NAME ______________________________ DATE ____________ PERIOD _____

1-1 Enrichment

Using a Reference Point

There are many times when you need to make an estimate in relation to a *reference point*. For example, at the right there are prices listed for some school supplies. You might wonder if $5 is enough money to buy a small spiral notebook and a pen. This is how you might estimate, using $5 as the reference point.

- The notebook costs $1.59 and the pen costs $3.69.
- $1 + $3 = $4. I have $5 − $4, or $1, left.
- $0.59 and $0.69 are each more than $0.50, so $0.59 + $0.69 is more than $1.

So $5 will not be enough money.

Spiral Notebook
Large $2.29
Small $1.59

Three-Ring
Binder
$4.75

Filler Paper
Pack of 100
$1.29

Ball-Point
Pen
$3.69

Pencils
Pack of 10
$2.39

Eraser
$0.55

Use the prices at the right to answer each question.

1. Jamaal has $5. Will that be enough money to buy a large spiral notebook and a pack of pencils?

2. Andreas wants to buy a three-ring binder and two packs of filler paper. Will $7 be enough money?

3. Rosita has $10. Can she buy a large spiral notebook and a pen and still have $5 left?

4. Kevin has $10 and has to buy a pen and two small spiral notebooks. Will he have $2.50 left to buy lunch?

5. What is the greatest number of erasers you can buy with $2?

6. What is the greatest amount of filler paper that you can buy with $5?

7. Lee bought three items and spent exactly $8.99. What were the items?

8. Select five items whose total cost is as close as possible to $10, but not more than $10.

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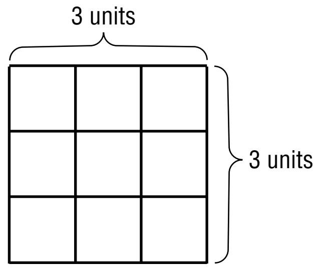
1-2 Enrichment

Making Models for Numbers

Have you wondered why we read the number 3^2 as three squared? The reason is that a common model for 3^2 is a square with sides of length 3 units. As you see, the figure that results is made up of 9 square units.

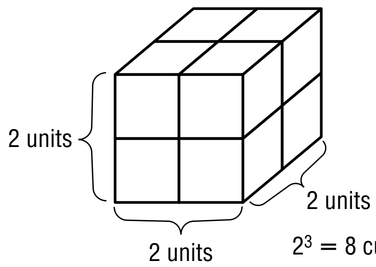


$3^2 = 9$ square units

Make a model for each expression.

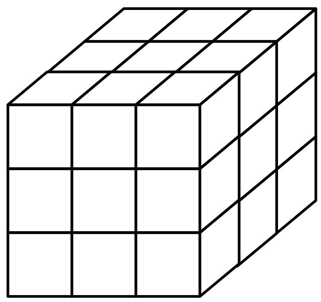
1. 2^2 **2.** 4^2 **3.** 1^2 **4.** 5^2

Since we read the expression 2^3 as *two cubed*, you probably have guessed that there is also a model for this number. The model, shown at the right, is a cube with sides of length 2 units. The figure that results is made up of 8 *cubic units*.



$2^3 = 8$ cubic units

Exercises 5 and 6 refer to the figure at the right.



5. What expression is being modeled?

6. Suppose that the entire cube is painted red. Then the cube is cut into small cubes along the lines shown.

a. How many small cubes are there in all?

b. How many small cubes have red paint on exactly three of their faces?

c. How many small cubes have red paint on exactly two of their faces?

d. How many small cubes have red paint on exactly one face?

e. How many small cubes have no red paint at all?

7. **CHALLENGE** In the space at the right, draw a model for the expression 4^3.

1-3 Enrichment

The Sieve of Erathosthenes

Erathosthenes was a Greek mathematician who lived from about 276 B.C. to 194 B.C. He devised the **Sieve of Erathosthenes** as a method of identifying all the prime numbers up to a certain number. Using the chart below, you can use his method to find all the prime numbers up to 120. Just follow these numbered steps.

1. The number 1 is not prime. Cross it out.
2. The number 2 is prime. Circle it. Then cross out every second number—4, 6, 8, 10, and so on.
3. The number 3 is prime. Circle it. Then cross out every third number—6, 9, 12, and so on.
4. The number 4 is crossed out. Go to the next number that is not crossed out.
5. The number 5 is prime. Circle it. Then cross out every fifth number—10, 15, 20, 25, and so on.
6. Continue crossing out numbers as described in Steps 2–5. The numbers that remain at the end of this process are prime numbers.
7. **CHALLENGE** Look at the prime numbers that are circled in the chart. Do you see a pattern among the prime numbers that are greater than 3? What do you think the pattern is?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120

1-4 Enrichment

Operations Puzzles

Now that you have learned how to evaluate an expression using the order of operations, can you work backward? In this activity, the value of the expression will be given to you. It is your job to decide what the operations or the numbers must be in order to arrive at that value.

Fill in each □ with +, −, ×, or ÷ to make a true statement.

1. 48 □ 3 □ 12 = 12
2. 30 □ 15 □ 3 = 6
3. 24 □ 12 □ 6 □ 3 = 4
4. 24 □ 12 □ 6 □ 3 = 18
5. 4 □ 16 □ 2 □ 8 = 24
6. 45 □ 3 □ 3 □ 9 = 3
7. 36 □ 2 □ 3 □ 12 □ 2 = 0
8. 72 □ 12 □ 4 □ 8 □ 3 = 0

Fill in each □ with one of the given numbers to make a true statement. Each number may be used only once.

9. 6, 12, 24
   □ ÷ □ × □ = 12
10. 4, 9, 36
   □ − □ ÷ □ = 0
11. 6, 8, 12, 24
   □ ÷ □ + □ − □ = 4
12. 2, 5, 10, 50
   □ − □ ÷ □ + □ = 50
13. 2, 4, 6, 8, 10
   □ ÷ □ × □ + □ − □ = 0
14. 1, 3, 5, 7, 9
   □ ÷ □ + □ − □ ÷ □ = 1
15. **CHALLENGE** Fill in each □ with one of the digits from 1 through 9 to make a true statement. Each digit may be used only once.
   □ ÷ □ × □ + □ × □ × □ ÷ □ + □ × □ = 100

Lesson 1–4

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1-5 Enrichment

Using Formulas

A formula is an equation that can be used to solve certain kinds of problems. Formulas often have algebraic expressions. Here are some common formulas used to solve geometry problems. The variables in geometric formulas represent dimensions of the geometric figures.

Area (A)
- of a rectangle: $A = \ell \times w$
- of a square: $A = s^2$
- of a triangle: $A = \frac{1}{2}bh$
- of a square: $P = 4s$

Volume (V)
- of a rectangular prism: $V = \ell \times w \times h$

Perimeter (P)
- of a rectangle: $P = 2(w + \ell)$

b = base h = height ℓ = length s = side w = width

Write the formula that would be used to solve each problem.

1. Jack wants to put a fence around his garden to keep rabbits out. Jack's garden is square shape. Which formula can Jack use to find how much fence he needs to buy?

2. Diane's mother will replace the carpeting in their living room. The living room is rectangular in shape. Which formula can Diane's mother use to determine how much carpeting she will need to order for her living room?

3. Victor is cleaning his aquarium, which is shaped like a rectangular prism. After he empties the aquarium and cleans the sides, he will refill the aquarium. Which formula can Victor use to determine how much water he will put back in the aquarium?

4. Joann is making a triangular flag for a school project. Which formula can she use to determine how much material she needs to buy to make the flag?

Solve each problem.

5. A tablecloth is 8 feet long and 5 feet wide. What is the area of the tablecloth?

6. Jessica wants to frame a square picture that has sides of 6 inches. How many inches of wood will she need to make the frame?

7. How many cubic centimeters of packing peanuts will fit in a cardboard box that is 9 centimeters long, 8 centimeters wide, and 3 centimeters high?

8. Joaquin is painting a mural on one wall of the school's gymnasium. Part of the mural is a triangle with a base of 20 ft and a height of 8 feet. What is the area of the triangle?

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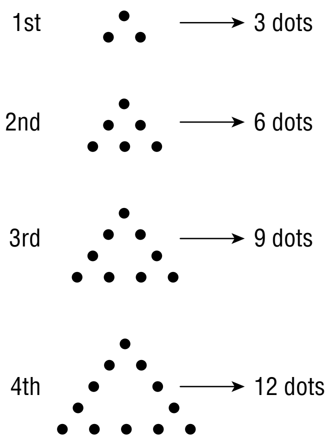
1-6 Enrichment

Function Rules and Dot Patterns

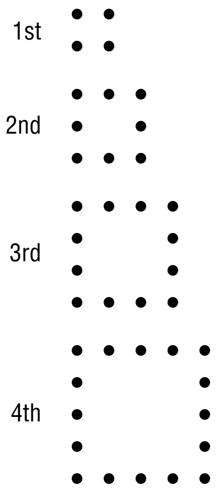
Function rules are often used to describe geometric patterns. In the pattern at the right, for example, do you see this relationship?

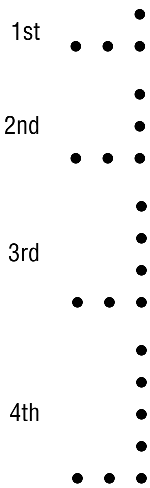
1st figure: $3 \times 1 = 3$ dots
2nd figure: $3 \times 2 = 6$ dots
3rd figure: $3 \times 3 = 9$ dots
4th figure: $3 \times 4 = 12$ dots

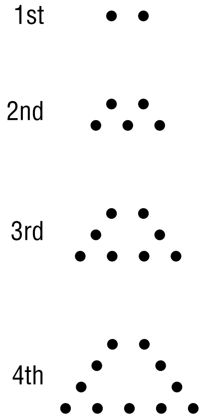
So the "nth" figure in this pattern would have $3 \times n$, or $3n$, dots. A function rule that describes the pattern is $3n$.

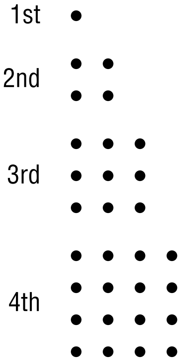


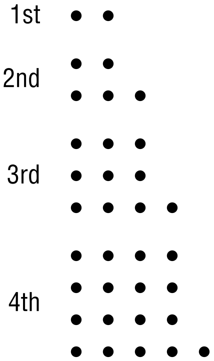
Write a function rule to describe each dot pattern.

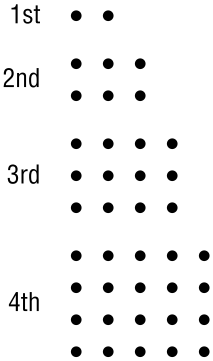
1.


2.


3.


4.


5.


6.


7. CHALLENGE Create your own dot pattern. Then exchange patterns with a classmate. Try to find the function rule for each other's patterns.

1-8 Enrichment

Equation Chains

In an **equation chain**, you use the solution of one equation to help you find the solution of the next equation in the chain. The last equation in the chain is used to check that you have solved the entire chain correctly.

Complete each equation chain.

1. $5 + a = 12$, so $a =$ ______.

$ab = 14$, so $b =$ ______.

$16 \div b = c$, so $c =$ ______.

$14 - d = c$, so $d =$ ______.

$e \div d = 3$, so $e =$ ______.

$a + e = 25$ ← **Check:**

2. $9f = 36$, so $f =$ ______.

$g = 13 - f$, so $g =$ ______.

$63 \div g = h$, so $h =$ ______.

$h + i = 18$, so $i =$ ______.

$j - i = 9$, so $j =$ ______.

$j \div f = 5$ ← **Check:**

3. $m \div 4 = 8$, so $m =$ ______.

$m - n = 12$, so $n =$ ______.

$np = 100$, so $p =$ ______.

$q = 40 + p$, so $q =$ ______.

$p + q - 10 = r$, so $r =$ ______.

$r - m = 8$ ← **Check:**

4. $18 = v - 12$, so $v =$ ______.

$v \div w = 3$, so $w =$ ______.

$80 = wx$, so $x =$ ______.

$w + x = 2y$, so $y =$ ______.

$xy - z = 40$, so $z =$ ______.

$z - v = 2$ ← **Check:**

5. CHALLENGE Create your own equation chain using these numbers for the variables: $a = 10$, $b = 6$, $c = 18$, and $d = 3$.

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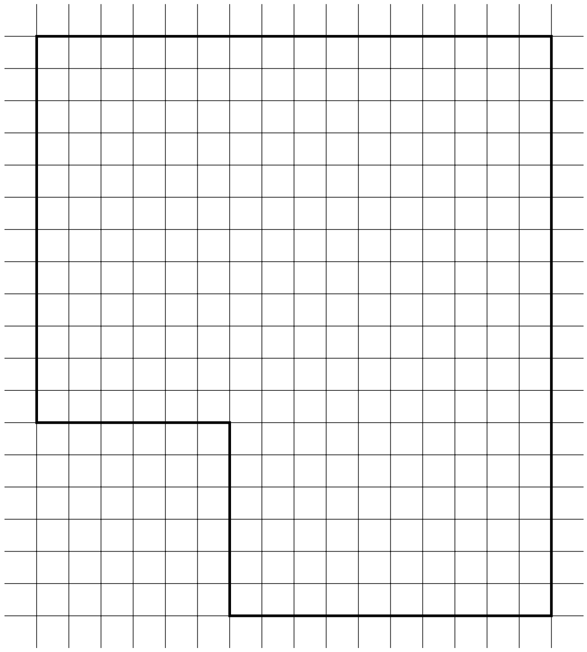
1-9 Enrichment

Tiling a Floor

The figure at the right is the floor plan of a family room. The plan is drawn on grid paper, and each square of the grid represents one square foot. The floor is going to be covered completely with tiles.



1. What is the area of the floor?

2. Suppose each tile is a square with a side that measures one foot. How many tiles will be needed?

3. Suppose each tile is a square with a side that measures one inch. How many tiles will be needed?

4. Suppose each tile is a square with a side that measures six inches. How many tiles will be needed?

Use the given information to find the total cost of tiles for the floor.

5. tile: square, 1 foot by 1 foot
cost of one tile: $3.50

6. tile: square, 6 inches by 6 inches
cost of one tile: $0.95

7. tile: square, 4 inches by 4 inches
cost of one tile: $0.50

8. tile: square, 2 feet by 2 feet
cost of one tile: $12

9. tile: square, 1 foot by 1 foot
cost of two tiles: $6.99

10. tile: rectangle, 1 foot by 2 feet
cost of one tile: $7.99

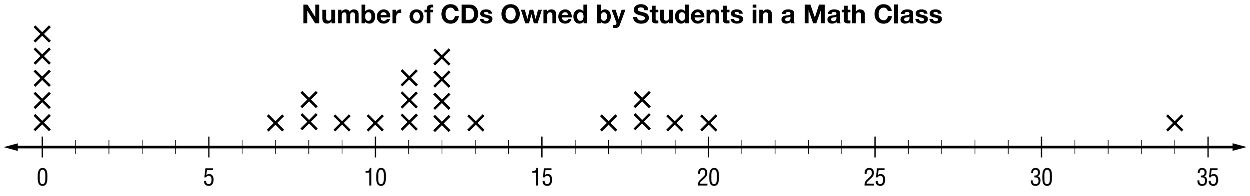
11. Refer to your answers in Exercises 5-10. Which way of tiling the floor costs the least? the most?

2-2 Enrichment

Line Plots

In a **line plot**, data are pictured on a number line. An × is used to represent each item of data. For example, the figure below is a line plot that pictures data about the number of CDs owned by the students in a math class.

Number of CDs Owned by Students in a Math Class



Use the line plot above to answer each question.

1. How many students own exactly eighteen CDs?

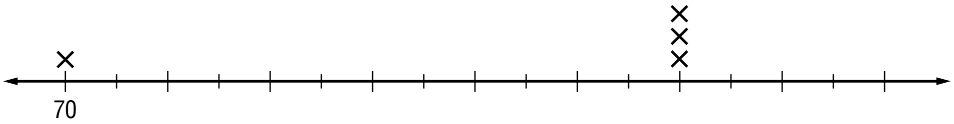
2. What number of CDs is owned by exactly three students?

3. A data item that is far apart from the rest of the data is called an outlier. Is there an outlier among these data? What is it?

4. What would you say is the number of CDs owned by the "typical" student in this class?

5. Use the data in the table to complete the line plot below. Four data points have been graphed for you.

Number of Seconds for 24 Sixth Graders to Run 200 Meters											
130	100	85	120	100	100	110	150	90	100	110	130
125	105	100	70	125	85	95	130	105	90	105	100



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2-3 Enrichment

Graphs and Decision Making

Just as important as knowing how to make a graph, is deciding what type of graph to use. Here are some guidelines to help you make that decision.

- A **bar graph** compares data that fall into distinct categories, such as the populations of several cities compare in one year.
- A **line graph** shows changes in data over a period of time, such as the population of one city changing over several years.
- A **histogram** uses bars to represent the frequency of numerical data organized in intervals.

Would you use a bar graph, line graph, or histogram to show these data?

1. average temperatures in Sacramento for each month of the year

2. land area of continents

3. number of CD players purchased each year from 1999 through 2005

4. number of babies that weighed between 5 lb and 5 lb 15 oz, 6 lb and 6 lb 15 oz, 7 lb and 7 lb 15 oz, 8 lb and 8 lb 15 oz, or 9 lb and 9 lb 15 oz

Make an appropriate graph for each set of data.

5. **Taxis in Use**

Year	Number (millions)
1999	135
2000	136
2001	142
2002	148

6. **Aircraft Capacity**

Model	Number of Seats
B747	405
DC-10	288
L-1011	296
MD-80	142

7. **Video Games Owned**

Number of Games	Number of Students
0–2	5
3–5	4
6–8	9
9–11	6

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2-4 Enrichment

A **back-to-back stem-and-leaf plot** is used to compare two sets of data. In this type of plot, the leaves for one set of data are on one side of the stems, and the leaves for the other set of data are on the other side of the stems. Two keys to the data are needed.

ELECTIONS Use the back-to-back stem-and-leaf plot of the electoral votes cast by each state and the District of Columbia for the Democratic and Republican candidates for U.S. president in 2004.

Democrat	Stem	Republican
3 3 3 4 4 4 4 7 7 9	0	3 3 3 3 3 4 5 5 5 5 5 6 6 6 7 7 8 8 9 9 9
0 0 1 2 5 7	1	0 1 1 1 3 5 5
1 1	2	0 7
1	3	4
	4	
5	5	

3 | 0 = 3 votes   *0 | 3 = 3 votes*

Source: infoplease.com

1. What is the greatest number of electoral votes cast by a state for the Democratic candidate? the greatest number of electoral votes cast by a state for the Republican candidate?

2. Which candidate received votes from the greater number of states?

3. Which candidate received the greater number of total votes?

4. What is the difference between the number electoral votes cast for the candidates?

5. Write a sentence or two comparing the number of electoral votes cast for the two candidates.

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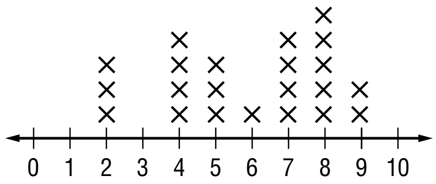
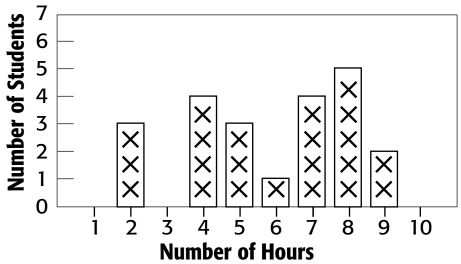
2-5 Enrichment

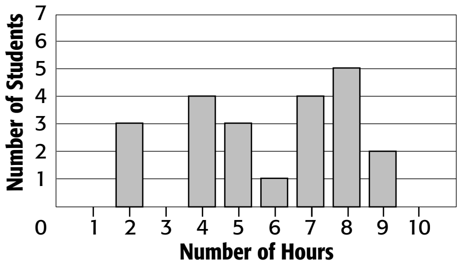
Line Plots and Bar Graphs

A line plot is a version of a bar graph. Look at the line plot on the right. It shows the results of a survey about TV viewing habits. Twenty-two students were asked how many hours of television they watch in one week. Three students said they watch 2 hours of television each week. Two students said they watch 9 hours of television per week.



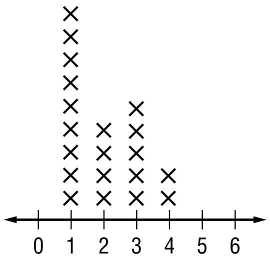
A bar graph is another way to display data. You can use a line plot to create a bar graph. First, draw a vertical line up from zero to form the *y*-axis. Decide on an interval for the *y*-axis. Draw horizontal lines across from the numbers. Draw bars over the *x*'s and shade them in. Label the *y*-axis "Number of Students" and the *x*-axis "Number of Hours."



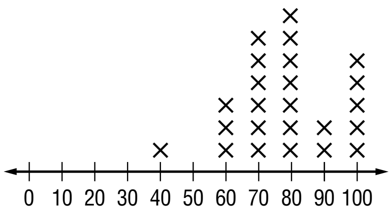


Change each line plot into a bar graph.

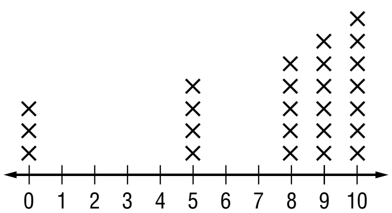
1. **Number of Siblings Students Have**



2. **Class Scores on a Math Quiz**



3. **Number of Hours Playing Sports per Week**



2-6 Enrichment

Mean, Median, or Mode?

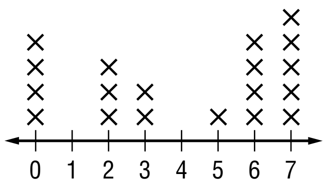
When most people hear the word "average," they think about what mathematicians call arithmetic mean. But the three measures of central tendency, mean, median and mode, are all different types of averages. Average is not a mathematical word. In mathematics, it is necessary to specify which type of average you are using.

1. The prices of seven homes for sale in Sunnydale are $151,000; $148,500; $163,000; $180,500; $151,000; $172,000; $189,000. Find the mean, median, and mode for the price of the homes for sale.

2. A real estate agent is writing an advertisement for a newspaper. She writes, "The average price of a home in Sunnydale is $151,000." Which average did she use? Explain why she chose to use this particular average. Is this average misleading?

3. Which type of average should be used to best represent the "average" price of a home in Sunnydale?

A candy company is having a special promotion for which it includes special blue colored candies in its packages. The line plot shows how many blue candies were found in each of 19 packages.



Sam, Matt, and Carla solve to find the average number of blue candies per package. None of the students finds the same answer. Sam has the highest value, then Carla, and Matt's answer is has the lowest value. Their teacher tells them that each one has a correct answer.

4. Determine which average each student found.

5. Find the mean, median, and mode for the line plot.

6. Matt looks at the line plot and notices that the number he found as average was never plotted. Matt decides that since that number of candies was never found in the bags, it can't be the average. Explain why the number is still considered an average.

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2-7 Enrichment

Puzzling Over Data

Each puzzle on this page contains an incomplete set of data. The clues give you information about the mean, median, mode, or range of the data. Working from these clues, you can decide what the missing data items must be. For example, this is how you might solve the data puzzle at the right.

Clue: mean = 18

Data: 12, 17, 18, 19, 19, [ ]

There are 6 items of data.
The mean is 18, so the sum of the data must be 6 × 18 = 108.
Add the given data: 12 + 17 + 18 + 19 + 19 = 85.
Subtract from 108: 108 − 85 = 23.

So the complete set of data is: 12, 17, 18, 19, 19, [23].

Lesson 2–7

Find the missing data. (Assume that the data items are listed in order from least to greatest.)

1. *Clue*: mode = 8

Data: 7, 7, 8, [ ], [ ], 14

2. *Clue*: median = 54.5

Data: 36, 40, 49, [ ], 65, 84

3. *Clues*: mean = 27
mode = 30

Data: 10, 25, 27, [ ], 30, [ ]

4. *Clues*: median = 120
range = 46

Data: 110, 112, [ ], 124, 136, [ ]

5. *Clues*: mean = 13
median = 13
range = 13

Data: [ ], 9, 12, [ ], 18, [ ]

6. *Clues*: mean = 7
median = 8.5
mode = 10

Data: [ ], 4, 8, [ ], [ ], [ ]

7. *Clues*: mean = 60
mode = 52
range = 28

Data: [ ], 52, [ ], [ ], 72, 78

8. *Clues*: median = 24
mode = 28
range = 24

Data: 6, 15, [ ], [ ], [ ], [ ]

2-8 Enrichment

Choosing a Representative Sample

Statisticians often use **samples** to represent larger groups. For example, television ratings are based on the opinions of a few people who are surveyed about a program. The people surveyed are just part of the whole group of people who watched the program. When using samples, people taking surveys must make sure that their samples are representative of the larger group in order to ensure that their conclusions are not misleading.

ADVERTISING A company that makes athletic shoes is considering hiring a professional basketball player to appear in its commercials. Before hiring him, they are doing research to see if he is popular with teens. Would they get good survey results from taking a survey about the basketball player from each of these surveys?

1. 200 teens at a basketball game of the basketball player's team

2. 25 teens at a shopping mall

3. 500 students at a number of different middle and high schools

Decide whether each location is a good place to find a representative sample for the selected survey. Justify your answer.

4. number of hours of television watched in a month at a shopping mall

5. favorite kind of entertainment at a movie theater

6. whether families own pets in an apartment complex

7. taste test of a soft drink at a grocery store

8. favorite teacher in a school cafeteria

9. teenagers' favorite magazine at five different high schools

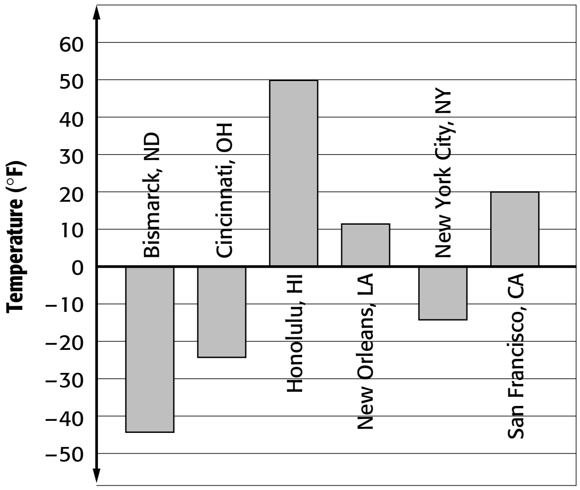
2-9 Enrichment

Graphs with Integers

Statistical graphs that display temperatures, elevations, and similar data often involve negative quantities. On graphs like these, the scale usually will have a zero point and will include both positive and negative numbers.

For Exercises 1–6, use the bar graph at the right to answer each question.

Lowest Recorded Temperatures in Selected Cities



1. In which cities is the record low temperature greater than 0°F?

2. In which cities is the record low temperature less than 0°F?

3. In which city is the record low temperature about −25°F?

4. Estimate the record low temperature for New York City.

5. In which cities is the record low temperature less than twenty degrees from 0°F?

6. How many degrees are between the record low temperatures for Bismarck and Honolulu?

7. In the space at the right, make a bar graph for the data below.

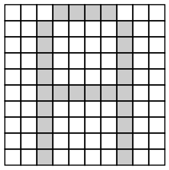
Altitudes of Some California Locations Relative to Sea Level

Location	Altitude (ft)
Alameda	30
Brawley	−112
Calexico	7
Death Valley	−282
El Centro	−39
Salton City	−230

NAME ______________________________ DATE ____________ PERIOD _____

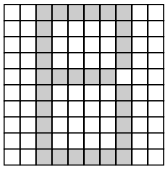
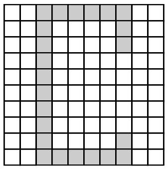
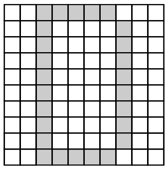
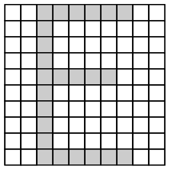
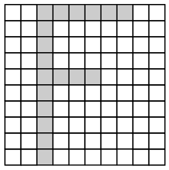
3-1 Enrichment

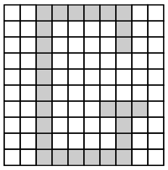
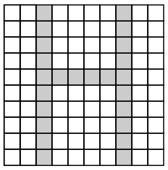
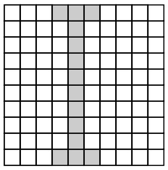
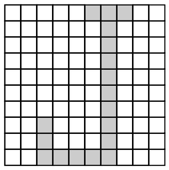
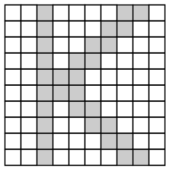
Decimal Letters

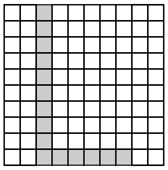
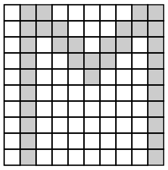
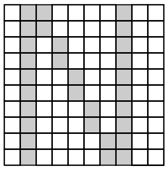
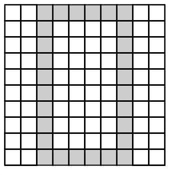
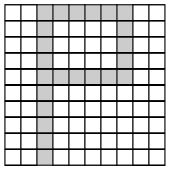


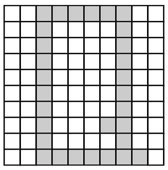
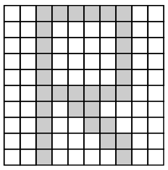
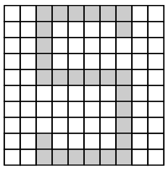
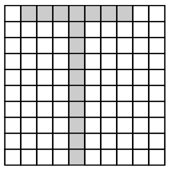
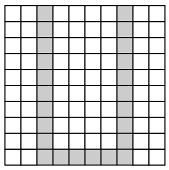
The letter A at the right was created by shading part of a hundreds square. There are 26 parts shaded, so the *value* of the letter A is 26 hundredths, or 0.26.

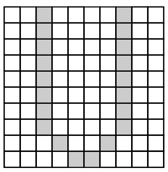
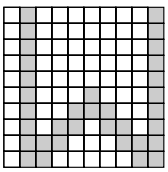
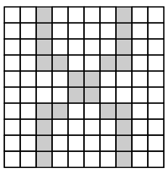
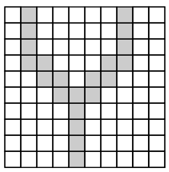
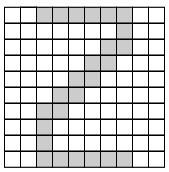
Find the value of each letter.

1.  **2.**  **3.**  **4.**  **5.** 

6.  **7.**  **8.**  **9.**  **10.** 

11.  **12.**  **13.**  **14.**  **15.** 

16.  **17.**  **18.**  **19.**  **20.** 

21.  **22.**  **23.**  **24.**  **25.** 

26. **CHALLENGE** Use the values of the 26 letters as a set of data. What is the frequency of the value 0.26? Which value is the mode?

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3-2 Enrichment

A Look at Nutrients

The table below gives data about a few of the nutrients in an average serving of some common foods.

Food	Protein (grams)	Fat (grams)	Carbohydrates (grams)	Vitamins (milligrams)			Minerals* (milligrams)		
				B	B-1	B-2	Na	K	Ca
apple (medium)	0.3	0.5	21.1	8	0.02	0.02	1	159	10
chocolate bar (1.02 oz)	2.2	9.4	16.5	0	0.02	0.08	29	119	55
cola (12 fl oz)	0.0	0.0	40.7	0	0.00	0.00	20	7	11
hamburger (1 medium)	21.8	14.5	0.0	0	0.13	0.15	40	382	6
orange juice (8 fl oz)	1.7	0.1	26.8	97	0.20	0.05	2	474	22
peas (1/2 cup)	4.5	0.4	10.8	19	0.22	0.09	128	137	17
wheat bread (1 slice)	2.3	1.0	11.3	0	0.11	0.08	129	33	30
whole milk (8 fl oz)	8.0	8.2	11.4	2	0.09	0.40	120	370	291

*Na = sodium, K = potassium, Ca = calcium

Use the data in the table to answer each question.

1. Is there more potassium in one apple or in one serving of peas?

2. Does one serving of milk contain more fat or more carbohydrates?

3. Which foods contain less than 0.05 milligram of vitamin B-2?

4. Which foods contain an amount of carbohydrates between 15 grams and 25 grams?

5. Which food contains the least amount of calcium?

6. Which food contains the greatest amount of vitamin B-1?

7. List the foods in order of their protein content from least to greatest.

8. List the foods in order of their fat content from greatest to least.

9. Make up two questions about the data in the table. Exchange questions with a classmate. Then answer your classmate's questions.

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3-3 Enrichment

Everybody into the Pool!

Answer each question using the "decimal pool" below.

1. Which decimal when rounded to the nearest hundredth is 0.03?

2. Which decimal when rounded to the nearest thousandth is 0.003?

3. Which two decimals when rounded to the nearest hundredth are 0.02?

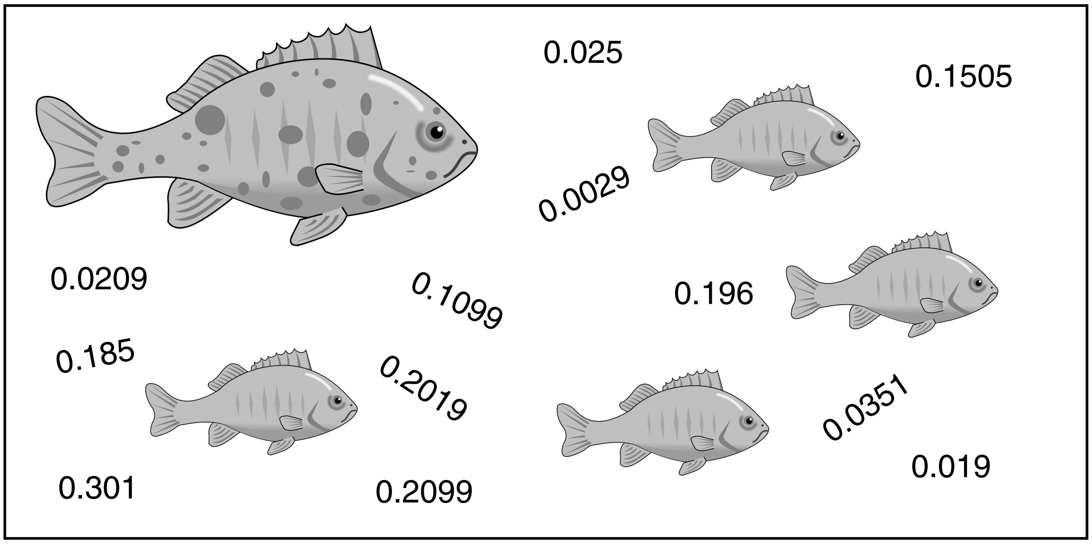
4. Which five decimals when rounded to the nearest tenth are 0.2?

5. Which decimal when rounded to the nearest thousandth is 0.210?

6. Which two decimals when rounded to the nearest hundredth are 0.20?

7. Add to the pool four different decimals that when rounded to the nearest thousandth are 0.301.

8. Add to the pool a three-place decimal that when rounded to the nearest tenth is 1.0.



9. **CHALLENGE** Suppose that you are rounding decimals to the nearest hundredth. How many three-place decimals round to 0.05? List them. How many four-place decimals do you think round to 0.05?

Lesson 3–3

3-4 Enrichment

Horizontal Estimation

Many times an addition problem is given to you in *horizontal form*, with the addends written from left to right. To estimate the sum, you don't have to rewrite the addition vertically in order to line up the decimal points. Just use place value to figure out which digits are most important. Here is an example.

3.11 + 0.4639 + 8.205

The most important digits are in the ones place.

3 + 0 + 8 = 11

The next group of important digits are in the tenths place.

1 tenth + 4 tenths + 2 tenths = 7 tenths

Add to make your estimate: 11 + 7 tenths → about 11.7

Estimate each sum.

1. 7.44 + 0.2193

2. 0.4015 + 9.3 + 3.264

3. 0.4208 + 0.16

4. 0.52 + 0.1 + 0.308 + 0.0294

5. 10.2 + 0.519

6. 12.004 + 1.5 + 4.32 + 0.1009

7. 6.72 + 0.5037

8. 0.805 + 1.006 + 0.4 + 2.0305

9. 1.208 + 3.1 + 0.04 + 6.143 + 0.3075

10. 0.9005 + 5.03 + 7.108 + 0.004 + 10.7

This same method works when you need to estimate a sum of much greater numbers. Estimate each sum.

11. 53,129 + 420,916

12. 6,048 + 2,137 + 509

13. 723 + 4,106 + 4,051 + 318

14. 7,095 + 12,402 + 3,114 + 360

15. 650,129 + 22,018 + 107,664 + 10,509

NAME _______________________________ DATE ____________ PERIOD _____

3-5 Enrichment

Currency

The currency used in the United States is the US dollar. Each dollar is divided into 100 cents. Most countries have their own currencies. On January 1, 2002, 12 countries in Europe converted to a common monetary unit that is called the *euro*.

The symbol, €, is used to indicate the euro.

The exchange rate between dollars and euros changes every day.

$1.00 is worth about 0.85€.

EXERCISES Add or subtract to solve each problem.

1. Henry bought a pair of shoes for €34.75 and a pair of pants for €21.49. How much money did he spend?

2. Louis receives €10.50 a week for doing his chores. His sister is younger and has fewer chores. She receives €5.25. How much money do Louis and his sister receive together in one week?

3. A gallon of Brand A of vanilla ice cream costs €5.49. A gallon of Brand B vanilla ice cream costs €4.87. How much money will Luca save if he buys Brand A instead of Brand B?

4. Michael passed up a pair of jeans that cost €29.50 and decided to buy a pair that were only €15.86. How much money did he save by buying the less expensive jeans?

5. Jesse's favorite magazine costs €1.75 at the store. If he buys a subscription, each issue is only 0.37€. How much money will Jesse save on each issue if he buys a subscription?

6. Layla wants to buy a CD for €11.99 and a book for €6.29. She has €15.00. How much more money does she need to buy the CD and book?

7. **CHALLENGE** Lynne's lunch came to €4.00. Her drink was €1.50. How much did she spend total? What would be the equivalent dollar amount?

8. **CHALLENGE** At the grocery store, Jaden purchased a box of cereal for $3.55 and a gallon of milk for $2.89. He gave the cashier $10.00. How much change did he receive? What would be the equivalent euro amount?

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4-1 Enrichment

GCFs By Successive Division

Here is a different way to find the greatest common factor (GCF) of two numbers. This method works well for large numbers.

Find the GCF of 848 and 1,325.

Step 1 Divide the smaller number into the larger.

$$\begin{array}{r} 1 \text{ R}477 \\ 848\overline{)1{,}325} \\ \underline{848} \\ 477 \end{array}$$

Step 2 Divide the remainder into the divisor.
Repeat this step until you get a remainder of 0.

$$\begin{array}{r} 1 \text{ R}371 \\ 477\overline{)848} \\ \underline{477} \\ 371 \end{array} \qquad \begin{array}{r} 1 \text{ R}106 \\ 371\overline{)477} \\ \underline{371} \\ 106 \end{array} \qquad \begin{array}{r} 3 \text{ R}53 \\ 106\overline{)371} \\ \underline{318} \\ 53 \end{array} \qquad \begin{array}{r} 2 \text{ R}0 \\ 53\overline{)106} \\ \underline{106} \\ 0 \end{array}$$

Step 3 The last divisor is the GCF of the two original numbers.
The GCF of 848 and 1,325 is 53.

Use the method above to find the GCF for each pair of numbers.

1. 187; 578

2. 161; 943

3. 215; 1,849

4. 453; 484

5. 432; 588

6. 279; 403

7. 1,325; 3,498

8. 9,840; 1,751

9. 3,484; 5,963

10. 1,802; 106

11. 45,787; 69,875

12. 35,811; 102,070

4-2 Enrichment

Fraction Mysteries

Here is a set of mysteries that will help you sharpen your thinking skills. In each exercise, use the clues to discover the identity of the mystery fraction.

1. My numerator is 6 less than my denominator.
   I am equivalent to $\frac{3}{4}$.

2. My denominator is 5 more than twice my numerator.
   I am equivalent to $\frac{1}{3}$.

3. The GCF of my numerator and denominator is 3.
   I am equivalent to $\frac{2}{5}$.

4. The GCF of my numerator and denominator is 5.
   I am equivalent to $\frac{4}{6}$.

5. My numerator and denominator are prime numbers.
   My numerator is one less than my denominator.

6. My numerator and denominator are prime numbers.
   The sum of my numerator and denominator is 24.

7. My numerator is divisible by 3.
   My denominator is divisible by 5.
   My denominator is 4 less than twice my numerator.

8. My numerator is divisible by 3.
   My denominator is divisible by 5.
   My denominator is 3 more than twice my numerator.

9. My numerator is a one-digit prime number.
   My denominator is a one-digit composite number.
   I am equivalent to $\frac{8}{32}$.

10. My numerator is a prime number.
    The GCF of my numerator and denominator is 2.
    I am equivalent to $\frac{1}{5}$.

11. **CHALLENGE** Make up your own mystery like the ones above. Be sure that there is only one solution. To check, have a classmate solve your mystery.

4-3 Enrichment

Recipes

It is common to see mixed fractions in recipes. A recipe for a pizza crust may ask for $1\frac{1}{2}$ cups of flour. You could measure this amount in two ways. You could fill a one-cup measuring cup with flour and a one-half-cup measuring cup with flour or you could fill a half-cup measuring cup three times, because $1\frac{1}{2}$ is the same as $\frac{3}{2}$.

In the following recipes, some mixed numbers have been changed to improper fractions and other fractions may not be written in simplest form. Rewrite each recipe as you would expect to find it in a cookbook.

Quick Pizza Crust	
$\frac{3}{2}$ cups flour	
$\frac{2}{4}$ cup water	
$\frac{9}{4}$ teaspoons yeast	
$\frac{2}{2}$ teaspoon salt	
$\frac{4}{4}$ teaspoon sugar	
$\frac{8}{8}$ tablespoon oil	

Apple Crunch	
$\frac{3}{2}$ cups white sugar	
$\frac{3}{2}$ cups brown sugar	
$\frac{4}{2}$ cups of flour	
$\frac{4}{2}$ cups oatmeal	
$\frac{8}{3}$ sticks margarine	
$\frac{2}{2}$ teaspoon salt	

Granola	
$\frac{4}{3}$ cups sesame seeds	
$\frac{4}{2}$ cups coconut	
$\frac{3}{2}$ cups sunflower seeds	
$\frac{8}{2}$ cups rolled oats	
$\frac{2}{2}$ cup honey	
$\frac{4}{4}$ tablespoon brown sugar	

Chocolate Treats	
$\frac{4}{6}$ cup butter	
$\frac{9}{4}$ cups brown sugar	
$\frac{6}{2}$ eggs	
$\frac{11}{4}$ cups flour	
$\frac{5}{2}$ teaspoons baking powder	
$\frac{6}{3}$ cups chocolate chips	

Lesson 4–3

4-5 Enrichment

Perfect!

A **proper factor** of a number is any factor of the number except the number itself. You can use proper factors to classify numbers.

A number is **abundant** if the sum of its proper factors is greater than the number itself.

Proper factors of 12: 1, 2, 3, 4, 6
$1 + 2 + 3 + 4 + 6 = 16$, and
$16 > 12$. So, 12 is *abundant*.

A number is **deficient** if the sum of its proper factors is less than the number itself.

Proper factors of 16: 1, 2, 4, 8
$1 + 2 + 4 + 8 = 15$, and $15 < 16$.
So, 16 is *deficient*.

Now you can probably guess the definition of a perfect number. A number is **perfect** if the sum of its proper factors is equal to the number itself.

Proper factors of 6: 1, 2, 3
$1 + 2 + 3 = 6$
So, 6 is *perfect!*

Tell whether each number is *abundant*, *deficient*, or *perfect*.

1. 8

2. 9

3. 15

4. 18

5. 20

6. 24

7. 25

8. 28

9. 30

10. 35

11. What is the least whole number that is abundant?

12. Is it possible for a prime number to be perfect? Explain.

13. Is it possible for the sum of two deficient numbers to be an abundant number? Explain.

14. **CHALLENGE** Show why 496 is a perfect number.

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4-6 Enrichment

Developing Fraction Sense

If someone asked you to name a fraction between $\frac{4}{7}$ and $\frac{6}{7}$, you probably would give the answer $\frac{5}{7}$ pretty quickly. But what if you were asked to name a fraction between $\frac{4}{7}$ and $\frac{5}{7}$? At the right, you can see how to approach the problem using "fraction sense." So, one fraction between $\frac{4}{7}$ and $\frac{5}{7}$ is $\frac{9}{14}$.

$$\frac{4}{7} = \frac{\bullet}{14} \rightarrow \frac{4}{7} = \frac{8}{14}$$
$$\frac{5}{7} = \frac{\bullet}{14} \rightarrow \frac{5}{7} = \frac{10}{14}$$

Use your fraction sense to solve each problem.

1. Name a fraction between $\frac{1}{3}$ and $\frac{2}{3}$.

2. Name a fraction between $\frac{3}{5}$ and $\frac{4}{5}$.

3. Name five fractions between $\frac{1}{2}$ and 1.

4. Name five fractions between 0 and $\frac{1}{4}$.

5. Name a fraction between $\frac{1}{4}$ and $\frac{1}{2}$ whose denominator is 16.

6. Name a fraction between $\frac{2}{3}$ and $\frac{3}{4}$ whose denominator is 10.

7. Name a fraction between 0 and $\frac{1}{6}$ whose numerator is 1.

8. Name a fraction between 0 and $\frac{1}{10}$ whose numerator is *not* 1.

9. Name a fraction that is halfway between $\frac{2}{9}$ and $\frac{5}{9}$.

10. Name a fraction between $\frac{1}{4}$ and $\frac{3}{4}$ that is closer to $\frac{1}{4}$ than $\frac{3}{4}$.

11. Name a fraction between 0 and $\frac{1}{2}$ that is less than $\frac{3}{10}$.

12. Name a fraction between $\frac{1}{2}$ and 1 that is less than $\frac{3}{5}$.

13. Name a fraction between $\frac{1}{2}$ and $\frac{3}{4}$ that is greater than $\frac{4}{5}$.

14. How many fractions are there between $\frac{1}{4}$ and $\frac{1}{2}$?

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4-7 Enrichment

Estimating with Decimals and Fractions

Often you only need to give a fractional estimate for a decimal. To make fractional estimates, it helps to become familiar with the fraction-decimal equivalents shown in the chart at the right. You also should be able to identify the fraction as an *overestimate* or *underestimate*. Here's how.

The decimal 0.789 is a little less than 0.8, so it is a little less than $\frac{4}{5}$. Write $\frac{4^-}{5}$.

The decimal 1.13 is a little more than 1.125, so it is a little more than $1\frac{1}{8}$. Write $1\frac{1^+}{8}$.

0.1	$= \frac{1}{10}$
0.125	$= \frac{1}{8}$
0.2	$= \frac{1}{5}$
0.25	$= \frac{1}{4}$
0.3	$= \frac{3}{10}$
0.375	$= \frac{3}{8}$
0.4	$= \frac{2}{5}$
0.5	$= \frac{1}{2}$
0.6	$= \frac{3}{5}$
0.625	$= \frac{5}{8}$
0.7	$= \frac{7}{10}$
0.75	$= \frac{3}{4}$
0.8	$= \frac{4}{5}$
0.875	$= \frac{7}{8}$
0.9	$= \frac{9}{10}$

Write a fractional estimate for each decimal. Be sure to identify your estimate as an overestimate or an underestimate.

1. 0.243 **2.** 0.509 **3.** 0.429

4. 0.741 **5.** 0.88 **6.** 0.63

7. 0.09 **8.** 0.57 **9.** 1.471

10. 2.76 **11.** 1.289 **12.** 5.218

13. The scale in the delicatessen shows 0.73 pound. Write a fractional estimate for this weight.

14. Darnell ordered a quarter pound of cheese. The scale shows 0.23 pound. Is this more or less than he ordered?

15. On the stock market, prices are listed as halves, fourths, and eighths of a dollar. Yesterday the price of one share of a stock was $25.61. Write a fractional estimate for this amount.

16. Charlotte used a calculator to figure out how many yards of ribbon she needed for a craft project. The display shows 2.53125. Write a fractional estimate for this length.

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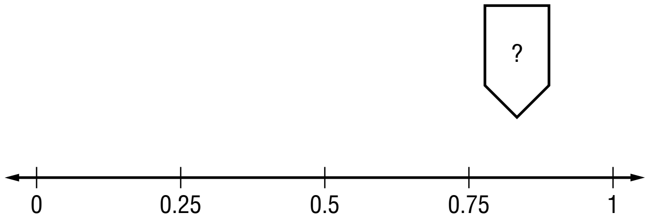
4-8 Enrichment

Tagging Along

Which of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{9}{10}$ belongs in the “tag” on the number line at the right? The tag is to the right of 0.75, so the fraction must be greater than 0.75. Express each fraction as a decimal.

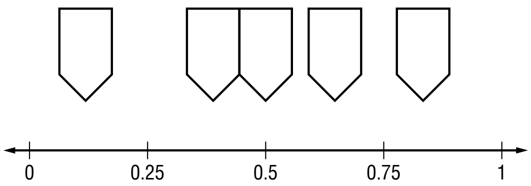


$\frac{2}{3} = 0.\overline{6}$, $\frac{3}{4} = 0.75$, $\frac{4}{5} = 0.8$, $\frac{9}{10} = 0.9$

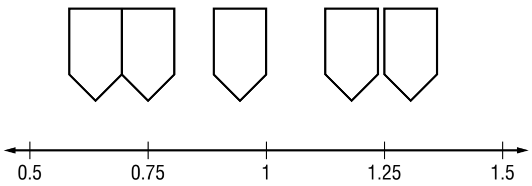
Only 0.8 and 0.9 are greater than 0.75, and 0.9 is much closer to 1 than to 0.75. Choose 0.8, which is equal to $\frac{4}{5}$.

On each number line, fill in the tags using the given fractions.

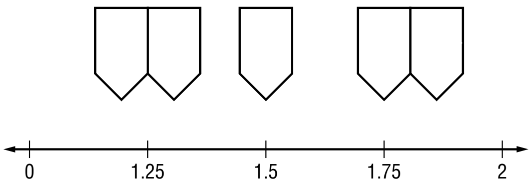
1. $\frac{3}{8}, \frac{1}{2}, \frac{2}{3}, \frac{1}{9}, \frac{7}{8}$



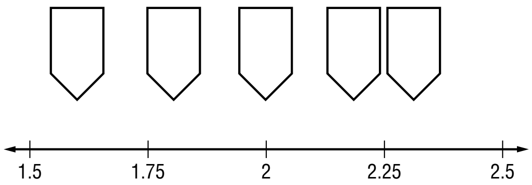
2. $\frac{4}{3}, \frac{3}{4}, \frac{6}{5}, \frac{5}{8}, \frac{15}{16}$



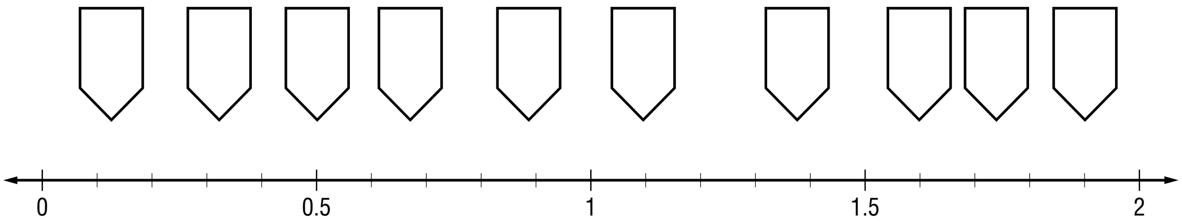
3. $\frac{7}{4}, \frac{6}{5}, \frac{15}{8}, \frac{3}{2}, \frac{4}{3}$



4. $\frac{9}{5}, \frac{7}{3}, \frac{8}{5}, \frac{13}{6}, \frac{8}{4}$



5. Write a fraction in simplest form for each tag on this number line. Use only the denominators 2, 3, 4, 5, 8, and 10. Express numbers greater than 1 as improper fractions.



Lesson 4–8

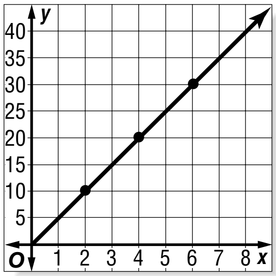
4-9 Enrichment

Investigating Coordinate Grids

You can use coordinate grids to display sets of ordered pairs. You can also find new ordered pairs by looking at the line that the plotted ordered pairs make.

The table below lists the cost of tickets to a play. The data from the table are plotted on the grid.

Number of Tickets	Total Cost
2	$10.00
4	$20.00
6	$30.00
8	$40.00

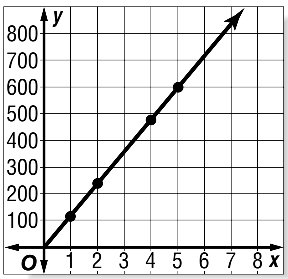


The table shows the cost of 2, 4, 6, and 8 tickets. To find the cost of 5 tickets, you can use the grid to find the ordered pair that fits the table.

- Start at the origin and move to 5 on the x-axis. This is the x-coordinate.
- Move up until you meet the line. Then follow across to the left to the y-axis to find the corresponding y-coordinate. The value is 25.
- The ordered pair is (5, 25). This ordered pair means 5 tickets cost $25.

EXERCISES Use the data plotted on the coordinate grid to answer the questions.

Time (in hours)	Distance
2	240
3	360
5	600
8	960



1. How many miles did the airplane travel in 1 hour?

2. How many miles did the airplane travel in 2 hours?

3. How many miles did the airplane travel in 5 hours?

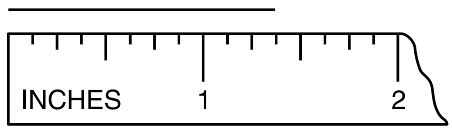
4. How long did it take the airplane to travel 720 miles?

5. How long did it take the airplane to travel 360 miles?

5-1 Enrichment

Greatest Possible Error

When you measure a quantity, your measurement is more **precise** when you use a smaller unit of measure. But no measurement is ever exact—there is always some amount of error. The **greatest possible error (GPE)** of a measurement is one half the unit of measure.

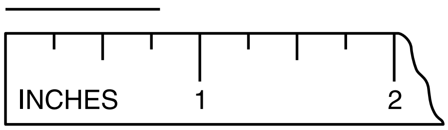


At the right, you see how the GPE for the ruler shown is calculated as $\frac{1}{16}$ inch. Since $1\frac{3}{8} = 1\frac{6}{16}$, the actual measure of the line segment may range anywhere from $1\frac{5}{16}$ inches to $1\frac{7}{16}$ inches.

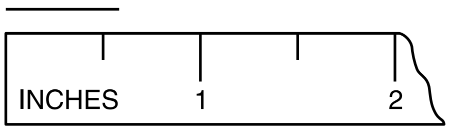
length of line segment:
$1\frac{3}{8}$ inches, to the nearest $\frac{1}{8}$ inch
unit of measure: $\frac{1}{8}$ inch
GPE: half of $\frac{1}{8}$ inch, or $\frac{1}{16}$ inch

Use the GPE to give a range for the measure of each line segment.

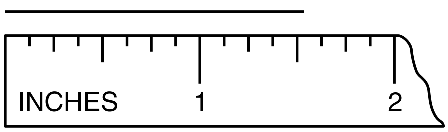
1.



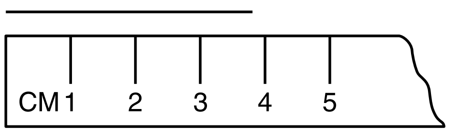
2.



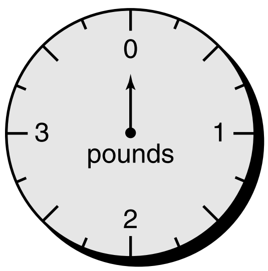
3.



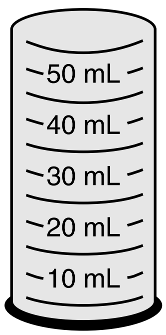
4.



5. Using this scale, the weight of a bag of potatoes is measured as 3 pounds. What is the range for the actual weight of the potatoes?



6. Using this container, the amount of a liquid is measured as 20 milliliters. What is the range for the actual amount of the liquid?



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5-2 Enrichment

Using 1 as a Benchmark

When you estimate sums of proper fractions, it often helps to use the number 1 as a *benchmark*, like this.

Two halves make a whole, so $\frac{1}{2} + \frac{1}{2} = 1$.

If two fractions are each less than $\frac{1}{2}$, their sum is less than 1.

$$\frac{3}{8} + \frac{4}{9} < 1$$

If two fractions are each greater than $\frac{1}{2}$, their sum is greater than 1.

$$\frac{5}{8} + \frac{7}{9} > 1$$

Fill in each $\square$ with < or > to make a true statement.

1. $\frac{2}{3} + \frac{5}{8} \square 1$ **2.** $\frac{2}{5} + \frac{3}{7} \square 1$ **3.** $\frac{3}{10} + \frac{5}{11} \square 1$

4. $\frac{27}{50} + \frac{7}{10} \square 1$ **5.** $\frac{50}{99} + \frac{38}{75} \square 1$ **6.** $\frac{24}{49} + \frac{32}{65} \square 1$

Fill in each $\square$ with one of the given fractions to make a true statement.

7. $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{1}{2} + \square > 1$ $\frac{1}{2} + \square < 1$

8. $\frac{8}{11}$ $\frac{7}{11}$ $\frac{6}{11}$ $\frac{5}{11}$ $\frac{1}{2} + \square > 1$ $\frac{1}{2} + \square < 1$

9. $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{9}{16} + \square > 1$ $\frac{9}{16} + \square < 1$

10. $\frac{1}{25}$ $\frac{12}{25}$ $\frac{13}{25}$ $\frac{24}{25}$ $\frac{6}{13} + \square > 1$ $\frac{6}{13} + \square < 1$

Fill in each $\square$ with < or > to make a true statement.

11. $1\frac{5}{8} - 1\frac{1}{2} \square \frac{1}{2}$ **12.** $1 - \frac{5}{11} \square \frac{1}{2}$ **13.** $1 - \frac{10}{19} \square \frac{1}{2}$

14. $1 - \frac{49}{99} \square \frac{1}{2}$ **15.** $4\frac{3}{7} + \frac{1}{3} \square 5$ **16.** $3 - \frac{4}{7} \square 2\frac{1}{2}$

5-4 Enrichment

Fraction Puzzles

In the puzzles below, the sum of the fractions in each row is the same as the sum of the fractions in each column. Use your knowledge of adding and subtracting fractions to find the missing fractions. Hint: Remember to check for like denominators before adding.

$\frac{3}{20}$	$\frac{9}{20}$		
	$\frac{2}{20}$		$\frac{2}{20}$
$\frac{2}{20}$	$\frac{4}{20}$		$\frac{7}{20}$
	$\frac{3}{20}$	$\frac{6}{20}$	

$\frac{9}{15}$		$\frac{3}{15}$	$\frac{2}{15}$
$\frac{4}{15}$		$\frac{0}{15}$	
$\frac{2}{15}$		$\frac{7}{15}$	
$\frac{1}{15}$	$\frac{2}{15}$		$\frac{7}{15}$

$\frac{6}{25}$	$\frac{3}{25}$	$\frac{11}{25}$	
			$\frac{2}{25}$
$\frac{2}{25}$			$\frac{6}{25}$
$\frac{3}{25}$	$\frac{4}{25}$	$\frac{1}{25}$	$\frac{12}{25}$

$\frac{8}{16}$	$\frac{1}{16}$		$\frac{1}{8}$
	$\frac{7}{16}$		$\frac{1}{8}$
$\frac{3}{16}$			$\frac{1}{8}$
$\frac{0}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$

CHALLENGE Create your own fraction puzzle using a box of 5 rows and 5 columns.

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5-5 Enrichment

Unit Fractions

A **unit fraction** is a fraction with a numerator of 1 and a denominator that is any counting number greater than 1.

unit fractions: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{10}$

A curious fact about unit fractions is that each one can be expressed as a sum of two distinct unit fractions. (*Distinct* means that the two new fractions are different from one another.)

$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$ $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$ $\frac{1}{10} = \frac{1}{11} + \frac{1}{110}$

Did you know?

The *Rhind Papyrus* indicates that fractions were used in ancient Egypt nearly 4,000 years ago. If a fraction was not a unit fraction, the Egyptians wrote it as a sum of unit fractions. The only exception to this rule seems to be the fraction $\frac{2}{3}$.

1. The three sums shown above follow a pattern. What is it?

2. Use the pattern you described in Exercise 1. Express each unit fraction as a sum of two distinct unit fractions.

a. $\frac{1}{4}$ **b.** $\frac{1}{5}$ **c.** $\frac{1}{12}$ **d.** $\frac{1}{100}$

Does it surprise you to know that other fractions, such as $\frac{5}{6}$, can be expressed as sums of unit fractions? One way to do this is by using equivalent fractions. Here's how.

$\frac{5}{6} = \frac{10}{12}$ → $\frac{10}{12} = \frac{6}{12} + \frac{4}{12} = \frac{1}{2} + \frac{1}{3}$ → $\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$

3. Express each fraction as a sum of two distinct unit fractions.

a. $\frac{2}{3}$ **b.** $\frac{4}{15}$ **c.** $\frac{5}{9}$ **d.** $\frac{2}{5}$

4. Express $\frac{4}{5}$ as the sum of *three* distinct unit fractions.

5. **CHALLENGE** Show two different ways to express $\frac{1}{2}$ as the sum of three distinct unit fractions.

5-6 Enrichment

Equations with Fractions and Decimals

Sometimes an equation involves both fractions and decimals. To solve an equation like this, you probably want to work with numbers in the same form. One method of doing this is to start by expressing the decimals as fractions. The example at the right shows how you might solve the equation $m + \frac{2}{5} = 0.6$.

$$m + \frac{2}{5} = 0.6$$

$$m + \frac{2}{5} = \frac{3}{5} \quad \leftarrow \text{Write 0.6 as a fraction.}$$

$$m = \frac{3}{5} - \frac{2}{5}$$

$$m = \frac{1}{5}$$

Name the number that is a solution of the given equation.

1. $z = \frac{1}{8} + 0.375;\ \frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$

2. $0.75 - \frac{3}{4} = b;\ 0, \frac{1}{4}, 1, 1\frac{1}{4}$

3. $c + 0.6 = \frac{4}{5};\ \frac{1}{5}, \frac{3}{5}, 1\frac{1}{5}, 1\frac{2}{5}$

4. $0.6 = j - \frac{1}{5};\ \frac{1}{5}, \frac{4}{5}, 1, 1\frac{2}{5}$

5. $\frac{1}{4} + r = 0.75;\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

6. $d - 0.1 = \frac{7}{10};\ \frac{1}{2}, \frac{3}{5}, \frac{4}{5}, \frac{9}{10}$

Solve each equation. If the solution is a fraction or a mixed number, be sure to express it in simplest form.

7. $\frac{2}{5} + 0.4 = k$

8. $s = \frac{7}{8} - 0.125$

9. $0.6 - n = \frac{2}{5}$

10. $t + 0.2 = \frac{4}{5}$

11. $0.375 + g = \frac{5}{8}$

12. $y - 0.25 = \frac{3}{4}$

13. $0.8 - \frac{1}{5} = x$

14. $q + 0.125 = \frac{5}{8}$

15. $w = \frac{1}{8} + 0.375 + \frac{5}{8}$

16. $0.7 + \frac{1}{10} - 0.3 = a$

17. $p + \frac{1}{5} = 0.8 - \frac{3}{5}$

18. $k - 0.875 = 0.375 + \frac{1}{8}$

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5-7 Enrichment

A Fraction of an Inch

Fractions are important in measurement. When you go to the doctor, your height is not measured to the nearest inch. It is measured to fractions of an inch. You may be 4 feet, $10\frac{1}{2}$ inches tall. How much taller are you than a friend who is 4 feet, $6\frac{3}{4}$ inches tall?

Solve each problem by renaming mixed numbers and subtracting.

1. Janelle is cutting a piece of wood that is $15\frac{1}{2}$ inches long for a picture frame. If she is cutting it from a piece of wood that is $20\frac{1}{8}$ inches long, what is the length of wood that will be left over?

2. The winning high jump in a track meet was 4 feet, 8 inches. The second place jump was 4 feet, $6\frac{1}{2}$ inches. How many inches higher than the second place jump was the winning jump?

3. A carpenter needs to cut a piece of wood that is $21\frac{5}{8}$ inches long from a board that is $32\frac{1}{4}$ inches long. How long is the piece of wood that is left after it is cut?

4. Evie is cutting ribbons $8\frac{1}{3}$ feet long for a sewing project. If the original ribbon is $36\frac{1}{4}$ feet long, how long is it after she cuts her first ribbon?

5. Fabric is sold by the yard. Derek wants $3\frac{3}{8}$ yards of a particular kind of fabric. There is only to be $4\frac{1}{4}$ yards of the fabric left on the bolt. Derek buys what is left. How much extra did he buy?

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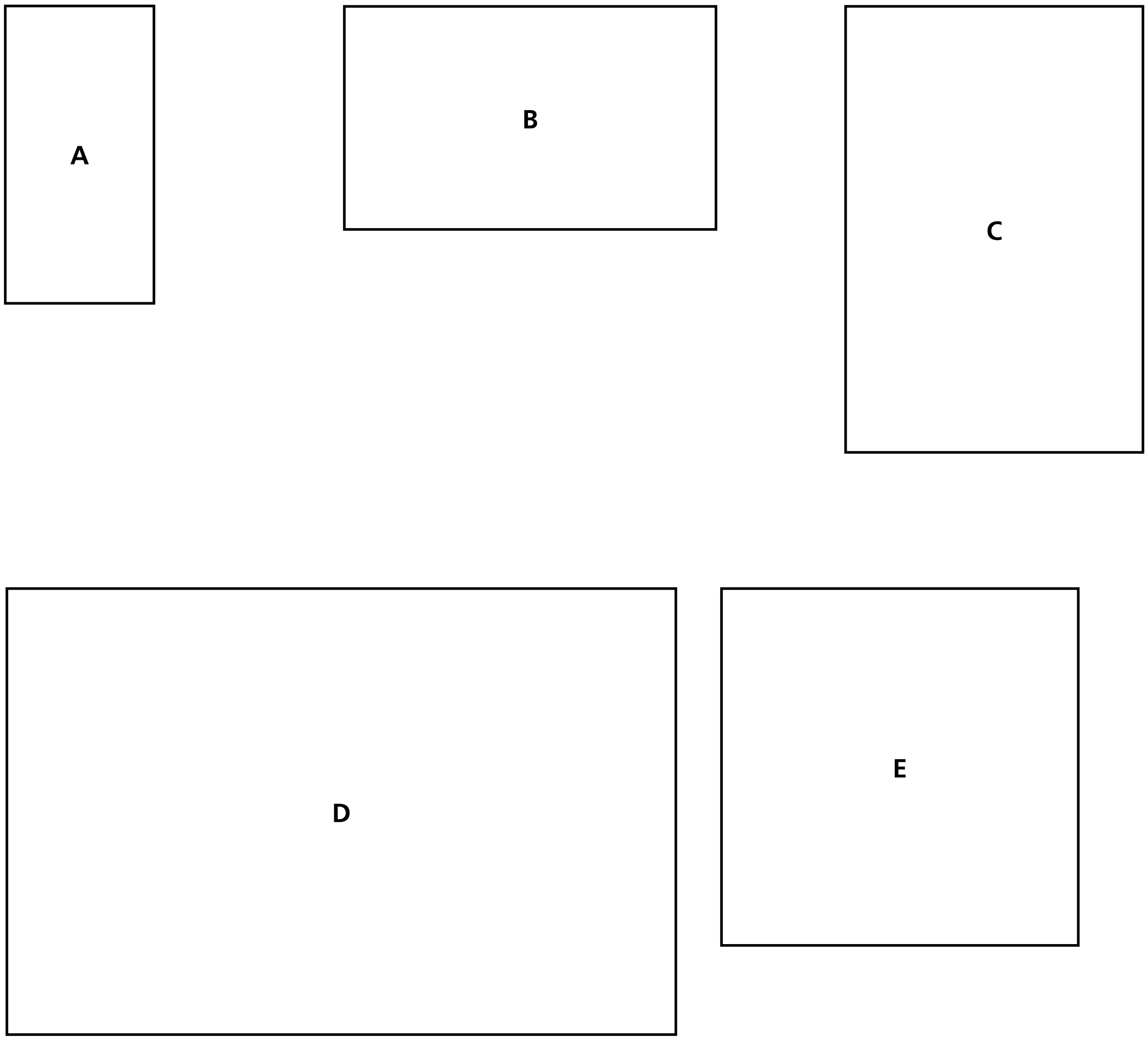
6-1 Enrichment

Ratios and Rectangles

1. Use a centimeter ruler to measure the width and the length of each rectangle. Then express the ratio of the width to the length as a fraction in simplest form.



2. Similar figures have the same shape, but not necessarily the same size. Two rectangles are similar if the ratio of the width to the length is the same for each. Which rectangles in Exercise 1 are similar?

3. For centuries artists and architects have used a shape called the **golden rectangle** because people seem to find it most pleasant to look at. In a golden rectangle, the ratio of the width to the length is a little less than $\frac{5}{8}$. Which rectangle in Exercise 1 is most nearly a golden rectangle?

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6-2 Enrichment

Business Planning

In order to have a successful business, the manager must plan ahead and decide how certain actions will affect the business. The first step is to predict the financial impact of business decisions. Julie has decided that she wants to start a brownie business to make extra money over the summer. Before she can ask her parents for money to start her business, she needs to have some information about how many batches of brownies she can make in a day and for how much she must sell the brownies to make a profit.

1. Julie can bake 3 batches of brownies in 2 hours. Her goal is to bake 12 batches of brownies each day. Use the table to find how many hours Julie will need to bake to reach her goal.

Batches of Brownies	3			12
Hours	2			

2. Each batch of brownies will be sold for $2.00. How much money will Julie make if she sells 6 batches of brownies?

Batches of Brownies	1				6
Cost	$2				

3. If Julie works for 10 hours a day, how many batches of brownies can she bake?

Batches of Brownies	3	
Hours	2	10

4. Julie hires a friend to help. Together, they can bake 24 batches of brownies in 8 hours. How long does it take for the two of them to bake 6 batches of brownies?

Batches of Brownies	6			24
Hours				8

5. If Julie and her friend can bake 24 batches of brownies in 8 hours, and they both work 40 hours in one week, how many batches of brownies can they bake that week? If Julie still charges $2.00 a batch, how much money will they make that week?

Hours	8				40
Batches of Brownies	24				

Batches of Brownies	1	
Cost	$2	

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6-3 Enrichment

"Liberty Enlightening the World"

On July 4, 1889, in gratitude to the French for the gift of the Statue of Liberty, Americans from Paris gave to the French a miniature Statue of Liberty. The statue is made of bronze and is approximately one fourth the size of the original. This smaller-scale copy is found near the Grenelle Bridge on the Île des Cygnes, an island in the Seine River about one mile south of the Eiffel Tower.

1. If the original Statue of Liberty is approximately 150 feet tall, about how tall is the replica?

2. Complete the table. The first one is done for you.

	Original Statue of Liberty	Replica
Length of hand	16 ft	4 ft
Length of nose	4.5 ft	
Length of right arm	42 ft	
Head thickness from ear to ear		2.5 ft
Width of mouth		9 in.
Thickness of waist	35 ft	
Distance from heel to the top of her head	111 ft	
Length of index finger	8 ft	
Circumference of the second joint	3.5 ft	

3. The fingernail on the index finger of the original weighs 1.5 kilograms. How much does the fingernail on the replica in France weigh?

4. The dimensions of the tablet that Lady Liberty is holding are 23.6 feet by 13.6 feet by 2 feet. What are the dimensions of the smaller-scale tablet in France?

5. **CHALLENGE** The fingernail on the index finger is 13 inches long and 10 inches wide. What will be the area of the fingernail on the replica in France?

6-4 Enrichment

Ada

Did you know that a woman wrote the first description of a computer programming language? She was the daughter of a famous English lord and was born in 1815. She had a deep understanding of mathematics and was fascinated by calculating machines. Her interests led her to create the first algorithm. In 1843, she translated a French version of a lecture by Charles Babbage. In her notes to the translation, she outlined the fundamental concepts of computer programming. She died in 1852. In 1979, the U.S. Department of Defense named the computer language *Ada* after her.

To find out this woman's full name, solve the proportion for each letter.

1. $\frac{7}{A} = \frac{28}{40}$

2. $\frac{5}{4} = \frac{B}{36}$

3. $\frac{1}{3} = \frac{C}{15}$

4. $\frac{5}{D} = \frac{35}{63}$

5. $\frac{2}{5} = \frac{E}{20}$

6. $\frac{2}{18} = \frac{L}{27}$

7. $\frac{6}{N} = \frac{12}{14}$

8. $\frac{9}{11} = \frac{O}{44}$

9. $\frac{2}{8} = \frac{R}{4}$

10. $\frac{5}{V} = \frac{25}{30}$

11. $\frac{7}{4} = \frac{Y}{28}$

Now look for each solution below. Write the corresponding letter on the line above the solution. If you have calculated correctly, the letters will spell her name.

___ ___ ___   ___ ___ ___ ___ ___
10 9 10   45 49 1 36 7

___ ___ ___ ___ ___ ___ ___ ___
3 36 6 8 3 10 5 8

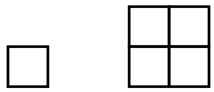
6-6 Enrichment

Geometric Sequences

A geometric sequence is one in which the ratio between the two terms is constant.

1. SQUARE NUMBERS A square number can be modeled by using an area model to create an actual square.

a. Draw the next two terms in the sequence and determine the fourth term.



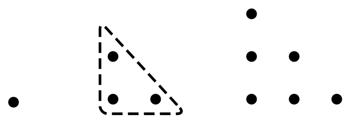
1   4   9   ___?___

b. The function that describes square numbers is n^2. Write this function using multiplication.

c. Complete the table by finding the missing position and the missing value of the term for square numbers.

Position	3			11	13	15	25
Value of Term	9	64	100			225	625

2. TRIANGULAR NUMBERS A triangular number can be modeled by using manipulatives or objects to create triangles. The first three triangular numbers are 1, 3, and 6.



1   3   6

a. Draw the next three terms in the sequence in the space above.

b. What is the ninth term?

c. The function that describes the triangular number sequence is $n \times \frac{(n + 1)}{2}$. Complete the table by finding either the missing position or missing value of the term for triangular numbers.

Position	3		8	10	15	20	100
Value of Term	6	10			120	210	

6-7 Enrichment

Enchanted Rock

Enchanted Rock is a pink granite dome located in Enchanted Rock State Natural Area in Central Texas. It is of the largest batholiths in the United States. A batholith is made of igneous rock and is the result of volcanic activity. The Enchanted Rock dome rises 425 feet above the ground and is 1825 feet above sea level.

The entrance fee to Enchanted Rock State Natural Area is $5.00 per person.

1. Complete the table to find the entrance cost for groups of different sizes.

Input, *x*	1	2	3	4	5	6	7	8
Output, *y*	$5.00	$10.00						

2. Write an equation to represent the function displayed in the table.

3. If the park has 290 visitors, how much money did they collect in entrance fees?.

4. A local environmental group is planning to hike up Enchanted Rock. The group will cover each member's entrance fee and will provide lunch for its members. The group budgets $75.00 for lunch, regardless of the number of people on the hike. Complete the table to show the total expenses of the group based on the number of people on the hike.

Input, *x*	5	10	15	20	25	30
Output, *y*	$100.00	$125.00				

5. Write an equation to represent the function displayed in the table.

6. Write an equation to represent the function displayed in the table.

7. The group will hike up the dome at a rate of 1500 feet per hour. What is their hiking speed per minute?

8. Complete the table to show the progression of their hike.

Input (min), *x*	1	3	5	8	10	12	15	
Output (feet), *y*	25	75						425

9. Write an equation that represents the function displayed in the table.

10. At the rate given, how long will it take the group to reach the top of Enchanted Rock?

NAME ______________________________ DATE ____________ PERIOD _____

7-1 Enrichment

It's On Sale!

Stores have sales to attract people to buy their merchandise or to sell off seasonal merchandise at the end of a season. They may advertise 20% off the regular price of an item or $\frac{1}{2}$ off the regular price. Sometimes, stores will offer an extra sale on top of the sale price.

Stores usually advertise the sale price as a percentage or a fraction off the original price. Savvy shoppers know how percentages and fractions compare to know which is a better deal.

Write a fraction representing how much off the regular price is the store offering.

1. 25% off all kitchen items!

2. 50% off ELECTRONICS

3. 20% off all outerwear

Write each fraction as a percent.

4. *Sale Today $\frac{1}{2}$ off*

5. $\frac{1}{5}$ off with your Rewards Card

6. $\frac{1}{4}$ off all winter jackets

Which is the better deal?

7. HUGE CLEARANCE! 45% OFF!

$\frac{1}{2}$ off every purchase with coupon

8. Sale Today $\frac{1}{3}$ off all shoes!

Save 40% on all shoes!

NAME ________________________________ DATE ____________ PERIOD _____

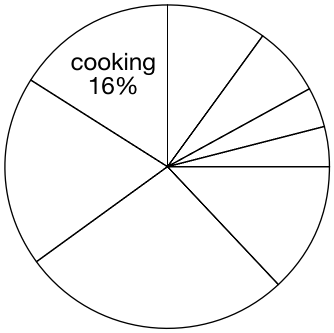
7-2 Enrichment

A Circle Graph Mystery

The circle graph below was drawn to show the leading causes of fire in the United States. However, all the labels except one have mysteriously disappeared.

Use the clues below to decide what the labels should be and where they belong. Then complete the graph. (Remember: Each label must include a word or phrase and a percent.)

Causes of Fires



Clue 1 Most fires are caused by *heating equipment*.

Clue 2 Fires caused by *electrical wiring* and fires caused by *heating equipment* together make up 46% of all fires.

Clue 3 The percent of fires caused by *children playing* is 12% less than the percent of fires caused by *cooking*.

Clue 4 The percent of fires caused by *open flames* is equal to the percent of fires caused by *children playing*.

Clue 5 The percent of the fires caused by *cooking* and the percent of fires caused by *arson* are together just 1% less than the percent of fires caused by *heating equipment*.

Clue 6 The percent of the fires caused by *electrical wiring* is 15% greater than the percent caused by *children playing*.

Clue 7 Fires caused by *smoking* and fires caused by *arson* together make up 17% of all fires.

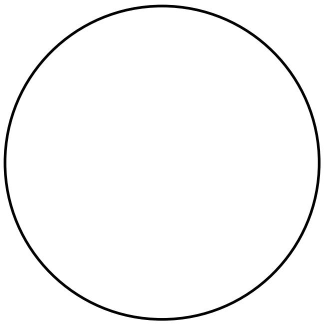
Clue 8 Fires that result from other causes are listed in a category called *other*.

Exercise

How well can you picture data? In the space at the right, sketch a circle graph to show the data below.

Americans' Region of Residence, 2000	
Northeast	19%
Midwest	23%
South	35%
West	23%

Americans' Region of Residence, 2000



7-3 Enrichment

Percent and Per Mill

A **percent** is a ratio that compares a number to 100.

$$\frac{83}{100} = 83 \text{ percent} = 83\% = 0.83$$

A ratio that compares a number to 1,000 is called a **per mill**. Just like percent, the ratio *per mill* has a special symbol, ‰.

$$\frac{83}{1{,}000} = 83 \text{ per mill} = 83‰ = 0.083$$

Throughout the world, the ratio that is used most commonly is percent. However, in some countries, you will find both ratios in use.

Express per mill as a decimal.

1. 325‰

2. 71‰

3. 6‰

4. 900‰

5. 20‰

6. 100‰

Express each per mill as a fraction in simplest form.

7. 47‰

8. 400‰

9. 100‰

10. 25‰

11. 150‰

12. 30‰

Express each fraction as a per mill.

13. $\frac{729}{1{,}000}$

14. $\frac{58}{100}$

15. $\frac{7}{10}$

16. $\frac{1}{2}$

17. $\frac{3}{4}$

18 $\frac{5}{8}$

19. $\frac{4}{5}$

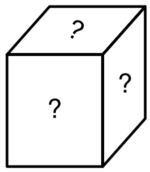
20. $\frac{17}{20}$

21. $\frac{1}{3}$

22. **CHALLENGE** In the United States, you will sometimes find the **mill** used as a monetary unit. What amount of money do you think is represented by 1 mill?

7-4 Enrichment

Working Backward with Probabilities



Suppose that you are given this information about rolling a number cube.

$P(1) = \frac{1}{2}$ $\quad P(3) = \frac{1}{3}$ $\quad P(5) = \frac{1}{6}$

Can you tell what numbers are marked on the faces of the cube? Work backward. Since a cube has six faces, express each probability as a fraction whose denominator is 6.

$P(1) = \frac{3}{6}$ $\quad P(3) = \frac{2}{6}$ $\quad P(5) = \frac{1}{6}$

So, the cube must have three faces marked with the number 1, two faces marked 3, and one face marked 5.

Each set of probabilities is associated with rolling a number cube. What numbers are marked on the faces of each cube?

1. $P(2) = \frac{1}{3}$

$P(4) = \frac{1}{3}$

$P(6) = \frac{1}{3}$

2. $P(1) = \frac{1}{6}$

$P(4) = \frac{1}{6}$

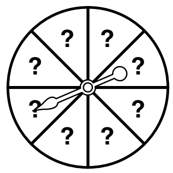
$P(\text{factor of } 4) = 1$

3. $P(1 \text{ or } 2) = \frac{5}{6}$

$P(2 \text{ or } 3) = \frac{2}{3}$

$P(1, 2, \text{ or } 3) = 1$

Each set of probabilities is associated with the spinner shown at the right. How many sections of each color are there?



4. $P(\text{red}) = \frac{1}{2}$

$P(\text{blue}) = \frac{1}{4}$

$P(\text{green}) = \frac{1}{8}$

$P(\text{black}) = \frac{1}{8}$

5. $P(\text{yellow or purple}) = \frac{5}{8}$

$P(\text{purple or white}) = \frac{3}{4}$

$P(\text{green or blue}) = 0$

$P(\text{yellow, purple, or white}) = 1$

6. Suppose that you are given this information about pulling a marble out of a bag.

$P(\text{green}) = \frac{1}{4}$ $\quad P(\text{blue}) = \frac{1}{6}$ $\quad P(\text{red}) = \frac{3}{8}$

$P(\text{yellow}) = \frac{1}{24}$ $\quad P(\text{white}) = \frac{1}{24}$ $\quad P(\text{black}) = \frac{1}{8}$

If the bag contains 48 marbles, how many marbles of each color are there?

7-5 Enrichment

Listing Outcomes in a Table

Suppose that you spin the two spinners below. What is the probability that the sum of the numbers you spin is 5?

First Spinner

Second Spinner

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

To find this probability, you first need to count the outcomes. One way to do this is to use a table of sums like the one at the right. From the table, it is easy to see that there are 24 outcomes. It is also easy to see that, in 4 of these outcomes, the sum of the numbers is 5. So, the probability that the sum of the numbers is 5 is $\frac{4}{24}$, or $\frac{1}{6}$.

Use the spinners and the table above. Find each probability.

1. *P*(sum is 8)
2. *P*(sum is 12)
3. *P*(sum is greater than 6)
4. *P*(sum is less than or equal to 10)

Suppose you roll two number cubes. Each cube is marked with 1, 2, 3, 4, 5, and 6 on its faces. Find each probability. (*Hint*: On a separate sheet of paper, make a chart like the one above.)

5. *P*(sum is 9)
6. *P*(sum is 3)
7. *P*(sum is an even number)
8. *P*(sum is a multiple of 3)
9. *P*(sum is a prime number)
10. *P*(sum is a factor of 12)
11. *P*(sum is greater than 12)
12. *P*(sum is less than 6)

13. **CHALLENGE** Here is a set of probabilities associated with two spinners.

$P(\text{sum is 4}) = \frac{1}{6}$ $P(\text{sum is 6}) = \frac{1}{3}$

$P(\text{sum is 8}) = \frac{1}{3}$ $P(\text{sum is 10}) = \frac{1}{6}$

In the space at the right, sketch the two spinners.

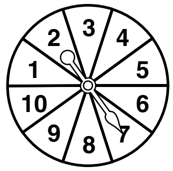
7-6 Enrichment

Odds

People who play games of chance often talk about **odds**. You can find the *odds in favor* of an event by using this formula.

$$\text{odds in favor} = \frac{\text{number of ways an event can occur}}{\text{number of ways the event cannot occur}}$$

With the spinner shown at the right, for example, this is how you would find the odds in favor of the event *prime number*.



There are four prime numbers (2, 3, 5, 7). → $\frac{4}{6} = \frac{2}{3}$
Six numbers are not prime (1, 4, 6, 8, 9, 10). →

The odds in favor of the event *prime number* are $\frac{2}{3}$ or 2 to 3.

Suppose that you spin the spinner shown above. Find the odds in favor of each event.

1. number greater than 3

2. number less than or equal to 6

3. even number

4. odd number

5. multiple of 3

6. factor of 10

To find the *odds against* an event, you use this formula.

$$\text{odds against} = \frac{\text{number of ways an event cannot occur}}{\text{number of ways the event can occur}}$$

Suppose that you roll a number cube with 1, 2, 3, 4, 5, and 6 marked on its faces. Find the odds against each event.

7. number less than 5

8. number greater than or equal to 2

9. even number

10. odd number

11. number divisible by 3

12. factor of 12

13. **CHALLENGE** The probability of an event is $\frac{2}{3}$. What are the odds in favor of the event? the odds against the event?

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7-8 Enrichment

Using 100%, 10%, and 1%

Many people think of 100%, 10%, and 1% as *key percents*.

100% is the **whole**. 100% of 24 = 1 × 24, or 24.

10% is **one tenth** of the whole. 10% of 24 = 0.1 × 24, or 2.4.

1% is **one hundredth** of the whole. 1% of 24 = 0.01 × 24, or 0.24.

Find the percent of each number.

1. 100% of 8,000

2. 10% of 8,000

3. 1% of 8,000

4. 10% of 640

5. 100% of 720

6. 1% of 290

7. 1% of 50

8. 100% of 33

9. 10% of 14

10. 100% of 2

11. 1% of 9

12. 10% of 7

This is how you can use the key percents to make some computations easier.

3% of 610 = __?__.
1% of 610 = 6.1,
so 3% of 610 = 3 × 6.1, or 18.3.

5% of 24 = __?__.
10% of 24 = 2.4,
so 5% of 24 = $\frac{1}{2}$ of 2.4, or 1.2.

Find the percent of each number.

13. 2% of 140

14. 8% of 2,100

15. 4% of 9

16. 20% of 233

17. 70% of 90

18. 30% of 4,110

19. 5% of 160

20. 5% of 38

21. 50% of 612

22. 25% of 168

23. 2.5% of 320

24. 2.5% of 28

NAME ________________________ DATE ____________ PERIOD _____

8-1 Enrichment

Estimating Lengths

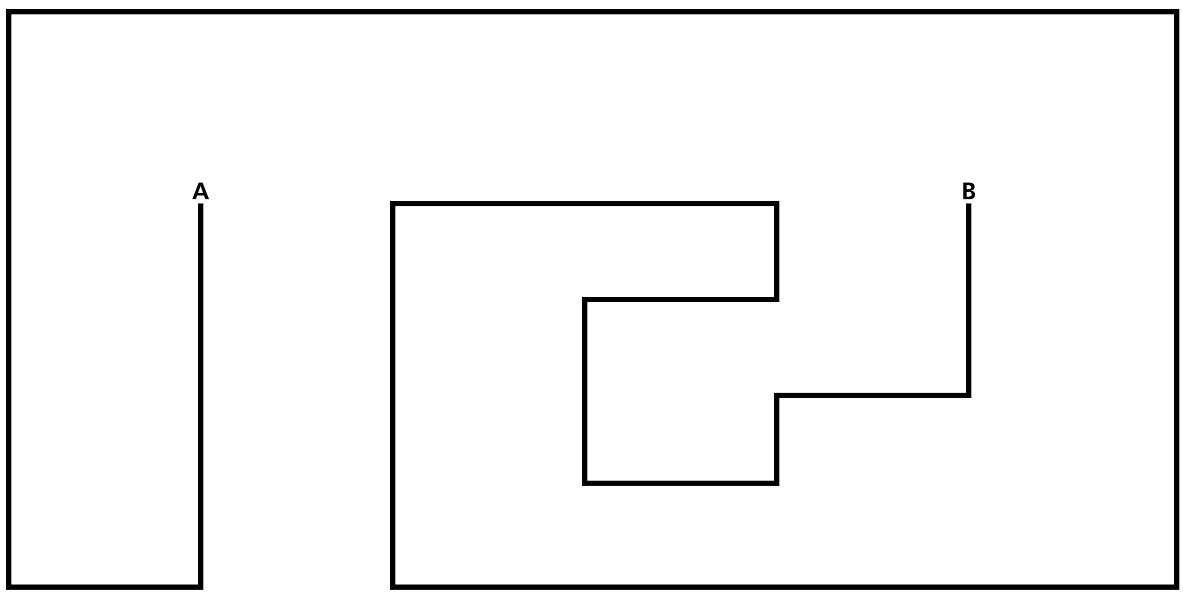
Many people estimate lengths using *rules of thumb* like those you see at the right.

An **inch** is about the width of a quarter.
A **foot** is about the length of a sheet of notebook paper.
A **yard** is about the distance from the floor to a doorknob.
A **mile** is about the length of ten city blocks.

Use the rules of thumb to estimate. Circle the most reasonable measure.

1.	length of a bus	40 in.	40 ft	40 yd
2.	length of a baseball bat	15 in.	1 ft	1 yd
3.	height of a flagpole	30 in.	3 ft	5 yd
4.	height of a table	36 in.	10 ft	2 yd
5.	distance across a street	20 ft	200 yd	1 mi
6.	length of one city block	30 ft	180 yd	$\frac{1}{2}$ mi
7.	width of a door	15 in.	15 ft	1 yd
8.	height of the world's tallest building	50 ft	100 yd	$\frac{1}{4}$ mi

9. Estimate the length of the path from A to B. Then measure. How close was your estimate?



8-2 Enrichment

Relating Customary and Metric Units

Both customary and metric measurements are used in the United States. Therefore, it is a good idea to develop some sense of the relationships between the two systems. Here are some *rules of thumb* that are commonly used.

An **inch** is about equal to 2.5 centimeters.
A **yard** is a little less than a meter.
A **mile** is a little more than 1.5 kilometers.
A **kilogram** is a little more than 2 pounds.
A **liter** is a little more than 1 quart.

Use the relationships given above. Tell whether each statement is true or false.

1. A length of 4 meters is longer than 4 yards.

2. A weight of 10 pounds is more than 5 kilograms.

3. A capacity of 1 gallon is more than 4 liters.

4. A length of 1 foot is about the same as 30 centimeters.

5. A kilometer is more than half a mile.

6. A pound is a little more than half a kilogram.

7. On a road in Canada, the posted speed limit is 45 kilometers per hour. Aimée is driving at a speed of 40 miles per hour. Is this above or below the speed limit?

8. Sean has a recipe that calls for 0.25 L of milk. He has a one-cup container of milk in the refrigerator. Is this enough milk for the recipe?

9. The posted load limit for a bridge is 5 tons. The mass of Darryl's truck is 1,500 kilograms, and it is holding cargo that weighs a half ton. Can Darryl drive his truck across the bridge?

10. Leah is pouring paint from a 5-gallon can into some jars. She has twelve jars that each hold 1 liter and six jars that each hold 1.25 liters. Does she have enough jars for all the paint?

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8-3 Enrichment

Metric System

The metric system was developed in the 18th century as a standardized form of measurement based on powers of ten. The 18th century metric system is different from the metric system we use now. The current metric system is called The *Système International* (SI) or the International System of Units. SI was created in 1960 and scientists all over the world have agreed to use this as their major unit system. Each SI unit has its own prefix to indicate its relative size.

Prefix	Decimal Equivalent	Exponential Equivalent
Pico-	0.000000000001	10^{-12}
Nano-	0.000000001	10^{-9}
Micro-	0.000001	10^{-6}
Milli-	0.001	10^{-3}
Centi-	0.01	10^{-2}
Deci-	0.1	10^{-1}

Prefix	Decimal Equivalent	Exponential Equivalent
(no prefix)	1	10^{0}
Deka-	10	10^{1}
Hecto-	100	10^{2}
Kilo-	1000	10^{3}
Mega-	1,000,000	10^{6}
Giga-	1,000,000,000	10^{9}

Source: http://members.aol.com/profchm/metric.html#end

1. Another prefix is *femto-*. The exponential equivalent for femto- is written as 10^{-15}. Find the decimal equivalent for femto-.

2. *Zepto-* is also a prefix in the SI. The decimal equivalent for zepto- is 0.000000000000000000001. Find the exponential equivalent.

3. Still another prefix is *tera-*. The decimal equivalent for tera- is 1,000,000,000,000. Find the exponential equivalent.

4. *Yotta-* is a prefix for very large numbers. The exponential equivalent for yotta- is written as 10^{24}. Find the decimal equivalent for yotta-.

5. Which amount is greater, 7 nanograms or 7 gigagrams? Explain your answer.

6. Which amount is the least, 9 kiloliters, 12 deciliters, or 18 microliters? Explain your answer.

7. The SI prefixes allow for measurements that are both very large and very small. When might a scientist use the prefix giga-? When might a scientist use prefix pico-?

8-4 Enrichment

Length, Mass, or Capacity?

When you encounter a problem about measurement, you won't necessarily see or hear one of the words *length*, *mass*, or *capacity*. Often you need to decide what type of measurement is involved, then choose the best unit of measure.

Tell whether each question most likely involves length, mass, or capacity.

1. Do I have enough milk to make this recipe?

2. Do I have enough string to tie around this package?

3. Will this punch bowl fit inside that box?

4. Will this amount of punch fit inside that bowl?

5. Is that tunnel high enough for this truck to drive through it?

6. Is that bridge strong enough for this truck to drive over it?

Circle the most reasonable measure for each object.

7. height of a doorway

2 g   2 kg   2 L   2 mL   2 m   2 cm

8. load limit of an elevator

1,000 g   1,000 kg   1,000 L   1,000 mL   1,000 m   1,000 cm

9. amount of water in a bathtub

150 g   1.5 kg   150 L   15 mL   1.5 m   150 cm

10. amount of cereal in a cereal box

400 g   4 kg   4,000 mL   4 L   0.4 m   400 cm

Name an item that you think has the given measure.

11. about 2 kg

12. about 250 mL

13. about 30 cm

14. about 25 g

8-6 Enrichment

Other Metric Units

Meters, millimeters, centimeters, and kilometers are the most commonly used metric units of length. But did you know that there are other units, like *decimeters*, *dekameters*, and *hectometers*? This table shows how all these units are related to the meter.

Unit	Number of Meters
kilometer (km)	1,000 m
hectometer (hm)	100 m
dekameter (dam)	10 m
meter (m)	1 m
decimeter (dm)	0.1 m
centimeter (cm)	0.01 m
millimeter (mm)	0.001 m

Each unit in the table is ten times as large as the unit below it. So, 1 km = 10 hm, and 1 hm = 10 dam. It follows that 1 km = (10×10) dam, or 1 km = 100 dam.

Use the table to complete each statement.

1. 1 dm = ______ cm

2. 1 dm = ______ mm

3. 5 hm = ______ dam

4. 12 km = ______ dam

5. 8.5 km = ______ hm

6. 3.2 dam = ______ dm

7. 1 m = ______ dm = ______ cm = ______ mm

8. 1 km = ______ hm = ______ dam = ______ m

Complete each table, modeling it on the table above.

9.

Unit	Number of Grams
kilogram (kg)	1,000 g
gram (g)	1 g
milligram (mg)	0.001 g

10.

Unit	Number of Liters
liter (L)	1 L

8-7 Enrichment

Aztec Calendars

The calendar used in the United States is the Gregorian calendar. It has $365\frac{1}{4}$ days in each year. The ancient Aztecs used a calendar that had been invented by the Mayas. The Aztec calendar had two systems. One was the *xiuhpohualli,* which had 365 days, like our Gregorian calendar. Each year in the *xiuhophualli* calendar had 18 months of 20 days each plus five additional days, for a total of 365 days.

The other system was the *tonalpohualli*, or the day-count. The *tonalpohualli* had 13 days and 20 symbols representing different gods. The calendar worked like two wheels. The wheels turned and matched a number with a symbol. After 260 days, the wheels returned to their starting positions and the day-counting started over.

Answer these questions about the xiuhophualli calendar.

1. How many fewer days on average does each month of the xiuhophualli calendar have than the months in the Gregorian calendar?

2. If the first day of the first month of the Aztec year corresponds to January 1, what date would the first day of the second month correspond to?

3. What date in the Gregorian calendar would correspond to the first day of the sixth month in the xiuhophualli calendar?

4. How many months in the xiuhophualli calendar would correspond to six months in the Gregorian calendar?

5. The Aztec divided the year into four seasons. How many months in the xiuhophualli calendar do you think would be in each season?

6. **CHALLENGE** Write the month and day of your birthday. Then count the days to determine in which month of the xiuhophualli calendar your birthday would be?

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8-8 Enrichment

Absolute Zero

Temperature is most often measured in degrees Fahrenheit or degrees Celsius. The temperature of a substance tells how fast the atoms or molecules in the substance are moving. The higher the temperature, the faster the atoms or molecules are moving. When water is at a temperature below 32°F or 0°C, it forms ice, and the molecules move very little.

Another temperature scale, called the Kelvin scale, is sometimes used by scientists. William Thomson Kelvin proposed this new scale in 1848, called an *absolute* scale, and 0K became *absolute zero*. Absolute zero is the coldest possible temperature in the universe. No place in our galaxy has ever reached the temperature of absolute zero.

The Kelvin scale has the same intervals as the Celsius scale. However, the degree mark is most commonly omitted when using the Kelvin scale. 0K equals −273.15°C. To convert from degrees Celsius to degrees Kelvin, you use the formula, K = °C + 273.15

Convert the temperatures from degrees Celsius to Kelvin and from Kelvin to degrees Celsius.

1. 272K = ________°C

2. 373.15K = ________°C

3. 88°C = ________K

4. 176.85°C = ________K

5. 30.15°C = ________K

What is a reasonable estimate of the temperature in Kelvins for each activity.

6. water skiing

7. ice fishing

8. temperature at the top of Mt. Everest

9. the record high temperature in Texas

10. **CHALLENGE** Nitrogen condenses to a liquid at 77K. What is the temperature in degrees Celsius?

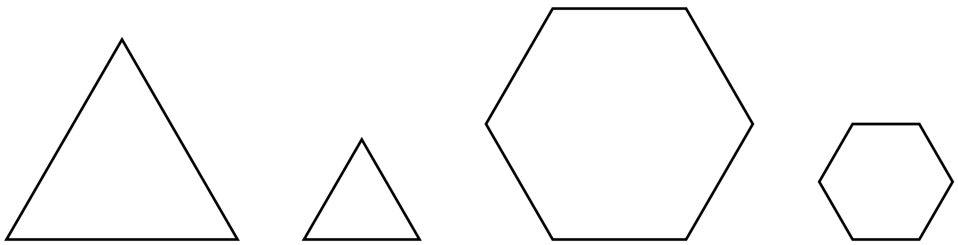
11. **CHALLENGE** Use the formula $\frac{9}{5}(K - 273.15) + 32$ to convert Kelvin degrees to degrees Fahrenheit. Helium condenses to a liquid at 4.2K. What is the temperature in degrees Fahrenheit?

9-1 Enrichment

Angles in Regular Polygons

The Department of Defense headquarters in Washington, D.C., is the Pentagon. This building was constructed in 1943 and is one of the world's largest office buildings. The Pentagon gets its name from the actual shape of the building. It is a regular pentagon so that all of the sides are the same length. The angles in a regular polygon are related in a special way.

1. Use a protractor to measure each angle in the regular polygons below.



_______________ _______________ _______________ _______________

2. What do you notice about the measures of the angles in the two triangles?

3. What do you notice about the measures of the angles in the two hexagons?

4. What can you conclude about the angles inside a regular polygon?

5. You can find the measure of an interior angle of a regular polygon with n-sides by using the formula $m = \frac{(n - 2)(180°)}{n}$. Find the measure of an interior angle of a stop sign.

6. If Sabrina builds a pen with 144° interior angles for her turkeys, and all the sides are of equal length, how many sides are on Sabrina's pen?

7. Draw a regular nonagon. Use a protractor to measure the angles. Use a ruler to measure the sides to make sure that they are equal.

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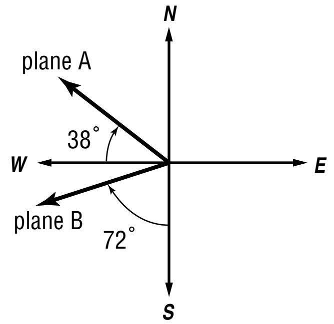
9-2 Enrichment

Compass Directions

When a plane is in flight, its direction is expressed as an angle measure. One method of doing this is to give the measure of the angle formed by the plane's flight path and one of the directions of the compass—north, east, south, or west. For example, this is how you express the two flight paths shown in the figure at the right.

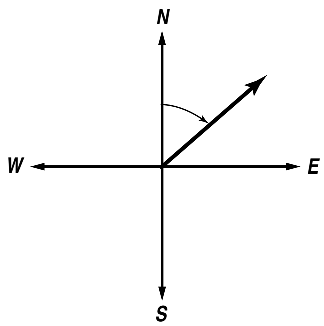


plane A: west 38° north, or W 38° N

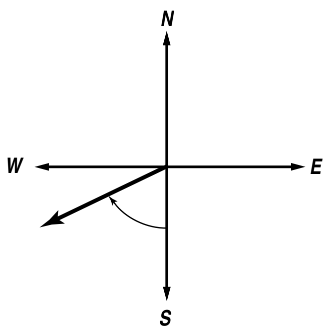
plane B: south 72° west, or S 72° W

Write an expression for the direction of each flight path. (You will need to measure the angle with your protractor.)

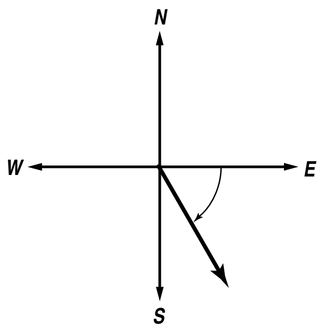
1.



2.

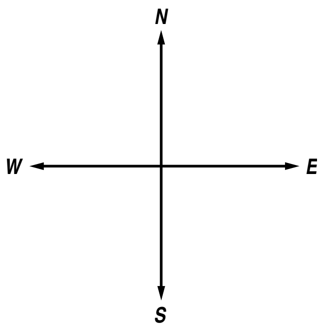


3.

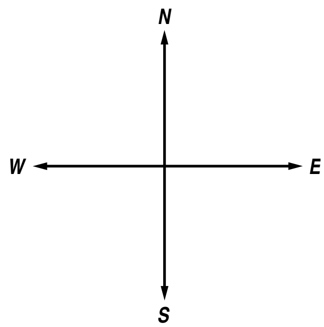


Use your protractor to draw each flight path.

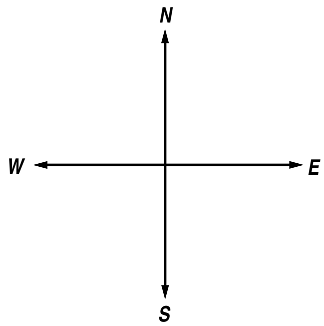
4. E 70° S



5. E 51° N



6. W 75° N



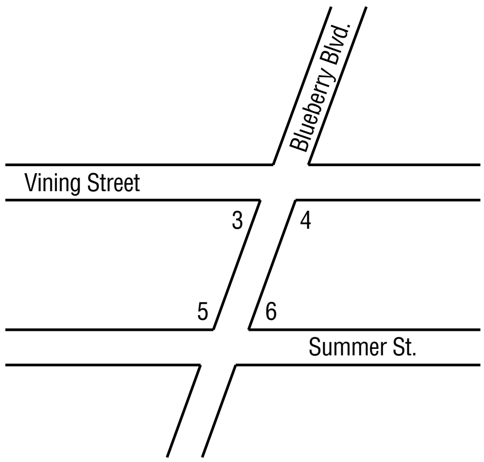
7. CHALLENGE The **bearing** of a plane is the measure of the angle between its flight path and due north, measured in a clockwise direction. For example, in the figure at the top of the page, the bearing of plane B is 90° + 90° + 72° = 252°. Give the bearing for each flight path in Exercises 1–6.

9-3 Enrichment

Parallel Lines and Interior Angles

Parallel lines are always the same distance apart and never meet. A line that intersects two parallel lines is called a transversal. A transversal forms angles with the parallel lines that are related.

On the map, Vining Street is parallel to Summer Street. Blueberry Boulevard is a transversal.



The angles between the two parallel lines are called **interior angles**. **Alternate interior angles** are on opposite sides of the transversal.

$\angle 3$ and $\angle 6$ are alternate interior angles.

$\angle 4$ and $\angle 5$ are alternate interior angles.

Alternate interior angles are congruent, so

$m\angle 3 = m\angle 6$ and $m\angle 4 = m\angle 5$.

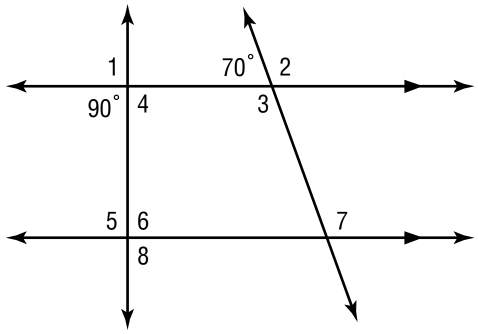
Interior angles on the same side of the transversal are supplementary.

$m\angle 4 + m\angle 6 = 180°$

$m\angle 3 + m\angle 5 = 180°$

You can find the measures of other angles in the diagram by remembering that opposite angles formed by intersecting lines are congruent.

Find the measure of the angle in the figure.



1. $m\angle 5$

3. $m\angle 8$

5. $m\angle 7$

7. $m\angle 6$

2. $m\angle 1$

4. $m\angle 2$

6. $m\angle 3$

8. $m\angle 4$

9-4 Enrichment

African Weaving

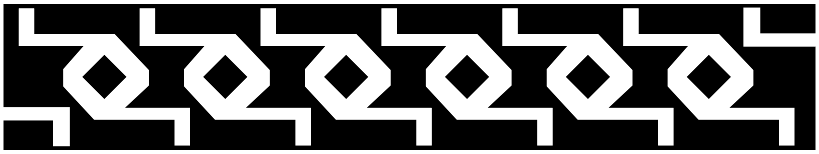
For the people of Africa, weaving is a form of art. They have woven intricate and beautiful designs into fabric for many centuries. As with so many other art forms, the beauty of their designs is based on geometric principles.

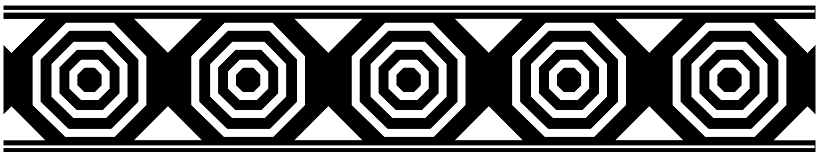
The designs on this page were created more than one hundred years ago in the region of Africa that today is Zaire. They are examples of *strip patterns*, which were repetitive patterns used as decorative borders on clothing. In the exercises below, you will take a closer look at the geometry of these patterns.

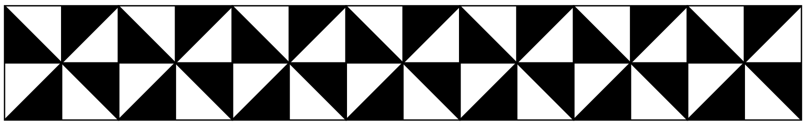
In a strip pattern, the *pattern unit* is the basic design that is repeated along the strip. For each of these patterns:

a. Identify the pattern unit and make a sketch of it in the space at the right.

b. Name any shapes you recognize that could be used to make the pattern unit.

1. 

2. 

3. 

4. 

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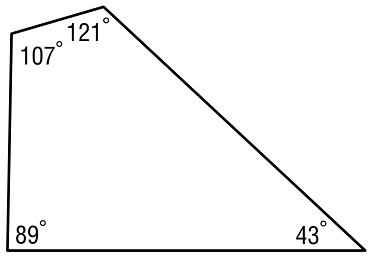
9-5 Enrichment

Making Conjectures

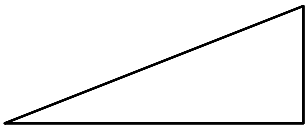
A **conjecture** is an educated guess or an opinion. Mathematicians and scientists often make conjectures when they observe patterns in a collection of data. On this page, you will be asked to make a conjecture about polygons.



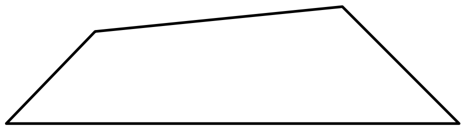
$89^\circ + 107^\circ + 121^\circ + 43^\circ = 360^\circ$

Use a protractor to measure the angles of each polygon. Then find the sum of the measures. (Use the quadrilateral at the right as an example.)

1.



2.



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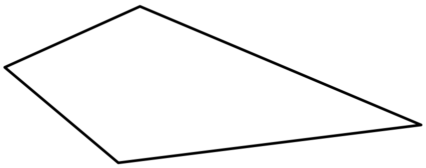

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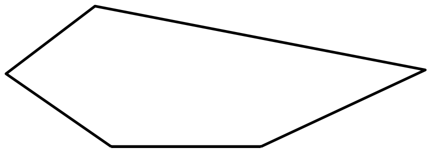
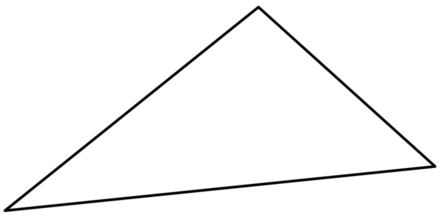
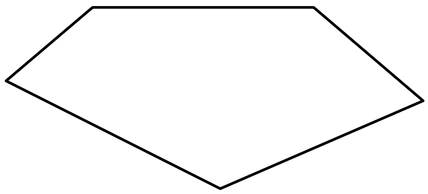


5.



6.



7. Make a conjecture. How is the sum of the angle measures of a polygon related to the number of sides?

8. Test your conjecture. On a clean sheet of paper, use a straightedge to draw a hexagon. What do you guess is the sum of the angle measures? Measure each angle and find the sum. Was your conjecture true?

9-7 Enrichment

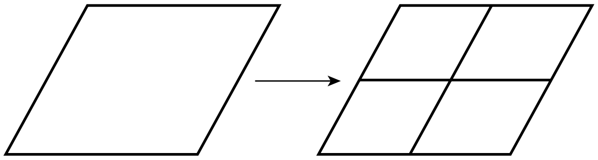
Rep-Tiles

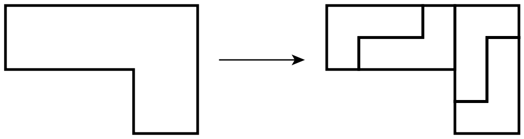
The word **rep-tiles** stands for repeating tiles. A geometric figure is a rep-tile if it can be divided into smaller parts according to these rules.

1. All the smaller parts must be *congruent* to each other.

2. All the smaller parts must be *similar* to the original tile.

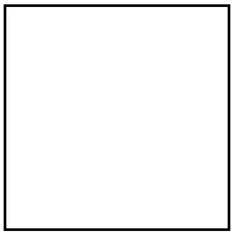
Here are two examples of figures that are rep-tiles.





Divide each rep-tile into four congruent parts.

1.



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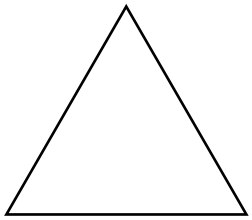

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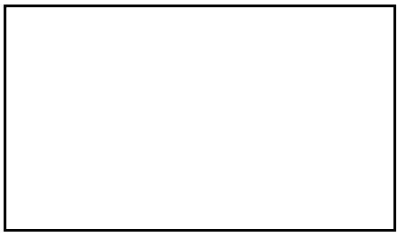

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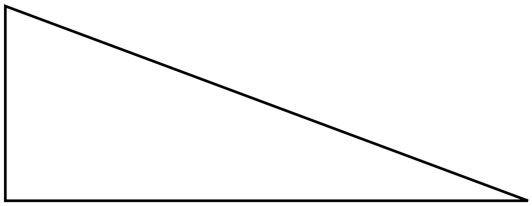

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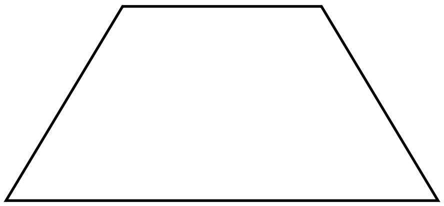

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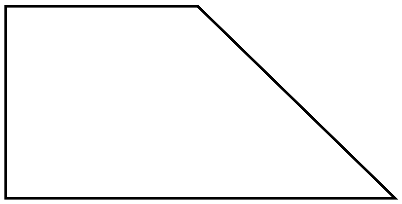
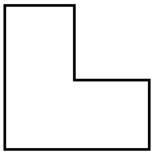


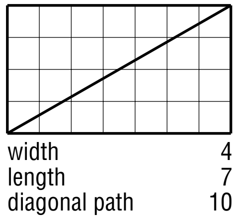
7. **CHALLENGE** Show how to use four figures like the one at the right to make a rep-tile.



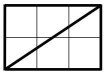
10-1 Enrichment

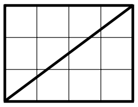
Getting From Here to There

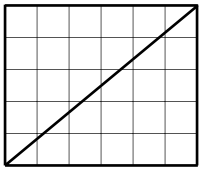
At the right, you see a rectangle on a grid of squares. The rectangle is 4 units wide and 7 units long. The *diagonal path* of this rectangle crosses 10 squares of the grid.

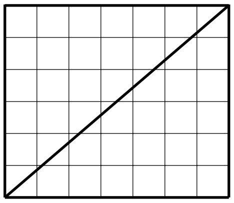


For each rectangle, record the width, the length, and the diagonal path.

1. 

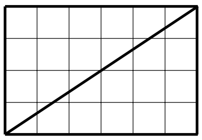
2. 

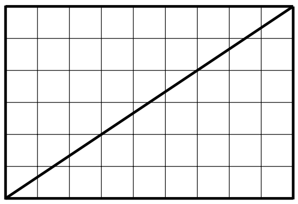
3. 

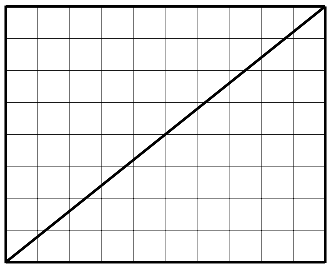
4. 

5. Refer to your answers to Exercises 1–4. What is the pattern?

Now record the width, length, and diagonal path for each of these rectangles.

6. 

7. 

8. 

9. Refer to your answers to Exercises 6–8. Does the pattern that you found in Exercise 5 still hold?

10. What is the difference between the rectangles in Exercises 1–4 and the rectangles in Exercises 6–8?

Predict the diagonal path for each rectangle.

11. 4 units by 9 units

12. 10 units by 21 units

13. 20 units by 30 units

14. 20 units by 24 units

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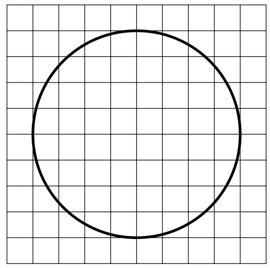
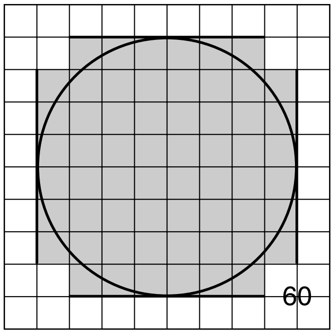
10-2 Enrichment

Estimating the Area of a Circle

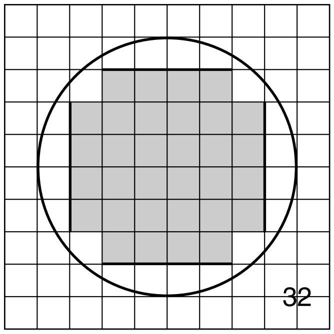
You have learned that area is the number of square units needed to cover a surface. Counting square units on a circular surface can be challenging. Here is a counting method that gives a fairly good *estimate* of the area of a circle.



Count the squares that cover any part of the circular region.



Count the squares that are entirely within the circle.



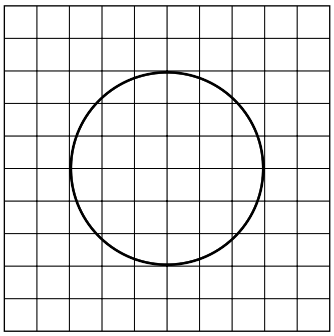
Find the mean of the two numbers.

$$\frac{60 + 32}{1} = \frac{92}{2} = 46$$

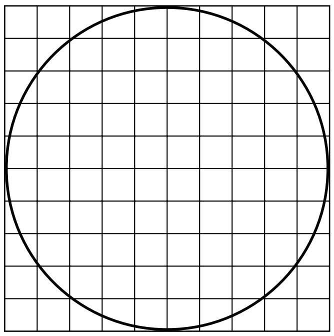
So the area of the circle is about 46 square units.

Estimate the area of each circle or oval.

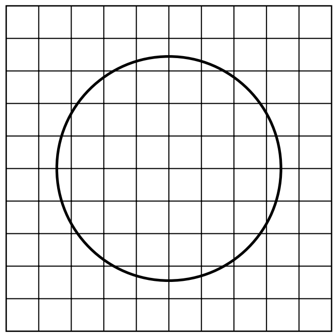
1.



2.



3.



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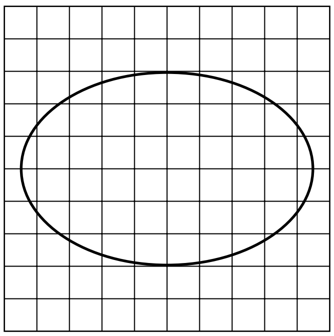

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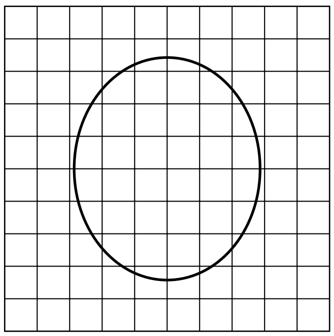
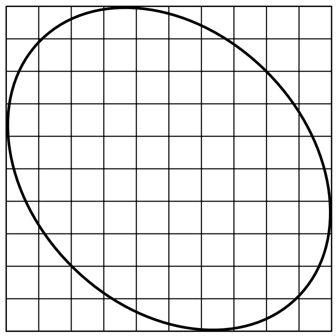


6.

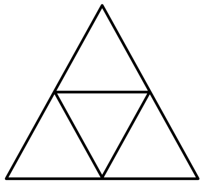


Lesson 10–2

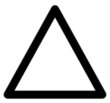
10-3 Enrichment

You Can Count On It!

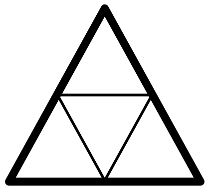
How many triangles are there in the figure at the right?
How many parallelograms?



When counting shapes in a figure like this, you usually have to think of different sizes.



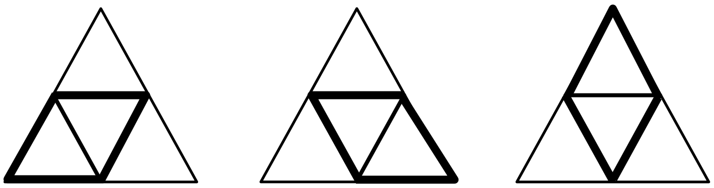
There are four small triangles.



There is one large triangle.

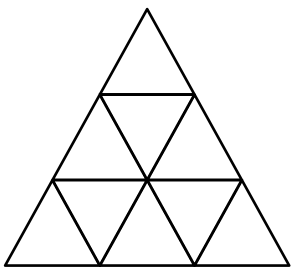
There are five triangles in all.

You also have to think of different positions.

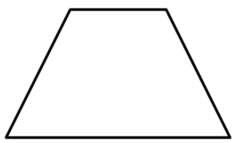


There are three parallelograms in all.

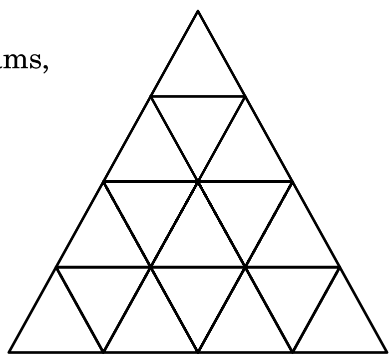
1. Now it's your turn. How many triangles are in the figure below? How many parallelograms? Use the space at the right to organize your counting.



2. A trapezoid is a quadrilateral with only one pair of sides parallel, as shown at the right. How many trapezoids are in the figure in Exercise 1?



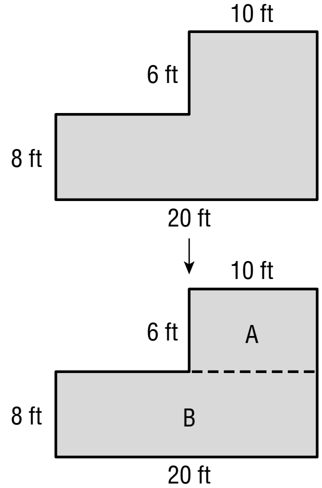
3. **CHALLENGE** How many triangles, parallelograms, and trapezoids are in the figure at the right?



10-4 Enrichment

Area of Composite Figures

A **composite figure** is made up, or composed, of other figures. For example, the L-shaped figure at the right is composed of two rectangles. To find the area of the L-shape, find the area of each rectangle, then add.

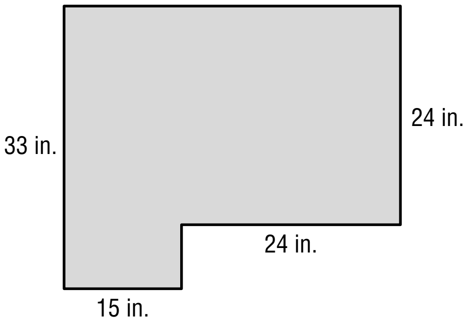


Area of A	**Area of B**
$A = \ell \times w$	$A = \ell \times w$
$A = 10 \times 6$	$A = 20 \times 8$
$A = 60$	$A = 160$

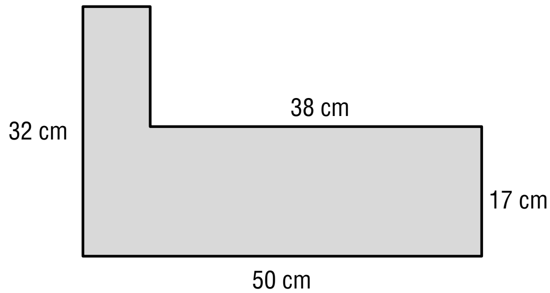
So the area of the L-shaped figure is 60 ft^2 + 160 ft^2, or 220 ft^2.

Find the area of each composite figure.

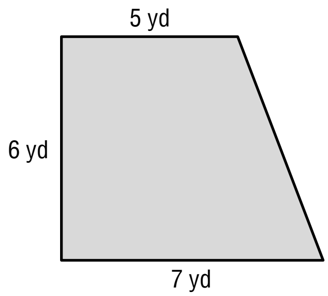
1.



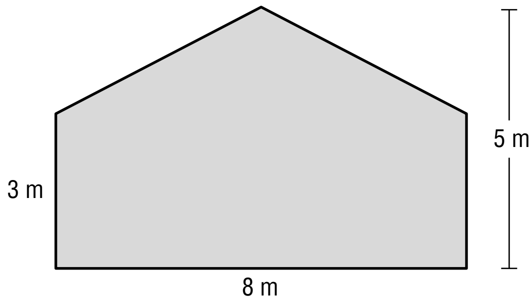
2.



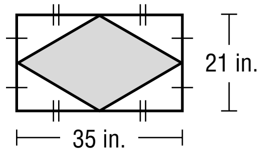
3.



4.



5. CHALLENGE Find the area of the shaded region in the figure at the right.



10-6 Enrichment

Volume and Liquid Capacity

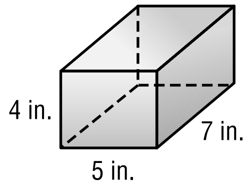
Volume and Liquid Capacity
Metric
$1 \text{ cm}^3 = 1 \text{ mL}$
$1 \text{ m}^3 = 1{,}000 \text{ L}$
Customary
$1 \text{ in}^3 \approx 0.544 \text{ fl oz}$
$1 \text{ ft}^3 \approx 7.481 \text{ gal}$

The volume of a three-dimensional figure is the amount of *space* it contains. Volume is measured in cubic units—cubic meters, cubic inches, and so on.

The liquid capacity of a container is the amount of *liquid* it can hold. Liquid capacity generally is measured in units like liters, milliliters, cups, pints, quarts, and gallons.

The chart at the right shows the relationship between volume and liquid capacity. If a container were shaped like the rectangular prism below the chart, this is how you would find its liquid capacity.

Volume	**Liquid Capacity**
$V = \ell wh$	$1 \text{ in}^3 \approx 0.544 \text{ fl oz}$
$V = 7 \times 5 \times 4$	$140 \text{ in}^3 \approx (140 \times 0.544) \text{ fl oz}$
$V = 140$	$140 \text{ in}^3 \approx 76.16 \text{ fl oz}$



So the liquid capacity of the container is about 76 fluid ounces.

For Exercises 1–4, find the liquid capacity of a container shaped like a rectangular prism with the given dimensions. If necessary, round to the nearest whole number.

1. length, 8 cm
width, 4 cm
height, 6 cm

2. length, 7 ft
width, 2 ft
height, 3 ft

3. length, 4 m
width, 2 m
height, 5 m

4. length, 5 in.
width, 1 in.
height, 3 in.

5. An aquarium is 36 inches long, 18 inches wide, and 18 inches tall. It is filled with water to a height of 12 inches. How many gallons of water are in the aquarium? (Round to the nearest gallon.)

6. **CHALLENGE** How many cubic inches of space are occupied by one quart of water? Round to the nearest whole number.

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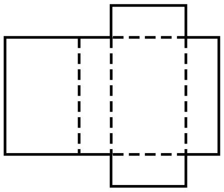
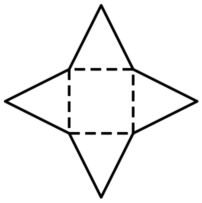
10-7 Enrichment

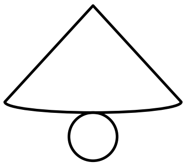
Nets

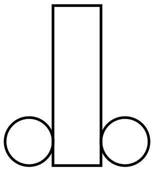
A **net** is a two-dimensional pattern that can be folded to form a three-dimensional figure. For example, the figure at the right is a net for a rectangular prism.

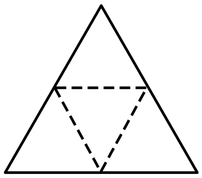


Identify the figure that would be formed by folding each net.

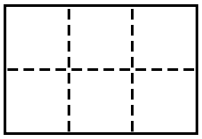
1. 

2. 

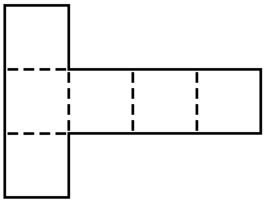
3. 

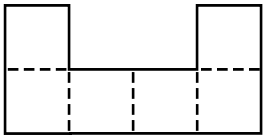
4. 

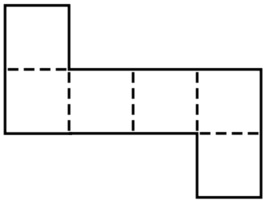
A **cube** is a rectangular prism in which all the edges have the same length. A net for a cube is made up of six squares. However, not every pattern of six squares is a net for a cube. For example, it would be impossible to fold the pattern at the right to form a cube.

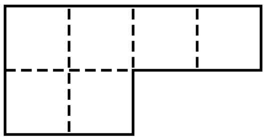


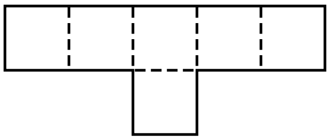
Tell whether each of these patterns is a net for a cube.

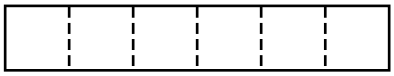
5. 

6. 

7. 

8. 

9. 

10. 

11. **CHALLENGE** In all, there are eleven different patterns of six squares that form a net for a cube. Sketch the eleven patterns in the space below.

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11-1 Enrichment

Multiplying by 10, 100, and 1,000

Can you see a pattern in these multiplications?

$$\begin{array}{r} 5.931 \\ \times \quad 10 \\ \hline 59.310 \end{array} = 59.31 \qquad \begin{array}{r} 5.931 \\ \times \quad 100 \\ \hline 593.100 \end{array} = 593.1 \qquad \begin{array}{r} 5.931 \\ \times 1{,}000 \\ \hline 5{,}931.000 \end{array} = 5{,}931$$

When you multiply a number by 10, 100, or 1,000, the product contains the same digits as the original number. However, the decimal point "moves" according to these rules.

multiply by 10 ⟶ move to the right one place
multiply by 100 ⟶ move to the right two places
multiply by 1,000 ⟶ move to the right three places

Many people use this fact as a mental math strategy.

Find each product mentally.

1. 10×7.402

2. 100×7.402

3. $1{,}000 \times 7.402$

4. 10×0.84

5. $1{,}000 \times 0.5362$

6. 100×3.83

7. 24.07×10

8. $1.918 \times 1{,}000$

9. 0.075×100

10. 6.1×10

11. 0.0046×100

12. $0.005 \times 1{,}000$

Now you can use this mental math strategy to estimate some products. The secret is to recognize when one of the factors is fairly close to 10, 100, or 1,000. An example is shown at the right.

$$\begin{array}{r} 32.83 \\ \times 97 \\ \hline \end{array} \longrightarrow \begin{array}{r} 32.83 \\ \times 100 \\ \hline 3{,}283 \end{array}$$

So, 32.83×97 is about 3,283.

Estimate by rounding one number to 10, 100, or 1,000.

13. 6.57×9

14. 14.32×96

15. $1{,}225 \times 3.548$

16. 0.6214×11.05

17. 98.04×26.331

18. 0.0358×9.3145

19. **CHALLENGE** Find the product $1{,}000 \times 16.5$ mentally.
How is this different from the other exercises on this page?

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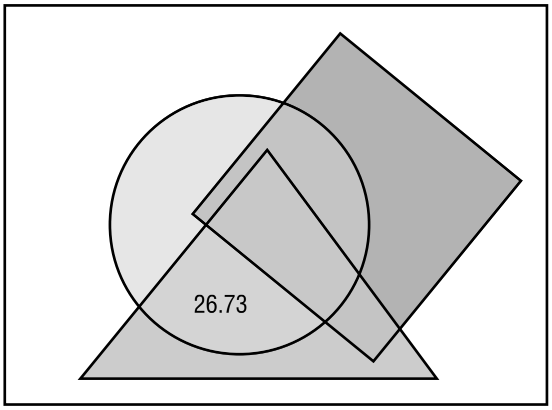
11-2 Enrichment

A Logic Puzzle

Here is a puzzle that will help you brush up on your logical thinking skills.



The product 3.3×8.1 is in both the circle and the triangle, but not in the square. Place the product in the diagram at the right.

$$\begin{array}{r} 8.1 \\ \times\ 3.3 \\ \hline 2\,4\,3 \\ 2\,4\,3\ \ \\ \hline 2\,6.7\,3 \end{array}$$

Write 26.73 in the correct region of the diagram.

Use the given information to place the product in the diagram above.

1. The product 14.19×1.3 is in both the triangle and the square, but not in the circle.

2. The product 0.08×2.7 is in the triangle, but not in the circle or the square.

3. The product 1.24×0.16 is not in the circle, the square, or the triangle.

4. The product 2.2×0.815 is in both the square and the circle, but not in the triangle.

5. The product 0.02×0.03 is in the circle, but not the triangle or the square.

6. The product 21.7×0.95 is in the circle, the square, and the triangle.

7. The product 2.5×12.8 is in the square, but not the circle or triangle.

8. If you did all the calculations correctly, the sum of all the numbers in the diagram should be a "nice" number. What is the sum?

11-3 Enrichment

Unit Pricing

The **unit price** of an item is the cost of the item given in terms of one *unit* of the item. The unit might be something that you count, like jars or cans, or it might be a unit of measure, like ounces or pounds. You can find a unit price using this formula.

TUNA 89¢ 6 ounce can

unit price = cost of item ÷ number of units

For example, you find the unit price of the tuna in the ad at the right by finding the quotient 0.89 ÷ 6. The work is shown below the ad. Rounding the quotient to the nearest cent, the unit price is $0.15 *per ounce*.

$$\begin{array}{r} 0.148 \\ 6\overline{)0.890} \\ \underline{6} \\ 29 \\ \underline{24} \\ 50 \\ \underline{48} \\ 2 \end{array}$$

Find a unit price for each item.

1.

5-pound bag CARROTS $1.29

2.

18-ounce jar PEANUT BUTTER $2.49

3.

Grade A Jumbo EGGS Dozen $1.59

Give two different unit prices for each item.

4.

Frozen BURRITOS 5-ounce pkg 2 for $1.39

5.

Purr-fect CAT FOOD 3/$1 3-ounce can

6.

Old Tyme SPAGHETTI SAUCE 12-ounce jars 2/$3

Circle the better buy.

7.

Mozarella Cheese 3/$4 10-ounce pkg	Mozarella Cheese 2/$3 18-ounce pkg

8.

Dee-light Chicken Wings $9.99 5-pound bag	Top Q Chicken Wings $2.29 18-ounce bag

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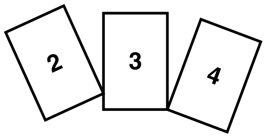
11-4 Enrichment

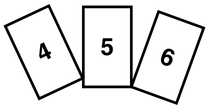
It's in the Cards

Below each set of cards, a quotient is given. Use the digits on the cards to form a division sentence with that quotient. Use as many zeros as you need to get the correct number of decimal places. For example, this is how to find a division for the cards at the right.

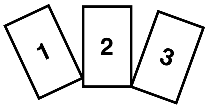


Quotient: 0.0008

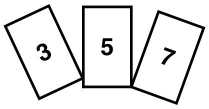
You know that 24 ÷ 3 = 8.
So, one division is 0.0024 ÷ 30 = 0.0008.

1.


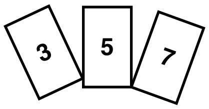
Quotient: 0.009

2.


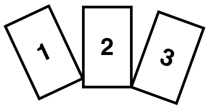
Quotient: 0.04

3.


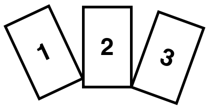
Quotient: 0.0005

4.


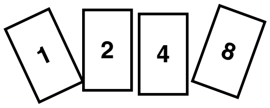
Quotient: 0.0074

5.


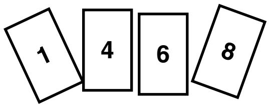
Quotient: 0.0155

6.


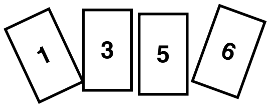
Quotient: 0.0025

7.


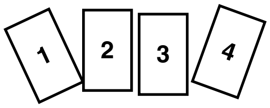
Quotient: 0.0004

8.


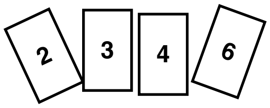
Quotient: 0.03

9.


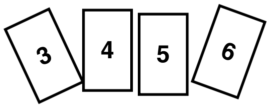
Quotient: 0.005

10.


Quotient: 20.65

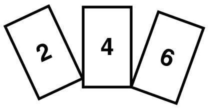
11.


Quotient: 0.0208

12.


Quotient: 0.08

13. **CHALLENGE** Use the cards at the right. Write four *different* divisions that have the quotient 0.4.



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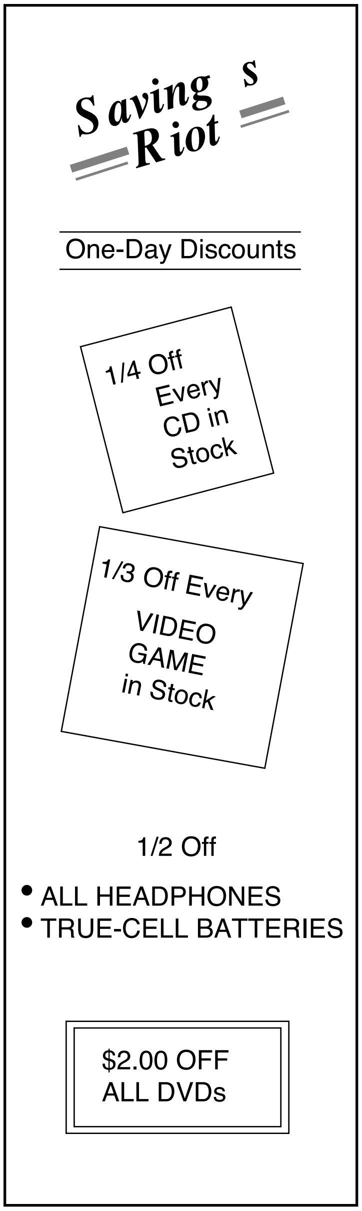
11-6 Enrichment

Shopping with Compatible Numbers

Suppose that you are meeting a friend for lunch and come across the sale advertised at the right. For weeks, you have wanted to buy a set of CDs that is regularly priced at $31.98. Here is how compatible numbers can help you find the sale price of the set.

- $\frac{1}{4}$ of $31.98 is about $\frac{1}{4}$ of $32, or $8.
- "$\frac{1}{4}$ off" means that you pay $1 - \frac{1}{4}$, or $\frac{3}{4}$.
- Since $\frac{1}{4}$ of $32 = $8, $\frac{3}{4}$ of $32 = $24.

The sale price is about $24.



Each exercise gives the regular price of one or more items. Use the information at the right to estimate the sale price.

1. video game: $23.95

2. CD: $15.95

3. headphones: $10.98

4. three packs of TRUE-CELL batteries; $5.98 per pack

5. one CD: $20.95
one video game: $27.99

6. one set of headphones: $15.79
two video games: $17.55 and $15.50

7. one CD: $16.95
one set of headphones: $14.50
one DVD: $19.98

8. two CDs: $14.95 and $12.95
one video game: $20.99
two DVDs: $14.95 each

11-7 Enrichment

Word Sums

Can one sixth plus one third equal one? It is possible if the fractions are fractions of *words*! Here is an example.

Find the last one sixth of the word TOMATO: O
Find the middle one third of the word FINEST: NE
Add the letters O + NE = ONE!

Match each word sum in the first column with its definition in the second column.

1. the first one fourth of CHECKERS + the last one half of AFFAIR	**a.** purchased
2. the first one half of CLOSET + the last one fourth of DOWNTOWN	**b.** clock sound
3. the first one fifth of BACKGROUND + the middle one third of WONDER	**c.** capital of Georgia
4. the middle one third of ADVENTURE + the last one third of LEADER	**d.** to come into a room
5. the middle one third of BUGLER + the last one fourth of SATISFACTORY	**e.** where we live
6. the first two thirds of TICKET + the last four fifths of STOCK	**f.** honor
7. the middle one half of SEAT + the last one half of FOURTH	**g.** circus act
8. the first two fifths of BOARD + the middle one half of DAUGHTER	**h.** place to sit
9. the first one half of MARBLE + the last three fifths of SUGAR + the last one fourth of CLARINET	**i.** woman's name
10. the last two thirds of EAT + the first one third of LANDSLIDE + the first one fifth of TABLESPOON	**j.** music makers

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11-8 Enrichment

Mixed Numbers and Mental Math

Sometimes you can multiply a whole number and a mixed number in your head. Think of the mixed number in two parts—the whole number and the fraction.

Find each product mentally.

Example Think: 3×10 Think: $\frac{1}{2}$ of 10

$3\frac{1}{2} \times 10 =$ **30** + **5** = **35**

1. $7\frac{1}{2} \times 6 =$ ________ + ________ = ________

2. $4 \times 9\frac{1}{2} =$ ________ + ________ = ________

3. $4\frac{1}{3} \times 6 =$ ________ + ________ = ________

4. $5\frac{1}{4} \times 8 =$ ________ + ________ = ________

5. $15 \times 2\frac{1}{5} =$ ________ + ________ = ________

6. $12 \times 4\frac{1}{6} =$ ________ + ________ = ________

7. $1\frac{2}{3} \times 6 =$ ________ + ________ = ________

8. $5\frac{3}{4} \times 20 =$ ________ + ________ = ________

Now you can use this mental math technique to make better estimates. Here's how.

Estimate the product: $4\frac{1}{2} \times 11\frac{7}{9}$

$4\frac{1}{2} \times 11\frac{7}{9} \rightarrow 4\frac{1}{2} \times 12$

$4\frac{1}{2} \times 12 = 4 \times 12 + \frac{1}{2}$ of 12

$= 48 + 6$

$= 54$

So, $4\frac{1}{2} \times 11\frac{7}{9}$ is about 54.

Estimate by rounding only one factor.

9. $6\frac{1}{2} \times 4\frac{2}{11}$

10. $5\frac{1}{3} \times 8\frac{9}{10}$

11. $11\frac{15}{16} \times 2\frac{1}{4}$

12. $5\frac{7}{10} \times 4\frac{1}{6}$

13. $1\frac{2}{3} \times 14\frac{6}{7}$

14. $19\frac{2}{7} \times 1\frac{3}{4}$

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11-9 Enrichment

Operations with Fractions and Decimals

Sometimes an operation involves both fractions and decimals. To perform the operation, you need to express all the numbers in the same form. Here are two examples.

$$\frac{1}{5} \div 0.\overline{3} = \frac{1}{5} \div \frac{1}{3} \quad \longleftarrow \text{Express the decimal as a fraction.}$$
$$= \frac{1}{5} \times \frac{3}{1}$$
$$= \frac{3}{5}$$

$$\frac{3}{4} + 0.115 = 0.75 + 0.115 \quad \longleftarrow \text{Express the fraction as a decimal.}$$
$$= 0.865$$

Perform the operation. Express the answer as a fraction or mixed number in simplest form.

1. $\frac{5}{16} \div 0.25$

2. $0.\overline{6} \div \frac{7}{9}$

3. $0.125 \times \frac{4}{11}$

4. $1\frac{1}{5} \times 0.\overline{3}$

5. $0.8 - \frac{3}{5}$

6. $1\frac{3}{8} - 0.875$

Perform the operation. Express the answer as a decimal.

7. $0.34 \div \frac{1}{5}$

8. $\frac{1}{8} \div 0.005$

9. $0.001 \times \frac{3}{5}$

10. $6.39 + \frac{7}{8}$

11. $9.1 - \frac{1}{4}$

12. $\frac{3}{8} + 0.709 + \frac{2}{5}$

13. Kevin is making one recipe that calls for $1\frac{1}{4}$ pounds of hamburger and another that calls for 2 pounds. In the store, he finds a family pack of hamburger that is labeled 3.75 pounds. Is this more or less than he needs? How much more or less?

14. Daneesha needs $1\frac{1}{2}$ yards of material to make a jacket and $1\frac{3}{4}$ yards of material to make a skirt. The material costs $7.50 per yard. What is the total cost of the material for the skirt and jacket? Round your answer to the nearest cent.

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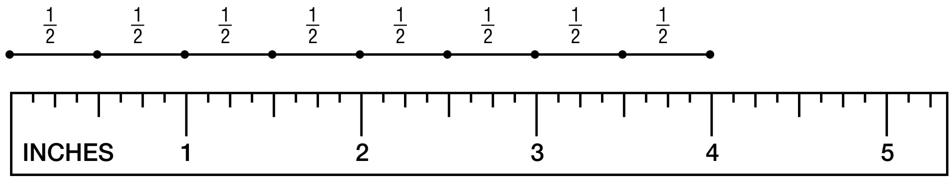
11-10 Enrichment

Modeling Division of Fractions on a Ruler

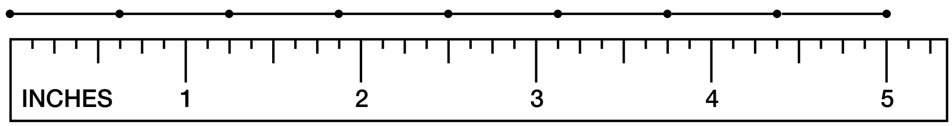
How many half-inch lengths are in 4 inches? When you look at a ruler, it is easy to see that the answer is 8.



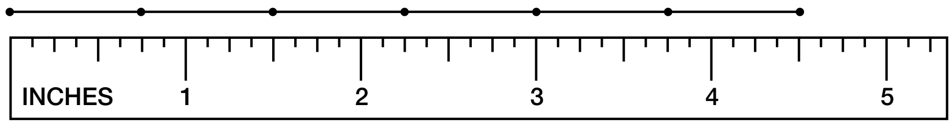
So, this diagram is also a model for the division $4 \div \frac{1}{2} = 8$.

Write the division that is modeled in each diagram.

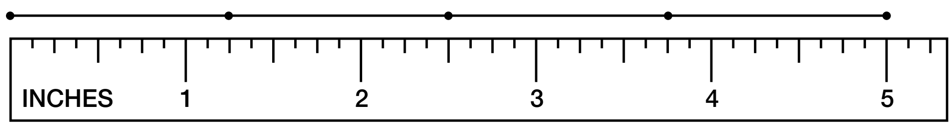
1.



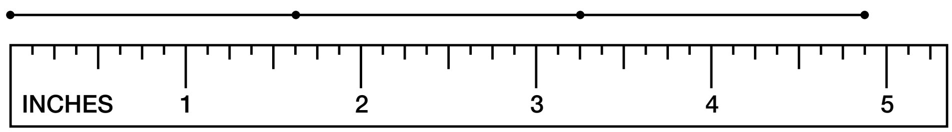
2.



3.



4.



5. Use the ruler below. Create a model for the division $4\frac{2}{3} \div \frac{2}{3} = 7$.

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12-1 Enrichment

A Number Puzzle

Order each set of numbers from least to greatest. If they are arranged correctly, the letters that correspond to each number will spell a word that has to do with comparing numbers. Write the word on the blanks to the right of each set. Then arrange the letters in the circles to discover another word.

1.

−2	5	17	3	−15
E	S	T	A	L

___ ___ ___ ___ (___)

2.

65	122	−12	30	−4
E	R	O	D	R

___ (___) ___ ___ ___

3.

−8	−20	8	13	7	−28	−18	17
A	E	I	V	T	N	G	E

(___) ___ ___ ___ ___ ___ ___ ___

4.

99	87	64	−49	56	55	−58	−65
T	S	E	E	T	A	R	G

(___) ___ ___ ___ ___ ___ ___ ___

5.

−11	10	−9	8	7	−6	5	−4
P	E	O	V	I	S	T	I

___ ___ ___ (___) ___ ___ ___ ___

6.

−2	2	11	0	9	−17
U	B	R	M	E	N

___ ___ ___ ___ (___) ___

7.

$\frac{1}{2}$	$\frac{-1}{4}$	$\frac{3}{4}$	$\frac{5}{6}$
I	L	N	E

___ ___ ___ (___)

___ ___ ___ ___ ___ ___ ___

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12-2 Enrichment

Speedy Addition

How would you perform an addition like this?

$$-4 + 7 + (-1) + 4 + (-7) + (-5)$$

Some people add all the positive integers, add all the negative integers, then add the results.

$$-4 + (-1) + (-7) + (-5) = -17$$
$$7 + 4 = 11$$
$$\rightarrow -17 + 11 = -6$$

Other people find it easier to first group all the zero pairs.

$$\underbrace{-4 + 4}_{0} + \underbrace{7 + (-7)}_{0} + \underbrace{(-1) + (-5)}_{-6} \rightarrow -6$$

Which method do you think you would prefer? Check it out by finding each of these sums.

1. $-9 + 5 + 3 + 9 + (-3)$

2. $-16 + 9 + (-11) + 16 + 11 + (-12) + (-9)$

3. $10 + (-8) + (-4) + (-2)$

4. $-6 + 14 + (-11) + (-8) + 7 + 11$

5. $-15 + 6 + (-12) + 3 + 9 + (-3)$

6. $20 + (-13) + (-5) + 13 + (-10) + 16 + (-5)$

7. $19 + (-7) + (-9) + (-9) + 15 + (-10) + 16$

8. $-4 + 17 + (-8) + 5 + (-17) + (-13) + 8 + (-12)$

9. $16 + (-11) + 4 + (-2) + 11 + (-14) + 5 + (-9)$

10. $-21 + 3 + (-7) + (-4) + (-8) + 15 + 6 + 12 + 15$

11. Which method(s) did you use in Exercises 1–10? Did you choose from the methods above, or did you use a different method? Explain.

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12-3 Enrichment

Windchill Temperatures

When you go outside on a windy day, it usually *feels* much colder than the actual temperature on the thermometer. This happens because the wind causes you to lose more heat from the surface of your skin than you would lose if the air were still. The temperature you feel is called the **windchill temperature.** The table below lists some of the windchill temperatures that have been calculated by the National Weather Service.

Windchill Temperatures (degrees Fahrenheit)

Wind Speed (miles per hour)	Actual Temperature								
	20	15	10	5	0	−5	−10	−15	−20
5	13	7	1	−5	−11	−16	−22	−28	−34
10	9	3	−4	−10	−16	−22	−28	−35	−41
15	6	0	−7	−13	−19	−26	−32	−39	−45
20	4	−2	−9	−15	−22	−29	−35	−42	−48
25	3	−4	−11	−17	−24	−31	−37	−44	−51
30	1	−5	−12	−19	−26	−33	−39	−46	−53
35	0	−7	−14	−21	−27	−34	−41	−48	−55
40	−1	−8	−15	−22	−29	−36	−43	−50	−57
45	−2	−9	−16	−23	−30	−37	−44	−51	−58

Use the table above to answer each question.

1. If the wind speed is 10 miles per hour and the actual temperature is 0°F, what is the windchill temperature?

2. Suppose that the actual temperature is −5°F and the wind speed is 15 miles per hour. How much colder than −5°F does it feel?

Describe the change in the windchill temperature.

3. The wind speed remains constant at 10 miles per hour, but the actual temperature rises from −5° to 20°F.

4. The actual temperature remains constant at −10°F, but the wind speed increases from 5 miles per hour to 35 miles per hour.

Estimate the windchill temperature in each situation.

5. The actual temperature is 8°F and the wind speed is 22 miles per hour.

6. The actual temperature is −10°F and the wind speed is 55 miles per hour.

Lesson 12–3

12-4 Enrichment

Integer Patterns

Many number patterns involve integers. When you work with patterns like these, you need to pay special attention to the sign of each number in the pattern. Here are two examples.

$1, \quad -2, \quad 4, \quad -8, \quad 16, \quad -32, \quad 64, \ldots$ ← Multiply by −2.

$\times(-2) \ \times(-2) \ \times(-2) \ \times(-2) \ \times(-2) \ \times(-2)$

$1, \quad 3, \quad 0, \quad 2, \quad -1, \quad 1, \quad -2, \ldots$ ← Add 2, subtract 3, add 2, and so on.

$+2 \ \ -3 \ \ +2 \ \ -3 \ \ +2 \ \ -3$

Write the next five numbers in each pattern shown above.

1. 1, −2, 4, −8, 16, −32, 64, _____, _____, _____, _____, _____

2. 1, 3, 0, 2, −1, 1, −2, _____, _____, _____, _____, _____

For each set of numbers, identify the pattern. Then write the next three numbers in the pattern.

3. −1, 3, −9, 27, −81, _____, _____, _____

4. 5, −1, −7, −13, −19, _____, _____, _____

5. −11, −8, −5, −2, 1, _____, _____, _____

6. −2, −10, −50, −250, −1,250, _____, _____, _____

7. 12, 7, 8, 3, 4, _____, _____, _____

8. −15, −10, −12, −7, −9, _____, _____, _____

9. 7, −7, −2, 2, 7, _____, _____, _____

10. 3, 6, −2, −4, −12, _____, _____, _____

11. −4, 8, 6, −12, −14, 28, _____, _____, _____

12. **CHALLENGE** 1, 2, 0, 3, −1, _____, _____, _____

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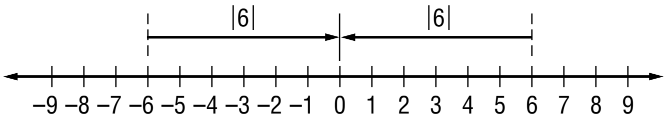
12-6 Enrichment

Absolute Value

Every integer has an **absolute value**. This is the value of the integer, whether the integer is positive or negative. You can think of absolute value as the distance from 0 that an integer is on a number line. For example, the absolute value of 6 is 6. The absolute value of −6 is also 6 because −6 is 6 units from 0 on the number line. The absolute value of a number is written , where x is any integer.



You can use absolute values when you multiply and divide positive and negative integers. You can multiply or divide the absolute values of the integers. Then, you look at the signs of the integers to decide what sign the product or quotient should have. If the signs are the same, the product or quotient is always positive. If the signs are different, the product or quotient is always negative.

Find the absolute value of each integer.

1. $|5|$ **2.** $|-17|$ **3.** $|24|$

4. $|18|$ **5.** $|-68|$ **6.** $|-11|$

7. $|-2|$ **8.** $|-7|$ **9.** $|256|$

10. $\left|\frac{1}{2}\right|$ **11.** $\left|\frac{-1}{4}\right|$ **12.** $\left|\frac{3}{5}\right|$

Find each product or quotient.

1. $24 \div (-8)$ **2.** -21×7 **3.** $-36 \times (-6)$

4. -5×3 **5.** $28 \div (-7)$ **6.** $42 \times (-4)$

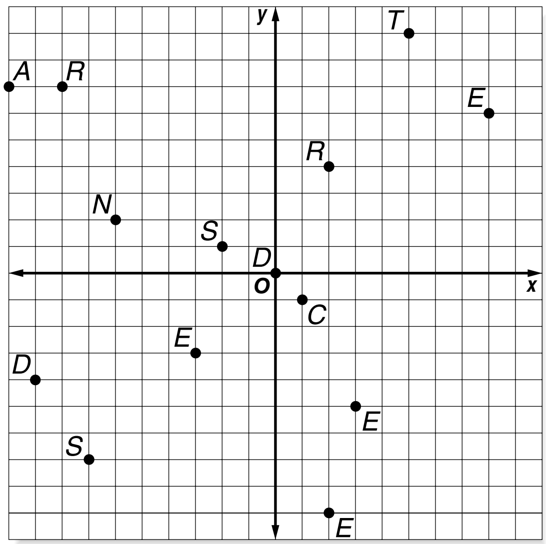
7. $-30 \div (-3)$ **8.** $45 \div 5$ **9.** 25×15

12-7 Enrichment

The Cartesian Plane

Another name for the coordinate plane is the **Cartesian plane**. Its name comes from a French mathematician and philosopher who lived in the 1600s. He invented the coordinate plane. Although it is likely not true, a story is told that this mathematician first came up with the idea of the coordinate plane while lying in bed looking at the ceiling. His ceiling was made of tiles. As he watched a fly crawling on the ceiling, he realized he could describe the fly's location using the tiles on the ceiling. From that, he created the coordinate plane and a system by which to describe locations on the coordinate plane.

Identify the letter that corresponds to the ordered pairs listed below. The letters spell the name of the Frenchman who invented the coordinate plane.



First Name

______ ______ ______ ______

(2, 4) (3, −5) (−6, 2) (−3, −3)

Last Name

______ ______ ______ ______ ______ ______ ______ ______ ______

(−9, −4) (8, 6) (−7, −7) (1, −1) (−10, 7) (−8, 7) (5, 9) (2, −9) (−2, 1)

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12-8 Enrichment

Deposits and Withdrawals

People deposit money into or withdraw money from bank accounts. Deposits are written as positive numbers and withdrawals are written as negative numbers. Transactions are recorded in an account register.

Abby recently opened an account to keep the money she is earning as a Mother's helper. She deposited her first pay of $200.00 into her account. She needs to keep track of all of her deposits and withdrawals in her account register from the past two months. Record all of Abby's transactions listed. How much does she have in he account at the end of July?

Date	Transaction	Withdrawal	Deposit	Balance
6/12	Initial deposit		200.00	200.00

1. Abby went to the movies with her friends on June 15. She took $20.00 out of her account.
2. Abby bought 2 CDs and a video game on June 19. She spent $75.00.
3. Abby deposited $200.00, her earnings from two weeks of work, on June 26
4. Abby went shopping with her friends on June 30. She took out $50. Then they had pizza, so Abby took out another $15.00
5. The bank charged Abby $2.00 service fee on her account on June 30.
6. Abby deposited $100.00, her earnings from one week of work, on July 2.
7. Abby went on vacation with her family from July 2 to July 12. She took $100.00 with her.
8. Abby went to the video arcade with her friends on July 15. She took out $20.00.
9. Abby bought her sister a birthday present on July 23. She spent $15.00
10. Abby deposited $200.00, her earnings from two weeks of work, on July 28.

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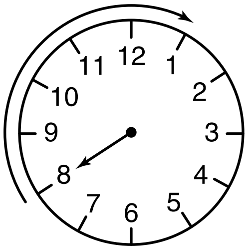
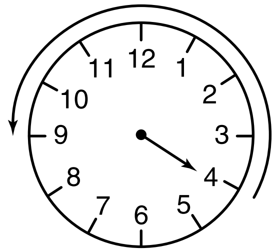
12-9 Enrichment

Clock Arithmetic

Did you realize that, when you work with elapsed time, you use a special kind of arithmetic that is called **clock arithmetic**? In clock arithmetic, you use the symbols $\oplus$ for addition and $\ominus$ for subtraction. Here are two examples.



8 o'clock plus 5 hours is 1 o'clock.
$8 \oplus 5 = 1$



4 o'clock minus 7 hours is 9 o'clock.
$4 \ominus 7 = 9$

Add or subtract using the 12-hour clock above.

1. $11 \oplus 3$ **2.** $7 \oplus 9$ **3.** $3 \ominus 10$ **4.** $7 \ominus 8$

5. $2 \oplus 12$ **6.** $2 \ominus 12$ **7.** $4 \oplus 6$ **8.** $9 \ominus 4$

To solve "clock equations" involving the 12-hour clock, use inverse operations.

$$d \oplus 5 = 2$$
$$d \oplus 5 \ominus 5 = 2 \ominus 5$$
$$d = 9$$

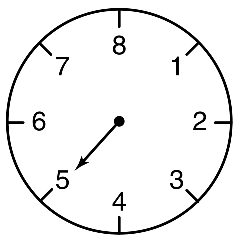
$$j \ominus 4 = 10$$
$$j \ominus 4 \oplus 4 = 10 \oplus 4$$
$$j = 2$$

Solve each equation using the 12-hour clock above.

9. $r \oplus 7 = 5$ **10.** $x \ominus 9 = 11$ **11.** $b \oplus 6 = 7$

12. $t \ominus 12 = 4$ **13.** $n \ominus 4 = 3$ **14.** $y \oplus 6 = 1$

CHALLENGE In clock arithmetic, you often work with clocks that have different numbers of hours. For example, the clock shown at the right is an *8-hour clock.*



Solve each equation using the 8-hour clock at the right.

15. $m \oplus 5 = 2$ **16.** $z \ominus 4 = 7$ **17.** $p \oplus 8 = 1$

18. $c \ominus 8 = 6$ **19.** $w \ominus 4 = 8$ **20.** $k \oplus 6 = 3$

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12-10 Enrichment

Patterns in Equations

On this page, you will explore patterns of change in equations.

For each table:
a. Describe how the equation changes from row to row.
b. Complete the Solution column.
c. Describe how the solution changes from row to row.

1.

Equation	Solution
$t + 3 = 4$	
$t + 3 = 3$	
$t + 3 = 2$	
$t + 3 = 1$	

2.

Equation	Solution
$3x = 6$	
$3x = 3$	
$3x = 0$	
$3x = -3$	

3.

Equation	Solution
$r - 3 = -2$	
$r - 3 = -1$	
$r - 3 = 0$	
$r - 3 = 1$	

4.

Equation	Solution
$m + 8 = 7$	
$m + 7 = 7$	
$m + 6 = 7$	
$m + 5 = 7$	

5.

Equation	Solution
$\frac{1}{5}j = 1$	
$\frac{1}{5}j = 0$	
$\frac{1}{5}j = -1$	
$\frac{1}{5}j = -2$	

6.

Equation	Solution
$\frac{1}{4}c = -1$	
$\frac{1}{3}c = -1$	
$\frac{1}{2}c = -1$	
$\frac{1}{1}c = -1$	