1-1 Enrichment

7MR2.3

Logical Reasoning

When planning how to solve problems, it is helpful to be familiar with a number of problem-solving strategies. When a problem presents a large amount of information, one strategy that can be effective is logical reasoning combined with the use of a table to organize the information.

Holly, Keisha, Sandra, and Jamal are Bexley Middle School's student council officers. The offices they hold are president, vice-president, secretary, and treasurer. Sandra is the president, Holly is not the treasurer, and Keisha is the vice-president. What office does Jamal hold?

	Holly	Keisha	Sandra	Jamal
President	N	N	Y	N
Vice President	N	Y	N	N
Secretary		N	N	
Treasurer		N	N	

Using the table, mark Y for relationships that are true, and N for relationships that are not true. For example, since you know that Sandra is the president, put a Y in that cell and put an N in each of the other cells of that column and in the president row. Fill in the remaining cells to show that Jamal must be the treasurer.

Use the process of logical reasoning and the table below to answer the following question.

Five male athletes won events in the district track and field meet. Each boy won exactly one event. From the clues below, find each boy's name, school name, and the event he won.

- Two boys competed in the field events and three boys competed in the three track events.
- No boy participated in both track events and field events.
- The athlete from North Middle School, who is not Mitch, placed last in the 100-meter.
- The 100-meter winner lost to the South Middle School student in another event.
- The boy from Wilson Academy, who placed second in the discus throw, was not in any event with Mitch or Rob.
- The South Middle School boy and Kyle, who is not from Wilson, were not in any of the same events.
- The student from Taft Junior High did not participate in any field events.
- In one event Nick beat the student from North Middle School and the 400-meter winner.

		Vine	North	South	Taft	Shot Put	Discus	400-m	100-m	Hurdles
	Mitch									
	Kyle									
	Rob									
	Nick									
	Cory									
Field	Shot Put									
Field	Discus									
Track	400-m									
Track	100-m									
Track	Hurdles									

NAME ____ DATE ______________ PERIOD _____

1-2 Enrichment

7AF1.3

Division by Zero?

Some interesting things happen when you try to divide by zero. For example, look at these two equations.

$$\frac{5}{0} = x \qquad \frac{0}{0} = y$$

Because multiplication "undoes" division, you can write two equivalent equations for the ones above.

$$0 \cdot x = 5 \qquad 0 \cdot y = 0$$

There is no number that will make the left equation true. This equation has no solution. For the right equation, *every* number will make it true. The solution set for this equation is "all numbers."

Because division by zero leads to impossible situations, it is not a "legal" step in solving a problem. People say that division by zero is undefined, or not possible, or simply not allowed.

Explain what is wrong with each of these "proofs."

1. Step 1 $0 \cdot 1 = 0$ and $0 \cdot 2 = 0$

Step 2 Therefore, $\frac{0}{0} = 1$ and $\frac{0}{0} = 2$.

Step 3 Therefore, $1 = 2$.

But, $1 = 2$ is a contradiction.

2. Step 1 Assume $a \neq b$.

Step 2 $0 \cdot a = 0$ and $0 \cdot b = 0$

Step 3 Therefore, $\frac{0}{0} = a$ and $\frac{0}{0} = b$.

Step 4 Therefore, $a = b$.

But, $a = b$ contradicts $a \neq b$.

Describe the solution set for each equation.

3. $4x = 0$

4. $x \cdot 0 = 0$

5. $x \cdot 0 = x$

6. $\frac{0}{x} = 0$

7. $\frac{0}{x} = x$

8. $\frac{0}{x} = \frac{0}{y}$

1-3 Enrichment

7AF1.3

When You Want to Be Negative

Many symbols and signs use a slash mark such as /, \, or | to mean *is not* or *no*. For example, the symbol $\neq$ means *is not equal to*.

Which of the symbols, $\neq$, $\ngtr$, and $\nless$ will make the statement true? Some problems have more than one correct answer.

1. -2 ____ 0

2. $|-4|$ ____ $|4|$

3. For any number x, $|x|$ ____ x.

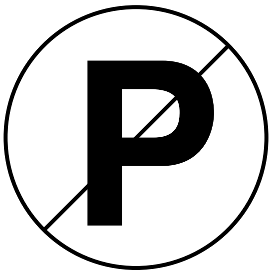
4. For any nonzero integer n, $-n$ ____ n.

5. A number x is either greater than 0 or less than 0. So, x ____ 0.

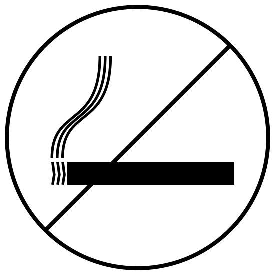
6. ____ means the same as $\leq$.

What does each of these signs mean?

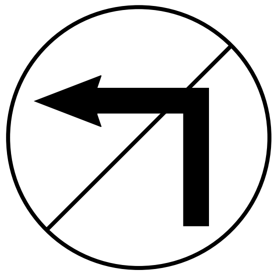
7.



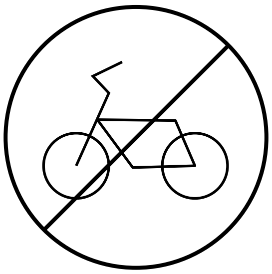
8.



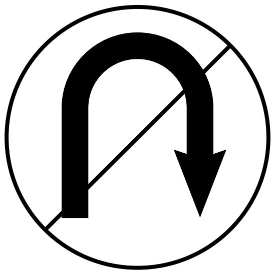
9.



10.



11.



12.

13. Create three different "no" signs of your own.

Lesson 1–3

1-4 Enrichment

7NS1.2

Adding Integers

Listed below are the scores for a game of cards in which the highest score wins. The three players recorded their scores for each hand, but did not total the scores until they were done playing.

Hand	Micah	Juanita	Taylor
1	125	−30	68
2	72	54	0
3	−15	105	95
4	0	−5	−20
5	146	37	110
6	82	−15	62
7	−25	130	47
8	40	0	−12

Refer to the table above to answer the following questions.

1. Who had the highest total score after round 3? How many points did this player have?

2. Who had the lowest score after round 5? What was his or her score at this point in the game?

3. What was each player's score after round 6?

4. Who was in second place after round 7? How many points did this player have?

5. Who won the game?

6. What was each player's final score at the end of the game?

NAME ______________________________ DATE ____________ PERIOD _____

1-5 Enrichment

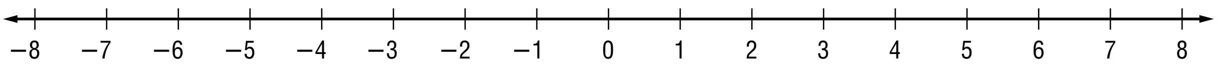
7NS2.5

Distance on the Number Line

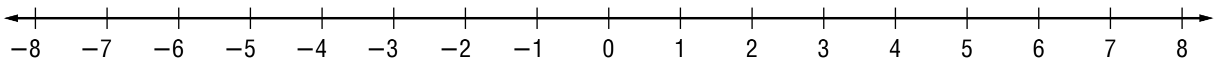
The **absolute value** of the difference between two integers can be interpreted as the distance between two points on a number line. That is, if point A has a as a coordinate and point B has b as a coordinate, then $|a - b|$ is the distance between points A and B.

Graph each pair of points on the number line. Then write an expression using absolute value to find the distance between the points.

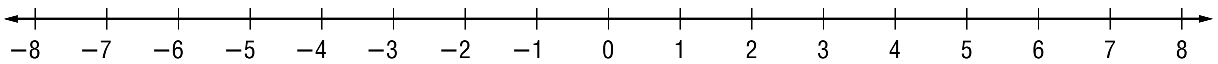
1. H at -4 and G at 2



2. X at -7 and Y at -1

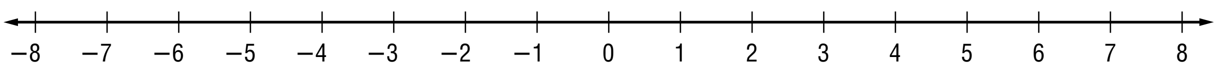


3. A at 5 and B at -5

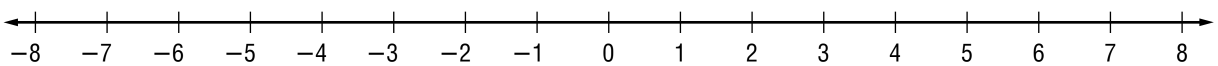


Use the number lines to solve the problems.

4. Graph two points, M and N, that are each 5 units from -2. Make $M > N$.



5. Graph the two solutions to the equation $|y - 2| = 3$. Call the points y_1 and y_2.



NAME ________________________ DATE ____________ PERIOD _____

1-6 Enrichment

7NS1.2, 7AF1.3

Doubles and Halves

Most numbers are easy to double or halve mentally. And, many types of multiplication problems can be done mentally by using doubling and halving. In working problems of this type, it is helpful to remember that 5 equals 10 divided by 2. And, dividing by 2 is the same as multiplying by one-half. So, multiplying by 5 is the same as first multiplying by 10 and then halving.

$5 = \frac{10}{2}$; $5 = 10 \times \frac{1}{2}$; $87 \times 5 = 87 \times 10 \div 2 = 870 \div 2 = 435$

Double each number. Use mental math.

1. 13 **2.** −214 **3.** 48 **4.** 2,512

5. −57 **6.** 609 **7.** −383 **8.** −6,523

Halve each number. Use mental math.

9. −64 **10.** 468 **11.** 690 **12.** −1,484

13. 72 **14.** −56 **15.** −38 **16.** 54

17. −116 **18.** 364 **19.** −5,296 **20.** 7,436

Compute mentally.

21. -5×126 **22.** 5×234 **23.** $5 \times (-872)$

24. 20×93 **25.** -20×361 **26.** 20×317

Transform each product into an expression that uses doubling or halving. Change only the second factor.

27. 256×20 **28.** 613×5

29. -472×50 **30.** 57×40

31. -138×25 **32.** 93×125

1-7 Enrichment

7AF1.1

Writing Equations to Describe Sequences

A **sequence** can be extended by finding the pattern, describing it, and then applying the description to produce successive **terms**. To describe the pattern in words, we could write, "Add four to the previous term to find the next term." Determine the pattern rule for the sequence below. What are the next three terms?

Position	1	2	3	4	5	6	7	8
Term	**4**	**8**	**12**	**16**	**20**			

Pattern +4 +4 +4 +4

Describe the pattern in words and write the next three terms in each of the following sequences.

A. 2, 4, 6, 8, 10, 12, ___, ___, ___

B. −3, −6, −9, −12, −15, −18, ___, ___, ___

C. 3, 5, 7, 9, 11, 13, ___, ___, ___

D. 3, 9, 27, 81, 243, ___, ___, ___

E. 6,230, 623, 62.3, 6.23, 0.623, ___, ___, ___

F. 1, 4, 9, 16, 25, 36, ___, ___, ___

The **rule** of a sequence can be generalized into an equation so that it is possible to find the 10th term, 100th term, or nth term without writing out of the terms in between. The rule of the sequence shows the relationship between a term and its position number.

Look again at the beginning example. The rule is *multiply the position number by four*. If we call the position numbers n, the algebraic expression for the rule is $4n$. For each term $t = 4n$.

Write an equation rule for each of the sequences in exercises 1–6. Be careful that your rule gives the correct first term.

1. Sequence A

2. Sequence B

3. Sequence C

4. Sequence D

5. Sequence E

6. Sequence F

Write an equation rule for each of the sequences below. Then use the equation to find the 100th term.

7. 4, 7, 10, 13, 16, …

8. 2, 5, 10, 17, 26, …

9. 0, 2, 4, 6, 8, …

10. 0.75, 1.5, 2.25, 3, 3.75, …

11. 11, 12, 13, 14, 15, …

12. Write your own sequence rule and find the first 5 terms.

1-9 Enrichment

7AF1.1

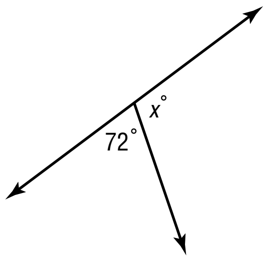
Geometric Equations

Equations are often used to solve geometric problems. To work the problems on this page, you will need to use the following facts.

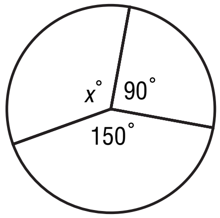
Angles are **complementary** if their measures add to 90°. If their measures add to 180°, they are **supplementary.** The total number of degrees in the measures of the central angles of a circle is 360°. The sum of the measures of the angles in a triangle is 180°. A **straight angle** measures 180°.

Match each equation in the chart at the bottom of the page with a figure that could be used to solve for the missing angle measurement. Then solve for that measurement.

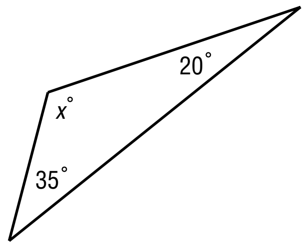
A.



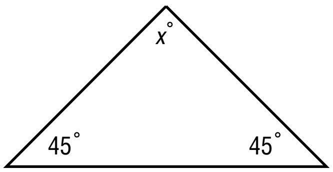
B.



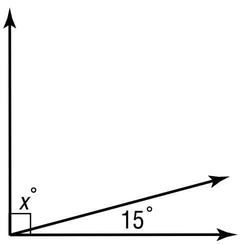
C.



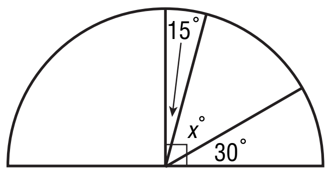
D.



E.



F.



Equation	Letter of Figure	Angle Measurement ($x°$)
$35° + 20° + x° = 180°$		
$90° - x° = 15°$		
$x° + 72° = 180°$		
$360° = x° + 150° + 90°$		
$2(45°) + x° = 180°$		
$30° + x° + 15° = 90°$		

1-10 Enrichment

7AF1.1

Consecutive Integers

Equations can be used to solve problems that involve consecutive integers. In solving these problems, you will need to translate certain phrases into algebraic expressions. Here are some examples.

Phrase A "five consecutive integers"
Expression A $n, n + 1, n + 2, n + 3, n + 4$

Phrase B "five consecutive even integers"
Expression B $n, n + 2, n + 4, n + 6, n + 8$

Phrase C "five consecutive odd integers"
Expression C $n, n + 2, n + 4, n + 6, n + 8$

Use Expressions A, B, and C for these problems.

1. What five consecutive integers does Expression A produce when $n = 8$?

2. What five consecutive even integers does Expression B produce when $n = 0$?

3. What five consecutive odd integers does Expression C produce when $n = 9$?

Write an equation to solve each problem.

4. Find the three consecutive integers that have a sum of -12.

5. Find the four consecutive odd integers with a sum of -80.

6. The larger of two consecutive even integers is 6 less than 3 times the smaller. Find the integers.

7. Find four consecutive even integers such that the largest is twice the smallest.

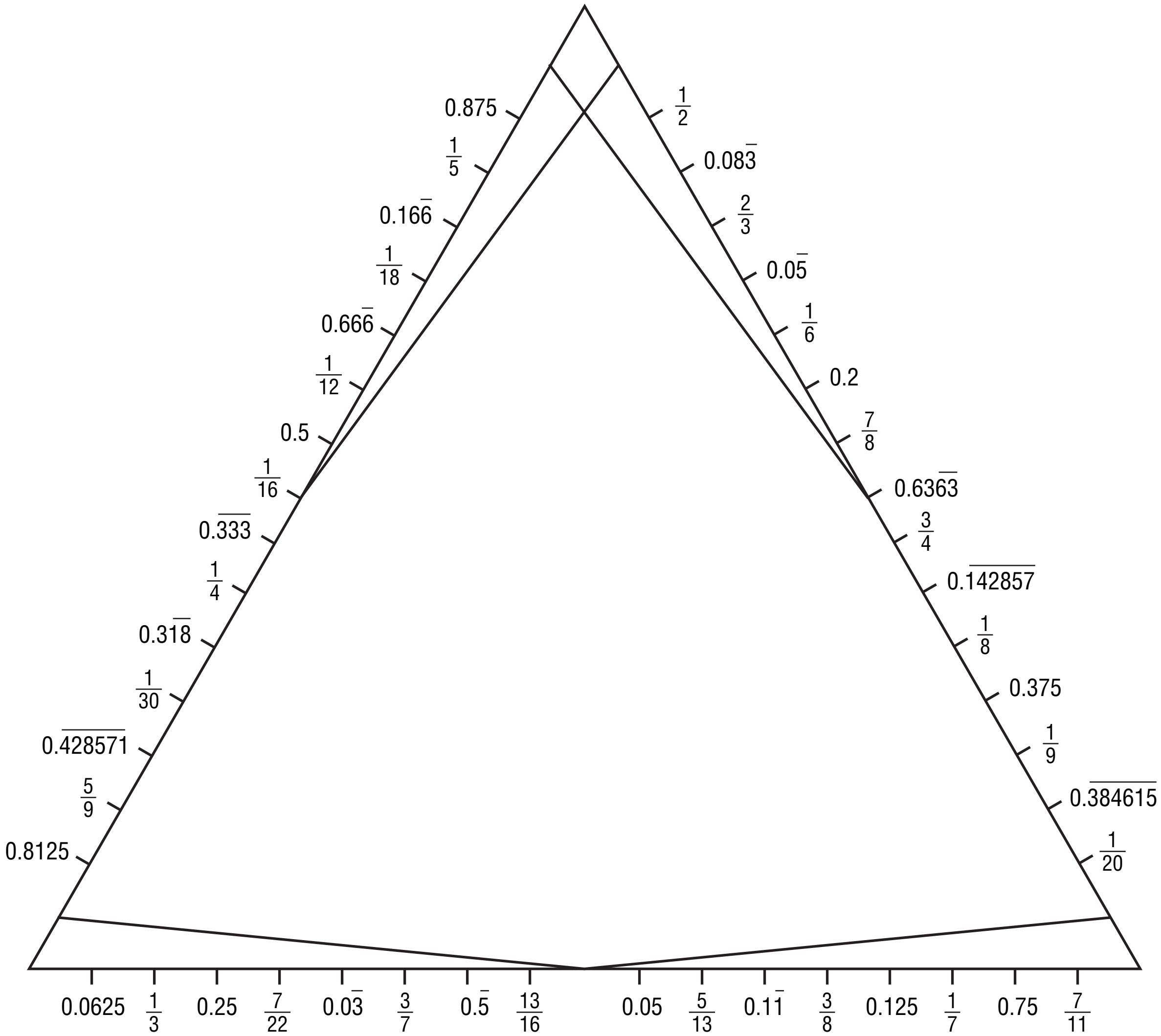
NAME ____ DATE ______________ PERIOD _____

2-1 Enrichment

7NS1.3

A Triangular Line Design

Connect each pair of equivalent rational numbers with a straight line segment. Although you will draw only straight lines, the finished design will appear curved!



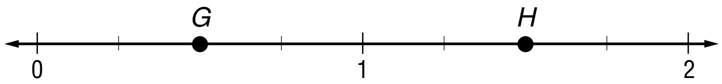
NAME ______________________________ DATE ______________ PERIOD _____

2-2 Enrichment

7NS2.5

A Famous Line-Up

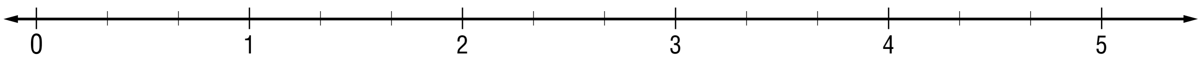
A number line can be used to graph a mixed number or an improper fraction.



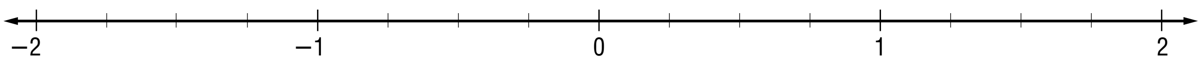
The number line above shows the graph of two points. Point G is at $\frac{1}{2}$ and point H is at $\frac{3}{2}$.

Graph each set of points on the number line. When you are finished, the letters will spell the last names of some famous people.

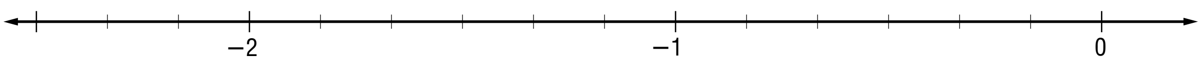
1. point R at $\frac{10}{3}$, point A at $1\frac{1}{3}$, point N at $4\frac{2}{3}$, point G at $\frac{6}{3}$, point G at $\frac{1}{3}$, point I at $\frac{13}{3}$, and point A at $2\frac{2}{3}$



2. point R at 1, point E at $-\frac{3}{4}$, point S at -2, point D at $\frac{3}{2}$, point A at $\frac{1}{2}$, point H at $-\frac{5}{4}$, and point P at $-\frac{1}{4}$



3. point G at $-2\frac{1}{6}$, point M at $-\frac{1}{6}$, point S at $-\frac{5}{6}$, point S at $-1\frac{1}{3}$, point R at $-\frac{11}{6}$, point O at $-\frac{1}{3}$, and point I at $-\frac{5}{3}$



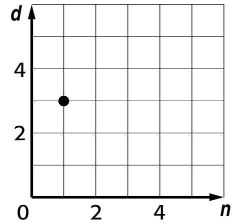
4. Why are these three people famous?

2-3 Enrichment

7AF3.3

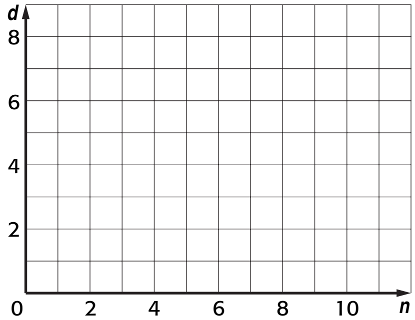
Rational Numbers as Ordered Pairs

If you think of a rational number as an ordered pair, it can be located on a coordinate system. The example graph shows the number $\frac{1}{3}$. The horizontal axis is used for the numerator and the vertical axis for the denominator.

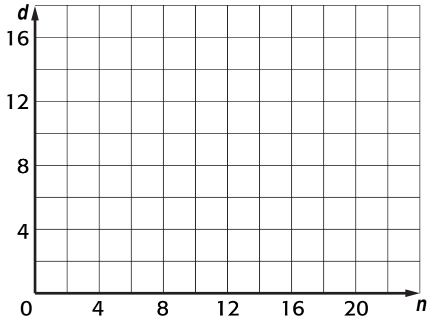


Graph the rational numbers as ordered pairs.

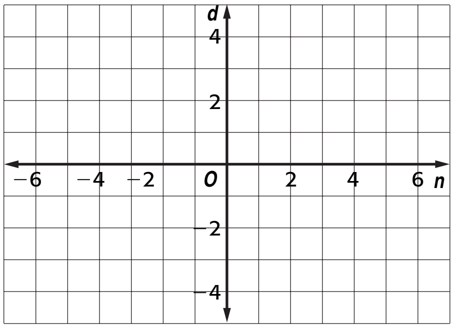
1. $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$



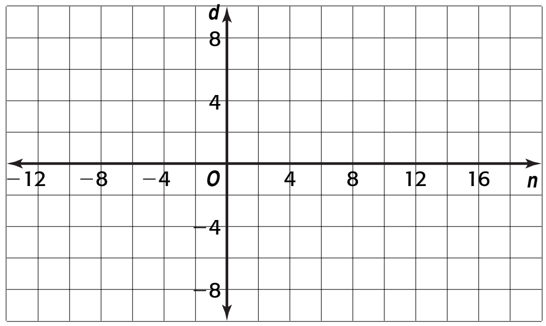
2. $\frac{4}{3}$ $\frac{8}{6}$ $\frac{12}{9}$ $\frac{16}{12}$ $\frac{20}{15}$



3. $\frac{-3}{2}$ $\frac{-6}{4}$ $\frac{3}{-2}$ $\frac{6}{-4}$



4. $\frac{-5}{2}$ $\frac{-10}{4}$ $\frac{5}{-2}$ $\frac{10}{-4}$ $\frac{15}{-6}$



5. Complete this generalization: A rational number $\frac{a}{b}$ is shown on a coordinate system using the ordered pair (a, b). Using this model, equivalent rational numbers will ______________________________.

6. Show that this generalization is false: A rational number $\frac{a}{b}$ is shown on a coordinate system using the ordered pair (a, b). All ordered pairs on the same line stand for equivalent rational numbers.

2-4 Enrichment

7NS2.2

Continued Fractions

The expression at the right is an example of a *continued fraction*. The example shows how to change an improper fraction into a continued fraction.

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9}}}$$

Example **Write $\frac{72}{17}$ as a continued fraction.**

$$\frac{72}{17} = 4 + \frac{4}{17}$$

$$= 4 + \cfrac{1}{\cfrac{17}{4}}$$

$$= 4 + \cfrac{1}{4 + \cfrac{1}{4}}$$

Notice that each fraction must have a numerator of 1 before the process is complete.

Exercises

Change each improper fraction to a continued fraction.

1. $\frac{13}{10}$

2. $\frac{17}{11}$

3. $\frac{25}{13}$

4. $\frac{17}{6}$

Write each continued fraction as an improper fraction.

5. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$

6. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}$

7. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5}}}$

NAME ______________________ DATE ____________ PERIOD _____

2-5 Enrichment

7MR1.1

Extending Problems

When examining the solution of a problem, good problem solvers look for ways to extend the problem. The questions on this page show you a way to examine and extend the following pattern.

Row 1:	$\frac{1}{2} =$	$\frac{1}{2} =$	$\frac{1}{2}$
Row 2:	$\frac{1}{2} + \frac{1}{4} =$	$\frac{2}{4} + \frac{1}{4} =$	$\frac{3}{4}$
Row 3:	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} =$	$\frac{4}{8} + \frac{2}{8} + \frac{1}{8} =$	$\frac{7}{8}$
Row 4:	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} =$	$\frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16} =$	$\frac{15}{16}$

1. What is the relationship between the denominators of the fractions in the first column?

2. What is the relationship between the numerators of the fractions in the second column?

3. In the space below, write Row 5 of the pattern.

4. What would be the fraction at the end of Row 6? Row 9?

5. Now complete the following pattern.

Row 1:	$\frac{1}{3} =$	$\frac{1}{3} =$	$\frac{1}{3}$
Row 2:	$\frac{1}{3} + \frac{1}{9} =$	$\frac{3}{9} + \frac{1}{9} =$	______
Row 3:	$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} =$	______________________	______
Row 4:	______________________	______________________	______
Row 5:	______________________	______________________	______

6. **CHALLENGE** Find this sum: $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1{,}024} + \frac{1}{4{,}096}$.

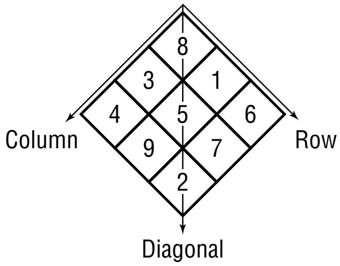
NAME ______________________________ DATE ____________ PERIOD _____

2-6 Enrichment

7NS1.2

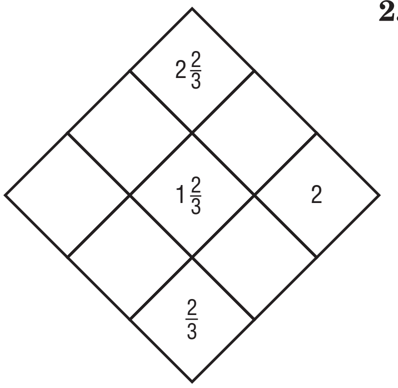
Magic Squares

A **magic square** is an arrangement of numbers such that the rows, columns, and diagonals all have the same sum. In this magic square, the *magic sum* is 15.

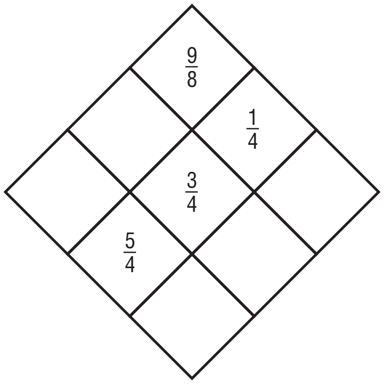


Find the magic sum for each square in Exercises 1–5. Then fill in the empty cells.

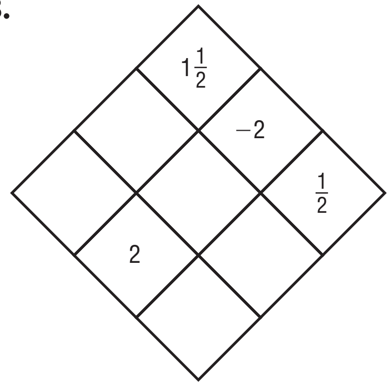
1.



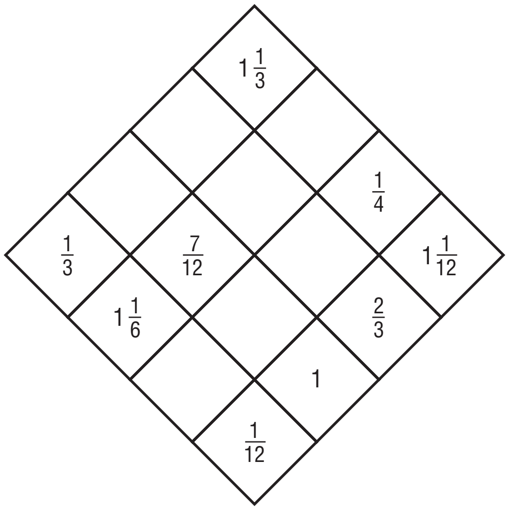
2.



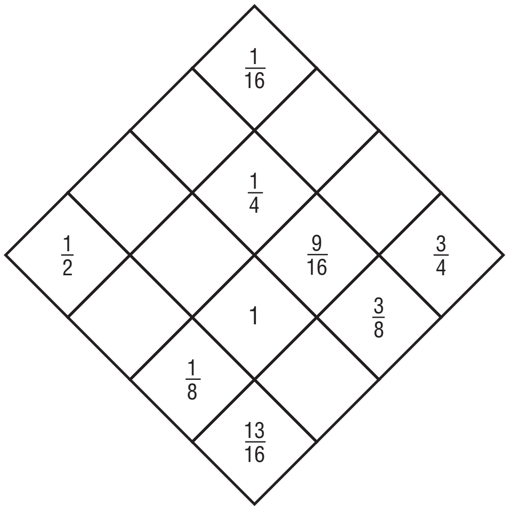
3.



4.



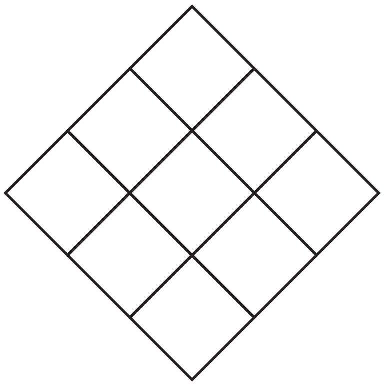
5.



6. Arrange these numbers to make a magic square.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$

$\frac{1}{6}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{7}{12}$



NAME ______________________________ DATE ____________ PERIOD _____

2-7 Enrichment

7AF1.2

Equation Hexa-Maze

To solve the maze, start with the number in the center. This number must be the solution of the equation in the next cell. The number in the new cell will then be the solution to the equation in the next cell. At each move, you may only move to an adjacent cell. Each cell is used only once.

$n + 3.7 = 7$ 11		$19 = n + 17.9$ 1.5		$40 = 40 - n$ **End**
	$3.3n = 36.3$ 1.1		$\frac{n}{4} - \frac{1}{3} = \frac{2}{3}$ 0	
$90 - \frac{3}{2}n = 30$ 3.3		$43 = n + 41.5$ 4		$n + 11 = 16$ 1.5
	$21 + 12n = 13$ 40		$14 = 0.5n - 6$ 5	
$0.7n - 4 = 0.9$ $-\frac{2}{3}$		**Start Here** 40		$5.2 = n + 3.7$ 7
	$\frac{n}{2} + 0.1 = 0$ 7		$\frac{2}{3} - \frac{n}{12} = -\frac{1}{4}$ -0.4	
$\frac{9}{2} = 6 - n$ $\frac{9}{2}$		$5n = -2$ $\frac{2}{3}$		$29.2 = 36.2 - n$ 11
	$9 = n + \frac{9}{2}$ -0.2		$-6n + 5 = 1$ 100	
$3 = 4.5 - n$ $\frac{3}{2}$		$\frac{n}{0.3} = 66\frac{2}{3}$ 5.5		$-15 = 5 - \frac{n}{5}$ $-\frac{2}{3}$
	$n + 4.5 = 10$ 1.5		$75n = -50$ 20	

2-9 Enrichment

7NS1.2, 7AF2.1

A-Mazing Exponents

Solve the following puzzle by finding the correct path through the boxes. The solution is a famous quote from United States history.

Starting with Box 1, draw an arrow to the box next or diagonal to Box 1 with the expression of the least value. The arrow cannot go to a box that has already been used. The first arrow has been drawn to get you started.

When you have finished drawing your path through the boxes, write the box numbers on the lines below. Put the numbers in the order in which they are connected. Then use the chart at the right to convert each box number to a letter.

1	G
2	M
3	E
4	E
5	R
6	E
7	I
8	V
9	B
10	T
11	D
12	L
13	I
14	Y
15	R
16	E
17	E
18	E
19	O
20	G
21	T
22	A
23	M
24	V
25	I

1: 5^3	2: 13^2	3: $4^3 + 3^4$	4: 17^2	5: $4^5 - 9^3$
6: 6^3	7: 2^7	8: $2^4 \cdot 3^2$	9: $2^5 \cdot 3^2$	10: 18^2
11: $4^4 + 16^2$	12: 3^5	13: 4^4	14: 7^3	15: $5^3 + 3^5$
16: $3^6 - 6^3$	17: $8^3 - 1^8$	18: $16^2 + 6^3$	19: 19^2	20: $2^8 + 11^2$
21: $2^9 + 9^2$	22: 23^2	23: $3^6 - 3^5$	24: 21^2	25: $2^3 \cdot 7^2$

Box Number	1	7											
Letter	G	I											

Box Number													
Letter													H

Lesson 2-9

NAME ______________________________ DATE ____________ PERIOD _____

2-10 Enrichment

7NS1.1

Scientific Notation and Space

What travels faster than jets, spaceships, and sound waves? Light does. The **speed of light** is about 3×10^8 meters per second (3×10^5 kilometers per second). Because distances in space are so large, they are often discussed in terms of **light years**, or the **distance** a photon of light would travel in a year.

1 light year = speed of light in meters per second × number of seconds in a year.

There are $365 \times 24 \times 60 \times 60 = 31{,}536{,}000 \approx 3.15 \times 10^7$ seconds in a year.

1 light year $\approx (3 \times 10^8) \times (3.15 \times 10^7) = 9.45 \times 10^{15}$ **meters** $= 9.45 \times 10^{12}$ **kilometers**

When performing operations with numbers in scientific notation, it is often helpful to consider the decimal part and the power of ten separately.

$$\begin{aligned}(2.3 \times 10^3)(1.4 \times 10^2) &= (2.3 \times 1.4) \times (10^3 \times 10^2)\\ &= 3.22 \times (10 \times 10 \times 10) \times (10 \times 10)\\ &= 3.22 \times 10^5\end{aligned}$$

Use the information above and the following tables to answer Exercises 1–6 below.

Planet	Distance from Sun (km)	Diameter (km)
Mercury	5.7×10^7	5.9×10^3
Venus	1.07×10^8	1.2×10^4
Earth	1.5×10^8	1.3×10^4
Mars	2.3×10^8	6.8×10^3
Jupiter	7.8×10^8	1.43×10^5
Saturn	1.4×10^9	1.2×10^5
Uranus	2.9×10^9	5.1×10^4
Neptune	4.5×10^9	5.0×10^4
Pluto	5.9×10^9	2.4×10^3

Source: wikipedia.com

Object	Distance from Earth (light-years)
Alpha Centauri	4.27
Sirius (Dog star)	8.7
Arcturus	36
Pleiades Cluster	400
Betelgeuse	520
Deneb	1,600
Crab Nebula	4,000
Center of Milky Way	38,000

Source: pbs.org

1. How long does it take a photon of light to travel from the Sun to Earth?

2. How long does it take a photon of light to travel from the Sun to Pluto?

3. How far is Alpha Centauri from Earth in kilometers?

4. The Pleiades Cluster is about how many times as far from Earth as Alpha Centauri?

5. If you see Sirius in the night sky, how long ago was that light emitted from the star?

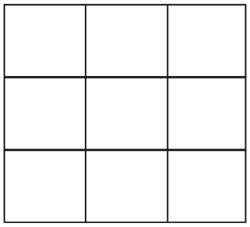
6. The diameter of Jupiter is how many times the diameter of Earth?

3-1 Enrichment

7AF2.2

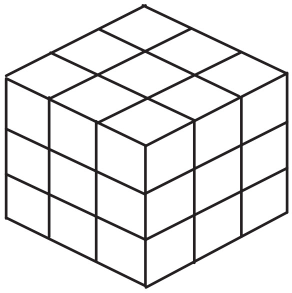
Cube Roots

A square root is just one of many kinds of roots. Another kind of root is the cube root. Just as the number 9 is a perfect square because it is a square of a whole number, the number 27 is a perfect cube because it is the cube of a whole number.



Square Root
The square root of 9 is 3 because
$3 \times 3 = 9$.
In symbols, we can write:
$\sqrt{9} = 3$.

Cube Root
A cube root of 27 is 3 because
$3 \times 3 \times 3 = 27$.
In symbols, we can write:
$\sqrt[3]{27} = 3$.



For Exercises 1 and 2, discuss the problems with a classmate before writing your answer.

1. What does $\sqrt{-9}$ mean? Does this make sense? Explain.

2. What does $\sqrt[3]{-27}$ mean? Does this make sense? Explain.

3. Complete the following table to list some perfect cubes and their cube roots.

Perfect Cube		−1		8	64	125				1,000
Cube Root	−2		1				6	7	8	

Find each cube root.

4. $\sqrt[3]{343}$

5. $\sqrt[3]{\frac{27}{64}}$

6. $\sqrt[3]{0.000008}$

Solve each equation. Check your solution.

7. $\sqrt[3]{x} = -5$

8. $y^3 = 216$

9. $z^3 = -0.512$

10. **PACKAGING** FunTime Woodworkers manufacture letter blocks to be used by young children. Each block is a cube measuring 1 inch on each side. FunTime wants to package the blocks in containers that are perfect cubes, and their marketing research recommends that each package contain at least three full sets of 36 blocks. What is the smallest perfect cube box that will fit all the blocks?

NAME ______________________________ DATE ____________ PERIOD _____

3-2 Enrichment

7MG2.1

Heron's Formula

A formula named after Heron of Alexandria can be used to find the area of a triangle if you know the lengths of the sides.

Step 1

Find s, the semi-perimeter. For a triangle with sides a, b, and c, the semi-perimeter is:

$$s = \frac{a + b + c}{2}.$$

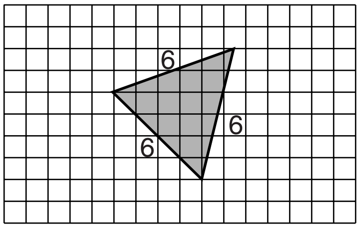
Step 2

Substitute s, a, b, and c into Heron's Formula to find the area, A.

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

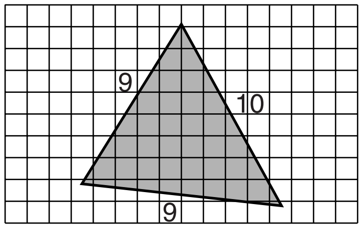
Estimate the area of each triangle by counting squares. Then use Heron's Formula to compute a more exact area. Give each answer to the nearest tenth of a unit.

1.



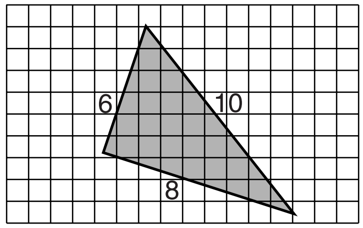
Estimated area:
Computed area:

2.



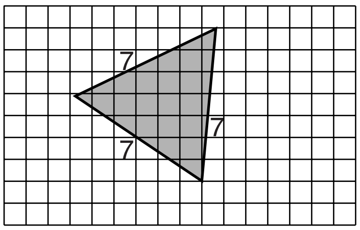
Estimated area:
Computed area:

3.



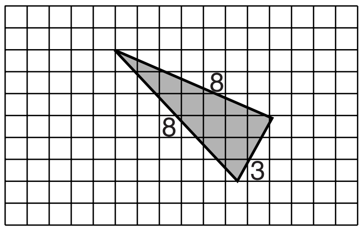
Estimated area:
Computed area:

4.



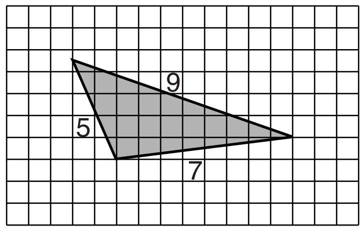
Estimated area:
Computed area:

5.



Estimated area:
Computed area:

6.



Estimated area:
Computed area:

7. Why would it be foolish to use Heron's Formula to find the area of a right triangle?

3-4 Enrichment

7AF1.3

The Closure Property

The **Real Number System** contains properties that can help you solve problems. These include the *Commutative Property,* the *Distributive Property,* and the *Associative Property*.

Another property of real numbers is the *Closure Property.*

Closure Property of Real Numbers
A set of numbers is closed under a particular operation if performing the operation on any number in the set results in a number in the set.

Is the set of whole numbers closed under addition?

Yes. The sum cannot contain a decimal part because none of the addends has a decimal part. Also, the sum cannot be negative because none of the addends is negative. The sum must be a whole number.

Answer each of the following questions about the Closure Property in the real number system. If the answer is yes, explain how you know. If the answer is no, give a counterexample to show.

1. Is the set of rational numbers closed under multiplication?

2. Is the set of whole numbers closed under subtraction?

3. Is the set of integers closed under division?

4. Is the set of rational numbers closed under division?

5. Is the set of irrational numbers closed under subtraction?

6. Is the set of integers closed under subtraction?

NAME ____________________ DATE ____________ PERIOD _____

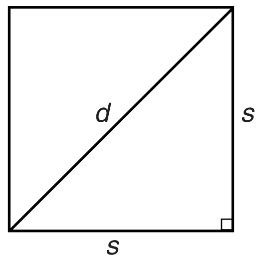
3-5 Enrichment

7MG3.3

Geometric Relationships

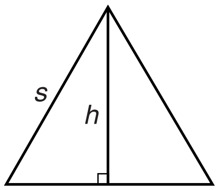
The Pythagorean Theorem can be used to express relationships between parts of geometric figures. The example shows how to write a formula for the length of the diagonal of a square in terms of the length of the side.

$$d^2 = s^2 + s^2$$
$$d^2 = 2s^2$$
$$d = \sqrt{2}s$$

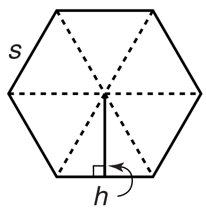


Develop a formula for each problem. The dashed lines have been included to help you.

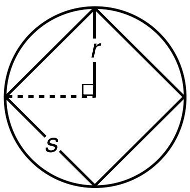
1. An equilateral triangle has three sides of the same length. Express the altitude h in terms of the side s.



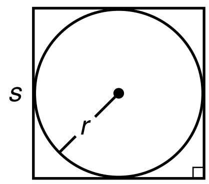
2. A regular hexagon has six sides of the same length. Express the height h in terms of the length of the side s.



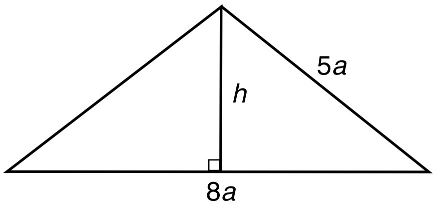
3. A circle is circumscribed about a square. Express the radius r of the circumscribed circle in terms of the side s of the square.



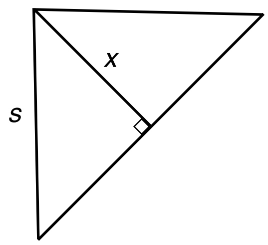
4. A circle is inscribed in a square. Express the radius r of the inscribed circle in terms of the side s of the square.



5. Use the isosceles triangle below. Express the altitude h in terms of the quantity a.



6. Use the isosceles right triangle below. Express x in terms of s.

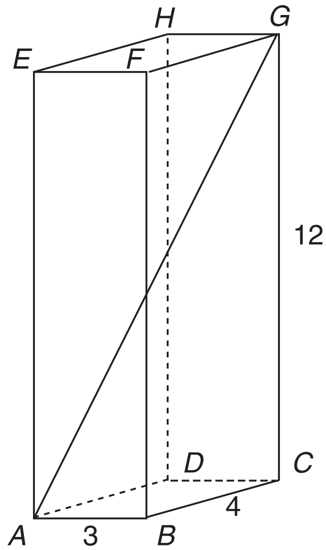


3-6 Enrichment

7MG3.3

Pythagorean Theorem in 3-D

At the right, you see an illustration of a rectangular prism. The prism is 3 units wide, 4 units deep, and 12 units high. Use the Pythagorean Theorem to first find the length of diagonal AC, and then to find the length of diagonal AG.

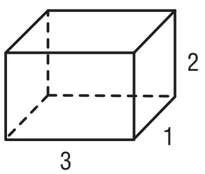


$AC^2 = 3^2 + 4^2$	$AG^2 = 5^2 + 12^2$
$AC^2 = 9 + 16$	$AG^2 = 25 + 144$
$AC^2 = 25$	$AG^2 = 169$
$AC = \sqrt{25}$	$AG = \sqrt{169}$
$AC = 5$	$AG = 13$

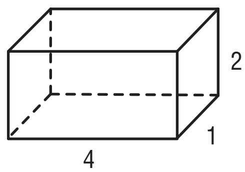
The diagonal of the rectangular prism is 13 units.

For each rectangular prism, find the length of a diagonal. Round to the nearest hundredth.

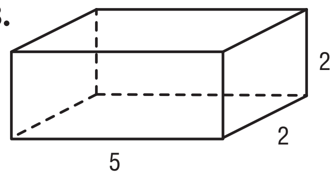
1.



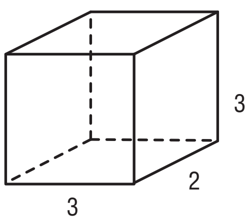
2.



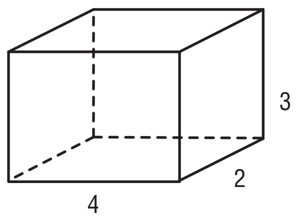
3.



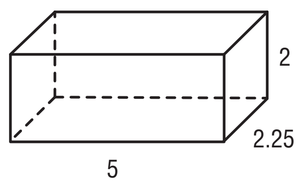
4.



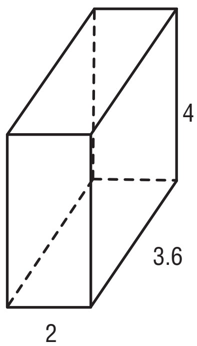
5.



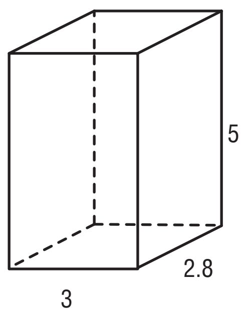
6.



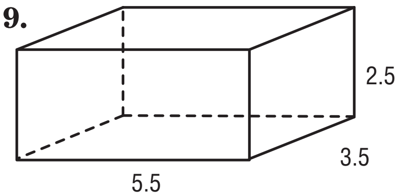
7.



8.



9.



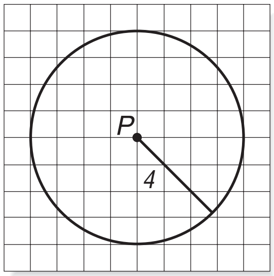
3-7 Enrichment

7MG2.1

Circles on the Coordinate Plane

A **circle** is defined as *the set of all points in a plane that lie a given distance from a fixed point.*

This is a sketch of the set of all points a distance of 4 units from point *P*.



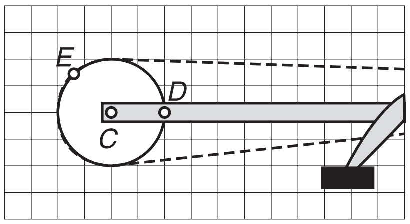
Use your knowledge of distance on the coordinate plane to answer the following questions about circles.

1. The distance from the center of a circle to any point on the circle is called the ______.

2. The diameter of a circle has endpoints (−1, 2) and (9, −4). Find the coordinates of the center of the circle.

3. The diameter of a circle has endpoints (2, 3) and (10, 6). Find the length of the radius.

4. A diagram of a circular gear is shown on the coordinate plane below. Point C is the center of the gear and has coordinates (5, 4). Point D is on the pedal crank shaft and has coordinates (7, 4). After the crank has completed $\frac{3}{8}$ of a rotation, the crank shaft will be located at point E. Find the distance between points D and E.



5. The center of a circle has coordinates (1, 3). Points A and B are located on the circle. The point A is located at (5, 1), and the distance between points A and B is $2\sqrt{10}$. Name all possible locations of point B.

Lesson 3-7

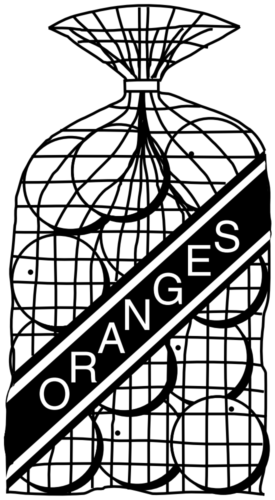
4-1 Enrichment

7AF4.2

Bargain Hunting

Rates are useful and meaningful when expressed as a unit rate. For example, which is the better buy—one orange for $0.29 or 12 oranges for $3.00?

To find the unit rate for 12 oranges, divide $3.00 by 12. The result is $0.25 per orange. If a shopper needs to buy at least 12 oranges, then 12 oranges for $3.00 is the better buy.



For each exercise below, rates are given in Column A and Column B. In the blank next to each exercise number, write the letter of the column that contains the better buy.

	Column A	Column B
1. ______	1 apple for $0.19	3 apples for $0.59
2. ______	20 pounds of pet food for $14.99	50 pounds of pet food for $37.99
3. ______	A car that travels 308 miles on 11 gallons of gasoline	A car that travels 406 miles on 14 gallons of gasoline
4. ______	10 floppy discs for $8.99	25 floppy discs for $19.75
5. ______	1-gallon can of paint for $13.99	5-gallon bucket of paint for $67.45
6. ______	84 ounces of liquid detergent for $10.64	48 ounces of liquid detergent for $6.19
7. ______	5,000 square feet of lawn food for $11.99	12,500 square feet of lawn food for $29.99
8. ______	2 compact discs for $26.50	3 compact discs for $40.00
9. ______	8 pencils for $0.99	12 pencils for $1.49
10. ______	1,000 sheets of computer paper for $8.95	5,000 sheets of computer paper for $41.99

4-2 Enrichment

7AF4.2

Saving with Larger Orders

Stores are able to make a profit by buying in bulk. They are charged less per item because they agree to buy a large number of items. When the unit price of bulk items is less than the unit price for a small order, the rates are **nonproportional.**

For each exercise below, the number of units and the total cost of the units is given in Column A and Column B. In the blank next to each exercise number, write whether the columns are *proportional* or *nonproportional*.

	Column A	Column B
1. ________	100 T-shirts printed for $428.00	3,000 T-shirts printed for $12,180.00
2. ________	3 compact discs for $23.34	10,000 compact discs for $32,900.00
3. ________	8 books for $43.20	250 books for $1,350.00
4. ________	$546.00 earned in 42 hours	$31,200.00 earned in 2,400 hours
5. ________	36 photos printed for $7.20	800 photos printed for $96.00
6. ________	1 bus token for $1.75	20 bus tokens for $34.00
7. ________	$3,840.00 earned in 160 hours	$52,000 earned in 2,000 hours
8. ________	$730.00 earned in 40 hours	$9,125.00 earned in 500 hours
9. ________	1 bagel for $0.65	13 bagels for $7.80
10. _______	a series of 5 concert tickets for $187.50	a series of 20 concert tickets for $685.00

4-3 Enrichment

7AF4.2

People of the United States

A national census is taken every ten years. The 1990 census revealed that there were about 250,000,000 people in the United States, and that about 8 out of 100 of these people were 5–13 years old. To find the number of people in the United States that were 5–13 years old, use the ratio of people 5–13 years old and create a proportion.

$$\frac{8}{100} = \frac{n}{250{,}000{,}000}$$

To solve the proportion, find cross products.

$8 \times 250{,}000{,}000 = 2{,}000{,}000{,}000$ and $n \times 100 = 100n$

Then divide: $2{,}000{,}000{,}000 \div 100 = 20{,}000{,}000$.

In 1990, about 20,000,000 people in the United States were 5–13 years old.

Use the approximate ratios in each exercise to create a proportion, given that there were about 250,000,000 people in the United States. Then solve and choose the correct answer from the choices at the right.

1. The United States is a diverse collection of different races and ethnic origins. Asians or Pacific Islanders accounted for about $\frac{3}{100}$ of the population of the United States. About how many people of Asian or Pacific-Island origin lived in the United States? ________ **A.** 200,000,000

2. African-Americans accounted for about $\frac{3}{25}$ of the population of the United States. About how many African-American people lived in the United States? ________ **B.** 22,500,000

3. People of Hispanic origin accounted for about $\frac{9}{100}$ of the population of the United States. About how many people of Hispanic origin lived in the United States? ________ **C.** 7,500,000

4. Caucasian people accounted for about $\frac{4}{5}$ of the population of the United States. About how many people of white or Caucasian origin lived in the United States? ________ **D.** 2,000,000

5. People of American-Indian, Eskimo, or Aluet origin accounted for about $\frac{8}{1{,}000}$ of the population of the United States. About how many people of American-Indian, Eskimo, or Aluet origin lived in the United States? ______ **E.** 30,000,000

NAME ________________________________ DATE ____________ PERIOD _____

4-5 Enrichment

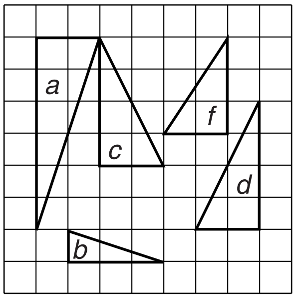
7MG3.4

Similar and Congruent Figures

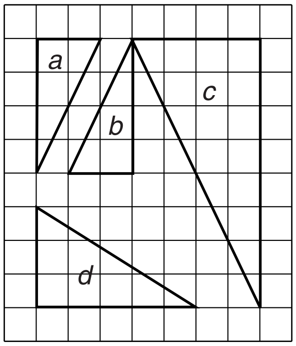
If a 4-inch by 5-inch photograph is enlarged to an 8-inch by 10-inch photograph, the photographs are said to be **similar.** They have the same shape, but they do not have the same size. If a 4-inch by 5-inch photograph is duplicated and a new 4-inch by 5-inch photograph is made, the photographs are said to be **congruent.** They have the same size and shape.

In each exercise, identify which of the triangles are congruent to each other and identify which of the shapes are similar to each other. Use the symbol for $\cong$ congruence and the symbol $\sim$ for similarity.

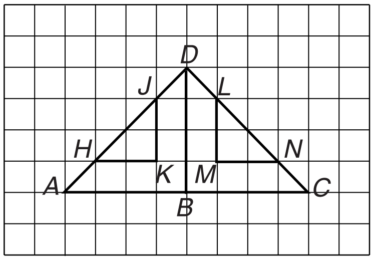
1.



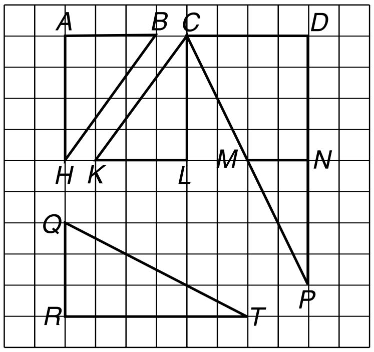
2.



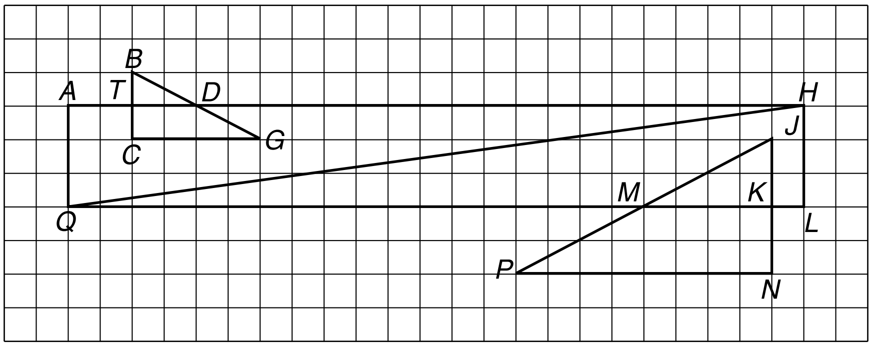
3.



4.



5.

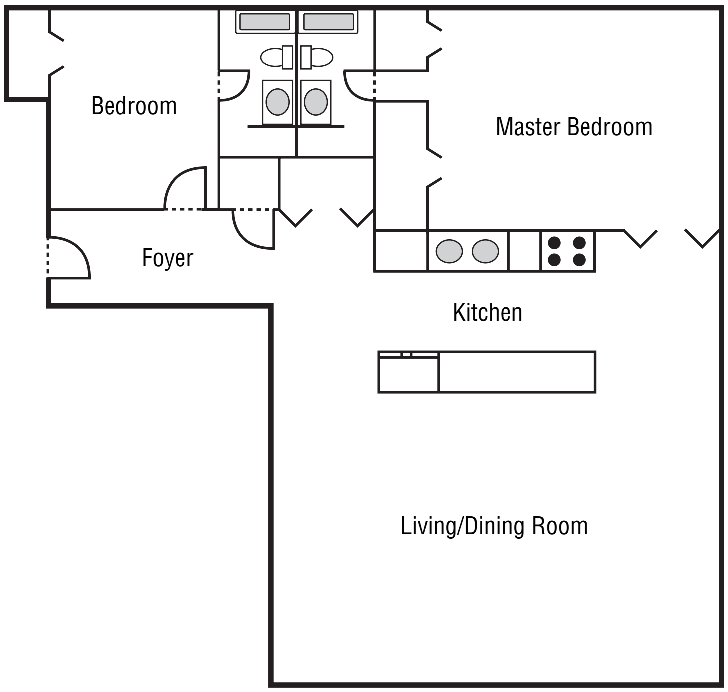


4-6 Enrichment

7MG1.1

Scale Drawings

The map below shows the floor plan for a two-bedroom apartment. The length of the living room is 6 m. On the plan the living room is 6 cm long.



Answer each question.

1. What is the scale ratio represented on the floor plan?

2. On the floor plan, how many feet are represented by a centimeter?

3. On the floor plan, measure the dimensions of the master bedroom in centimeters (including the closets and master bathroom). What are the actual dimensions in feet?

4. How long, in feet, would a rug need to be to stretch across the foyer?

5. Measure the length and width of your bedroom. Make a scale drawing of your bedroom using an appropriate scale. Then complete the following table based on your measurements.

Room Proportions		
	Measurement (ft)	**Measurement (m)**
Width of Room		
Length of Room		

4-7 Enrichment

7MG2.4

Similar Shapes and Solids

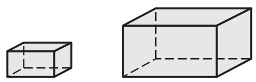
If two shapes are similar, the ratio of their areas is proportional to the square of the scale factor between them. $\frac{\text{area of shape A}}{\text{area of shape B}} = \left(\frac{a}{b}\right)^2$

Example 1 A rectangle has an area of 60 square meters. If the rectangle is reduced to one-fourth its original size, what is the area of the new rectangle?

$\frac{60}{A} = \left(\frac{4}{1}\right)^2$ So, the area of the smaller rectangle is 3.75 square meters.

If two solids are similar, the ratio of their volumes is proportional to the cube of the scale factor between them. $\frac{\text{volume of solid A}}{\text{volume of solid B}} = \left(\frac{a}{b}\right)^3$

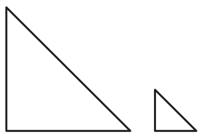
Example 2 A rectangular prism has a volume of 36 cubic feet. Suppose the dimensions are doubled. What is the volume of the new prism?



$\frac{36}{V} = \left(\frac{1}{2}\right)^3$ So, the volume of the larger prism is 288 cubic feet.

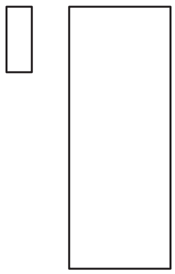
Find the area of the similar shapes.

1.



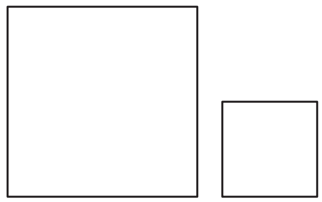
$A = 24$ cm^2
Scale Factor $= \frac{1}{3}$

2.



$A = 7$ mm^2
Scale Factor $= 4$

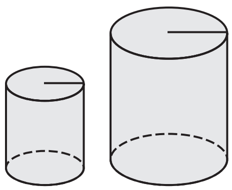
3.



$A = 64$ in^2
Scale Factor $= \frac{1}{2}$

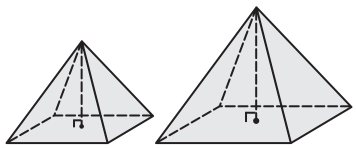
Find the volume of the similar solids.

4.



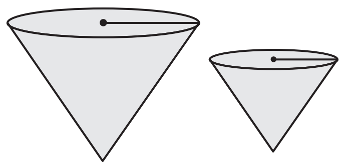
$V = 10$ m^3
Scale Factor $= \frac{3}{2}$

5.



$V = 9$ mm^3
Scale Factor $= \frac{4}{3}$

6.



$V = 27$ ft^3
Scale Factor $= \frac{2}{3}$

Lesson 4–7

NAME ______________________ DATE ____________ PERIOD _____

4-8 Enrichment

7MG I.2

Scale Drawings

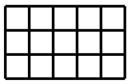
The figure at the right has an area of 6 square units. If the figure represented a map, and was drawn to a scale of 1 unit = 3 feet, the lengths of the sides would be 6 ft and 9 ft. So, the figure would represent an area of 54 square feet.

The ratio of the actual area of the figure to the scale area of the figure can be expressed as a ratio.

$$\frac{\text{actual area}}{\text{scale area}} = \frac{6}{54} = \frac{1}{9} \text{ or 1 to 9}$$

Find the actual area and the scale area of these figures. Then determine the ratio of actual area to scale area.

1. Scale: 1 unit = 4 ft

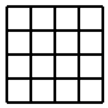


actual area ________

scale area ________

ratio ________

2. Scale: 1 unit = 50 cm

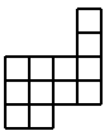


actual area ________

scale area ________

ratio ________

3. Scale: 1 unit = 8 mi

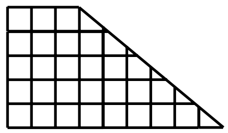


actual area ________

scale area ________

ratio ________

4. Scale: 1 unit = 12 m

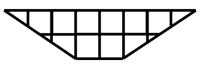


actual area ________

scale area ________

ratio ________

5. Scale: 1 unit = 18 in.

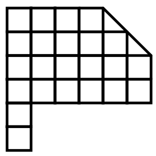


actual area ________

scale area ________

ratio ________

6. Scale: 1 unit = 6 km



actual area ________

scale area ________

ratio ________

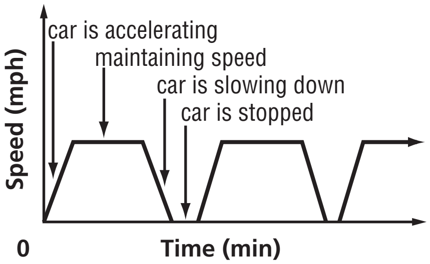
4-9 Enrichment

7AF3.4

Analyzing Graphs

A graph can be used to represent many real-life situations. Graphs such as these often have time as the dimension on the horizontal axis. By analyzing the rate of change of different parts of a graph, you can draw conclusions about what was happening in the real-life situation at that time.

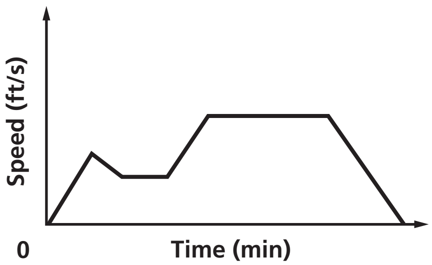
TRAVEL **The graph at the right represents the speed of a car as it travels along the road. Describe what is happening in the graph.**



- At the origin, the car is stopped.
- Where the line shows a fast, positive rate of change, the car is speeding up.
- Then the car is going at a constant speed, shown by the horizontal part of the graph.
- The car is slowing down where the graph shows a negative rate of change.
- The car stops and stays still for a short time. Then it speeds up again. The starting and stopping process repeats continually.

Analyze each graph.

1. Ashley is riding her bicycle along a scenic trail. Describe what is happening in the graph.



2. The graph shows the depth of water in a pond as you travel out from the shore. Describe what is happening in the graph.

NAME ________________________ DATE ____________ PERIOD _____

4-10 Enrichment

7AF3.4

Chien-Shiung Wu

American physicist Chien-Shiung Wu (1912–1997) was born in Shanghai, China. In 1936, she came to the United States to further her studies in science. She received her doctorate in physics in 1940 from the University of California, and became known as one of the world's leading physicists. In 1975, she was awarded the National Medal of Science.

Wu is most famous for an experiment that she conducted in 1957. The outcome of the experiment was considered the most significant discovery in physics in more than seventy years. The exercise below will help you learn some facts about it.

The points given in each table lie on a line. Find the rate of change of the line. The word or phrase following the solution will complete the statement correctly.

1.

x	0	1	2	3
y	1	4	7	10

At the time of the experiment, Wu was a professor at _____?_____.

rate of change = 3: Columbia University

rate of change = 1: Stanford University

2.

x	−2	−1	0	1
y	3	3	3	3

The site of the experiment was the _____?_____ in Washington, D.C.

rate of change = 0: National Bureau of Standards

rate of change = 3: Smithsonian Institution

3.

x	0	2	4	6
y	2	4	6	8

The experiment involved a substance called _____?_____.

rate of change = 2: carbon 14

rate of change = 1: cobalt 60

4.

x	−2	1	4	7
y	−3	1	5	9

In the experiment, the substance was cooled to _____?_____.

rate of change = $\frac{4}{3}$: −273°C

rate of change = $\frac{3}{4}$: −100°C

NAME ______________________ DATE ____________ PERIOD _____

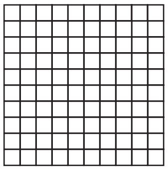
5-1 Enrichment

7NS1.3

Visualizing Percent

Shade each grid to show the given ratio. Write the percent of the grid that is shaded and the percent that is not shaded.

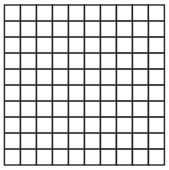
1. $\frac{14}{100}$



Shaded ____________

Not shaded ____________

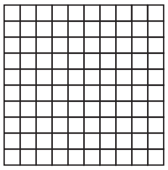
2. $\frac{16}{25}$



Shaded ____________

Not shaded ____________

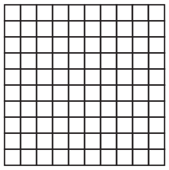
3. $\frac{3}{10}$



Shaded ____________

Not shaded ____________

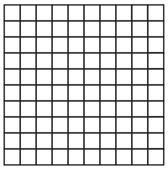
4. $\frac{3}{5}$



Shaded ____________

Not shaded ____________

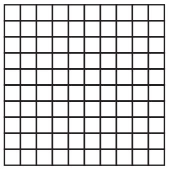
5. $\frac{23}{25}$



Shaded ____________

Not shaded ____________

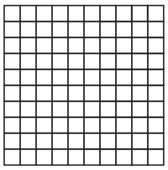
6. $\frac{7}{20}$



Shaded ____________

Not shaded ____________

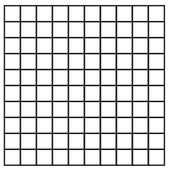
7. $\frac{111}{200}$



Shaded ____________

Not shaded ____________

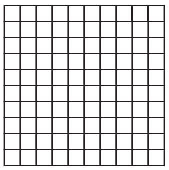
8. $\frac{1}{2}$



Shaded ____________

Not shaded ____________

9. $\frac{1}{4}$



Shaded ____________

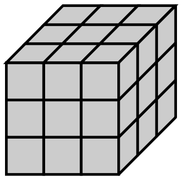
Not shaded ____________

5-2 Enrichment

7NS1.3

Block Party

1. This model is made up of 27 cubes and has a length of 3 cubes, a width of 3 cubes, and a height of 3 cubes. The entire model will be painted yellow, then cut apart into individual cubes.



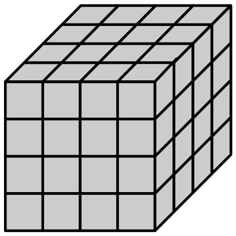
What percent of the cubes will be painted yellow on:

0 sides? __________ 1 side? __________ 2 sides? __________

3 sides? __________ 4 sides? __________ 5 sides? __________

6 sides? __________

2. This model is made up of 64 cubes and has a length of 4 cubes, a width of 4 cubes, and a height of 4 cubes. The entire model will be painted orange, then cut apart into individual cubes.



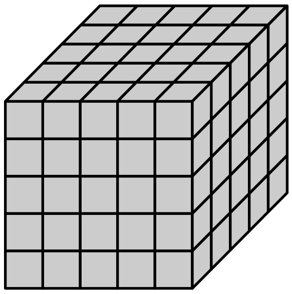
What percent of the cubes will be painted orange on:

0 sides? __________ 1 side? __________ 2 sides? __________

3 sides? __________ 4 sides? __________ 5 sides? __________

6 sides? __________

3. This model is made up of 125 cubes and has a length of 5 cubes, a width of 5 cubes, and a height of 5 cubes. The entire model will be painted purple, then cut apart into individual cubes.



What percent of the cubes will be painted purple on:

0 sides? __________ 1 side? __________ 2 sides? __________

3 sides? __________ 4 sides? __________ 5 sides? __________

6 sides? __________

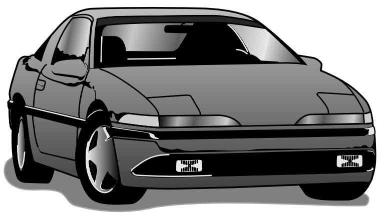
5-3 Enrichment

7NS1.6

The Cost of Using

The percent proportion can be used to describe the percent by which something depreciates, or loses its value during the course of time.

Example 1 **In 2006, the average purchase price of a new compact automobile was $17,000. One year later, the compact automobile that was purchased new in 2006 typically was worth $11,500. By what percent did the automobile depreciate during the first year of ownership?**



First find the amount of decrease.

$17,000 − $11,500 = $5,500

Then write and solve the percent proportion.

$5,500 is what percent of $17,000?

$$\frac{5{,}500}{17{,}000} = \frac{r}{100}$$

$$550{,}000 = 17{,}000r$$

$$32.4 \approx r$$

The automobile depreciated about 32.4%, or about $\frac{1}{3}$ of its value, during the first year of ownership.

Exercises

Advertisements such as these regularly appear in the classified pages of newspapers. Find the percent of depreciation to the nearest tenth in each advertisement.

1. For sale: Venus 2 dr, 5 speed, air, sport wheels, mint. Bought new 8 months ago for $9,600, yours for $7,200. Must sell—baby on way.

2. For sale: Washer/dryer combo. Antique white, electric, like new—18 mos. old. Paid $950, asking $500.

3. For sale: Mars 4 dr, auto, air, ABS, the works. 1 year old—12,400 mi. Purchase price—$16,000. Your price—$12,800 firm.

4. For sale: Dishwasher, 2 racks, full factory warranty. Never used—won in contest. List: $840. I will deliver for $588 cash.

5. For sale: Mountain bike. 20-inch frame. 26 × 1.60 new tires. Used but not abused. Bought last summer for $320. $150 or best offer.

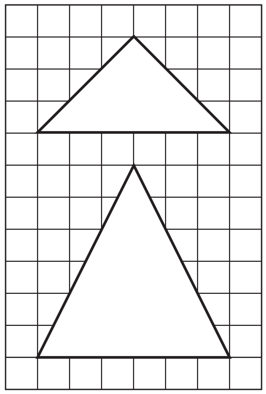
6. For sale: Personal Computer. Intex processor. 8.0 GB hard drive. 56X CD-ROM drive. 17-inch monitor. Color inkjet printer with lots of software. $2,200 new—yours for $895.

NAME ______________________ DATE ____________ PERIOD _____

5-4 Enrichment

7NS1.3

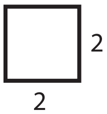
Using Percent to Compare Areas

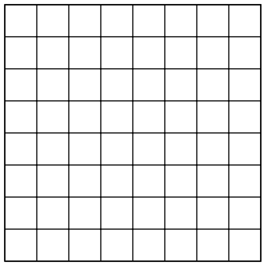


The area of the smaller triangle at the right is 50% of the area of the larger triangle. The smaller triangle has a base of 6 units and a height of 3 units; its area is 9 square units.

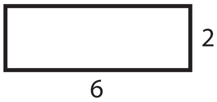
Since the area of the smaller triangle is 50% of the area of the larger triangle, the area of the larger triangle is 18 square units.

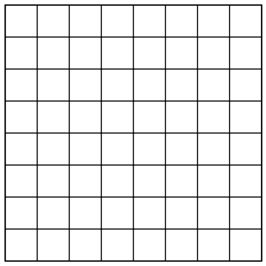
1. The area of this square is 25% of the area of a larger square. Determine the dimensions and draw the larger square.



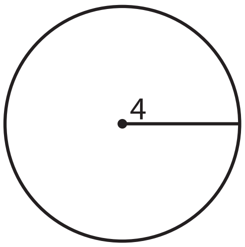


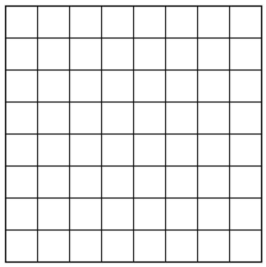
2. The area of this rectangle is 150% of the area of a smaller rectangle. Determine the dimensions and draw the smaller rectangle.



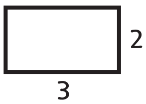


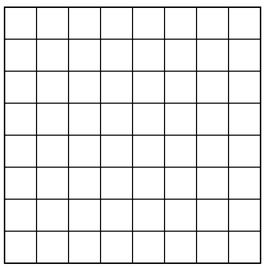
3. The area of this circle is 1,600% of the area of a smaller circle. Determine the dimensions and draw the smaller circle.





4. The area of this rectangle is 60% of the area of a larger rectangle. Determine the dimensions and draw the larger rectangle.





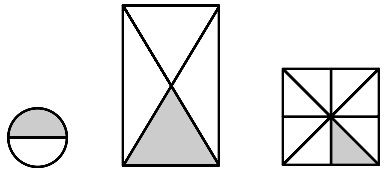
NAME ______________________ DATE ____________ PERIOD _____

5-6 Enrichment

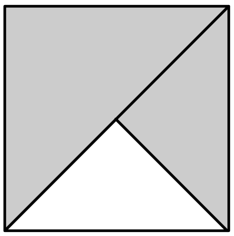
7NS1.3

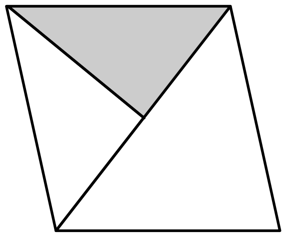
Shaded Regions

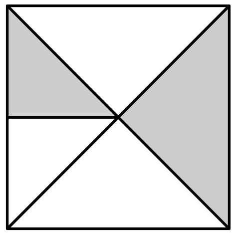
In these figures, each interior segment bisects, or divides, a region into two equal parts. One-half, or 50% of the circle is shaded, $\frac{1}{4}$ or 25% of the rectangle is shaded, and $\frac{1}{8}$ or 12.5% of the square is shaded.

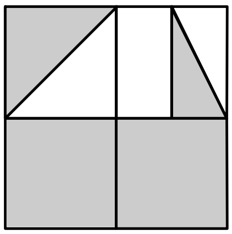


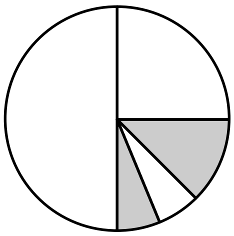
Match the area of the shaded region in each figure with a choice given at the bottom of the page.

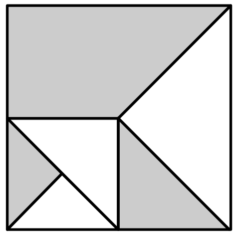
1.  _____

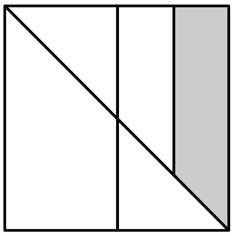
2.  _____

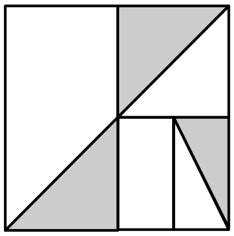
3.  _____

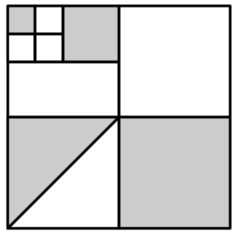
4.  _____

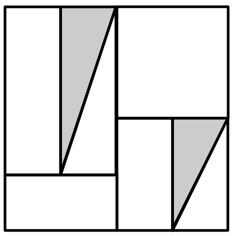
5.  _____

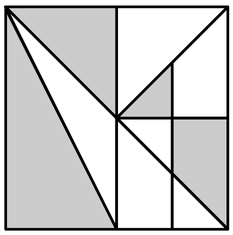
6.  _____

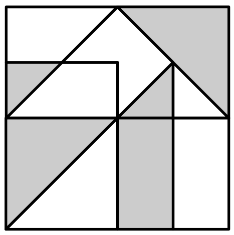
7.  _____

8.  _____

9.  _____

10.  _____

11.  _____

12.  _____

a. 18.75% **b.** $\frac{5}{32}$ **c.** 25% **d.** $\frac{5}{16}$ **e.** $21\frac{7}{8}\%$ **f.** $\frac{3}{4}$

g. 50% **h.** $\frac{11}{16}$ **i.** 43.75% **j.** $\frac{29}{64}$ **k.** 56.25% **l.** 37.5%

Lesson 5–6

5-7 Enrichment

7NS1.3

Making Estimates

Estimates often vary from person to person.

Example **Estimate 52% of 1,045.**

Estimate 1	**Estimate 2**
52% ≈ 50%	$52\% \approx \frac{1}{2}$
1,045 ≈ 1,000	1,045 ≈ 1,050
50% of 1,000 is 500.	$\frac{1}{2}(1{,}050) = 525$

Both estimates are good because both are close to the exact answer, which is 543.4.

Exercises

From the list at the right, choose a good estimate for each exercise. Record the letter of your choice in the blank for each exercise and in the box or boxes at the bottom of the page. The message describes something astronomers use estimates for.

1. Estimate 25% of 97. ____
2. Estimate the percent for 31 out of 59. ____
3. Estimate 80% of 62. ____
4. Estimate the percent for 6 out of 25. ____
5. Estimate 48% of 180. ____
6. Estimate the percent for 31 out of 42. ____
7. Estimate 21% of 39. ____
8. Estimate the percent for 4.2 out of 6. ____
9. Estimate $\frac{1}{3}$ of 238. ____
10. Estimate 145% of 398. ____

10.	9.	2.	6.	3.	1.	6.	4.

7.	3.	8.	5.	7.

a. 67%
b. 10%
c. 600
d. 32
e. 125%
f. 13%
g. 25%
h. 10
i. 25
j. 5%
k. 4
l. 175%
m. 450
n. 75%
o. 80
p. 17
q. 40%
r. 90
s. 8
t. 50
u. 50%
v. 150%
w. 350
x. 1%
y. 225
z. 100%

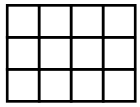
Lesson 5–7

5-8 Enrichment

7NS1.3

Shady Deals

The rectangle at the right has an area of 12 square units. If 75% of the rectangle needs to be shaded, the percent equation can help determine how many units to shade.



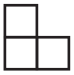
What number is 75% of 12?

$n = 0.75 \times 12$

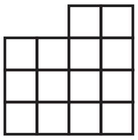
$n = 9$ Shading 9 units would shade 75% of the rectangle.

Shade the indicated region of each grid.

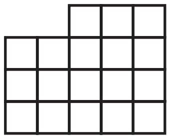
1. Shade 75%.



2. Shade $41\frac{2}{3}$%.

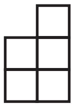


3. Shade 28%.

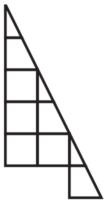


Shade the indicated region of each grid. If necessary, divide the grid into smaller units.

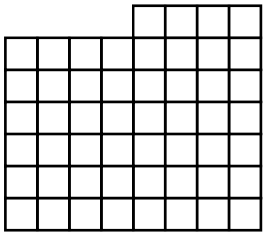
4. Shade 25%.



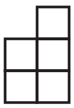
5. Shade 22.2%.



6. Shade 18.75%.

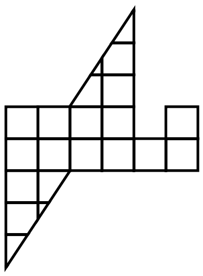


7. Shade 62.5%.



8. Shade 90.625%.

9. Shade 16.25%.



NAME ______________________________ DATE ____________ PERIOD _____

5-9 Enrichment

7NS1.6

Mountain Climbing

Changes in elevation experienced by mountain climbers can be described using percents. To describe a change in elevation, determine the amount of increase or decrease, then use the percent equation.

Example 1 **An expedition began a day's climb at 4,000 feet and made camp at the end of the day at 6,200 feet. By what percent did the elevation of the climbers increase?**

$6{,}200 - 4{,}000 = 2{,}200$ — Find the amount of increase.

$\text{percent of change} = \dfrac{\text{amount of change}}{\text{original amount}}$ — Write a ratio to compare the change to the original.

$= \dfrac{2{,}200}{4{,}000}$ — Substitute.

$= 0.55 \text{ or } 55\%$ — The elevation increased by 55%.

Example 2 **An expedition ended a day's descent at 9,600 feet after beginning the day's descent at 12,000 feet. By what percent did the elevation of the climbers decrease?**

$12{,}000 - 9{,}600 = 2{,}400$ — Find the amount of increase.

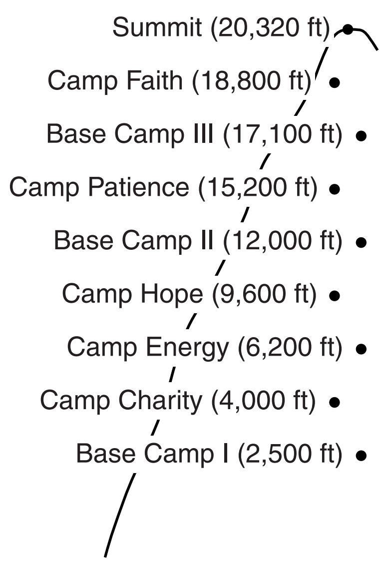
$\text{percent of change} = \dfrac{\text{amount of change}}{\text{original amount}}$ — Write a ratio to compare the change to the original.

$= \dfrac{2{,}400}{12{,}000}$ — Substitute.

$= 0.20$ — The elevation decreased by 20%.

This chart contains information about a climbing expedition. Use the information from the mountain pictured here, the percent equation, and your calculator to complete the chart. When necessary, round your answer to the nearest whole percent.

	Starting Location	Ending Location	Change in Elevation
1.	Base Camp I	Camp Charity	
2.	Base Camp III	Camp Patience	
3.	Camp Hope	Base Camp II	
4.	Camp Energy	Base Camp I	
5.	Camp Faith	Base Camp III	
6.	Camp Faith	Summit	
7.	Camp Patience	Base Camp II	
8.	Camp Hope	Camp Energy	



NAME ______________________________ DATE ____________ PERIOD _____

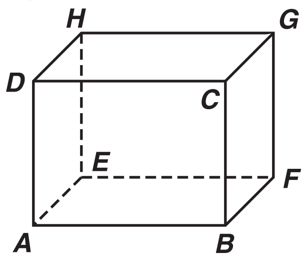
6-1 Enrichment

7MG3.1

Lines and Angles in Space

In a plane two lines are either parallel or intersecting. In *space,* there are three possibilities: **parallel**, **intersecting**, or **skew**. Imagine holding two yardsticks in the air and that the lines created by the sticks extend forever in both directions. You could hold the sticks so that the lines meet or do not meet. If the lines ever meet, they are intersecting. If they do not intersect, they are either parallel or skew. If they are oriented in the same direction, they are parallel. If lines do not intersect and are not parallel, they are skew.

Imagine that the figure to the right is a cubic room with a floor, ceiling, and four walls. Each corner is labeled with a letter for reference. The line segments that form the edges of the room are each contained in a line.



$\overleftrightarrow{AB}$ and $\overleftrightarrow{HG}$ are parallel.
$\overleftrightarrow{BC}$ and $\overleftrightarrow{HG}$ are skew.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are intersecting.

Refer to the figure above for Exercises 1–14. Determine if the lines are *parallel, intersecting,* or *skew*.

1. $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$
2. $\overleftrightarrow{CD}$ and $\overleftrightarrow{DH}$
3. $\overleftrightarrow{FG}$ and $\overleftrightarrow{AB}$
4. $\overleftrightarrow{EH}$ and $\overleftrightarrow{FG}$
5. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EH}$
6. $\overleftrightarrow{GH}$ and $\overleftrightarrow{AD}$
7. $\overleftrightarrow{EH}$ and $\overleftrightarrow{AE}$
8. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$

Find the measure of each angle.

9. $\angle DAB$
10. $\angle AFB$
11. $\angle CHE$

CHALLENGE **Determine if the given lines would be *parallel, intersecting,* or *skew*.**

12. $\overleftrightarrow{CE}$ and $\overleftrightarrow{GA}$
13. $\overleftrightarrow{GB}$ and $\overleftrightarrow{DE}$
14. $\overleftrightarrow{FH}$ and $\overleftrightarrow{BD}$

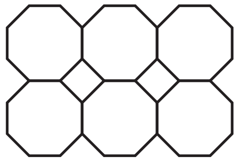
NAME ______________________________________ DATE ______________ PERIOD _____

6-3 Enrichment

7MG3.2

M.C. Escher

Maurits Cornelis Escher (1898-1972) was a Dutch graphic and mathematical artist. Some of his most famous pieces used **tessellations**, or repeated tiling of one or more shapes. His designs range from artfully simple to extremely intricate. You can see examples of his work at www.mcescher.com.

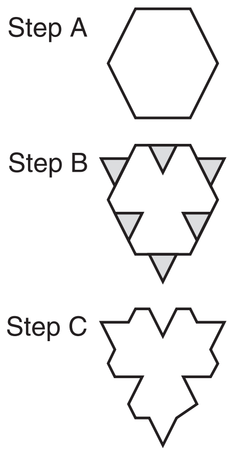


A regular polygon will tessellate a plane if the measure of one of its interior angles is a factor of 360°. Other combinations of polygons tessellate if the sum of the measures of the adjoining angles equals 360°. The tessellation at the right is made of regular octagons and squares. At any vertex the sum of the measures of the angles is 90° + 135° + 135° or 360°.

1. Make a list of all regular polygons that will tessellate.

2. Explain why you know there are no other regular polygons that will tessellate.

For Steps A–E, you will create your own tessellation on a separate piece of paper.



Step A Start with a polygon that will tessellate. Trace it, and cut it out. Grid paper or isometric dot paper may help you accurately draw your shape.

Step B Cut a portion of the figure on one side, slide it to the *opposite* side, and tape it on. (This was done three times in the example at right.)

Step C Use the modified shape as a tracing template. Trace the template on another sheet of paper.

Step D Slide, reflect, and/or rotate the shape so that it fits with your first tracing. Trace the template where it fits with the previous tracing. Repeat the process to cover the page.

Step E Color each polygon in the tessellation. Escher often decorated the shapes so that they resembled objects or animals.

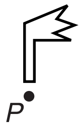
6-4 Enrichment

7MG3.2

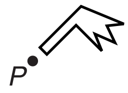
Order of Rotational Symmetry

The two examples below show how to create designs with rotational symmetry. In both cases, point *P* is the center of the rotation.

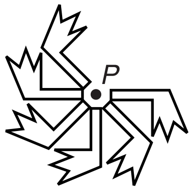
Notice that the *order of rotational symmetry* is the number of times the original figure maps onto itself in a complete rotation. The first step in making the design is to divide 360° by the order of rotation.



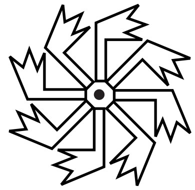
360° ÷ 8 = 45°



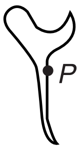
Rotate 45° and trace



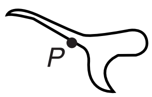
Repeat 6 times.



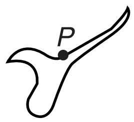
Rotational Symmetry Order 8

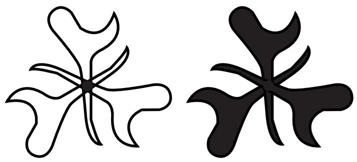


360° ÷ 3 = 120°



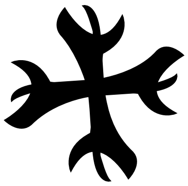
Rotate 120° Repeat 1 time. and trace

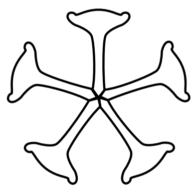


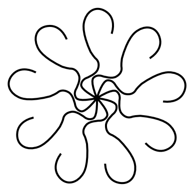


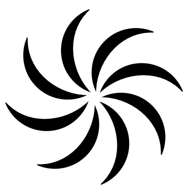
Rotational Symmetry Order 3

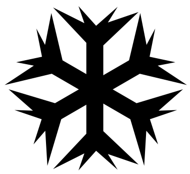
Write the order of rotational symmetry for each design.

1. 

2. 

3. 

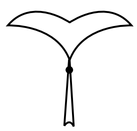
4. 

5. 

Use a shape like the one shown to create a design with the given center and order of rotational symmetry.

6. Order 8

7. Order 3



8. Order 5

NAME ______________________________ DATE ____________ PERIOD _____

6-5 Enrichment

7MG3.2

Ambigrams

During the 1970s, a graphic artist named John Langdon began to experiment with a special way to write words as **ambigrams**, or writing that could be read more than one way. One example of his work is the word "wordplay," displayed at the right. Read the word, and then try turning this page upside-down to read it again.

1. What kind of symmetry is demonstrated by the *wordplay* image? Explain and choose one letter as an example.

There are several types of ambigrams, including those that have *rotational symmetry* and those that have *mirror symmetry* or *reflection symmetry*. Identify the kind of symmetry shown by each figure.

2. 

3. 

4. 

Typographers who create ambigrams study the alphabet carefully to identify letters that have rotational and mirror symmetry. Notice that a letter and its image can be useful in an ambigram both if the letter and its image represent the same letter, and if the letter and its image represent different letters.

5. Show at least three more examples in each category of letters that can be helpful in creating ambigrams. Remember that you can vary how you write letters.

	Same Letter	Different Letter
Rotation	S, S	u, n
Reflection	Y, Y	p, q

6. Write an ambigram of the word "bed" that uses mirror symmetry.

bed

7. Write an ambigram of the word "unfun" that uses rotational symmetry.

unfun

8. Write an ambigram of a word of your choice using mirror symmetry.

9. Write an ambigram of a word of your choice using rotational symmetry.

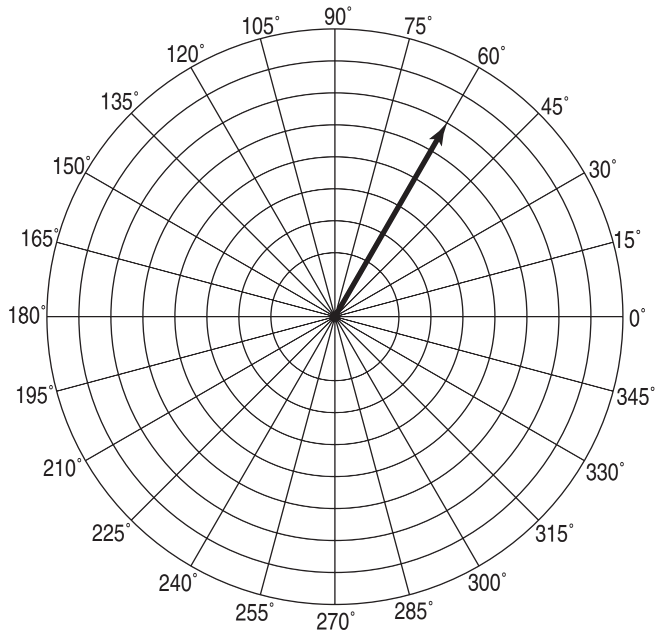
6-6 Enrichment

7MG3.2

Reflecting Vectors

Navigators of aircraft and ocean vessels use **vectors** to communicate *distance* or *speed* and *direction*. On a Cartesian Coordinate Plane, a point is located by giving its horizontal and vertical direction. The coordinates of a vector give a distance in units from a central point and an angle (direction). Vectors are often plotted on a circular grid called a *polar* grid. A polar grid allows a point to be located by its distance and direction from one point, or pole.

The vector (6, 60º) is drawn on the figure. The arrow is 6 units long and measures 60º counterclockwise from 0º. The distance is 6 units from center. The direction is 60º counterclockwise from 0º.



Draw each vector and label it with its capital letter name.

1. **A:** (8, 120º)
2. **B:** (4, 300º)
3. **C:** The reflection of **B** over the 0º vector.
4. **D:** The reflection of **A** over the 180º vector.
5. **E:** (6, 345º)
6. **F:** (8, 0º)

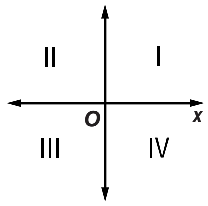
6-7 Enrichment

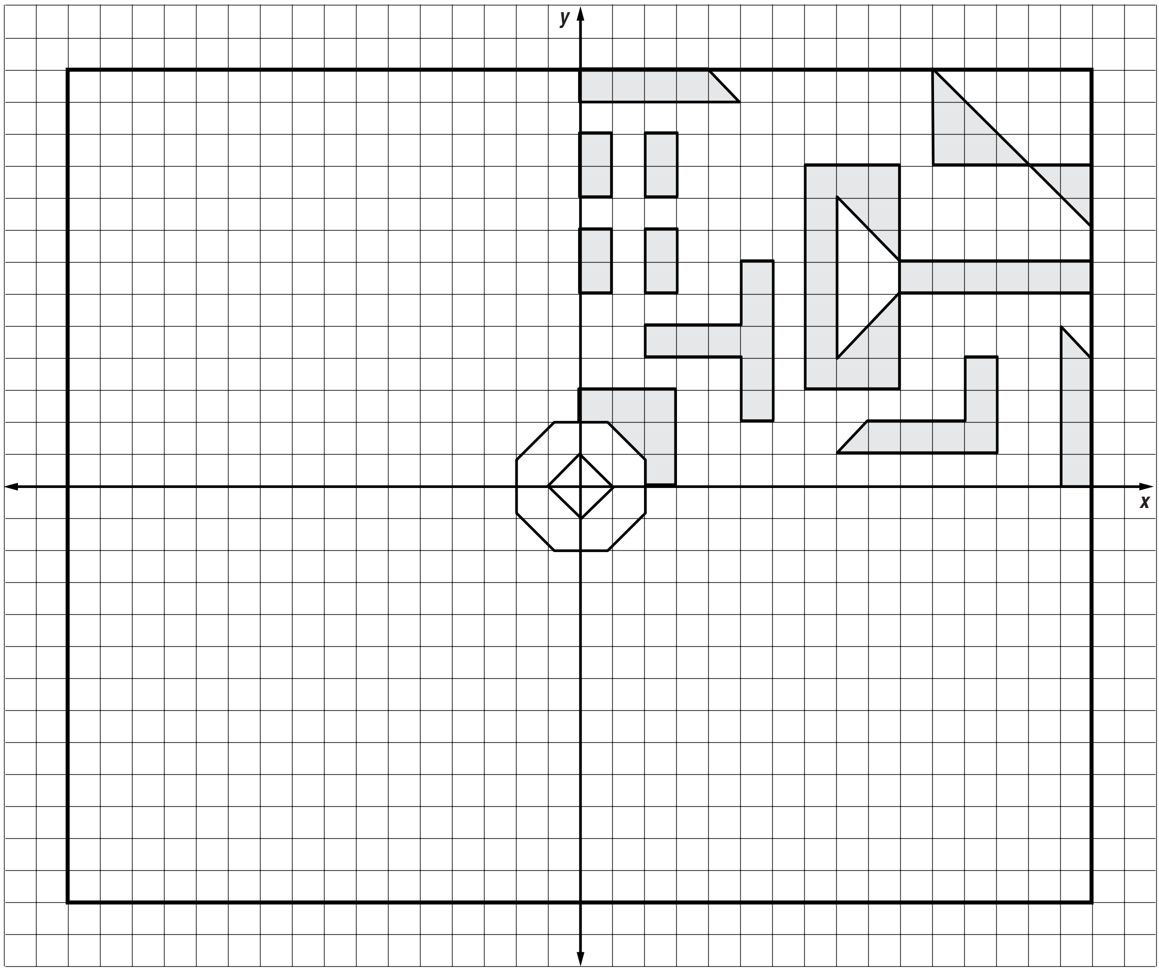
7MG3.2

Reflections

Reflections can create many interesting patterns and shapes. Reflect the pattern in Quadrant I over the vertical y-axis into Quadrant II. Then reflect the pattern in Quadrants I and II over the horizontal x-axis into Quadrants III and IV.

Quadrants





NAME ______________________________ DATE ____________ PERIOD _____

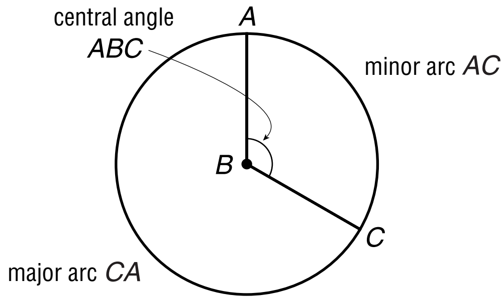
7-1 Enrichment

7MG3.1

Circumference and Area of Circles

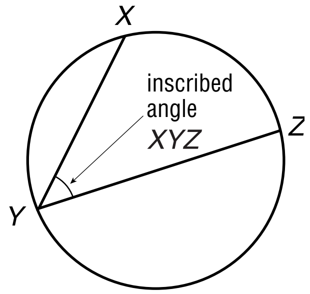
Angles and Arcs

A **central angle** is an angle that intersects a circle in two points and has its vertex at the center of the circle. It separates a circle into a **major arc** and a **minor arc**.



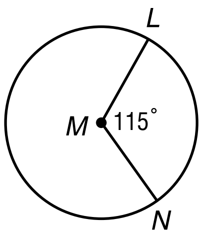
- The degree measure of a minor arc is the degree measure of the central angle. In circle B,
  $m = m\angle ABC$.
- The degree measure of a major arc is 360 minus the degree measure of the central angle. In circle B,
  $m = 360° - m\angle ABC$.

An **inscribed angle** has its vertex on the circle and sides that contain chords. The measure of an inscribed angle equals one-half the measure of its intercepted arc. In the circle shown at the right, $m\angle XYZ = \frac{1}{2}m$. Thus, $m = 2 \cdot m\angle XYZ$.



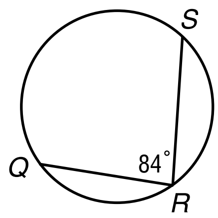
Find the measure of each arc.

1.



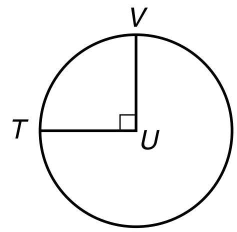
minor arc *LN*

2.



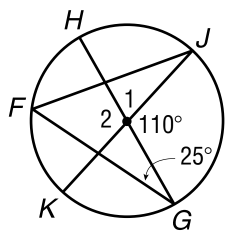
minor arc *QS*

3.



major arc *VT*

Refer to the diagram at the right. Find the measure of each of the following angles or arcs.



4. minor arc *JG*

5. $\angle 1$

6. major arc *GJ*

7. $\angle 2$

8. minor arc *KH*

9. minor arc *GK*

10. minor arc *FH*

11. $\angle FJK$

12. $\angle JFG$

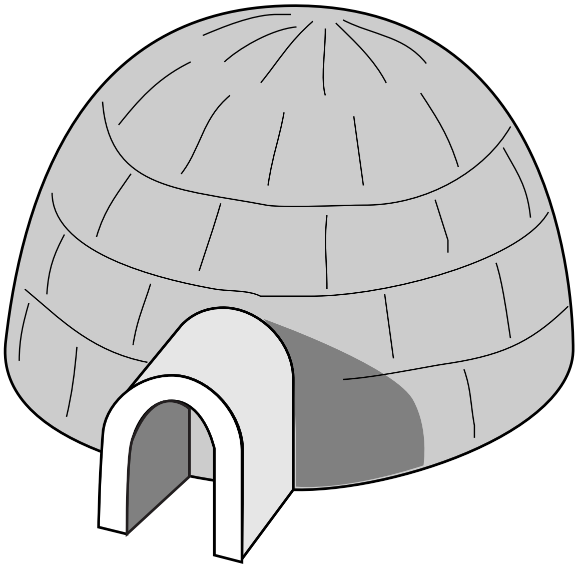
13. arc *HJG*

7-3 Enrichment

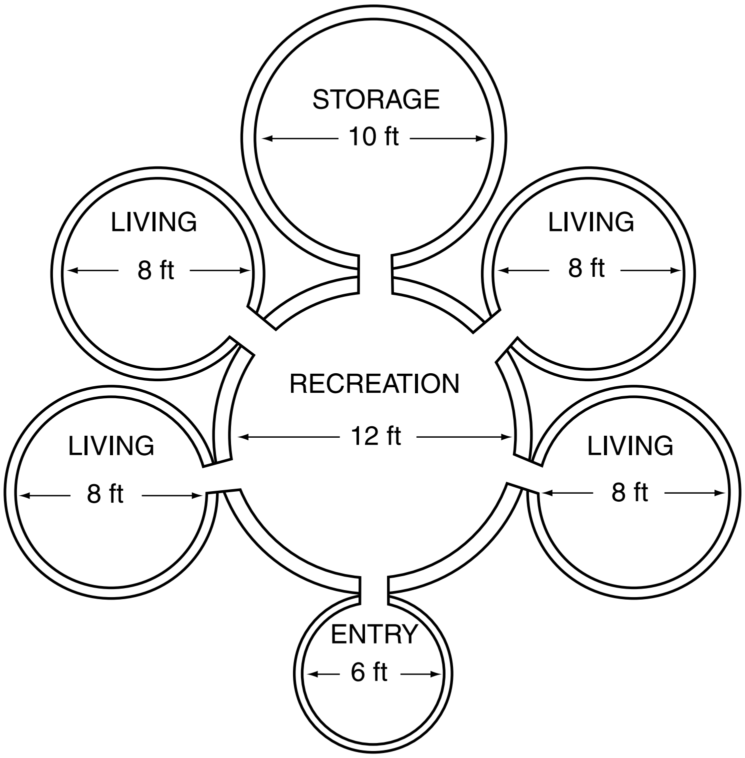
7MG2.1, 7MG2.2

Inuit Architecture

The Inuit are a Native American people who live primarily in the arctic regions of Alaska, Canada, Siberia, and Greenland. The Inuit word *iglu* means "winter house," and it originally referred to any permanent structure used for shelter in the winter months. In the nineteenth century, however, the term came to mean a domed structure built of snow blocks, as shown in the figure at the right.



An iglu could shelter a family of five or six people. Sometimes several families built a *cluster* of iglus that were connected by passageways and shared storage and recreation chambers. The figure below is a drawing of such a cluster. Use the drawing to answer each of the following questions. When appropriate, round answers to the nearest whole number.



1. What is the circumference of the entry chamber?

2. What is the circumference of one of the living chambers?

3. Estimate the distance from the front of the entry chamber to the back of the storage chamber.

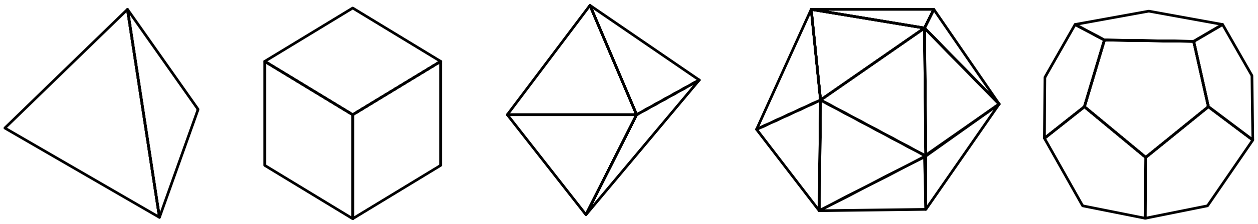
4. An iglu is a *hemisphere*, or half a sphere. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. Estimate the volume of the storage chamber.

7-4 Enrichment

7MG3.5

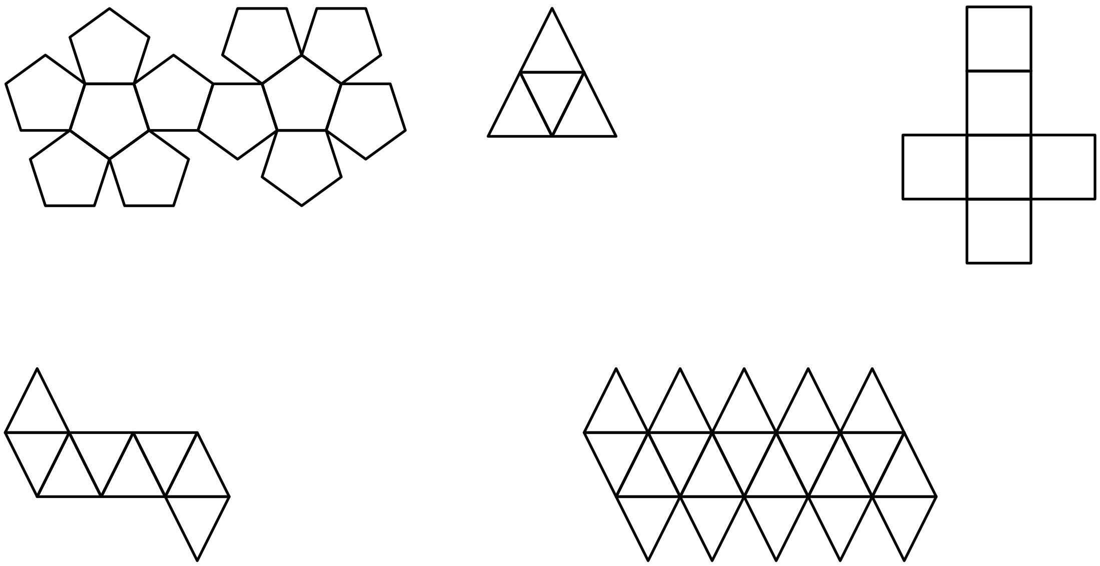
The Five Platonic Solids

There are only five regular convex solids. They are called the *Platonic Solids* and are shown here.



tetrahedron hexahedron octahedron icosahedron dodecahedron

1. Write the name of each Platonic Solid under its net.



2. Complete this chart for the Platonic Solids.

Solid	Tetrahedron	Hexahedron	Octahedron	Icosahedron	Dodecahedron
Number of Faces					
Number of Edges					
Number of Vertices					

3. Write an equation relating the number of faces, edges, and vertices of the Platonic Solids. This equation is called Euler's Formula and is true for all simple polyhedra.

NAME ______________________ DATE ____________ PERIOD _____

7-5 Enrichment

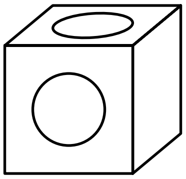
7MG3.5

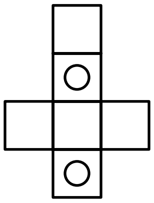
Lesson 7-5

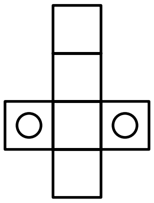
Puzzling Patterns

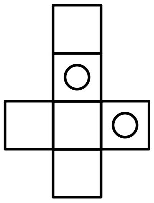
In these visual puzzles, the challenge is to choose the one pattern that could be folded up into the box shown. You are not allowed to make any extra cuts in the patterns. The trick is that the six faces of the box must be arranged in the correct order.

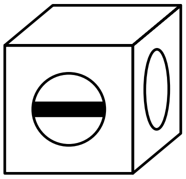
Circle the letter of the pattern that could be used to make each box.

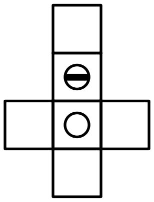
1. 

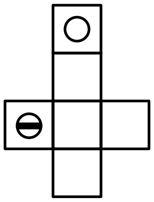
A. 

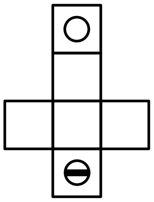
B. 

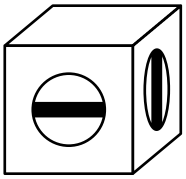
C. 

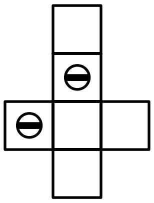
2. 

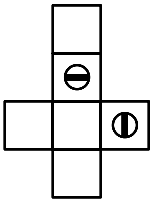
A. 

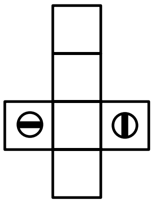
B. 

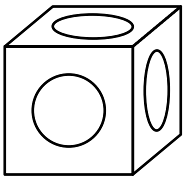
C. 

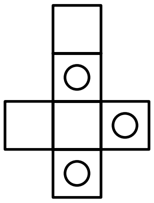
3. 

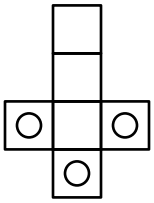
A. 

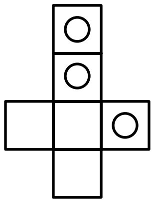
B. 

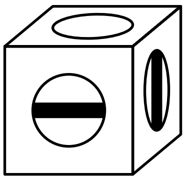
C. 

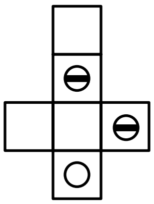
4. 

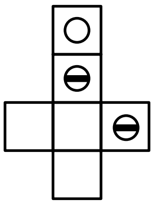
A. 

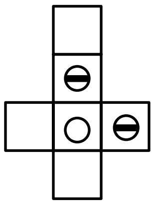
B. 

C. 

5. 

A. 

B. 

C. 

NAME ______________________ DATE __________ PERIOD _____

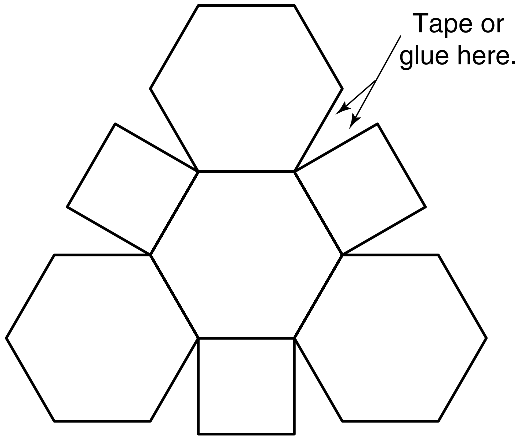
7-6 Enrichment

7MG2.1, 7MG3.5

Two Truncated Solids

To create a truncated solid, you could start with an ordinary solid and then cut off the corners. Another way to make such a shape is to use the patterns on this page.

The Truncated Octahedron

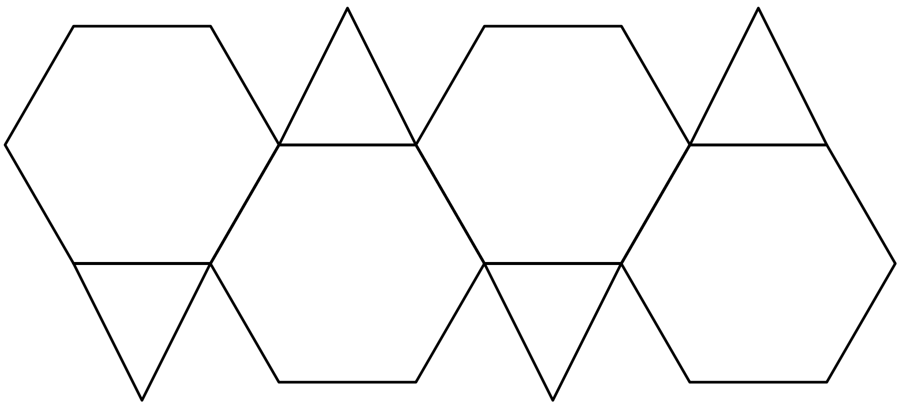


1. Two copies of the pattern at the right can be used to make a *truncated octahedron*, a solid with 6 square faces and 8 regular hexagonal faces.

   Each pattern makes half of the truncated octahedron. Attach adjacent faces using glue or tape to make a cup-shaped figure.

The Truncated Tetrahedron

2. The pattern below will make a *truncated tetrahedron*, a solid with 8 polygonal faces:
   4 hexagons and 4 equilateral triangles.



Solve.

3. Find the surface area of the truncated octahedron if each polygon in the pattern has sides of 3 inches.

4. Find the surface area of the truncated tetrahedron if each polygon in the pattern has sides of 3 inches.

Area Formulas for Regular Polygons
(s is the length of one side)

triangle	$A = \frac{s^2}{4}\sqrt{3}$
hexagon	$A = \frac{3s^2}{2}\sqrt{3}$
octagon	$A = 2s^2(\sqrt{2} + 1)$

7-7 Enrichment

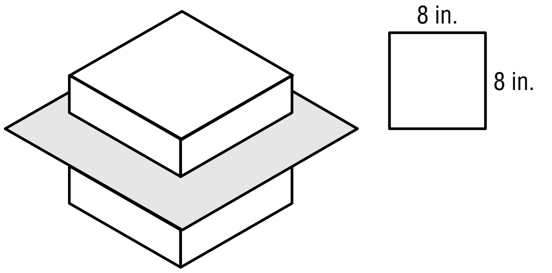
7MG2.1

Sliced Solids

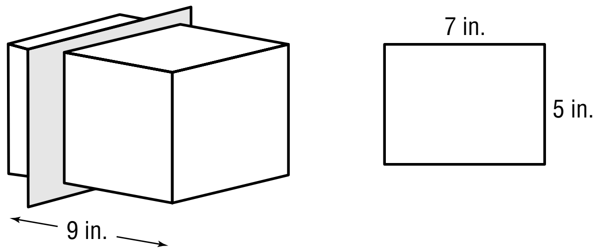
In the diagrams on this page, a plane slices through a solid figure. The intersection of the plane with the solid is called a *cross section*. The drawings for each problem show a sliced solid and the dimensions of the resulting cross section.

Find the surface areas of the two solids that result from the slice. Round to the nearest tenth.

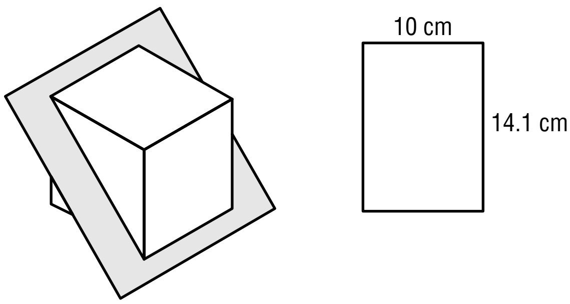
1. One-fourth of the cube is sliced off the top.



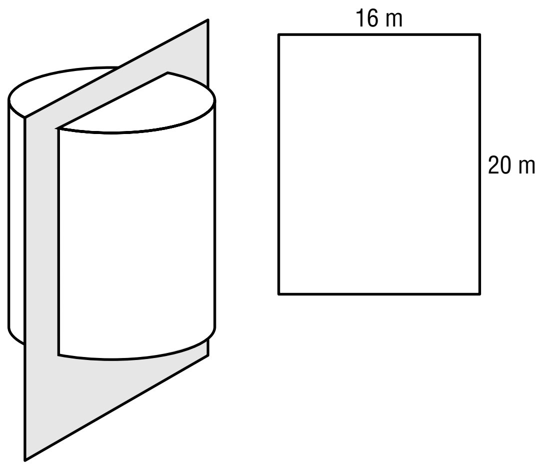
2. One-third of the prism is sliced off the back.



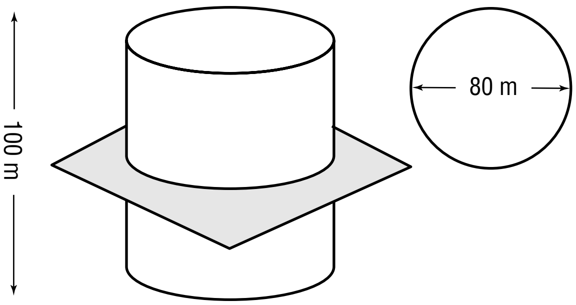
3. The cube is sliced in half.



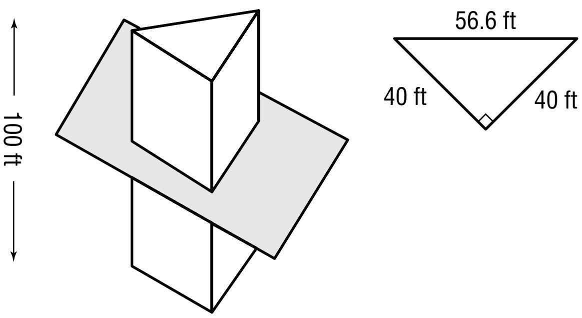
4. The cylinder is sliced in half.



5. The cylinder is sliced in half.



6. The prism is sliced in half.



Lesson 7-7

7-8 Enrichment

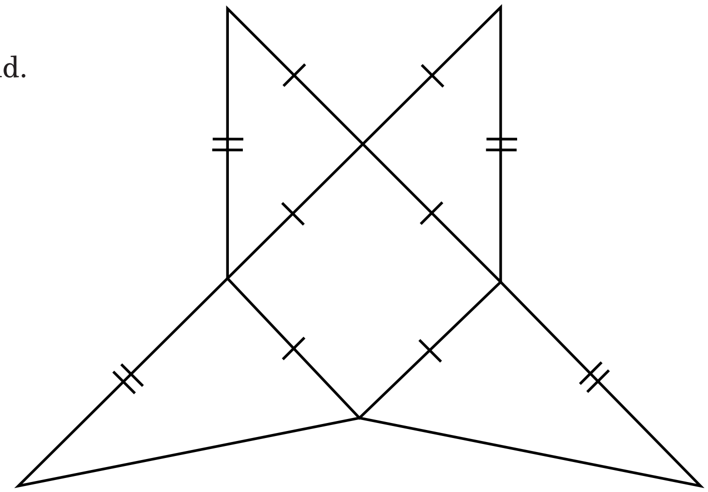
7MG2.1, 7MG3.5

Two Three-Dimensional Puzzles

In the nets on this page, segments of equal length are marked in the same way.

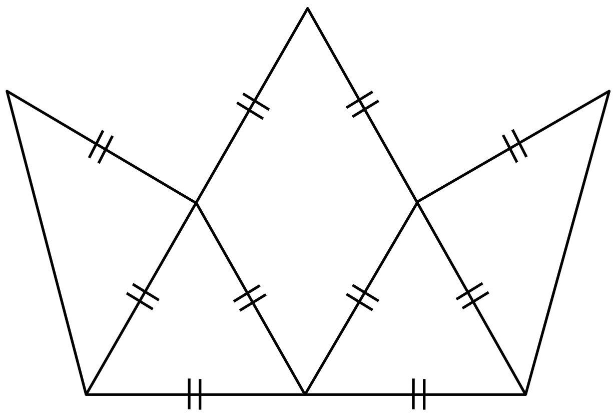
1. Make three copies of this pattern. Use 2 inches for each side of the central square.

Fold each pattern to make a pyramid. Put the three pyramids together to make a cube. Make a sketch of the result.



2. Make four copies of this pattern. Use 6 inches for the base of the figure.

Fold each pattern to make a solid. Put the four solids together to make a regular tetrahedron. Make a sketch of the result.



Solve.

3. Find the surface area of the cube in Exercise 1.

4. Find the volume of each of the three pyramids in Exercise 1.

5. Find the surface area of the tetrahedron in Exercise 2. You will need to measure an altitude for one of the faces.

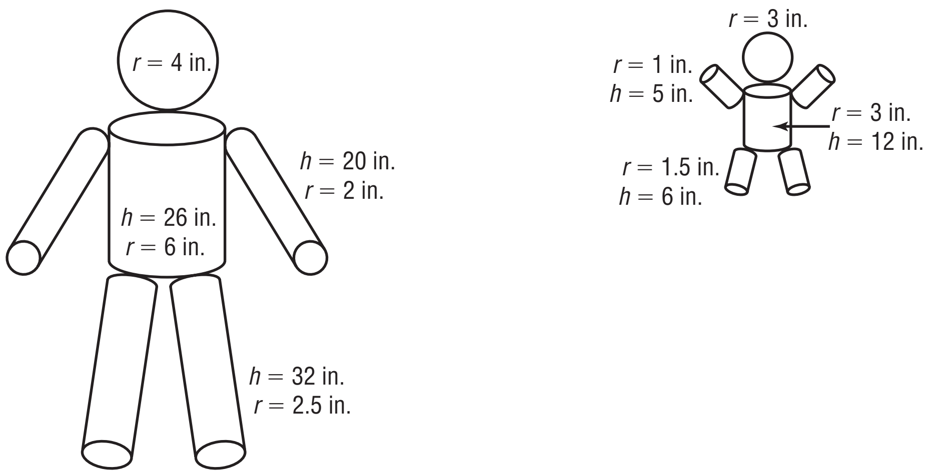
7-9 Enrichment

7MG2.1

Ratios of Surface Area

Parents often keep their babies bundled up with hats, blankets and extra layers of clothing to avoid a dangerous drop in body temperature due to heat loss. Babies are small and they are more likely to suffer problems in very cold temperatures than adults. To understand this better, medical researchers study the relationship between body surface area and body weight. For simplicity, we will approximate the surface area of infants and adults using a sphere for the head and cylinders for the legs, arms, and torso.

1. Consider an infant who weighs 18 pounds and an adult who weighs 170 pounds. Suppose the arms, legs, torso, and head can be approximated with the given solids and dimensions below. Find the total surface area of both the infant and adult models. Use 3.14 as an approximation for π.



2. What is the ratio of the total surface area to body weight of both the infant and the adult? How are the two ratios related?

3. Based on your findings, why do you think it is important that parents bundle up their babies when they are out in the cold? Write two or three sentences to explain your reasoning.

8-1 Enrichment

7AF1.3

Olga Taussky-Todd

Olga Taussky-Todd (1906–1995) had a rich and varied career as a research mathematician, mathematics professor, and author and editor of mathematical texts. Born in eastern Europe, she lived and worked in Austria, Germany, England, and the United States. She served as the consultant in mathematics for the National Bureau of Standards in Washington, D.C., for ten years. In 1957, she became the first woman appointed to the mathematics department of the California Institute of Technology.

Dr. Taussky-Todd made contributions in many areas of mathematics and physics. The exercises below will help you learn some more about her life.

Find each product. Circle the correct solution. The phrase following the solution will complete the statement correctly.

1. $(x + 3)(x + 7)$ Her paper on sums of squares won the Ford Prize of the Mathematical Association of America in __?__

   $x^2 + 10x + 21$: 1971

   $x^2 + 21x$: 1981

2. $(x + 4)(2x + 1)$ In 1978, she was elected __?__ of the Austrian Academy of Sciences.

   $3x + 5$: Correspondent

   $2x^2 + 9x + 4$: Corresponding Member

3. $(2x + 1)(x + 7)$ In 1978, the government of Austria awarded her the __?__.

   $2x^2 + 15x + 7$: Cross of Honor in Science and Arts, First Class

   $3x^2 + 8x + 7$: Purple Cross

4. $(x + 6)(x + 6)$ In 1988, the University of Southern California awarded her an __?__.

   $2x + 12$: honorary science degree

   $x^2 + 12x + 36$: honorary Doctor of Science degree

8-2 Enrichment

7AF4.1

Rational Numbers

Systems of Equations

Sometimes it takes more than one equation to solve a problem. A group of such equations is called a **system of simultaneous equations**. Here is one system of equations.

$$y = x + 2$$

$$3x - 5 = 16$$

The solution of this system must be a pair of numbers x and y such that the numbers make *both* equations true.

To solve a system of this type, solve the equation with one variable. Then substitute that answer into the second equation and solve for the other variable. For the system above, $x = 7$ and $y = 9$.

Example **Solve the system of equations.**
$\boldsymbol{c = d - 2}$
$\boldsymbol{3d - 1 = 17}$

Solve $3d - 1 = 17$ for d.

$3d - 1 = 17$	Write the equation.
$3d - 1 + 1 = 17 + 1$	Add 1 to both sides.
$3d = 18$	Simplify.
$\frac{3d}{3} = \frac{18}{3}$	Divide both sides by 3.
$d = 6$	Simplify.

Substitute 6 for d in $c = d - 2$.

$c = 6 - 2$

$c = 4$

The solution is $c = 4$ and $d = 6$.

Solve each system of equations.

1. $40 - 2t = 10$
$3t - s = 35$

2. $4a + 2b = 22$
$25 = 11a - 8$

3. $82.5 = 1.5s$
$d = 3s + 35$

4. $\frac{m}{5} + 1.5 = 2$
$7m + n = 17.5$

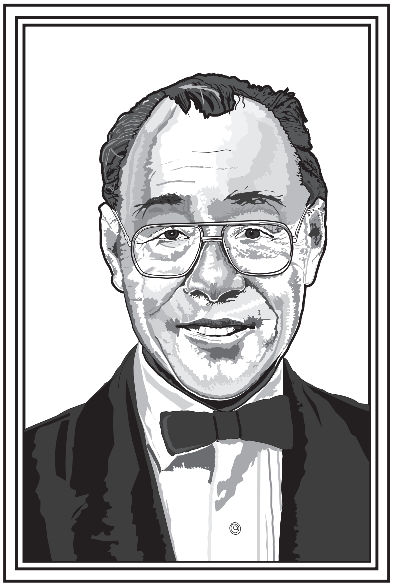
5. $6x + \frac{y}{2} = 43$
$22 + 3x = 43$

6. $\frac{c}{5} + p = 4$
$20 = 4p - 3$

8-3 Enrichment

7AF4.1

An Wang



An Wang (1920–1990) was an Asian American who became one of the pioneers of the computer industry in the United States. In 1948, he invented a magnetic-pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word-processing systems.

In 1988, Wang received a special honor for his contributions to the advancement of the computer industry. To find out what the honor was, solve each equation. If the solution appears at the bottom of this page, write the variable on the line directly above the solution *each time it appears*. If you have solved the equations correctly, the variables will spell out the honor.

1. $3V = -24$

2. $\frac{R}{-3} = 2$

3. $-\frac{3}{8}M = \frac{3}{4}$

4. $\frac{S}{4} = -2.5$

5. $H - 1.3 = 4.7$

6. $-3.2D = -16$

7. $\frac{2}{3}F + 6.5 = 4.5$

8. $\frac{O}{1.5} = -8$

9. $\frac{W}{1.4} - 3.5 = 1.5$

10. $-\frac{1}{4}T = -2.5$

11. $-\frac{2}{5}I = 1\frac{3}{5}$

12. $\frac{5}{8}A - \frac{1}{2} = \frac{3}{4}$

13. $\frac{4}{3}E - \frac{1}{6} = 3\frac{5}{6}$

14. $\frac{N}{-1.6} = -5$

15. $1.25 = 3.5 - \frac{9}{4}L$

___ ___   ___ ___ ___   ___ ___ ___ ___ ___
6 3   7 2 −10   8 2 −2 3 5

___ ___   ___ ___ ___   ___ ___ ___ ___ ___ ___ ___ ___
10 −12   10 6 3   8 2 10 −4 −12 8 2 1

___ ___ ___ ___ ___ ___ ___ ___ ___
−4 8 −8 3 8 10 −12 −6 −10

___ ___ ___ ___   ___ ___   ___ ___ ___ ___.
6 2 1 1   −12 −3   −3 2 −2 3

Lesson 8–3

8-4 Enrichment

7AF1.1

Famous Scientific Equations

Many important laws or principles in physical science are described by equations. You may have already studied some of the equations on this page, or you may learn about them in future science classes.

Match each statement with its equation. Then write the variables for the quantities listed.

SCIENTIFIC PRINCIPLE

1. **Law of the Lever**
   A lever will balance if the mass of object 1 times its distance from the fulcrum equals the mass of object 2 times its distance from the fulcrum.

2. **Newton's Second Law of Motion**
   The acceleration on an object equals the applied force divided by the object's mass.

3. **Ohm's Law**
   The amount of current in an electrical circuit equals the voltage divided by the resistance.

4. **Boyle's Law**
   For a gas at a constant temperature, the product of the pressure and the volume remains constant.

5. **Law of Universal Gravitation**
   To compute the force of gravity between two objects, multiply their masses by the gravitational constant and then divide by the square of the distance between the objects.

EQUATION AND VARIABLES

_______ $P_1V_1 = P_2V_2$

pressure at first time =

volume at first time =

pressure at second time =

volume at second time =

_______ $I = \frac{V}{R}$

voltage =

resistance =

current =

_______ $A = \frac{F}{m}$

applied force =

mass of object =

acceleration =

_______ $F = G\frac{m_1m_2}{d^2}$

mass of first object =

mass of second object =

distance between objects =

gravitational constant =

force of gravity =

_______ $m_1d_1 = m_2d_2$

mass of first object =

distance of first object from fulcrum =

mass of second object =

distance of second object from fulcrum =

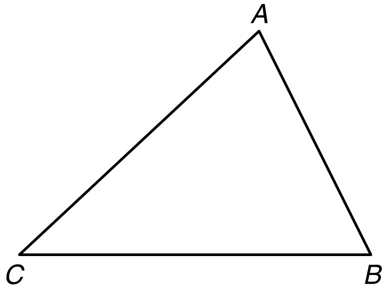
NAME ______________________________ DATE ______________ PERIOD _____

8-6 Enrichment

7AF4.1

A Triangle Inequality

A well-known inequality in geometry relates the measures of the three sides of any triangle. Here are two different statements of this inequality.



> **The Triangle Inequality**
>
> The sum of the measures of any two sides of a triangle is greater than the measure of the third side.
>
> In any $\triangle ABC$, $AB + BC > AC$.

Solve.

1. Consider three line segments with the measures 3, 4, and 8. Write a statement using the < symbol to show that these three segments do not satisfy the triangle inequality.

2. Try to draw a triangle using the three segments in Problem 1. Describe what happens.

Can the three measures be used to make a triangle? Write *yes* or *no*.

3. 6 m, 2 m, 7 m

4. 5 cm, 8 cm, 11 cm

5. 5 in., 14 in., 7 in.

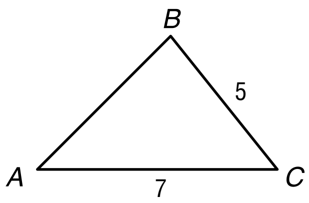
6. 9 cm, 5 cm, 4 cm

7. 10 yd, 10 yd, 10 yd

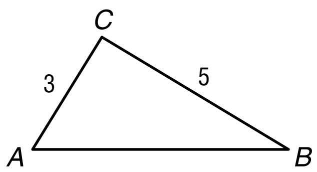
8. 4 ft, 10 ft, 5 ft

For each triangle, describe the possible measures of side AB.

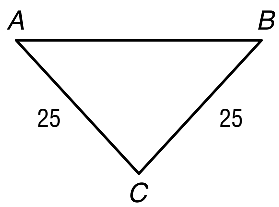
9.



10.



11.



8-7 Enrichment

IA1.0

Closure

A set of numbers and an operation may possess a property called *closure.* If the result of the operation on the numbers of the set is always a member of the original set, the set is said to be **closed** under that operation.

For example, the set of whole numbers is not closed under subtraction because the result of 3 − 6 is not a whole number.

The set of fractions is closed under multiplication. The product of any two fractions is always another member of the set of fractions.

Is each set closed under addition? Write *yes* or *no.* If your answer is *no,* give an example.

1. even whole numbers

2. odd whole numbersno;

3. the factors of 24

4. the multiples of 10

5. all fractions

6. fractions less than 1

Is each set closed under division? Write *yes* or *no.* If your answer is *no,* give an example.

7. all whole numbers

8. all fractions

9. the whole numbers excluding 0

10. the fractions excluding 0

Is the set of positive fractions less than 1 closed under each operation? Write *yes* or *no.* If your answer is *no,* give an example.

11. addition

12. subtraction

13. multiplication

14. division

Is the set of fractions greater than 1 closed under each operation? Write *yes* or *no.* If your answer is *no,* give an example.

15. addition

16. subtraction

17. multiplication

18. division

8-8 Enrichment

7AF4.1

Al-Khwarizmi

In the ninth century A.D., the Arabian mathematician Muhammed ibn-Musa al-Khwarizmi wrote a book called *Hisab al-jabr w' al muqabalah.* In this book, he detailed much of what was known at that time about the process of solving equations. Although some of the methods he described had been used in the Middle East for nearly 2,500 years, it was through his book that the methods came to Europe. The book became known simply as *Al-jabr,* and it is from this phrase that we get the word "algebra."

One method described in al-Khwarizmi's writings is called the *rule of false position.* To solve an equation by this method, first choose any number as a trial value of the variable. Substitute this number for the variable in the equation, then make a correction based on the result. At the right, there are two examples of how this method can be used.

$c + 9 = 47$ trial value: 31 If $c = 31$, then $c + 9 = 40$. $40 \boxed{+ 7} = 47$ So, $c = 31 \boxed{+ 7}$ or 38.
$3y = 84$ trial value: 7 If $y = 7$, then $3y = 21$. $21 \boxed{\times 4} = 84$ So, $y = 7 \boxed{\times 4}$ or 28.

Solve each equation by the rule of false position, using the given trial value for the variable.

1. $m + 58 = 125$
trial value: 42

2. $4k = 60$
trial value: 5

3. $r - 34 = 79$
trial value: 100

4. $a \div 5 = 32$
trial value: 20

Solve each equation by the rule of false position. This time, make your own choice for the trial value of the variable.

5. $y + 16 = 51$

6. $3t = 48$

7. $d - 42 = 88$

8. $z \div 4 = 18$

9. Write a sentence or two relating this method to inequalities.

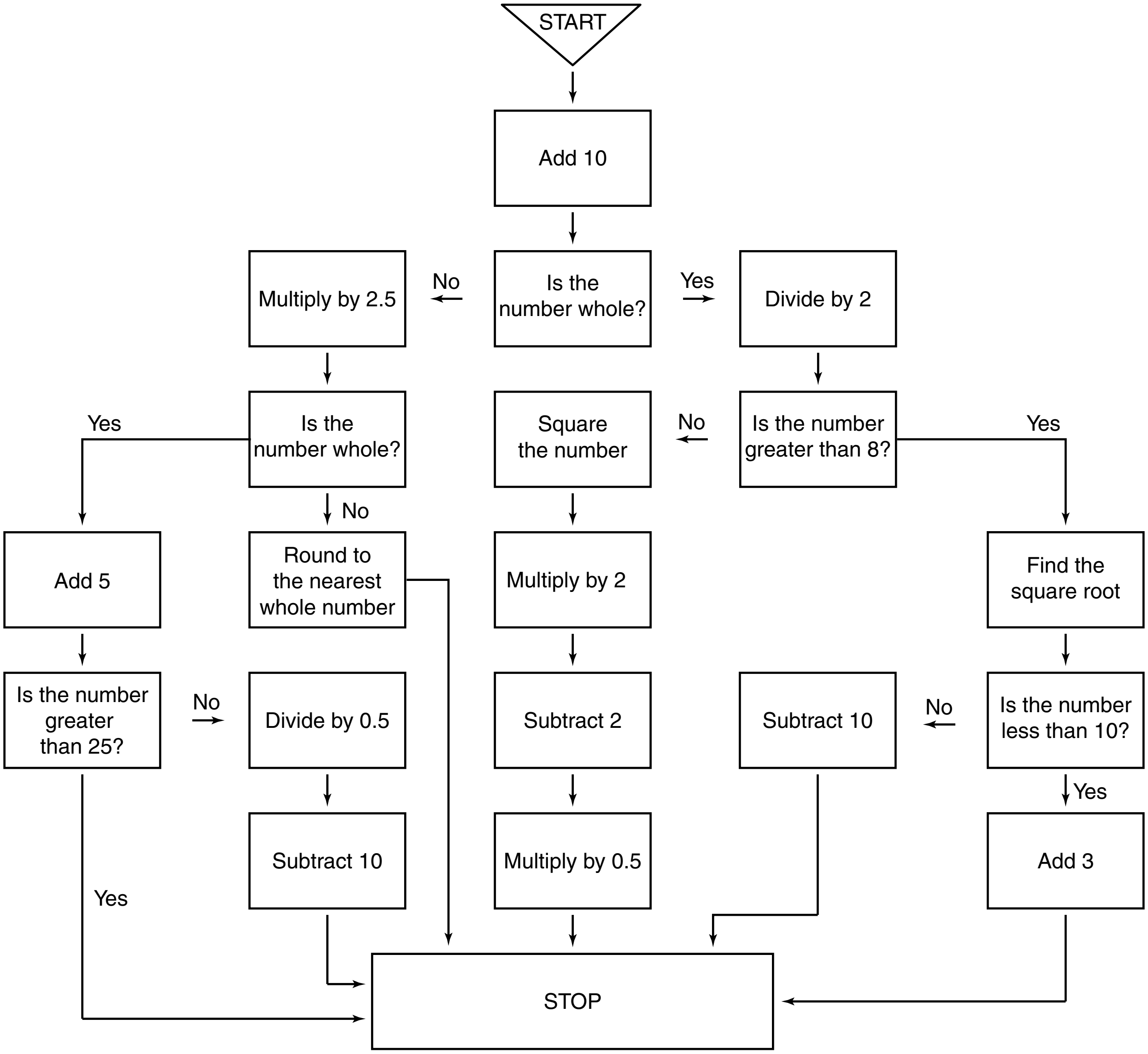
9-1 Enrichment

IA13.0

Going with the Flow

A mathematical function results when one or more operations are performed on a number. A function flowchart and ten numbers are given below. Input each number into the flowchart at the place marked START, then follow the number through the flowchart. When you reach the place marked STOP, record the output number.

Input n	4	790	−1.2	1	0.2	−6.8	278	88	−5.2	0
Output $f(n)$										



9-2 Enrichment

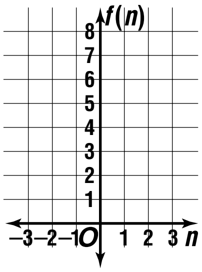
7AF3.1

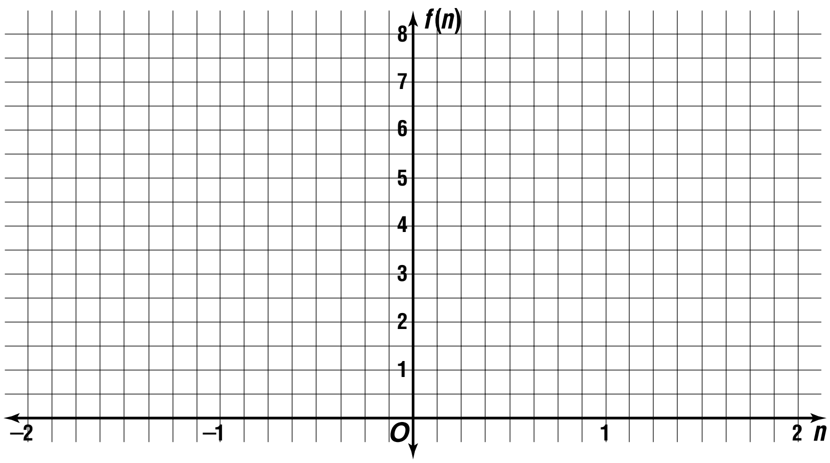
Graphing Functions

Depending on the scale of a graph, the graphed shape of a function can be made to appear very different from one graph to another.

Given the function $f(n) = 2n^2$, find the values of $f(n)$ for each value in the table. Write the ordered pairs, then draw the graph of the function on each grid.

n	$2n^2$	$f(n)$	$(n, f(n))$
-2	$2(-2)^2$	8	$(-2, 8)$
$-\frac{3}{2}$			
-1			
$-\frac{1}{2}$			
0			
$\frac{1}{2}$			
1			
$\frac{3}{2}$			
2			





1. Examine each graph of the function $f(n) = 2n^2$. What do you notice about the graphs?

2. Explain why the graphs are different.

Lesson 9–2

9-3 Enrichment

7AF3.3

Olga Taussky-Todd

Olga Taussky-Todd (1906–1995) had a rich and varied career as a research mathematician, mathematics professor, and author and editor of mathematical texts. Born in eastern Europe, she lived and worked in Austria, Germany, England, and the United States. She served as the consultant in mathematics for the National Bureau of Standards in Washington, D.C., for ten years. In 1957, she became the first woman appointed to the mathematics department of the California Institute of Technology.

Dr. Taussky-Todd made contributions in many areas of mathematics and physics. The exercises below will help you learn some more about her life.

Find the slope of the line that passes through each pair of points. The phrase following the slope will complete the statement correctly.

1. $A(0, 4)$, $B(2, 10)$

Her paper on sums of squares won the Ford Prize of the Mathematical Association of America in __?__.

3: 1971

6: 1981

2. $C(-2, 1)$, $D(3, -9)$

In 1975, she was elected __?__ of the Austrian Academy of Sciences.

$-\frac{1}{2}$: Correspondent

-2: Corresponding Member

3. $E(-1, 3)$, $F(8, 4)$

In 1978, the government of Austria awarded her the __?__.

$\frac{1}{9}$: Cross of Honor in Science and Arts, First Class

-9: Purple Cross

4. $G(8, -3)$, $H(5, -3)$

In 1988, the University of Southern California awarded her an __?__.

3: honorary science degree

0: honorary Doctor of Science degree

NAME ____ DATE ______________ PERIOD _____

9-4 Enrichment

7AF3.4, 7AF4.2

Finding a Formula and Inverse Variation

If you know the general form of direct variation equations and the value of the constant of variation, you can write an equation that describes the data points of a direct variation. Look at the table at the right which shows a relationship where *c* varies as *r*.

r	c
1	6.28
2	12.56
3	18.84
4	25.12
5	31.4

1. What is the constant of variation?

2. What familiar relationship do you recognize in the direct variation?

3. Write an equation for finding the circumference of a circle with radius, *r*.

4. If *r* is doubled, what happens to the value of *c*?

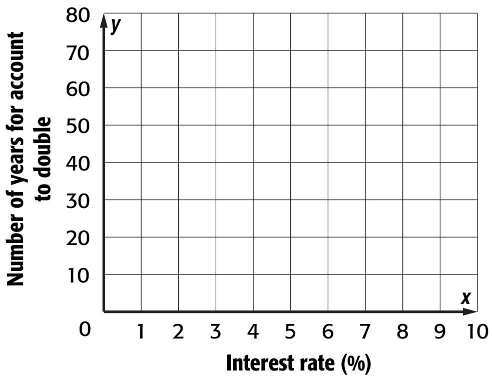
Inverse Variation For the variables x and y, with any constant k, y is said to vary inversely (or indirectly) as x when $y = \frac{k}{x}$.

For Exercises 5–8, refer to the following information.

INVESTMENTS Economists use inverse variation to approximate how fast the balance of an account will double when it is invested at a given compound interest rate. The number of years *y* it takes for an investment to double varies inversely as the annual interest rate *r*, expressed as a percent (not a decimal value).

5. If you invest $1,000 at a 6% compound interest rate, it will take 12 years to double your money. Find the constant of variation, *k*, and write the equation for the variation.

6. Find and graph at least six data points within the domain $0 < r \leq 10$. What do you notice about the shape of the graph?



7. If *r* is doubled, what happens to the value of *y*?

8. Why do you think investors often refer to this relationship as "The rule of 72?"

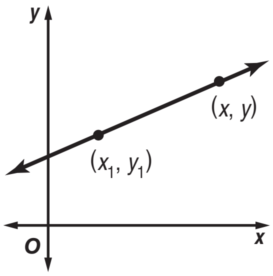
9-5 Enrichment

7AF3.3

Point-Slope Form

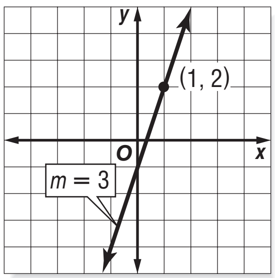
You have learned to write the equation of a line given the slope and the y-intercept. You can use what you know to write the equation of a line given the slope and any point on the line. To do so, use the **point-slope form** of a linear equation.

The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

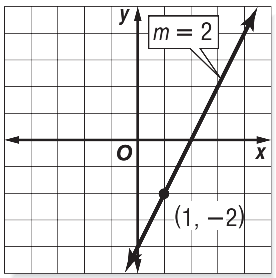


Write the point-slope form of an equation for the line that passes through each point with the given slope.

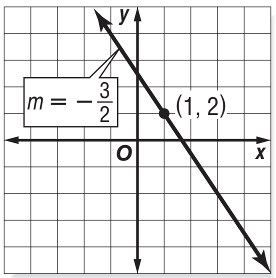
1.



2.



3.



4. $(6, 2), m = 3$

5. $(4, -1), m = 5$

6. $(3, 1), m = -2$

7. $(-4, 1), m = 1$

8. $(-9, -2), m = \frac{1}{2}$

9. $(-5, -6), m = -\frac{3}{4}$

10. $(2, 0), m = 3$

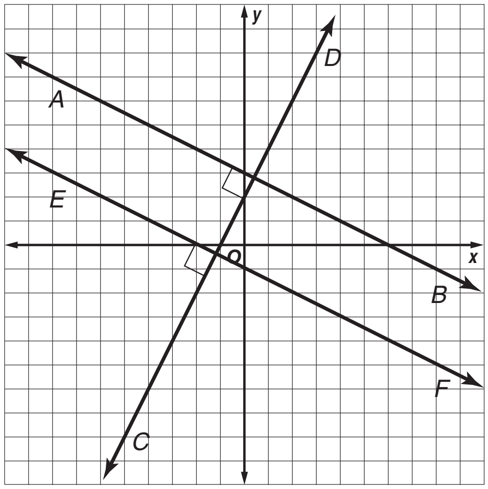
11. $(1, -3), m = 0$

12. $(0, -2), m = -3$

9-6 Enrichment

I A8.0

Parallel and Perpendicular Lines



Parallel lines are lines that never intersect just as train tracks are a constant distance apart and do not intersect. Parallel lines have the same slope.

For example, the slope of line *AB* is $-\frac{1}{2}$. The slope of *EF* is also $-\frac{1}{2}$.

Perpendicular lines are lines that intersect at a right angle. Perpendicular lines have slopes that are the **negative reciprocal** of one another.

For example, the slope of line *AB* is $-\frac{1}{2}$. The slope of *CD* is 2.

Write an equation for the line that satisfies the following conditions.

1. perpendicular to the line $y = -x - 7$; y-intercept $= -2$

2. parallel to a line with slope = 3; passes through $(-1, 3)$

3. perpendicular to line $y = -3x + 10$; passes through $(0, 0)$

4. parallel to the $y = 2x - 5$; passes through $(-1, -8)$

You can write the equation of a line without graphing and without finding the y-intercept using the *point-slope form.*

Point-Slope Form of a Linear Equation

$y - y_1 = m(x - x_1)$

(x_1, y_1) is a point on the line

m is the slope of the line

Write an equation in point-slope form for the line that satisfies the following conditions.

5. passes through $(5, 2)$ with slope $= -3$

6. passes through $(-2, -2)$ and $(3, 8)$

7. parallel to $-\frac{2}{3}x + 4$; passes through $(1, 3)$

8. perpendicular to $y = 2x - 1$; passes through $(3, 4)$

9-8 Enrichment

7MR1.1

Latin Squares

Suppose that an experimenter was comparing four new kinds of bicycle tires. The experimenter might choose 4 different kinds of bicycles and 4 different riders. Then, 64 test rides would be needed to check all the possible combinations.

One way to reduce the number of needed trials is to use a *Latin square*. In this example, the four tires are labeled T_1, T_2, T_3, and T_4.

The four types of tires must be arranged in the Latin square so that each type appears only once in a row or a column.

Now the number of test rides is just 16, one for each cell of the Latin square.

	R_1	R_2	R_3	R_4
B_1	T_1	T_2	T_3	T_4
B_2	T_2	T_1	T_4	T_3
B_3	T_3	T_4	T_1	T_2
B_4	T_4	T_3	T_2	T_1

Make two 4-by-4 Latin squares that are different from the example.

1.

	R_1	R_2	R_3	R_4
B_1				
B_2				
B_3				
B_4				

2.

	R_1	R_2	R_3	R_4
B_1				
B_2				
B_3				
B_4				

Make three different 3-by-3 Latin squares.

3.

	R_1	R_2	R_3
B_1			
B_2			
B_3			

4.

	R_1	R_2	R_3
B_1			
B_2			
B_3			

5.

	R_1	R_2	R_3
B_1			
B_2			
B_3			

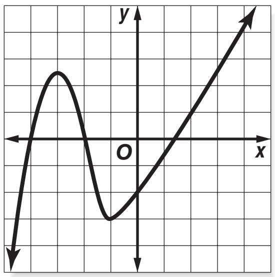
10-1 Enrichment

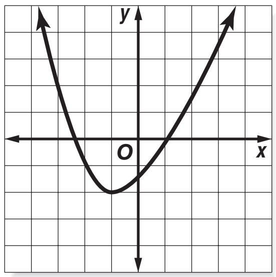
7AF3.1, 7AF3.3

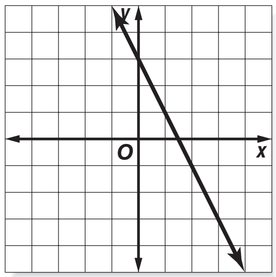
Linear and Nonlinear Functions

The graphs of linear functions are straight lines. This is because they have a constant rate of change. The graphs of other functions that do not have constant rates of change are not straight lines.

Determine whether each graph represents a *linear* or *nonlinear* function. Explain.

1.


2.


3.


Determine whether each equation represents a *linear* or *nonlinear* function. Explain.

4. $y = \frac{1}{4}x + 3$

5. $y = 2x^2 - 5$

6. $x = y + 6$

Determine whether the graph of the data for each situation would most likely represent a *linear* or *nonlinear* function. Explain.

7. the distance and speed a car travels between two red traffic lights

8. the amount of time spent studying in relationship to test scores

9. the height of a child from age 2 to 10

10. the temperature each hour of a given day

NAME ____ DATE ______________ PERIOD _____

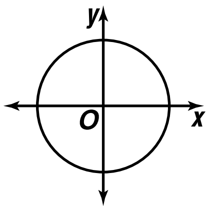
10-2 Enrichment

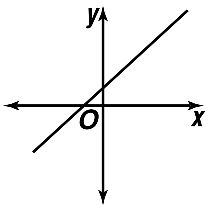
I A I 8.0

Vertical-Line Test

Not every graph that appears in mathematics and elsewhere is necessarily the graph of a function. A vertical-line test is used to determine if a graph is the graph of a function.

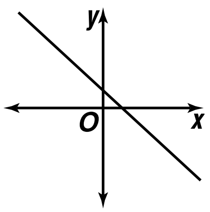
A graph is the graph of a function if any vertical line intersects the graph at no more than one point. For example, the circle at the right is not the graph of a function because a vertical line can intersect the graph at more than one point. However, since any vertical line would intersect the straight line at only one point, the straight line is the graph of a function.



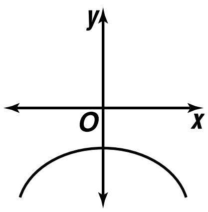


Using the vertical-line test, determine whether each of the following graphs is the graph of a function. If a graph is the graph of a function, write *yes*; if it is not the graph of a function, write *no*.

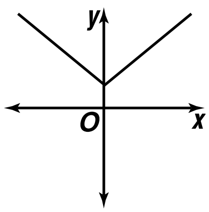
1.



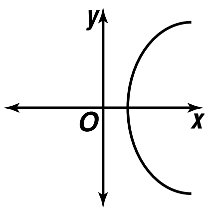
2.



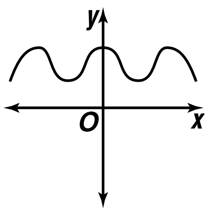
3.



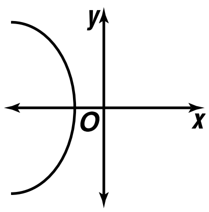
4.



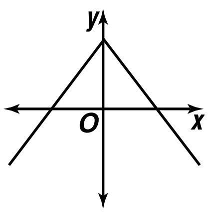
5.



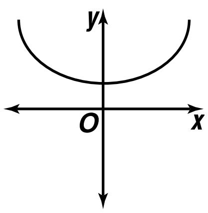
6.



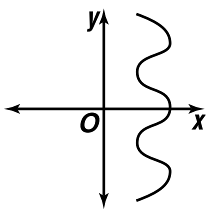
7.



8.



9.



10. Which letters of the alphabet given below could represent the graph of a function?
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

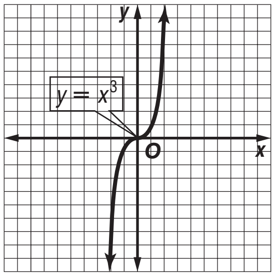
NAME ______________________ DATE ____________ PERIOD _____

10-4 Enrichment

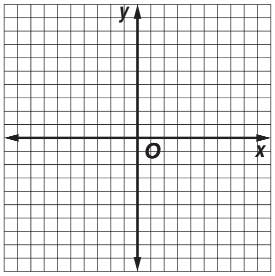
7AF3.1

Graphing Cubic Functions

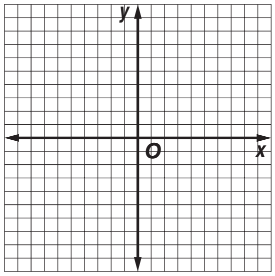
The graph of x^3 is shown below.



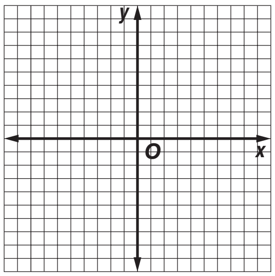
1. Graph $x^3 + 1$. What happens when you add a positive number to x^3?



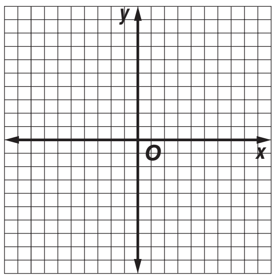
2. Graph $x^3 - 1$. What happens when you subtract a positive number from $x3$?



3. Graph $2x^3$. What happens when you multiply x^3 by a positive number?



4. Graph $-3x^3$. What happens when you multiply x^3 by a negative number?



NAME ______________________ DATE ____________ PERIOD _____

10-5 Enrichment

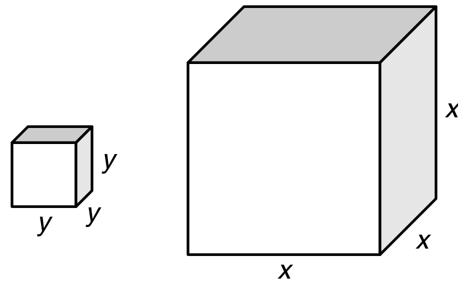
IA10.0

Lesson 10–5

Polynomials and Volume

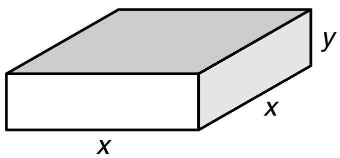
The volume of a rectangular prism can be written as the product of three polynomials. Recall that the volume equals the length times the width times the height.

The two volumes at the right represent the cube of y and the cube of x.

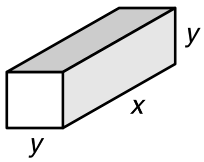


Multiply to find the volume of each prism. Write each answer as an algebraic expression.

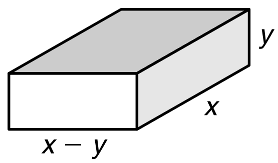
1.



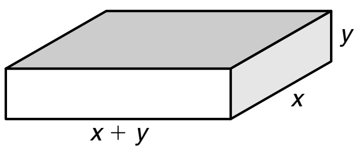
2.



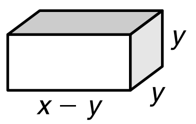
3.



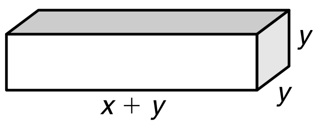
4.



5.

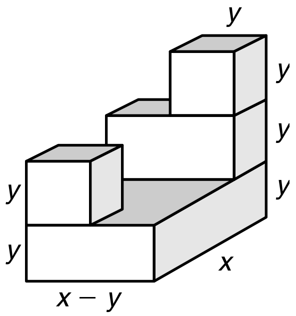


6.

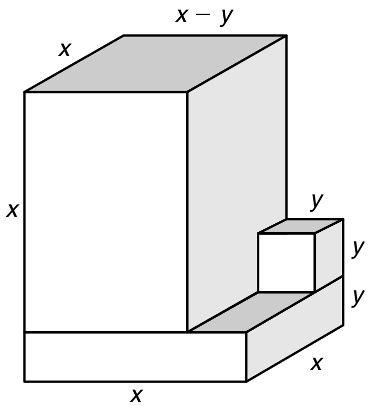


Multiply, then add to find each volume. Write the answer as an algebraic expression.

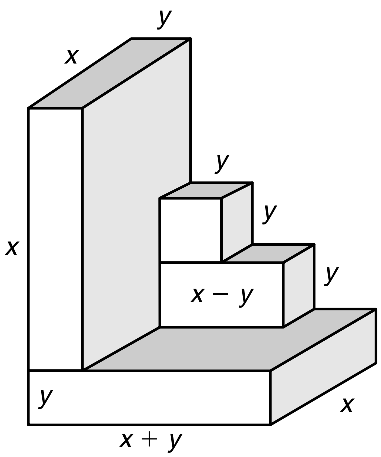
7.



8.



9.



NAME ____ DATE ______________ PERIOD _____

10-6 Enrichment

7AF2.2

Dividing Powers with Different Bases

Some powers with different bases can be divided. First, you must be able to write both as powers of the same base. An example is shown below.

$$\frac{3^9}{81^2} = \frac{3^9}{(3^4)^2}$$ To find the power of a power, multiply the exponents.

$$= \frac{3^9}{3^8}$$

$$= 3^1 \text{ or } 3$$

This method could not have been used to divide $\frac{3^9}{80^2}$, since 80 cannot be written as a power of 3 using integers.

Lesson 10–6

Simplify each fraction using the method shown above. Express the solution without exponents.

1. $\frac{2^5}{8^2}$

2. $\frac{64^3}{8^5}$

3. $\frac{125^2}{25^3}$

4. $\frac{32^4}{16^4}$

5. $\frac{343^3}{7^5}$

6. $\frac{81^4}{3^4}$

7. $\frac{10^6}{1{,}000^3}$

8. $\frac{6^6}{216^2}$

9. $\frac{27^5}{9^4}$

10. $\frac{8^2}{2^2}$

11. $\frac{9^3}{3^3}$

12. $\frac{16^4}{8^3}$

10-7 Enrichment

IA10.0

Simplifying Polynomials

A **monomial** is a number, a variable, or a product of numbers and/or variables. An algebraic expression that is the sum or difference of one or more monomials is called a **polynomial**.

You can simplify polynomials by combining like terms. Like terms must have the same variable and the same power. Thus, $4x^2$ and $-x^2$ are like terms, while $2x^2$ and $6x$ are not.

EXAMPLE 1 **Simplify $8a + 3b - 5b + a$.**

The monomials $8a$ and a are like terms, and $3b$ and $-5b$ are like terms.

$8a + 3b - 5b + a$	Write the polynomial.
$= 8a + 3b + (-5b) + a$	Definition of subtraction
$= (8a + a) + [3b + (-5b)]$	Group like terms.
$= 9a + (-2b)$ or $9a - 2b$	Simplify by combining like terms.

EXAMPLE 2 **Simplify $x^2 - 2x + 5 + x^2 - 1$.**

$x^2 (-2x) + 5 + x^2 (-1)$	Write the polynomial.
$= [x^2 + x^2] + (-2x) + [5 + (-1)]$	Group like terms.
$= 2x^2 + (-2x) + 4$	Simplify by combining like terms.
$= 2x^2 - 2x + 4$	

Exercise

Simplify each polynomial. If the polynomial cannot be simplified, write in *simplest* form.

1. $2s + 3d + 3s + 2d$

2. $3f - 5g - f - 2g$

3. $-3h + 6k + 2 - 2k$

4. $2e^2 - 3e + 6e$

5. $2u^2 + 5u + 9 - 8u$

6. $3r + 2r^2 - 4r^2$

7. $7a^2 + 5a + 1 + 3a^2 - 8a$

8. $2s^2 - 7s - 6 - 2s + 4$

9. $-3d^2 + 5d - 8 + 2d^2 - d + 5$

10. $3x + 5 + 2x + 4y$

Lesson 10–7

NAME ______________________________ DATE ____________ PERIOD _____

10-8 Enrichment

7AF2.2

Roots of Monomials

Simplify expressions with roots of monomials.

Example 1 **Simplify $\sqrt{64a^4b^8} \cdot \sqrt{25a^8b^{12}}$.**

$$\sqrt{64a^4b^8} \cdot \sqrt{25a^8b^{12}} = \left(\sqrt{64} \cdot \sqrt{a^4} \cdot \sqrt{b^8}\right) \cdot \left(\sqrt{25} \cdot \sqrt{a^8} \cdot \sqrt{b^{12}}\right)$$
$$= 8 \cdot a^2 \cdot b^4 \cdot 5 \cdot a^4 \cdot b^6$$
$$= 40a^6b^{10}$$

Example 2 **Simplify $\sqrt[3]{27m^6n^{12}} \cdot \sqrt[3]{216m^9n^{15}}$.**

$$\sqrt[3]{27m^6n^{12}} \cdot \sqrt[3]{216m^9n^{15}} = \left(\sqrt[3]{27} \cdot \sqrt[3]{m^6} \cdot \sqrt[3]{n^{12}}\right) \cdot \left(\sqrt[3]{216} \cdot \sqrt[3]{m^9} \cdot \sqrt[3]{n^{15}}\right)$$
$$= 3 \cdot m^2 \cdot n^4 \cdot 6 \cdot m^3 \cdot n^5$$
$$= 18m^6n^{20}$$

Exercises

1. $\sqrt{16w^8x^{12}} \cdot \sqrt{36w^4x^8y^{16}}$

2. $\sqrt{121m^{20}n^{12}} \cdot \sqrt{81m^4n^8}$

3. $\sqrt{1.69s^{12}t^{16}u^8} \cdot \sqrt{2.56s^4u^{20}}$

4. $\sqrt{0.49f^{12}g^{16}} \cdot \sqrt{0.36g^{20}h^8}$

5. $\sqrt[3]{216a^9b^{21}} \cdot \sqrt[3]{64a^{12}b^{18}}$

6. $\sqrt[3]{512t^{24}u^9} \cdot \sqrt[3]{27u^{15}v^{27}}$

7. $\sqrt[3]{125x^{21}y^{18}z^{12}} \cdot \sqrt[3]{64y^9z^3}$

8. $\sqrt[3]{0.729j^3k^{12}m^{18}} \cdot \sqrt[3]{0.125k^6m^{15}}$

9. $\sqrt[3]{27r^{15}s^{12}} \cdot \sqrt{100r^8s^{16}}$

10. $\sqrt{144g^8h^{12}} \cdot \sqrt[3]{125g^{21}h^{18}}$

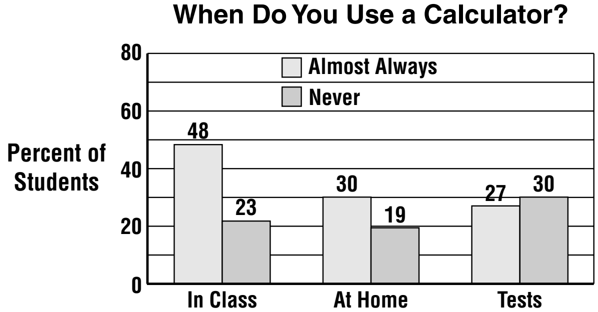
NAME ______________________ DATE ____________ PERIOD _____

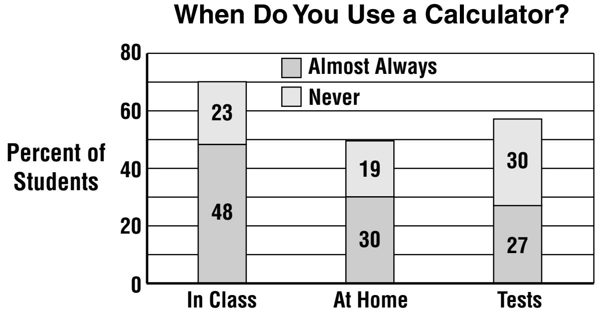
11-2 Enrichment

7SDAP1.1

Double and Stacked Bar Graphs

Bar graphs and histograms are often used to compare data sets. Two types of comparison graphs are shown—a double-bar graph and a stacked-bar graph. The same information is presented on both graphs.

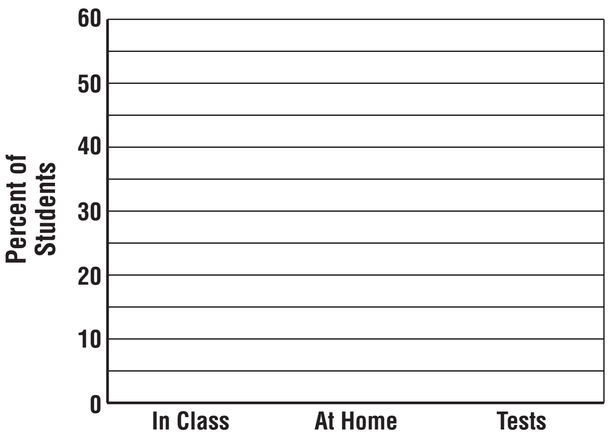




1. Complete the double-bar graph. Then draw a stacked-bar graph on a separate sheet of paper.

Percents Reporting "Almost Always"

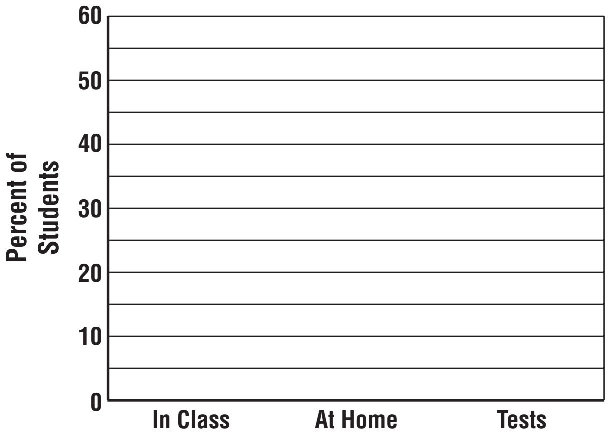
Type of Use	Female	Male
In Class	46	50
At Home	32	29
Tests	27	27



2. Complete the stacked-bar graph. Then draw a double-bar graph on a separate sheet of paper.

Percents Reporting "Never"

Type of Use	Female	Male
In Class	26	20
At Home	18	19
Tests	33	26



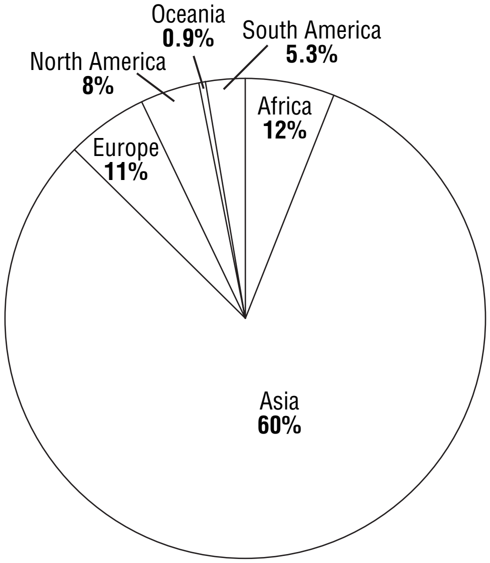
11-3 Enrichment

7SDAP1.1

The Population of the World

In 2006, there were about 6,536,262,000 people living in the world. The circle graph shows the continents in which these people were living.

World Population in 2006



Determine the approximate number of people living in:

1. Africa ____________
2. Asia ____________
3. Europe ____________
4. North America ____________
5. Oceania ____________
6. South America ____________

Given that there are 360° in a circle, find the number of degrees of each section of the circle graph. Round to the nearest whole degree.

7. Africa ____
8. Asia ____
9. Europe ____
10. North America ____
11. Oceania ____
12. South America ____

11-4 Enrichment

7SDAP1.3

Median and Mean of Grouped Data

To find the median, add a column for the cumulative frequency. This is the total of the frequencies up to and including the frequency in a given row.

The last number in the cumulative frequency column will equal the number of data items. In this example, there are 75 data items. So, the median will be the 38th item. The median age is in the interval 30–39.

To find the mean, multiply the frequency of each interval by the midpoint of the interval. Then, divide by the total number of data items.

$$\frac{(20 \times 14.5) + (17 \times 24.5) + (23 \times 34.5) + (15 \times 44.5)}{75} = 28.9$$

People Responding to Radio Station Survey

Ages	Frequency	Cumulative Frequency
10–19	20	20
20–29	17	37
30–39	23	60
40–49	15	75

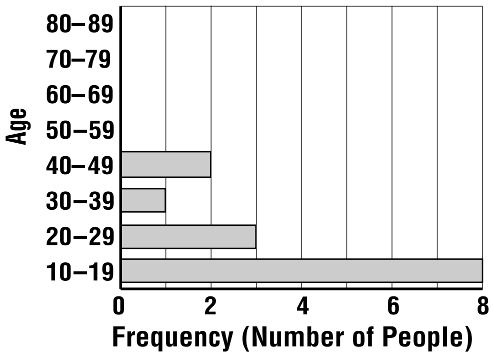
Find the interval for the median and the mean to the nearest tenth.

1. Add this data to the chart in the example: ages 50–59, 11 people; ages 60–69, 16 people; ages 70–79, 19 people; ages 80–89, 4 people.

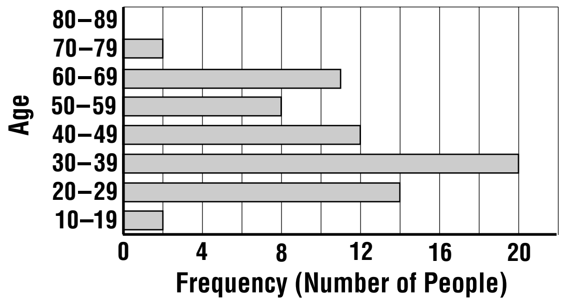
2. **People Who Prefer "All Talk" Radio**

Age	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
Frequency	4	10	14	5	6	5	4	2

3. **Listen to Radio While Doing Homework**



4. **Listen to Radio While Driving to Work**



11-5 Enrichment

7SDAP1.1

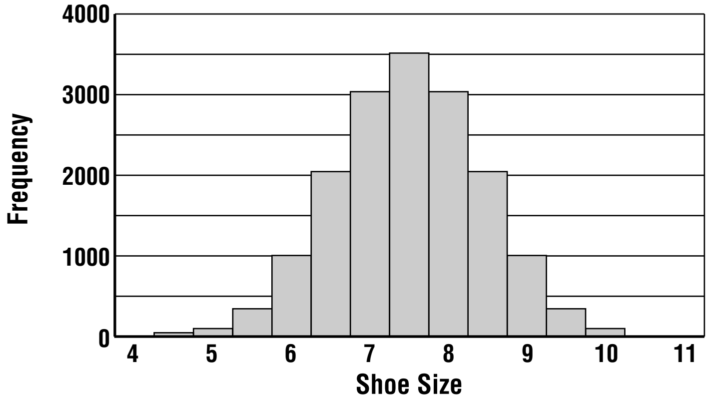
The Bell Curve and Pascal's Triangle

Shoe Size	4	4.5	5	5.5	6	6.5	7
Frequency	1	14	91	364	1,001	2,002	3,003

Shoe Size	7.5	8	8.5	9	9.5	10	10.5	11
Frequency	3,432	3,003	2,002	1,001	364	91	14	1

Graphing the shoe size data in the tables above results in a histogram with a bell-shaped outline.

This type of frequency distribution is called the **bell curve** or the **normal distribution curve**. Many data sets have normal distributions.



On a separate sheet of paper, make a histogram for each data set. Both sets have normal distributions.

1.

Shoe Size	4	4.5	5	5.5	6	6.5	7
Frequency	1	6	15	20	15	6	1

2.

Shoe Size	4	5	6	7	8	9	10	11
Frequency	1	7	21	35	35	21	7	1

The numbers at the right show the first five rows in Pascal's Triangle. Each number equals the sum of the two numbers above it.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

3. Find the next three rows of the triangle. What pattern do you notice?

4. How does Pascal's Triangle relate to the shoe-size data used at the top of this page?

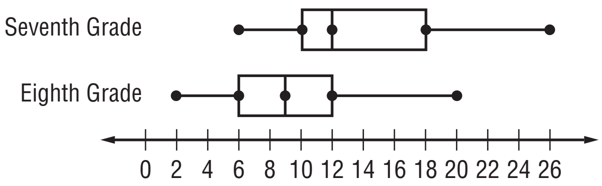
NAME ______________________________________ DATE ______________ PERIOD _____

11-6 Enrichment

7SDAP1.1

Double Box-and-Whisker Plots

Box-and-whisker plots allow data to be interpreted easily. Double box-and-whisker plots, which use the same scale, are a useful tool for comparing sets of data. The number of hours seventh and eighth graders from one school spent watching television per week is shown below.



Use the double box-and-whisker plot above.

1. How many hours do most seventh and eighth graders spend watching television per week?

2. Half of the seventh graders watch television at least how many hours or more per week?

3. Half of the eighth graders watch television no more than how many hours per week?

4. What are the least and greatest number of hours spent watching television for each grade level?

5. Make a conclusion about how the amount of time seventh graders spend watching television compares to eighth graders.

6. Use a newspaper or the Internet to find two sets of data. Construct a double box-and-whisker plot for the data.

NAME ______________________ DATE ____________ PERIOD _____

11-7 Enrichment

7SDAP1.1

U.S. Presidents

The political parties in our country have changed over time. At the time of our nation's founding, the Federalist and Democratic-Republican parties were the nationally prominent parties. In recent years, all U.S. Presidents have been from either the Republican or Democratic Party.

Republican	Democrat	Federalist
Abraham Lincoln	Andrew Jackson	George Washington
Ulysses S. Grant	Martin Van Buren	John Adams
Rutherford B. Hayes	James K. Polk	
James A. Garfield	Franklin Pierce	**Democratic-Republican**
Chester A. Arthur	James Buchanan	Thomas Jefferson
Benjamin Harrison	Grover Cleveland	James Madison
William McKinley	Woodrow Wilson	James Monroe
Theodore Roosevelt	Franklin D. Roosevelt	John Quincy Adams
William Howard Taft	Harry S Truman	
Warren G. Harding	John F. Kennedy	**Whig**
Calvin Coolidge	Lyndon B. Johnson	William Henry Harrison
Herbert Hoover	Jimmy Carter	John Tyler
Dwight D. Eisenhower	Bill Clinton	Zachary Taylor
Richard Nixon		Millard Fillmore
Gerald Ford		
Ronald Reagan		**Union**
George Bush		Andrew Johnson
George W. Bush		

1. Make a table to display the data for the number of U.S. Presidents from each political party.

2. Display the data from the table in a stem-and-leaf plot.

3. What is the difference in the number of presidents from the party with the most presidents and the party with the fewest presidents?

4. What is another type of information about U.S. Presidents that could be displayed using a stem-and-leaf plot?

Lesson 11–7

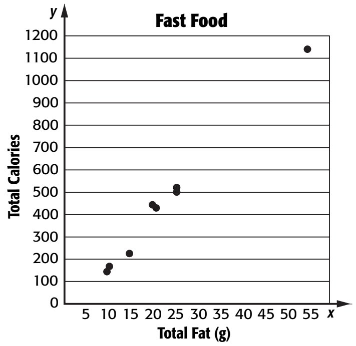
11-8 Enrichment

7SDAP1.2

Graphing Two Variables

In statistics, sets of ordered pairs are often used to determine if a set of data points is related in such a way that one can be predicted from another. When these sets of ordered pairs are plotted on a graph to look for a relationship, it is called a **scatter plot**.

Food	Total Fat (g)	Total Calories
Large hamburger	21	420
Taco	14	220
Chicken burrito	19	430
French fries (large)	25	520
Crispy chicken sandwich	25	500
Chicken nuggets (4)	10	170
Meatball sub	54	1,140
Cobb salad	9	150



From the table alone it is difficult to notice much about the data. However, from the scatter plot it is easy to see that the total Calories in a food item increase as the total fat in the food item increases.

Create a scatter plot for the data set. Make some observations about the data.

1. The following table shows data collected on graduation rates and average ACT scores of incoming freshman at 7 of the Big Ten universities.

ACT Score	Graduation %
27	76.2
24	57.6
24	55.4
23	59.7
28	86
22	46.2
23	66.7

2. The following table shows the scores that students in Ms. Carlson's class earned, and the average number of hours each student watches TV after school.

TV Hours	Grade	TV Hours	Grade
1	87	5	52
1.5	89	0.5	92
3	72	2	85
2	71	4	68
3	65	2.5	79

NAME ______________________________ DATE ____________ PERIOD _____

12-1 Enrichment

7MR2.5

Diagrams

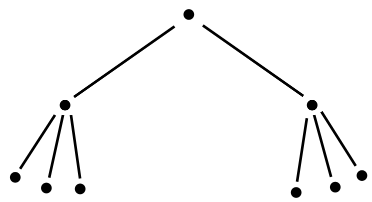
Stories can sometimes travel quickly. Let's say someone told a story to two friends, and each of these two friends in turn told the story to three other friends. How many people in all have been told the story?

Diagrams are useful problem-solving tools. Indicate the original storyteller with a dot.

The original storyteller told two friends; extend the diagram with two new dots.

Each of these two friends told three other friends; extend the diagram to show six more dots.

The diagram helps us see that eight people have been told the story.



Draw a diagram and answer the following questions.

1. A football coach wanted to change the time of football practice, so the coach told two players of the time change. Each of these two players were to tell three other players of the time change. Each of those three players were to tell four other players of the time change. How many players were to be told of the time change?

2. In December, some people enjoy singing a song called "The Twelve Days of Christmas." In the song, a person receives a gift of a partridge in a pear tree on the first day. On the second day, the person receives two turtle doves and a partridge in a pear tree. On the third day, three French hens, two turtle doves, and a partridge in a pear tree are received. This pattern continues for a total of 12 days. After 12 days, how many gifts are received?

3. A creature in no particular hurry and with nothing better to do is at the bottom of a 15-foot well. Each day the creature climbs up three feet toward the top of the well, then slides back down one foot toward the bottom of the well each night. At that rate, how many days will it take the creature to climb out of the well? (Think before you answer!)

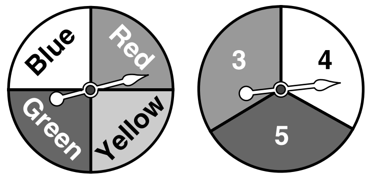
NAME ______________________________ DATE ____________ PERIOD _____

12-2 Enrichment

7MR2.2

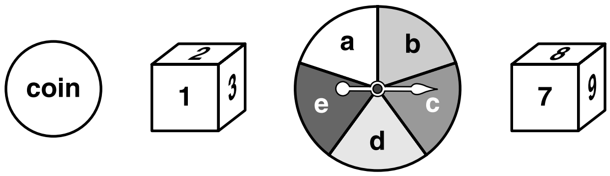
Independent Events

The probability of two independent events is found by multiplying the probability of the first event by the probability of the second event.



With the spinners at the right, the probability of obtaining red with the first spinner and an even number with the second spinner is $\frac{1}{4} \cdot \frac{1}{3}$ or $\frac{1}{12}$.

The objects below consist of a coin with faces of heads and tails, a cube numbered 1–6, a spinner labeled a–e, and a cube numbered 7–12. Match each probability to its event.



Event	Probability
1. Coin: tails Spinner: vowel	________ $\frac{3}{10}$
2. Cube 1–6: even number Cube 7–12: odd number	________ $\frac{1}{12}$
3. Spinner: consonant Coin: heads	________ $\frac{1}{15}$
4. Cube 1–6: prime number Cube 7–12: prime number	________ $\frac{1}{6}$
5. Coin: heads or tails Spinner: a, b, or c	________ $\frac{1}{1}$
6. Cube 1–6: multiple of 2 Cube 7–12: multiple of 5	________ $\frac{3}{5}$
7. Cube 1–6: composite number Cube 7–12: composite number	________ $\frac{1}{4}$
8. Spinner: vowel Cube 1–6: odd number Cube 7–12: number divisible by 4	________ $\frac{1}{20}$
9. Coin: heads Spinner: a, c, or e Cube 1–6: non-prime and non-composite number	________ $\frac{1}{5}$
10. Coin: heads or tails Spinner: a, b, c, d, or e Cube 1–6: any number divisible by 1 Cube 7–12: 7, 8, 9, 10, 11, or 12	________ $\frac{2}{9}$

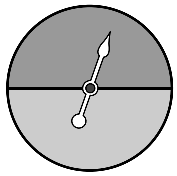
Lesson 12–2

12-3 Enrichment

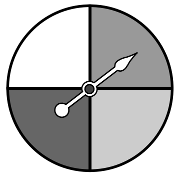
7MR1.3

Probability Regions

If a circular spinner is divided by a diameter into two equal regions, the theoretical probability of the pointer landing in any region is $\frac{1}{2}$.

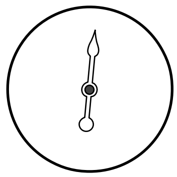


If the circular spinner containing one diameter is divided by another diameter perpendicular to the first, the spinner is divided into 4 equal regions. The theoretical probability of the pointer landing in any region is $\frac{1}{4}$.

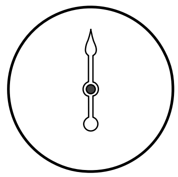


Read each description of a spinner. Using a ruler and protractor, divide each spinner into the indicated regions.

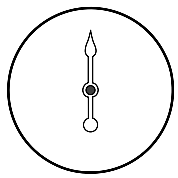
1. Divide this circular spinner into three regions so that the probability of landing in any region is $\frac{1}{3}$.



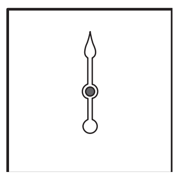
2. Divide this circular spinner into three regions so that the probability of landing in one region is $\frac{1}{2}$ and another region is $\frac{1}{4}$.



3. Divide this circular spinner into three regions so that the probability of landing in one region is $\frac{1}{4}$ and another region is $\frac{1}{8}$.



4. Divide this square spinner into three regions so that the probability landing in one region is $\frac{1}{8}$ and another region is $\frac{5}{8}$.

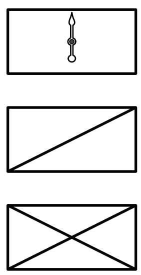


5. This rectangular spinner must be divided into four regions of equal probability.

   It has been divided in half by one diagonal.

   It has been divided in half again by the other diagonal.

   Is the spinner divided into four regions of equal probability? Explain your reasoning.



NAME ______________________ DATE ____________ PERIOD ____

12-5 Enrichment

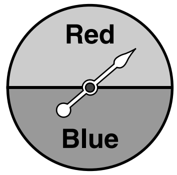
7MR2.5

Probability Predictions

The outcomes of an experiment using a spinner and 20 trials are:

Red	Red	Red	Red	Red	Red	Red	Red	Red	Red
Blue	Blue	Blue	Blue	Blue	Blue	Blue	Blue	Blue	Red

There were 11 red outcomes and 9 blue outcomes. If we assume that these outcomes are close to the theoretical probability of the spinner, we can predict that one-half of the spinner is red and one-half of the spinner is blue, and that it might look like the spinner at the right.



Use the outcomes of each experiment to describe what the spinner or cube used in each experiment might look like. Assume all outcomes are close to the theoretical probability of each spinner or cube.

1. A spinner is spun 100 times.

Outcome	Frequency
Red	48 times
Yellow	24 times
Green	28 times

Description of spinner:

2. A cube is rolled 60 times.

Outcome	Frequency
1	22 times
2	18 times
3	20 times

Description of cube:

3. A spinner is spun 80 times.

Outcome	Frequency
Orange	23 times

Description of spinner:

4. A cube is rolled 120 times.

Outcome	Frequency
1	19 times
2	22 times
3	19 times

Description of cube:

5. A cube is rolled 60 times.

Outcome	Frequency
2	18 times
4	11 times
6	9 times

Description of cube: