart from the origin.



5. Write a formula that gives the horizontal distance from the center of the dartboard.

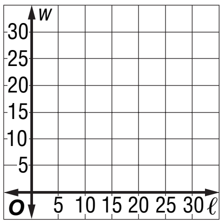
6. Write a formula using the greatest integer function that can be used to find the person's score.

NAME ____ DATE______________ PERIOD _____

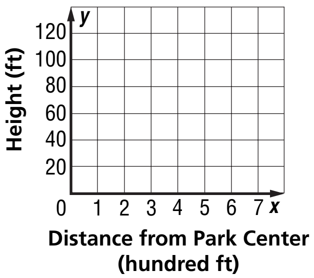
2-7 Word Problem Practice

Graphing Inequalities

1. FRAMES The dimensions of a rectangular frame that can be made from a 50 inch plank of wood are limited by the inequality $\ell + w \leq 25$. Graph this inequality.

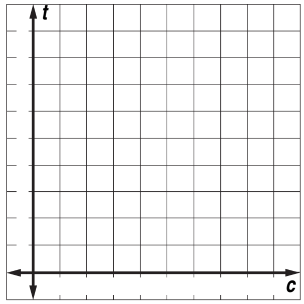


2. BUILDING CODE A city has a building code that limits the height of buildings around the central park. The code says that all buildings must be less than $0.1x$ in height where x is the distance of the building from the center of the park. Assume that the park center is located at 0. Graph the inequality that represents the building code.



3. LIVESTOCK During the winter, a horse requires about 36 liters of water per day and a sheep requires about 3.6 liters per day. A farmer is able to supply his horses and sheep with a total of 300 liters of water each day. Write an inequality that represents the possible number of horses and sheep this farmer can keep.

4. WEIGHT A delivery crew is going to load a truck with tables and chairs. The trucks weight limitations are represented by the inequality $200t + 60c < 1200$, where t is the number of tables and c is the number of chairs. Graph this inequality.

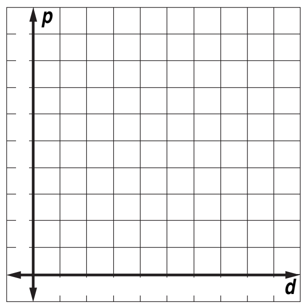


ART For Exercises 5–7, use the following information.

An artist can sell each drawing for $100 and each painting for $400. He hopes to make at least $2000 every month.

5. Write an inequality that expresses how many paintings and/or drawings the artist needs to sell each month to reach his goal.

6. Graph the inequality.



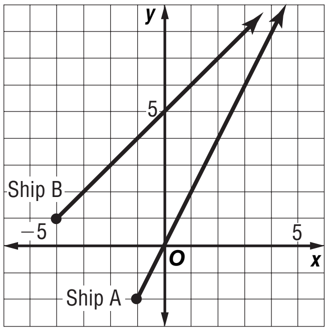
7. If David sells three paintings one month, how many drawings would he have to sell in the same month to reach $2000?

3-1 Word Problem Practice

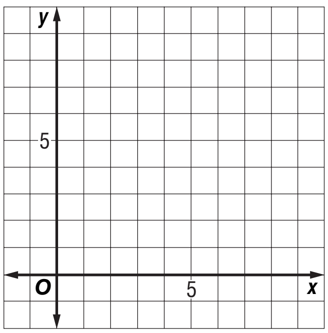
Solving Systems of Equations By Graphing

1. STREETS Andrew is studying a map and notices two streets that run parallel to each other. He computes the equations of the lines that represent the two roads. Are these two equations *consistent* or *inconsistent*? If they are consistent, are they *independent* or *dependent*? Explain.

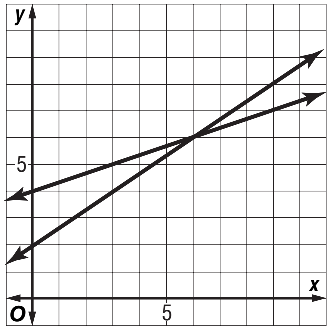
2. SPOTLIGHTS Ship A has coordinates $(-1, -2)$ and Ship B has coordinates $(-4, 1)$. Both ships have their spotlights fixated on the same lifeboat. The light beam from Ship A travels along the line $y = 2x$. The light beam from Ship B travels along the line $y = x + 5$. What are the coordinates of the lifeboat?



3. LASERS A machine heats up a single point by shining several lasers at it. The equations $y = x + 1$ and $y = -x + 7$ describe two of the laser beams. Graph both of these lines to find the coordinates of the heated point



4. PATHS The graph shows the paths of two people who took a walk in a park. Where did their paths intersect?



PHONE SERVICE For Exercises 5–7, use the following information.
Beth is deciding between two telephone plans. Plan A charges \$15 per month plus 10 cents per minute. Plan B charges \$20 per month plus 5 cents per minute.

5. Write a system of equations that represent the monthly cost of each plan.

6. Graph the equations.

7. For how many minutes per month do the two phone plans cost the same amount?

3-2 Word Problem Practice

Solving Systems of Equations Algebraically

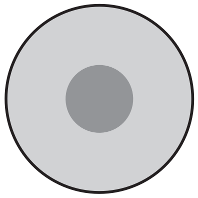
1. **SUPPLIES** Kirsta and Arthur both need pens and blank CDs. The equation that represents Kirsta's purchases is $y = 27 - 3x$. The equation that represents Arthur's purchases is $y = 17 - x$. If x represents the price of the pens, and y represents the price of the CDs, what are the prices of the pens and the CDs?

2. **WALKING** Amy is walking a straight path that can be represented by the equation $y = 2x + 3$. At the same time Kendra is walking the straight path that has the equation $3y = 6x + 6$. What is the solution to the system of equations that represents the paths the two girls walked? Explain.

3. **CAFETERIA** To furnish a cafeteria, a school can spend $5200 on tables and chairs. Tables cost $200 and chairs cost $40. Each table will have 8 chairs around it. How many tables and chairs will the school purchase?

4. **PRICES** At a store, toothbrushes cost x dollars and bars of soap cost y dollars. One customer bought 2 toothbrushes and 1 bar of soap for $11. Another customer bought 6 toothbrushes and 5 bars of soap for $38. Both amounts do not include tax. Write and solve a system of equations for x and y.

GAMES For Exercises 5–7, use the following information.
Mark and Stephanie are playing a game where they toss a dart at a game board that is hanging on the wall. The points earned from a toss depends on where the dart lands. The center area is worth more points than the surrounding area. Each player tosses 12 darts.



5. Stephanie earned a total of 66 points with 6 darts landing in each area. Mark earned a total of 56 points with 4 darts landing in the center area, and 8 darts landing in the surrounding area. Write a system of equations that represents the number of darts each player tossed into each section. Use x for the inner circle, and y for the outer circle.

6. How many points is the inner circle worth? How many points is the outer circle worth?

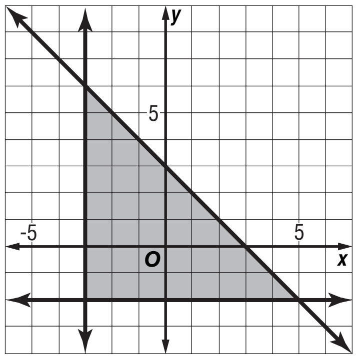
7. If a player gets 10 darts in the inner circle and 2 in the outer circle the total score is doubled. How many points would the player earn if he or she gets exactly 10 darts in the center?

NAME ____ DATE______________ PERIOD _____

3-3 Word Problem Practice

Solving Systems of Inequalities by Graphing

1. BIRD BATH Melissa wants to put a bird bath in her yard at point (x, y), and wants it to be is inside the enclosed shaded area shown in the graph.



First, she checks that $x \geq -3$ and $y \geq -2$. What linear inequality must she check to conclude that (x, y) is inside the triangle?

2. SQUARES Matt finds a blot of ink covering his writing in his notes for math class. He sees "A square is defined by $|x| \leq 8$ and _". Write an inequality that completes this sentence.

3. HOLIDAY Amanda received presents and cards from friends over the holiday season. Every present came with one card and none of her friends sent her more than one card. Less than 10 of her friends sent only a card. Describe this situation using inequalities.

4. DECK The Wrights are building a deck. The deck is defined by the inequalities $x \leq 5$, $0.25x + y \geq -4.75$, $y \leq 5$, and $4.5x + y \geq -17.5$. Graph the inequalities and find the coordinates of the deck's corners?

TICKETS For Exercises 5 and 6, use the following information.
A theater charges \$10 for adults and \$5 for children 12 or under. The theater makes a profit if they can sell more than \$600 worth of tickets. The theater has seating for 100 people.

5. Write a system of linear inequalities that describes the situation.

6. Graph the solution to the inequalities. Can the theater make a profit if no adults come to the performance?

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3-4 Word Problem Practice

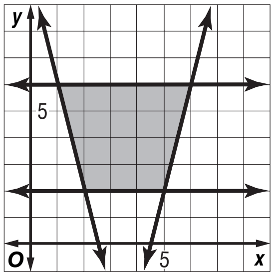
Linear Programming

1. REGIONS A region in the plane is formed by the equations $x - y < 3$, $x - y > -3$, and $x + y > -3$. Is this region bounded or unbounded? Explain.

2. MANUFACTURING Eighty workers are available to assemble tables and chairs. It takes 5 people to assemble a table and 3 people to assemble a chair. The workers always make at least as many tables as chairs because the tables are easier to make. If x is the number of tables and y is the number of chairs, the system of inequalities that represent what can be assembled is $x > 0$, $y > 0$, $y \leq x$, and $5x + 3y \leq 80$. What is the maximum total number of chairs and tables the workers can make?

3. FISH An aquarium is 2000 cubic inches. Nathan wants to populate the aquarium with neon tetras and catfish. It is recommended that each neon tetra be allowed 50 cubic inches and each catfish be allowed 200 cubic inches of space. Nathan would like at least one catfish for every 4 neon tetras. Let n be the number of neon tetra and c be the number of catfish. The following inequalities form the feasible region for this situation: $n > 0$, $c > 0$, $4c \geq n$, and $50n + 200c \leq 2000$. What is the maximum number of fish Nathan can put in his aquarium?

4. ELEVATION A trapezoidal park is built on a slight incline. The function for the ground elevation above sea level is $f(x, y) = x - 3y + 20$ feet. What are the coordinates of the highest point in the park?



CERAMICS For Exercises 5–7, use the following information.
Josh has 8 days to make pots and plates to sell at a local fair. Each pot weighs 2 pounds and each plate weighs 1 pound. Josh cannot carry more than 50 pounds to the fair. Each day, he can make at most 5 plates and at most 3 pots. He will make $12 profit for every plate and $25 profit for every pot that he sells.

5. Write linear inequalities to represent the number of pots p and plates a Josh may bring to the fair.

6. List the coordinates of the vertices of the feasible region.

7. How many pots and how many plates should Josh make to maximize his potential profit?

NAME ____ DATE______________ PERIOD _____

3-5 Word Problem Practice

Solving Systems of Equations in Three Variables

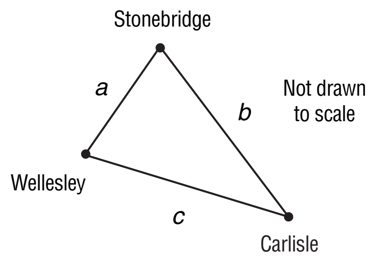
1. **SIBLINGS** Amy, Karen, and Nolan are siblings. Their ages in years can be represented by the variables A, K, and N, respectively. They have lived a total of 22 years combined. Karen has lived twice as many years as Amy, and Nolan has lived 6 years longer than Amy. Use the equations $A + K + N = 22$, $K = 2A$, and $N = A + 6$ to find the age of each sibling.

2. **HOCKEY** Bobby Hull scored G goals, A assists, and P points in his NHL career. By definition, $P = G + A$. He scored 50 more goals than assists. Had he scored 15 more goals and 15 more assists, he would have scored 1200 points. How many goals, assists, and points did Bobby Hull score?

3. **EXERCISE** Larry, Camille, and Simone are keeping track of how far they walk each day. At the end of the week, they combined their distances and found that they had walked 34 miles in total. They also learned that Camille walked twice as far as Larry, and that Larry walked 2 more miles than Simone. How far did each person walk?

DISTANCES For Exercises 5 and 6, use the following information.
Let c be the distance between Carlisle and Wellesley, let b be the distance between Carlisle and Stonebridge, and let a be the distance between Wellesley and Stonebridge.

- If you did a circuit, traveling from Carlisle to Wellesley to Stonebridge and back to Carlisle, you would travel 73 miles.
- Stonebridge is 12 miles farther than Wellesley is from Carlisle.
- If you drove from Stonebridge to Carlisle and back to Stonebridge, and then continued to Wellesley then back to Stonebridge, you would travel 102 miles.



5. Write a system of linear equations to represent the situation.

6. Solve the system of equations. Explain the meaning of the solution in the context of the situation.

NAME ____ DATE______________ PERIOD _____

4-1 Word Problem Practice

Introduction to Matrices

1. HAWAII The table shows the population and area of some of the islands in Hawaii. What would be the dimensions of a matrix that represented this information?

Island	Population	Area
Hawaii	120,317	4,038
Maui	91,361	729
Oahu	836,231	594
Kauai	50,947	549
Lanai	2,426	140

Source: www.vthawaii.com

2. LAUNDRY Carl is looking for a Laundromat. SuperWash has 20 small washers, 10 large washers, and 20 dryers. QuickClean has 40 small washers, 5 large washers, and 50 dryers. ToughSuds has 15 small washers, 40 large washers, and 100 dryers. Write a matrix to organize this information.

3. CITY DISTANCES The incomplete matrix shown gives the approximate distances between Chicago, Los Angeles, and New York City. Complete the matrix.

$$\begin{array}{r c} & \begin{array}{ccc} \text{NYC} & \text{Chicago} & \text{Los Angeles} \end{array} \\ \begin{array}{r} \text{NYC} \\ \text{Chicago} \\ \text{Los Angeles} \end{array} & \begin{bmatrix} \mathbf{0} & \quad & \mathbf{2790} \\ \mathbf{810} & & \mathbf{2050} \\ & & \end{bmatrix} \end{array}$$

4. INVENTORY A store manager records the number of light bulbs in stock for 3 different brands over a five-day period. The manager decides to make a matrix of this information. Each row represents a different brand, and each column represents a different day. The entry in column N represents the inventories at the beginning of day N.

$$\begin{bmatrix} 25 & 24 & 22 & 20 & 19 \\ 30 & 27 & 25 & 22 & 21 \\ 28 & 25 & 21 & 19 & 19 \end{bmatrix}$$

Assuming that the inventories were never replenished, which brand holds the record for most light bulbs sold on a given day?

SHOE SALES For Exercises 5 and 6, use the following information.
A shoe store manager keeps track of the amount of money made by each of three salespeople for each day of a workweek. Monday through Friday, Carla made \$40, \$70, \$35, \$50, and \$20. John made \$30, \$60, \$20, \$45, and \$30. Mary made \$35, \$90, \$30, \$40, and \$30.

5. Organize this data in a 3 by 5 matrix.

6. Which salesperson made the most money that week?

4-2 Word Problem Practice

Operations with Matrices

1. FARES The matrix below gives general admission and planetarium fares at a science museum.

$$\begin{array}{r c} & \begin{array}{cc}\textbf{Child} & \textbf{Adult}\end{array} \\ \begin{array}{r}\textbf{General Admission} \\ \textbf{Planetarium}\end{array} & \begin{bmatrix} 5 & 10 \\ 4 & 8 \end{bmatrix}\end{array}$$

What can you do to this matrix in order to create another matrix that represents fares for 5 people?

2. NEGATION Two engineers need to negate all the entries of a matrix. One engineer tries to do this by multiplying the matrix by -1. The other engineer tries to do this by subtracting twice the matrix from itself. Which engineer, if any, will get the correct result?

3. PLANE FARES The airfares for travel between New York, Chicago, and Los Angeles are organized in the matrix on the left. The matrix on the right gives the tax surcharges for corresponding flights.

$$\begin{array}{r c} & \begin{array}{ccc}\text{NYC} & \text{Chicago} & \text{Los Angeles}\end{array} \\ \begin{array}{r}\text{NYC} \\ \text{Chicago} \\ \text{Los Angeles}\end{array} & \begin{bmatrix} 0 & 440 & 700 \\ 460 & 0 & 660 \\ 850 & 700 & 0 \end{bmatrix}\end{array} \quad \begin{array}{r c} & \begin{array}{ccc}\text{NYC} & \text{Chicago} & \text{Los Angeles}\end{array} \\ \begin{array}{r}\text{NYC} \\ \text{Chicago} \\ \text{Los Angeles}\end{array} & \begin{bmatrix} 0 & 40 & 70 \\ 46 & 0 & 60 \\ 85 & 70 & 0 \end{bmatrix}\end{array}$$

Write a matrix that represents the full cost for travel between these cities.

4. SUNFLOWERS Matrix H is a 3 by 1 matrix that contains the initial heights of three sunflowers. Matrix G is a 3 by 1 matrix that contains the numbers of inches the corresponding sunflowers grow in a week. What does matrix $H + 4G$ represent?

DINNER For Exercises 5–7, use the following information.
The menu shows prices for some dishes at a restaurant.

Il Ristorante Menu

	Regular	Half-portion
Lamb	$17.00	$9.00
Chicken	$14.00	$7.00
Steak	$22.00	$11.00

5. Make a 3 by 2 matrix to organize these data.

6. Let M be the matrix you wrote for Exercise 5. Write an expression involving M that would give prices that include an additional 20% to cover tax and tip.

7. Compute the matrix you described in Exercise 6.

Lesson 4-2

NAME ______________________________ DATE______________ PERIOD _____

4-3 Word Problem Practice

Multiplying Matrices

1. FIND THE ERROR Both A and B are 2 by 2 matrices. Maggie made the following derivation. Is this derivation valid? If not, what error did she make?

a. $(A + B)^2 = (A + B)(A + B)$

b. $= (A + B)A + (A + B)B$

c. $= AA + BA + AB + BB$

d. $= A^2 + BA + AB + B^2$

e. $= A^2 + AB + AB + B^2$

f. $= A^2 + 2AB + B^2$

2. EXAM SCORES Mr. Farey recorded the exam scores of his students in a 20 by 3 matrix. Each row listed the scores of a different student. The first exam scores were listed in the first column, and the second exam scores were listed in the second column. The final exam scores were listed in the third column. Mr. Farey needed to create a 20 by 1 matrix that contained the weighted scores of each student. The first two exams account for 25% of the weighted score, and the final exam counted 50%. To make the matrix of weighted scores, what matrix can Mr. Farey multiply his 20 by 3 matrix by on the right?

3. SPECIAL MATRICES Mandy has a 3 by 3 matrix M. She notices that for any 3 by 3 matrix X, $MX = X$. What must M be?

4. POWERS Thad just learned about matrix multiplication. He began to wonder what happens when you take powers of a matrix. He computed the first few powers of the matrix $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and noticed a pattern. What is M^n?

COST COMPARISONS For Exercises 5 and 6, use the following information.

Barbara and Lance need to buy pens, pencils, and erasers. They make a 2 by 3 matrix that represents the numbers of each item they would like to purchase.

$$\begin{array}{r} \\ \textbf{Barbara} \\ \textbf{Lance} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Pens} & \textbf{Pencils} & \textbf{Erasers} \end{array} \\ \begin{bmatrix} 10 & 15 & 3 \\ 5 & 20 & 5 \end{bmatrix} \end{array}$$

They call this matrix M. Barbara and Lance find two stores that sell the items at different prices and record this information in a second matrix that they call P.

$$\begin{array}{r} \\ \textbf{Pens} \\ \textbf{Pencils} \\ \textbf{Erasers} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{Store 1} & \textbf{Store 2} \end{array} \\ \begin{bmatrix} 2.20 & 1.90 \\ 0.85 & 0.95 \\ 0.60 & 0.65 \end{bmatrix} \end{array}$$

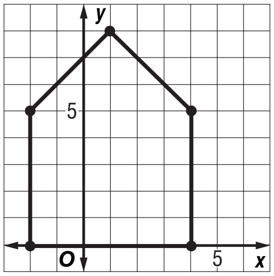
5. Compute MP.

6. What do the entries in MP mean?

4-4 Word Problem Practice

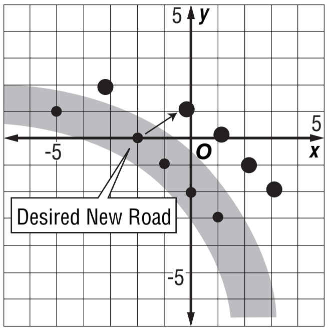
Transformations with Matrices

1. **ICONS** Louis needs to perform many matrix transformations to the basic house icon shown in the graph.



What is the vertex matrix for this image?

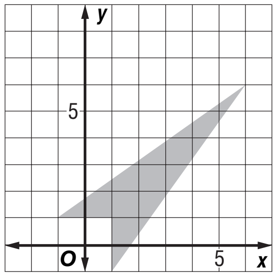
2. **RELOCATION** City planners are making a new road. Unfortunately, the road will pass through five ancient trees indicated by the small dots. The planners decide to move the trees to the locations indicated by the large dots. What matrix represents this translation?



3. **MIRROR SYMMETRY** A detective found only half of an image with mirror symmetry about the line $y = x$. The vertex matrix of the visible part is $\begin{bmatrix} 4 & 5 & -2 \\ 2 & -5 & -4 \end{bmatrix}$. What are the coordinates of the hidden vertices?

4. **PHOTOGRAPHY** Alejandra used a digital camera to take a picture. Because she held the camera sideways, the image on her computer screen appeared sideways. In order to transform the picture, she needed to perform a 90° clockwise rotation. What matrix represents this transformation?

ARROWS For Exercises 5–6, use the following information.
A compass arrow is pointing Northeast.



5. What is the vertex matrix for the arrow?

6. What would the vertex matrix be for the arrow if it were pointing Northwest? (Hint: Rotate 90° around the origin.)

Lesson 4-4

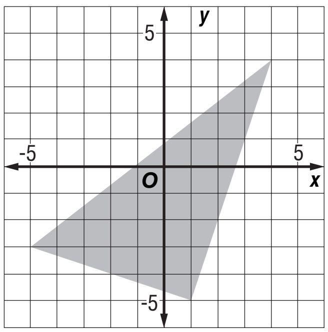
4-5 Word Problem Practice

Determinants

1. FIND THE ERROR Mark's determinant computation has sign errors. Circle the signs that must be reversed.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5)(9) - 2(6)(7) + 3(4)(8) - 3(5)(7) + 1(6)(8) - 2(4)(9)$$

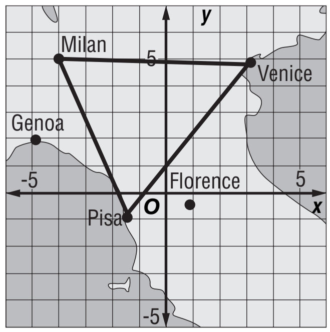
2. POOL An architect has a pool in the floor plans for a home. Set up a determinant that gives the unit area of the pool.



3. HALF-UNIT TRIANGLES For a school art project, students had to decorate a pegboard by looping strings around the pegs. Ronald wanted to make triangles with areas of one half square unit. Because Ronald had studied determinants, he knew that this was essentially the same as finding the coordinates of the vertices of a triangle (a, b), (c, d) and (e, f), so that the determinant $\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$ is 1 or -1.

Give an example of such a triangle.

4. ITALY The figure shows a map of Italy overlaid on a graph. The coordinates of Milan, Venice, and Pisa are about $(-4, 5)$, $(3.25, 4.8)$, and $(-1.4, -0.8)$, respectively. Each square unit on the map represents about 400 square miles.



What is the area of the triangular region? Round your answer to the nearest square mile.

ARROWS For Exercises 5 and 6, use the following information.
Kyle is making a triangle with vertices at $(-6, 0)$, $(0, -x)$, and $(0, x)$, and $x > 0$. He plans to make the triangle using a material that costs $2 for every square unit.

5. Write the determinant that gives the area of this triangle.

6. Evaluate the determinant you wrote for Exercise 5 and determine the value of x that results in a $60 triangle.

4-6 Word Problem Practice

Cramer's Rule

1. **USING CRAMER'S RULE** Lucy is solving the following system of linear equations using Cramer's Rule.

$$2x + 3y = 5$$
$$x + y = 2$$

Write the three determinants she will have to compute.

2. **IMPLICATIONS OF CRAMER'S RULE** Cramer's Rule gives the solutions of systems of linear equations in terms of their coefficients. The formula involves addition, subtraction, multiplication, and division of those coefficients. Is it possible for an irrational number to be part of the solution of a system of linear equations whose coefficients are all rational numbers?

3. **SHOPPING** Sheets cost $18.59 each and pillowcases cost $7.24 at Carol's Linens. If Agatha buys x sheets and y pillowcases at Carol's Linens, she'll spend $210.75. On the other hand, at Save-n-Sleep, sheets cost $15.79 and pillowcases cost $8.19. If Agatha buys x sheets and y pillows cases at Save-n-Sleep, she'll spend $191.25. Use Cramer's Rule to determine how many sheets and pillow cases Agatha wants to buy.

4. **BRICKS** Linus owns three different types of brick that differ only in length. If he lines up 2 short, 1 medium, and 2 long bricks, the total length will be 45 inches. If he lines up 1 short, 2 medium, and 3 long bricks, the total length will be 59 inches. If he lines up 5 short, 1 medium, and 1 long brick, the total length will be 53 inches. Use Cramer's Rule to determine how long the different types of brick are.

PROMOTIONS For Exercises 5–7, use the following information.
A local zoo was trying to increase attendance by offering $2 for every child that came. However, the zoo insisted that there be at least 1 adult for every 8 children. A school decided to take advantage of the situation by sending 1 adult for every 8 children. Let c be the number of children and let a be the number of adults. Admission for adults was d dollars. The total cost of admission for everyone was $13.50.

5. Write a system of equations that describes the situation.

6. Is it possible that $d = 16$? Explain in terms of Cramer's Rule.

7. If adults were charged $20.50 for admission, how many adults and children went? Use Cramer's Rule to solve.

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4-7 Word Problem Practice

Identity and Inverse Matrices

1. **ROTATIONS** Suppose R represents a counterclockwise rotation about the origin by an angle of 45°. For what values of n is R^n equal to the inverse of R?

2. **SPECIAL MATRICES** Norman only likes working with matrices whose determinant is 1. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is such a matrix, what is its inverse?

3. **CRYPTOGRAPHY** A friend sends you a secret message that was coded using the coding matrix $C = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and the alphabet table.

CODE		
A 65	J 74	S 83
B 66	K 75	T 84
C 67	L 76	U 85
D 68	M 77	V 86
E 69	N 78	W 87
F 70	O 79	X 88
G 71	P 80	Y 89
H 72	Q 81	Z 90
I 73	R 82	–91

The message is 567|354|620|388. What is the decoded message?

4. **SELF-INVERSES** Phillip notices that any matrix with ones and negative ones on the diagonal and zeroes everywhere else has the property that it is its own inverse. Give an example of a 2 by 2 matrix that is its own inverse but has at least 1 nonzero number off the diagonal.

MATRIX OPERATIONS For Exercises 5–7, use the following information.
Garth is studying determinants and inverses of matrices in math class. His teacher suggests that there are some matrices with unique properties, and challenges the class to find such matrices and describe the properties found. Garth is curious about the matrix $G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

5. What is the determinant of G?

6. Does the inverse of G exist? Explain.

7. Determine a matrix operation that could be used to transform G into its Additive Identity matrix.

4-8 Word Problem Practice

Using Matrices to Solve Systems of Equations

1. **TEACHING** Paula is explaining matrices to her father. She writes down the following system of equations.

$$2x + y = 4$$
$$3x + y = 5.$$

Next, Paula shows her father the matrices that correspond to this system of equations. What are the matrices?

2. **FIND THE ERROR** Paula proceeds to solve the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

First, she finds the inverse.

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

Then she computes the answer.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \end{bmatrix}$$

When she checked her answer, she found that it was not correct. Where did she make a mistake?

3. **AGES** Hank, Laura, and Ned are ages h, l, and n, respectively. The sum of their ages is 15 years. Laura is one year younger than the sum of Hank and Ned's ages. Ned is three times as old as Hank. Use matrices to determine the age of each sibling.

4. **ANIMALS** Quinton takes care of dogs and chickens. There are a total of 28 animals, and altogether they have 68 legs. Use matrices to determine the number of dogs and the number of chickens in Quinton's care.

SALES For Exercises 5 and 6, use the following information.
The school film society is selling only granola bars and oranges to raise money at their movie review. They sell oranges for $1 and granola bars for $1.50. The person selling snacks recorded the total cost and number of items in each sale. The manager wants to know how many of each kind of snack each person bought.

5. Suppose a person spent d dollars to buy n items. Write a system of linear equations that relate d and n to the number of oranges r and granola bars g that the person purchased.

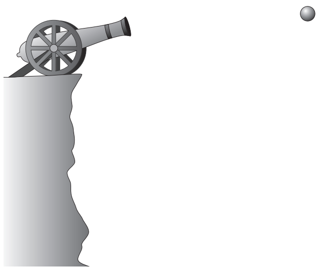
6. One recorded sale showed that 10 items were purchased for $13.00. How many oranges and granola bars were purchased for this sale?

Lesson 4-8

5-1 Word Problem Practice

Graphing Quadratic Functions

1. TRAJECTORIES A cannonball is launched from a cannon at the top of a cliff. If the path of the cannonball is traced on a piece of graph paper aligned so that the cannon is situated on the y-axis, the equation that describes the path is



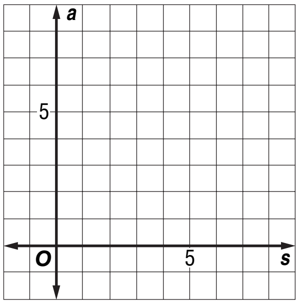
$$y = -\frac{1}{1600}x^2 + \frac{1}{2}x + 47,$$

where x is the horizontal distance from the cliff and y is the vertical distance above the ground in feet. How high above the ground is the cannon?

2. TICKETING The manager of a symphony computes that the symphony will earn $-40P^2 + 1100P$ dollars per concert if they charge P dollars for tickets. What ticket price should the symphony charge in order to maximize its profits?

3. ARCHES An architect decides to use a parabolic arch for the main entrance of a science museum. In one of his plans, the top edge of the arch is described by the graph of $y = -\frac{1}{4}x^2 + \frac{5}{2}x + \frac{75}{4}$. What are the coordinates of the vertex of this parabola?

4. FRAMING A frame company offers a line of square frames. If the side length of the frame is s, then the area of the opening in the frame is given by the function $a(s) = s^2 - 10s + 24$. Graph $a(s)$.



WALKING For Exercises 5–7, use the following information.

Canal Street and Walker Street are perpendicular to each other. Evita is driving south on Canal Street and is currently 5 miles north of the intersection with Walker Street. Jack is at the intersection of Canal and Walker Streets and heading east on Walker. Jack and Evita are both driving 30 miles per hour.

5. When Jack is x miles east of the intersection, where is Evita?

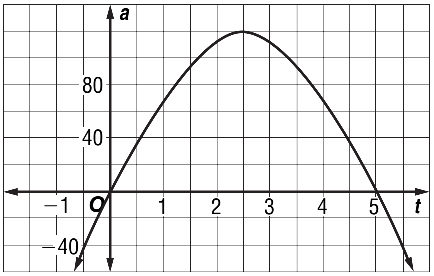
6. The distance between Jack and Evita is given by the formula $\sqrt{x^2 + (5 - x)^2}$. For what value of x are Jack and Evita at their closest? (Hint: Minimize the square of the distance.)

7. What is the distance of closest approach?

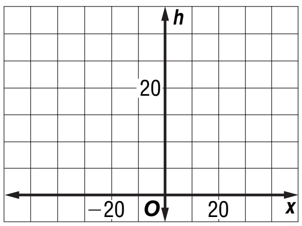
5-2 Word Problem Practice

Solving Quadratic Equations by Graphing

1. TRAJECTORIES David threw a baseball into the air. The function of the height of the baseball in feet is $h = 80t - 16t^2$, where t represents the time in seconds after the ball was thrown. Use this graph of the function to determine how long it took for the ball to fall back to the ground.

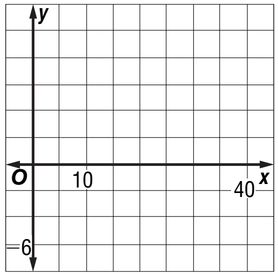


2. BRIDGES The main support for a bridge is a large parabolic arch. The height of the arch above the ground is given by the function $h = 32 - \frac{1}{50}x^2$, where h is the height in meters and x is the distance in meters from the center of the bridge. Graph this equation and describe where the arch touches the ground.



3. LOGIC Wilma is thinking of two numbers. The sum is 2 and the product is −24. Use a quadratic equation to find the two numbers.

4. RADIO TELESCOPES The cross-section of a large radio telescope is a parabola. The dish is set into the ground. The equation that describes the cross-section is $d = \frac{2}{75}x^2 - \frac{4}{3}x - \frac{32}{3}$, where d gives the depth of the dish below ground and x is the distance from the control center, both in meters. If the dish does not extend above the ground level, what is the diameter of the dish? Solve by graphing.

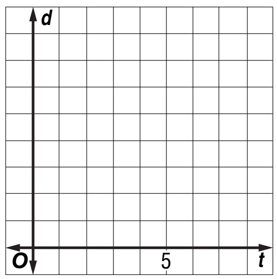


BOATS For Exercises 5 and 6, use the following information.
The distance between two boats is

$$d = \sqrt{t^2 - 10t + 35},$$

where d is distance in meters and t is time in seconds.

5. Make a graph of d^2 versus t.



6. Do the boats ever collide?

5-3 Word Problem Practice

Solving Quadratic Equations by Factoring

1. **FLASHLIGHTS** When Dora shines her flashlight on the wall at a certain angle, the edge of the lit area is in the shape of a parabola. The equation of the parabola is $y = 2x^2 + 2x - 60$. Factor this quadratic equation.

2. **SIGNS** David was looking through an old algebra book and came across this equation.

x^2 [redacted] $6x + 8 = 0$

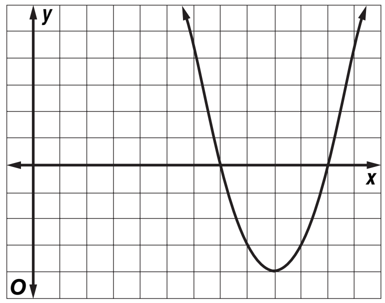
The sign in front of the 6 was blotted out. How does the missing sign depend on the signs of the roots?

3. **ROOTS** In the same algebra book that he was looking through in Exercise 2, David found another partially blotted out equation.

$x^2 + 21x$ [redacted] $0 = 0$

The book claims that one of the roots of the equation is 4. What must the other root be and what number is covered by the blot?

4. **PROGRAMMING** Ray is a computer programmer. He needs to find the quadratic function of this graph for an algorithm related to a game involving dice. Provide such a function.



ANIMATION For Exercises 5–7, use the following information.
A computer graphics animator would like to make a realistic simulation of tossed ball. The animator wants the ball to follow the parabolic trajectory represented by the quadratic equation $f(x) = -0.2(x + 5)(x - 5)$.

5. What are the solutions of $f(x) = 0$?

6. Write $f(x)$ in standard form.

7. If the animator changes the equation to $f(x) = -0.2x^2 + 20$, what are the solutions of $f(x) = 0$?

5-4 Word Problem Practice

Complex Numbers

1. SIGN ERRORS Jennifer and Jessica come up with different answers to the same problem. They had to multiply $(4 + \boldsymbol{i})(4 - \boldsymbol{i})$ and give their answer as a complex number. Jennifer claims that the answer is 15 and Jessica claims that the answer is 17. Who is correct? Explain.

2. COMPLEX CONJUGATES You have seen that the product of complex conjugates is always a real number. Show that the sum of complex conjugates is also always a real number.

3. PYTHAGOREAN TRIPLES If three integers a, b, and c, satisfy $a^2 + a^2 = c^2$, then they are called a *Pythagorean Triple*. Suppose that a, b, and c are a Pythagorean triple. Show that the real and imaginary parts of $(a + b\boldsymbol{i})^2$, together with the number c^2, form another Pythagorean triple.

4. ROTATIONS Complex numbers can be used to perform rotations in the plane. For example, if (x, y) are the coordinates of a point in the plane, then the real and imaginary parts of $\boldsymbol{i}(x + y\boldsymbol{i})$ are the horizontal and vertical coordinates of the 90° counterclockwise rotation of (x, y) about the origin. What are the real and imaginary parts of $\boldsymbol{i}(x + y\boldsymbol{i})$?

ELECTRICAL ENGINEERING For Exercises 5–7, use the following information.

Complex numbers can be used to describe the alternating current (AC) in an electric circuit like the one used in your home. Z, the impedance in an AC circuit, is related to the voltage V and the current I by the formula $Z = \frac{V}{I}$.

5. Find Z if $V = 5 + 2i$ and $I = 3i$.

6. Find Z if $V = 2 + 3i$ and $I = -3i$.

7. Find V if $Z = \frac{2 - 3i}{3}$ and $I = 3i$.

5-5 Word Problem Practice

Completing the Square

1. COMPLETING THE SQUARE Samantha needs to solve the equation

$$x^2 - 12x = 40.$$

What must she do to each side of the equation to complete the square?

2. SQUARE ROOTS Evan is asked to solve the equation $x^2 + 8x + 16 = 25$. He recognizes that the left-hand side of the equation is a perfect square trinomial. Factor the left-hand side.

3. COMPOUND INTEREST Nikki invested \$1000 in a savings account with interest compounded annually. After two years the balance in the account is \$1210. Use the compound interest formula $A = P(1 + r)^t$ to find the annual interest rate.

4. REACTION TIME Lauren was eating lunch when she saw her friend Jason approach. The room was crowded and Jason had to lift his tray to avoid obstacles. Suddenly, a glass on Jason's lunch tray tipped and fell off the tray. Lauren lunged forward and managed to catch the glass just before it hit the ground. The height h, in feet, of the glass t seconds after it was dropped is given by $h = -16t^2 + 4.5$. Lauren caught the glass when it was six inches off the ground. How long was the glass in the air before Lauren caught it?

5. PARABOLAS A parabola is modeled by $y = x^2 - 10x + 28$. Jane's homework problem requires that she find the vertex of the parabola. She uses the completing square method to express the function in the form $y = (x - h)^2 + k$, where (h,k) is the vertex of the parabola. Write the function in the form used by Jane.

AUDITORIUM SEATING For Exercises 6–8, use the following information.
The seats in an auditorium are arranged in a square grid pattern. There are 45 rows and 45 columns of chairs. For a special concert, organizers decide to increase seating by adding n rows and n columns to make a square pattern of seating $45 + n$ seats on a side.

6. How many seats are there after the expansion?

7. What is n if organizers wish to add 1000 seats?

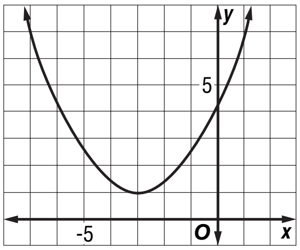
8. If organizers do add 1000 seats, what is the seating capacity of the auditorium?

NAME ____ DATE______________ PERIOD _____

5-6 Word Problem Practice

The Quadratic Formula and the Discriminant

1. PARABOLAS The graph of a quadratic equation of the form $y = ax^2 + bx + c$ is shown below.



Is the discriminant $b^2 - 4ac$ positive, negative, or zero?

2. TANGENT Kathleen is trying to find b so that the x-axis is tangent to the parabola $y = x^2 + bx + 4$. She finds one value that works, $b = 4$. Is this the only value that works? Explain.

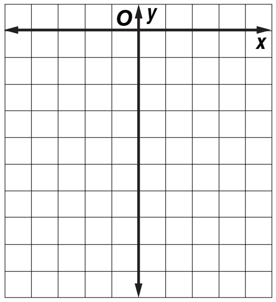
3. AREA Conrad has a triangle whose base has length $x + 3$ and whose height is $2x + 4$. What is the area of this triangle? For what values of x is this area equal to 210? Do all the solutions make sense?

4. EXAMPLES Give an example of a quadratic function $f(x)$ that has the following properties.

I. The discriminant of f is zero.

II. There is no real solution of the equation $f(x) = 10$.

Sketch the graph of $x = f(x)$.



TANGENTS For Exercises 5 and 6, use the following information.
The graph of $y = x^2$ is a parabola that passes through the point at (1, 1). The line $y = mx - m + 1$, where m is a constant, also passes through the point at (1, 1).

5. To find the points of intersection between the line $y = mx - m + 1$ and the parabola $y = x^2$, set $x^2 = mx - m + 1$ and then solve for x. Rearranging terms, this equation becomes $x^2 - mx + m - 1 = 0$. What is the discriminant of this equation?

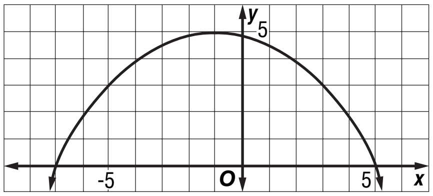
6. For what value of m is there only one point of intersection? Explain the meaning of this in terms of the corresponding line and the parabola.

NAME ____ DATE______________ PERIOD _____

5-7 Word Problem Practice

Analyzing Graphs of Quadratic Functions

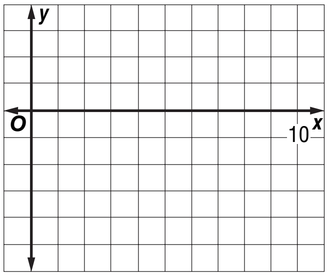
1. ARCHES A parabolic arch is used as a bridge support. The graph of the arch is shown below.



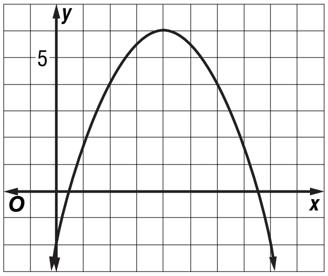
If the equation that corresponds to this graph is written in the form $y + a(x - h)^2 + k$, what are h and k?

2. TRANSLATIONS For a computer animation, Barbara uses the quadratic function $f(x) = -42(x - 20)^2 + 16800$ to help her simulate an object tossed on another planet. For one skit, she had to use the function $f(x + 5) - 8000$ instead of $f(x)$. Where is the vertex of the graph of $y = f(x + 5) - 8000$?

3. MIRRORS The cross-section of a reflecting telescope mirror is described by the parabola $y = \frac{1}{10}(x - 5)^2 - \frac{5}{2}$. Graph this parabola.



4. WATER JETS The graph shows the path of a jet of water.



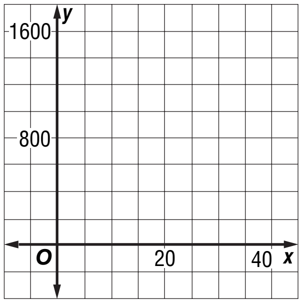
The equation corresponding to this graph is $y = a(x - h)^2 + k$. What are a, h, and k?

PROFIT For Exercises 5–7, use the following information.
A theater operator predicts that the theater can make $-4x^2 + 160x$ dollars per show if tickets are priced at x dollars.

5. Rewrite the equation $y = -4x^2 + 16x$ in the form $y = a(x - h)^2 + k$.

6. What is the vertex of the parabola and what is its axis of symmetry?

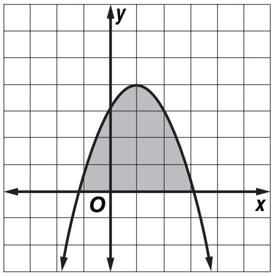
7. Graph the parabola.



5-8 Word Problem Practice

Graphing and Solving of Quadratic Inequalities

1. HUTS The space inside a hut is shaded in the graph. The parabola is described by the equation $y = -\frac{4}{5}(x - 1)^2 + 4$.



Write an inequality that describes the shaded region.

2. DISCRIMINANTS Consider the equation $ax^2 + bx + c = 0$. Assume that the discriminant is zero and that a is positive. What are the solutions of the inequality $ax^2 + bx + c \le 0$?

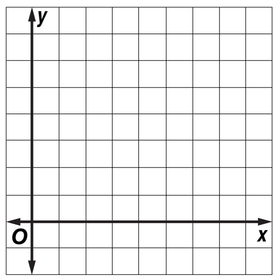
3. TOSSING Gail and Veronica are fixing a leak in a roof. Gail is working on the roof and Veronica is tossing up supplies to Gail. When Gail tosses up a tape measure, the height h, in feet, of the object above the ground t seconds after Gail tosses it is $h = -16t^2 + 32t + 5$. Gail can catch the object any time it is above 17 feet. How much time does Gail have to try to catch the tape measure?

4. KIOSKS Caleb is designing a kiosk by wrapping a piece of sheet metal with dimensions $x + 5$ inches by $4x + 8$ inches into a cylindrical shape. Ignoring cost, Caleb would like a kiosk that has a surface area of at least 4480 square inches. What values of x satisfy this condition?

TUNNELS For Exercises 5 and 6, use the following information.
An architect wants to use a parabolic arch as the entrance of a tunnel. She sketches the plan on a piece of graph paper. She would like the maximum height of the tunnel to be located at (4, 4), and she would like the origin to be on the parabola as well.

5. Write an equation for the desired parabola.

6. Write an inequality that describes the region above the parabola, part of which will be filled in with concrete. Graph this inequality.



Lesson 5-8

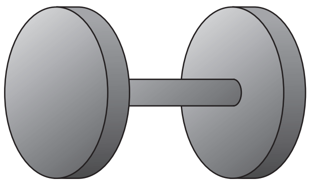
NAME ______________________________ DATE ____________ PERIOD _____

6-1 Word Problem Practice

Properties of Exponents

1. MASS Joseph operates a forklift. He is able to lift 4.72×10^3 kilograms with the forklift. There are 10^3 grams in 1 kilogram. How many grams is 4.72×10^3 kilograms? Express your answer in scientific notation.

2. DENSITY The density of an object is equal to its mass divided by its volume. A dumbbell has a mass of 9×10^3 grams and a volume of 1.2×10^3 cubic centimeters.



What is the density of the dumbbell?

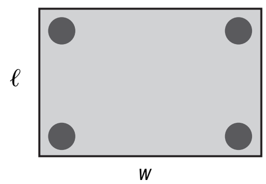
3. THE EARTH Earth's diameter is approximately 1.2756×10^4 kilometers. The surface area of a sphere can be found using the formula $SA = 4\pi r^2$.

What is the approximate surface area of Earth? Express your answer in scientific notation.

4. POPULATION As of November 2004, the United States Census Bureau estimated the population of the United States as 297,681,499 and the world population as 6,479,541,872. Write the ratio of the United States population to the world population in scientific notation.

GLASS TABLES For Exercises 5 and 6, use the following information.

Evan builds rectangular glass coffee tables. The area A of the tabletop is given by $A = \ell w$, where ℓ is the length of the table and w is the width of the table.



5. The larger the table surface, the thicker the glass must be. For this reason, the cost of the table glass is proportional to A^2. What is A^2 in terms of ℓ and w? Express your answer without using parentheses.

6. The cost per unit length is proportional to $\frac{A^2}{\ell}$. Express the cost per unit length in terms of ℓ and w. Express your answer in simplest form.

6-2 Word Problem Practice

Operations with Polynomials

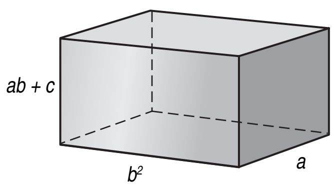
1. ROLLER COASTERS A roller coaster has a section of track that can be described mathematically by the expression

$$\frac{1}{50}(x^3 - x).$$

Is this a polynomial?

2. JUGGLING When balls are being juggled, the paths of the balls can be described mathematically. For a short period of time, the altitudes of two balls are described by the polynomials $-16t^2 + 7t + 4$ and $-16t^2 + 14t + 4$, where t represents time. What is the difference in altitudes between these two balls?

3. VOLUME The volume of a rectangular prism is given by the product of its length, width, and height. Samantha has a rectangular prism that has a length of b^2 units, a width of a units, and a height of $ab + c$ units.

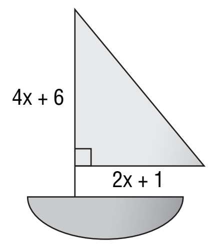


What is the volume of Samantha's rectangular prism? Express your answer in simplified form.

4. CONSTRUCTION A rectangular deck is built around a square pool. The pool has side length s. The length of the deck is 5 units longer than twice the side length of the pool. The width of the deck is 3 units longer than the side length of the pool. What is the area of the deck in terms of s?

SAIL BOATS For Exercises 5–7, use the following information.

Tamara requests a custom-made sail for her sailboat. The base of her triangular sail is $2x + 1$ and the height is $4x + 6$.



5. Find the area of the sail.

6. If Tamara wants a different color on each side of her sail, write a polynomial to represent the total amount of fabric she will need to make the sail.

7. Tamara decides she also wants a special trim for the hypotenuse of her triangular sail. Write an expression that describes the amount of trim she will need.

Lesson 6-2

NAME ____ DATE______________ PERIOD _____

6-3 Word Problem Practice

Dividing Polynomials

1. REMAINDERS Jordan divided the polynomial $x^4 + x - 6$ into the polynomial $p(x)$ yesterday. Today his work is smudged and he cannot read $p(x)$ or most of his answer. The only part he could read was the remainder $x + 4$. His teacher wants him to find $p(-3)$. What is $p(-3)$?

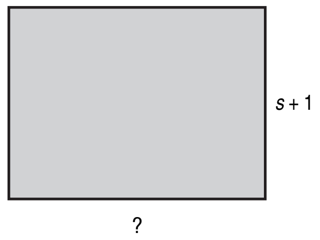
2. LONG DIVISION Dana used long division to divide $x^4 + x^3 + x^2 + x + 1$ by $x + 2$. Her work is shown below with three numbers missing.

$$
\begin{array}{r}
x^3 \; - \; x^2 \; + \; 3x \; - \; 5 \\
x + 2 \overline{)\, x^4 \; + \; x^3 \; + \; x^2 \; + \; x \; + \; 1} \\
\underline{(-)x^4 \; + \; 2x^3} \\
-x^3 \; + \; A \\
\underline{(-)\; -x^3 \; - \; 2x^2} \\
3x^2 \; + \; x \\
\underline{(-)\; 3x^2 \; + \; B} \\
-5x \; + \; 1 \\
\underline{(-)\; -5x \; - \; 10} \\
C
\end{array}
$$

What are A, B, and C?

3. AVERAGES Shelby is a statistician. She has a list of $n + 1$ numbers and she needs to find their average. Two of the numbers are n^3 and 2. Each of the other $n - 1$ numbers are all equal to 1. What is the average of these numbers?

4. AREA The area of a large rectangular sheet is $s^3 + 3s^2 + 4s + 1$ square inches.



If the length of the sheet is $s + 1$ inches, what is the width of the sheet?

NUMBER THEORY For Exercises 5-6, use the following information.

Mr. Collins has his class working with bases and polynomials. He wrote on the board that the number 1111 in base B has the value $B^3 + B^2 + B + 1$. The class was then given the following questions to answer.

5. The number 11 in base B has the value $B + 1$. What is 1111 (in base B) divided by 11 (in base B)?

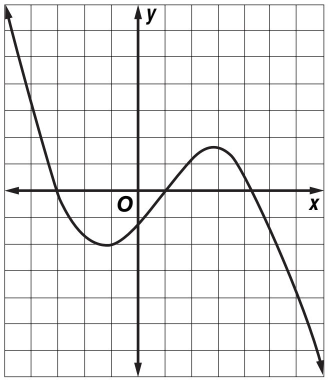
6. The number 111 in base B has the value $B^2 + B + 1$. What is 1111 (in base B) divided by 111 (in base B)?

6-4 Word Problem Practice

Polynomial Functions

1. MANUFACTURING A metal sheet is curved according to the shape of the graph of $f(x) = x^4 - 9x^2$. What is the degree of this polynomial?

2. GRAPHS Kendra graphed the polynomial $f(x)$ shown below.



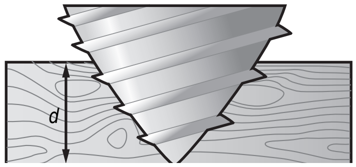
From this graph, describe the end behavior, degree, and sign of the leading coefficient.

3. PENTAGONAL NUMBERS The nth pentagonal number is given by the expression

$$\frac{n(3n - 1)}{2}.$$

What is the degree of this polynomial? What is the seventh pentagonal number?

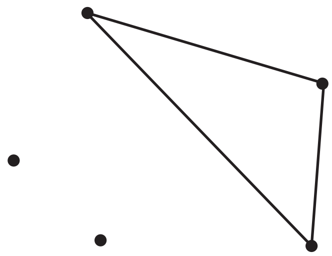
4. DRILLING A drill bit is shaped like a cone and used to make conical indentations in a piece of wood. The volume of wood removed depends on the depth of the indentation. If the depth is d millimeters, then the volume V of wood removed is $V = \pi d^3$. The formula for the depth d of the indentation being created is $d = t^2$, where t is the amount of time that it takes to reach the depth.



What is the volume of wood removed as a function of time t?

TRIANGLES For Exercises 5 and 6, use the following information.

Dylan drew n dots on a piece of paper making sure that no line contained 3 of the dots. The number of triangles that can be made using the dots as vertices is equal to $f(n) = \frac{1}{6}(n^3 - 3n^2 + 2n)$.



5. What is the degree of f?

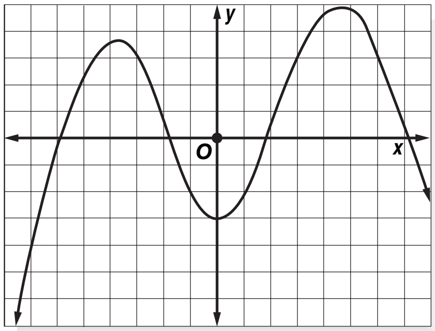
6. If Dylan drew 15 dots, how many triangles can be made?

NAME ____ DATE______________ PERIOD _____

6-5 Word Problem Practice

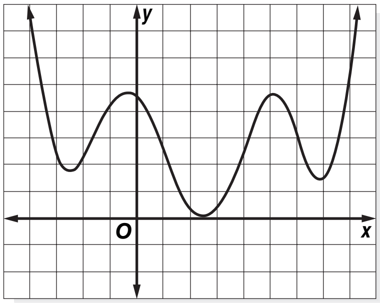
Analyze Graphs of Polynomial Functions

1. LANDSCAPES Jalen uses a fourth-degree polynomial to describe the shape of two hills in the background of a video game that he is helping to write. The graph of the polynomial is shown below.



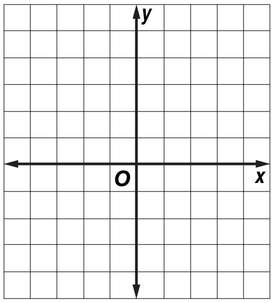
Estimate the x-coordinates at which the relative maxima and relative minima occur.

2. CANYONS The graph shows the cross section of an underwater canyon.



If you were to model this graph by a polynomial, what is the smallest degree that the equation could have?

3. VALUE A banker models the expected value of a company in millions of dollars by the formula $n^3 - 3n^2$, where n is the number of years in business. Sketch a graph of $v = n^3 - 3n^2$.

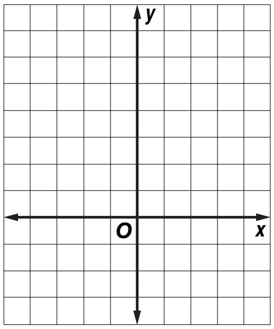


CONSECUTIVE NUMBERS For Exercises 4 and 5, use the following information.

Ms. Sanchez asks her students to write expressions to represent five consecutive integers. One solution is $x - 2, x - 1, x, x + 1$, and $x + 2$. The product of these five consecutive integers is given by the fifth degree polynomial $f(x) = x^5 - 5x^3 + 4x$.

4. For what values of x is $f(x) = 0$?

5. Sketch the graph of $y = f(x)$.



6-6 Word Problem Practice

Solving Polynomial Equations

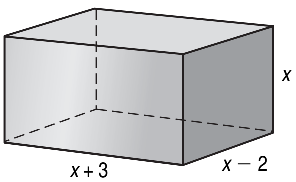
1. CODES Marisa has been trying to discover the secret code for a lock. After a long investigation, she discovers that the numbers in the secret code are solutions of the polynomial equation $x^4 - 68x^3 + 1557x^2 - 13770x + 37800 = 0$. After more work, Marisa found that $x^4 - 68x^3 + 1557x^2 - 13770x + 37800 = (x - 5)(x - 12)(x - 21)(x - 30)$. What are the numbers in the secret code?

2. OUTPUT Eduardo is a mechanical engineer. For one of his projects, he had to solve the polynomial equation

$$m^6 + 5m^3 - 10 = 0.$$

Write the polynomial $m^6 + 5m^3 - 10$ in quadratic form.

3. VOLUME Jacob builds a wooden box. The box is x inches high. The width is 3 inches more than the height, and the length is 2 inches less than the height. The volume of the box is 3 times the width.



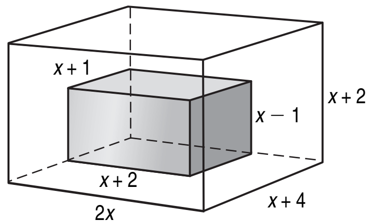
What is x?

4. ROBOTS A robot explorer's distance from its starting location is given by the polynomial $t^5 - 29t^3 + 100t$, where t is time measured in hours.

Factor this polynomial.

PACKAGING For Exercises 5-8, use the following information.

A small box is placed inside a larger box. The dimensions of the small box are $x + 1$ by $x + 2$ by $x - 1$. The dimensions of the larger box are $2x$ by $x + 4$ by $x + 2$.



5. Write an expression for the volume of the space inside the larger box but outside the smaller box.

6. If the volume of the space inside the larger box but outside the smaller box is equal to $33x + 162$ cubic units, what is x?

7. What is the volume of the smaller box?

8. What is the volume of the larger box?

NAME ____ DATE______________ PERIOD _____

6-7 Word Problem Practice

The Remainder and Factor Theorems

1. HEIGHT A ball tossed into the air follows a parabolic trajectory. Its height after t seconds is given by a polynomial of degree two with leading coefficient -16. Using synthetic substitution, Norman found that the polynomial evaluates to 0 for the values $t = 0$ and $t = 4$. What is the polynomial that describes the ball's height as a function of t?

2. SYNTHETIC SUBSTITUTION Branford evaluates the polynomial $p(x) = x^3 - 5x^2 + 3x + 5$ for a factor using synthetic substitution. Some of his work is shown below. Unfortunately, the factor and the solution have ink spots over it.

	1	−5	3	5
		11	66	759
	1	6	69	

What is the factor he solved for? What is the hidden solution?

3. PROFIT The profits of Clyde's Corporation can be modeled by the polynomial $P(y) = y^4 - 4y^3 + 2y^2 + 10y - 200$, where y is the number of years after the business was started. The chief financial officer wants to know the value of $P(10)$. Use synthetic substitution to determine $P(10)$. Show your work.

4. EXPONENTIALS The exponential function $t = e^x$ is a special function that you will learn about later. It is not a polynomial function. However, for small values of x, the value of e^x is very closely approximated by the polynomial function

$$e(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1.$$

Use synthetic substitution to determine $e(0.1)$. Show your work.

VOLUME For Exercises 5-7, use the following information.

The Jackson family just had a pool installed in their backyard. The volume of the pool is given by the polynomial

$$v(x) = x^3 + 10x^2 + 31x + 30.$$

5. Use synthetic division to show that $x + 2$ is a factor of $v(x)$. Show your work.

6. Factor $v(x)$ completely.

7. Determine $v(18)$ using any method you wish.

6-8 Word Problem Practice

Roots and Zeros

1. TABLES Li Pang made a table of values for the polynomial $p(x)$. Her table is shown below.

x	$p(x)$
−4	−3
−3	−1
−2	0
−1	2
0	0
1	4
2	0
3	2
4	5

Name three roots of $p(x)$.

2. ROOTS Ryan is an electrical engineer. He often solves polynomial equations to work out various properties of the circuits he builds. For one circuit, he must find the roots of a polynomial $p(x)$. He finds that $p(2 - 3\boldsymbol{i}) = 0$. Give two different roots of $p(x)$.

3. REAL ROOTS Madison is studying the polynomial $f(x) = x^6 - 14x^4 + 49x^2 - 36$. She knows that all of the roots of $f(x)$ are real. How many positive and how many negative roots are there? How are the set of positive roots and negative roots related to each other? Explain.

4. COMPLEX ROOTS Eric is a statistician. During the course of his work, he had to find something called the "eigenvalues of a matrix," which was basically the same as finding the roots of a polynomial. The polynomial was $x^4 + 6x^2 + 25$. One of the roots of this polynomial is $1 + 2i$. What are the other 3 roots? Explain.

QUADRILATERALS For Exercises 5-7, use the following information.

Shayna plotted the four vertices of a quadrilateral in the complex plane and then encoded the points in a polynomial $p(x)$ by making them the roots of $p(x)$. The polynomial $p(x)$ is $x^4 - 9x^3 + 27x^2 + 23x - 150$.

5. The polynomial $p(x)$ has one positive real root, and it is an integer. Find the integer.

6. Find the negative real root(s) of $p(x)$.

7. Find the complexl roots of $p(x)$.

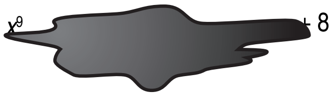
Lesson 6-8

NAME ____ DATE______________ PERIOD _____

6-9 Word Problem Practice

Rational Zero Theorem

1. ROOTS Paul was examining an old algebra book. He came upon a page about polynomial equations and saw the polynomial below.

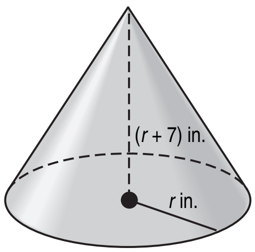


As you can see, all the middle terms were blotted out by an ink spill. What are all the possible rational roots of this polynomial?

2. IRRATIONAL CONSTANTS Cherie was given a polynomial whose constant term was $\sqrt{2}$. Is it possible for this polynomial to have a rational root? If it is not, explain why not. If it is possible, give an example of such a polynomial with a rational root.

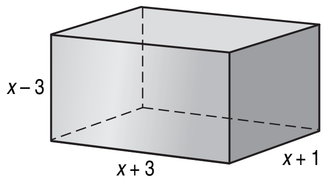
3. MARKOV CHAINS Tara is a mathematician who specializes in probability. In the course of her work, she needed to find the roots of the polynomial $p(x) = 288x^4 - 288x^3 + 106x^2 - 17x + 1$. What are the roots of $p(x)$?

4. PYRAMIDS Pedro made a pyramid out of construction paper. The base of the pyramid is a square with side length $3x$. The height of the pyramid is $x + 12$. The function for the volume of the pyramid is $V(x) = 3x^3 + 36x^2$. The actual volume of the pyramid is 168 cubic centimeters. What is the length of the sides of the base and height of the pyramid?



BOXES For Exercises 5–7, use the following information.

Devon made a box with length $x + 1$, width $x + 3$, and height $x - 3$.



5. What is the volume of Devon's box as a function of x?

6. What is x if the volume of the box is equal to 1001 cubic inches?

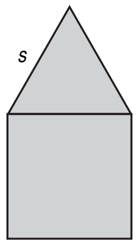
7. What is x if the volume of the box is equal to $14\frac{5}{8}$ cubic inches?

NAME ____ DATE_____________ PERIOD _____

7-1 Word Problem Practice

Operations on Functions

1. AREA Bernard wants to know the area of a figure made by joining an equilateral triangle and square along an edge. The function $f(s) = \frac{\sqrt{3}}{4}s^2$ gives the area of an equilateral triangle with side s. The function $g(s) = s^2$ gives the area of a square with side s. What function $h(s)$ gives the area of the figure as a function of its side length s?



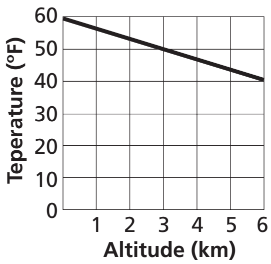
2. PRICING A computer company decides to continuously adjust the pricing of and discounts to its products in an effort to remain competitive. The function $P(t)$ gives the sale price of its Super2000 computer as a function of time. The function $D(t)$ gives the value of a special discount it offers to valued customers. How much would valued customers have to pay for one Super2000 computer?

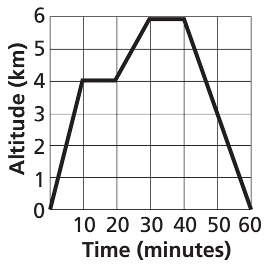
3. LAVA A freshly ejected lava rock immediately begins to cool down. The temperature of the lava rock in degrees Fahrenheit as a function of time is given by $T(t)$. Let $C(F)$ be the function that gives degrees Celsius as a function of degrees Fahrenheit. What function gives the temperature of the lava rock in degrees Celsius as a function of time?

4. ENGINEERING A group of engineers is designing a staple gun. One team determines that the speed of impact s of the staple (in feet per second) as a function of the handle length ℓ (in inches) is given by $s(\ell) = 40 + 3\ell$. A second team determines that the number of sheets N that can be stapled as a function of the impact speed is given by $N(s) = \frac{s - 10}{3}$. What function gives N as a function of ℓ?

HOT AIR BALLOONS For Exercises 5 and 6, use the following information.

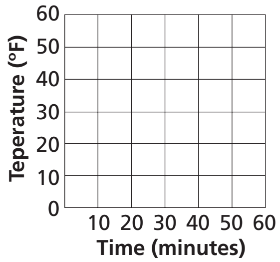
Hannah and Terry went on a one-hour hot air balloon ride. Let $T(A)$ be the outside air temperature as a function of altitude and let $A(t)$ be the altitude of the balloon as a function of time.





5. What function describes the air temperature Hannah and Terry felt at different times during their trip?

6. Sketch a graph of the function you wrote for Exercise 5 based on the graphs for $T(A)$ and $A(t)$ that are given.



NAME ____ DATE______________ PERIOD _____

7-2 Word Problem Practice

Inverse Functions and Relations

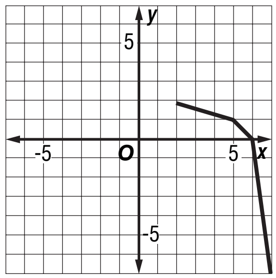
1. **VOLUME** Jason wants to make a spherical water cooler that can hold half a cubic meter of water. He knows that $V = \frac{4}{3}\pi r^3$, but he needs to know how to find r given V. Find this inverse function.

2. **EXERCISE** Alex began a new exercise routine. To gain the maximum benefit from his exercise, Alex calculated his maximum target heart rate using the function. $f(x) = 0.85(220 - x)$ where x represents his age. Find the inverse of this function.

3. **ROCKETS** The altitude of a rocket in feet as a function of time is given by $f(t) = 49t^2$, where $t \geq 0$. Find the inverse of this function and determine the times when the rocket will be 10, 100, and 1000 feet high. Round your answers to the nearest hundredth of a second.

4. **SELF-INVERTIBLE** Karen finds the incomplete graph of a function in the back of her engineering handbook. The function is graphed in the figure below.

   Karen knows that this function is its own inverse. Armed with this knowledge, extend the graph for values of x between -7 and 2.



PLANETS For Exercises 5 and 6, use the following information.

The approximate distance of a planet from the Sun is given by $d = T^{\frac{2}{3}}$ where d is distance in astronomical units and T is Earth years. An astronomical unit is the distance of the Earth from the Sun.

5. Solve for T in terms of d.

6. Pluto is about 39.44 times as far from the Sun as the Earth. About how many years does it take Pluto to orbit the Sun?

7-3 Word Problem Practice

Square Root Functions and Inequalities

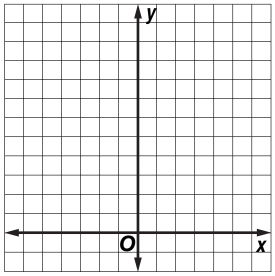
1. **SQUARES** Cathy is building a square roof for her garage. The roof will occupy 625 square feet. What are the dimensions of the roof?

2. **PENDULUMS** The period of a pendulum, the time it takes to complete one swing, is given by the formula $p = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum and g is acceleration due to gravity, 9.8 m/s^2. Find the period of a pendulum that is 0.65 meters long. Round to the nearest tenth.

3. **REFLEXES** Rachel and Ashley are testing one another's reflexes. Rachel drops a ruler from a given height so that it falls between Ashley's thumb and index finger. Ashley tries to catch the ruler before it falls through her hand. The time required to catch the ruler is given by $t = \frac{\sqrt{d}}{4}$ where d is measured *in feet*. Complete the table. Round your answers to the nearest hundredth.

Distance (in.)	Reflex Time (seconds)
3 in.	
6 in.	
9 in.	
12 in.	

4. **DISTANCE** Lance is standing at the side of a road watching a cyclist go by. The distance between Lance and the cyclist as a function of time is given by $d = \sqrt{9 + 36t^2}$. Graph this function. Find the distance between Lance and the cyclist after 3 seconds.



STARS For Exercises 5-7, use the following information.

The intensity of the light from an object varies inversely with the square of the distance. In other words, $I = \frac{k}{d^2}$.

5. Solve the equation to find d in terms of I.

6. Two stars give off the same amount of light. However, from Earth their intensities differ. Let I_1 and I_2 be their intensities and let d_1 and d_2 be their respective distances from Earth. What is the ratio of d_2 to d_1?

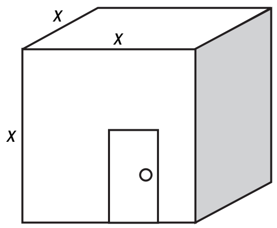
7. If one star appears 9 times as intense as the other, how much closer is it to Earth?

Lesson 7-3

7-4 Word Problem Practice

*n*th *Roots*

1. CUBES Cathy is building a cubic storage room. She wants the volume of the space to be 1728 cubic feet. What should the dimensions of the cube be?



2. ASTRONOMY A special form of Kepler's Third Law of Planetary Motion is given by $a = \sqrt[3]{P^2}$ where a is the average distance of an object from the Sun in AU (astronomical units) and P is the period of the orbit in years. If an object is orbiting the Sun with a period of 12 years, what is its distance from the Sun?

3. TUNING Two notes are an octave apart if the frequency of the higher note is twice the frequency of the lower note. Casey is experimenting with an instrument that has 6 notes tuned so that the frequency of each successive note increases by the same factor and the first and last note are an octave apart. By what factor does the frequency increase from note to note?

4. MARKUPS A wholesaler manufactures a part for D dollars. The wholesaler sells the part to a dealer for a P percent markup. The dealer sells the part to a retailer at an additional P percent markup. The retailer in turn sells the part to its customers marking up the price yet another P percent. What is the price that customers see? If the customer buys the part for $80 and the markup is 40%, what approximately was the original cost to make the part?

PENDULUMS For Exercises 5 and 6, use the following information.

Mr. Topalian's physics class is experimenting with pendulums. The class learned the formula $T = 2\pi\sqrt{\frac{L}{g}}$ which relates the time T that it takes for a pendulum to swing back and forth based on gravity g equal to 32 feet per second squared, and the length of the pendulum L in feet.

5. One group in the class made a 2-foot long pendulum. Use the formula to determine how long it will take for their pendulum to swing back and forth.

6. Another group decided they wanted to make a pendulum that took about 1.76 seconds to go back and forth. Approximately how long should their pendulum be?

7-5 Word Problem Practice

Operations with Radical Expressions

1. CUBES Cathy has a rectangular box with dimensions 20 inches by 35 inches by 40 inches. She would like to replace it with a box in the shape of a cube but with the same volume. What should the length of a side of the cube be? Express your answer as a radical expression in simplest form.

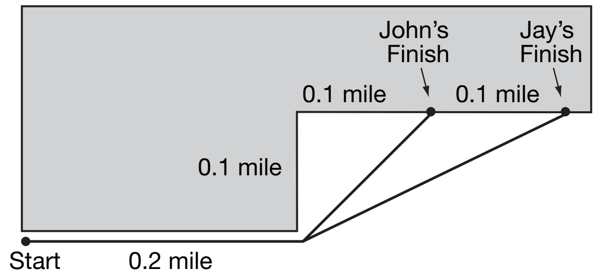
2. PHYSICS The speed of a wave traveling over a string is given by $\frac{\sqrt{t}}{\sqrt{u}}$, where t is the tension of the string and u is the density. Rewrite the expression in simplest form by rationalizing the denominator.

3. TUNING With each note higher on a piano, the frequency of the pitch increases by a factor of $\sqrt[12]{2}$. What is the ratio of the frequencies of two notes that are 6 steps apart on the piano? What is the ratio of the frequencies of two notes that are 9 steps apart on the piano? Express your answers in simplest form.

4. LIGHTS Suppose a light has a brightness intensity of I_1 when it is at a distance of d_1 and a brightness intensity of I_2 when it is at a distance of d_2. These quantities are related by the equation $\frac{d_2}{d_1} = \sqrt{\frac{I_1}{I_2}}$. Suppose $I_1 = 50$ units and $I_2 = 24$ units. What would $\frac{d_2}{d_1}$ be? Express your answer in simplest form.

RACING For Exercises 5 and 6, use the following information and express your answers in simplest form.

John is Jay's younger brother. They like to race and, after many races, they found that the fairest race was to run slightly different distances. They both start at the same place and run straight for 0.2 miles. Then they head for different finishes. In the figure, John and Jay's finishing paths are marked.



This time, they tied. Both of them finished the race in exactly 4 minutes.

5. If John and Jay continued at their average paces during the race, exactly how many minutes would it take them each to run a mile? Express your answer as a radical expression in simplest form.

6. Exactly how many times as fast did John run as Jay?

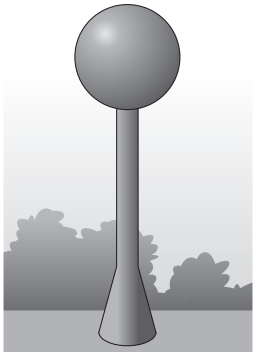
Lesson 7-5

7-6 Word Problem Practice

Rational Exponents

1. SQUARING THE CUBE A cube has side length s. What is the side length of the square that has an area equal to the volume of this cube? Write your answer using rational exponents.

2. WATER TOWER A large water tower stores drinking water in a big spherical tank. The mayor of the town decides that the water tower must be replaced with a larger tank. Town residents insist that the new tower be a sphere. If the new tank will hold 10 times as much water as the old tank, how many times long should the radius of the new tank be compared to the old tank? Write your answer using rational exponents.



3. BALLOONS A spherical balloon is being inflated faster and faster. The volume of the balloon as a function of time is $9\pi t^2$. What is the radius of the balloon as a function of time? Write your answer using rational exponents.

4. INTEREST Rita opened a bank account that accumulated interest at the rate of 1% compounded annually. Her money accumulated interest in that account for 8 years. She then took all of her money out of that account and placed it into another account that paid 5% interest compounded annually. After 4 years, she took all of her money out of that account. What single interest rate when compounded annually would give her the same outcome for those 12 years? Round your answer to the nearest hundredth of a percent.

CELLS For Exercises 5-7, use the following information.

The number of cells in a cell culture grows exponentially. The number of cells in the culture as a function of time is given by the expression $N\left(\frac{6}{5}\right)^t$ where t is measured in hours and N is the initial size of the culture.

5. After 3 hours, there were 1728 cells in the culture. What is N?

6. How many cells were in the culture after 20 minutes? Express your answer in simplest form.

7. How many cells were in the culture after 2.5 hours? Express your answer in simplest form.

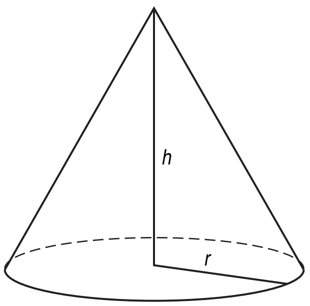
NAME ____ DATE______________ PERIOD _____

7-7 Word Problem Practice

Rational Equations and Inequalities

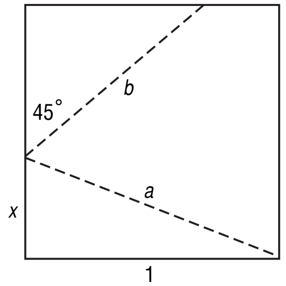
1. SIGNS A sign painter must spend $\$8n^{\frac{2}{3}} + 400$ to make n signs. How many signs can the painter make for \$1200?

2. LATERAL AREA The lateral area of a cone with base radius r and height h is given by the formula $L = \pi r\sqrt{r^2 + h^2}$. A cone has a lateral area of 65π square units and a base radius of 5 units.

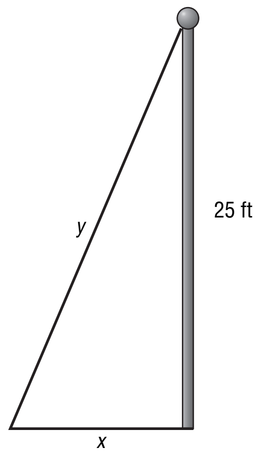


What is the height of the cone?

3. ORIGAMI Georgia wants to fold a square piece of paper into an equilateral triangle. She wants to locate the distance x up the side of the square where she can make the fold indicated by the dashed line in the figure so that $a = b$. From geometry class, she knows that $a = \sqrt{1 + x^2}$ and $b = \sqrt{2}(1 - x)$. So the equation she must solve is $\sqrt{1 + x^2} = \sqrt{2}(1 - x)$. What is x?



4. TETHERS A tether is being attached to a 25-foot pole in such a way that $x + y = 50$. By the Pythagorean Theorem, the distance $y = \sqrt{x^2 + 25^2}$. What must x be?



RANGE For Exercises 5 and 6, use the following information.

An asteroid is passing near Earth. If Earth is located at the origin of a coordinate plane, the path that the asteroid will trace out is given by $y = \frac{17}{x}$, $x > 0$. One unit corresponds to one million miles. Carl learns that he will be able to see the asteroid with his telescope when the asteroid is within $\frac{145}{12}$ million miles of Earth.

5. Write an expression that gives the distance of the asteroid from Earth as a function of x.

6. For what values of x will the asteroid be in range of Carl's telescope?

NAME ______________________ DATE ____________ PERIOD _____

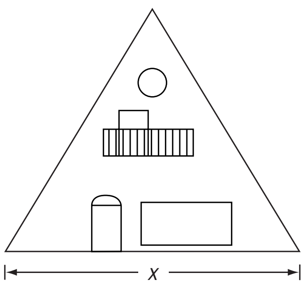
8-1 Word Problem Practice

Multiplying and Dividing Rational Expressions

1. **JELLY BEANS** A large vat contains G green jelly beans and R red jelly beans. A bag of 100 red and 100 green jelly beans is added to the vat. What is the new ratio of red to green jelly beans in the vat?

2. **MILEAGE** Beth's car gets 15 miles per gallon in the city and 26 miles per gallon on the highway. Beth uses C gallons of gas in the city and H gallons of gas on the highway. Write an expression for the average number of miles per gallon that Beth gets with her car in terms of C and H.

3. **HEIGHT** The front face of a Nordic house is triangular. The surface area of the face is $x^2 + 3x + 10$ where x is the base of the triangle.



What is the height of the triangle in terms of x?

4. **OIL SLICKS** David was moving a drum of oil around his circular outdoor pool when the drum cracked, and oil spilled into the pool. The oil spread itself evenly over the surface of the pool. Let V denote the volume of oil spilled and let r be the radius of the pool. Write an equation for the thickness of the oil layer.

RUNNING For Exercises 5 and 6, use the following information.

Harold runs to the local food mart to buy a gallon of soy milk. Because he is weighed down on his return trip, he runs slower on the way back. He travels S_1 feet per second on the way to the food mart and S_2 feet per second on the way back. Let d be the distance he has to run to get to the food mart. Remember: distance = rate $\times$ time.

5. Write an equation that gives the total time Harold spent running for this errand.

6. What speed would Harold have to run if he wanted to maintain a constant speed for the entire trip yet take the same amount of time running?

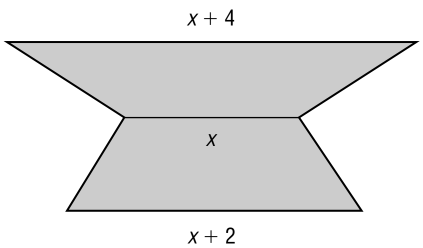
8-2 Word Problem Practice

Adding and Subtracting Rational Expressions

1. SQUARES Susan's favorite perfect square is s^2 and Travis' is t^2, where s and t are whole numbers. What perfect square is guaranteed to be divisible by both Susan's and Travis' favorite perfect squares regardless of their specific value?

2. ELECTRIC POTENTIAL The electrical potential function between two electrons is given by a formula that has the form $\frac{1}{r} + \frac{1}{1-r}$. Simplify this expression.

3. TRAPEZOIDS The cross section of a stand consists of two trapezoids stacked one on top of the other.



The total area of the cross section is x^2 square units. Assuming the trapezoids have the same height, write an expression for the height of the stand in terms of x. Put your answer in simplest form. (Recall that the area of a trapezoid with height h and bases b_1 and b_2 is given by $\frac{1}{2}h(b_1 + b_2)$.)

4. FRACTIONS In the seventeenth century, Lord Brouncker wrote down a most peculiar mathematical equation:

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{\ddots}}}}$$

This is an example of a continued fraction. Simplify the continued fraction

$$n + \cfrac{1}{n + \cfrac{1}{n}}.$$

RELAY RACE For Exercises 5-7, use the following information.

Mark, Connell, Zack, and Moses run the 4 by 400 meter relay together. Their average speeds were s, $s + 0.5$, $s - 0.5$, and $s + 1$ meters per second, respectively.

5. What were their individual times for their own legs of the race?

6. Write an expression for their time as a team. Write your answer as a ratio of two polynomials.

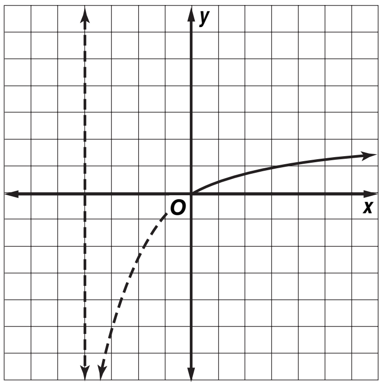
7. If s was 6 meters per second, what was the team's time? Round your answer to the nearest second.

Lesson 8-2

8-3 Word Problem Practice

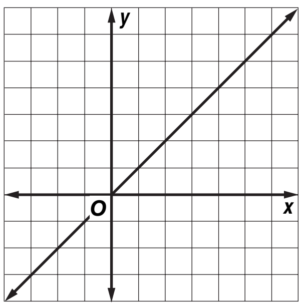
Graphing Rational Expressions

1. **ROAD TRIP** Robert and Sarah start off on a road trip from the same house. During the trip, Robert's and Sarah's cars remain separated by a constant distance. The graph shows the ratio of the distance Sarah has traveled to the distance Robert has traveled. The dotted line shows how this graph would be extended to hypothetical negative values of x. What does the x-coordinate of the vertical asymptote represent?



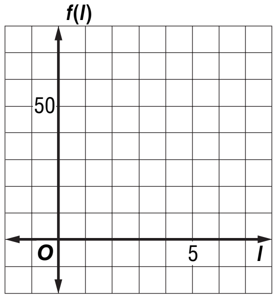
2. **GRAPHS** Alma graphed the function

$f(x) = \frac{x^2 - 4x}{x - 4}$ below.



There is a problem with her graph. Explain how to correct it.

3. **FINANCE** A quick way to get an idea of how many years before a savings account will double at an interest rate of I percent compounded annually, is to divide I into 72. Sketch a graph of the function $f(I) = \frac{72}{I}$.



4. **NEWTON** Sir Isaac Newton studied the rational function

$$f(x) = \frac{ax^3 + bx^2 + cx + d}{x}.$$

Assuming that $d \neq 0$, where will there be a vertical asymptote to the graph of this function?

BATTING AVERAGES For Exercises 5 and 6, use the following information.

Josh has made 26 hits in 80 at bats for a batting average of .325. Josh goes on a hitting streak and makes x hits in the next $2x$ at bats.

5. What function describes Josh's batting average during this streak?

6. What is the equation of the horizontal asymptote to the graph of the function you wrote for Exercise 5? What is its meaning?

8-4 Word Problem Practice

Direct, Joint, and Inverse Variation

1. DIVING The height that a diver leaps above a diving board varies directly with the amount that the tip of the diving board dips below its normal level. If a diver leaps 44 inches above the diving board when the diving board tip dips 12 inches, how high will the diver leap above the diving board if the tip dips 18 inches?

2. PARKING LOT DESIGN As a general rule, the number of parking spaces in a parking lot for a movie theater complex varies directly with the number of theaters in the complex. A typical theater has 30 parking spaces for each theater. A businessman wants to build a new cinema complex on a lot that has enough space for 210 parking spaces. How many theaters should the businessman build in his complex?

3. RENT An apartment rents for m dollars per month. If n students share the rent equally, how much would each student have to pay? How does the cost per student vary with the number of students? If 2 students have to pay \$700 each, how much money would each student have to pay if there were 5 students sharing the rent?

4. PAINTING The cost of painting a wall varies directly with the area of the wall. Write a formula for the cost of painting a rectangular wall with dimensions ℓ by w. With respect to ℓ and w, does the cost vary directly, jointly, or inversely?

HYDROGEN For Exercises 5-7, use the following information.

The cost of a hydrogen storage tank varies directly with the volume of the tank. A laboratory wants to purchase a storage tank shaped like a block with dimensions L by W by H.

5. Fill in the missing spaces in the following table from a brochure of various tank sizes.

Hydrogen Tank Dimensions (inches)			Cost
L	*W*	*H*	
36	36	36	
18		24	\$150
24	24	72	\$800

6. The hydrogen tank must fit in a shelf that has a fixed height and depth. How does the cost of the hydrogen storage tank vary with the width of tank with fixed depth and height?

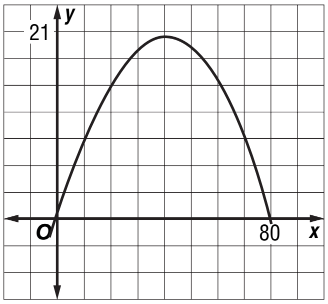
7. How much would a spherical tank of radius 24 inches cost? (Recall that the volume of a sphere is given by $\frac{4}{3}\pi r^3$, where r is the radius.)

8-5 Word Problem Practice

Classes of Functions

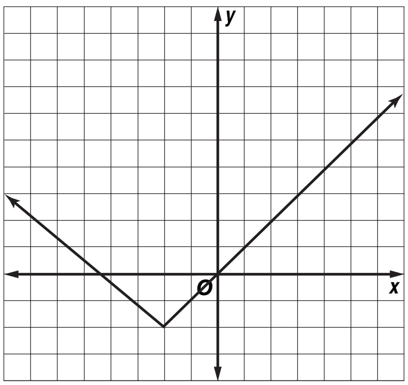
1. **STAIRS** What type of a function has a graph that could be used to model a staircase?

2. **GOLF BALLS** The trajectory of a golf ball hit by an astronaut on the moon is described by the function $f(x) = -0.0125(x - 40)^2 + 20$.



Describe the shape of this trajectory.

3. **RAVINE** The graph shows the cross-section of a ravine.



What kind of function is represented by the graph? Write the function.

4. **LEAKY FAUCETS** A leaky faucet leaks 1 milliliter of water every second. Write a function that gives the number of milliliters leaked in t seconds as a function of t. What type of function is it?

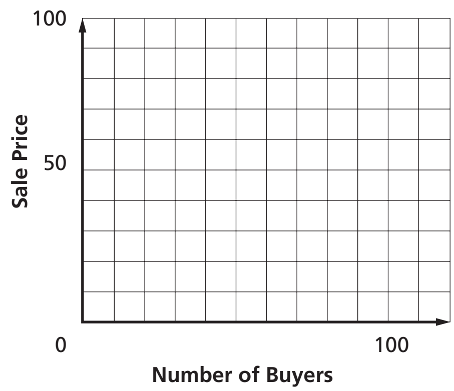
PUBLISHING For Exercises 5-8, use the following information.

Kate has just finished writing a book that explains how to sew your own stuffed animals. She hopes to make $1000 from sales of the book because that is how much it would cost her to go to the European Sewing Convention. Each book costs $2 to print and assemble. Let P be the selling price of the book. Let N be the number of people who will buy the book.

5. Write an equation that relates P and N if she earns exactly $1,000 from sales of the book.

6. Solve the equation you wrote for Exercise 5 for P in terms of N.

7. What kind of function is P in terms of N? Sketch a graph of P as a function of N.



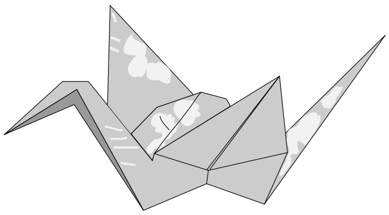
8. If Kate thinks that 125 people will buy her book, how much should she charge for the book?

8-6 Word Problem Practice

Solving Rational Equations and Inequalities

1. HEIGHT Serena can be described as being 8 inches shorter than her sister Malia, or as being 12.5% shorter than Malia. In other words, $\frac{8}{H + 8} = \frac{1}{8}$, where H is Serena's height in inches. How tall is Serena?

2. CRANES For a wedding, Paula wants to fold 1000 origami cranes.



She does not want to make anyone fold more than 15 cranes. In other words, if N is the number of people enlisted to fold cranes, Paula wants $\frac{1000}{\mathrm{N}} \leq 15$. What is the minimum number of people that will satisfy this inequality?

3. RENTAL Carlos and his friends rent a car. They split the \$200 rental fee evenly. Carlos, together with just two of his friends, decide to rent a portable DVD player as well, and split the \$30 rental fee for the DVD player evenly among themselves. Carlos ends up spending \$50 for these rentals. Write an equation involving N, the number of friends Carlos has, using this information. Solve the equation for N.

4. PROJECTILES A projectile target is launched into the air. A rocket interceptor is fired at the target. The ratio of the altitude of the rocket to the altitude of the projectile t seconds after the launch of the rocket is given by the formula $\frac{333t}{-32t^2 + 420t + 27}$. At what time are the target and interceptor at the same altitude?

FLIGHT TIME For Exercises 5 and 6, use the following information.

The distance between New York City and Los Angeles is about 2500 miles. Let S be the airspeed of a jet. The wind speed is 100 miles per hour. Because of the wind, it takes longer to fly one way than the other.

5. Write an equation for S if it takes 2 hours and 5 minutes longer to fly between New York and Los Angeles against the wind versus flying with the wind.

6. Solve the equation you wrote in Exercise 5 for S.

7. Write an equation and find how much longer to fly between New York and Los Angeles if the wind speed increases to 150 miles per hour and the airspeed of the jet is 525 miles per hour.

9-1 Word Problem Practice

Exponential Functions

1. **GOLF BALLS** A golf ball manufacturer packs 3 golf balls into a single package. Three of these packages make a gift box. Three gift boxes make a value pack. The display shelf is high enough to stack 3 value packs one on top of the other. Three such columns of value packs make up a display front. Three display fronts can be packed in a single shipping box and shipped to various retail stores. How many golf balls are in a single shipping box?

2. **FOLDING** Kay folds a piece of paper in half over and over until it is at least 25 layers thick. How many times does she fold the paper in half and how many layers are there?

3. **SUBSCRIPTIONS** Subscriptions to an online arts and crafts club have been increasing by 20% every year. The club began with 40 members. Make a graph of the number of subscribers over the first 5 years of the club's existence.

4. **TENNIS SHOES** The cost of a pair of tennis shoes increases about 5.1% every year. About how much would a $50 pair of tennis shoes cost 25 years from now?

MONEY For Exercises 5–7, use the following information.

Sam opened a savings account that accrues compound interest at a rate of 3% annually. Let P be the initial amount Sam deposited and let t be the number of years the account has been open.

5. Write an equation to find A, the amount of money in the account after t years. Assume that Sam made more additional deposits and no withdrawals.

6. If Sam opened the account with $500 and made no deposits or withdrawals, how much is in the account 10 years later?

7. What is the least number of years it would take for such an account to double in value?

9-2 Word Problem Practice

Logarithms and Logarithmic Functions

1. FISH The population of silver carp has been growing in the Mississippi River. About every 3 years, the population doubles. Write logarithmic expression that gives the number of years it will take for the population to increase by a factor of ten.

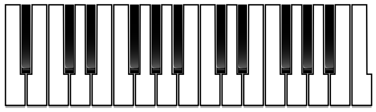
2. POWERS Haley tries to solve the equation $\log_4 2x = 32$. She got the wrong answer. What was her mistake? What should the correct answer be?

1.	$\log_4 2x = 5$
2.	$2x = 4^5$
3.	$x = 2^5$
4.	$x = 32$

3. DIGITS A computer programmer wants to write a formula that tells how many digits there are in a number n, where n is a positive integer. For example, if $n = 343$, the formula should evaluate to 3 and if $n = 10{,}000$, the formula should evaluate to 5. Suppose $8 \leq \log_{10} n < 9$. How many digits does n have?

4. LOGARITHMS Pauline knows that $\log_b x = 3$ and $\log_b y = 5$. She knows that this is the same as knowing that $b^3 = x$ and $b^5 = y$. Multiply these two equations together and rewrite it as an equation involving logarithms. What is $\log_b xy$?

MUSIC For Exercises 5 and 6, use the following information.



The first note on a piano keyboard corresponds to a pitch with a frequency of 27.5 cycles per second. With every successive note you go up the white and black keys of a piano, the pitch multiplies by a factor of $\sqrt[12]{2}$. The formula for the frequency of the pitch sounded when the nth note up the keyboard is played is given by

$$n = 1 + 12 \log_2 \frac{f}{27.5}.$$

5. The pitch that orchestras tune to is the A above middle C. It has a frequency of 440 cycles per second. How many notes up the piano keyboard is this A?

6. Another pitch on the keyboard has a frequency of 1760 cycles per second. How many notes up the keyboard will this be found?

9-3 Word Problem Practice

Properties of Logarithms

1. **MENTAL COMPUTATION** Jessica has memorized $\log_5 2 \approx 0.4307$ and $\log_5 3 \approx 0.6826$. Using this information, to the nearest thousandth, what power of 5 is equal to 6?

2. **POWERS** A chemist is formulating an acid. The pH of a solution is given by

$$-\log_{10} C,$$

where C is the concentration of hydrogen ions. If the concentration of hydrogen ions is increased by a factor of 100, what happens to the pH of the solution?

3. **LUCKY MATH** Frank is solving a problem involving logarithms. He does everything correctly except for one thing. He mistakenly writes

$$\log_2 a + \log_2 b = \log_2 (a + b).$$

However, after substituting the values for a and b in his problem, he amazingly still gets the right answer! The value of a was 11. What must the value of b have been?

4. **LENGTHS** Charles has two poles. One pole has length equal to $\log_7 21$ and the other has length equal to $\log_7 25$. Express the length of both poles joined end to end as the logarithm of a single number.

SIZE For Exercises 5-7, use the following information.

Alicia wanted to try to quantify the terms *puny, tiny, small, medium, large, big, huge,* and *humongous.* She picked a number of objects and classified them with these adjectives of size. She noticed that the scale seemed exponential. Therefore, she came up with the following definition. Define S to be $\frac{1}{3}\log_3 V$, where V is volume in cubic feet.

Then use the following table to find the appropriate adjective.

S satisfies	Adjective
$-2 \le S < -1$	tiny
$-1 \le S < 0$	small
$0 \le S < 1$	medium
$1 \le S < 2$	large
$2 \le S < 3$	big
$3 \le S < 4$	huge

5. Derive an expression for S applied to a cube in terms of ℓ where ℓ is the side length of a cube.

6. How many cubes, each one foot on a side, would have to be put together to get an object that Alicia would call "big"?

7. How likely is it that a large object attached to a big object would result in a huge object, according to Alicia's scale.

Lesson 9-3

9-4 Word Problem Practice

Common Logarithms

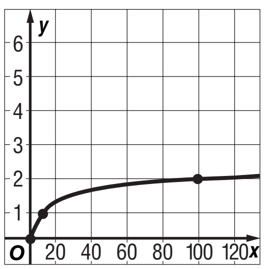
1. OTHER BASES Jamie needs to figure out what $\log_2 3$ is, but she only has a table of common logarithms. In the table, she finds that $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.4771$. Using this information, to the nearest thousandth, what is $\log_2 3$?

2. PH The pH of a solution is given by

$$-\log_{10} C,$$

where C is the concentration of hydrogen ions in moles per liter. A solution of baking soda creates a hydrogen ion concentration 5×10^{-9} of mole per liter. What is the pH of a solution of baking soda? Round your answer to the nearest tenth.

3. GRAPHING The graph of $y = \log_{10} x$ is shown below. Use the fact that $\frac{1}{\log_{10} 2} \approx 3.32$ to sketch a graph of $y = \log_2 x$ on the same graph.



4. SCIENTIFIC NOTATION When a number n is written in scientific notation, it has the form $n = s \times 10^p$, where s is a number greater than or equal to 1 and less than 10 and p is an integer. Show that $p \leq \log_{10} n < p + 1$.

LOG TABLE For Exercises 5 and 6, use the following information.

Marjorie is looking through some old science books owned by her grandfather. At the back of one of them, there is a table of logarithms base 10. However, the book is worn out and some of the entries are unreadable.

Table of Common Logarithms (4 decimal places of accuracy)	
x	$\log_{10} x$
2	0.3010
3	0.4771
4	?
5	0.6989
6	?

5. Approximately what are the missing entries in the table? Round off your answers to the nearest thousandth.

6. How can you use this table to determine $\log_{10} 1.5$?

9-5 Word Problem Practice

Base e and Natural Logarithms

1. **INTEREST** Horatio opens a bank account that pays 2.3% annual interest compounded continuously. He makes an initial deposit of 10,000. What will be the balance of the account in 10 years? Assume that he makes no additional deposits and no withdrawals.

2. **INTEREST** Janie's bank pays 2.8% annual interest compounded continuously on savings accounts. She placed $2000 in the account. How long will it take for her initial deposit to double in value? Assume that she makes no additional deposits and no withdrawals. Round your answer to the nearest quarter year.

3. **BACTERIA** A bacterial population grows exponentially, doubling every 72 hours. Let P be the initial population size and let t be time in hours. Write a formula using the natural base exponential function that gives the size of the population as a function of P and t.

4. **POPULATION** The equation $A = A_0e^{rt}$ describes the growth of the world's population where A is the population at time t, A_0 is the population at $t = 0$, and r is the annual growth rate. How long will take a population of 6.5 billion to increase to 9 billion if the annual growth rate is 2%?

MONEY MANAGEMENT For Exercises 5–7, use the following information.

Linda wants to invest $20,000. She is looking at two possible accounts. Account A is a standard savings account that pays 3.4% annual interest compounded continuously. Account B would pay her a fixed amount of $200 every quarter.

5. If Linda can invest the money for 5 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?

6. If Linda can invest the money for 10 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?

7. If Linda can invest the money for 20 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?

9-6 Word Problem Practice

Exponential Growth and Decay

1. **PROGRAMMING** For reasons having to do with speed, a computer programmer wishes to model population size using a natural base exponential function. However, the programmer is told that the users of the program will be thinking in terms of the annual percentage increase. Let r be the percentage that the population increases each year. Find the value for k in terms of r so that

$$e^k = 1 + r.$$

2. **CARBON DATING** Archeologists uncover an ancient wooden tool. They analyze the tool and find that it has 22% as much carbon–14 compared to the likely amount that it contained when it was made. Given that the half-life of carbon–14 is about 5730 years, about how old is the artifact? Round your answer to the nearest 100 years.

3. **POPULATION** The doubling time of a population is d years. The population size can be modeled by an exponential equation of the form Pe^{kt}, where P is the initial population size and t is time. What is k in terms of d?

4. **POPULATION** Louisa read that the population of her town has increased steadily at a rate of 2% each year. Today, the population of her town has grown to 68,735. Based on this information, what was the population of her town 100 years ago?

CONSUMER AWARENESS For Exercises 5–7, use the following information.

Jason wants to buy a brand new high-definition (HD) television. He could buy one now because he has \$7000 to spend, but he thinks that if he waits, the quality of HD televisions will improve. His \$7000 earns 2.5% interest annually compounded continuously. The television he wants to buy costs \$5000 now, but the cost increases each year by 7%.

5. Write a natural base exponential function that gives the value of Jason's account as a function of time t.

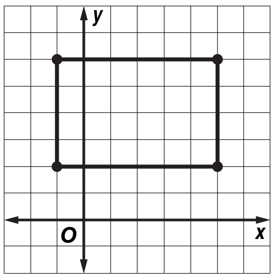
6. Write a natural base exponential function that gives the cost of the television Jason wants as a function of time t.

7. In how many years will the cost of the television exceed the value of the money in Jason's account? In other words, how much time does Jason have to decide whether he wants to buy the television? Round your answer to the nearest tenth of a year.

10-1 Word Problem Practice

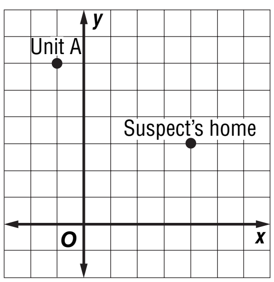
Midpoint and Distance Formulas

1. EXHIBITS Museum planners want to place a statue directly in the center of their Special Exhibits Room. Suppose the room is placed on a coordinate plane as shown. What are the coordinates of the center of this room?



2. WALKING Laura starts at the origin. She walks 8 units to the right and then 12 units up. How far away from the origin is she? Round your answer to the nearest tenth.

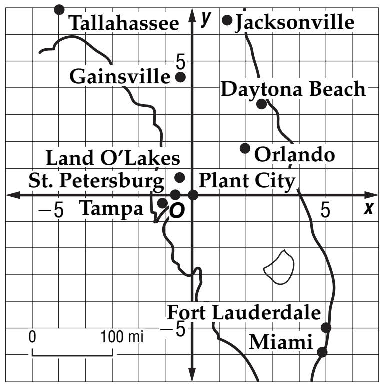
3. SURVEILLANCE A grid is superimposed on a map of the area directly surrounding the home of a suspect. Detectives want to position themselves on opposite sides of the suspect's house. Coordinates are assigned to the suspect's home. Unit A is positioned at $(-1, 6)$ on the coordinate plane. Where should Unit B be located so that the suspect's home is centered between the two units?



4. AIRPLANES A grid is superimposed on a map of Texas. Dallas has coordinates (200, 5) and Amarillo has coordinates $(-100, 208)$. If each unit represents 1 mile, how long will it take a plane flying at an average speed of 410 miles per hour to fly directly from Dallas to Amarillo? Round your answer to the nearest tenth of an hour.

TRAVEL For Exercises 5 and 6, use the following information and the figure below.

The Martinez family is planning a trip from their home in Fort Lauderdale to Tallahassee. They plan to stop overnight at a location about halfway between the two cities.



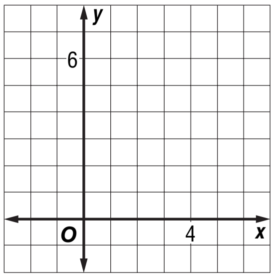
5. What are the coordinates of the point halfway between Tallahassee and Fort Lauderdale? Which of the cities on the map is closest to this point?

6. How many miles is it from Fort Lauderdale to Tallahassee? Round your answer to the nearest mile.

10-2 Word Problem Practice

Parabolas

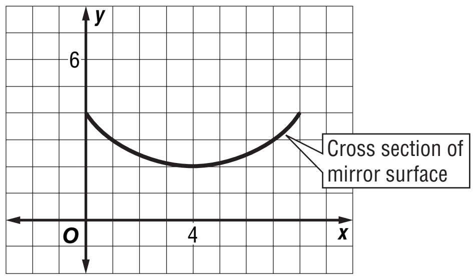
1. **PROJECTILE** A projectile follows the graph of the parabola $y = -\frac{3}{2}x^2 + 6x$. Sketch the path of the projectile by graphing the parabola.



2. **COMMUNICATION** David has just made a large parabolic dish whose cross section is based on the graph of the parabola $y = 0.25x^2$. Each unit represents one foot and the diameter of his dish is 4 feet. He wants to make a listening device by placing a microphone at the focus of the parabola. Where should the microphone be placed?

3. **BRIDGES** A bridge is in the shape of a parabola that opens downward. The equation of the parabola to model the arch of the bridge is given by $y = -\frac{x^2}{24} + \frac{5}{6}x + \frac{11}{6}$, where each unit is equivalent to 1 yard. The x-axis is the ground level. What is the maximum height of the bridge above the ground?

4. **TELESCOPES** An astronomer is working with a large reflecting telescope. The reflecting mirror in the telescope has the parabolic cross section shown in the graph whose equation is given by $y = \frac{1}{8}(x - 4)^2 + 2$. Each unit represents 1 meter. The astronomer is standing at the origin. How far from the focus of the parabola is the point on the mirror directly over the astronomer's head?



BRIDGES For Exercises 5 and 6, use the following information.

Part of the Sydney Harbor Bridge in Sydney, Australia, can be modeled by a parabolic arch. If each unit corresponds to 10 meters, the arch would pass through the points at $(-25, 5)$, $(0, 10)$, and $(25, 5)$.

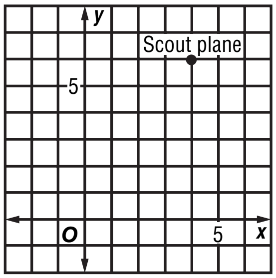
5. Write the equation of the parabola to model the arch.

6. Identify the coordinates of the focus of this parabola.

10-3 Word Problem Practice

Circles

1. RADAR A scout plane is equipped with radar. The boundary of the radar's range is given by the circle $(x - 4)^2 + (y - 6)^2 = 4900$. Each unit corresponds to one mile. What is the maximum distance that an object can be from the plane and still be detected by its radar?



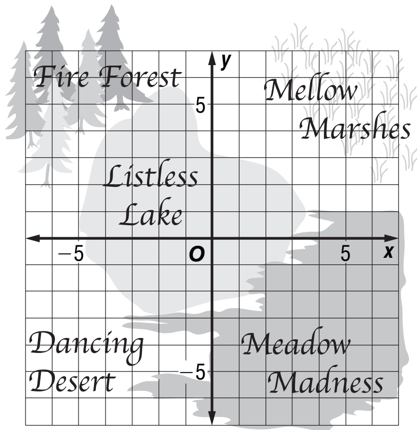
2. STORAGE An engineer uses a coordinate plane to show the layout of a side view of a storage building. The y-axis represents a wall and the x-axis represents the floor. A 10-meter diameter cylinder rests on its side flush against the wall. On the side view, the cylinder is represented by a circle in the first quadrant that is tangent to both axes. Each unit represents 1 meter. What is the equation of this circle?

3. FERRIS WHEEL The Texas Star, the largest Ferris wheel in North America, is located in Dallas, Texas. It weighs 678,554 pounds and can hold 264 riders in its 44 gondolas. The Texas Star has a diameter of 212 feet. Use the rectangular coordinate system with the origin on the ground directly below the center of the wheel and write the equation of the circle that models the Texas Star.

4. POOLS The pool on an architectural floor plan is given by the equation $x^2 + 6x + y^2 + 8y = 0$. What point on the edge of the pool is farthest from the origin?

TREASURE For Exercises 5 and 6, use the following information.

A mathematically inclined pirate decided to hide the location of a treasure by marking it as the center of a circle given by an equation in non-standard form.



The secret circle can be represented by:

$$x^2 + y^2 - 2x + 14y + 49 = 0.$$

5. Rewrite the equation of the circle in standard form.

6. Draw the circle on the map. Where is the treasure?

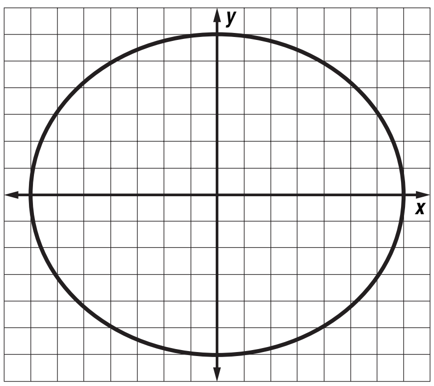
10-4 Word Problem Practice

Ellipses

1. PERSPECTIVE A graphic designer uses an ellipse to draw a circle from the horizontal perspective. The equation used is $\frac{x^2}{25} + y^2 = 1$. Graph this ellipse.

2. ECHOES The walls of an elliptical room are given by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Two people want to stand at the foci of the ellipse so that they can whisper to each other without anybody else hearing. What are the coordinates of the foci?

3. FLASHLIGHTS Daniella ended up doing her math homework late at night. To avoid disturbing others, she worked in bed with a pen light. One problem asked her to draw an ellipse. She noticed that her pen light created an elliptical patch of light on her paper, so she simply traced the outline of the patch of light. The outline of the ellipse is shown below. What is the equation of this ellipse in standard form?



4. ASTRONOMY The orbit of an asteroid is given by the equation $\frac{x^2}{400} + \frac{y^2}{441} = 1$, where each unit represents one astronomical unit (i.e. the distance from Sun to Earth). What are the lengths of the major and minor axes of the orbit?

MODELING For Exercises 5 and 6, use the following information.

James wants to try to make an ellipse using a piece of string 26 inches long. He tacks the two ends down 10 inches apart. He then takes a pen and pulls the string taut. He keeps the string taut and pulls the pen around the tacks. By doing this, he creates an ellipse.

5. Determine the lengths of the major and minor axes of the ellipse that James drew.

6. If a coordinate grid is overlaid on the ellipse so that the tacks are located at (5, 0) and (−5, 0), what is the equation of the ellipse in standard form?

10-5 Word Problem Practice

Hyperbolas

1. **LIGHTHOUSES** The location of a lighthouse is represented by the origin of a coordinate plane. A boat in the distance appears to be on a collision course with the lighthouse. However, the boat veers off and turns away at the last moment, avoiding the rocky shallows. The path followed by the boat is modeled by a branch of the hyperbola with equation $\frac{x^2}{900} - \frac{y^2}{400} = 1$. If the unit length corresponds to a yard, how close did the boat come to the lighthouse?

2. **FIND THE ERROR** Curtis was trying to write the equation for a hyperbola with a vertical transverse axis of length 10 and conjugate axis of length 6. The equation he got was $\frac{y^2}{9} - \frac{x^2}{25} = 1$. Did he make a mistake? If so, what did he do wrong?

3. **MIRROR** At a carnival, designers are planning a funhouse. They plan to put a large hyperbolic mirror inside this funhouse. They design the mirror's hyperbolic cross section on graph paper using a hyperbola with a horizontal transverse axis. The asymptotes are to be $y = 9x$ and $y = -9x$ so the mirror is somewhat shallow. They also want the vertices to be 1 unit from the origin. What equation should they use for the hyperbola?

4. **ASTRONOMY** Astronomers discover a new comet. They study its path and discover that it can be modeled by a branch of a hyperbola with equation $4x^2 - 40x - 25y^2 = 0$. Rewrite this equation in standard form and find the center of the hyperbola.

LIGHTNING For Exercises 5–7, use the following information.

Brittany and Kirk were talking on the phone when Brittany heard the thunder from a lightning bolt outside. Eight seconds later, she could hear the same thunder over the phone. Brittany and Kirk live 2 miles apart and sounds travels about 1 mile every 5 seconds.

5. On a coordinate plane, assume that Brittany is located at $(-1, 0)$ and Kirk is located at $(1, 0)$. Write an equation using the Distance Formula that describes the possible locations of the lightning strike.

6. Rewrite the equation you wrote for Exercise 5 so it is in the standard form for a hyperbola.

7. Which branch of the hyperbola corresponds to the places where the lightning bolt might have struck?

10-6 Word Problem Practice

Conic Sections

1. **MISSING INFORMATION** Rick began reading a book on conic sections. He came to this passage and discovered an inkblot covering part of an equation.

> For example, although it may not be obvious, the equation below describes a circle.
>
> $$7x^2 - 12x + \quad y^2 - 16y - 94 = 0$$
>
> To see that it is a circle, observe that the

Based on the information in the passage and your own knowledge of conic sections, what number is being covered by the inkblot?

2. **HEADLIGHTS** The light from the headlight of a car is in the shape of a cone. The axis of the cone is parallel to the ground. What shape does the edge of the lit region form on the road, assuming that the road is flat and level?

3. **REASONING** Jason has been struggling with conic sections. He decides he needs more practice, but he needs to have a way of making practice equations. He decides to use an equation of the form

$$Ax^2 + By^2 = 1,$$

where A and B are determined by rolling a pair of dice. After several rolls, he begins to realize that this system is not good enough because some conic sections never appear. Which types of conic section cannot occur using his method?

4. **MIRROR** A painter used a can of spray paint to make an image. The boundary of the image is described by the equation

$$4x^2 - 16x + y^2 - 6y + 21 = 0.$$

Put this equation into standard form and describe whether the curve is a circle, ellipse, parabola, or hyperbola.

NONSTANDARD EQUATIONS

For Exercises 5–7, use the following information.

Consider the equation $xy = 1$.

5. Are there any solutions of this equation that lie on the x- or y-axis?

6. Sketch a graph of the solutions of the equation.

7. Assuming that the equation represents a conic section, based on the graph, which type of conic section is it?

10-7 Word Problem Practice

Solving Quadratic Systems

1. GRAPHIC DESIGN A graphic designer is drawing an ellipse and a line. The ellipse is drawn so that it appears on top of the line. In order to determine if the line is partially covered by the ellipse, the program solves for simultaneous solutions of the equations of the line and the ellipse. Complete the following table.

No. of Intersections	Covered? Y/N
0	
1	
2	

2. ORBITS An asteroid travels in an elliptical orbit. If the orbit of Earth is also an ellipse, what is the maximum number of times the asteroid could cross the orbit of Earth?

3. CIRCLES An artist is commissioned to complete a painting of only circles. She wants to include all possible ways circles can relate. What are the possible numbers of intersection points between two circles? For each case, sketch two distinct circles that intersect with the corresponding number of points. Explain why more intersections are not possible.

4. COLLISION AVOIDANCE An object is traveling along a hyperbola given by the equation $\frac{x^2}{9} - \frac{y^2}{36} = 1$. A probe is launched from the origin along a straight-line path. Mission planners want the probe to get closer and closer to the object, but never hit it. There are two straight lines that meet their criteria. What are they?

TANGENTS For Exercises 5 and 6, use the following information.

An architect wants a straight path to run from the origin of a coordinate plane to the edge of an elliptically shaped patio so that the pathway forms a tangent to the ellipse. The ellipse is given by the equation

$$\frac{(x-6)^2}{12} + \frac{y^2}{96} = 1.$$

5. Using the equation $y = mx$ to describe the path, substitute into the equation for the ellipse to get a quadratic equation in x.

6. Solve for m in the equation you found for Exercise 5.

11-1 Word Problem Practice

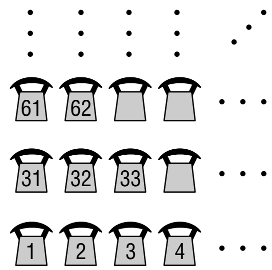
Arithmetic Sequences

1. ALLOWANCES Mark has saved $370 for a scooter and continues to save his weekly allowance of $10. Find the amount Mark will have saved after 7 weeks.

2. GRAPHS A financial officer is making a graph of a company's financial performance for the month. The vertical axis is labeled "Monthly Profit." The values range from 5400 to 6900. There is not enough space along the vertical axis to write all the numbers between 5400 and 6900, so the financial officer decides to write only 7 numbers, evenly spaced, starting at 5400 and ending at 6900. What should the numbers along the vertical axis be?

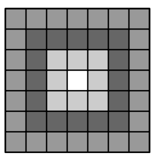
3. BIKING City planners want to mark a bike trail with posts that give the distance along the trail to City Hall. The trail begins 37.2 miles from City Hall and ends at City Hall. Write a formula for the number of miles on the nth post if posts are placed every half mile starting at 37.2 miles and decreasing along the way to City Hall.

4. SEATING Kay is trying to find her seat in a theater. The seats are numbered sequentially going left to right. Each row has 30 seats.



The figure shows some of the chairs in the left corner near the stage. Kay is at seat 129, but she needs to find seat 219. She notices that the seat numbers in a fixed column form an arithmetic sequence. What are the numbers of the next 4 seats in the same column as seat 129 going away from the stage? Where does Kay have to go to find her seat? In what row and column is her seat?

RINGS For Exercises 5-7, use the figure of expanding square rings.



5. How many small squares are in the first few square rings in the figure?

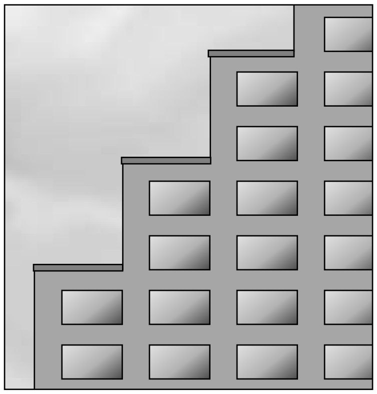
6. If the pattern is continued, write a formula for the number of squares in the nth ring.

7. What is the side length of the nth ring?

11-2 Word Problem Practice

Arithmetic Series

1. WINDOWS A side of an apartment building is shaped like a steep staircase. The windows are arranged in columns. The first column has 2 windows, the next has 4, then 6, and so on. How many windows are on the side of the apartment building if it has 15 columns?

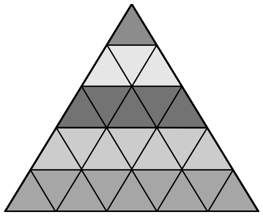


2. WEIGHTS Nathan has a collection of barbells for his home gym. He has 2 barbells for every 5 pounds starting at 5 pounds and going up to 80 pounds. What is the total weight of all his barbells?

3. TRAINING Matthew is training to run a marathon. He runs 20 miles his first week of training. Each week, he increases the number of miles he runs by 4 miles. How many total miles did he run in 8 weeks of training?

4. VOLUNTEERING Maryland Public Schools requires all high school students to complete 75 hours of volunteer service as a condition for graduation. One school includes grades 1-12, with 50 students in each grade. The school decides that students in grade g will volunteer $0.25g$ hours per week of their time. How many hours will all the school's students collectively donate to charity each week?

TRIANGLES For Exercises 5-7, use the following information.



A triangle is made of congruent equilateral triangles as shown in the figure.

5. Starting from the top, each colored row of triangles has more and more triangles. Write a formula for the number of triangles in row n.

6. If the large triangle consists of N rows of small triangles, how many small triangles are there in the large triangle? Write your answer using sigma notation.

7. Evaluate the sum you wrote for Exercise 6.

Lesson 11-2

11-3 Word Problem Practice

Geometric Sequences

1. INVESTMENT Beth deposits $1500 into a retirement account that pays an APR of 8% compounded yearly. Assuming Beth makes no withdrawals, how much money will she have in her account after 20 years?

2. CAKE Lauren has a piece of cake. She decides she wants to save some for later, so she eats half of it. Each time she returns to what remains, she only eats half of what is left. After her nth serving of ever smaller portions of cake, how much of the piece remains?

3. MOORE'S LAW Gordon Moore, co-founder of Intel, suggested that the number of transistors on a square inch of integrated circuit in a computer chip would double every 18 months. Assuming Moore's law is true, how many times as many transistors would you expect on a square inch of integrated circuit every 18 months for the next 6 years?

4. MONGEESE A population of mongeese has been growing by 20% every year. If the initial population size was 5000 mongeese, what is the size of the mongoose population after n years? How many years will it take, roughly, for the mongoose population to exceed 10,000 mongeese?

BIOLOGY For Exercises 5–7, use the following information.

Mitosis is a process of cell division that results in two identical daughter cells from a single parent cell. The table illustrates the number of cells produced after each of the first 5 cell divisions.

Division Number	0	1	2	3	4	5
Number of Cells	1	2	4	8	16	32

5. Do the entries in the "Number of Cells" row form a geometric series? If so, find r.

6. Write an expression to find the nth term of the sequence.

7. Find the number of cells after 100 divisions.

Lesson 11-3

11-4 Word Problem Practice

Geometric Series

1. BASE 10 When the common ratio of a geometric series is 10, the sum is sometimes easier to compute because we use a decimal number system. For example, what is the sum of $1 + 10 + 10^2 + 10^3 + 10^4 + 10^5$?

2. INVITATIONS Amanda wants to host a party. She invites 3 friends and tells each of them to invite 3 of their friends. The 3 friends do invite 3 others and ask each of them to invite 3 more people. This invitation process goes on for 5 generations of invitations. Including herself, how many people can Amanda expect at her party?

3. TRAINING Arnold lifts weights. He does three bench press workouts each week. For each workout, he lifts a weight 12 times. The first week he starts with 50 pounds. Each week he increases the amount that he lifts by 10%. After 10 weeks, what is the total amount of weight that Arnold has lifted during his bench press workouts? Round your answer to the nearest pound.

4. TEACHING A teacher teaches 8 students how to fold an origami model. Each of these students goes on to teach 8 students of their own how to fold the same model. If this teaching process goes on for n generations, how many people will know how to fold the origami model?

Generation	1	2	3	4	5	*n*
Number of People Taught	1	8	64	512	4096	?

CAREERS For Exercises 5-7, use the following information.

Mary begins her new career as a professor. She begins with a salary of $50,000. Every year, her salary increases by 7%.

5. What is Mary's salary for her nth year?

6. Use sigma notation to give an expression for the total income she will receive from the university after N years.

7. What will be her total income from the university after 20 years?

11-5 Word Problem Practice

Infinite Geometric Series

1. PARADOX If the formula for the sum of a geometric series is applied to the series whose first term is 1 and common ratio is 2, the result is the equation $-1 = 1 + 2 + 2^2 + 2^3 + \ldots$. Is this equality really true? Explain.

2. BOUNDS Can the sum of an infinite geometric series whose first term is 1 be as large as we wish?

3. BASES The infinite repeating decimal 0.999… is equal to 1. This can be shown by using the sum of a geometric series with common ratio $\frac{1}{10}$ and first term $\frac{9}{10}$. In a similar vein, compute the sum of the infinite geometric series $\frac{b-1}{b} + \frac{b-1}{b^2} + \frac{b-1}{b^3} + \cdots$, where b is a positive integer greater than 1. How is this sum related to the fact that $0.999\ldots = 1$?

4. CLIMBING A robot is designed to climb a wall each time a button is pressed. The first time the button is pressed, it climbs 10 feet. Each time after, the robot climbs only 75% of what it climbed the last time. What is the smallest upper limit on how high the robot can climb?

INSTALLMENTS For Exercises 5-7, use the following information.

Jade lends Jack a 100-pound chunk of pure gold for one year. After one year, she wants to start getting the gold back. One year later, Jack begins returning the gold, by giving Jade 1 pound of gold. The next day, Jack gives her 0.99 pounds of gold. The next day, Jack gives her $(0.99)^2$ or 0.9801 pounds of gold. Each successive day, Jack gives 0.99 times as much gold as the previous day.

5. How much gold does Jade get back on the nth day that Jack begins returning the gold.

6. How much gold has Jade received after 10 days? 100 days? Infinitely many days? Round your answers to the nearest hundredth of a pound.

7. Will Jade have all her gold back at any specific date in the future? Explain.

11-6 Word Problem Practice

Recursion and Special Sequences

1. GEOMETRIC SEQUENCES The geometric sequence with first term a and common ratio r goes like this: a, ar, ar^2, ar^3, etc. It happens that this sequence can also be seen from the point of view of iterative sequences. What function $f(x)$ can be used to define the geometric sequence above iteratively?

2. BACTERIA All the bacteria in a bacterial culture divide in two every hour. Also, every hour, 1,000 bacteria are removed from the culture. If the initial population consisted of 1,100 bacteria, what are the population sizes every hour for the next four hours?

3. WORK The company that Robert works for has a policy where the number of hours you have to work one week depends on the number of hours worked the previous week. If you worked h hours one week, then the next week you must work at least $80 - h$ hours. Robert worked 20 hours his first week with the company. From then on, he always worked the minimum number of hours required of him. Describe the number of hours Robert worked from week to week.

4. GEOMETRY A sequence of triangular shapes is made using squares as shown in the figure.

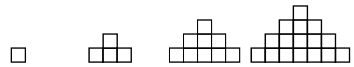
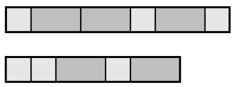


Figure 1 Figure 2 Figure 3 Figure 4

Let x_n be the number of squares to make the nth figure. Write a recursive formula for x_n.

PATHS For Exercises 5 and 6, use the following information.

Gregory makes walking paths out of two different rectangles. One is a 1-yard by 1-yard square and the other is a 1-yard by 2-yard rectangle. He makes paths by lining up the squares and rectangles as shown in the figure.



Gregory wants to know how many different paths he can make of a fixed length. Let a_n denote the number of paths he can make of length n yards.

5. What are the first 5 values of a_n?

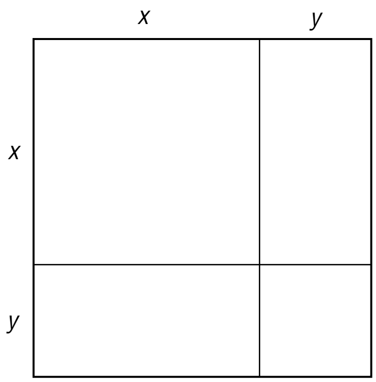
6. Write a recursive formula for a_n. Explain.

NAME ____ DATE______________ PERIOD _____

11-7 Word Problem Practice

The Binomial Theorem

1. AREA The square shown has a side length of $x + y$. The area must therefore be $(x + y)^2 = x^2 + xy + xy + y^2$. Each of these four terms corresponds to a different part of the area. Place each term in the corresponding region of the square.



2. POWERS The binomial theorem states that

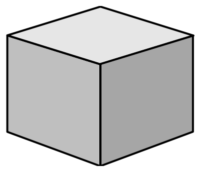
$$(x + y)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^{n-k}y^k.$$

Explain what this implies about powers of 2 if you substitute $x = y = 1$ into the equation.

3. COMBINATIONS Helen left home and went to the bank, the library, the post office, the pet store, the supermarket, and then returned home. Of the six paths of her journey, she took the bus 3 times and walked the other 3. How many different sequences of walking and riding the bus might she have taken?

4. SYMMETRY Each row of Pascal's triangle is like a palindrome. That is, the numbers read the same left to right as they do right to left. Explain why this is the case.

VOLUME For Exercises 5 and 6, use the following figure of a cube.



5. Expand $(x + y)^3$ using the binomial Theorem.

6. Make a picture similar to the one used in Exercise 1 for the cube. For the three-dimensional cube, it helps to make a blow-up version of the drawing.

11-8 Word Problem Practice

Proof and Mathematical Induction

1. **AREA** Cathy claims that there are only 4 pairs of consecutive odd prime numbers, namely, (3, 5), (5, 7), (11, 13), and (17, 19). Is this true or false? If it is true, prove it. If it is false, give a counterexample.

2. **PROOFS** Mrs. Smith has written the following "proof" on the board. Mrs. Smith asks her students to verify her work.
   For all positive integer n,
   $2 + 4 + 6 + \ldots + (2n) = n^2 + n + 1$.
   Mrs. Smith's proof goes like this:
   "Assume that the identity is true for $n = k$, that is $2 + 4 + 6 + \ldots + (2k) = k^2 + k + 1$. Add $2k + 2$ to both sides.
   $2 + 4 + 6 + \ldots + (2k) + (2k + 2)$
   $= k^2 + k + 1 + (2k + 2)$
   $= k^2 + 2k + 1 + k + 1 + 1$
   $= (k + 1)^2 + (k + 1) + 1$.
   The last equality shows that the identity holds for $n = k + 1$ as well. Therefore, by induction, the identity is true for all n."
   What response should the students give?

3. **INDUCTION** Luke is trying to prove that something is true for all positive integers n. He succeeds in proving the statement for $n = 1$. However, instead of proving the $n = k$ implies $n = k + 1$, he proves that $n = k$ implies $n = 2k$ AND $n = k$ implies $n = k – 1$. Is the statement true for all positive integers n?

4. **PARITY** Numbers can be either odd or even. If they are divisible by 2, they are even. Otherwise, they are odd. One fact about parity is that $n^2 - n$ is even for all positive integers n. Note that $12 - 1 = 0$, so the statement is obvious for $n = 1$. Assume that the statement is true for $n = k$ and prove that it is then also true for $n = k + 1$.

VOLUME For Exercises 5 and 6, use the following information.

Let F_n be the Fibonacci numbers. In other words, $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for $n > 1$. You will prove by induction that

$$F_1 + F_3 + F_5 + \ldots + F_{2n+1} = F_{2n+2},$$

for all positive integers n.

5. Show that the identity holds for $n = 1$.

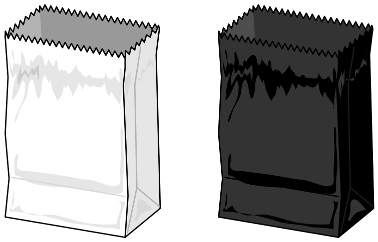
6. Assume the identity for $n = k$. Show that the identity holds for $n = k + 1$.

Lesson 11-8

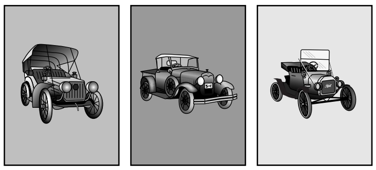
12-1 Word Problem Practice

The Counting Principle

1. **CANDY** Amy, Bruce, and Carol can choose one piece of candy from either a white or black bag. The white bag contains various chocolates. The black bag contains small bags of jelly beans. Amy picks from the white bag and Bruce and Carol both pick from the black bag. Describe whether each of the picks is related as dependent or independent events.



2. **PHOTOS** Morgan has three pictures that she would like to display side by side.



In how many different ways can the pictures be displayed?

3. **COMBINATION LOCKS** Eric uses a combination lock for his locker. The lock uses a three number secret code. Each number ranges from 1 to 35, inclusive. How many different combinations are possible with Eric's lock?

4. **TRUE OR FALSE** Faith is preparing a true or false quiz for her biology class. How many different answer keys can there be for a 10 question true or false quiz?

WEBSITES For Exercises 5-8, use the following information.

Greg is registering to use a website. The website requires him to choose an 8 character alphanumeric password that is not case-sensitive. In other words, for each character, he can choose one of the 26 letters A through Z or one of the 10 digits 0 through 9.

5. How many different passwords are possible?

6. Greg decides to use a password that does not contain any repeated characters. How many different passwords are possible with this constraint?

7. Suppose Greg chooses to use only letters with possible repeats. How many different passwords would be possible?

8. If Greg's password begins with his first name and ends with his birth month and date, how many possibilities would need to be checked to find his password?

12-2 Word Problem Practice

Permutations and Combinations

1. WAITING IN LINE When the 12 students in Mr. Jaybird's class go to lunch, they form a single file line. Does forming a line involve a permutation or a combination of the students?

2. ART Isabel needs to select three different colors of construction paper to make a flag for a school project. She can choose from a selection of 15 different colors. In how many ways can she pick her colors?

3. SUDOKU A popular game called "Sudoku" involves square arrays of numbers. In a game of Sudoku, every entry is an integer between 1 and 9, inclusive. No number appears twice in any row or column.

7	1	8	6	9	4	2	3	5
9	2	5	7	3	1	6	4	8
4	6	3	8	5	2	7	9	1
5	9	2	1	7	3	4	8	6
8	3	1	4	6	5	9	2	7
9	7	4	2	8	9	5	1	3
3	4	9	5	1	7	8	6	2
2	8	7	3	4	6	1	5	9
1	5	6	9	2	8	3	7	4

For a game of Sudoku, how many different possibilities are there for the first row of numbers?

4. NAMES Hannah is curious to know how many different 6 letter sequences she can make using each of the letters of her name exactly once. For example, "HANNAH," "AAHHNN," and "NAHNAH" are all possible sequences. How many total sequences are possible?

METEORITES For Exercises 5 and 6, use the following information.

Over the course of several years, Kendra managed to collect 7 meteorites. Each one is unique.

5. For a school science fair, Kendra displays her meteorites in a row. How many ways are there to order the meteorites?

6. She decides to trade three of her meteorites for a telescope after the fair. How many ways can she pick out 3 meteorites from her collection?

Lesson 12-2

12-3 Word Problem Practice

Probability

1. **ART** The letters "A", "R", and "T" are written on three different pieces of paper. The pieces of paper are then put in a bag and mixed up. Logan picks each letter without looking and places them side by side. What is the probability that the letters spell "ART"?

2. **AGE** There are 24 students in Miss Mason's third grade class, all born on different days. Eleven students are boys. In the morning, the classroom is empty. One student arrives followed by another. What is the probability that when the first two students arrive, one is a boy and the other a girl?

3. **DICE** Jamal rolls two six-sided dice, one after the other. What is the probability that the second die shows a number larger than the first die?

4. **LANGUAGES** Noah cannot decide whether to learn French, German, Italian, Russian, or Chinese. He assigns each language a different number from 0 to 4. He then takes four fair coins and flips them. He decided to take the language corresponding to the number of coins that come up heads. Does Noah's method for choosing a language give each language the same chance of being chosen? Explain.

ICE CREAM For Exercises 5-7, use the following information.

A survey of the students in Mr. Orr's fifth grade class asked each student to name their favorite flavor of ice cream. The results are shown in the table below.

Flavor	Number of Students
Vanilla	10
Chocolate	9
Butternut	5
Strawberry	4
Banana	1
Coffee	1

5. A student from Mr. Orr's class is selected at random. What is the probability that the student's favorite flavor of ice cream is chocolate?

6. A student from Mr. Orr's class is selected at random. What is the probability that the student's favorite flavor of ice cream is banana?

7. A student from Mr. Orr's class is selected at random. Is it more likely that the student prefers either butternut or strawberry or that the student prefers either chocolate or banana?

Lesson 12-3

NAME ____ DATE______________ PERIOD _____

12-4 Word Problem Practice

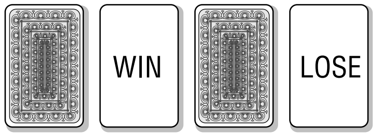
Multiplying Probabilities

1. **BUSSING** Portia and Quinton use the same bus stop when they go to work. They arrive at the bus stop independently of each other. The probability that Portia catches the 7:45 A.M. bus is $\frac{3}{5}$. The probability that Quinton catches the 7:45 A.M. bus is $\frac{1}{2}$. What is the probability that they both catch the 7:45 A.M. bus on the same day?

2. **GOODY BAGS** Ryan and Sophia are given goody bags with identical contents. The probability of reaching into either of these goody bags and pulling out a stick of chewing gum is $\frac{1}{10}$. Ryan and Sophia each reach into their own goody bag and randomly pull something out. What is the probability that they both pulled out a stick of chewing gum?

3. **PENCILS** A box of pencils contains 11 type 2 pencils and 5 type 3 pencils. Tara picks out a pencil from the box without looking and keeps it. Then, Upton picks out a pencil from the box without looking. What is the probability that Tara picks a type 2 pencil and Upton picks a type 3 pencil?

4. **GUESSING GAMES** Valerie is playing a guessing game. Four cards are placed face down before her. The hidden side of each card shows either the word "LOSE" or "WIN". Only one card is labeled "WIN". Valerie is given two chances to find the card labeled "WIN".



What is the probability that she does not pick the "win" card on her first try but does find it with her second?

WALLETS For Exercises 5 and 6, use the following information.

Wayne has 1 ten-dollar bill, 2 five-dollar bills, and 5 one-dollar bills in his wallet.

5. Wayne randomly chooses a bill from his wallet, puts it back, then picks another bill, and puts that one back too. What is the probability that both were five-dollar bills?

6. Wayne randomly pulls out a bill from his wallet, and then, without putting it back, randomly pulls a second bill from his wallet. He then puts both bills back into the wallet. What is the probability that both of the bills pulled out were five-dollar bills?

NAME ______________________________ DATE______________ PERIOD _____

12-5 Word Problem Practice

Adding Probabilities

1. **PICK-UP** When Tina's parents pick her up from school, there is a $\frac{1}{5}$ chance that she will be in the library, a $\frac{1}{2}$ chance that she will be on the playground, and a $\frac{3}{10}$ chance that she will be in her classroom. What is the probability that when Tina's parents pick her up, she is found in her classroom or on the playground?

2. **TRAVEL** John is randomly selected to be given a chance to win a new car. He must choose a red or yellow marble from a bag containing 1 red, 2 yellow, 10 green, and 12 blue marbles. What is the probability he will win the car?

3. **DICE** Alexis rolls two identical dice. What is the probability that the sum of the numbers rolled is odd? What is the probability that the sum of the numbers rolled is greater than 7? What is the probability that the sum of the numbers rolled is odd or greater than 7?

4. **CLASSES** At Jackson High School, 56 of the eleventh graders take physics and 70 of them take biology. There are 400 eleventh graders in total at the school. An eleventh grader is chosen at random from among all the eleventh graders at the high school. The probability that the selected student takes physics and biology is $\frac{11}{40}$. How many students at the high school take physics or biology?

PASSENGERS For Exercises 5 and 6, use the following information.

On an airplane flight, some passengers travel with carry-on luggage while others travel with a suitcase. Some passengers travel with carry-on luggage and a suitcase. Everyone travels with some form of luggage.

5. On one flight, there was no passenger with both carry-on luggage and a suitcase. On this flight are the events of picking a passenger with carry-on luggage and picking a passenger with a suitcase mutually exclusive?

6. On another flight, there are 120 passengers. Of those 120 passengers, 80 have carry-on luggage and 70 have a suitcase. What is the probability that a passenger has both carry-on luggage and a suitcase?

NAME ____ DATE______________ PERIOD _____

12-6 Word Problem Practice

Statistical Measures

1. **SPORTS** The table below shows the number of times some teams in the National Football League have won the Super Bowl.

NFL Team	Number of Super Bowl Victories
New England	3
Baltimore	2
Kansas City	1
St. Louis	1
Denver	2
Green Bay	1
Dallas	5
San Francisco	5
Oakland	2
Pittsburgh	5
Miami	2
Washington	3
NY Giants	2
NY Jets	1
Chicago	1

Source: www.pubquizhelp.34sp.com

Which statistical measure represents the team(s) with the least Super Bowl victories?

2. **SALARIES** The median salary in a small company is $10.20 per hour. What percentage of the employees at the company earns more than $10.20 per hour?

3. **RANDOM GENERATORS** Samuel has written a computer program to generate a random selection of the following two-digit numbers.

25, 67, 54, 99, 41, 87, 90, 18, 32

Find the mean, median, and mode of this data.

4. **HEIGHTS** The following table lists the heights of some of the great NBA players.

Player	Height (in inches)
Kareem Abdul-Jabbar	86
Larry Bird	81
Shaquille O'Neal	85
Wilt Chamberlain	85
Michael Jordan	78

Source: www.sidwell.edu

Find the mean and standard deviation of the data in the table. Round your answer to the nearest hundredth.

METEORS For Exercises 5-8, use the following information.

Arlene stayed up late one night to watch the Perseid meteor shower. She recorded the number of meteors she saw every ten minutes starting at 1 A.M. and going until 4 A.M. Her data are shown below.

8, 7, 8, 12, 17, 15, 22, 28, 29,
31, 28, 23, 29, 28, 25, 23, 15, 12

5. What is the mean of this data set?

6. What is the median of this data set?

7. What is the mode of this data set?

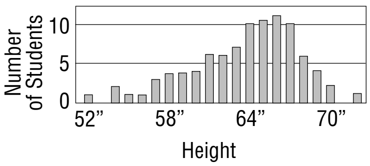
8. What is the standard deviation of this data set? Round your answer to the nearest hundredth.

12-7 Word Problem Practice

The Normal Distribution

1. **PARKING** Over several years, Bertram conducted a study of how far into parking spaces people tend to park by measuring the distance from the end of a parking space to the front fender of a car parked in the space. He discovered that the distribution of the data closely approximated a normal distribution with mean 8.5 inches. He found that about 5% of cars parked more than 11.5 inches away from the end of the parking space. What percentage of cars would you expect parked less than 5.5 inches away from the end of the parking space?

2. **HEIGHT** Chandra's graph of the number of tenth grade students of different heights is shown below.



Is the data positively skewed, negatively skewed, or normally distributed?

3. **OVENS** An oven manufacturer tries to make the temperature setting on its ovens as accurate as possible. However, if one measures the actual temperatures in the ovens when the temperature setting is 350°F, they will differ slightly from 350°F. The set of actual temperatures for all the ovens is normally distributed around 350°F with a standard deviation of 0.5°F. About what percentage of ovens will be between 350°F and 351°F when their temperature setting is 350°F?

4. **LIGHT BULBS** The time that a certain brand of light bulb will last before burning out is normally distributed. About 2.5% of the bulbs last longer than 6800 hours and about 16% of the bulbs last longer than 6500 hours. How long does the average bulb last?

DOGS For Exercises 5-8, use the following information.

The weights of adult greyhound dogs are normally distributed. The mean weight is about 69 pounds and the standard deviation is about 10 pounds.

5. Approximately what percentage of adult greyhound dogs would you expect weigh between 59 and 79 pounds?

6. Approximately what percentage of adult greyhound dogs would you expect weigh more than 99 pounds?

7. Approximately what percentage of adult greyhound dogs would you expect weigh less than 49 pounds?

8. What would you expect an adult greyhound dog to weigh if it weighed less than 0.5% of an average adult greyhound?

12-8 Word Problem Practice

Binomial Experiments

1. GENETICS Dagmar is conducting a genetic experiment. Before she performs the experiment, she would like to compute theoretically probabilities for some of the outcomes. One of these computations involves expanding $(p + q)^4$. What is this expansion?

2. GAMES The probability that Kendra will win a card game is $\frac{2}{3}$. If Kendra plays 7 games what is the probability she wins exactly 4 games? Round your answer to the nearest thousandth.

3. DEFECTS An electronics parts manufacturer produces capacitors for electronic circuits. The probability that a capacitor comes out defective is 1 in 1,000. In a batch of 10,000 capacitors, write an expression for the probability that 10 of the capacitors are defective.

4. SUBWAYS Fiona uses the subway to commute to work. During the morning commute, the trains run frequently and there is a 1 in 8 chance that she will find a train waiting for her as soon as she gets to the platform. Over the course of a five-day work week, what is the probability that she found a train waiting for her at least twice? Round your answer to the nearest thousandth.

5. SOCCER The boys varsity soccer team at Lincoln High School has a 75% probability of winning each of their 17 games this season. What is the probability that the team will win at least 13 games this season? Round your answer to the nearest thousandth.

CHESS For Exercises 6-8, use the following information.

Gary and Howard play chess. Gary's chess rating is 2050 and Howard's chess rating is 1948. This means that whenever they play, Gary has a 64% chance of defeating Howard. One day, Gary and Howard play three games against each other. Round your answers to the nearest thousandth.

6. What is the probability that Gary will win all three of the matches?

7. What is the probability that Gary will win at least two of the three matches?

8. What is the probability that Gary will win only one of the matches?

12-9 Word Problem Practice

Sampling and Error

1. **COMICS** Isaac would like to know if people prefer reading comic books or novels. He decides to wait outside of a bookstore and ask people exiting whether they purchased comics or novels. Discuss whether this method of acquiring data would produce a biased or unbiased sample.

2. **PARKING** A town wants to find out if people are happy with a proposal to tear down a section of a park and replace it with a parking lot. The town council decides to conduct a random survey of the town's citizens. They send a person to the location in the park where the proposed parking lot will be and have that person ask all passersby whether they would like to see a parking lot built at the location. Discuss whether or not this would produce a random sample.

3. **PROMS** A poll asked 50 random seniors at a high school whether they would like to have the senior prom at a nearby hotel or at a local convention hall. Sixteen students responded that they would prefer the hotel. What is the margin of sampling error? Round your answer to the nearest percent.

4. **AIRPORTS** In a large city, a random survey found that 18% of the city's population want a new runway built at the city airport. The survey had a margin of error of 5%. About how many people were surveyed?

INTERNET USE For Exercises 5-7, use the following information.

Two surveys were conducted to find out if people think that Americans are becoming more knowledgeable about the Internet. One survey polled 500 people and found that 395 of them felt that Americans are becoming more Internet savvy. A second survey concluded that 79% of those polled think that Americans are becoming more Internet savvy with a margin of error of 2%.

5. What was the margin of error for the first survey? Round your answer to the nearest percent.

6. About how many people were polled in the second survey?

7. Based on the results of the second survey, between what two percentages would you estimate is the true percentage of people who think that Americans are more Internet savvy, with 95% confidence?