Glencoe McGraw-Hill

California Mathematics

Grade 7

Study Guide

and Intervention Workbook

Contents Include:
99 worksheets—
one for each lesson

TO THE STUDENT This *Study Guide and Intervention Workbook* gives you additional examples and problems for the concept exercises in each lesson. The exercises are designed to aid your study of mathematics by reinforcing important mathematical skills needed to succeed in the everyday world. The materials are organized by chapter and lesson, with one *Study Guide and Intervention* worksheet for every lesson in *Glencoe California Mathematics, Grade 7.*

Always keep your workbook handy. Along with your textbook, daily homework, and class notes, the completed *Study Guide and Intervention Workbook* can help you review for quizzes and tests.

TO THE TEACHER These worksheets are the same as those found in the Chapter Resource Masters for *Glencoe California Mathematics, Grade 7.* The answers to these worksheets are available at the end of each Chapter Resource Masters booklet as well as in your Teacher Wraparound Edition interleaf pages.

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1-1 Study Guide and Intervention

7MR1.1, 6AF2.3

A Plan for Problem Solving

You can always use the four-step plan to solve a problem.

Explore	Determine what information is given in the problem and what you need to find.
Plan	Select a strategy including a possible estimate.
Solve	Solve the problem by carrying out your plan.
Check	Examine your answer to see if it seems reasonable.

Example 1 **Plant A and Plant B are two new experimental apple trees being grown in a laboratory. The table displays their heights, in millimeters, when they are 5 to 10 days old.**

Day	5	6	7	8	9	10
Plant A	36	39	42	45	48	51
Plant B	32	36	40	44	48	52

Estimate the height of each plant on day 12.

Explore You know their heights for days 5 to 10. You need to determine their heights in two more days.

Plan Determine whether there is a pattern and extend that pattern to day 12.

Solve Comparing each plant's heights on consecutive days, we see that Plant A's height increases by 3 millimeters each day, while Plant B's height increases by 4 millimeters each day. To estimate Plant A's height on day 12, assume that it will grow 3 millimeters each day past day 10, so it will be 51 + 3 + 3 or 57 millimeters. To estimate Plant B's height on day 12, assume that it will grow 4 millimeters each day past day 10, so it will be 52 + 4 + 4 or 60 millimeters.

Check Given what we know about each plant's height and how plants grow in general, both estimates seem reasonable.

Exercises

Use the four-step plan to solve each problem.

1. **MOVIES** A movie ticket costs $3.50. A large popcorn costs $3.75 and a large soda costs $3.00. How much will it cost two friends to go to a movie if they share a popcorn and each has a large soda?

2. **FLOUR BEETLES** The population of a flour beetle doubles in about a week. How long would it take for the population to grow to eight times its original size?

1-2 Study Guide and Intervention

7AF1.2, 7AF1.3, 7AF1.4

Variables, Expressions, and Properties

When finding the value of an expression with more than one operation, perform the operations in the order specified by the order of operations.

Order of Operations

1. Perform all operations within grouping symbols first; start with the innermost grouping symbols.
2. Evaluate all powers before other operations.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example 1 **Evaluate the expression $(5 + 7) \div 2 \times 3 - (8 + 1)$.**

$(5 + 7) \div 2 \times 3 - (8 + 1) = 12 \div 2 \times 3 - (8 + 1)$	Add inside the left parentheses.
$= 12 \div 2 \times 3 - 9$	Add inside the remaining parentheses.
$= 6 \times 3 - 9$	Divide.
$= 18 - 9$	Multiply.
$= 9$	Subtract.

Example 2 **Evaluate the expression $3x^2 - 4y$ if $x = 3$ and $y = 2$.**

$3x^2 - 4y = 3(3)^2 - 4(2)$	Replace x with 3 and y with 2.
$= 3(9) - 4(2)$	Evaluate the power first.
$= 27 - 8$	Do all multiplications.
$= 19$	Subtract.

Exercises

Evaluate each expression.

1. $4 \times 5 + 8$

2. $16 - 12 \div 4$

3. $14 \div 2 + 3(5)$

4. $5 - 6 \times 2 \div 3$

5. $2 \cdot 3^2 + 10 - 14$

6. $2^2 + 32 \div 8 - 5$

7. $(10 + 5) \div 3$

8. $5^2 \cdot (8 - 6)$

9. $(17 - 5)(6 + 5)$

10. $3 + 7(14 - 8 \div 2)$

11. $5[24 - (6 + 8)]$

12. $\frac{14}{3^2 - 2}$

Evaluate each expression if $a = 3$, $b = 5$, and $c = 6$.

13. $a + 3b$

14. $4b - 3c$

15. $2a - b + 5c$

16. $(ab)^2$

17. $a(b + c)$

18. $3(bc - 8) \div a$

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1-3 Study Guide and Intervention

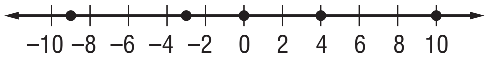
7NS2.5

Integers and Absolute Value

A number line can help you order a set of integers. When graphed on a number line, the smaller of two integers is always to the left of the greater integer.

Example 1 **Order the set of integers {10, −3, −9, 4, 0} from least to greatest.**

Graph each integer on a number line.



The numbers from left to right are {−9, −3, 0, 4, 10}.

The absolute value of a number is the distance of that number from 0 on a number line.

Example 2 **Evaluate the expression $|-20| + |10|$.**

$|-20| + |10|$	$= 20 + |10|$	The absolute value of −20 is 20.
	$= 20 + 10$	The absolute value of 10 is 10.
	$= 30$	Simplify.

Exercises

Order each set of integers in each set from least to greatest.

1. {3, 0, −5, 1, 4}

2. {−6, −8, 3, −1, −4}

3. {2, 13, −11, −21, 5}

4. {31, 0, −34, −9, 7}

Evaluate each expression.

5. $|-13|$

6. $|21|$

7. $|-3| + |-5|$

8. $|9| + |-8|$

9. $|-13| + |15|$

10. $|21 - 18|$

11. $|-11| - |-5|$

12. $|4| - |-4|$

13. $|23 + 15|$

Evaluate each expression if $a = -6$, $b = 4$, and $c = 5$.

14. $|a| + 14$

15. $|c - b|$

16. $b + |c|$

17. $|3b|$

18. $2|a| + c$

19. $|2b + c|$

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1-4 Study Guide and Intervention

7NS1.2, 7AF1.3

Adding Integers

To add integers with the same sign, add their absolute values. The sum has the same sign as the integers.

Example 1 **Find $-3 + (-4)$.**

$-3 + (-4) = -7$ Add $|-3| + |-4|$. Both numbers are negative, so the sum is negative.

To add integers with different signs, subtract their absolute values. The sum has the same sign as the integer with the greater absolute value.

Example 2 **Find $-16 + 12$.**

$-16 + 12 = -4$ Subtract $|12|$ from $|-16|$. The sum is negative because $|-16| > |12|$.

Exercises

Add.

1. $9 + 16$

2. $-10 + (-10)$

3. $18 + (-26)$

4. $-23 + (-15)$

5. $-45 + 35$

6. $39 + (-38)$

7. $-55 + 81$

8. $-61 + (-39)$

9. $-74 + 36$

10. $5 + (-4) + 8$

11. $-3 + 10 + (-6)$

12. $-13 + (-8) + (-12)$

13. $3 + (-10) + (-16) + 11$

14. $-17 + 31 + (-14) + 26$

Evaluate each expression if $x = 4$ and $y = -3$.

15. $11 + y$

16. $x + (-6)$

17. $y + 2$

18. $|x + y|$

19. $|x| + y$

20. $x + |y|$

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1-5 Study Guide and Intervention

7NS1.2

Subtracting Integers

To subtract an integer, add its opposite or additive inverse.

Example 1 **Find 8 − 15.**

$8 - 15 = 8 + (-15)$ **To subtract 15, add −15.**

$= -7$ **Add.**

Example 2 **Find 13 − (−22).**

$13 - (-22) = 13 + 22$ **To subtract −22, add 22.**

$= 35$ **Add.**

Exercises

Subtract.

1. $-3 - 4$

2. $5 - (-2)$

3. $-10 - 8$

4. $-15 - (-12)$

5. $-23 - (-28)$

6. $16 - 9$

7. $9 - 16$

8. $-21 - 16$

9. $28 - 37$

10. $-34 - (-46)$

11. $65 - (-6)$

12. $19 - |29|$

Evaluate each expression if $a = -7$, $b = -3$, and $c = 5$.

13. $a - 8$

14. $20 - b$

15. $a - c$

16. $c - b$

17. $b - a - c$

18. $c - b - a$

1-6 Study Guide and Intervention

7NS1.2, 7AF1.3

Multiplying and Dividing Integers

Use the following rules to determine whether the product or quotient of two integers is positive or negative.

- The product of two integers with different signs is negative.
- The product of two integers with the same sign is positive.
- The quotient of two integers with different signs is negative.
- The quotient of two integers with the same sign is positive.

Example 1 **Find $7(-4)$.**

$7(-4) = -28$ **The factors have different signs. The product is negative.**

Example 2 **Find $-5(-6)$.**

$-5(-6) = 30$ **The factors have the same sign. The product is positive.**

Example 3 **Find $15 \div (-3)$.**

$15 \div (-3) = -5$ **The dividend and divisor have different signs. The quotient is negative.**

Example 4 **Find $-54 \div (-6)$.**

$-54 \div (-6) = 9$ **The dividend and divisor have the same sign. The quotient is positive.**

Exercises

Multiply or divide.

1. $8(-8)$ **2.** $-3(-7)$ **3.** $-9(4)$ **4.** $12(8)$

5. $33 \div (-3)$ **6.** $-25 \div 5$ **7.** $48 \div 4$ **8.** $-63 \div (-7)$

9. $(-4)^2$ **10.** $\frac{-75}{15}$ **11.** $-6(3)(-5)$ **12.** $\frac{-143}{-13}$

Evaluate each expression if $a = -1$, $b = 4$, and $c = -7$.

13. $3c + b$ **14.** $a(b + c)$ **15.** $c^2 - 5b$ **16.** $\frac{a-6}{c}$

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1-7 Study Guide and Intervention

7AF1.1, 7AF1.4

Writing Equations

The table shows several verbal phrases for each algebraic expression.

Phrases	Expression	Phrases	Expression
8 more than a number the sum of 8 and a number x plus 8 x increased by 8	$x + 8$	the difference of r and 6 6 subtracted from a number 6 less than a number r minus 6	$r - 6$
Phrases	**Expression**	**Phrases**	**Expression**
4 multiplied by n 4 times a number the product of 4 and n	$4n$	a number divided by 3 the quotient of z and 3 the ratio of z and 3	$\frac{z}{3}$

The table shows several verbal sentences that represent the same equation.

Sentences	Equation
9 less than a number is equal to 45. The difference of a number and 9 is 45. A number decreased by 9 is 45. 45 is equal to a number minus 9.	$n - 9 = 45$

Exercises

Write each verbal phrase as an algebraic expression.

1. the sum of 8 and t

2. the quotient of g and 15

3. the product of 5 and b

4. p increased by 10

5. 14 less than f

6. the difference of 32 and x

Write each verbal sentence as an algebraic equation.

7. 5 more than a number is 6.

8. The product of 7 and b is equal to 63.

9. The sum of r and 45 is 79.

10. The quotient of x and 7 is equal to 13.

11. The original price decreased by $5 is $34.

12. 5 shirts at $$d$ each is $105.65.

1-8 Study Guide and Intervention

7MR1.1, 7NS1.2

Problem-Solving Investigation: Work Backward

You may need to work backward to solve a problems.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

Example 1

Mari put money in her savings account each week. She put a certain amount of money in the bank on the first week. On the second week she put twice as much money in the bank as the first week. On the third week, she put $40 less in the bank than on the second week. On the fourth week, she put $20 more in the bank than on the third week. Mari put $200 in the bank on the fourth week. How much money did Mari put in the bank on the first week?

Explore You know that Mari put $200 in the bank on the fourth week. You need to know how much money she put in the bank on the first week.

Plan Start with the amount she put in the bank on the last week and work backward.

Solve Start with the $200 Mari put in the bank on the fourth week.

Fourth Week		Third Week		Second Week		First Week
$200	−$20	$180	+$40	$220	÷ 2	$110
This is $20 more than the third week.	Work backward. Subtract $20.	This is $40 less than the second week.	Work backward. Add $40.	This is twice as much as the first week.	Work backward. Divide by 2.	

Check Start with $110 for the first week and work forward. On the second week she deposited twice as much money in the bank than on the first week, which is $220. On the third week, she deposited $40 less than the second week, which is $180. On the fourth week she deposited $20 more than on the third week, or $200. This is what you know she deposited on the fourth week.

Exercises

Use the work backward strategy to solve each problem.

1. **SHOPPING** Jack spent a total of $87.58 when he went shopping for camping supplies. He spent $36.89 on food, $23.24 on a sleeping bag, and bought lunch. When he got home, he had $15.70. How much did he spend on lunch?

2. **AGE** Sam is 4 years older than Eliot. Eliot is 9 years younger than Xing. Xing is 3 years older than Damien. If Damien is 15 years old, how old are each of the other boys?

1-9 Study Guide and Intervention

6AF1.1

Solving Addition and Subtraction Equations

You can use the following properties to solve addition and subtraction equations.

- *Addition Property of Equality* — If you add the same number to each side of an equation, the two sides remain equal.
- *Subtraction Property of Equality* — If you subtract the same number from each side of an equation, the two sides remain equal.

Example 1 **Solve $w + 19 = 45$. Check your solution.**

	$w + 19 = 45$	**Write the equation.**
	$w + 19 - 19 = 45 - 19$	**Subtract 19 from each side.**
	$w = 26$	**$19 - 19 = 0$ and $45 - 19 = 26$. w is by itself.**
Check	$w + 19 = 45$	**Write the original equation.**
	$26 + 19 \stackrel{?}{=} 45$	**Replace w with 26. Is this sentence true?**
	$45 = 45$ ✓	**$26 + 19 = 45$**

Example 2 **Solve $h - 25 = -76$. Check your solution.**

	$h - 25 = -76$	**Write the equation.**
	$h - 25 + 25 = -76 + 25$	**Add 25 to each side.**
	$h = -51$	**$-25 + 25 = 0$ and $-76 + 25 = -51$. h is by itself.**
Check	$h - 25 = -76$	**Write the original equation.**
	$-51 - 25 \stackrel{?}{=} -76$	**Replace h with -51. Is this sentence true?**
	$-76 = -76$ ✓	$-51 - 25 = -51 + (-25)$ or -76

Exercises

Solve each equation. Check your solution.

1. $s - 4 = 12$

2. $d + 2 = 21$

3. $h + 6 = 15$

4. $x + 5 = -8$

5. $b - 10 = -34$

6. $f - 22 = -6$

7. $17 + c = 41$

8. $v - 36 = 25$

9. $y - 29 = -51$

10. $19 = z - 32$

11. $13 + t = -29$

12. $55 = 39 + k$

13. $62 + b = 45$

14. $x - 39 = -65$

15. $-56 = -47 + n$

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1-10 Study Guide and Intervention

6AF1.1

Solving Multiplication and Division Equations

You can use the following properties to solve multiplication and division equations.

- *Multiplication Property of Equality* — If you multiply each side of an equation by the same number, the two sides remain equal.
- *Division Property of Equality* — If you divide each side of an equation by the same nonzero number, the two sides remain equal.

Example 1 **Solve 19*w* = 114. Check your solution.**

$19w = 114$ **Write the equation.**

$\frac{19w}{19} = \frac{114}{19}$ **Divide each side of the equation by 19.**

$1w = 6$ **19 ÷ 19 = 1 and 114 ÷ 19 = 6.**

$w = 6$ **Identity Property; 1*w* = *w***

Check $19w = 114$ **Write the original equation.**

$19(6) \stackrel{?}{=} 114$ **Replace *w* with 6.**

$114 = 114$ ✓ **This sentence is true.**

Example 2 **Solve $\frac{d}{15} = -9$. Check your solution.**

$\frac{d}{15} = -9$

$\frac{d}{15}(15) = -9(15)$ Multiply each side of the equation by 15.

$d = -135$

Check $\frac{d}{15} = -9$ **Write the original equation.**

$\frac{-135}{15} \stackrel{?}{=} -9$ **Replace *d* with −135.**

$-9 = -9$ ✓ **−135 ÷ 15 = −9**

Exercises

Solve each equation. Check your solution.

1. $\frac{r}{5} = 6$ **2.** $2d = 12$ **3.** $7h = -21$

4. $-8x = 40$ **5.** $\frac{f}{8} = -6$ **6.** $\frac{x}{-10} = -7$

7. $17c = -68$ **8.** $\frac{h}{-11} = 12$ **9.** $29t = -145$

10. $125 = 5z$ **11.** $13t = -182$ **12.** $117 = -39k$

2-1 Study Guide and Intervention

7NS1.3, 7NS1.5

Rational Numbers

To express a fraction as a decimal, divide the numerator by the denominator.

Example 1 **Write $\frac{3}{4}$ as a decimal.**

$\frac{3}{4}$ means $3 \div 4$.

The fraction $\frac{3}{4}$ can be written as 0.75, since $3 \div 4 = 0.75$.

Example 2 **Write −0.16 as a fraction.**

$-0.16 = -\frac{16}{100}$ 0.16 is 16 hundredths.

$= -\frac{4}{25}$ Simplify.

The decimal −0.16 can be written as $-\frac{4}{25}$.

Example 3 **Write $8.\overline{2}$ as a mixed number.**

Let $N = 8.\overline{2}$ or $8.222\ldots$.

Then $10N = 82.222\ldots$.

Subtract.

$10N = 82.222\ldots$

$-1N = 8.222\ldots$ $N = 1N$

$9N = 74$ $10N - 1N = 9N$

$\frac{9N}{9} = \frac{74}{9}$ Divide each side by 9.

$N = 8\frac{2}{9}$ Simplify.

The decimal $8.\overline{2}$ can be written as $8\frac{2}{9}$.

Exercises **Write each fraction or mixed number as a decimal.**

1. $\frac{2}{5}$ **2.** $\frac{3}{10}$ **3.** $\frac{7}{8}$ **4.** $2\frac{16}{25}$

5. $-\frac{2}{3}$ **6.** $-1\frac{2}{9}$ **7.** $6\frac{2}{3}$ **8.** $-4\frac{3}{11}$

Write each decimal as a fraction or mixed number in simplest form.

9. 0.8 **10.** −0.15 **11.** $0.\overline{1}$ **12.** $1.\overline{7}$

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2-2 Study Guide and Intervention

7NS1.1

Comparing and Ordering Rational Numbers

When comparing two or more rational numbers, either write the numbers as fractions with the same denominator or write the numbers as decimals.

Example 1 **Replace ● with <, >, or = to make $\frac{4}{5}$ ● $\frac{7}{10}$ a true sentence.**

Write as fractions with the same denominator. The least common denominator is 10.

$\frac{4}{5} = \frac{4 \cdot 2}{5 \cdot 2}$ or $\frac{8}{10}$

$\frac{7}{10} = \frac{7 \cdot 1}{10 \cdot 1}$ or $\frac{7}{10}$

Since $\frac{8}{10} > \frac{7}{10}$, $\frac{4}{5} > \frac{7}{10}$.

Example 2 **Order the set of rational numbers -3.25, $-3\frac{1}{3}$, $-3\frac{2}{5}$, and $-3.2\overline{5}$ from least to greatest.**

Write $-3\frac{1}{3}$ and $-3\frac{2}{5}$ as decimals.

$\frac{1}{3} = 0.\overline{3}$, so $-3\frac{1}{3} = -3.\overline{3}$.

$\frac{2}{5} = 0.4$, so $-3\frac{2}{5} = -3.4$.

Since $-3.4 < -3.\overline{3} < -3.2\overline{5} < -3.25$, the numbers from least to greatest are $-3\frac{2}{5}$, $-3\frac{1}{3}$, $-3.2\overline{5}$, and -3.25.

Exercises

Replace each ● with <, >, or = to make a true sentence.

1. $\frac{5}{6}$ ● $\frac{2}{3}$

2. $\frac{4}{5}$ ● $\frac{13}{15}$

3. $\frac{1}{9}$ ● $\frac{1}{8}$

4. $-\frac{2}{3}$ ● $-\frac{7}{10}$

5. $3\frac{7}{10}$ ● $3\frac{4}{5}$

6. $-2\frac{3}{7}$ ● $-2\frac{4}{9}$

7. 2.6 ● $2\frac{5}{8}$

8. $4\frac{1}{6}$ ● $4.1\overline{6}$

9. $-4.5\overline{8}$ ● $-4.\overline{58}$

Order each set of rational numbers from least to greatest.

10. $0.5, 0.1, \frac{1}{4}, \frac{2}{3}$

11. $2.4, 2\frac{4}{7}, 2.13, 1\frac{9}{10}$

12. $\frac{1}{5}, -0.7, 0.25, -\frac{3}{5}$

13. $1\frac{2}{9}, 1\frac{2}{3}, 1.45, 1.67$

14. $-2\frac{1}{4}, -2.28, -2.7, -2\frac{4}{5}$

15. $4\frac{2}{3}, 4\frac{5}{6}, 4.6, 5.3$

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2-3 Study Guide and Intervention

7NS1.2, 7MG1.3

Multiplying Positive and Negative Fractions

To multiply fractions, multiply the numerators and multiply the denominators.

Example 1 **Find $\frac{3}{8} \cdot \frac{4}{11}$. Write in simplest form.**

$\frac{3}{8} \cdot \frac{4}{11} = \frac{3}{\cancel{8}_2} \cdot \frac{\cancel{4}^1}{11}$ Divide 8 and 4 by their GCF, 4.

$= \frac{3 \cdot 1}{2 \cdot 11}$ Multiply the numerators and denominators.

$= \frac{3}{22}$ Simplify.

To multiply mixed numbers, first rewrite them as improper fractions.

Example 2 **Find $-2\frac{1}{3} \cdot 3\frac{3}{5}$. Write in simplest form.**

$-2\frac{1}{3} \cdot 3\frac{3}{5} = -\frac{7}{3} \cdot \frac{18}{5}$ $-2\frac{1}{3} = -\frac{7}{3}$, $3\frac{3}{5} = \frac{18}{5}$

$= -\frac{7}{\cancel{3}_1} \cdot \frac{\cancel{18}^6}{5}$ Divide 18 and 3 by their GCF, 3.

$= -\frac{7 \cdot 6}{1 \cdot 5}$ Multiply the numerators and denominators.

$= -\frac{42}{5}$ Simplify.

$= -8\frac{2}{5}$ Write the result as a mixed number.

Exercises

Multiply. Write in simplest form.

1. $\frac{2}{3} \cdot \frac{3}{5}$

2. $\frac{4}{7} \cdot \frac{3}{4}$

3. $-\frac{1}{2} \cdot \frac{7}{9}$

4. $\frac{9}{10} \cdot \frac{2}{3}$

5. $\frac{5}{8} \cdot \left(-\frac{4}{9}\right)$

6. $-\frac{4}{7} \cdot \left(-\frac{2}{3}\right)$

7. $2\frac{2}{5} \cdot \frac{1}{6}$

8. $-3\frac{1}{3} \cdot 1\frac{1}{2}$

9. $3\frac{3}{7} \cdot 2\frac{5}{8}$

10. $-1\frac{7}{8} \cdot \left(-2\frac{2}{5}\right)$

11. $-1\frac{3}{4} \cdot 2\frac{1}{5}$

12. $2\frac{2}{3} \cdot 2\frac{3}{7}$

2-4 Study Guide and Intervention

7NS1.2, 7MG1.3

Dividing Positive and Negative Fractions

Two numbers whose product is 1 are **multiplicative inverses**, or **reciprocals**, of each other.

Example 1 **Write the multiplicative inverse of $-2\frac{3}{4}$.**

$-2\frac{3}{4} = -\frac{11}{4}$ Write $-2\frac{3}{4}$ as an improper fraction.

Since $-\frac{11}{4}\left(-\frac{4}{11}\right) = 1$, the multiplicative inverse of $-2\frac{3}{4}$ is $-\frac{4}{11}$.

To divide by a fraction or mixed number, multiply by its multiplicative inverse.

Example 2 **Find $\frac{3}{8} \div \frac{6}{7}$. Write in simplest form.**

$\frac{3}{8} \div \frac{6}{7} = \frac{3}{8} \cdot \frac{7}{6}$ Multiply by the multiplicative inverse of $\frac{6}{7}$, which is $\frac{7}{6}$.

$= \frac{\overset{1}{\cancel{3}}}{8} \cdot \frac{7}{\underset{2}{\cancel{6}}}$ Divide 6 and 3 by their GCF, 3.

$= \frac{7}{16}$ Simplify.

Exercises

Write the multiplicative inverse of each number.

1. $\frac{3}{5}$

2. $-\frac{8}{9}$

3. $\frac{1}{10}$

4. $-\frac{1}{6}$

5. $2\frac{3}{5}$

6. $-1\frac{2}{3}$

7. $-5\frac{2}{5}$

8. $7\frac{1}{4}$

Divide. Write in simplest form.

9. $\frac{1}{3} \div \frac{1}{6}$

10. $\frac{2}{5} \div \frac{4}{7}$

11. $-\frac{5}{6} \div \frac{3}{4}$

12. $1\frac{1}{5} \div 2\frac{1}{4}$

13. $3\frac{1}{7} \div \left(-3\frac{2}{3}\right)$

14. $-\frac{4}{9} \div 2$

15. $\frac{6}{11} \div (-4)$

16. $5 \div 2\frac{1}{3}$

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2-5 Study Guide and Intervention

7NS1.2

Adding and Subtracting Like Fractions

Fractions that have the same denominator are called **like fractions**. To add like fractions, add the numerators of the fractions and write the sum over the denominator.

Example 1 **Find $\frac{1}{5} + \left(-\frac{4}{5}\right)$. Write in simplest form.**

$\frac{1}{5} + \left(-\frac{4}{5}\right) = \frac{1 + (-4)}{5}$ Add the numerators. The denominators are the same.

$= \frac{-3}{5}$ or $-\frac{3}{5}$ Simplify.

To subtract like fractions, subtract the numerators of the fractions and write the difference over the denominator.

Example 2 **Find $-\frac{4}{9} - \frac{7}{9}$. Write in simplest form.**

$-\frac{4}{9} - \frac{7}{9} = \frac{-4 - 7}{9}$ Subtract the numerators. The denominators are the same.

$= \frac{-11}{9}$ or $-1\frac{2}{9}$ Rename $\frac{-11}{9}$ as $-1\frac{2}{9}$.

To add or subtract mixed numbers, first write the mixed numbers as improper fractions. Then add or subtract the improper fractions and simplify the result.

Example 3 **Find $2\frac{3}{7} + 6\frac{5}{7}$. Write in simplest form.**

$2\frac{3}{7} + 6\frac{5}{7} = \frac{17}{7} + \frac{47}{7}$ Write the mixed numbers as improper fractions.

$= \frac{17 + 47}{7}$ Add the numerators. The denominators are the same.

$= \frac{64}{7}$ or $9\frac{1}{7}$ Rewrite $\frac{64}{7}$ as $9\frac{1}{7}$.

Exercises

Add or subtract. Write in simplest form.

1. $\frac{4}{7} + \frac{2}{7}$

2. $\frac{1}{10} + \frac{5}{10}$

3. $\frac{5}{9} + -\frac{1}{9}$

4. $\frac{1}{6} + -\frac{5}{6}$

5. $-\frac{3}{8} + \frac{7}{8}$

6. $\frac{5}{11} - \left(-\frac{4}{11}\right)$

7. $-\frac{4}{5} - \frac{3}{5}$

8. $-\frac{9}{13} + \left(-\frac{6}{13}\right)$

9. $2\frac{1}{4} + 1\frac{1}{4}$

10. $3\frac{5}{7} + 2\frac{3}{7}$

11. $3\frac{5}{8} - 1\frac{3}{8}$

12. $4\frac{3}{5} - 2\frac{4}{5}$

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2-6 Study Guide and Intervention

7NS1.2, 7NS2.2

Adding and Subtracting Unlike Fractions

Fractions with unlike denominators are called **unlike fractions**. To add or subtract unlike fractions, rename the fractions using the least common denominator. Then add or subtract as with like fractions.

Example 1 **Find $\frac{3}{5} + \frac{2}{3}$. Write in simplest form.**

$\frac{3}{5} + \frac{2}{3} = \frac{3}{5} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{5}{5}$	The LCD is 5 · 3 or 15.
$= \frac{9}{15} + \frac{10}{15}$	Rename each fraction using the LCD.
$= \frac{9 + 10}{15}$	Add the numerators. The denominators are the same.
$= \frac{19}{15}$ or $1\frac{4}{15}$	Simplify.

Example 2 **Find $-3\frac{1}{2} - 1\frac{5}{6}$. Write in simplest form.**

$-3\frac{1}{2} - 1\frac{5}{6} = -\frac{7}{2} - \frac{11}{6}$	Write the mixed numbers as improper fractions.
$= -\frac{7}{2} \cdot \frac{3}{3} - \frac{11}{6}$	The LCD is 2 · 3 or 6.
$= -\frac{21}{6} - \frac{11}{6}$	Rename $\frac{7}{2}$ using the LCD.
$= \frac{-21 - 11}{6}$	Subtract the numerators.
$= -\frac{32}{6}$ or $-\frac{16}{3}$ or $-5\frac{1}{3}$	Simplify.

Exxercises

Add or subtract. Write in simplest form.

1. $\frac{2}{5} + \frac{3}{10}$

2. $\frac{1}{3} + \frac{2}{9}$

3. $\frac{5}{9} + \left(-\frac{1}{6}\right)$

4. $-\frac{3}{4} - \frac{5}{6}$

5. $\frac{4}{5} - \left(-\frac{1}{3}\right)$

6. $1\frac{2}{3} - \left(-\frac{4}{9}\right)$

7. $-\frac{7}{10} - \left(-\frac{1}{2}\right)$

8. $2\frac{1}{4} + 1\frac{3}{8}$

9. $3\frac{3}{4} - 1\frac{1}{3}$

10. $-1\frac{1}{5} - 2\frac{1}{4}$

11. $-2\frac{4}{9} - \left(-1\frac{1}{3}\right)$

12. $3\frac{3}{5} - 2\frac{2}{3}$

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2-7 Study Guide and Intervention

7AF1.1, 7NS1.2

Solving Equations with Rational Numbers

The Addition, Subtraction, Multiplication, and Division Properties of Equality can be used to solve equations with rational numbers.

Example 1 **Solve $x - 2.73 = 1.31$. Check your solution.**

	$x - 2.73 = 1.31$	Write the equation.
	$x - 2.73 + 2.73 = 1.31 + 2.73$	Add 2.73 to each side.
	$x = 4.04$	Simplify.
Check	$x - 2.73 = 1.31$	Write the original equation.
	$4.04 - 2.73 \stackrel{?}{=} 1.31$	Replace x with 4.04.
	$1.31 = 1.31$ ✓	Simplify.

Example 2 **Solve $\frac{4}{5}y = \frac{2}{3}$. Check your solution.**

	$\frac{4}{5}y = \frac{2}{3}$	Write the equation.
	$\frac{5}{4}\left(\frac{4}{5}y\right) = \frac{5}{4} \cdot \frac{2}{3}$	Multiply each side by $\frac{5}{4}$.
	$y = \frac{5}{6}$	Simplify.
Check	$\frac{4}{5}y = \frac{2}{3}$	Write the original equation.
	$\frac{4}{5}\left(\frac{5}{6}\right) \stackrel{?}{=} \frac{2}{3}$	Replace y with $\frac{5}{6}$.
	$\frac{2}{3} = \frac{2}{3}$ ✓	Simplify.

Exercises

Solve each equation. Check your solution.

1. $t + 1.32 = 3.48$

2. $b - 4.22 = 7.08$

3. $-8.07 = r - 4.48$

4. $h + \frac{4}{9} = \frac{7}{9}$

5. $-\frac{5}{8} = x - \frac{1}{4}$

6. $-\frac{2}{3} + f = \frac{3}{5}$

7. $3.2c = 9.6$

8. $-5.04 = 1.26d$

9. $\frac{3}{5}x = 6$

10. $-\frac{2}{3} = \frac{3}{4}t$

11. $\frac{w}{2.5} = 4.2$

12. $1\frac{3}{4}r = 3\frac{5}{8}$

2-8 Study Guide and Intervention

7MR2.4, 7NS1.2

Problem-Solving Investigation: Look for a Pattern

You may need to look for a pattern to solve a problem.

Explore Determine what information is given in the problem and what you need to find.

Plan Select a strategy including a possible estimate.

Solve Solve the problem by carrying out your plan.

Check Examine your answer to see if it seems reasonable.

Example

Three people board the subway train at the first stop. Five people board the train at the second stop. Seven people board the train at the third stop. If this pattern continues and no one gets off the train, how many people are on the subway train when it reaches the seventh and final stop?

Explore You know that 3 people boarded the subway train at the first stop. At each subsequent stop, 2 more people board the train than at the previous stop.

Plan Look for a pattern and use the pattern to find how many people boarded the train in all.

Solve Complete the information for the first, second, and third stops. Continue the pattern to solve the problem.

First Stop	Second Stop	Third Stop	Fourth Stop	Fifth Stop	Sixth Stop	Seventh Stop
3	5	7	9	11	13	15
3 people on the train	3 + 5 = 8 people on the train	8 + 7 = 15 people on the train	15 + 9 = 24 people on the train	24 + 11 = 35 people on the train	35 + 13 = 48 people on the train	48 + 15 = 63 people on the train

At the seventh and final stop there were 63 people on the subway train.

Check Check your pattern to make sure the answer is correct.

Exercises

Look for a pattern. Then use the pattern to solve each problem.

1. **COOKING** A muffin recipe calls for $2\frac{1}{2}$ cups of flour for every $\frac{2}{3}$ cup of sugar. How many cups of flour should be used when 4 cups of sugar are used?

2. **FUNDRAISER** There were 256 people at a fundraiser. When the event was over, half of the people who remained left every 5 minutes. How long after the event ended did the last person leave?

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2-9 Study Guide and Intervention

7NS1.2, 7NS2.1, 7AF2.1

Powers and Exponents

Expressions containing repeated factors can be written using exponents.

Example 1 **Write $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ using exponents.**

Since 7 is used as a factor 5 times, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^5$.

Example 2 **Write $p \cdot p \cdot p \cdot q \cdot q$ using exponents.**

Since p is used as a factor 3 times and q is used as a factor 2 times, $p \cdot p \cdot p \cdot q \cdot q = p^3 \cdot q^2$.

Any nonzero number to the zero power is 1. Any nonzero number to the negative *n* power is the multiplicative inverse of *n*th power.

Example 3 **Evaluate 6^2.**

$6^2 = 6 \cdot 6$ Definition of exponents

$= 36$ Simplify.

Example 4 **Evaluate 5^{-3}.**

$5^{-3} = \frac{1}{5^3}$ Definition of negative exponents

$= \frac{1}{125}$ Simplify.

Exercises

Write each expression using exponents.

1. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

2. $4 \cdot 4 \cdot 4 \cdot 4$

3. $a \cdot a \cdot a \cdot a \cdot a \cdot a$

4. $g \cdot g \cdot g \cdot g \cdot g \cdot g \cdot g$

5. $5 \cdot 5 \cdot 9 \cdot 9 \cdot 5 \cdot 9 \cdot 5 \cdot 5$

6. $s \cdot w \cdot w \cdot s \cdot s \cdot s$

Evaluate each expression.

7. 4^2

8. 5^3

9. 13^2

10. $2^3 \cdot 3^2$

11. 8^{-2}

12. $2^4 \cdot 5^2$

13. 3^{-4}

14. $3^4 \cdot 7^2$

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2-10 Study Guide and Intervention

7NS1.1

Scientific Notation

A number in scientific notation is written as the product of a factor that is at least one, but less than ten, and a power of ten.

Example 1 **Write 8.65×10^7 in standard form.**

$8.65 \times 10^7 = 8.65 \times 10{,}000{,}000$ — $10^7 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ or 10,000,000

$= 86{,}500{,}000$ — Move the decimal point 7 places to the right.

Example 2 **Write 9.2×10^{-3} in standard form.**

$9.2 \times 10^{-3} = 9.2 \times \frac{1}{10^3}$ — $10^{-3} = \frac{1}{10^3}$

$= 9.2 \times 0.001$ — $\frac{1}{10^3} = \frac{1}{1{,}000}$ or 0.001

$= 0.0092$ — Move the decimal point 3 places to the left.

Example 3 **Write 76,250 in scientific notation.**

$76{,}250 = 7.625 \times 10{,}000$ — The decimal point moves 4 places.

$= 7.625 \times 10^4$ — The exponent is positive.

Example 4 **Write 0.00157 in scientific notation.**

$0.00157 = 1.57 \times 0.001$ — The decimal point moves 3 places.

$= 1.57 \times 10^{-3}$ — The exponent is negative.

Exercises

Write each number in standard form.

1. 5.3×10^1

2. 9.4×10^3

3. 7.07×10^5

4. 2.6×10^{-3}

5. 8.651×10^{-2}

6. 6.7×10^{-6}

Write each number in scientific notation.

7. 561

8. 14

9. 56,400,000

10. 0.752

11. 0.0064

12. 0.000581

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3-1 Study Guide and Intervention

7NS2.4

Square Roots

The square root of a number is one of two equal factors. The radical sign $\sqrt{\ }$ is used to indicate a square root.

Examples **Find each square root.**

1. $\sqrt{1}$ — Since $1 \cdot 1 = 1$, $\sqrt{1} = 1$.
2. $-\sqrt{16}$ — Since $4 \cdot 4 = 16$, $-\sqrt{16} = -4$.
3. $\sqrt{0.25}$ — Since $0.5 \cdot 0.5 = 0.25$, $\sqrt{0.25} = 0.5$.
4. $\sqrt{\frac{25}{36}}$ — Since $\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$, $\sqrt{\frac{25}{36}} = \frac{5}{6}$.

Example 5 **Solve $a^2 = \frac{4}{9}$.**

$a^2 = \frac{4}{9}$ — Write the equation.

$\sqrt{a^2} = \sqrt{\frac{4}{9}}$ — Take the square root of each side.

$a = \frac{2}{3}$ or $-\frac{2}{3}$ — Notice that $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$ and $\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = \frac{4}{9}$.

The equation has two solutions, $\frac{2}{3}$ and $-\frac{2}{3}$.

Exercises

Find each square root.

1. $\sqrt{4}$

2. $\sqrt{9}$

3. $-\sqrt{49}$

4. $-\sqrt{25}$

5. $\sqrt{0.01}$

6. $-\sqrt{0.64}$

7. $\sqrt{\frac{9}{16}}$

8. $-\sqrt{\frac{1}{25}}$

ALGEBRA Solve each equation.

9. $x^2 = 121$

10. $a^2 = 3{,}600$

11. $p^2 = \frac{81}{100}$

12. $t^2 = \frac{121}{196}$

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3-2 Study Guide and Intervention

7NS2

Estimating Square Roots

Most numbers are not perfect squares. You can estimate square roots for these numbers.

Example 1 **Estimate $\sqrt{204}$ to the nearest whole number.**

- The first perfect square less than 204 is 14.
- The first perfect square greater than 204 is 15.

$196 < 204 < 225$ Write an inequality.

$14^2 < 204 < 15^2$ $196 = 14^2$ and $225 = 15^2$

$14 < \sqrt{204} < 15$ Take the square root of each number.

So, $\sqrt{204}$ is between 14 and 15. Since 204 is closer to 196 than 225, the best whole number estimate for $\sqrt{204}$ is 14.

Example 2 **Estimate $\sqrt{79.3}$ to the nearest whole number.**

- The first perfect square less than 79.3 is 64.
- The first perfect square greater than 79.3 is 81.

$64 < 79.3 < 81$ Write an inequality.

$8^2 < 79.3 < 9^2$ $64 = 8^2$ and $81 = 9^2$

$8 < \sqrt{79.3} < 9$ Take the square root of each number.

So, $\sqrt{79.3}$ is between 8 and 9. Since 79.3 is closer to 81 than 64, the best whole number estimate for $\sqrt{79.3}$ is 9.

Exercises

Estimate to the nearest whole number.

1. $\sqrt{8}$

2. $\sqrt{37}$

3. $\sqrt{14}$

4. $\sqrt{26}$

5. $\sqrt{62}$

6. $\sqrt{48}$

7. $\sqrt{103}$

8. $\sqrt{141}$

9. $\sqrt{14.3}$

10. $\sqrt{51.2}$

11. $\sqrt{82.7}$

12. $\sqrt{175.2}$

3-3 Study Guide and Intervention

7MR2.5, 7NS1.2

Problem Solving Investigation: Use a Venn Diagram

You may need to use a Venn diagram to solve some problems.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

Example

Of the 25 skiers on the ski team, 13 signed up to race in the Slalom race, and 8 signed up for the Giant Slalom race. Six skiers signed up to ski in both the Slalom and the Giant Slalom races. How many skiers did not sign up for any races?

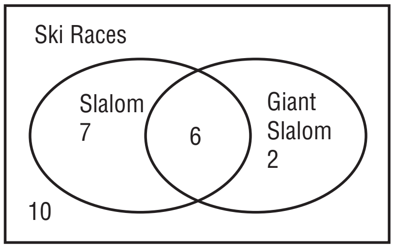
Explore You know how many skiers signed up for each race and how many signed up for both races. You need to organize the information.

Plan You can use a Venn diagram to organize the information.

Solve Draw two overlapping circles to represent the two different races. Place a 6 in the section that is a part of both circles. Use subtraction to determine the number for each other section.

only the Slalom race: $13 - 6 = 7$
only the Giant Slalom race: $8 - 6 = 2$
neither the Slalom or the Giant Slalom race:
$25 - 7 - 2 - 6 = 10$

There were 10 skiers who did not sign up for either race.

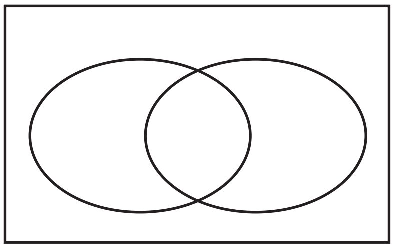


Check Check each circle to see if the appropriate number of students is represented.

Exercise

Use a Venn diagram to solve the problem.

SPORTS The athletic club took a survey to find out what sports students might participate in next fall. Of the 80 students surveyed, 42 wanted to play football, 37 wanted to play soccer, and 15 wanted to play both football and soccer. How many students did not want to play either sport in the fall?



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3-4 Study Guide and Intervention

7NS1.4

The Real Number System

Numbers may be classified by identifying to which of the following sets they belong.

Whole Numbers	0, 1, 2, 3, 4, …
Integers	…, −2, −1, 0, 1, 2, …
Rational Numbers	numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$
Irrational Numbers	numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Examples **Name all sets of numbers to which each real number belongs.**

1. 5 — whole number, integer, rational number

2. 0.666… — Decimals that terminate or repeat are rational numbers, since they can be expressed as fractions. $0.666\ldots = \frac{2}{3}$

3. $-\sqrt{25}$ — Since $-\sqrt{25} = -5$, it is an integer and a rational number.

4. $-\sqrt{11}$ — $\sqrt{11} \approx 3.31662479\ldots$ Since the decimal does not terminate or repeat, it is an irrational number.

To compare real numbers, write each number as a decimal and then compare the decimal values.

Example 5 **Replace ● with <, >, or = to make $2\frac{1}{4}$ ● $\sqrt{5}$ a true sentence.**

Write each number as a decimal.

$2\frac{1}{4} = 2.25$

$\sqrt{5} \approx 2.236067\ldots$

Since 2.25 is greater than 2.236067…, $2\frac{1}{4} > \sqrt{5}$.

Exercises

Name all sets of numbers to which each real number belongs.

1. 30

2. -11

3. $5\frac{4}{7}$

4. $\sqrt{21}$

5. 0

6. $-\sqrt{9}$

7. $\frac{6}{3}$

8. $-\sqrt{101}$

Replace each ● with <, >, or = to make a true sentence.

9. 2.7 ● $\sqrt{7}$

10. $\sqrt{11}$ ● $3\frac{1}{2}$

11. $4\frac{1}{6}$ ● $\sqrt{17}$

12. $3.\overline{8}$ ● $\sqrt{15}$

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3-5 Study Guide and Intervention

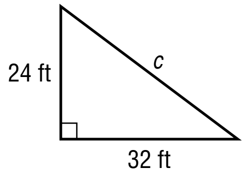
7MG3.3, 7MR3.2

The Pythagorean Theorem

The **Pythagorean Theorem** describes the relationship between the lengths of the legs of any right triangle. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. You can use the Pythagorean Theorem to find the length of a side of a right triangle if the lengths of the other two legs are known.

Examples **Find the missing measure for each right triangle. Round to the nearest tenth if necessary.**

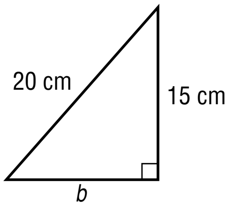
1



$$c^2 = a^2 + b^2$$
$$c^2 = 24^2 + 32^2$$
$$c^2 = 576 + 1{,}024$$
$$c^2 = 1{,}600$$
$$c = \pm\sqrt{1{,}600}$$
$$c = 40 \text{ or } -40$$

Length must be positive, so the length of the hypotenuse is 40 feet.

2



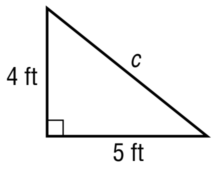
$$c^2 = a^2 + b^2$$
$$20^2 = 15^2 + b^2$$
$$400 = 225 + b^2$$
$$400 - 225 = 225 + b^2 - 225$$
$$175 = b^2$$
$$\sqrt{175} = \sqrt{b^2}$$
$$13.2 \approx b$$

The length of the other leg is about 13.2 centimeters.

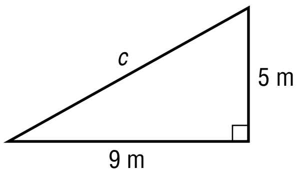
Exercises

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

1.



2.



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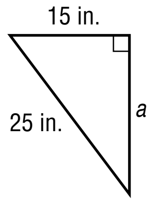


4. $a = 7$ km, $b = 12$ km

5. $a = 10$ yd, $c = 25$ yd

6. $b = 14$ ft, $c = 20$ ft

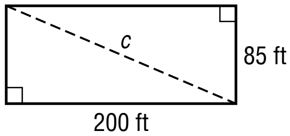
3-6 Study Guide and Intervention

7MG3.3

Using the Pythagorean Theorem

You can use the Pythagorean Theorem to help you solve problems.

Example 1 **A professional ice hockey rink is 200 feet long and 85 feet wide. What is the length of the diagonal of the rink?**



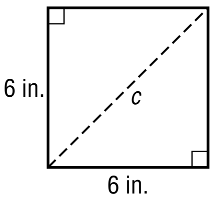
$c^2 = a^2 + b^2$	The Pythagorean Theorem
$c^2 = 200^2 + 85^2$	Replace *a* with 200 and *b* with 85.
$c^2 = 40{,}000 + 7{,}225$	Evaluate 200^2 and 85^2.
$c^2 = 47{,}225$	Simplify.
$\sqrt{c^2} = \sqrt{47{,}225}$	Take the square root of each side.
$c \approx 217.3$	Simplify.

The length of the diagonal of an ice hockey rink is about 217.3 feet.

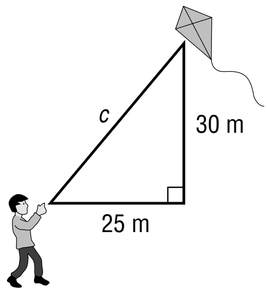
Exercises

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

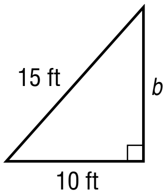
1. What is the length of the diagonal?



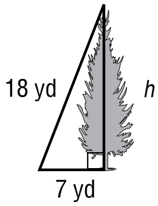
2. How long is the kite string?



3. How high is the ramp?



4. How tall is the tree?



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3-7 Study Guide and Intervention

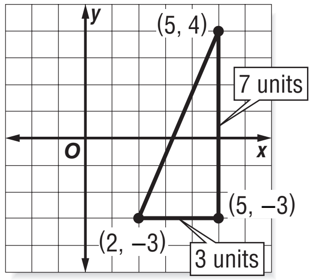
7MG3.2

Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

Example **Find the distance between points (2, −3) and (5, 4).**

Graph the points and connect them with a line segment. Draw a horizontal line through (2, −3) and a vertical line through (5, 4). The lines intersect at (5, −3).



Count units to find the length of each leg of the triangle. The lengths are 3 units and 7 units. Then use the Pythagorean Theorem to find the hypotenuse.

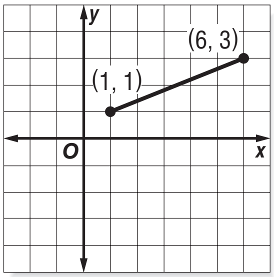
$c^2 = a^2 + b^2$	The Pythagorean Theorem
$c^2 = 3^2 + 7^2$	Replace a with 3 and b with 7.
$c^2 = 9 + 49$	Evaluate 3^2 and 7^2.
$c^2 = 58$	Simplify.
$\sqrt{c^2} = \sqrt{58}$	Take the square root of each side.
$c \approx 7.6$	Simplify.

The distance between the points is about 7.6 units.

Exercises

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth if necessary.

1.



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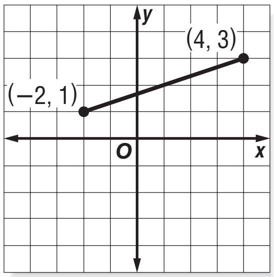
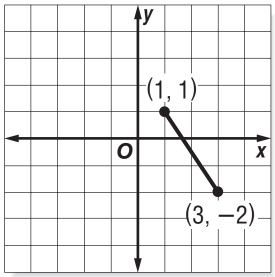
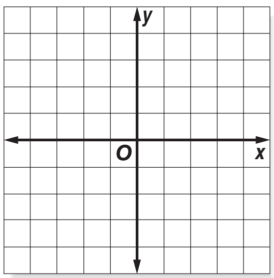


3.

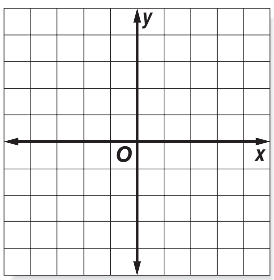


Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

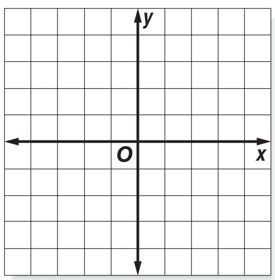
4. (4, 5), (0, 2)



5. (0, −4), (−3, 0)



6. (−1, 1), (−4, 4)



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4-1 Study Guide and Intervention

7AF4.2, 7MG1.3

Ratios and Rates

A **ratio** is a comparison of two numbers by quantities. Since a ratio can be written as a fraction, it can be simplified.

Example 1 **Express *35 wins to 42 losses* in simplest form.**

$\frac{35}{42} = \frac{5}{6}$ Divide the numerator and denominator by the greatest common factor, 7.

The ratio in simplest form is $\frac{5}{6}$ or 5:6.

Example 2 **Express *1 foot to 3 inches* in simplest form.**

To simplify a ratio involving measurements, both quantities must have the same unit of measure.

$\frac{\text{1 foot}}{\text{3 inches}} = \frac{\text{12 inches}}{\text{3 inches}}$ Convert 1 foot to 12 inches.

$= \frac{\text{4 inches}}{\text{1 inch}}$ Divide the numerator and denominator by 3.

The ratio in simplest form is $\frac{4}{1}$ or 4:1.

A **rate** is a ratio that compares two quanitities with different types of units. A unit rate is a rate with a denominator of 1.

Example 3 **Express *309 miles in 6 hours* as a unit rate.**

$\frac{\text{309 miles}}{\text{6 hours}} = \frac{\text{51.5 miles}}{\text{1 hour}}$ Divide the numerator and denominator by 6 to get a denominator of 1.

The unit rate is 51.5 miles per hour.

Exercises

Express each ratio in simplest form.

1. 3 out of 9 students

2. 8 passengers:2 cars

3. 5 out of 10 dentists

4. 35 boys:60 girls

5. 18 red apples to 42 green apples

6. 50 millimeters to 1 meter

Express each rate as a unit rate.

7. 12 waves in 2 hours

8. 200 miles in 4 hours

9. 21 gallons in 2.4 minutes

10. $12 for 4.8 pounds

11. $49,500 in 12 months

12. 112 feet in 5 seconds

4-2 Study Guide and Intervention

7AF3.4

Proportional and Nonproportional Relationships

Two related quantities are **proportional** if they have a constant ratio between them. If two related quantities do not have a constant ratio, then they are **nonproportional**.

Example 1 **The cost of one CD at a record store is $12. Create a table to show the total cost for different numbers of CDs. Is the total cost proportional to the number of CDs purchased?**

Number of CDs	1	2	3	4
Total Cost	$12	$24	$36	$48

$$\frac{\text{Total Cost}}{\text{Number of CDs}} = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \frac{48}{4} = \$12 \text{ per CD}$$

Divide the total cost for each by the number of CDs to find a ratio. Compare the ratios.

Since the ratios are the same, the total cost is proportional to the number of CDs purchased.

Example 2 **The cost to rent a lane at a bowling alley is $9 per hour plus $4 for shoe rental. Create a table to show the total cost for each hour a bowling lane is rented if one person rents shoes. Is the total cost proportional to the number of hours rented?**

Number of Hours	1	2	3	4
Total Cost	$13	$22	$31	$40

$$\frac{\text{Total Cost}}{\text{Number of Hours}} \rightarrow \frac{13}{1} \text{ or } 13 \quad \frac{22}{2} \text{ or } 11 \quad \frac{31}{3} \text{ or } 10.34 \quad \frac{40}{4} \text{ or } 10$$

Divide each cost by the number of hours.

Since the ratios are not the same, the total cost is nonproportional to the number of hours rented with shoes.

Exercises

Use a table of values to explain your reasoning.

1. **PICTURES** A photo developer charges $0.25 per photo developed. Is the total cost proportional to the number of photos developed?

2. **SOCCER** A soccer club has 15 players for every team, with the exception of two teams that have 16 players each. Is the number of players proportional to the number of teams?

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4-3 Study Guide and Intervention

7AF4.2

Solving Proportions

A **proportion** is an equation that states that two ratios are equivalent. To determine whether a pair of ratios forms a proportion, use cross products. You can also use cross products to solve proportions.

Example 1 **Determine whether the pair of ratios $\frac{20}{24}$ and $\frac{12}{18}$ forms a proportion.**

Find the cross products.

$\frac{20}{24} \stackrel{?}{=} \frac{12}{18}$ → $24 \cdot 12 = 288$
→ $20 \cdot 18 = 360$

Since the cross products are not equal, the ratios do not form a proportion.

Example 2 **Solve $\frac{12}{30} = \frac{k}{70}$.**

$\frac{12}{30} = \frac{k}{70}$ **Write the equation.**

$12 \cdot 70 = 30 \cdot k$ **Find the cross products.**

$840 = 30k$ **Multiply.**

$\frac{840}{30} = \frac{30k}{30}$ **Divide each side by 30.**

$28 = k$ **Simplify.** The solution is 28.

Exercises

Determine whether each pair of ratios forms a proportion.

1. $\frac{17}{10}, \frac{12}{5}$

2. $\frac{6}{9}, \frac{12}{18}$

3. $\frac{8}{12}, \frac{10}{15}$

4. $\frac{7}{15}, \frac{13}{32}$

5. $\frac{7}{9}, \frac{49}{63}$

6. $\frac{8}{24}, \frac{12}{28}$

7. $\frac{4}{7}, \frac{12}{71}$

8. $\frac{20}{35}, \frac{30}{45}$

9. $\frac{18}{24}, \frac{3}{4}$

Solve each proportion.

10. $\frac{x}{5} = \frac{15}{25}$

11. $\frac{3}{4} = \frac{12}{c}$

12. $\frac{6}{9} = \frac{10}{r}$

13. $\frac{16}{24} = \frac{z}{15}$

14. $\frac{5}{8} = \frac{s}{12}$

15. $\frac{14}{t} = \frac{10}{11}$

16. $\frac{w}{6} = \frac{2.8}{7}$

17. $\frac{5}{y} = \frac{7}{16.8}$

18. $\frac{x}{18} = \frac{7}{36}$

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4-4 Study Guide and Intervention

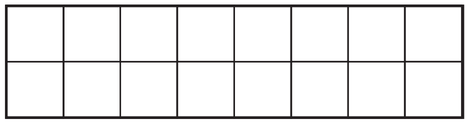
7MR2.5, 7AF4.2

Problem-Solving Investigation: Draw a Diagram

Example **It takes a worker 4 minutes to stack 2 rows of 8 boxes in a warehouse. How long will it take to stack 8 rows of 8 boxes? Use the draw a diagram strategy to solve the problem.**

Explore After 4 minutes, a worker has stacked a 2 rows of 8 boxes. At this rate, how long would it take to stack 8 rows of boxes?

Plan Draw a diagram showing the level of boxes after 4 minutes.



Solve 2 rows of 8 boxes = 4 minutes
8 rows = 4×2 rows, so multiply the time by 4.
4×4 minutes = 16 minutes

Check

8 boxes × 2 rows of boxes = 16 boxes	Multiply to find the total number of boxes in the stack.
4 minutes ÷ 16 boxes = 0.25 min. per box	Divide the number of minutes by the number of boxes.
8 boxes × 8 rows of boxes = 64 boxes	Multiply to find the number of boxes in the new stack.
64 boxes × 0.25 min. = 16 minutes	Multiply the number of boxes by the time per box.

It will take 16 minutes to stack an 8×8 wall of boxes.

Exercises

For Exercises 1–4, use the draw a diagram strategy to solve the problem.

1. **GAS** A car's gas tank holds 16 gallons. After filling it for 20 seconds, the tank contains 2.5 gallons. How many more seconds will it take to fill the tank?

2. **TILING** It takes 96 tiles to fill a 2-foot by 3-foot rectangle. How many tiles would it take to fill a 4-foot by 6-foot rectangle?

3. **BEVERAGES** Four juice cartons can fill 36 glasses of juice equally. How many juice cartons are needed to fill 126 glasses equally?

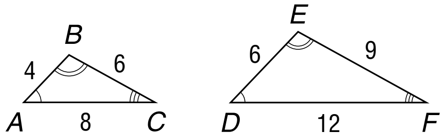
4. **PACKAGING** It takes 5 large shipping boxes to hold 120 boxes of an action figure. How many action figures would 8 large shipping boxes hold?

4-5 Study Guide and Intervention

Similar Polygons

Two polygons are **similar** if their corresponding angles are congruent and their corresponding side measures are proportional.

Example 1 **Determine whether $\triangle ABC$ is similar to $\triangle DEF$. Explain your reasoning.**



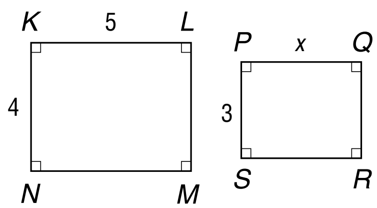
$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$,

$\frac{AB}{DE} = \frac{4}{6}$ or $\frac{2}{3}$, $\frac{BC}{EF} = \frac{6}{9}$ or $\frac{2}{3}$, $\frac{AC}{DF} = \frac{8}{12}$ or $\frac{2}{3}$

The corresponding angles are congruent, and the corresponding sides are proportional.

Thus, $\triangle ABC$ is similar to $\triangle DEF$.

Example 2 **Given that polygon $KLMN \sim$ polygon $PQRS$, write a proportion to find the measure of $\overline{PQ}$. Then solve.**

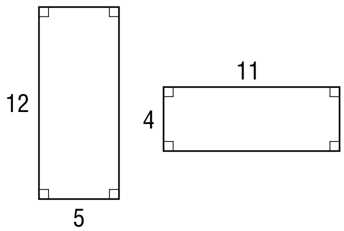


The ratio of corresponding sides from polygon $KLMN$ to polygon $PQRS$ is $\frac{4}{3}$. Write a proportion with this scale factor. Let x represent the measure of $\overline{PQ}$.

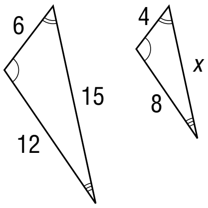
$\frac{KL}{PQ} = \frac{4}{3}$	$\overline{KL}$ corresponds to $\overline{PQ}$. The scale factor is $\frac{4}{3}$.
$\frac{5}{x} = \frac{4}{3}$	$KL = 5$ and $PQ = x$
$5 \cdot 3 = x \cdot 4$	Find the cross products.
$\frac{15}{4} = \frac{4x}{4}$	Multiply. Then divide each side by 4.
$3.75 = x$	Simplify.

Exercises

1. Determine whether the polygons below are similar. Explain your reasoning.



2. The triangles below are similar. Write a proportion to find each missing measure. Then solve.



4-6 Study Guide and Intervention

7MG1.1

Converting Measures

A **unit ratio** is a ratio in which the denominator is 1 and the numerator and denominator are equivalent so that the value of the ratio is 1.

Example 1 **Convert $2\frac{1}{2}$ meters to centimeters.**

$$2\frac{1}{2}\text{ m} = 2\frac{1}{2}\text{ m} \cdot \frac{100\text{ cm}}{1\text{ m}}$$
$$= 2\frac{1}{2} \cdot 100\text{ cm or }250\text{ cm}$$

Example 2 **Convert 5.4 liters to gallons.**

Use 1 gal ≈ 3.7854 L

$$5.4\text{ L} \approx 5.4\text{ L} \cdot \frac{1\text{ gal}}{3.7854\text{ L}}$$
$$\approx 5.4 \cdot \frac{1\text{ gal}}{3.7854}\text{ or }1.43\text{ gal}$$

Example 3 DRIVING **While driving out of town for a business trip, Mr. Johansen averaged a speed of 65 miles per hour. How fast is this in meters per second?**

$$\frac{65\text{ mi}}{1\text{ hr}} \cdot \frac{1\text{ km}}{0.6214\text{ mi}} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}}$$
$$= \frac{65\text{ mi}}{1\text{ hr}} \cdot \frac{1\text{ km}}{0.6214\text{ mi}} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}}$$
$$= \frac{65000\text{ m}}{2237.04\text{ sec}}$$
$$= \frac{29.06\text{ m}}{1\text{ sec}}$$

Mr. Johansen averaged 29.06 meters per second.

Exercises **Complete each conversion. Round to the nearest hundredth if necessary.**

1. 3 T = ■ lb

2. $5\frac{1}{2}$ c = ■ fl oz

3. 4.75 m = ■ mm

4. 2,473 m = ■ km

5. 8.5 yd = ■ ft

6. $1\frac{1}{4}$ L = ■ mL

7. 4 oz ≈ ■ g

8. 5.5 yd ≈ ■ m

9. 15 pt ≈ ■ L

10. RESTAURANT During the dinner rush, a certain restaurant cooks about 25 pounds of spaghetti per hour. How many grams of spaghetti do they cook per second?

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4-7 Study Guide and Intervention

7MG1.1, 7MG2.4

Converting Square and Cubic Units of Measure

Some of the units of area in the customary system are in^2, ft^2, yd^2, and mi^2.
Some of the units of area in the metric system are cm^2 and m^2.

Example 1 **Convert 5 square yards to square feet.**

$$5\ yd^2 = 5 \cdot \cancel{yd} \cdot \cancel{yd} \cdot \frac{3\ ft}{1\ \cancel{yd}} \cdot \frac{3\ ft}{1\ \cancel{yd}}$$
$$= 45\ ft^2$$

Example 2 **Convert 2.5 square meters to square centimeters.**

$$2.5\ m^2 = 2.5 \cdot \cancel{m} \cdot \cancel{m} \cdot \frac{100\ cm}{1\ \cancel{m}} \cdot \frac{100\ cm}{1\ \cancel{m}}$$
$$= 25{,}000\ cm^2$$

Some of the units of volume in the customary system are in^3, ft^3, yd^3, and mi^3.
Some of the units of volume in the metric system are cm^3 and m^3.

Example 3 **Convert 1,500 cubic centimeters to cubic meters.**

$$1{,}500\ cm^3 = 1{,}500 \cdot \cancel{cm} \cdot \cancel{cm} \cdot \cancel{cm} \cdot \frac{1\ m}{100\ \cancel{cm}} \cdot \frac{1\ m}{100\ \cancel{cm}} \cdot \frac{1\ m}{100\ \cancel{cm}}$$
$$= \frac{1{,}500\ m^3}{1{,}000{,}000}$$
$$= \frac{65{,}000\ m}{2{,}237.04\ sec}$$
$$= 0.0015\ m^3$$

Convert 30 square feet to square meters.

$$30\ ft^2 = 30 \cdot \cancel{ft} \cdot \cancel{ft} \cdot \frac{0.3048\ m}{1\ \cancel{ft}} \cdot \frac{0.3048\ m}{1\ \cancel{ft}}$$
$$\approx 2.79\ m^2$$

Example 4 **Complete each conversion. Round to the nearest hundredth if necessary.**

1. $6\ ft^2 = ■\ in^2$

2. $0.25\ m^2 = ■\ cm^2$

3. $18\ ft^2 = ■\ yd^2$

4. $189\ ft^3 = ■\ yd^3$

5. $2\ m^3 = ■\ cm^3$

6. $3{,}456\ in^3 = ■\ ft^3$

7. $24\ cm^2 \approx ■\ in^2$

8. $15\ ft^3 \approx ■\ m^3$

9. $7\ in^3 \approx ■\ cm^3$

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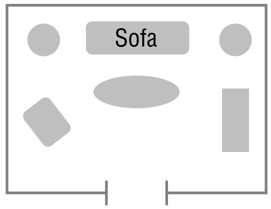
4-8 Study Guide and Intervention

7MG1.2

Scale Drawings and Models

Distances on a scale drawing or model are proportional to real-life distances. The **scale** is determined by the ratio of a given length on a drawing or model to its corresponding actual length.

Example 1 INTERIOR DESIGN **A designer has made a scale drawing of a living room for one of her clients. The scale of the drawing is 1 inch = $1\frac{1}{3}$ feet. On the drawing, the sofa is 6 inches long. Find the actual length of the sofa.**



Let x represent the actual length of the sofa. Write and solve a proportion.

Drawing Scale → $\frac{1 \text{ in.}}{1\frac{1}{3} \text{ ft}}$ (drawing distance → 1 in.; actual distance → $1\frac{1}{3}$ ft)

Actual Length → $\frac{6 \text{ in.}}{x \text{ ft}}$ (← drawing length; ← actual length)

$$\frac{1 \text{ in.}}{1\frac{1}{3} \text{ ft}} = \frac{6 \text{ in.}}{x \text{ ft}}$$

$1 \cdot x = 1\frac{1}{3} \cdot 6$ Find the cross products.

$x = 8$ Simplify.

The actual length of the sofa is 8 feet.

To find the scale factor for scale drawings and models, write the ratio given by the scale in simplest form.

Example 2 **Find the scale factor for the drawing in Example 1.**

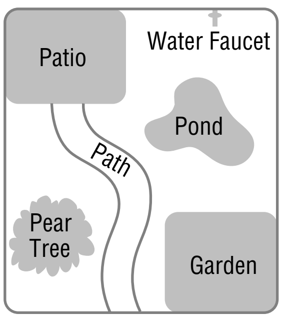
Write the ratio of 1 inch to $1\frac{1}{3}$ feet in simplest form.

$\frac{1 \text{ in.}}{1\frac{1}{3} \text{ ft}} = \frac{1 \text{ in.}}{16 \text{ in.}}$ **Convert $1\frac{1}{3}$ feet to inches.**

The scale factor is $\frac{1}{16}$ or 1:16. This means that each distance on the drawing is $\frac{1}{16}$ the actual distance.

Exercises

LANDSCAPING **Yutaka has made a scale drawing of his yard. The scale of the drawing is 1 centimeter = 0.5 meter.**



1. The length of the patio is 4.5 centimeters in the drawing. Find the actual length.
2. The actual distance between the water faucet and the pear tree is 11.2 meters. Find the corresponding distance on the drawing.
3. Find the scale factor for the drawing.

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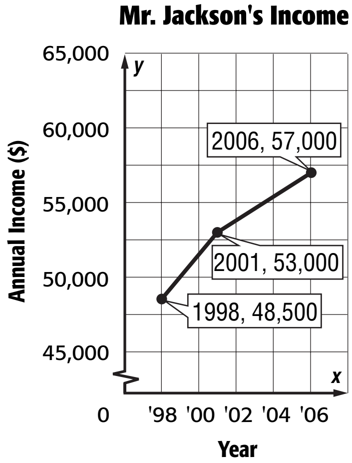
4-9 Study Guide and Intervention

7AF3.4

Rate of Change

To find the rate of change between two data points, divide the difference of the *y*-coordinates by the difference of the *x*-coordinates. The rate of change between (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

Example INCOME **The graph shows Mr. Jackson's annual income between 1998 and 2006. Find the rate of change in Mr. Jackson's income between 1998 and 2001.**



Use the formula for the rate of change.
Let $(x_1, y_1) = (1998, 48{,}500)$ and $(x_2, y_2) = (2001, 53{,}000)$.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{53{,}000 - 48{,}500}{2001 - 1998}$ Write the formula for rate of change.

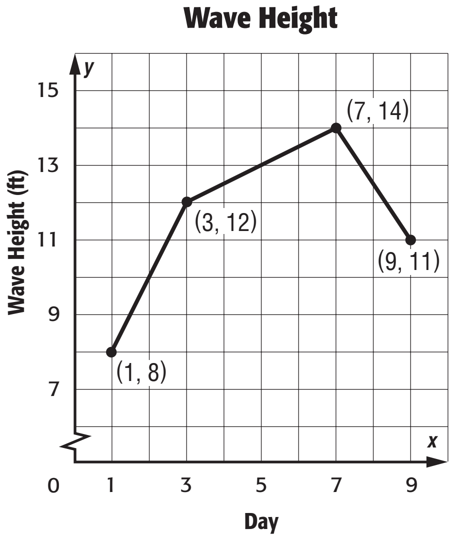
$= \frac{4{,}500}{3}$ Simplify.

$= \frac{1{,}500}{1}$ Express this rate as a unit rate.

Between 1998 and 2001, Mr. Jackson's income increased an average of $1,500 per year.

Exercises

SURF **For Exercises 1–3, use the graph that shows the average daily wave height as measured by an ocean buoy over a nine-day period.**



1. Find the rate of change in the average daily wave height between day 1 and day 3.

2. Find the rate of change in the average daily wave height between day 3 and day 7.

3. Find the rate of change in the average daily wave height between day 7 and day 9.

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4-10 Study Guide and Intervention

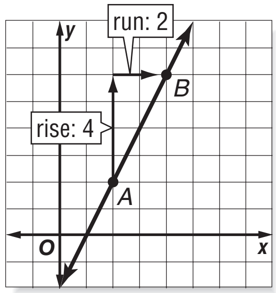
7AF3.4

Constant Rate of Change

The **slope** of a line is the ratio of the rise, or vertical change, to the run, or horizontal change.

Example 1 **Find the rate of change of the line in the graph if the *x*-axis represents seconds and the *y*-axis represents feet.**

Choose two points on the line. The vertical change from point *A* to point *B* is 4 units while the horizontal change is 2 units.



$\text{rate of change} = \frac{\text{feet}}{\text{seconds}}$ Definition of rate of change

$= \frac{4}{2}$ Difference in feet between two points divided by the difference in seconds for those two points

$= 2$ Simplify.

The rate of change is 2 feet per second.

Example 2 **The points in the table lie on a line. Find the rate of change in the line.**

− 4 − 4 − 4

seconds	5	1	−3	−7
feet	−2	1	4	7

+ 3 + 3 + 3

$\text{rate of change} = \frac{\text{feet}}{\text{seconds}}$ ← change in *y* / ← change in *x*

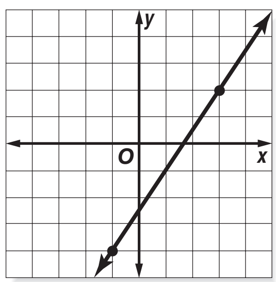
$= \frac{-4}{3} \text{ or } -\frac{4}{3}$

The rate of change is $-\frac{4}{3}$ feet per second.

Exercises

Find the rate of change for each line.

1.



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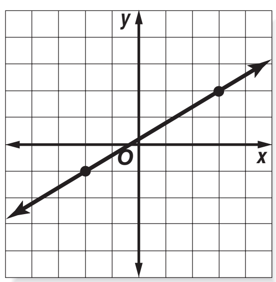
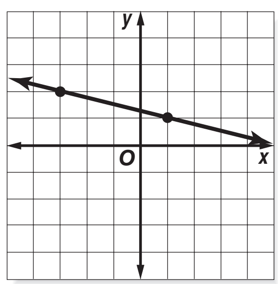


3.



The points given in each table lie on a line. Find the rate of change for the line.

4.

x	3	5	7	9
y	−1	2	5	8

5.

x	−5	0	5	10
y	4	3	2	1

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5-1 Study Guide and Intervention

7NS1.3

Ratios and Percents

A **percent** is a ratio that compares a number to 100. To write a fraction as a percent, find an equivalent fraction with a denominator of 100. If the denominator is a factor of 100, you can use mental math.

Examples **Write each ratio or fraction as a percent.**

1 **James made 65 out of 100 free throws.**

65 out of 100 = 65%

2 $\mathbf{\frac{1}{4}}$ **of all high school students are not taking physics.**

$$\frac{1}{4} \overset{\times 25}{=} \frac{25}{100} \quad (\times 25)$$

So, 1 out of 4 equals 25%.

You can express a percent as a fraction by writing it as a fraction with a denominator of 100. Then write the fraction in simplest form.

Example 3 **Write 35% as a fraction in simplest form.**

$35\% = \frac{35}{100}$ **Definition of percent.**

$= \frac{7}{20}$ **Simplify.**

So, $35\% = \frac{7}{20}$.

Exercises

Write each ratio or fraction as a percent.

1. 13 out of 100 **2.** 47 out of 100 **3.** 4:5

4. 11:20 **5.** $\frac{4}{25}$ **6.** $\frac{33}{50}$

Write each percent as a fraction in simplest form.

7. 21% **8.** 93% **9.** 10%

10. 60% **11.** 46% **12.** 88%

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5-2 Study Guide and Intervention

7NS1.1, 7NS1.3

Comparing Fractions, Decimals, and Percents

- To write a percent as a decimal, divide by 100 and remove the percent symbol.
- To write a decimal as a percent, multiply by 100 and add the percent symbol.
- To express a fraction as a percent, you can use a proportion. Alternatively, you can write the fraction as a decimal, and then express the decimal as a percent.

Example 1 **Write 56% as a decimal.**

$56\% = 56\%$ Divide by 100 and remove the percent symbol.

$= 0.56$

Example 2 **Write 0.17 as a percent.**

$0.17 = 0.17$ Multiply by 100 and add the percent symbol.

$= 17\%$

Example 3 **Write $\frac{7}{20}$ as a percent.**

Method 1 Use a proportion.

$\frac{7}{20} = \frac{x}{100}$ Write the proportion.

$7 \cdot 100 = 20 \cdot x$ Find cross products.

$700 = 20x$ Multiply.

$\frac{700}{20} = \frac{20x}{20}$ Divide each side by 20.

$35 = x$ Simplify.

Method 2 Write as a decimal.

$\frac{7}{20} = 0.35$ Convert to a decimal by dividing.

$= 35\%$ Multiply by 100 and add the percent symbol.

So, $\frac{7}{20}$ can be written as 35%.

Exercises

Write each percent as a decimal.

1. 10% **2.** 36% **3.** 82% **4.** 49.1%

Write each decimal as a percent.

5. 0.14 **6.** 0.59 **7.** 0.932 **8.** 1.07

Write each fraction as a percent.

9. $\frac{3}{4}$ **10.** $\frac{7}{10}$ **11.** $\frac{9}{16}$ **12.** $\frac{1}{40}$

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5-3 Study Guide and Intervention

7NS1.3

Algebra: The Percent Proportion

You can use a percent proportion to find a missing part, whole, or percent.

$$\frac{\text{part}}{\text{whole}} = \text{percent}$$

Example 1 **12 is what percent of 60?**

$\frac{\text{part} \to 12}{\text{whole} \to 60} = \frac{p}{100}$ } percent	Replace *a* with 12 and *b* with 60.
$12 \cdot 100 = 60 \cdot p$	Find the cross products.
$1{,}200 = 60p$	Multiply.
$\frac{1{,}200}{60} = \frac{60p}{60}$	Divide each side by 60.
$20 = p$	12 is 20% of 60.

Example 2 **What number is 40% of 55?**

$\frac{\text{part} \to a}{\text{whole} \to 55} = \frac{40}{100}$ } percent	Replace *p* with 40 and *b* with 55.
$a \cdot 100 = 55 \cdot 40$	Find the cross products.
$a = 22$	Use similar steps to solve for *a*.

So, 22 is 40% of 55.

Exercises

Write a percent proportion and solve each problem. Round to the nearest tenth if necessary.

1. 3 is what percent of 10?

2. What number is 15% of 40?

3. 24 is 75% of what number?

4. 86 is what percent of 200?

5. What number is 65% of 120?

6. 52 is 13% of what number?

7. 35 is what percent of 56?

8. What number is 12.5% of 88?

9. 161 is 92% of what number?

10. 45 is what percent of 66?

11. What number is 31.5% of 200?

12. 81 is 54% of what number?

5-4 Study Guide and Intervention

7NS1.3

Finding Percents Mentally

To find 1% of a number mentally, move the decimal point two places to the left. To find 10% of a number mentally, move the decimal point one place to the left.

Example 1 **Find 1% of 195.**

1% of 195 = 0.01 of 195 or 1.95

Example 2 **Find 10% of 3.9.**

10% of 3.9 = 0.1 of 3.9 or 0.39

When you compute with common percents like 50% or 25%, it may be easier to use the fraction form of the percent. It is a good idea to be familiar with the fraction form of some of the common percents.

$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$	$16\frac{2}{3}\% = \frac{1}{6}$	$12\frac{1}{2}\% = \frac{1}{8}$	$10\% = \frac{1}{10}$
$50\% = \frac{1}{2}$	$40\% = \frac{2}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$	$37\frac{1}{2}\% = \frac{3}{8}$	$30\% = \frac{3}{10}$
$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$	$62\frac{1}{2}\% = \frac{5}{8}$	$70\% = \frac{7}{10}$
	$80\% = \frac{4}{5}$	$83\frac{1}{3}\% = \frac{5}{6}$	$87\frac{1}{2}\% = \frac{7}{8}$	$90\% = \frac{9}{10}$

Example 3 **Find 25% of 68.**

25% of $68 = \frac{1}{4}$ of 68 or 17

Example 4 **Find $33\frac{1}{3}\%$ of 57.**

$33\frac{1}{3}\%$ of $57 = \frac{1}{3}$ of 57 or 19

Exercises

Compute mentally.

1. 20% of 50 **2.** 10% of 70 **3.** 50% of 34

4. 1% of 210 **5.** 60% of 25 **6.** 30% of 40

7. $66\frac{2}{3}\%$ of 33 **8.** $37\frac{1}{2}\%$ of 48 **9.** 75% of 36

10. 10% of 23 **11.** $83\frac{1}{3}\%$ of 24 **12.** 1% of 45

5-5 Study Guide and Intervention

7MR3.1, 7NS1.3

Problem-Solving Investigation: Reasonable Answers

In the four-step problem-solving plan, remember that the last step is to check for reasonable answers.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

Example **The cost of a guitar is $300. Margaret works at the music store and can buy the guitar for 65% of the price. Will she have to pay more or less than $200?**

Explore You know the cost of the guitar. Margaret can buy the guitar for 65% of the price. You want to know if the guitar will cost more or less than $200.

Plan Find a close estimate. 65% is close to 66.66% or $\frac{2}{3}$. Multiply the cost by the estimate.

$\$300 \times \frac{2}{3} = \$200$

Solve Think. $\$300 \times \frac{2}{3} = \$200$. 65% is less than 66.66%, so she will have to pay less than $200.

Check Find 65% of $300. $\$300.00 \times .65 = \$195$.

$\$195 < \$200.00$ The answer is reasonable.

Exercises

For Exercises 1–5, determine a reasonable answer.

1. **JOBS** Maxine is paid $9.25 an hour to work at the bookstore. If she is saving to buy a new video game system that costs $360, will she have to work 30, 40, or 50 hours?

2. **MONEY** Jeff brings $120 to purchase winter clothes. He buys a coat for $57.36. He wants to purchase a pair of jeans for $28.95 and a pair of boots for $54.98. Does he have enough money with him to make these two purchases?

3. **SURVEY** In a recent survey, 56% of students at Trenton Middle School work at part-time jobs during the school year. If there are 1,378 students in the school, is 550, 650, or 750 a reasonable estimate for the number of students who work part time during the school year?

4. **SHOPPING** Byron took $80 to the mall to buy gifts. He spent $28.73 on a video game. He wants to purchase a book for $13.89 and a laptop bag for $39.99. Does he have enough money with him to make these two purchases?

5. **ATTENDANCE** There are 1,200 students at Hillsboro Middle School. If 43% of the students attend an exhibit given by the art department, would the number of students who attended be 924, 516, or 430?

5-6 Study Guide and Intervention

7NS1.3

Percent and Estimation

You can use compatible numbers to estimate a percent of a number. **Compatible numbers** are two numbers that are easy to divide mentally.

Example 1 **Estimate 35% of 60.**

35% is about $33\frac{1}{3}\%$ or $\frac{1}{3}$. $\frac{1}{3}$ and 60 are compatible numbers.

$\frac{1}{3}$ of 60 is 20.

So, 35% of 60 is about 20.

Example 2 **Estimate what percent corresponds to 23 out of 59.**

$\frac{23}{59} \approx \frac{24}{60}$ or $\frac{2}{5}$ 23 is about 24, and 59 is about 60.

$\frac{2}{5} = 40\%$

So, 23 out of 59 is about 40%.

Exercises

Estimate.

1. 11% of 60

2. 24% of 36

3. 81% of 25

4. 19% of 41

5. 32% of 66

6. 67% of 44

Estimate each percent.

7. 7 out of 15

8. 6 out of 23

9. 5 out of 51

10. 8 out of 35

11. 13 out of 17

12. 17 out of 26

5-7 Study Guide and Intervention

7NS1.3, 7NS1.7

Algebra: The Percent Equation

A **percent equation** is an equivalent form of a percent proportion in which the percent is written as a decimal.

part = percent · whole

Example 1 **Find 22% of 245.**

The percent is 22%, and the whole is 245. Let n represent the part.

$n = 0.22(245)$ Write 22% as the decimal 0.22.
$n = 53.9$ Simplify.

So, 22% of 245 is 53.9.

Example 2 **600 is what percent of 750?**

The part is 600, and the whole is 750. Let n represent the percent.

$600 = n(750)$ Write the equation.

$\frac{600}{750} = \frac{750n}{750}$ Divide each side by 750.

$0.8 = n$ Simplify.

Since 0.8 = 80%, 600 is 80% of 750.

Example 3 45 is 90% of what number?

The part is 45, and the percent is 90%. Let n represent the whole.

$45 = 0.90 \cdot n$ Write 90% as the decimal 0.90.

$\frac{45}{0.90} = \frac{0.90n}{0.90}$ Divide each side by 0.90.

$50 = n$ Simplify.

So, 45 is 90% of 50.

Exercises

Solve each problem using the percent equation.

1. Find 30% of 70.
2. What is 80% of 65?
3. What percent of 56 is 14?
4. 36 is what percent of 40?
5. 80 is 40% of what number?
6. 65% of what number is 78?
7. What percent of 2,000 is 8?
8. 12 is what percent of 4,000?
9. What percent of 3,000 is 18?
10. What is 110% of 80?
11. Find 180% of 160.
12. 4% of what number is 11?

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5-8 Study Guide and Intervention

7NS1.6, 7NS1.7

Percent of Change

To find the percent of change, first find the amount of change. Then find the ratio of that amount to the original amount, and write the ratio as a percent.

Example **Two months ago, the bicycle shop sold 50 bicycles. Last month, 55 bicycles were sold. Find the percent of change. State whether the percent of change is an *increase* or a *decrease*.**

Step 1 Subtract to find the amount of change.

$55 - 50 = 5$

Step 2 Write a ratio that compares the amount of change to the original number of bicycles.

Step 3 Write the ratio as a percent.

$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$ Definition of percent of change

$= \frac{5}{50}$ The amount of change is 5. The original amount is 50.

$= 0.1 \text{ or } 10\%$ Divide. Write as a percent.

The percent of change is 10%. Since the new amount is greater than the original, it is a percent of increase.

Exercises

Find each percent of change. Round to the nearest tenth of a percent if necessary. State whether the percent of change is an *increase* or a *decrease*.

1. original: 4
new: 5

2. original: 10
new: 13

3. original: 15
new: 12

4. original: 30
new: 18

5. original: 60
new: 63

6. original: 160
new: 136

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5-9 Study Guide and Intervention

7NS1.7

Simple Interest

To find simple interest, use the formula $I = prt$. Interest I is the amount of money paid or earned. Principal p is the amount of money invested or borrowed. Rate r is the annual interest rate. Time t is the time in years.

Example 1 **Find the simple interest for $600 invested at 8.5% for 6 months.**

Notice the time is given in months. Six months is $\frac{6}{12}$ or $\frac{1}{2}$ year.

$I = prt$	Write the simple interest formula.
$I = 600 \cdot 0.085 \cdot \frac{1}{2}$	Replace p with 600, r with 0.085, and t with $\frac{1}{2}$.
$I = 25.50$	Simplify.

The simple interest is $25.50.

Example 2 **Find the total amount in an account where $136 is invested at 7.5% for 2 years.**

$I = prt$	Write the simple interest formula.
$I = 136 \cdot 0.075 \cdot 2$	Replace p with 136, r with 0.075, and t with 2.
$I = 20.40$	Simplify.

The simple interest is $20.40. The amount in the account is $136 + $20.40 = $156.40.

Exercises

Find the simple interest to the nearest cent.

1. $300 at 5% for 2 years
2. $650 at 8% for 3 years
3. $575 at 4.5% for 4 years
4. $735 at 7% for $2\frac{1}{2}$ years
5. $1,665 at 6.75% for 3 years
6. $2,105 at 11% for $1\frac{3}{4}$ years

Find the total amount in each account to the nearest cent.

7. $250 at 5% for 3 years
8. $425 at 6% for 2 years
9. $945 at 7.25% for 4 years
10. $1,250 at 7.4% for $2\frac{1}{4}$ years
11. $2,680 at 9.1% for $1\frac{3}{4}$ years
12. $4,205 at 4.5% for $3\frac{1}{2}$ years

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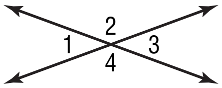
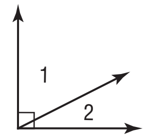
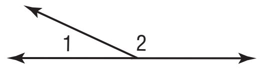
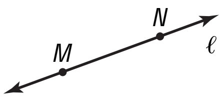
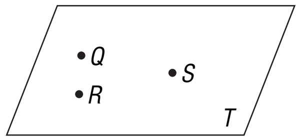
6-1 Study Guide and Intervention

7MG3.1

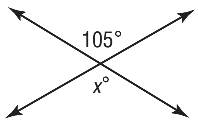
Line and Angle Relationships

Angle Relationships		
Vertical Angles	**Complementary Angles**	**Supplementary Angles**
		
$m\angle 1 = m\angle 3$ $m\angle 2 = m\angle 4$	$m\angle 1 + m\angle 2 = 90°$	$m\angle 1 + m\angle 2 = 180°$

Points, Lines, and Planes		
Point	**Line**	**Plane**
		

Example **Find the value of *x*.**

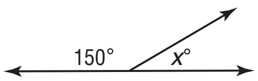
The angles are opposite each other and formed by intersecting lines, so they are vertical angles. Vertical angles are congruent.



Exercises

Find the value of *x* in each figure.

1.



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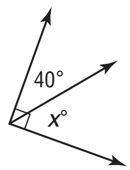
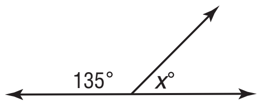
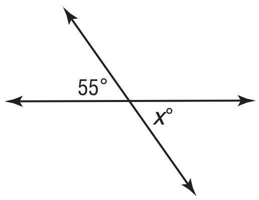


3.



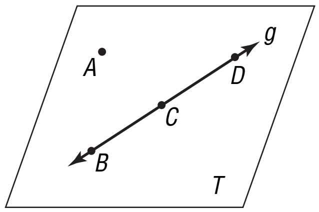
4.



For Questions 5 and 6, use the figure at the right.

5. Name the line in four different ways.

6. Name the plane in four different ways.



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6-2 Study Guide and Intervention

7MR1.2, 7NS1.3

Problem-Solving Investigation: Use Logical Reasoning

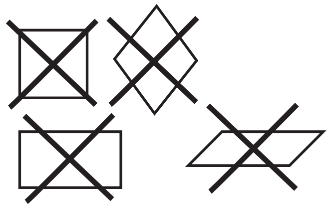
You may need to use logical reasoning to solve some problems.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

Example **A plane figure has four sides. The figure has only two congruent sides and two pairs of congruent angles. Is the figure a square, rectangle, parallelogram, rhombus, or trapezoid? Did you use *deductive* or *inductive* reasoning?**

Explore We know that a plane figure has four sides and the figure has only two congruent sides and two pairs of congruent angles. We need to see if the figure is a square, rectangle, parallelogram, rhombus, or trapezoid.

Plan Let's look at the characteristics of these different figures. A square or rhombus has *four* congruent sides. The figure is not a square or a rhombus. A rectangle or parallelogram has *two* pairs of congruent sides. The figure is not a rectangle or a parallelogram.



Solve An isosceles trapezoid can have two congruent sides and two pairs of congruent angles. The figure could be a trapezoid.

Check Since all choices but the trapezoid were eliminated, the figure is a trapezoid. Because you used existing rules about four-sided figures to make a decision, you used deductive reasoning.

Exercises

For Exercises 1–3, solve each problem using logical reasoning.

1. **GEOMETRY** Jennifer draws a square on a piece of paper and uses a ruler to draw one line through the square to create two shapes. What is the maximum number of sides that either of these shapes can have, and how would the line have to be drawn to create it?

2. **MODELS** You have 30 toothpicks. You can create two adjacent squares using 7 toothpicks if the adjacent square shares a toothpick for the side between them. How many total squares could be created this way with 30 toothpicks, if the squares are formed in a row?

3. **AGES** You and your grandfather have a combined age of 84 years. If your grandfather is 6 times as old as you are, how old are you? Explain.

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6-3 Study Guide and Intervention

7MR3.3, 7AF1.1

Polygons and Angles

An **interior angle** is an angle that lies inside a polygon and has sides that are adjacent to sides of the polygon. A **regular polygon** is a polygon whose sides and angles are congruent.

Example 1 **Find the sum of the measures of the interior angles of a tricontagon, which is a 30-sided polygon.**

$S = (n - 2)180°$	Write an equation.
$S = (30 - 2)180°$	Replace n with 30. Subtract.
$S = (28)180°$	Multiply.
$S = 5{,}040°$	

The sum of the measures of the interior angles of a tricontagon is 5,040°.

Example 2 **The defense department of the United States has its headquarters in a building called the Pentagon because it is shaped like a regular pentagon. What is the measure of an interior angle of a regular pentagon?**

$S = (n - 2)180°$	Write an equation.
$S = (5 - 2)180°$	Replace n with 5. Subtract.
$S = (3)180°$	Multiply.
$S = 540°$	
$540° \div 5 = 108°$	Divide by the number of interior angles to find the measure of one angle.

The measure of one interior angle of a regular pentagon is 108°.

Exercises

For Exercises 1–6, find the sum of the measures of the interior angles of the given polygon.

1. nonagon (9-sided)

2. 14-gon

3. 16-gon

4. hendecagon (11-sided)

5. 25-gon

6. 42-gon

For Exercises 7–12, find the measure of one interior angle of the given polygon. Round to the nearest hundredth if necessary.

7. hexagon

8. 15-gon

9. 22-gon

10. icosagon (20-sided)

11. 38-gon

12. pentacontagon (50-sided)

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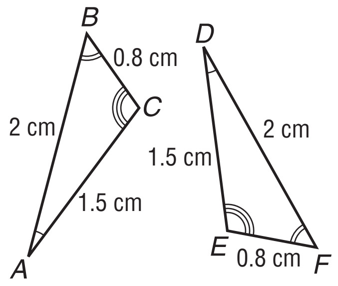
6-4 Study Guide and Intervention

7MG3.4

Congruent Polygons

Two polygons are **congruent** if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent. The letters identifying each polygon are written so that corresponding vertices appear in the same order.

Example 1 **Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.**

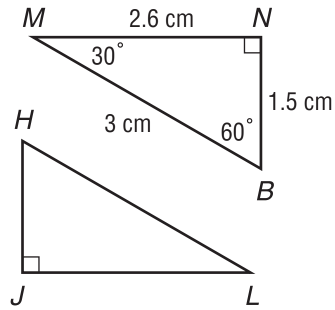


Angles The arcs indicate that $\angle A \cong \angle D$, $\angle B \cong \angle F$, and $\angle C \cong \angle E$.

Sides The side measures indicate that $\overline{AB} \cong \overline{DF}$, $\overline{BC} \cong \overline{FE}$, and $\overline{CA} \cong \overline{ED}$.

Since all pairs of corresponding sides and angles are congruent, the two triangles are congruent. One congruence statement is $\triangle ABC \cong \triangle DFE$.

Examples **In the figure, $\triangle MNB \cong \triangle LJH$.**



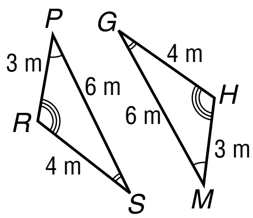
2 Find *LJ*.
$\overline{MN}$ corresponds to $\overline{LJ}$ So, $\overline{MN} \cong \overline{LJ}$.
Since $MN = 2.6$ centimeters, $LJ = 2.6$ centimeters.

3 Find $m\angle H$.
According to the congruence statement, $\angle B$ and $\angle H$ are corresponding angles. So, $\angle B \cong \angle H$. Since $m\angle B = 60°$, $m\angle H = 60°$.

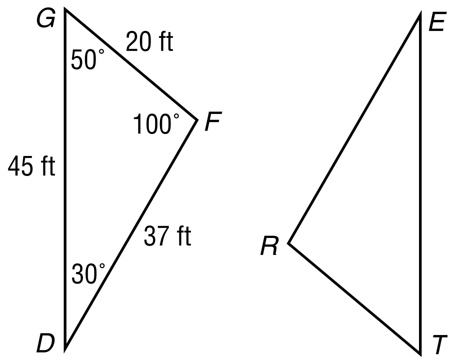
Exercises

Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.

1.



In the figure, $\triangle GFD \cong \triangle TRE$. Find each measure.



2. $m\angle R$

3. RT

4. $m\angle E$

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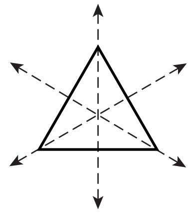
6-5 Study Guide and Intervention

7MG3.2

Symmetry

A figure has **line symmetry** if it can be folded over a line so that one half of the figure matches the other half. This fold line is called the **line of symmetry**. Some figures have more than one line of symmetry.

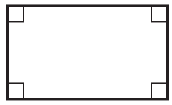
Example 1 **Determine whether the figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.**



This figure has three lines of symmetry.

A figure has **rotational symmetry** if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position. The degree measure of the angle through which the figure is rotated is called the **angle of rotation**.

Example 2 **Determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angles of rotation.**



Yes, this figure has rotational symmetry. It matches itself after being rotated 180°.

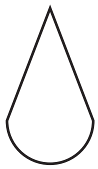
Exercises

For Exercises 1–6, complete parts a and b for each figure.

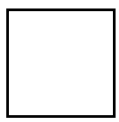
a. Determine whether the figure has line symmetry. If it does, draw all lines of symmetry. If not, write *none*.

b. Determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angles of rotation.

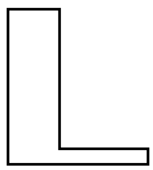
1.



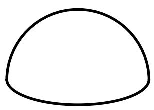
2.



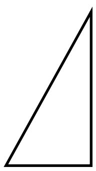
3.



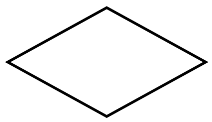
4.



5.



6.



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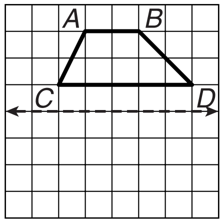
6-6 Study Guide and Intervention

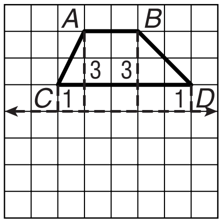
7MG3.2

Reflections

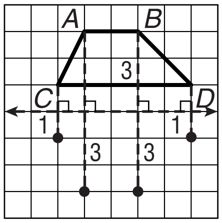
When a figure is reflected across a line, every point on the **reflection** is the same distance from the line of reflection as the corresponding point on the original figure. The image is congruent to the original figure, but the orientation of the image is different from that of the original figure.

Example 1 **Draw the image of quadrilateral *ABCD* after a reflection over the given line.**

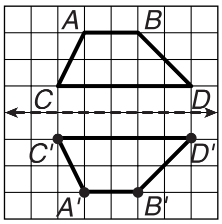




Step 1 Count the number of units between each vertex and the line of reflection.



Step 2 To find the corresponding point for vertex *A*, move along the line through vertex *A* perpendicular to the line of reflection until you are 3 units from the line on the opposite side. Draw a point and label it *A´*. Repeat for each vertex.



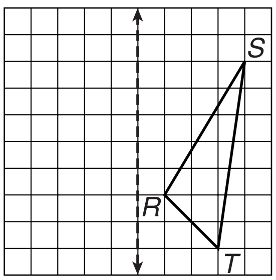
Step 3 Connect the new vertices to form quadrilateral *A´B´C´D´*.

Notice that if you move along quadrilateral *ABCD* from *A* to *B* to *C* to *D*, you are moving in the clockwise direction. However, if you move along quadrilateral *A´B´C´D´* from *A´* to *B´* to *C´* to *D´*, you are moving in the counterclockwise direction. A figure and its reflection have opposite orientations.

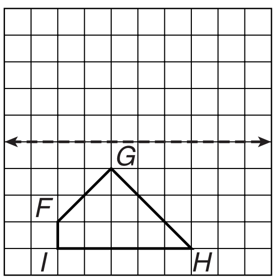
Exercises

Draw the image of the figure after a reflection over the given line.

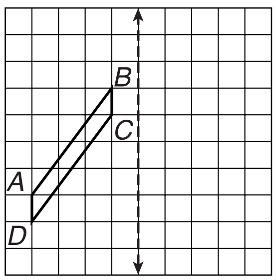
1.



2.



3.



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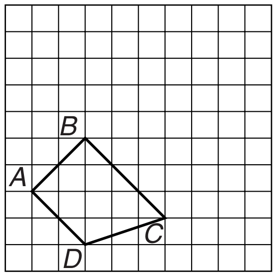
6-7 Study Guide and Intervention

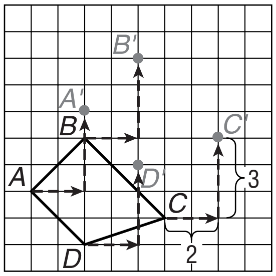
7MG3.2

Translations

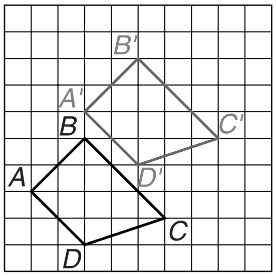
When a figure is translated, every point is moved the same distance and in the same direction. The translated figure is congruent to the original figure and has the same orientation.

Example **Draw the image of quadrilateral *ABCD* after a translation 2 units right and 3 units up.**





Step 1 To find the corresponding point for vertex A, start at A and move 2 units to the right along the horizontal grid line and then move up 3 units along the vertical grid line. Draw a point and label it A'. Repeat for each vertex.

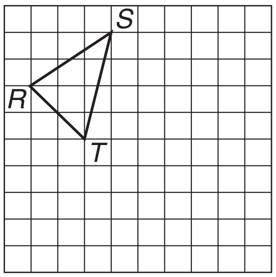


Step 2 Connect the new vertices to form quadrilateral $A'B'C'D'$.

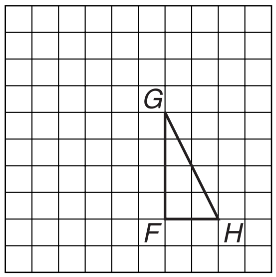
Exercises

Draw the image of the figure after the indicated translation.

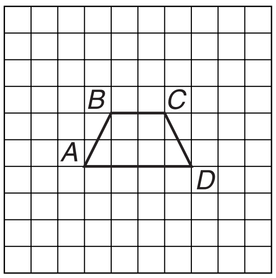
1. 5 units right and 4 units down



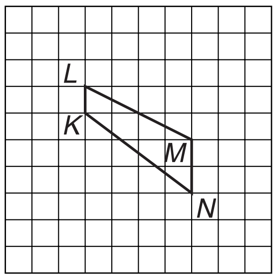
2. 3 units left and 2 units up



3. 2 units left and 3 units down



4. 2 units right and 1 unit up



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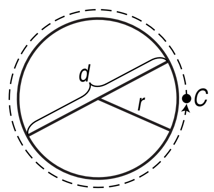
7-1 Study Guide and Intervention

7MG2.1, 7MG3.1

Circumference and Area of Circles

The **circumference** *C* of a circle is equal to its diameter *d* times π or 2 times the radius *r* times π, or $C = \pi d$ or $C = 2\pi r$.

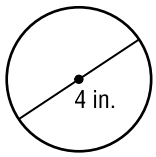
The **area** *A* of a circle is equal to π times the square of the radius *r*, or $A = \pi r^2$.



Examples

Find the circumference of each circle. Use 3.14 for π. Round to the nearest tenth.

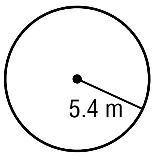
1



$C = \pi d$	Circumference of a circle
$C = \pi \cdot 4$	Replace *d* with 4.
$C = 4\pi$	This is the exact circumference.
$C \approx 4 \cdot 3.14$ or 12.6	Replace π with 3.14 and multiply.

The circumference is about 12.6 inches.

2

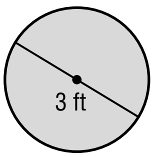


$C = 2\pi r$	Circumference of a circle
$C \approx 2 \cdot 3.14 \cdot 5.4$	Replace *r* with 5.4.
$C \approx 33.9$	Replace π with 3.14 and multiply.

The circumference is about 33.9 meters.

Example 3

Find the area of the circle. Use 3.14 for π. Round to the nearest tenth.



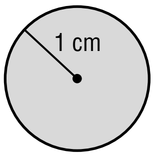
$A = \pi r^2$	Area of a circle
$A \approx 3.14(1.5)^2$	Replace π with 3.14 and *r* with half of 3 or 1.5.
$A \approx 3.14 \cdot 2.25$	Evaluate $(1.5)^2$.
$A \approx 7.1$	Multiply.

The area is about 7.1 square feet.

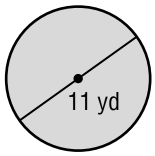
Exercises

Find the circumference and area of each circle. Use 3.14 for π. Round to the nearest tenth.

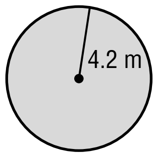
1.



2.



3.



4. The diameter is 9.3 meters.

5. The radius is 6.9 millimeter.

6. The diameter is 15.7 inches.

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7-2 Study Guide and Intervention

7MR1.3, 7MR2.2, 7AF4.2

Problem-Solving Investigation: Solve a Simpler Problem

Example 1 **Gift cards come in packages of 12 and envelopes come in packages of 15. Meagan needs to send 600 cards in envelopes. How many packages of each kind should she buy?**

Explore Meagan needs that same number of cards and envelopes.

Plan Find out how many packages are needed for 300 cards in envelopes.

Solve $12c = 300$ $\quad$ $15e = 300$

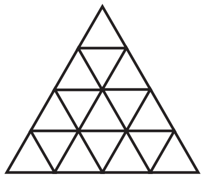
$c = 25$ $\quad$ $e = 20$

Multiply the answers by 2.

Check $2 \times 25 = 50$ packages of cards $\quad$ $2 \times 20 = 40$ packages of envelopes

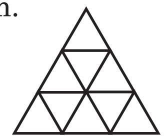
Meagan should buy 50 packages of cards and 40 packages of envelopes.

Example 2 **How many triangles of any size are in the figure at the right?**



Explore We need to find how many triangles are in the figure.

Plan Draw a simpler diagram.



Solve

9 Count the smallest triangles, which have 1 triangle per side.
3 Count the next largest triangles, which have 2 triangles per side.
1 Count the largest triangle, which has 3 triangles per side.
13 Add together to find the total triangles of any size.

Check Now repeat the steps for the original problem.

16 Count the smallest triangles, which have 1 triangle per side.
7 Count the next largest triangles, which have 2 triangles per side.
3 Count the next largest triangles, which have 3 triangles per side.
1 Count the largest triangle, which has 4 triangles per side.
27 Add together to find the total triangles of any size.

Exercises **For Exercises 1–3, solve a simpler problem.**

1. Hot dogs come in packages of 10 and buns come in packages of 8. How many packages of each will Mindy need to provide 640 hot dogs for a street fair?

2. Mark can plant 3 tree saplings in an hour and Randy can plant 5 tree saplings in an hour. Working together, how long will it take them to plant 80 tree saplings?

3. A restaurant has 18 square tables that can be pushed together to form one long table for large parties. Each square table can seat 2 people per side. How many people can be seated at the combined tables?

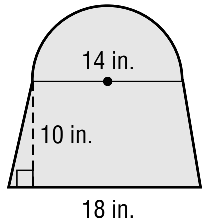
7-3 Study Guide and Intervention

7MG2.1, 7MG2.2

Area of Complex Figures

To find the area of a complex figure, separate the figure into shapes whose areas you know how to find. Then find the sum of these areas.

Example **Find the area of the complex figure.**



The figure can be separated into a semicircle and trapezoid.

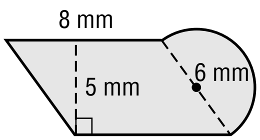
Area of semicircle	**Area of trapezoid**
$A = \frac{1}{2}\pi r^2$	$A = \frac{1}{2}h(b_1 + b_2)$
$A = \frac{1}{2}(3.14)(7)^2$	$A = \frac{1}{2} \cdot 10 \cdot (14 + 18)$
$A \approx 77.0$	$A = 160$

The area of the figure is about 77.0 + 160 or 237 square inches.

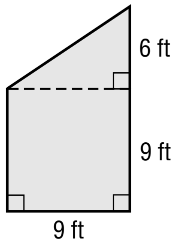
Exercises

Find the area of each figure. Use 3.14 for π. Round to the nearest tenth if necessary.

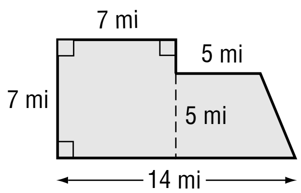
1.



2.



3.



4. What is the area of a figure formed using a triangle with a base of 6 meters and a height of 11 meters and a parallelogram with a base of 6 meters and a height of 11 meters?

5. What is the area of a figure formed using a semicircle with a diameter of 8 yards and a square with sides of a length of 6 yards?

6. What is the area of a figure formed using a rectangle with a length of 9 inches and a width of 3 inches and a triangle with a base of 4 inches and a height of 13 inches?

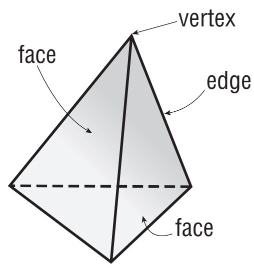
7-4 Study Guide and Intervention

7MG3.6

Three-Dimensional Figures

A **polyhedron** is a three-dimensional figure with flat surfaces that are polygons. A **prism** is a polyhedron with two parallel, congruent faces called **bases**. A **pyramid** is a polyhedron with one base that is a polygon and faces that are triangles. Prisms and pyramids are named by the shape of their bases.

Example **Identify the solid. Name the number and shapes of the faces. Then name the number of edges and vertices.**

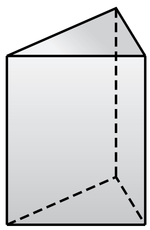


The figure has one base that is a triangle, so it is a triangular pyramid. The other faces are also triangles. It has a total of 4 faces, 6 edges, and 4 vertices.

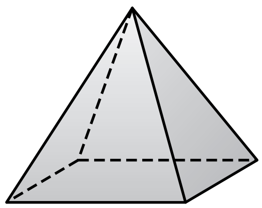
Exercises

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

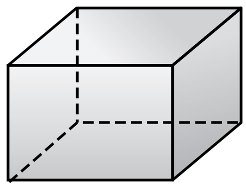
1.



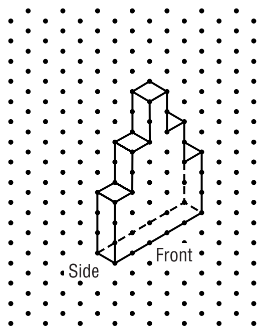
2.



3.



4. Draw and label the top, front, and side views of the three-dimensional drawing at the right.



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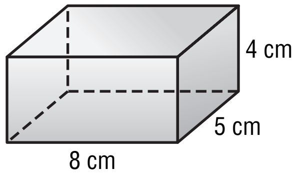
7-5 Study Guide and Intervention

7MG2.1, 7MG2.2

Volume of Prisms and Cylinders

The volume V of a prism or a cylinder is the area of the base B times the height h, or $V = Bh$.

Example 1 **Find the volume of the rectangular prism.**

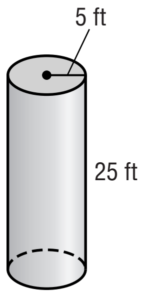


$V = Bh$	Volume of a prism
$V = (\ell \cdot w)h$	The base is a rectangle, so $B = \ell \cdot w$.
$V = (8 \cdot 5)4$	$\ell = 8$, $w = 5$, $h = 4$
$V = 160$	Simplify.

The volume is 160 cubic centimeters.

The volume V of a cylinder with radius r is the area of the base B times the height h, or $V = Bh$. Since the base is a circle, the volume can also be written as $V = \pi r^2 h$, where $B = \pi r^2$.

Example 2 **Find the volume of the cylinder. Use 3.14 for π. Round to the nearest tenth if necessary.**



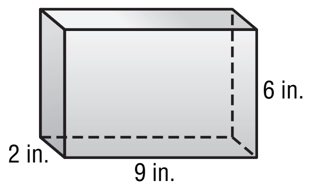
$V = \pi r^2 h$	Volume of a cylinder
$V = 3.14 \cdot 5^2 \cdot 25$	$\pi = 3.14$, $r = 5$, $h = 25$
$V \approx 1{,}962.5$	Simplify.

The volume is about 1,962.5 cubic feet.

Exercises

Find the volume of each solid. Use 3.14 for π. Round to the nearest tenth if necessary.

1.



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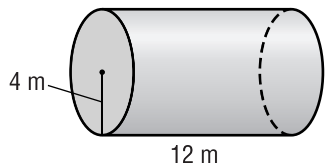

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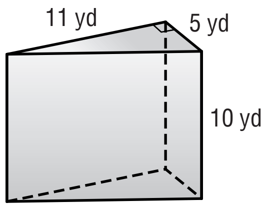

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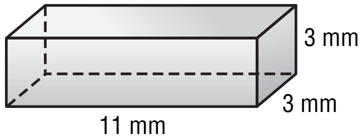

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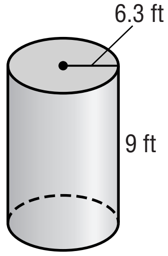

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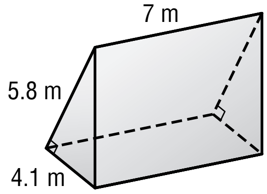

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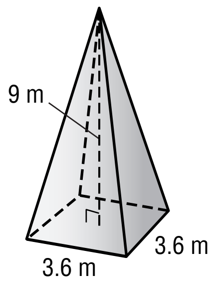
7-6 Study Guide and Intervention

7MG2.1

Volume of Pyramids and Cones

Volume Formulas	
Pyramid $V = \frac{1}{3}Bh$ V = volume, h = height, B = area of the base or ℓw	**Cone** $V = \frac{1}{3}Bh$ V = volume, h = height, B = area of the base or πr^2

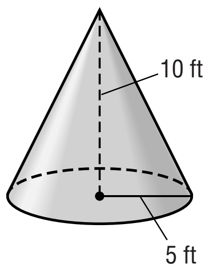
Example 1 **Find the volume of the pyramid.**



$V = \frac{1}{3}Bh$	Volume of a pyramid
$V = \frac{1}{3}s^2h$	The base is a square, so $B = s^2$.
$V = \frac{1}{3} \cdot (3.6)^2 \cdot 9$	$s = 3.6$, $h = 9$
$V = 38.88$	Simplify.

The volume is 38.88 cubic meters.

Example 2 **Find the volume of the cone. Use 3.14 for π.**



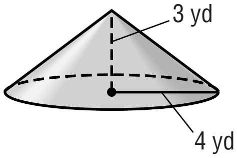
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \frac{1}{3} \cdot 3.14 \cdot 5^2 \cdot 10$	$\pi \approx 3.14$, $r = 5$, $h = 10$
$V \approx 261.7$	Simplify.

The volume is about 261.7 cubic feet.

Exercises

Find the volume of each solid. Use 3.14 for π. Round to the nearest tenth if necessary.

1.



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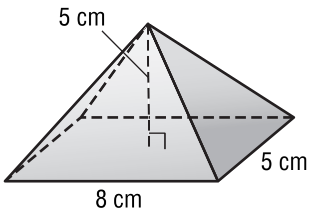

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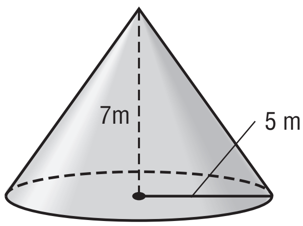

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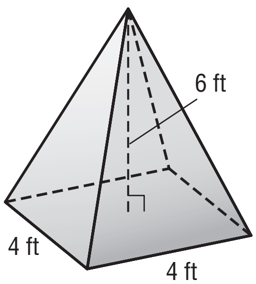

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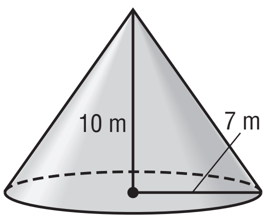

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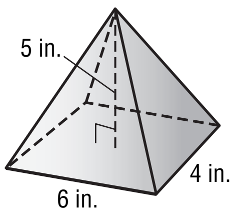

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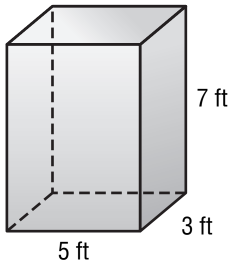
7-7 Study Guide and Intervention

7MG2.1, 7MG3.5

Surface Area of Prisms and Cylinders

The lateral area ℓ of a prism is the perimeter P of the base times the height h of the prism, or $\ell = Ph$. The total surface area S of a prism is the lateral surface area ℓ plus the area of the two bases $2B$, or $S = \ell + 2B$ or $S = Ph + 2B$.

Example 1 **Find the lateral and total surface areas of the rectangular prism.**



Perimeter of Base	**Area of Base**
$P = 2\ell + 2w$	$B = \ell w$
$P = 2(5) + 2(3)$	$B = 5(3)$
$P = 16$	$B = 15$

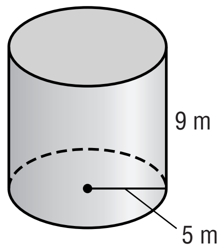
Use this information to find the lateral and total surface areas.

Lateral Surface Area	**Total Surface Area**
$L = Ph$	$S = L + 2B$
$L = 16(7)$ or 112	$S = 112 + 2(15)$ or 136

The lateral area is 112 square feet and the total surface area of the prism is 136 square feet.

The lateral area L of a cylinder with height h and radius r is the circumference of the base times the height, or $\ell = 2\pi rh$. The surface area S of a cylinder with height h and radius r is the lateral area plus the area of the two bases, or $S = \ell + 2\pi r^2$ or $S = 2\pi rh + 2\pi r^2$.

Example 2 **Find the surface area of the cylinder. Round to the nearest tenth.**

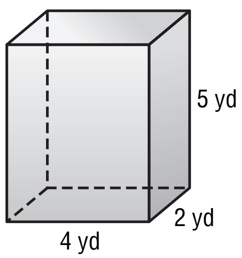


Lateral Surface Area	**Total Surface Area**
$L = 2\pi rh$	$S = L + 2\pi r^2$
$L = 2(3.14)(5)(9)$	$S = 282.6 + 2(3.14)(5)^2$
$L \approx 282.6$	$S \approx 439.6$

The lateral area is about 282.6 square meters, and the surface area of the cylinder is about 439.6 square meters.

Exercises **Find the lateral and total surface areas of each solid. Use 3.14 for π. Round to the nearest tenth if necessary.**

1.



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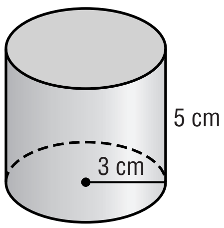

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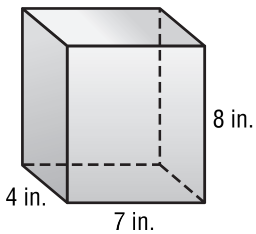

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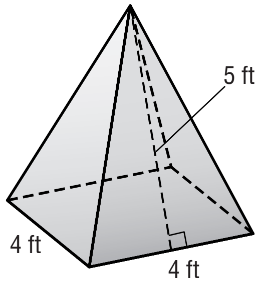
7-8 Study Guide and Intervention

7MG2.1

Surface Area of Pyramids

The lateral surface area L of a regular pyramid is half the perimeter P of the base times the slant height ℓ, or $L = \frac{1}{2}P\ell$. The total surface areas of a regular pyramid is the lateral area L plus the area of the base B, or $S = L + B$ or $S = \frac{1}{2}P\ell + B$.

Example **Find the lateral and total surface areas of the square pyramid.**



Lateral Surface Area

$L = \frac{1}{2} P\ell$

$L = \frac{1}{2}(16)(5)$ $P = 16$, $\ell = 5$

$L = 40$

Total Surface Area

$S = L + B$

$S = 40 + 4^2$

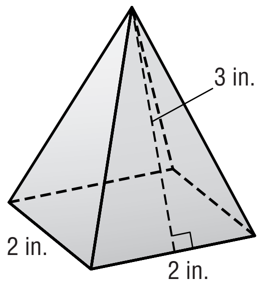
$S = 56$

The lateral surface area is 40 square feet, and the total surface area of the pyramid is 56 square feet.

Exercises

Find the surface area of each solid. Round to the nearest tenth if necessary.

1.



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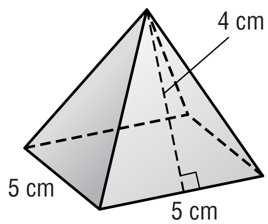

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7-9 Study Guide and Intervention

7MG2.3

Similar Solids

Similar solids have the same shape, their corresponding linear measures are proportional, and their corresponding faces are similar polygons.

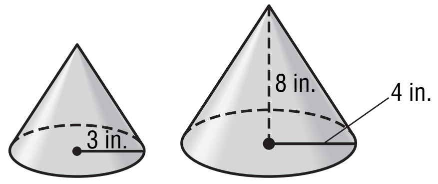
Example 1 **The cones at the right are similar. Find the height of cone A.**

$\frac{8}{x} = \frac{4}{3}$ Write a ratio.

$4x = 24$ Find the cross products.

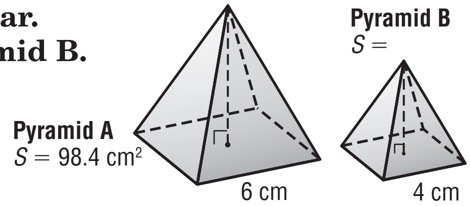
$x = 6$ Simplify.

The height of the smaller cone is 6 inches.



cone A cone B

Example 2 **The pyramids at the right are similar. Find the total surface area of pyramid B.**



The scale factor $\frac{a}{b}$ is $\frac{6}{4}$ or $\frac{3}{2}$.

$\frac{\text{surface area of pyramid A}}{\text{surface area of pyramid B}} = \left(\frac{a}{b}\right)^2$ Write a proportion.

$\frac{98.4}{S} = \left(\frac{3}{2}\right)^2$ Substitute the known values. Let S represent the surface area.

$\frac{98.4}{S} = \frac{9}{4}$ $\left(\frac{3}{2}\right)^2 = \frac{3}{2} \cdot \frac{3}{2}$ or $\frac{9}{4}$

$98.4 \cdot 4 = 9S$ Find the cross products.

$\frac{393.6}{9} = \frac{9S}{9}$ Divide each side by 9.

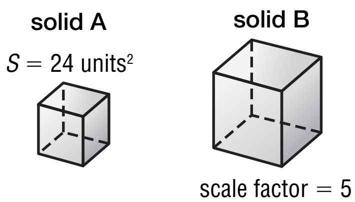
$43.7 \approx S$ Simplify.

The surface area of pyramid B is approximately 43.7 square centimeters.

Exercises

For Exercises 1 and 2, the solids in each pair are similar. Find the surface area of solid B.

1.



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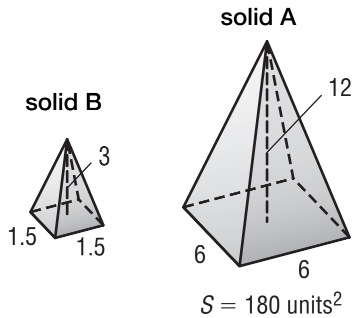
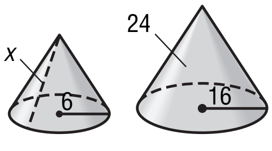


For Exercises 3 and 4, find the value of *x*.

3.



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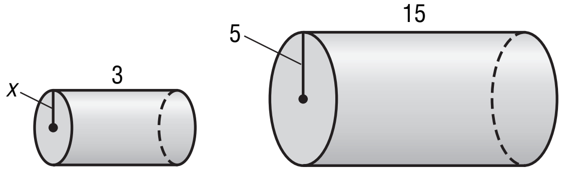

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8-1 Study Guide and Intervention

7AF1.1, 7AF1.3, 7AF1.4

Simplifying Algebraic Expressions

The **Distributive Property** can be used to simplify algebraic expressions.

Examples **Use the Distributive Property to rewrite each expression.**

1 $3(a + 5)$

$3(a + 5) = 3(a) + 3(5)$ Distributive Property

$= 3a + 15$ Simplify.

2 $-2(d - 3)$

$-2(d - 3) = -2[d + (-3)]$ Rewrite $d - 3$ as $d + (-3)$.

$= -2(d) + (-2)(-3)$ Distributive Property

$= -2(d) + 6$ Simplify.

When a plus sign separates an algebraic expression into parts, each part is called a **term**. In terms that contain a variable, the numerical part of the term is called the **coefficient** of the variable. A term without a variable is called a **constant. Like terms** contain the same variables, such as $3x$ and $2x$.

Example 3 **Identify the terms, like terms, coefficients, and constants in the expression $7x - 5 + x - 3x$.**

$7x - 5 + x - 3x = 7x + (-5) + x + (-3x)$ Definition of subtraction

$= 7x + (-5) + 1x + (-3x)$ Identity Property; $x = 1x$

The terms are $7x$, -5, x, and $-3x$. The like terms are $7x$, x, and $-3x$. The coefficients are 7, 1, and -3. The constant is -5.

An algebraic expression is in **simplest form** if it has no like terms and no parentheses.

Example 4 **Simplify the expression $-2m + 5 + 6m - 3$.**

$-2m$ and $6m$ are like terms. 5 and -3 are also like terms.

$-2m + 5 + 6m - 3 = -2m + 5 + 6m + (-3)$ Definition of subtraction

$= -2m + 6m + 5 + (-3)$ Commutative Property

$= (-2 + 6)m + 5 + (-3)$ Distributive Property

$= 4m + 2$ Simplify.

Exercises

Use the Distributive Property to rewrite each expression.

1. $2(c + 6)$

2. $-4(w + 6)$

3. $(b - 4)(-3)$

4. Identify the terms, like terms, coefficients, and constants in the expression $4m - 2 + 3m + 5$.

Simplify each expression.

5. $3d + 6d$

6. $2 + 5s - 4$

7. $2z + 3 + 9z - 8$

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8-2 Study Guide and Intervention

7AF4.1

Solving Two-Step Equations

A **two-step equation** contains two operations. To solve a two-step equation, undo each operation in reverse order.

Example 1 **Solve $-2a + 6 = 14$. Check your solution.**

Method 1 Vertical Method

$-2a + 6 = 14$	Write the equation.
$\underline{\quad -6 = -6}$	Subtract 6 from each side.
$-2a = 8$	Simplify.
$\frac{-2a}{-2} = \frac{8}{-2}$	Divide each side by -2.
$a = -4$	Simplify.

Method 2 Horizontal Method

$-2a + 6 = 14$

$-2a + 6 - 6 = 14 - 6$

$-2a = 8$

$\frac{-2a}{-2} = \frac{8}{-2}$

$a = -4$

Check $-2a + 6 = 14$	Write the equation.
$-2(-4) + 6 \stackrel{?}{=} 14$	Replace a with -4 to see if the sentence is true.
$14 = 14$ ✓	The sentence is true.

The solution is -4.

Sometimes it is necessary to combine like terms before solving an equation.

Example 2 **Solve $5 = 8x - 2x - 7$. Check your solution.**

$5 = 8x - 2x - 7$	Write the equation.
$5 = 6x - 7$	Combine like terms.
$5 + 7 = 6x - 7 + 7$	Add 7 to each side.
$12 = 6x$	Simplify.
$\frac{12}{6} = \frac{6x}{6}$	Divide each side by 6.
$2 = x$	Simplify.
The solution is 2.	Check this solution.

Exercises

Solve each equation. Check your solution.

1. $2d + 7 = 9$ **2.** $11 = 3z + 5$ **3.** $2s - 4 = 6$

4. $-12 = 5r + 8$ **5.** $-6p - 3 = 9$ **6.** $-14 = 3x + x - 2$

7. $5c + 2 - 3c = 10$ **8.** $3 + 7n + 2n = 21$ **9.** $21 = 6r + 5 - 7r$

10. $8 - 5b = -7$ **11.** $-10 = 6 - 4m$ **12.** $-3t + 4 = 19$

13. $2 + \frac{a}{6} = 5$ **14.** $-\frac{1}{3}q - 7 = -3$ **15.** $4 - \frac{v}{5} = 0$

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8-3 Study Guide and Intervention

7AF1.1

Writing Two-Step Equations

Some verbal sentences translate to two-step equations.

Example 1 **Translate each sentence into an equation.**

Sentence	Equation
Four more than three times a number is 19.	$3n + 4 = 19$
Five is seven less than twice a number.	$5 = 2n - 7$
Seven more than the quotient of a number and 3 is 10.	$7 + \frac{n}{3} = 10$

After a sentence has been translated into a two-step equation, you can solve the equation.

Example 2 **Translate the sentence into an equation. Then find the number. Thirteen more than five times a number is 28.**

Words Thirteen more than five times a number is 28.

Variable Let n = the number.

Equation

$5n + 13 = 28$ Write the equation.

$5n + 13 - 13 = 28 - 13$ Subtract 13 from each side.

$5n = 15$ Simplify.

$\frac{5n}{5} = \frac{15}{5}$ Divide each side by 5.

$n = 3$ Simplify.

Therefore, the number is 3.

Exercises

Translate each sentence into an equation. Then find each number.

1. Five more than twice a number is 7.

2. Fourteen more than three times a number is 2.

3. Seven less than twice a number is 5.

4. Two more than four times a number is -10.

5. Eight less than three times a number is -14.

6. Three more than the quotient of a number and 2 is 7.

8-4 Study Guide and Intervention

7AF1.1, 7AF4.1

Solving Equations with Variables on Each Side

Some equations, such as $3x - 9 = 6x$, have variables on each side of the equals sign. Use the Addition or Subtraction Property of Equality to write an equivalent equation with the variables on one side of the equals sign. Then solve the equation.

Example 1 **Solve $3x - 9 = 6x$. Check your solution.**

$3x - 9 = 6x$	Write the equation.
$3x - 3x - 9 = 6x - 3x$	Subtract $3x$ from each side.
$-9 = 3x$	Simplify.
$-3 = x$	Mentally divide each side by 3.

To check your solution, replace x with -3 in the original equation.

Check	$3x - 9 = 6x$	Write the equation.
	$3(-3) - 9 \stackrel{?}{=} 6(-3)$	Replace x with -3.
	$-18 = -18$ ✓	The sentence is true.

The solution is -3.

Example 2 **Solve $4a - 7 = 5 - 2a$.**

$4a - 7 = 5 - 2a$	Write the equation.
$4a + 2a - 7 = 5 - 2a + 2a$	Add $2a$ to each side.
$6a - 7 = 5$	Simplify.
$6a - 7 + 7 = 5 + 7$	Add 7 to each side.
$6a = 12$	Simplify.
$a = 2$	Mentally divide each side by 6.
The solution is 2.	Check this solution.

Exercises

Solve each equation. Check your solution.

1. $6s - 10 = s$

2. $8r = 4r - 16$

3. $25 - 3u = 2u$

4. $14t - 8 = 6t$

5. $k + 20 = 9k - 4$

6. $11m + 13 = m + 23$

7. $-4b - 5 = 3b + 9$

8. $6y - 1 = 27 - y$

9. $1.6h - 72 = 4h - 30$

10. $8.5 - 3z = -8z$

11. $10x + 8 = 5x - 3$

12. $16 - 7d = -3d + 2$

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8-5 Study Guide and Intervention

7MR2.8, 7AF1.1

Problem-Solving Investigation: Guess and Check

You may need to use the guess and check strategy to solve some problems.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

Example

The school booster club spent $776 on ski passes for the school ski trip. Adult tickets cost $25 each and student tickets cost $18 each. They bought four times as many student tickets as adult tickets. Find the number of adult and student tickets purchased.

Explore Adult tickets cost $25 each and student tickets cost $18 each. They bought four times more student tickets than adult tickets. The total amount paid for the tickets was $776.

Plan Make a guess and check to see if it is correct. Remember, the number you guess for the student tickets must be four times more than the number you guess for adult tickets.

Solve You need to find the combination that gives a total of $776. Make a list and use a to represent the number of adult tickets and s to represent the number of student tickets.

Guess	$\$25a + \$18s = \$776$	Check
If $a = 10$, then $s = 4(10) = 40$	$\$25(10) + \$18(40) = \$970$	too high
If $a = 5$, then $s = 4(5) = 20$	$\$25(5) + \$18(20) = \$485$	too low
If $a = 7$, then $s = 4(7) = 28$	$\$25(7) + \$18(28) = \$679$	still too low
If $a = 8$, then $s = 4(8) = 32$	$\$25(8) + \$18(32) = \$776$	correct

The booster club bought 8 adult tickets and 32 student tickets.

Check Thirty-two student tickets is 4 times more than the 8 adult tickets. Since the cost of 8 adult tickets, $200, plus the cost of 32 student tickets, $576, equals $776, the guess is correct.

Exercises **Use the guess and check strategy to solve each problem.**

1. **JEWELRY** Jana is making necklaces and bracelets. She puts 8 crystals on each necklace and 3 crystals on each bracelet. She needs to make 20 more necklaces than bracelets. She has 270 crystals. If she uses all the crystals, how many necklaces and bracelets can she make?

2. **GIFT BAGS** The ninth-grade class is filling gift bags for participants in a school fund-raiser. They put 2 raffle tickets in each child's bag and 4 raffle tickets in each adult's bag. They made twice as many adult bags as child bags. If they had 500 raffle tickets, how many child bags and adult bags did they make?

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8-6 Study Guide and Intervention

7AF1.1

Inequalities

A mathematical sentence that contains $<$ or $>$ is called an **inequality.** When used to compare a variable and a number, inequalities can describe a range of values. Some inequalities use the symbols $\leq$ or $\geq$. The symbol $\leq$ is read *is less than or equal to*. The symbol $\geq$ is read *is greater than or equal to*.

Examples **Write an inequality for each sentence.**

1 **SHOPPING** Shipping is free on orders of more than \$100.
Let c = the cost of the order.
$c > 100$

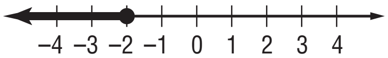
2 **RESTAURANTS** The restaurant seats a maximum of 150 guests.
Let g = the number of guests.
$g \leq 150$

Inequalities can be graphed on a number line. An open or closed circle is used to show where the solutions start, and an arrow pointing either left or right indicates the rest of the solutions. An open circle is used with inequalities having $>$ or $<$. A closed circle is used with inequalities having $\leq$ or $\geq$.

Examples **Graph each inequality on a number line.**

$d \leq -2$

Place a closed circle at -2. Then draw a line and an arrow to the left.



$d > -2$

Place an open circle at -2. Then draw a line and an arrow to the right.

Exercises

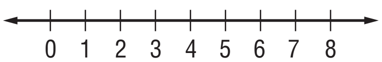
Write an inequality for each sentence.

1. **FOOD** Our delivery time is guaranteed to be less than 30 minutes.

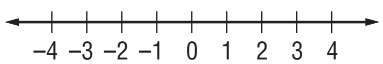
2. **DRIVING** Your speed must be at least 45 miles per hour on the highway.

Graph each inequality on a number line.

3. $r > 7$



4. $x \leq -1$



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8-7 Study Guide and Intervention

7AF1.1, 7AF4.1

Solving Inequalities by Adding or Subtracting

Solving an inequality means finding values for the variable that make the inequality true. You can use the Addition and Subtraction Properties of Inequality to help solve an inequality. When you add or subtract the same number from each side of an inequality, the inequality remains true.

Examples **Solve each inequality. Check your solution. Then graph the solution on a number line.**

1

$9 < r + 5$	Write the inequality.
$9 - 5 < r + 5 - 5$	Subtract 5 from each side.
$4 < r$ or $r > 4$	Simplify.

Check Solutions to the inequality should be greater than 4. Check this result by replacing r in the original inequality with two different numbers greater than 4. Both replacements should give true statements.

To graph the solution, place an open circle at 4 and draw a line and arrow to the right.

2

$x - 7 \geq -4$	Write the inequality.
$x - 7 + 7 \geq -4 + 7$	Add 7 to each side.
$x \geq 3$	Simplify.

Check Replace x in the original inequality with 3 and then with a number greater than 3. The solution is $x \geq 3$.

To graph the solution, place a closed circle at 3 and draw a line and arrow to the right.

Exercises

Solve each inequality. Check your solution.

1. $t - 4 > 2$

2. $b + 5 \leq 9$

3. $8 < r - 7$

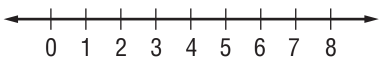
4. $6 < p - 6$

5. $2 > a + 7$

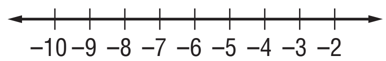
6. $4 + m \geq -6$

Solve each inequality and check your solution. Then graph the solution on a number line.

7. $s + 8 < 10$



8. $-11 \leq d - 3$



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8-8 Study Guide and Intervention

7AF1.1, 7AF4.1

Solving Inequalities by Multiplying or Dividing

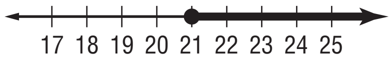
When you multiply or divide each side of an inequality by a positive number, the inequality remains true. However, when you multiply or divide each side of an inequality by a negative number, the direction of the inequality must be reversed for the inequality to remain true.

Example 1 **Solve $\frac{t}{-7} \leq -3$. Check your solution. Then graph the solution on a number line.**

$\frac{t}{-7} \leq -3$	Write the inequality.
$\frac{t}{-7}(-7) \leq -3(-7)$	Multiply each side by -7 and reverse the inequality symbol.
$t \geq 21$	Simplify.

The solution is t ≥ 21. You can check this solution by replacing *t* in the original inequality with 21 and a number greater than 21.

To graph the solution, place a closed circle at 21 and draw a line and arrow to the right.



Some inequalities involve more than one operation.

Example 2 **Solve $4x - 5 < 27$. Check your solution.**

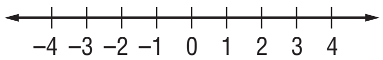
$4x - 5 < 27$	Write the inequality.
$4x - 5 + 5 < 27 + 5$	Add 5 to each side.
$4x < 32$	Simplify.
$\frac{4x}{4} < \frac{32}{4}$	Divide each side by 4.
$x < 8$	Simplify.

The solution is $x < 8$. You can check this solution by substituting numbers less than 8 into the original inequality.

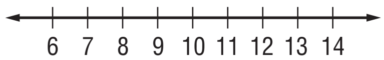
Exercises

Solve each inequality and check your solution. Then graph the solution on a number line.

1. $3a > 6$



2. $36 > 4r$



Solve each inequality. Check your solution.

3. $c + 2 \geq -2$

4. $13 > -2y - 3$

5. $\frac{h}{-5} - 6 < -10$

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9-1 Study Guide and Intervention

7AF3.3, 7MR2.5

Functions

A **function** connects an input number, x, to an output number, $f(x)$, by a rule. To find the value of a function for a certain number, substitute the number into the function value in place of x, and simplify.

Example 1 **Find $f(5)$ if $f(x) = 2 + 3x$.**

$f(x) = 2 + 3x$ — Write the function.
$f(5) = 2 + 3(5)$ or 17 — Substitute 5 for x into the function rule and simplify.
So, $f(5) = 17$.

You can organize the input, rule, and output of a function using a function table.

Example 2 **Complete the function table for $f(x) = 2x + 4$.**

Substitute each value of x, or input, into the function rule. Then simplify to find the output.

$f(x) = 2x + 4$
$f(-1) = 2(-1) + 4$ or 2
$f(0) = 2(0) + 4$ or 4
$f(1) = 2(1) + 4$ or 6
$f(2) = 2(2) + 4$ or 8

Input x	Rule $2x + 4$	Output $f(x)$
-1	$2(-1) + 4$	2
0	$2(0) + 4$	4
1	$2(1) + 4$	6
2	$2(2) + 4$	8

Exercises

Find each function value.

1. $f(1)$ if $f(x) = x + 3$

2. $f(6)$ if $f(x) = 2x$

3. $f(4)$ if $f(x) = 5x - 4$

4. $f(9)$ if $f(x) = -3x + 10$

5. $f(-2)$ if $f(x) = 4x - 1$

6. $f(-5)$ if $f(x) = -2x + 8$

Complete each function table.

7. $f(x) = x - 10$

x	$x - 10$	$f(x)$
-1		
0		
1		
2		

8. $f(x) = 2x + 6$

x	$2x + 6$	$f(x)$
-3		
-1		
2		
4		

9. $f(x) = 2 - 3x$

x	$2 - 3x$	$f(x)$
-2		
0		
3		
4		

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9-2 Study Guide and Intervention

7AF1.5

Representing Linear Functions

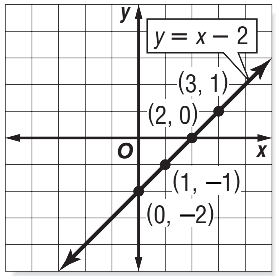
A function in which the graph of the solutions forms a line is called a **linear function**. A linear function can be represented by an equation, a table, a set of ordered pairs, or a graph.

Example **Graph $y = x - 2$.**

Step 1 Choose some values for x. Use these values to make a function table.

x	$x - 2$	y	(x, y)
0	$0 - 2$	-2	$(0, -2)$
1	$1 - 2$	-1	$(1, -1)$
2	$2 - 2$	0	$(2, 0)$
3	$3 - 2$	1	$(3, 1)$

Step 2 Graph each ordered pair on a coordinate plane. Draw a line that passes through the points. The line is the graph of the linear function.

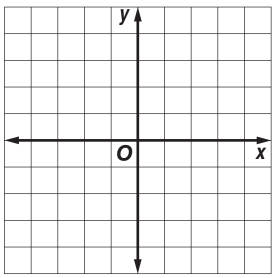


Exercises

Complete the function table. Then graph the function.

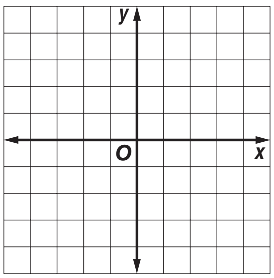
1. $y = x + 3$

x	$x + 3$	y	(x, y)
-2			
0			
1			
2			

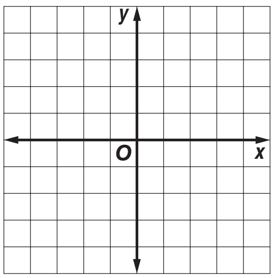


Graph each function.

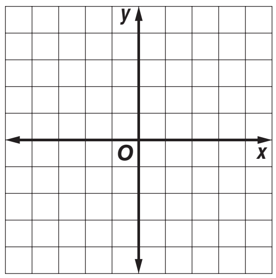
2. $y = 3x + 2$



3. $y = 2 - x$



4. $y = 3x - 1$



9-3 Study Guide and Intervention

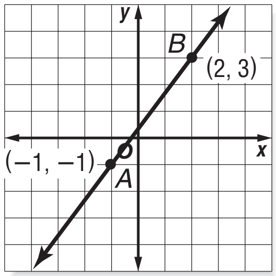
7AF3.3

Slope

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates. As an equation, the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

Example 1 **Find the slope of the line that passes through $A(-1, -1)$ and $B(2, 3)$.**



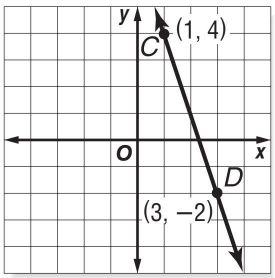
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{3 - (-1)}{2 - (-1)}$ $(x_1, y_1) = (-1, -1)$, $(x_2, y_2) = (2, 3)$

$m = \dfrac{4}{3}$ Simplify.

Check When going from left to right, the graph of the line slants upward. This is correct for a positive slope.

Example 2 **Find the slope of the line that passes through $C(1, 4)$ and $D(3, -2)$.**



$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{-2 - 4}{3 - 1}$ $(x_1, y_1) = (1, 4)$, $(x_2, y_2) = (3, -2)$

$m = \dfrac{-6}{2}$ or -3 Simplify.

Check When going from left to right, the graph of the line slants downward. This is correct for a negative slope.

Exercises

Find the slope of the line that passes through each pair of points.

1. $A(0, 1)$, $B(3, 4)$

2. $C(1, -2)$, $D(3, 2)$

3. $E(4, -4)$, $F(2, 2)$

4. $G(3, 1)$, $H(6, 3)$

5. $I(4, 3)$, $J(2, 4)$

6. $K(-4, 4)$, $L(5, 4)$

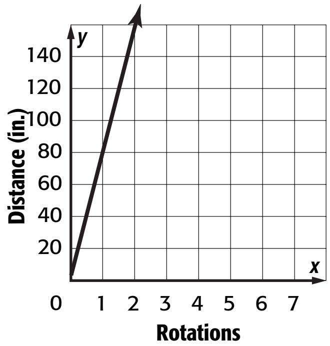
9-4 Study Guide and Intervention

7AF3.4, 7AF4.2

Direct Variation

When two variable quantities have a constant ratio, their relationship is called a **direct variation.**

Example 1 **The distance that a bicycle travels varies directly with the number of rotations that its tires make. Determine the distance that the bicycle travels for each rotation.**



Since the graph of the data forms a line, the rate of change is constant. Use the graph to find the constant ratio.

$\frac{\text{distance traveled}}{\text{\# of rotations}} \rightarrow \frac{80}{1} \quad \frac{160}{2} \text{ or } \frac{80}{1} \quad \frac{240}{3} \text{ or } \frac{80}{1} \quad \frac{320}{4} \text{ or } \frac{80}{1}$

The bicycle travels 80 inches for each rotation of the tires.

Example 2 **The number of trading cards varies directly as the number of packages. If there are 84 cards in 7 packages, how many cards are in 12 packages?**

Let x = the number of packages, y = the total number of cards, and z = the number of cards in each package.

$y = zx$	direct variation
$84 = z(7)$	$y = 84, x = 7$
$12 = z$	Simplify.
$y = 12x$	Substitute for $z = 12$.

Use the equation to find y when $x = 12$.

$y = 12x$	
$y = 12(12)$	$x = 12$
$y = 144$	Multiply.

There are 144 cards in 12 packages.

Exercises **Write an expression and solve the given situation.**

1. **TICKETS** Four friends bought movie tickets for \$41. The next day seven friends bought movie tickets for \$71.75. What is the price of one ticket?

2. **JOBS** Rick earns \$24.75 in three hours. If the amount that earns varies directly with the number of hours, how much would he earn in 20 hours?

3. **BAKING** A bread recipe calls for $2\frac{1}{2}$ cups of flour for 16 servings, and $3\frac{1}{8}$ cups of flour for 20 servings. How much flour is required to make bread to serve 12?

9-5 Study Guide and Intervention

7AF3.3

Slope-Intercept Form

Linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

Example 1 **State the slope and y-intercept of the graph of $y = x - 3$.**

$y = x - 3$ Write the original equation.

$y = 1x + (-3)$ Write the equation in the form $y = mx + b$.

$\uparrow \quad \uparrow$

$y = mx + b$ $m = 1$, $b = -3$

The slope of the graph is 1, and the y-intercept is -3.

You can use the slope-intercept form of an equation to graph the equation.

Example 2 **Graph $y = 2x + 1$ using the slope and y-intercept.**

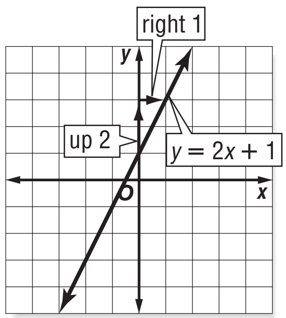
Step 1 Find the slope and y-intercept.
$y = 2x + 1$ slope $= 2$, y-intercept $= 1$.

Step 2 Graph the y-intercept 1.

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$m = \frac{2}{1}$ ← change in y : up 2 units; ← change in x : right 1 unit

Step 4 Draw a line through the two points.



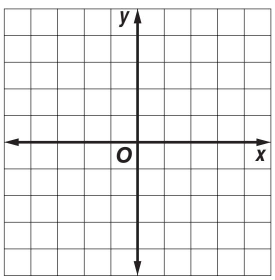
Exercises

State the slope and y-intercept of the graph of each equation.

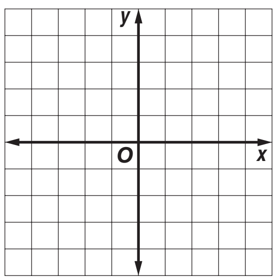
1. $y = x + 1$ **2.** $y = 2x - 4$ **3.** $y = \frac{1}{2}x - 1$

Graph each equation using the slope and y-intercept.

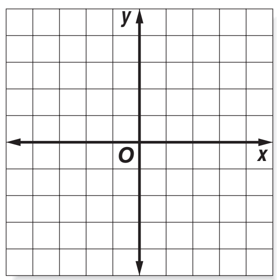
4. $y = 2x + 2$



5. $y = x - 1$



6. $y = \frac{1}{2}x + 2$



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9-6 Study Guide and Intervention

7AF1.1

Writing Systems of Equations and Inequalities

Together, the equations $y = x + 3$ and $y = 2x + 1$ are called a **system of equations.** There are two equations and two different unknowns, x and y. The solution of a system of equations is an ordered pair that satisfies each equation. A **system of inequalities** is similar to a system of equations except that it contains the symbol $<$, $\leq$, $>$, or $\geq$.

Example **Two students took a mathematics quiz. Their combined score was 172. Student A's score was 18 points higher than Student B's score. Write a system of equations to represent this situation.**

Step 1 *Choose variables to represent each student.*

Let a = Student A's score and let b = Student B's score

Step 2 *Select the different scenarios to be represented by equations with those variables.*

Their combined score was 172 and Student B's score was 18 points higher than Student A's.

Step 3 *Write a system of equations to represent the situation.*

$a + b = 172$ and $a = b + 18$

Step 4 *Write the system in standard form and line up the variables.*

$a + b = 172$
$a - b = 18$

Exercises

Write a system of equations or inequalities to represent each situation. Write the systems in standard form and line up the variables.

1. Bianca has a total of 356 CDs and DVDs. She has 24 more CDs than DVDs.

2. Tina doesn't have time to spend any more then thirty minutes total studying for math and history today. She also wants to spend at least five more minutes studying for history than for math.

3. Sixty-two people went to the movies. The price for adults was \$6.00 and the price for students was \$4.00. The total cost for the group was \$290.00.

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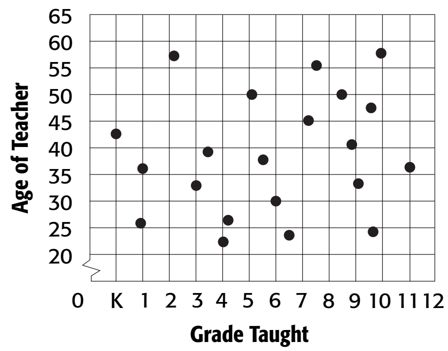
9-7 Study Guide and Intervention

7MR2.5, 7SDAP1.2

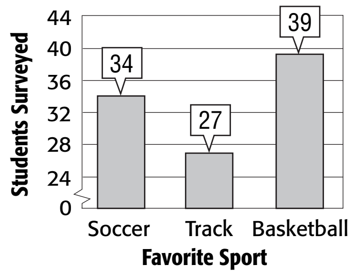
Problem-Solving Investigation: Use a Graph

Example 1 **The graph shows the results of a survey of teachers' ages and grade levels taught at school. Do the oldest teachers teach the highest grade level?**

Study the graph. The teachers who are oldest are plotted towards the top of the graph. The teachers who teach the highest grade levels are plotted towards the right of the graph. The graph shows that the points towards the top of the graph are spread out from left to right randomly. The graph shows that the oldest teachers teach all grade levels, not just the highest grade levels.



Example 2 **The graph shows the results of a survey of students' favorite sports. How many students were surveyed?**



Study the graph. Each bar on the graph represents the number of students who voted for that sport as their favorite. In order to find the number of students surveyed, add the amount from each sport.

soccer + track + basketball = total students surveyed	Write an equation.
34 + 27 + 39 = total students surveyed	Substitute.
100 = total students surveyed	Add.

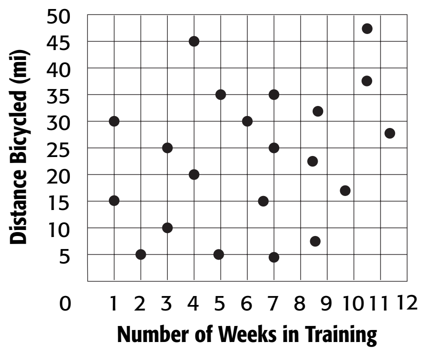
There were 100 students surveyed.

Exercises

Use the graph at the right. Each point on the graph represents one person in a group that is training for a long-distance bicycle ride. The point shows the number of miles that person cycles each day and the number of weeks that person has been in training.

1. What is the highest number of miles bicycled each day by any person in the group? How many weeks was this person in training?

2. Does the number of miles bicycled each day increase as the number of weeks in training increases?



9-8 Study Guide and Intervention

7SDAP1.2

Scatter Plots

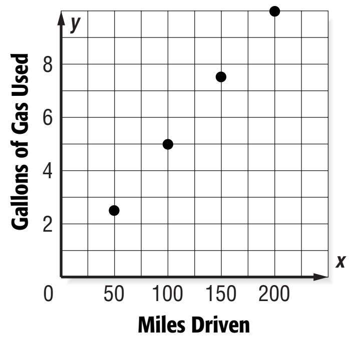
When you graph two sets of data as ordered pairs, you make a **scatter plot**. The pattern of the data points determines the relationship between the two sets of data.

- Data points that go generally upward show a *positive* relationship.
- Data points that go generally downward show a *negative* relationship.
- Data points with no clear pattern show *no* relationship between the data sets.

Examples **Explain whether the scatter plot of the data shows a *positive*, *negative*, or *no* relationship.**

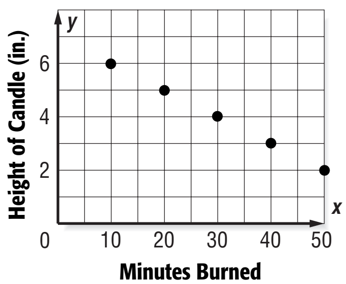
1 miles driven and gallons of gas used

As the number of miles driven increases, the amount of gas used increases. Therefore, the scatter plot will show a positive relationship.



2 number of minutes a candle burns and a candle's height

As the number of minutes increases, the height of the candle will decrease. Therefore, the scatter plot will show a negative relationship.



Exercises

Explain whether the scatter plot of the data for the following shows a *positive*, *negative*, or *no* relationship.

1. a student's age and the student's grade level in school

2. number of words written and amount of ink remaining in a pen

3. square feet of floor space and the cost of carpet for the entire floor

4. a person's height and the number of siblings the person has

5. length of time for a shower and the amount of hot water remaining

6. number of sides of a polygon and the area of the polygon

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10-1 Study Guide and Intervention

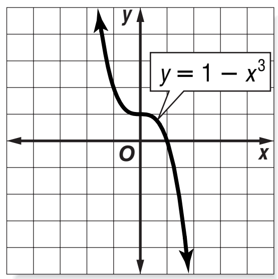
AF1.5

Linear and Nonlinear Functions

Linear functions, which have graphs that are straight lines, represent constant rates of change. The rate of change for nonlinear functions is not constant. Therefore, its graphs are not straight lines.

The equation for a linear function can always be written in the form $y = mx + b$, where m represents the constant rate of change. You can determine whether a function is linear by examining its equation. In a linear function, the power of x is always 1 or 0, and x does not appear in the denominator of a fraction.

Example 1 **Determine whether the graph represents a *linear* or *nonlinear* function. Explain.**



The graph is a curve, not a straight line. So, it represents a nonlinear function.

Example 2 **Determine whether $y = 2.5x$ represents a *linear* or *nonlinear* function. Explain.**

Since the equation can be written as $y = 2.5x + 0$, the function is linear.

A nonlinear function does not increase or decrease at the same rate. You can use a table to determine if the rate of change is constant.

Example 3 **Determine whether the table represents a *linear* or *nonlinear* function. Explain.**

+4 +4 +4

x	−2	2	6	10
y	8	3	−1	−4

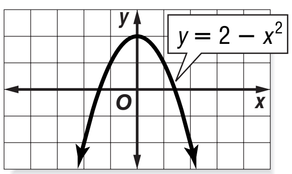
−5 −4 −3

As x increases by 4, y decreases by a different amount each time. The rate of change is not constant, so this function is nonlinear.

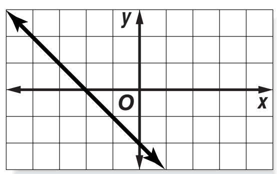
Exercises

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

1.



2.



3. $y = 2 - x^3$

4. $y = 5 - 2x$

5.

x	1	2	3	4
y	3	6	9	12

6.

x	0	2	4	6
y	5	3	0	−4

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10-2 Study Guide and Intervention

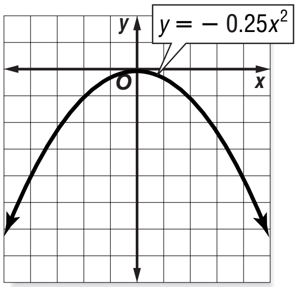
7AF1.5, 7AF3.1

Graphing Quadratic Functions

A **quadratic function**, such as $A = s^2$, is a function in which the greatest power of the variable is 2. Its graph is U-shaped, opening upward or downward.

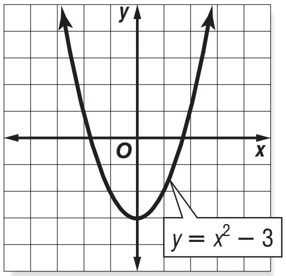
Example 1 **Graph $y = -0.25x^2$.**

x	$-0.25x^2$	y	(x, y)
-4	$-0.25(-4)^2 = -4$	-4	$(-4, -4)$
-2	$-0.25(-2)^2 = -1$	-1	$(-2, -1)$
0	$-0.25(0)^2 = 0$	0	$(0, 0)$
2	$-0.25(2)^2 = -1$	-1	$(2, -1)$
4	$-0.25(4)^2 = -4$	-4	$(4, -4)$



Example 2 **Graph $y = x^2 - 3$.**

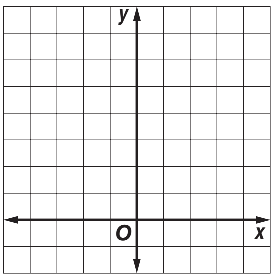
x	$x^2 - 3$	y	(x, y)
-2	$(-2)^2 - 3 = 1$	1	$(-2, 1)$
-1	$(-1)^2 - 3 = -2$	-2	$(-1, -2)$
0	$(0)^2 - 3 = -3$	-3	$(0, -3)$
1	$(1)^2 - 3 = -2$	-2	$(1, -2)$
2	$(2)^2 - 3 = 1$	1	$(2, 1)$



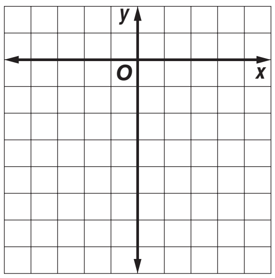
Exercises

Graph each function.

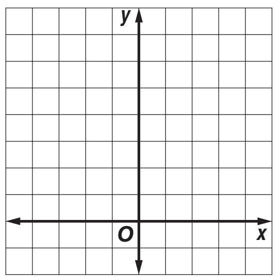
1. $y = 2x^2$



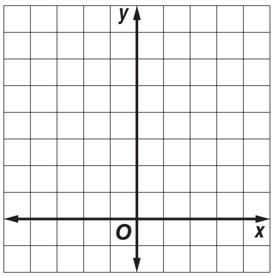
2. $y = -0.5x^2$



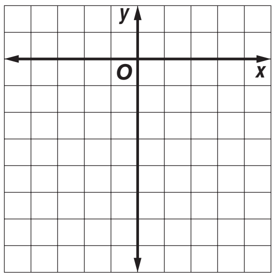
3. $y = x^2 - 1$



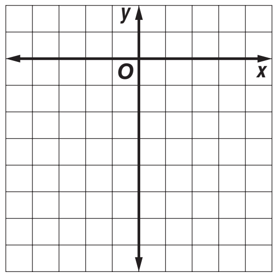
4. $y = 2x^2 + 4$



5. $y = -x^2 - 3$



6. $y = -3x^2 + 1$



10-3 Study Guide and Intervention

7MR2.5, 7AF1.1

Problem-Solving Investigation: Make a Model

You may need to use the make-a-model strategy to solve some problems.

You can always use the four-step plan to solve a problem.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

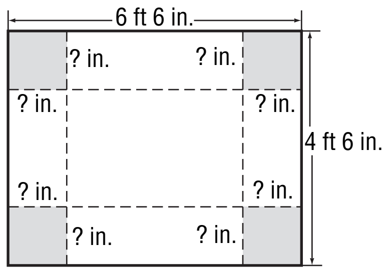
Example

Kisha is trying to make a box out of a piece of cardboard by cutting a square out of each corner. She will then fold up the sides and tape them together. The cardboard measures 4 feet 6 inches by 6 feet 6 inches. She wants the box to measure 3 feet wide by 5 feet long. What size squares should Kisha cut out of the corners to make the box?

Explore She wants to know what size squares to cut out of each corner to make a box which measures 3 feet by 5 feet by 9 inches.

Plan Start by making a model of the cardboard. Label the sides of the cardboard in feet. Draw lines to show the squares that will be cut out of the corners.

Solve



Subtract 5 feet from 6 feet 6 inches and divide by 2.
1 ft 6 in. = 18 in. 18 in. ÷ 2 = 9 in.

The square must have sides that are 9 inches long.

Check Check that the width of the box meets the specifications. Subtracting 18 inches or 1 foot 6 inches from 4 feet 6 inches yields 3 feet, which is the width required.

Exercises

Make a model to solve each problem.

1. **CONSTRUCTION** A chicken coop will be 20 feet long and 16 feet wide. One side that is 20 feet long will be formed by the barn. The other three sides will be made of wire fencing with posts at every corner and every 4 feet between each corner. How many feet of fencing and how many posts are needed to build the chicken coop?

2. **GEOMETRY** What is the fewest number of one-inch cubes needed to make a rectangular prism that measures 4 inches by 5 inches by 6 inches? (Hint: The prism can be hollow inside.)

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10-4 Study Guide and Intervention

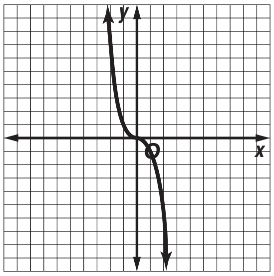
7AF3.1, 7AF3.2

Graphing Cubic Functions

A **cubic function,** such as $A = s^3$, is a function in which the greatest power of the variable is 3. Its graph is a curve. You can graph cubic functions by making a table of values.

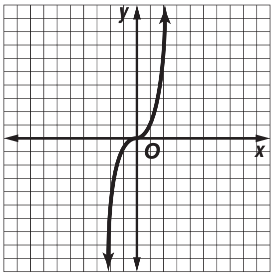
Example 1 **Graph $y = -x^3$.**

x	$y = x^3$	(x, y)
-2	$-(-2)^3$	$(-2, 8)$
-1	$-(-1)^3$	$(-1, 1)$
0	$-(0)^3$	$(0, 0)$
1	$-(1)^3$	$(1, -1)$
2	$-(2)^3$	$(2, -8)$



Example 2 **Graph $y = x^3$.**

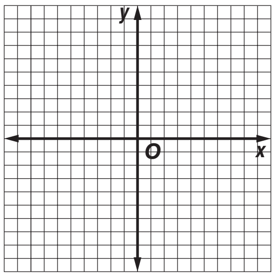
x	$y = x^3$	(x, y)
-1.5	$(-1.5)^3 \approx -3.4$	$(-1.5, -3.4)$
-1	$(-1)^3 = -1$	$(-1, 1)$
0	$-(0)^3 = 0$	$(0, 0)$
1	$(1)^3 = 1$	$(1, 1)$
1.5	$(1.5)^3 \approx 3.4$	$(1.5, 3.4)$



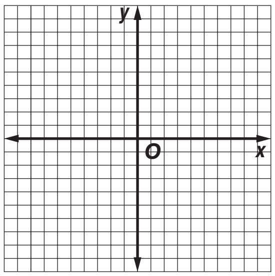
Exercises

Graph each function.

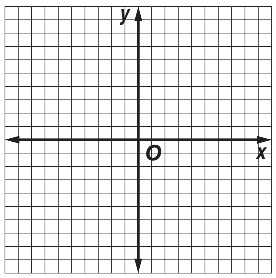
1. $x^3 + 1$



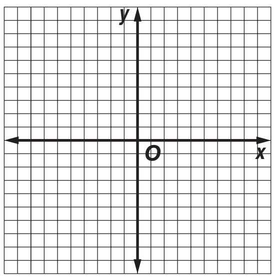
2. $x^3 - 2$



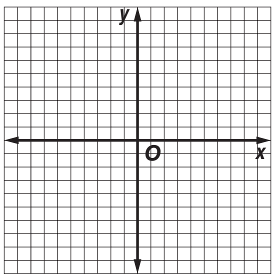
3. $2x^3$



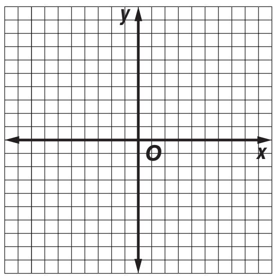
4. $2x^3 - 1$



5. $3x^3 + 1$



6. $-x^3 - 1$



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10-5 Study Guide and Intervention

7NS2.3, 7AF2.1, 7AF2.2

Multiplying Monomials

The **Product of Powers Property** states that to multiply powers that have the same base, add the exponents: $a^n \cdot a^m = a^{n+m}$.

Example **Multiply. Express using exponents.**

1 $\mathbf{2^3 \cdot 2^2}$

$2^3 \cdot 2^2 = 2^{3+2}$	The common base is 2.
$= 2^5$	Add the exponents.

2 $\mathbf{-2s^6(-7s^7)}$

$-2s^6(-7s^7) = (-2 \cdot -7)(s^6 \cdot s^7)$	Commutative and Associative Properties
$= (14)(s^{6+7})$	The common base is s.
$= 14s^{13}$	Add the exponents.

3 $\mathbf{n^5 \cdot n^{-3}}$

$n^5 \cdot n^{-3} = n^{5-3}$	The common base is *n*.
$= n^2$	Subtract the exponents.

Exercise

Multiply. Express using exponents.

1. $3^4 \cdot 3^1$

2. $5^2 \cdot 5^5$

3. $e^2 \cdot e^7$

4. $2a^5 \cdot 6a$

5. $-3t^3 \cdot 2t^8$

6. $4x^2(-5x^6)$

7. $-6t^4 \cdot -3t^5$

8. $\left(\frac{3}{4}\right)^{-3} \cdot \left(\frac{3}{4}\right)^6$

9. $-6m^2 \cdot 4m$

10. $3s^6(-9s^{-2}h^2)$

11. $9a^2(-6a^{-5})$

12. $-2e^4z^{-4}(6e^{-6})$

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10-6 Study Guide and Intervention

7NS2.3, 7AF2.1, 7AF2.2

Dividing Monomials

The Quotient of Powers Property states that to divide powers that have the same base, subtract the exponents: $a^n \div a^m = a^{n-m}$.

Example **Divide. Express using exponents.**

1 $\dfrac{k^8}{k}$

$\dfrac{k^8}{k} = k^{8-1}$ The common base is *k*.

$= k^7$ Subtract the exponents.

2 $\dfrac{28g^{12}}{-4g^3}$

$\dfrac{28g^{12}}{-4g^3} = \left(\dfrac{28}{-4}\right)\left(\dfrac{g^{12}}{g^3}\right)$ Commutative and Associative Properties

$= (-7)(g^{12-3})$ The common base is *g*.

$= -7g^9$ Subtract the exponents.

3 $\dfrac{5^8}{5^{-5}}$

$\dfrac{5^8}{5^{-5}} = 5^{8-(-5)}$ Quotient of Powers.

$= 5^{13}$ Simplify.

Exercise

Divide. Express using exponents.

1. $\dfrac{2^8}{2^6}$

2. $\dfrac{7^9}{7^3}$

3. $\dfrac{v^{14}}{v^6}$

4. $\dfrac{15w^7}{5w^2}$

5. $\dfrac{21z^{10}}{7z^9}$

6. $\dfrac{10m^8}{2m}$

7. $\dfrac{(-12)^3}{(-12)^3}$

8. $\dfrac{c^{20}}{c^{13}}$

9. $\dfrac{1^8}{1^6}$

10. $\dfrac{x^{-2}}{x^{-4}}$

11. $\dfrac{100^7}{100^6}$

12. $\dfrac{4^{-2}}{4^6}$

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10-7 Study Guide and Intervention

7AF2.2

Powers of Monomials

Rule: To find the power of a power, multiply the exponents.

Rule: To find the power of a product, find the power of each factor and multiply.

Example 1 **Simplify to find the power of the power:**

$(5^3)^6$

$(5^3)^6 = 5^{3 \cdot 6}$

$= 5^{18}$

Example 2 **Simplify to find the power of each factor.**

$(-3m^2n^4)^3$

$(-3m^2n^4)^3 = (-3)^3 \cdot m^{2 \cdot 3} \cdot n^{4 \cdot 3}$

$= -27m^6n^{12}$

Exercise

Simplify to find the power of the power.

1. $(4^3)^5$

2. $(4^2)^7$

3. $(9^2)^4$

4. $(k^4)^2$

5. $[(6^3)^2]^2$

6. $[(3^2)^2]^3$

Simplify to find the power of each product.

7. $(5q^4r^2)^5$

8. $(3y^2z^2)^6$

9. $(7a^4b^3c^7)^2$

10. $(-4d^3e^5)^2$

11. $(-5g^4h^9)^7$

12. $(0.2k^8)^2$

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10-8 Study Guide and Intervention

7AF2.2

Roots of Monomials

The **square root** of a monomial is one of the two equal factors of the monomial.

Example 1 **Simplify $\sqrt{25a^4}$.**

$\sqrt{25a^4} = \sqrt{25} \cdot \sqrt{a^4}$ Product Property of Square Roots

$= 5 \cdot a^2$ ← Absolute value is not necessary since the value of a^2 will never be negative.

Example 2 **Simplify $\sqrt{49y^6z^8}$.**

$\sqrt{49y^6z^8} = \sqrt{49} \cdot \sqrt{y^6} \cdot \sqrt{z^8}$ Product Property of Square Roots

$= 7 \cdot |y^3| \cdot z^4$ ← Use absolute value to indicate the positive value of y^3.

The **cube root** of a monomial is one of the three equal factors of the monomial.

Example 3 **Simplify $\sqrt[3]{d^6}$.**

$\sqrt[3]{d^6} = d^2$ $(d^2)^3 = d^6$

Example 4 **Simplify $\sqrt[3]{125m^9n^{12}}$.**

$\sqrt[3]{125m^9n^{12}} = \sqrt[3]{125} \cdot \sqrt[3]{m^9} \cdot \sqrt[3]{n^{12}}$ Product Property of Cube Roots

$= 5 \cdot m^3 \cdot n^4$ $(5)^3 = 125$; $(m^3)^3 = m^9$; and $(n^4)^3 = n^{12}$

Exercises

Simplify.

1. $\sqrt{c^2}$

2. $\sqrt{4s^6}$

3. $\sqrt{16a^8b^{12}}$

4. $\sqrt{64g^8h^{10}}$

5. $\sqrt{36r^2s^6}$

6. $\sqrt{121d^4e^{10}}$

7. $\sqrt[3]{p^6}$

8. $\sqrt[3]{27m^{15}}$

9. $\sqrt[3]{216a^9b^{21}}$

10. $\sqrt[3]{64y^{12}z^{24}}$

11. $\sqrt[3]{343t^{18}u^6}$

12. $\sqrt[3]{125p^{15}q^{27}}$

11-1 Study Guide and Intervention

7MR2.5, 7SDAP1.1

Problem-Solving Investigation: Make a Table

You may need to use the make a table strategy to solve some problems.

Explore	Determine what information is given in the problem and what you need to find out.
Plan	Select a strategy including a possible estimate.
Solve	Solve the problem by carrying out your plan.
Check	Examine your answer to see if it seems reasonable.

Example

For his science fair project, August decided to classify the 20 rocks and minerals in his collection by their hardness using the Mohs scale. After performing various tests for hardness, he recorded the hardness value of each rock or mineral in his collection in a list. Organize the data in a table using hardness intervals 1–2, 3–4, 5–6, 7–8, 9–10. What is the most common interval of rock hardness?

2	1	5	3	3	10	2	9	4	7	6	3	4	2	3	3	1	5	6	3

Explore You have a list of the hardness values for each rock or mineral. You need to know how many rocks have a hardness between 1 and 3, 4 and 6, and 7 and 10. Then you need to determine the most common interval of hardness.

Plan Make a frequency table with intervals to organize the information.

Hardness Interval	Tally	Frequency
1–2	𝍸	5
3–4	𝍸 |||	8
5–6	||||	4
7–8			1
9–10	||	2

Solve The most common interval of rock hardness is 1–3.

Check August tested 20 rocks for hardness. Since there are 20 values listed, the table seems reasonable.

Exercises

Make a table to solve each problem.

1. **BANKING** The list shows the amount of cash requested by each person that used a certain Automated Teller Machine (ATM) in one day. What is the most common amount of money requested by ATM users?

\$20	\$40	\$20	\$100	\$300	\$80	\$40	\$40	\$80	\$100	\$120	\$20
\$40	\$80	\$100	\$60	\$60	\$20	\$80	\$100	\$40	\$20	\$80	\$40

2. **COFFEE** The list shows the coffee sizes in ounces purchased in one hour at a local coffee house. What is the most commonly purchased size of coffee?

8	16	16	20	8	12	16	8	12	20
20	16	12	8	8	16	16	20	16	20

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11-2 Study Guide and Intervention

7SDAP1.1

Histograms

Data from a frequency table can be displayed as a histogram. A **histogram** is a type of bar graph used to display numerical data that have been organized into equal intervals. To make a histogram from a frequency table, use the following steps.

Step 1 Draw and label a horizontal and a vertical axis. Include a title.

Step 2 Show the intervals from the frequency table on the horizontal axis.

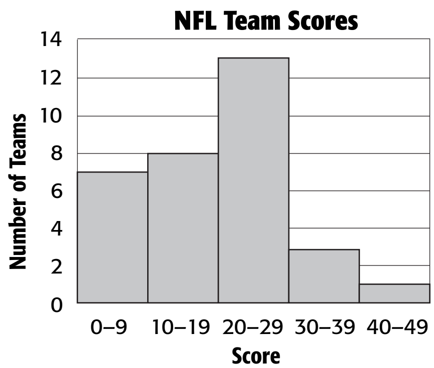
Step 3 For each interval on the horizontal axis, draw a bar whose height is given by the frequencies.

Example

FOOTBALL **The frequency table at the right shows the scores of all NFL teams in the first game of the 2005 season. Draw a histogram to represent the data.**

NFL Team Scores		
Score	**Tally**	**Frequency**
0–9	52	7
10–19	53	8
20–29	553	13
30–39	3	3
40–49	1	1

The histogram was created using the steps listed above. The horizontal axis is labeled "Score," the vertical axis is labeled "Number of Teams," and the histogram is titled "NFL Team Scores." The intervals are shown on the horizontal axis, and the frequencies are shown on the vertical axis. A bar is drawn in each interval to show the frequencies.

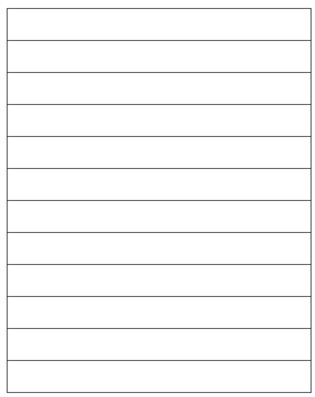


Exercise

TAXES The frequency table shows the tax on gasoline for the 50 states. Draw a histogram to represent the set of data.

Gas Tax for Each State		
Tax (cents/gal)	**Tally**	**Frequency**
8.1–12	2	2
12.1–16	5	5
16.1–20	55552	22
20.1–24	552	12
24.1–28	51	6
28.1–32	3	3

Gas Tax for Each State



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11-3 Study Guide and Intervention

7SDAP1.1

Circle Graphs

A **circle graph** can be used to represent data that compares parts of a set of data to the whole set.

Example BASEBALL **Make a circle graph using the information in the table at the right.**

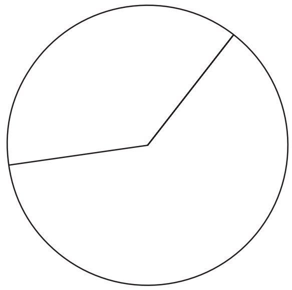
Batting in Major League Baseball	
Handedness	**Percent of Batters**
Right-Handed	62%
Left-Handed	26%
Switch Hitters /Unknown	12%

Step 1 There are 360° in a circle. So, multiply each percent by 360 to find the number of degrees for each section of the graph. Use a calculator.
Right-Handed: 62% of 360 = $0.62 \cdot 360$ or about 223°
Left-Handed: 26% of 360 = $0.26 \cdot 360$ or about 94°
Switch Hitters and Unknown: 12% of 360 = $0.12 \cdot 360$ or about 43°

Step 2 Use a compass to draw a circle and a radius. Then use a protractor to draw a 223° angle. This section represents right-handed hitters.

Step 3 From the new radius, draw a 94° angle. This section represents left-handed hitters. The remaining section represents switch hitters and players whose stance remains unknown. Now label each section. Then give the graph a title.

Batting in Major League Baseball



When the percents are not given, you must first determine what part of the whole each item represents.

Exercises

Make a circle graph for each set of data.

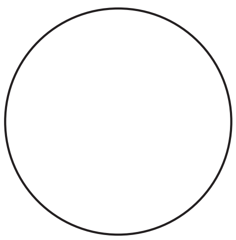
1.

U.S. Car and Truck Sales by Vehicle Size and Type, 2004	
Size and Type	**Percent**
Car	43%
Light Truck	54%
Heavy truck	3%

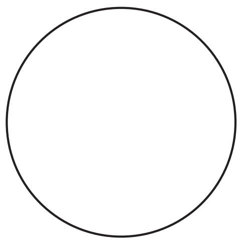
2.

Medals Won by the U.S. in the 2004 Summer Olympic Games	
Type	**Number**
Gold	35
Silver	39
Bronze	29

U.S. Car and Truck Sales by Vehicle Size and Type, 2004



Medals Won by the U.S. in the 2004 Summer Olympic Games



11-4 Study Guide and Intervention

7SDAP1.3

Measures of Central Tendency and Range

The most common measures of central tendency are mean, median, and mode. The range is also used to describe a set of data. To find the **mean** of a data set, find the sum of the data values then divide by the number of items in the set. To find the **median** of a data set, put the values in order from least to greatest, then find the middle number. If there are two middle numbers, add them together and divide by 2. The **mode** of a data set is the number or numbers that occur most often. If no number occurs more than once, the data set has no mode. The **range** of a data set is the difference between the greatest number and the least number in a set of data.

Example **Find the mean, median, mode, and range of the set of data. Round to the nearest tenth if necessary. The ages, in years, of relatives staying at your home are listed below.**
5, 14, 8, 2, 89, 14, 10, 2

Mean $\frac{5 + 14 + 8 + 2 + 89 + 14 + 10 + 2}{8} = 18$

The mean age is 18.

Median Arrange the numbers in order from least to greatest.
2 2 5 8 10 14 14 89

The middle numbers are 8 and 10. Since $\frac{8 + 10}{2} = 9$, the median age is 9.

Mode The numbers 2 and 14 each occur twice. The data set has two modes, 2 and 14.

Range 89–2 or 87

Different circumstances determine which measure of central tendency or range is most appropriate to describe a set of data. The mean is most useful when the data has no extreme values. The median is most useful when the data has a few extreme values with no big gaps in the middle of the data. The mode is most useful when the data has many identical numbers.

Exercises

Find the mean, median, mode, and range of each set of data. Round to the nearest tenth if necessary.

1. 2, 4, 5, 1, 3

2. 7, 5, 7, 7, 6, 4

3. 18, 14, 15, 11, 14, 12, 17

4. 19, 24, 22, 16, 15, 27, 22, 27

5. 2.3, 1.1, 1.5, 3.2, 1.7, 2.0, 2.4, 1.8

6. 36, 32, 34, 34, 35, 38, 36, 34

7. 30, 29, 30, 31, 30

8. 4.2, 5.2, 2.3, 4.0, 4.6, 6.0, 2.3, 5.3

11-5 Study Guide and Intervention

7SDAP1.3

Measures of Variation

The **lower quartile** or LQ is the median of the lower half of a set of data. The **upper quartile** or UQ is the median of the upper half of a set of data. The **interquartile range** is the difference between the upper quartile and the lower quartile.

Example 1 **Find the range, median, upper and lower quartiles, and interquartile range for the following set of data.**
13, 20, 18, 12, 21, 2, 18, 17, 15, 10, 14

The greatest number in the data set is 21. The least number is 2. The range is $21 - 2$ or 19.

To find the quartiles, arrange the numbers in order from least to greatest.
2 10 12 13 14 15 17 18 18 20 21

The median is 15. The numbers below 15 are 2, 10, 12, 13, and 14. The median of the numbers below 15 is 12, so the lower quartile is 12. The numbers above 15 are 17, 18, 18, 20, and 21. The median of the numbers above 15 is 18, so the upper quartile is 18. The interquartile range is $18 - 12$ or 6.

In some data sets, a few of the values are much greater than or less than the rest of the data. Data that are more than 1.5 times the value of the interquartile range beyond the quartiles are called **outliers**.

Example 2 **Find any outliers for the set of data given in Example 1.**

The interquartile range is $18-12$ or 6.
Multiply the interquartile range by 1.5. $6 \times 1.5 = 9$

Any data more than 9 above the upper quartile or below the lower quartile are outliers. Find the limits of the outliers.

Subtract 9 from the lower quartile. $12 - 9 = 3$
Add 9 to the upper quartile. $18 + 9 = 27$

The limits of the outliers are 3 and 27. The only data point outside this range is 2, so the only outlier is 2.

Exercises

Find the range, median, upper and lower quartiles, interquartile range, and any outliers for each set of data.

1. 14, 16, 18, 24, 19, 15, 13

2. 29, 27, 24, 28, 30, 51, 28

3. 57, 60, 43, 55, 46, 43, 62, 31

4. 91, 92, 88, 89, 93, 95, 65, 85, 91

5. 104, 116, 111, 108, 113, 127, 109, 122, 115, 105

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11-6 Study Guide and Intervention

7SDAP1.1

Box-and-Whisker Plots

A **box-and-whisker plot** uses a number line to show the distribution of a set of data. The *box* is drawn around the quartile values, and the *whiskers* extend from each quartile to the extreme data points that are not outliers.

Example **Use the data below to construct a box-and-whisker plot.**
12, 14, 8, 10, 1, 16, 10, 11, 10

Step 1 Put the data in order from least to greatest and find the median, lower quartile, upper quartile, and the least and greatest values that are not outliers.
Ordered data: 1, 8, 10, 10, 10, 11, 12, 14, 16

Least value: 1; Median: 10; Greatest value: 16;

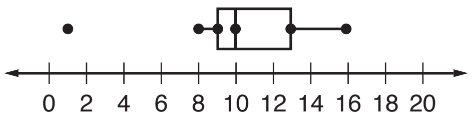
Lower quartile: $\frac{8 + 10}{2}$ or 9; Upper quartile: $\frac{12 + 14}{2}$ or 13;

Interquartile range: $13 - 9$ or 4;
Lower limit for outliers: $9 - 6$ or 3;
Upper limit for outliers: $13 + 6$ or 19;
Outliers: 1

Step 2 Draw a number line that includes the least and greatest numbers in the data.

Step 3 Mark the extremes, the median, and the upper and lower quartile above the number line. Since the data have an outlier, mark the least value that is not an outlier.

Step 4 Draw the box and the whiskers.

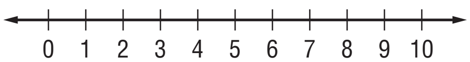


Box-and-whisker plots separate data into four parts. Even though the parts may differ in length, each part contains $\frac{1}{4}$ of the data.

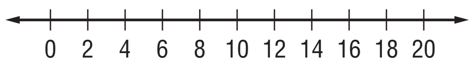
Exercises

Construct a box-and-whisker plot for each set of data.

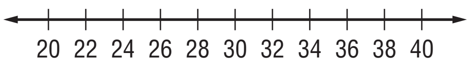
1. 4, 7, 5, 3, 9, 6, 4



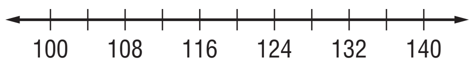
2. 13, 12, 17, 10, 6, 11, 14



3. 23, 36, 22, 34, 30, 29, 26, 27, 33



4. 108, 126, 110, 104, 106, 123, 140, 122, 114, 109



11-7 Study Guide and Intervention

7SDAP1.1

Stem-and-Leaf Plots

Stem-and-Leaf Plot

Words One way to organize and display data is to use a **stem-and-leaf plot**. In a stem-and-leaf plot, numerical data are listed in ascending or descending order.

Model

Stem	Leaf
2	0 1 1 2 3 5 5 6
3	1 2 2 3 7 9
4	0 3 4 8 8

3 | 7 = 37

The greatest place value of the data is used for the **stems**.

The next greatest place value forms the **leaves**.

Example ZOOS **Display the data shown at the right in a stem-and-leaf plot.**

Size of U. S. Zoos

Zoo	Size (acres)
Audubon (New Orleans)	58
Cincinnati	85
Dallas	95
Denver	80
Houston	55
Los Angeles	80
Oregon	64
St. Louis	90
San Francisco	75
Woodland Park (Seattle)	92

Step 1 The least and the greatest numbers are 55 and 95. The greatest place value digit in each number is in the tens. Draw a vertical line and write the stems from 5 to 9 to the left of the line.

Step 2 Write the leaves to the right of the line, with the corresponding stem. For example, for 85, write 5 to the right of 8.

Stem	Leaf
5	8 5
6	4
7	5
8	5 0 0
9	5 0 2

Step 3 Rearrange the leaves so they are ordered from least to greatest. Then include a key or an explanation.

Stem	Leaf
5	5 8
6	4
7	5
8	0 0 5
9	0 2 5

8 | 5 = 85 acres

Exercises

Display each set of data in a stem-and-leaf plot.

1. {27, 35, 39, 27, 24, 33, 18, 19}

2. {94, 83, 88, 77, 95, 99, 88, 87}

ROLLER COASTERS **For Exercises 3 and 4, use the stem-and-leaf plot shown.**

The Fastest Roller Coasters

Stem	Leaf
8	3 5
9	2 5
10	0

8 | 3 = 83 mph

3. What is the speed of the fastest roller coaster? The slowest?

4. What is the median speed?

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11-8 Study Guide and Intervention

7SDAP1.1

Select an Appropriate Display

There are many different ways to display data. Some of these displays and their uses are listed below.

Type of Display	Best Used to
Bar Graph	show the number of items in specific categories.
Box-and-Whisker Plot	show measures of variation for a set of data.
Circle Graph	compare parts of the data to the whole.
Histogram	show frequency of data divided into equal intervals.
Line Graph	show change over a period of time.
Line Plot	show how many times each number occurs in the data.
Stem-and-Leaf Plot	list all individual numerical data in condensed form.
Venn Diagram	show how elements among sets of data are related.

As you decide what type of display to use, ask the following questions.

- What type of information is this?
- What do I want my graph or display to show?

Remember, all data sets can be displayed in more than one way. And there is often more than one appropriate way to display a given set of data.

Example **Choose an appropriate type of display for each situation.**

1 the change in the winning times for the Kentucky Derby for the last 15 years

This data does not deal with categories or intervals. It deals with the change of a value over time. A line graph is a good way to show changes over time.

2 energy usage in the U.S., categorized by the type of user

In this case, there are specific categories. If you want to show the specific amount of energy used in each category, use a bar graph. If you want to show how each category is related to the whole, use a circle graph.

Exercises

Select an appropriate type of display for each situation. Justify your reasoning.

1. the cost of homeowners insurance over the past 10 years

2. the amount of federally owned land in each state, arranged in intervals

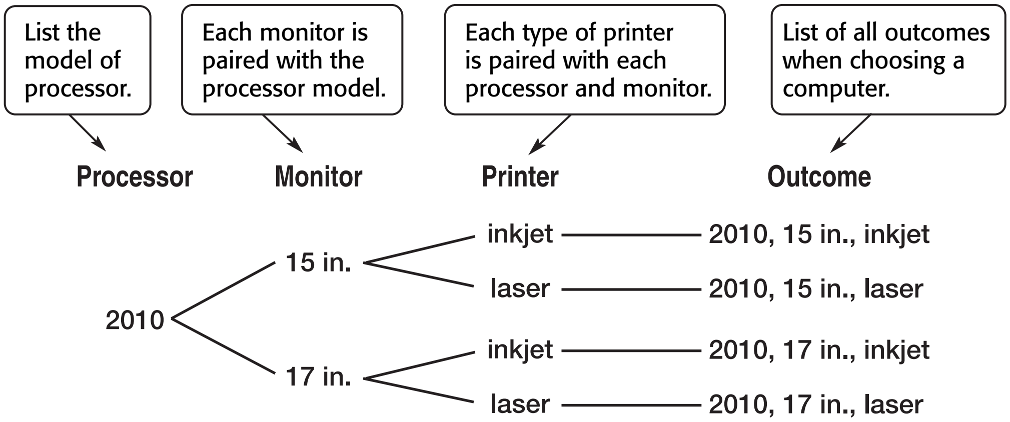
12-1 Study Guide and Intervention

6SDAP3.1

Counting Outcomes

An organized list of outcomes, called a **sample space,** can help you determine the total number of possible outcomes for an event.

Example 1 COMPUTERS **An electronics store offers a model 2010 processor with a choice of 2 monitors (15-inch and 17-inch) and 2 printers (inkjet and laser). Draw a tree diagram to determine how many different computer systems are available.**



There are 4 different computer systems available.

If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then the event *M* followed by the event *N* can occur in $m \cdot n$ ways. This principle is known as the **Fundamental Counting Principle**.

Example 2 LOCKS **A lock combination is made up of three numbers from 0 to 39. How many combinations are possible?**

Use the Fundamental Counting Principle. $40 \times 40 \times 40 = 64{,}000$

There are 64,000 possible lock combinations.

Exercises

1. A museum tour includes a box lunch which contains a ham, turkey, or cheese sandwich and an apple, a banana, an orange, or a pear. An equal number of all lunch combinations are available for each tour. Draw a tree diagram to determine the number of outcomes.

Use the Fundamental Counting Principle to find the number of possible outcomes.

2. A number cube is rolled twice.

3. Six coins are tossed.

12-2 Study Guide and Intervention

6SDAP3.1

Probability of Compound Events

The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

Example 1 **Two number cubes, one red and one blue, are rolled. What is the probability that the outcome of the red number cube is even and the outcome of the blue number cube is a 5?**

$P(\text{red number cube is even}) = \frac{1}{2}$

$P(\text{blue number cube is a 5}) = \frac{1}{6}$

$P(\text{red number cube is even and blue number cube is a 5}) = \frac{1}{2} \cdot \frac{1}{6} \text{ or } \frac{1}{12}$

The probability that the two events will occur is $\frac{1}{12}$.

If two events, *A* and *B*, are dependent, then the probability of both events occurring is the product of the probability of *A* and the probability of *B* after *A* occurs.

Example 2 **There are 6 black socks and 4 white socks in a drawer. If one sock is taken out without looking and then a second is taken out, what is the probability that they both will be black?**

$P(\text{first sock is black}) = \frac{6}{10} \text{ or } \frac{3}{5}$ — 6 is the number of black socks; 10 is the total number of socks.

$P(\text{second sock is black}) = \frac{5}{9}$ — 5 is the number of black socks after one black sock is removed; 9 is the total number of socks after one black sock is removed.

$P(\text{two black socks}) = \frac{3}{5} \cdot \frac{5}{9} \text{ or } \frac{1}{3}$

The probability of choosing two black socks is $\frac{1}{3}$.

Exercises

A card is drawn from a deck of 10 cards numbered 1 through 10 and a number cube is rolled. Find each probability.

1. *P*(10 and 3)

2. *P*(two even numbers)

3. *P*(two prime numbers)

4. *P*(9 and an odd number)

5. *P*(two numbers less than 4)

6. *P*(two numbers greater than 5)

There are 4 red, 6 green, and 5 yellow pencils in a jar. Once a pencil is selected, it is not replaced. Find each probability.

7. *P*(red and then yellow)

8. *P*(two green)

9. *P*(green and then yellow)

10. *P*(red and then green)

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12-3 Study Guide and Intervention

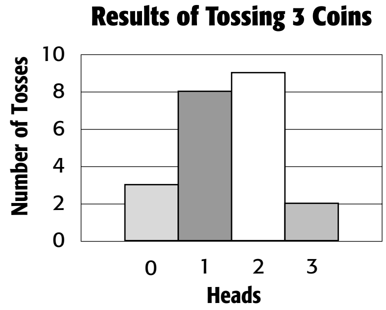
6SDAP3.2

Experimental and Theoretical Probability

Probabilities based on the outcomes obtained by conducting an experiment are called **experimental probabilities**. Probabilities based on known characteristics or facts are called **theoretical probabilities**. Theoretical probability tells you what *should* happen in an experiment.

Examples

Kuan is conducting an experiment to find the probability of getting 0, 1, 2, or 3 heads when tossing three coins on the floor. The results of his experiment are given at the right.



1 Based on the results in the bar graph, what is the probability of getting 3 heads on the next toss?

There were 22 tosses and 2 of those had 3 heads. The experimental probability is $\frac{2}{22}$ or $\frac{1}{11}$.

2 Based on the experimental probability, how many times should Kuan expect to get 3 heads in the next 55 tosses?

Kuan should expect to get 3 heads about $\frac{1}{11} \cdot 55$ or 5 times.

3 What is the theoretical probability of getting 3 heads on a toss?

The theoretical probability is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{8}$.

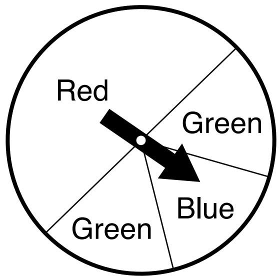
The experimental probability and the theoretical probability seem to be consistent.

Exercises

Use the table that shows the results of spinning a game spinner 50 times.

Color	Number of Times
green	18
red	24
blue	8

1. Based on the results in the table, what is the probability of spinning green?
2. Based on the results, how many green spins would you expect to occur in 300 spins?
3. What is the theoretical probability of spinning green?
4. Based on the theoretical probability, how many green spins would you expect to occur in 300 spins?
5. Compare the theoretical probability to the experimental probability.



12-4 Study Guide and Intervention

7MR2.5, 6SDAP3.2

Problem-Solving Investigation: Act It Out

Example **Michael has a red square tile, a blue square tile, a green square tile, and a yellow square tile. How many different ways can he arrange the tiles so that they form a larger square?**

Explore There are four tiles that can be arranged into a larger 2 by 2 square. How many different ways can the tiles be arranged into the larger square?

Plan Use letters to stand for each color tile. Arrange the tiles starting with each combination of tiles that has the red tile in the upper left corner. Then repeat this step for each of the other three colors.

Solve

RB GY	RB YG	RG BY	RG YB	RY BG	RY GB	There are 6 large squares with the red tile in the upper left.
BR GY	BR YG	BG RY	BG YR	BY RG	BY GR	There are 6 large squares with the blue tile in the upper left.
GR BY	GR YB	GB RY	GB YR	GY RB	GY BR	There are 6 large squares with the green tile in the upper left.
YR BG	YR GB	YB RG	YB GR	YG RB	YG BR	There are 6 large squares with the yellow tile in the upper left.

Check Each larger square with the red square in the upper left corner is shown for a total of six. Therefore there should be 6 sets for each color. $4 \times 6 = 24$. There are 24 ways that Michael can arrange the tiles into larger squares.

Exercises

For Exercises 1–3, solve each problem using the act it out strategy.

1. **GEOMETRY** How many different pairs of polygons can be made from 16 toothpicks with none left over?

2. **MONEY** Byron wants to buy a comic book that costs $0.65. If he uses exact change, how many different combinations of nickels, dimes, and quarters can he use?

3. **NUMBER LINE** In a math class game, players are using a number line on the floor. Grace starts at zero and moves forward 7 numbers on her first turn and moves backward 4 numbers on her second turn. If this pattern continues, how many turns will it take for her to move forward to 16?

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12-5 Study Guide and Intervention

6SDAP2.5

Using Sampling to Predict

Data gathered from a representative sample can be used to make predictions about a population. An **unbiased sample** is selected so that it is representative of the entire population. In a **biased sample,** one or more parts of the population are favored over others.

Examples **Describe each sample.**

1 **To determine the favorite dog breed of people who enter dog shows, every fifth person entering a dog show is surveyed.**

Since the people are selected according to a specific pattern, the sample is a systematic random sample. It is an unbiased sample.

2 **To determine what type of pet people prefer, the spectators at a dog show are surveyed.**

The spectators at a dog show probably prefer dogs. This is a biased sample. The sample is a convenience sample since all of the people surveyed are in one location.

Examples COOKIES **Students in the eighth grade surveyed 50 students at random about their favorite cookies. The results are in the table at the right.**

Flavor	Number
oatmeal	15
peanut butter	11
chocolate chip	16
sugar	8

3 **What percent of students prefer chocolate chip cookies?**

16 out of 50 students prefer chocolate chip cookies.
$16 \div 50 = 0.32$ 32% of the students prefer chocolate chip cookies.

4 **If the students order 500 boxes of cookie dough, how many boxes should be chocolate chip?**

Find 32% of 500.
$0.32 \times 500 = 160$ About 160 boxes of cookie dough should be chocolate chip.

Exercises

Describe the sample.

1. To determine if the tomatoes in 5 boxes stacked on a pallet are not spoiled, the restaurant manager checks 3 tomatoes from the top box.

A random survey of the students in eighth grade shows that 7 prefer hamburgers, 5 prefer chicken, and 3 prefer hot dogs.

2. What percent prefer hot dogs?
3. If 120 students will attend the eighth grade picnic, how many hot dogs should be ordered?

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