Glencoe McGraw-Hill

California Geometry

Noteables™

Interactive Study Notebook with Foldables™

Contributing Author
Dinah Zike

Foldables™

Consultant
Douglas Fisher, Ph.D.
Director of Professional Development
San Diego, CA

McGraw Hill Glencoe

New York, New York   Columbus, Ohio   Chicago, Illinois   Woodland Hills, California

Glencoe

The McGraw-Hill Companies

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 978-0-07-879364-6
MHID: 0-07-879364-5

Geometry (California Student Edition)
Noteables™: Interactive Study Notebook with Foldables™

1 2 3 4 5 6 7 8 9 10 005 16 15 14 13 12 11 10 09 08 07

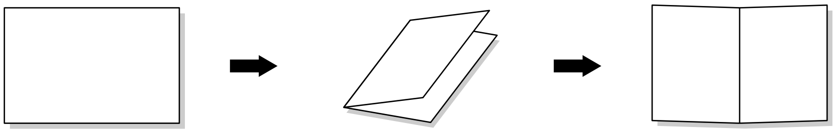
Contents

Contents

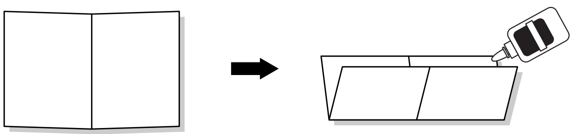
Organizing Your Foldables

FOLDABLES Make this Foldable to help you organize and store your chapter Foldables. Begin with one sheet of 11" × 17" paper.

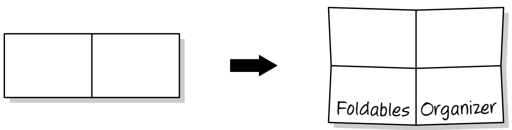
STEP 1 **Fold**
Fold the paper in half lengthwise. Then unfold.



STEP 2 **Fold and Glue**
Fold the paper in half widthwise and glue all of the edges.



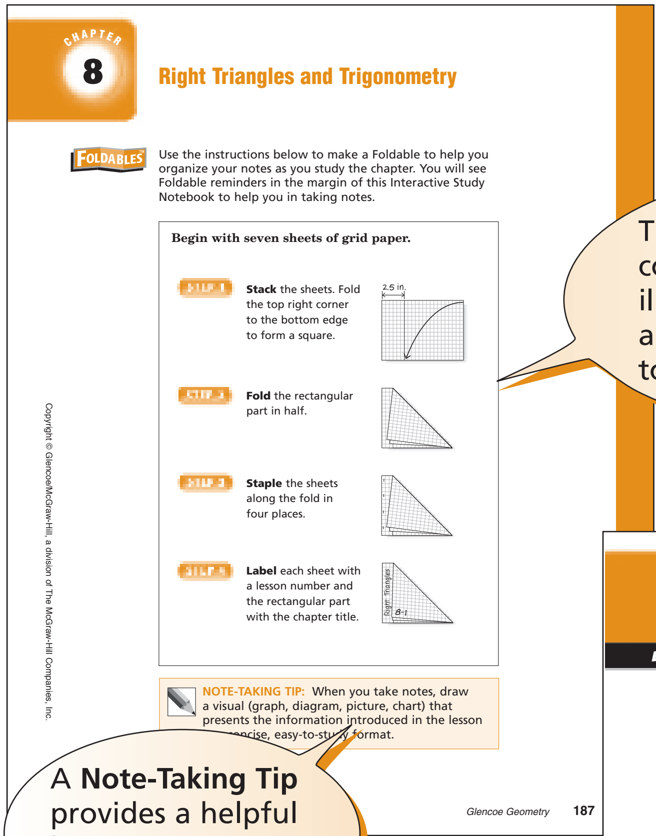
STEP 3 **Glue and Label**
Glue the left, right, and bottom edges of the Foldable to the inside back cover of your Noteables notebook.



Reading and Taking Notes As you read and study each chapter, record notes in your chapter Foldable. Then store your chapter Foldables inside this Foldable organizer.

Using Your Noteables™ with Foldables™
Interactive Study Notebook

This note-taking guide is designed to help you succeed in *Geometry*. Each chapter includes:



CHAPTER 8

Right Triangles and Trigonometry

FOLDABLES Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with seven sheets of grid paper.

STEP 1 **Stack** the sheets. Fold the top right corner to the bottom edge to form a square.

STEP 2 **Fold** the rectangular part in half.

STEP 3 **Staple** the sheets along the fold in four places.

STEP 4 **Label** each sheet with a lesson number and the rectangular part with the chapter title.

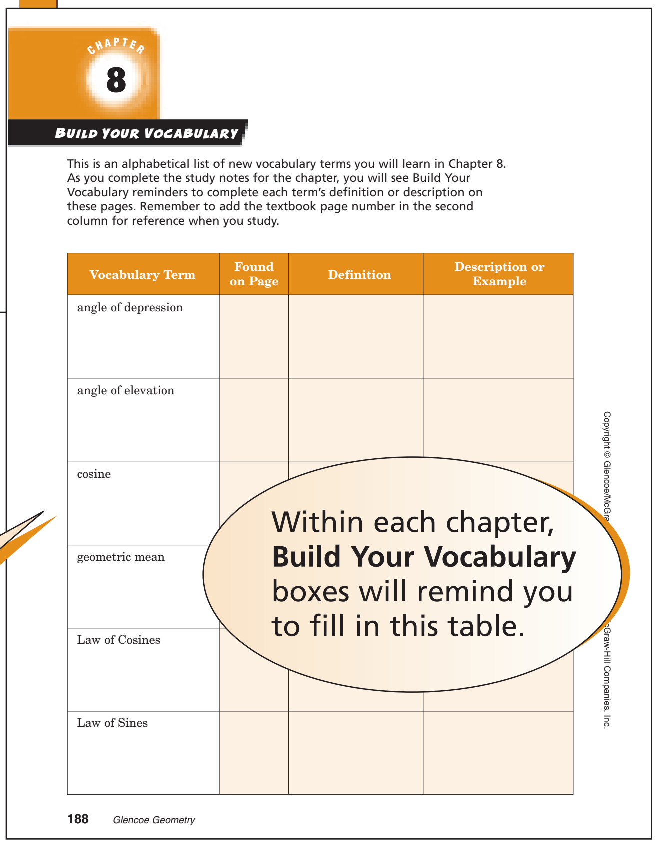
NOTE-TAKING TIP: When you take notes, draw a visual (graph, diagram, picture, chart) that presents the information introduced in the lesson …ncise, easy-to-stu…y format.

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

Glencoe Geometry 187

The **Chapter Opener** contains instructions and illustrations on how to make a Foldable that will help you to organize your notes.

A **Note-Taking Tip** provides a helpful hint you can use when taking notes.



CHAPTER 8

BUILD YOUR VOCABULARY

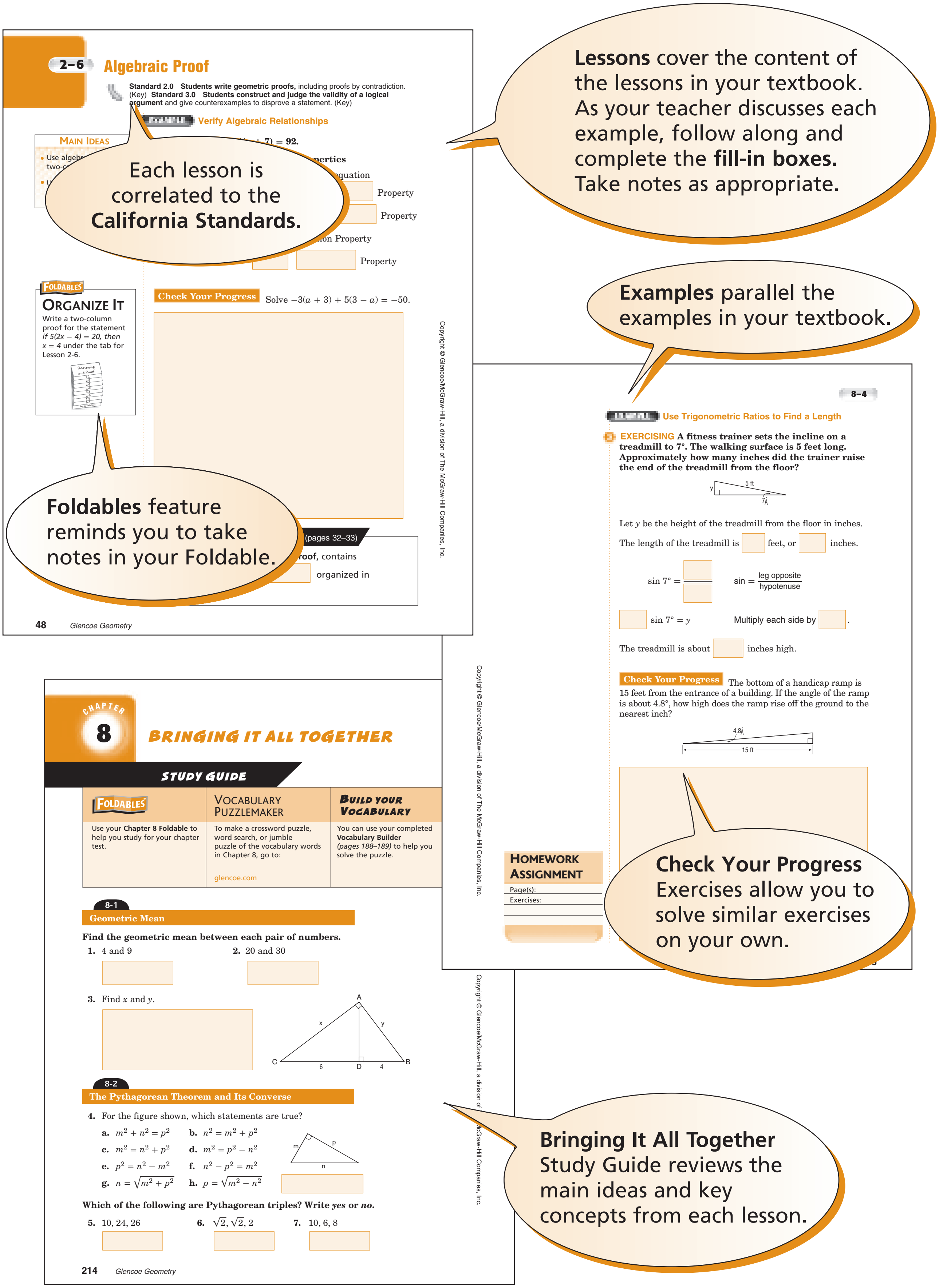
This is an alphabetical list of new vocabulary terms you will learn in Chapter 8. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
angle of depression			
angle of elevation			
cosine			
geometric mean			
Law of Cosines			
Law of Sines			

188 Glencoe Geometry

The **Build Your Vocabulary** table allows you to write definitions and examples of important vocabulary terms together in one convenient place.

Within each chapter, **Build Your Vocabulary** boxes will remind you to fill in this table.



Lessons cover the content of the lessons in your textbook. As your teacher discusses each example, follow along and complete the fill-in boxes. Take notes as appropriate.
Each lesson is correlated to the California Standards.
Examples parallel the examples in your textbook.
Foldables feature reminds you to take notes in your Foldable.
Check Your Progress Exercises allow you to solve similar exercises on your own.
Bringing It All Together Study Guide reviews the main ideas and key concepts from each lesson.
2–6 Algebraic Proof
Verify Algebraic Relationships
Organize It
Check Your Progress
8–4
Use Trigonometric Ratios to Find a Length
Homework Assignment
Chapter 8 Bringing It All Together
Study Guide
Vocabulary Puzzlemaker
Build Your Vocabulary
8-1 Geometric Mean
8-2 The Pythagorean Theorem and Its Converse

NOTE-TAKING TIPS

Your notes are a reminder of what you learned in class. Taking good notes can help you succeed in mathematics. The following tips will help you take better classroom notes.

- Before class, ask what your teacher will be discussing in class. Review mentally what you already know about the concept.
- Be an active listener. Focus on what your teacher is saying. Listen for important concepts. Pay attention to words, examples, and/or diagrams your teacher emphasizes.
- Write your notes as clear and concise as possible. The following symbols and abbreviations may be helpful in your note-taking.

Word or Phrase	Symbol or Abbreviation	Word or Phrase	Symbol or Abbreviation
for example	e.g.	not equal	$\neq$
such as	i.e.	approximately	$\approx$
with	w/	therefore	$\therefore$
without	w/o	versus	vs
and	+	angle	$\angle$

- Use a symbol such as a star (★) or an asterisk (*) to emphasize important concepts. Place a question mark (?) next to anything that you do not understand.
- Ask questions and participate in class discussion.
- Draw and label pictures or diagrams to help clarify a concept.
- When working out an example, write what you are doing to solve the problem next to each step. Be sure to use your own words.
- Review your notes as soon as possible after class. During this time, organize and summarize new concepts and clarify misunderstandings.

Note-Taking Don'ts

- **Don't** write every word. Concentrate on the main ideas and concepts.
- **Don't** use someone else's notes as they may not make sense.
- **Don't** doodle. It distracts you from listening actively.
- **Don't** lose focus or you will become lost in your note-taking.

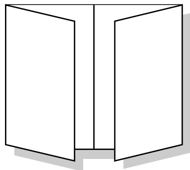
Tools of Geometry

FOLDABLES™

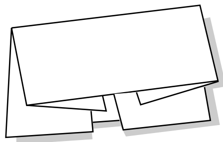
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11" × 17" paper.

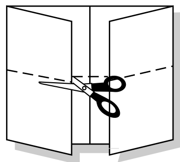
STEP 1 **Fold** the short sides to meet in the middle.



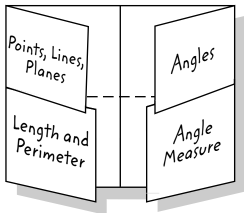
STEP 2 **Fold** the top to the bottom.



STEP 3 **Open.** Cut flaps along second fold to make four tabs.



STEP 4 **Label** the tabs as shown.



NOTE-TAKING TIP: When you take notes, listen or read for main ideas. Then record what you know and apply these concepts by drawing, measuring, and writing about the process.

CHAPTER 1

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 1. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
adjacent angles [uh-JAY-suhnt]			
angle			
angle bisector			
collinear [koh-LIN-ee-uhr]			
complementary angles			
congruent [kuhn-GROO-uhnt]			
coplanar [koh-PLAY-nuhr]			
degree			
line			
line segment			
linear pair			

Vocabulary Term	Found on Page	Definition	Description or Example
midpoint			
perpendicular			
plane			
point			
polygon [PAHL-ee-gahn]			
polyhedron			
precision			
ray			
segment bisector			
sides			
supplementary angles			
vertex			
vertical angles			

1–1 Points, Lines, and Planes

Standard 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning. (Key)

MAIN IDEAS

- Identify and model points, lines, and planes.
- Identify collinear and coplanar points and intersecting lines and planes in space.

BUILD YOUR VOCABULARY (page 2)

Points on the same ______ are **collinear**.

Points that lie on the same ______ are **coplanar**.

KEY CONCEPTS

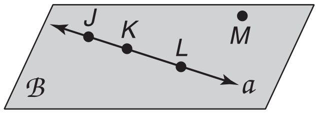
Point A point has neither shape nor size.

Line There is exactly one line through any two points.

Plane There is exactly one plane through any three noncollinear points.

EXAMPLE Name Lines and Planes

1 **Use the figure to name each of the following.**



a. a line containing point *K*

The line can be named as line ______.

There are three points on the line. Any two of the points can be used to name the line. The possible names are

______.

b. a plane containing point *L*

The plane can be named as plane ______.

You can also use the letters of any three

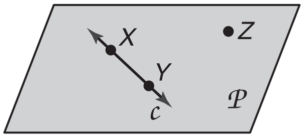
______ points to name the plane.

plane ______ plane ______ plane ______

Check Your Progress **Use the figure to name each of the following.**

a. a line containing point *X*

______



b. a plane containing point *Z*

______

EXAMPLE Model Points, Lines, and Planes

1 **Name the geometric term modeled by each object.**

a. the long hand on a clock

The long hand on a clock models a ______.

b. a 10 × 12 patio

The patio models a ______.

c. a water glass on the table

This models a ______.

Check Your Progress **Name the geometric shape modeled by each object.**

a. a colored dot on a map used to mark the location of a city ______

b. the ceiling of your classroom ______

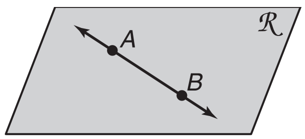
EXAMPLE Draw Geometric Figures

3 **Draw and label a figure for each situation.**

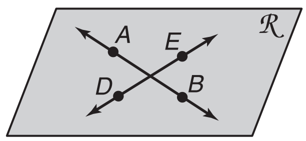
a. ALGEBRA **Plane $\mathcal{R}$ contains lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$, which intersect at point P. Add point C on plane $\mathcal{R}$ so that it is not collinear with $\overleftrightarrow{AB}$ or $\overleftrightarrow{DE}$.**

Draw a surface to represent ______ and label it.

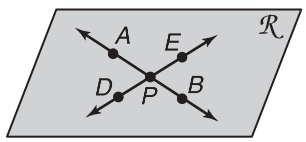
Draw a line anywhere on the plane and draw dots on the line for points A and B. Label the points.



Draw a line intersecting ______ and draw dots on the line for points D and E. Label the points.



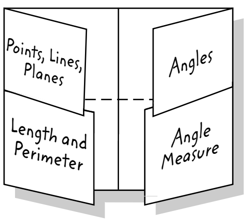
Label the intersection of the two lines as ______.



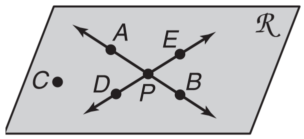
FOLDABLES™

ORGANIZE IT

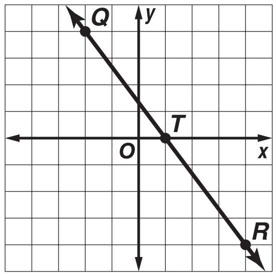
Draw and label a point *P*, a line *AB*, and a plane *XYZ* under the Points, Lines, and Planes tab.



Draw a dot for point C in plane $\mathcal{R}$ such that it will not lie on $\overleftrightarrow{AB}$ or $\overleftrightarrow{DE}$. Label the point.



b. $\overleftrightarrow{QR}$ on a coordinate plane contains $Q(-2, 4)$ and $R(4, -4)$. Add point T so that T is collinear with these points.



Graph each point and draw $\overleftrightarrow{QR}$.

There are an infinite number of points that are collinear with Q and R. In the graph, one such point is ______.

REMEMBER IT

The prefix *co-* means *together*. So, collinear means lying together on the same line. Coplanar means lying together in the same plane.

Check Your Progress **Draw and label a figure for each relationship.**

a. Plane $\mathcal{D}$ contains line a, line m, and line t, with all three lines intersecting at point Z. Add point F on plane $\mathcal{D}$ so that it is not collinear with any of the three given lines.

b. $\overleftrightarrow{BA}$ on a coordinate plane contains $B(-3, -2)$ and $A(3, 2)$. Add point M so that M is collinear with these points.

WRITE IT

Explain the different ways of naming a plane.

______

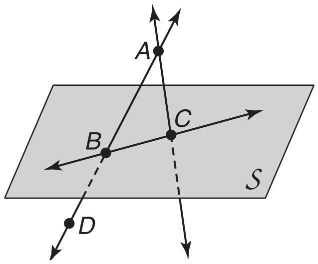
______

______

______

EXAMPLE Interpret Drawings

4 **Use the figure for parts a–d.**



a. How many planes appear in this figure?

b. Name three points that are collinear.

Points ___, ___, and ___ are collinear.

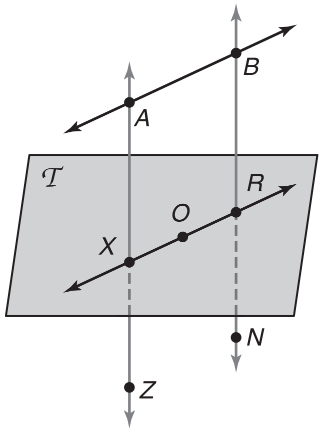
c. Are points *A, B, C,* and *D* coplanar? Explain.

Points *A, B, C,* and *D* all lie in ___, so they are coplanar.

d. At what point do $\overleftrightarrow{DB}$ and $\overleftrightarrow{CA}$ intersect?

The two lines intersect at point ___.

Check Your Progress **Refer to the figure.**



a. How many planes appear in this figure?

b. Name three points that are collinear.

c. Are points *X*, *O*, and *R* coplanar? Explain.

d. At what point do $\overleftrightarrow{BN}$ and $\overleftrightarrow{XO}$ intersect?

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–2 Linear Measure and Precision

Standard 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. (Key)

MAIN IDEAS

- Measure segments and determine accuracy of measurement.
- Compute with measures.

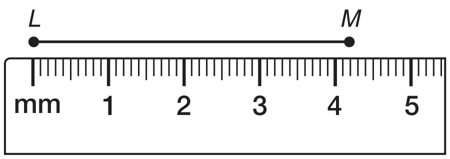
BUILD YOUR VOCABULARY (page 2)

A **line segment** can be measured because it has

two ________.

EXAMPLE Length in Metric Units

1 Find the length of $\overline{LM}$ using the ruler.



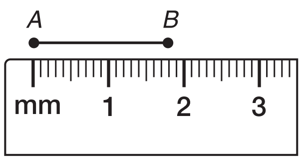
The long marks indicate ________, and the

shorter marks indicate ________.

$\overline{LM}$ is about ____ millimeters long.

Check Your Progress

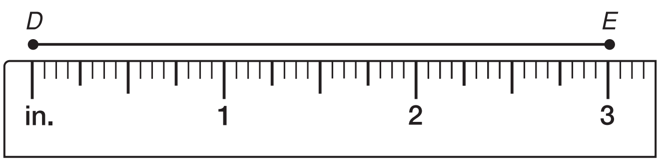
a. Find the length of $\overline{AB}$.



EXAMPLE Length in Customary Units

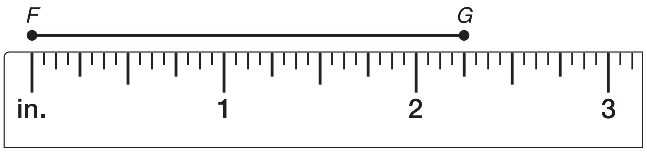
2 Find the length of each segment using each ruler.

a. $\overline{DE}$



Each inch is divided into ________. $\overline{DE}$ is about ____ inches long.

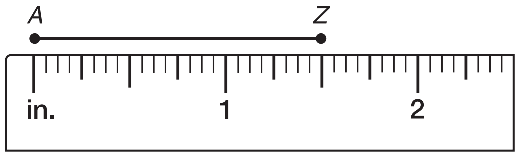
b. $\overline{FG}$



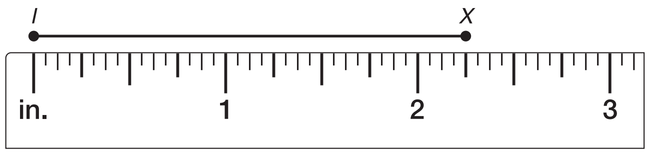
$\overline{FG}$ is about ______ inches long.

Check Your Progress

a. Find the length of $\overline{AZ}$.



b. Find the length of $\overline{IX}$.



EXAMPLE Precision

3 **Find the precision for each measurement. Explain its meaning.**

a. $32\frac{3}{4}$ **inches**

The measuring tool is divided into ______ increments.

Thus, the measurement is precise to within $\frac{1}{2}\left(\frac{1}{4}\right)$ or $\frac{1}{8}$ inch.

The measurement could be ______ to ______ inches.

b. **15 millimeters**

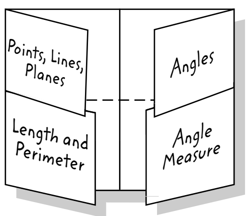
The measuring tool is divided into millimeters. Thus the measurement is precise to within $\frac{1}{2}$ of 1 millimeter. The measurement could be 14.5 to 15.5 millimeters.

Check Your Progress Find the precision for each measurement.

FOLDABLES™

ORGANIZE IT

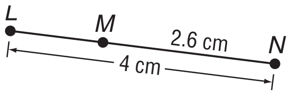
Explain how to find the precision of a measurement. Write this under the Length and Perimeter tab.



EXAMPLE Find Measurements

1 Find the measurement of each segment.

a. $\overline{LM}$



$LM + MN = LN$

$LM +$ ____ $=$ ____ Substitution

$LM +$ ____ $-$ ____ $=$ ____ $-$ ____ Subtract.

$LM =$ ____ Simplify.

$\overline{LM}$ is ____ centimeters long.

b. x **and** ST **if** T **is between** S **and** U**,** $ST = 7x$**,** $SU = 45$**, and** $TU = 5x - 3$**.**

$ST + TU = SU$

$7x +$ ____ $=$ ____ Substitute known values.

$7x + 5x - 3 + 3 = 45 + 3$ Add 3 to each side.

$12x = 48$ Simplify.

$\frac{12x}{12} = \frac{48}{12}$ Divide each side by 12.

____ $=$ ____ Simplify.

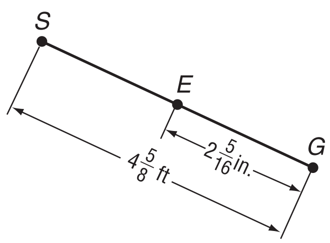
$ST = 7x$ Given

$= 7(4)$ $x = 4$

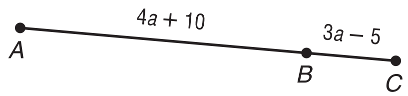
$=$ ____ Thus, $x = 4$, $ST =$ ____.

Check Your Progress

a. Find SE.



b. Find a and AB if $AB = 4a + 10$, $BC = 3a - 5$, and $AC = 19$.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–3 Distance and Midpoints

Preparation for Standard 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

MAIN IDEAS

- Find the distance between two points.
- Find the midpoint of a segment.

KEY CONCEPT

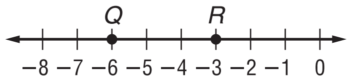
Distance Formulas

Number Line If P and Q have coordinates a and b, respectively, then the distance between P and Q is given by $PQ = |b - a|$ or $|a - b|$.

Coordinate Plane The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

EXAMPLE Find Distance on a Number Line

1 Use the number line to find QR.



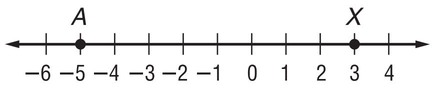
The coordinates of Q and R are ____ and ____.

$QR = |-6 - (-3)|$ Distance Formula

$= |-3|$ or 3 Simplify.

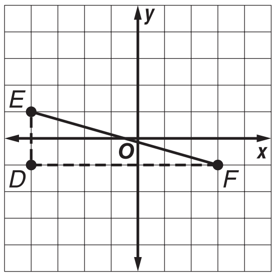
Check Your Progress Use the number line to find AX.

____



EXAMPLE Find Distance on a Coordinate Plane

2 Find the distance between $E(-4, 1)$ and $F(3, -1)$.



Method 1 Pythagorean Theorem

$(EF)^2 = (ED)^2 + (DF)^2$

$(EF)^2 = \text{____}^2 + \text{____}^2$

$(EF)^2 = 53$

$EF = \sqrt{53}$

Method 2 Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$EF = \sqrt{[3 - (-4)]^2 + (-1 - 1)^2}$ $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (3, -1)$

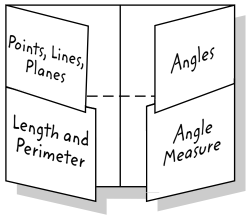
$EF = \sqrt{(\text{____})^2 + (\text{____})^2}$ Simplify.

$EF = \sqrt{53}$ Simplify.

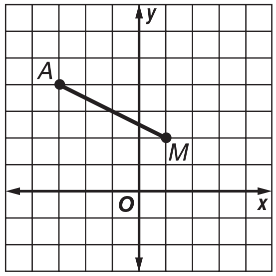
FOLDABLES™

ORGANIZE IT

Draw a coordinate system and show how to find the distance between two points under the Length and Perimeter tab.



Check Your Progress Find the distance between $A(-3, 4)$ and $M(1, 2)$.



BUILD YOUR VOCABULARY (page 3)

The **midpoint** of a segment is the point halfway between the ________ of the segment.

Any segment, line, or plane that intersects a segment at its ________ is called a **segment bisector**.

KEY CONCEPT

Midpoint The midpoint *M* of $\overline{PQ}$ is the point between *P* and *Q* such that *PM* = *MQ*.

EXAMPLE Find Coordinate of Endpoint

3 **Find the coordinates of *D* if *E*(–6, 4) is the midpoint of $\overline{DF}$ and *F* has coordinates (–5, –3).**

Let *F* be (x_2, y_2) in the Midpoint Formula.

$$E(-6, 4) = E\left(\frac{x_1 + (-5)}{2}, \frac{y_1 + (-3)}{2}\right) \qquad (x_2, y_2) = (-5, -3)$$

Write and solve two equations to find the coordinates of *D*.

$$-6 = \frac{x_1 + (-5)}{2} \qquad 4 = \frac{y_1 + (\square)}{2}$$

$$\square = \square \qquad 8 = y_1 - 3$$

$$\square = x_1 \qquad \square = y_1$$

The coordinates of *D* are ________.

Check Your Progress Find the coordinates of *R* if *N*(8, –3) is the midpoint of $\overline{RS}$ and *S* has coordinates (–1, 5).

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–4 Angle Measure

Standard 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. (Key)

MAIN IDEAS

- Measure and classify angles.
- Identify and use congruent angles and the bisector of an angle.

BUILD YOUR VOCABULARY (pages 2–3)

A **ray** is a part of a ______. A **ray** starts at a ______ on the line and extends endlessly in ______.

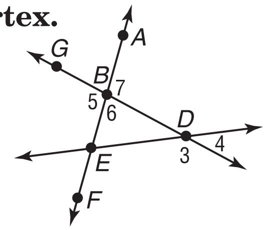
An **angle** is formed by two ______ rays that have a common endpoint.

KEY CONCEPT

Angle An angle is formed by two noncollinear rays that have a common endpoint.

EXAMPLE Angles and Their Parts

1 **Refer to the figure.**



a. Name all angles that have *B* as a vertex.

______

b. Name the sides of ∠5.

______ and ______ or ______ are the sides of ∠5.

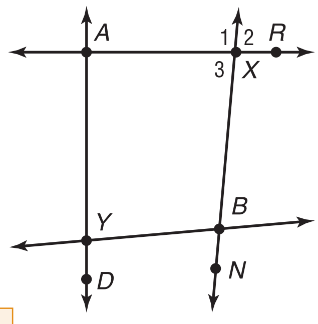
c. Write another name for ∠6.

______, ______, ______, and ______ are other names for ∠6.

Check Your Progress

a. Name all angles that have *X* as a vertex.

______



b. Name the sides of ∠3.

______

c. Write another name for ∠3.

______

WRITE IT

When can you use a single letter to name an angle?

______

______

______

______

Key Concepts

Classify Angles

A **right angle** has a degree measure of 90. An **acute angle** has a degree measure less than 90. An **obtuse angle** has a degree measure greater than 90 and less than 180.

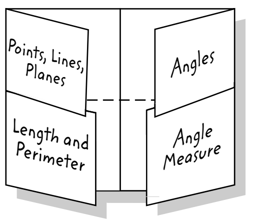
Congruent Angles

Angles that have the same measure are congruent angles.

FOLDABLES™

Organize It

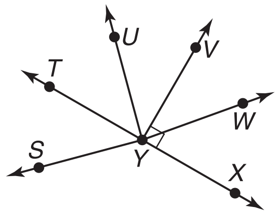
Draw and label ∠*RST* that measures 70° under the Angle Measure tab. Classify ∠*RST* as acute, right, or obtuse.



EXAMPLE **Measure and Classify Angles**

2 **Measure each angle named and classify it as *right*, *acute*, or *obtuse*.**

a. ∠*XYV*



∠*XYV* is marked with a right angle symbol, so measuring is not necessary. $m\angle XYV$ = ______,

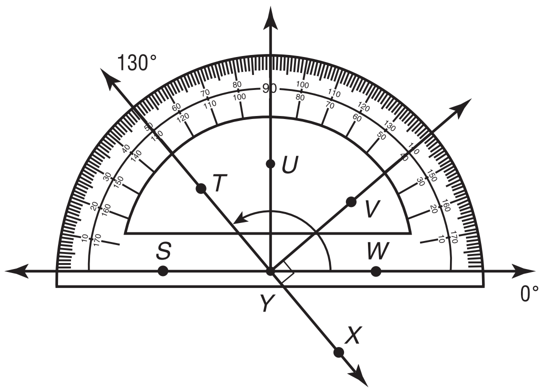
so ∠*XYV* is a(n) ______.

b. ∠*WYT*

Use a protractor to find that $m\angle WYT = 130$.

$180 > m\angle WYT > 90$, so ∠*WYT* is a(n)

______.

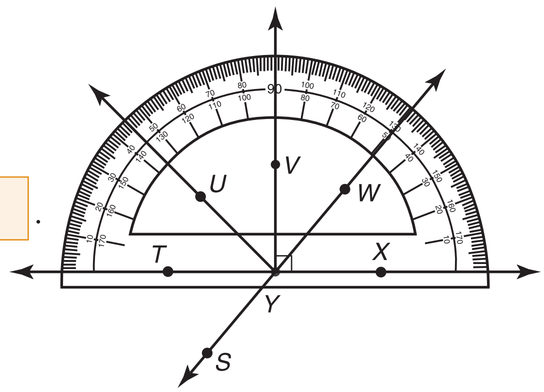


c. ∠*TYU*

Use a protractor to find that $m\angle TYU = 45$.

$45 < 90$, so ∠*TYU* is

a(n) ______.



Check Your Progress **Measure each angle named and classify it as *right*, *acute*, or *obtuse*.**

a. ∠*CZD*

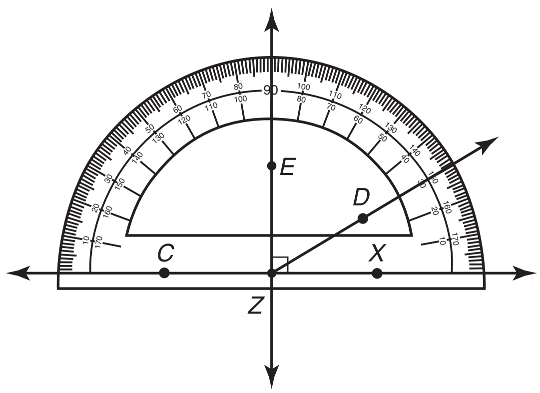
______

b. ∠*CZE*

______

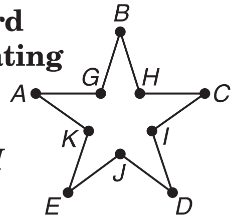
c. ∠*DZX*

______



EXAMPLE Use Algebra to Find Angle Measures

3 **INTERIOR DESIGN** **Wall stickers of standard shapes are often used to provide a stimulating environment for a young child's room. A five-pointed star sticker is shown with vertices labeled. Find $m\angle GBH$ and $m\angle HCI$ if $\angle GBH \cong \angle HCI$, $m\angle GBH = 2x + 5$, and $m\angle HCI = 3x - 10$.**



REMEMBER IT

If angles are congruent, then their measures are equal.

$\angle GBH \cong \angle HCI$	Given
$m\angle GBH = m\angle HCI$	______
$2x + 5 = 3x - 10$	Substitution
______ = ______	Add 10 to each side.
$15 = x$	Subtract $2x$ from each side.

Use the value of x to find the measure of one angle.

$m\angle GBH = 2x + 5$	Given
$= 2(15) + 5$	$x =$ ______
$=$ ______ $+$ ______ or 35	Simplify.

Since $m\angle GBH = m\angle HCI$, $m\angle HCI =$ ______.

Check Your Progress A railroad crossing sign forms congruent angles. In the figure, $m\angle WVX \cong m\angle ZVY$. If $m\angle WVX = 7a + 13$ and $m\angle ZVY = 10a - 20$, find the actual measurements of $m\angle WVX$ and $m\angle ZVY$.

______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

BUILD YOUR VOCABULARY (page 2)

A ______ that divides an angle into ______ congruent ______ is called an **angle bisector**.

1–5 Angle Relationships

Preparation for Standard 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

MAIN IDEAS

- Identify and use special pairs of angles.
- Identify perpendicular lines.

KEY CONCEPTS

Angle Pairs

Adjacent angles are two angles that lie in the same plane, have a common vertex, and a common side, but no common interior points.

Vertical angles are two nonadjacent angles formed by two intersecting lines.

A linear pair is a pair of adjacent angles whose noncommon sides are opposite rays.

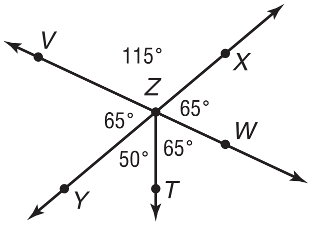
FOLDABLES™ Draw and label examples under the Angles tab.

EXAMPLE Identify Angle Pairs

1 Refer to the figure. Name two acute vertical angles.

There are four acute angles shown. There is one pair of vertical angles. The acute vertical angles are

______________.



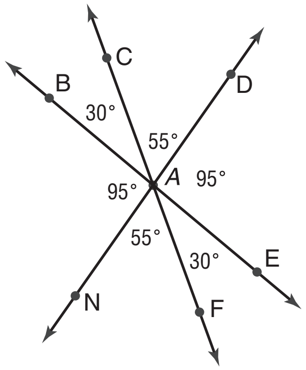
Check Your Progress **Name an angle pair that satisfies each condition.**

a. two angles that form a linear pair

____

b. two adjacent angles whose measures have a sum that is less than 90

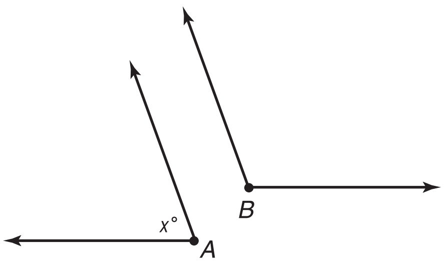
____



EXAMPLE Angle Measure

2 ALGEBRA Find the measures of two supplementary angles if the measure of one angle is 6 less than five times the other angle.

The sum of the measures of supplementary angles is ______.
Draw two figures to represent the angles.



If $m\angle A = x$, then $m\angle B =$ ______.

Key Concept

Angle Relationships

Complementary angles are two angles whose measures have a sum of 90°.

Supplementary angles are two angles whose measures have a sum of 180°.

$m\angle A + m\angle B = 180$ Given

______ + (______) $= 180$ $m\angle A = x$ and $m\angle B = 5x - 6$

$6x - 6 = 180$ Simplify.

$6x = 186$ Add ______ to each side.

$x = 31$ Divide each side by 6.

$m\angle A = x$ $m\angle B =$ ______

$m\angle A = 31$ $m\angle B = 5(31) - 6$ or 149

Check Your Progress Find the measures of two complementary angles if one angle measures six degrees less than five times the measure of the other.

Build Your Vocabulary (page 3)

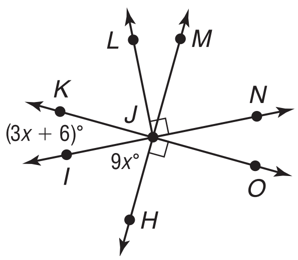
Lines that form ______ are **perpendicular**.

Key Concept

Perpendicular lines intersect to form four right angles. Perpendicular lines intersect to form congruent adjacent angles. Segments and rays can be perpendicular to lines or to other line segments and rays. The right angle symbol is used in figures to indicate that lines, rays, or segments are perpendicular.

EXAMPLE Perpendicular Lines

1 ALGEBRA **Find x so that $\overleftrightarrow{KO} \perp \overleftrightarrow{HM}$.**



If $\overleftrightarrow{KO} \perp \overleftrightarrow{HM}$, then $m\angle KJH =$ ______.

$m\angle KJH = m\angle KJI + m\angle IJH$

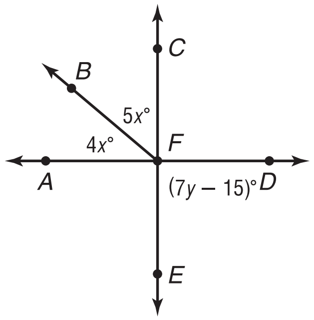
$90 = (3x + 6) + 9x$ Substitution

$90 =$ ______ Add.

$84 = 12x$ Subtract 6 from each side.

______ $= x$ Divide each side by 12.

Check Your Progress Find x and y so that $\overleftrightarrow{AD}$ and $\overleftrightarrow{CE}$ are perpendicular.

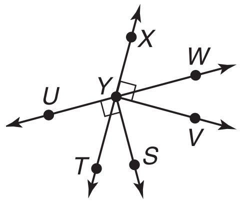


EXAMPLE Interpret Figures

4 **Determine whether each statement can be assumed from the figure below.**

a. $m\angle VYT = 90$

The diagram is marked to show $\overleftrightarrow{VY} \perp \overleftrightarrow{XT}$. From the definition of perpendicular, perpendicular lines intersect to form congruent adjacent angles.



______; $\overleftrightarrow{VY}$ and $\overleftrightarrow{TX}$ are perpendicular.

b. $\angle TYW$ **and** $\angle TYU$ **are supplementary.**

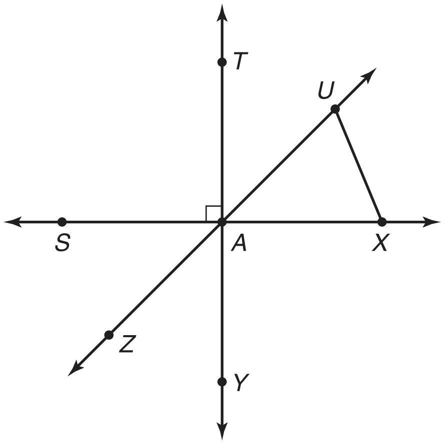
Yes; they form a ______ of angles.

c. $\angle VYW$ **and** $\angle TYS$ **are adjacent angles.**

No; they do not share a ______.

Check Your Progress **Determine whether each statement can be assumed from the figure below. Explain.**

a. $m\angle XAY = 90$



b. $\angle TAU$ and $\angle UAY$ are complementary.

c. $\angle UAX$ and $\angle UXA$ are adjacent.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–6 Two-Dimensional Figures

Main Ideas

- Identify and name polygons.
- Find perimeter or circumference and area of two-dimensional figures.

Build Your Vocabulary (page 3)

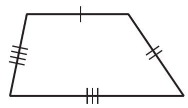
A **polygon** is a ______ figure whose ______ are all segments.

Key Concept

Polygon A polygon is a closed figure formed by a finite number of coplanar segments such that (1) the sides that have a common endpoint are noncollinear, and (2) each side intersects exactly two other sides, but only at their endpoints.

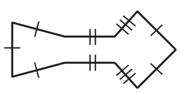
Example: Identify Polygons

1 **Name each polygon by the number of sides. Then classify it as *convex* or *concave*, *regular* or *irregular*.**

a. 

There are 4 sides, so this is a ______.

No line containing any of the sides will pass through the interior of the quadrilateral, so it is ______. The sides are not congruent, so it is ______.

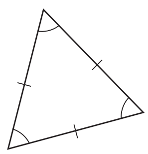
b. 

There are 9 sides, so this is a ______.

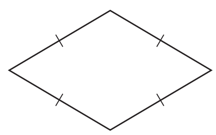
A line containing a side will pass through the interior of the nonagon, so it is ______.

The sides are not congruent, so it is ______.

Check Your Progress **Name each polygon by the number of sides. Then classify it as *convex* or *concave*, *regular* or *irregular*.**

a. 

______

b. 

______

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area **of common geometric figures.** (Key)

Standard 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids. (Key)

KEY CONCEPT

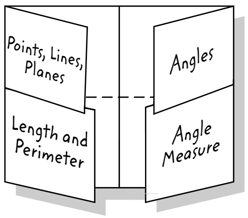
Perimeter The perimeter *P* of a polygon is the sum of the lengths of the sides of a polygon.

Area The area is the number of square units needed to cover a surface.

FOLDABLES™

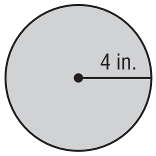
ORGANIZE IT

Draw a polygon and explain how to find the perimeter. Place this under the Length and Perimeter tab.



EXAMPLE Find Perimeter and Area

2 **Find the perimeter or circumference and area of the figure.**



$C = 2\pi r$	Circumference of a circle
$C = 2\pi(\ \ )$	$r =$ ____
$C =$ ____	Multiply.
$C =$ ____	Use a calculator.
$A = \pi r^2$	Area of a circle
$A = \pi(\ \ )^2$	$r =$ ____
$A =$ ____	Simplify.
$A =$ ____	Use a calculator.

The circumference of the circle is about ____ inches and the area is about ____ square inches.

Check Your Progress Find the perimeter or circumference and area of the figure.

EXAMPLE Largest Area

3 **TEST EXAMPLE** **Terri has 19 feet of tape to make an area in the classroom where the students can read. Which of these shapes has a perimeter or circumference that would use *most* or all of the tape?**

A square with side length of 5 feet

B circle with the radius of 3 feet

C right triangle with each leg length of 6 feet

D rectangle with a length of 8 feet and a width of 3 feet

Read the Test Item

You are asked to compare the perimeters of four different shapes.

Solve the Test Item

Find the perimeter of each shape.

Square
$P = 4s$
$P = 4(5)$
$P =$ ______ ft

Circle
$C = 2\pi r$
$C = 2\pi(3)$
$C = 6\pi$ or about ______ ft

Rectangle
$P = 2\ell + 2w$
$P = 2(8) + 2(3)$
$P =$ ______ ft

Right Triangle

Use the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = a^2 + b^2$
$c^2 = 6^2 + 6^2$
$c^2 = 72$
$c = 6\sqrt{2}$
$P = 6 + 6 + 6\sqrt{2}$
$P =$ ______ or about ______ ft

The shape that uses the most of the tape is the circle. The answer is ______.

Check Your Progress **Jason has 20 feet of fencing to make a pen for his dog. Which of these shapes encloses the largest area?**

A square with a side length of 5 feet

B circle with radius of 3 feet

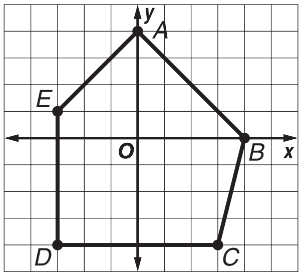
C right triangle with each leg about 6 feet

D rectangle with length of 4 feet and width of 6 feet

______

EXAMPLE Perimeter and Area on the Coordinate Plane

4 **Find the perimeter of pentagon *ABCDE* with *A*(0, 4), *B*(4, 0), *C*(3, –4), *D*(–3, –4), and *E*(–3, 1).**



Since $\overline{DE}$ is a vertical line segment, we can count the squares on the grid. The length of $\overline{DE}$ is ______ units. Likewise, since $\overline{CD}$ is a horizontal line segment, count the squares to find that the length is ______ units.

To find *AE*, *AB*, and *BC*, use the distance formula.

$AE = \sqrt{(0 - (-4))^2 + (4 - 1)^2}$ Substitution

$AE = \sqrt{\square^2 + \square^2}$ Subtract.

$AE = \square$ or 5 Simplify.

$AB = \sqrt{(0 - 4)^2 + (4 - 0)^2}$ Substitution

$AB = \sqrt{(\square)^2 + \square^2}$ Subtract.

$AB = \sqrt{32}$ or about ______ Simplify.

$BC = \sqrt{(4 - 3)^2 + (0 - (-4))^2}$ Substitution

$BC = \sqrt{\square^2 + \square^2}$ Subtract.

$BC = \square$ or about 4.1 Simplify.

To find the perimeter, add the lengths of each side.

$P = AB + BC + CD + DE + AE$

$P \approx 5.7 + 4.1 + 6 + 5 + 5$

$P \approx$ ______

The perimeter is approximately ______ units.

Check Your Progress Find the perimeter of quadrilateral *WXYZ* with *W*(2, 4), *X*(–3, 3), *Y*(–1, 0) and *Z*(3, –1).

______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

1–7 Three-Dimensional Figures

MAIN IDEAS

- Identify three-dimensional figures.
- Find surface area and volume.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, **volume, lateral area, and surface area of common geometric figures.** (Key)

Standard 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

BUILD YOUR VOCABULARY (page 3)

A solid with all ______ that enclose a single region of space is called a **polyhedron**.

A **prism** is a polyhedron with ______ congruent faces called bases.

A **regular prism** is a prism with ______ that are regular polygons.

A polyhedron with all faces (except for one) intersecting at ______ is a **pyramid**.

A polyhedron is a **regular polyhedron** if all of its faces are ______ and all of the ______ are congruent.

A **cylinder** is a solid with congruent ______ ______ in a pair of parallel planes.

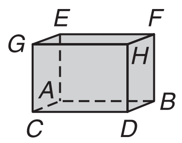
A **cone** has a ______ base and a ______ .

A **sphere** is a set of ______ in space that are a given distance from a given point.

EXAMPLE Identify Solids

1 Identify each solid. Name the bases, faces, edges, and vertices.

a.



The bases and faces are rectangles. This is a rectangular prism.

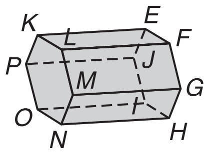
Bases: rectangles ______ and ______

Faces:

Edges:

Vertices:

b.



This figure has two faces that are hexagons. Therefore, it is a hexagonal prism.

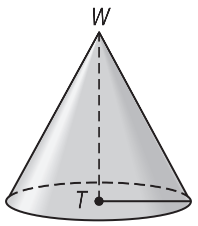
Bases: hexagons ______ and ______

Faces: rectangles *EFLK*, *FGML*, *GHMN*, *HNOI*, *IOPJ*, and *JPKE*

Edges: $\overline{FL}$, $\overline{GM}$, $\overline{HN}$, $\overline{IO}$, $\overline{JP}$, $\overline{EK}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HI}$, $\overline{IJ}$, $\overline{JE}$, $\overline{KL}$, $\overline{LM}$, $\overline{MN}$, $\overline{NO}$, $\overline{OP}$, and $\overline{PK}$

Vertices: *E*, *F*, *G*, *H*, *I*, *J*, *K*, *L*, *M*, *N*, *O*, *P*

c.

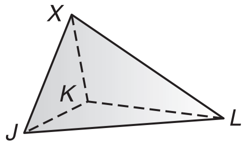


The base of the solid is a circle and the figure comes to a point. Therefore, it is a cone.

Base: ______ Vertex: ______

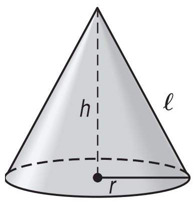
There are ______ faces or edges.

Check Your Progress Identify the solid. Name the bases, faces, edges, and vertices.



EXAMPLE Surface Area and Volume

2 **Refer to the cone with $r = 3$ cm, $h = 4$ cm, and $\ell = 5$ cm.**



a. Find the surface area of the cone.

$T = \pi r\ell + \pi r^2$	Surface area of a cone
$T = \pi(\square)(\square) + \pi(\square)^2$	Substitution
$T = 15\pi + 9\pi$	Simplify.
$T \approx \square$ cm^2	Use a calculator.

b. Find the volume of the cone.

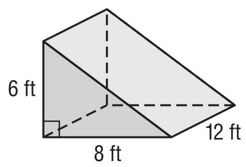
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \frac{1}{3}\pi(\square)^2(\square)$	Substitution
$V = 12\pi$	Simplify.
$V \approx \square$ cm^3	Use a calculator.

Homework Assignment

Page(s):

Exercises:

Check Your Progress Find the surface area and volume of the triangular prism.



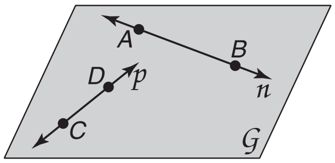
CHAPTER 1 BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 1 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 1, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 2–3)* to help you solve the puzzle.

1-1 Points, Lines, and Planes

Refer to the figure.



1. Name a point contained in line n.

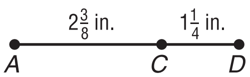
2. Name the plane containing lines n and p.

3. Draw a model for the relationship $\overleftrightarrow{AK}$ and $\overleftrightarrow{CG}$ intersect at point M in plane $\mathcal{T}$.

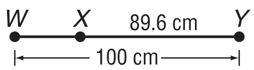
1-2 Linear Measure and Precision

Find the measure of each segment.

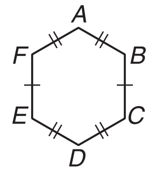
4. $\overline{AD}$



5. $\overline{WX}$

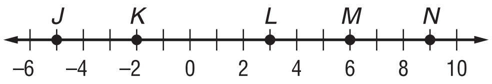


6. **CARPENTRY** Jorge used the figure at the right to make a pattern for a mosaic he plans to inlay on a tabletop. Name all of the congruent segments in the figure.



1-3 Distance and Midpoints

Use the number line to find each measure.



7. *LN*

8. *JL*

Find the distance between each pair of points.

9. $F(-3, -2)$, $G(1, 1)$

10. $Y(-6, 0)$, $P(2, 6)$

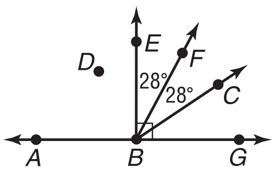
Find the coordinates of the midpoint of a segment having the given endpoints.

11. $A(3, 1)$, $B(5, 3)$

12. $T(-4, 9)$, $U(7, 5)$

1-4 Angle Measure

For Exercises 13–16, refer to the figure at the right.



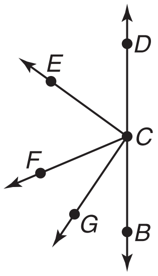
13. Name a right angle.

14. Name an obtuse angle.

15. Name a point in the interior of $\angle EBC$.

16. What is the angle bisector of $\angle EBC$?

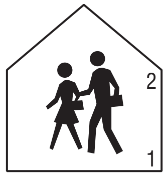
In the figure, $\overrightarrow{CB}$ and $\overrightarrow{CD}$ are opposite rays, $\overrightarrow{CE}$ bisects $\angle DCF$, and $\overrightarrow{CG}$ bisects $\angle FCB$.



17. If $m\angle DCE = 4x + 15$ and $m\angle ECF = 6x - 5$, find $m\angle DCE$.

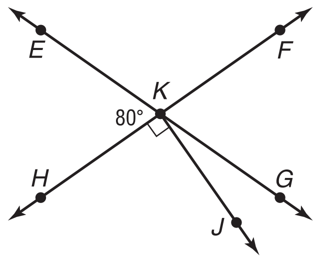
18. If $m\angle FCG = 9x + 3$ and $m\angle GCB = 13x - 9$, find $m\angle GCB$.

19. **TRAFFIC SIGNS** The diagram shows a sign used to warn drivers of a school zone or crossing. Measure and classify each numbered angle.



1-5 Angle Relationships

For Exercises 20–23, use the figure at the right.



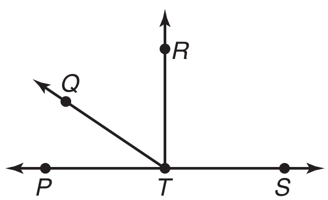
20. Name two acute vertical angles.

21. Name a linear pair.

22. Name two acute adjacent angles.

23. Name an angle supplementary to $\angle FKG$.

24. Find x so that $\overline{TR} \perp \overline{TS}$ if $m\angle RTS = 8x + 18$.

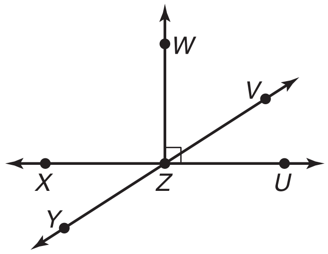


25. Find $m\angle PTQ$ if $\overline{TR} \perp \overline{TS}$ and $m\angle PTQ = m\angle RTQ - 18$.

Determine whether each statement can be assumed from the figure. Explain.

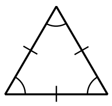
26. $\angle YZU$ and $\angle UZV$ are supplementary.

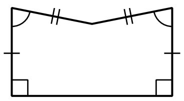
27. $\angle VZU$ is adjacent to $\angle YZX$.



1-6 Two-Dimensional Figures

Name each polygon by its number of sides and then classify it as *convex* or *concave* and *regular* or *irregular*.

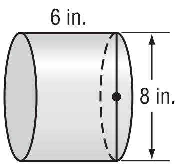
28. 

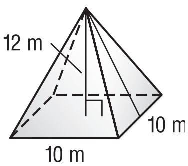
29. 

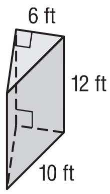
30. The length of a rectangle is 8 inches less than six times its width. The perimeter is 26 inches. Find the length of each side.

1-7 Three-Dimensional Figures

Find the surface area and volume of each solid.

31. 

32. 

33. 

CHAPTER 1

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 1.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 1 Practice Test on page 73 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 1 Study Guide and Review on pages 68–72 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 73 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 1 Foldable.
- Then complete the Chapter 1 Study Guide and Review on pages 68–72 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 1 Practice Test on page 73 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 2

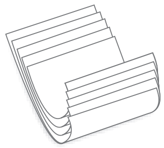
Reasoning and Proof

FOLDABLES™

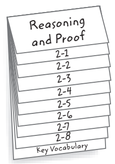
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of $8\frac{1}{2}$" × 11" plain paper.

STEP 1 **Stack** the sheets of paper with edges $\frac{3}{4}$-inch apart. Fold the bottom edges up to create equal tabs.



STEP 2 **Staple** along the fold. Label the top tab with the chapter title. Label the next 8 tabs with lesson numbers. The last tab is for Key Vocabulary.



NOTE-TAKING TIP: When you take notes, listen or read for main ideas. Then record concepts, define terms, write statement in if-then form, and write paragraph proofs.

CHAPTER 2

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 2. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
conclusion			
conditional statement			
conjecture [kuhn-JEK-chur]			
conjunction			
contrapositive			
converse			
counterexample			
deductive argument			
deductive reasoning			
disjunction			
hypothesis			
if-then statement			

Vocabulary Term	Found on Page	Definition	Description or Example
inductive reasoning			
inverse			
negation			
paragraph proof			
postulate			
related proof conditionals			
statement			
theorem			
truth table			
truth value			
two-column proof			

2–1 Inductive Reasoning and Conjecture

MAIN IDEAS

- Make conjectures based on inductive reasoning.
- Find counterexamples.

Standard 1.0 Students demonstrate understanding by identifying and **giving examples of** undefined terms, axioms, theorems, and **inductive** and deductive **reasoning**. (Key)
Standard 3.0 Students construct and judge the validity of a logical argument and **give counterexamples to disprove a statement. (Key)**

BUILD YOUR VOCABULARY (pages 32–33)

A **conjecture** is an educated   based on known information.

Inductive reasoning is reasoning that uses a number of specific examples to arrive at a plausible generalization or   .

EXAMPLE Patterns and Conjecture

1 Make a conjecture about the next number based on the pattern 2, 4, 12, 48, 240.

Find a Pattern: 2 → 4 → 12 → 48 → 240

Conjecture: The next number will be multiplied by 6.

So, it will be   · 240 or   .

Check Your Progress Make a conjecture about the next number based on the pattern $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$.

REMEMBER IT

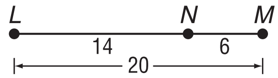
When looking for patterns in a sequence of numbers, test all fundamental operations. Sometimes two operations can be used.

EXAMPLE Geometric Conjecture

2 For points *L*, *M*, and *N*, *LM* = 20, *MN* = 6, and *LN* = 14. Make a conjecture and draw a figure to illustrate your conjecture.

Given: Points *L*, *M*, and *N*; *LM* = 20, *MN* = 6, and *LN* = 14. Since *LN* + *MN* = *LM*, the points can be collinear with point *N* between points *L* and *M*.

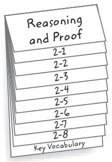
Conjecture: *L*, *M* and *N* are   .



FOLDABLES™

ORGANIZE IT

Write definitions for a conjecture and inductive reasoning under the tab for Lesson 2-1. Design a pattern and state a conjecture about the pattern.



Check Your Progress *ACE* is a right triangle with $AC = CE$. Make a conjecture and draw a figure to illustrate your conjecture.

BUILD YOUR VOCABULARY (pages 32–33)

A **counterexample** is one false example showing that a conjecture is not true.

EXAMPLE Find a Counterexample

3 UNEMPLOYMENT **Refer to the table. Find a counterexample for the following statement.**
The unemployment rate is highest in the cities with the most people.

City	Population	Rate
Armstrong	2163	3.7%
Cameron	371,825	7.2%
El Paso	713,126	7.0%
Hopkins	33,201	4.3%
Maverick	50,436	11.3%
Mitchell	9402	6.1%

Maverick has a population of ______ people, and it has a higher rate of unemployment than El Paso, which has a population of ______ people.

Check Your Progress Refer to the table. Find a counterexample for the following statement.
The unemployment rate is lowest in the cities with the least people.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–2 Logic

Standard 3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement. (Key)

BUILD YOUR VOCABULARY (pages 32–33)

A **statement** is any sentence that is either true or false, but not ______. The truth or falsity of a statement is called its **truth value**.

MAIN IDEAS

- Determine truth values of conjunctions and disjunctions.
- Construct truth tables.

KEY CONCEPTS

Negation If a statement is represented by *p*, then *not p* is the negation of the statement.

Conjunction A conjunction is a compound statement formed by joining two or more statements with the word *and*.

EXAMPLE Truth Values of Conjunctions

1 Use the following statements to write a compound statement for each conjunction. Then find its truth value.

***p*: One foot is 14 inches.**
***q*: September has 30 days.**
***r*: A plane is defined by three noncollinear points.**

a. *p* and *q*

One foot is 14 inches, and September has 30 days. p and q is ______ because ______ is false and q is ______.

b. $\sim p \wedge r$

A foot is not 14 inches, and a plane is defined by three noncollinear points. $\sim p \wedge r$ is ______, because $\sim p$ is ______ and r is ______.

Check Your Progress **Use the following statements to write a compound statement for each conjunction. Then find its truth value.**

***p*: June is the sixth month of the year.**
***q*: A square has five sides.**
***r*: A turtle is a bird.**

a. p and r ______

b. $\sim q \wedge \sim r$ ______

EXAMPLE Truth Values of Disjunctions

KEY CONCEPT

Disjunction A disjunction is a compound statement formed by joining two or more statements with the word *or*.

2 **Use the following statements to write a compound statement for each disjunction. Then find its truth value.**

p: **$\overline{AB}$ is proper notation for "segment *AB*."**

q: **Centimeters are metric units.**

r: **9 is a prime number.**

a. ***p*** **or** ***q***

$\overline{AB}$ is proper notation for "segment *AB*," or centimeters are metric units. *p* or *q* is ________ because *q* is ________.

It does not matter that ________ is false.

b. $q \vee r$

Centimeters are metric units, or 9 is a prime number. $q \vee r$ is ________ because *q* is ________. It does not matter that ________ is false.

Check Your Progress **Use the following statements to write a compound statement for each disjunction. Then find its truth value.**

p: **6 is an even number.**

q: **A cow has 12 legs.**

r: **A triangle has three sides.**

a. *p* or *r*

b. $\sim q \vee \sim r$

BUILD YOUR VOCABULARY (pages 32–33)

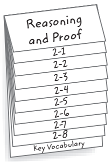
A convenient method for organizing the truth values of statements is to use a **truth table**.

EXAMPLE Use Venn Diagrams

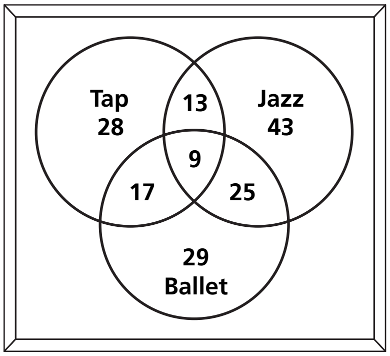
FOLDABLES™

ORGANIZE IT

Construct a truth table for the compound statement $\sim q \vee p$ and write it under the tab for Lesson 2-2.



3 DANCING **The Venn diagram shows the number of students enrolled in Monique's Dance School for tap, jazz, and ballet classes.**



a. How many students are enrolled in all three classes?

The number of students that are enrolled in all ______ classes is represented by the ______ of the three circles. There are ______ students enrolled in all three classes.

b. How many students are enrolled in tap or ballet?

The number of students enrolled in tap or ballet is represented by the union of the two sets. There are ______ or ______ students enrolled in tap or ballet.

c. How many students are enrolled in jazz and ballet, but not tap?

The number of students enrolled in ______ and ballet but not tap is represented by the intersection of the jazz and ______ sets. There are ______ students enrolled in ______ only.

Check Your Progress How many students are enrolled in ballet and tap, but not jazz? ______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–3 Conditional Statements

Standard 3.0 Students construct and **judge the validity of a logical argument** and give counterexamples to disprove a statement. (Key)

MAIN IDEAS

- Analyze statements in if-then form.
- Write the converse, inverse, and contrapositive of if-then statements.

KEY CONCEPT

If-Then Statement An if-then statement is written in the form *if p, then q*. The phrase immediately following the word *if* is called the **hypothesis**, and the phrase immediately following the word *then* is called the **conclusion**.

EXAMPLE Write a Conditional in If-Then Form

1 Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a. Distance is positive.

Hypothesis: ______

Conclusion: ______

If a distance is measured, then it is positive.

b. A five-sided polygon is a pentagon.

Hypothesis: ______

Conclusion: ______

If a polygon has ______, then it is a ______.

Check Your Progress Identify the hypothesis and conclusion of the statement.

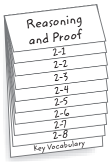
To find the distance between two points, you can use the Distance Formula.

EXAMPLE Truth Values of Conditionals

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 2-3, explain how to determine the truth value of a conditional. Be sure to include an example.



2 **Determine the truth value of the following statement for each set of conditions.** ***If Yukon rests for 10 days, his ankle will heal.***

a. Yukon rests for 10 days, and he still has a hurt ankle.

The hypothesis is ______, since he rested for 10 days. The conclusion is ______ since his ankle will heal. Thus, the conditional statement is ______.

b. Yukon rests for 10 days, and he does not have a hurt ankle anymore.

The hypothesis is ______ since Yukon rested for 10 days, and the conclusion is ______ because he does not have a hurt ankle. Since what was stated is true, the conditional statement is ______.

c. Yukon rests for 7 days, and he does not have a hurt ankle anymore.

The hypothesis is ______, and the conclusion is ______. The statement does not say what happens if Yukon only rests for 7 days. In this case, we cannot say that the statement is false. Thus, the statement is ______.

Check Your Progress **Determine the truth value of the following statement for each set of conditions.** ***If it rains today, then Michael will not go skiing.***

a. It rains today; Michael does not go skiing.

b. It rains today; Michael goes skiing.

EXAMPLE Related Conditionals

KEY CONCEPTS

Related Conditionals

Conditional Formed by given hypothesis and conclusion

Converse Formed by exchanging the hypothesis and conclusion of the conditional

Inverse Formed by negating both the hypothesis and conclusion of the conditional

Contrapositive Formed by negating both the hypothesis and conclusion of the converse statement

3 **Write the converse, inverse, and contrapositive of the statement *All squares are rectangles*. Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.**

Conditional: If a shape is a square, then it is a rectangle. The conditional statement is true.

Write the converse by switching the ______ and conclusion of the conditional.

Converse: If a shape is a rectangle, then it is a square. The converse is ______. A rectangle with $\ell = 2$ and $w = 4$ is not a square.

Inverse: If a shape is not a square, then it is not a rectangle. The inverse is ______. A rectangle with side lengths 2, 2, 4, and 4 is not a square.

The contrapositive is the ______ of the hypothesis and conclusion of the converse.

Contrapositive: If a shape is not a rectangle, then it is not a square. The contrapositive is ______.

Check Your Progress Write the converse, inverse, and contrapositive of the statement *The sum of the measures of two complementary angles is 90.* Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–4 Deductive Reasoning

MAIN IDEAS

- Use the Law of Detachment.
- Use the Law of Syllogism.

BUILD YOUR VOCABULARY (pages 32–33)

Deductive reasoning uses facts, rules, __________, or __________ to reach logical conclusions.

KEY CONCEPT

Law of Detachment If $p \rightarrow q$ is true and p is true, then q is also true.

EXAMPLE Determine Valid Conclusions

1 **The following is a true conditional. Determine whether the conclusion is valid based on the given information. Explain your reasoning.**

If two segments are congruent and the second segment is congruent to a third segment, then the first segment is also congruent to the third segment.

a. Given: $\overline{WX} \cong \overline{UV}$; $\overline{UV} \cong \overline{RT}$

Conclusion: $\overline{WX} \cong \overline{RT}$

The hypothesis states that __________ and __________. Since the conditional is true and the hypothesis is true, the conclusion is __________.

b. Given: $\overline{UV}$, $\overline{WX} \cong \overline{RT}$

Conclusion: $\overline{WX} \cong \overline{UV}$ **and** $\overline{UV} \cong \overline{RT}$

The hypothesis states that __________ and __________.

Not enough information is provided to reach the conclusion.

The conclusion is __________.

Standard 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and **deductive reasoning.** (Key)

Standard 3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement. (Key)

Check Your Progress The following is a true conditional. Determine whether the conclusion is valid based on the given information. Explain your reasoning.

If a polygon is a convex quadrilateral, then the sum of the measures of the interior angles is 360°.

Given: $m\angle X + m\angle N + m\angle O = 360°$

Conclusion: If you connect X, N, and O with segments, the figure will be a convex quadrilateral.

EXAMPLE Determine Valid Conclusions From Two Conditionals

KEY CONCEPT

Law of Syllogism If $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is also true.

2 PROM **Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements.**

a. (1) If Salline attends the prom, she will go with Mark.
(2) If Salline goes with Mark, Donna will go with Albert.

The conclusion of the first statement is the ______ of the second statement. Thus, if Salline attends the prom, Donna will go with ______.

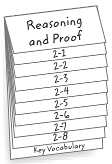
b. (1) If Mel and his date eat at the Peddler Steakhouse before going to the prom, they will miss the senior march.
(2) The Peddler Steakhouse stays open until 10 P.M.

There is ______. While both statements may be true, the conclusion of one statement is not used as the ______ of the other statement.

FOLDABLES™

ORGANIZE IT

Use symbols and words to write the Law of Detachment and the Law of Syllogism under the tab for Lesson 2-4.



Check Your Progress **Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements.**

a. (1) If you ride a bus, then you attend school.
(2) If you ride a bus, then you go to work.

b. (1) If your alarm clock goes off in the morning, then you will get out of bed.
(2) You will eat breakfast, if you get out of bed.

EXAMPLE Analyze Conclusions

3 **Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.**

(1) If the sum of the squares of two sides of a triangle is equal to the square of the third side, then the triangle is a right triangle.
(2) For $\triangle XYZ$, $(XY)^2 + (YZ)^2 = (ZX)^2$.
(3) $\triangle XYZ$ is a right triangle.

p: The ______ of the squares of the lengths of the two sides of a ______ is equal to the square of the length of the ______ side.

q: the triangle is a ______ triangle.

By the Law of ______, if $p \rightarrow q$ is true and *p* is true, then *q* is also true. Statement (3) is a ______ conclusion by the Law of Detachment.

Check Your Progress Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

(1) If a children's movie is playing on Saturday, Janine will take her little sister Jill to the movie.
(2) Janine always buys Jill popcorn at the movies.
(3) If a children's movie is playing on Saturday, Jill will get popcorn.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–5 Postulates and Paragraph Proofs

Main Ideas

- Identify and use basic postulates about points, lines, and planes.
- Write paragraph proofs.

Standard 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, **axioms, theorems,** and inductive and deductive reasoning. (Key)
Standard 2.0 Students write geometric proofs, including proofs by contradiction. (Key)
Standard 3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement. (Key)

BUILD YOUR VOCABULARY (pages 32–33)

A **postulate** is a statement that describes a fundamental relationship between the basic terms of ________.

Postulates are accepted as ________.

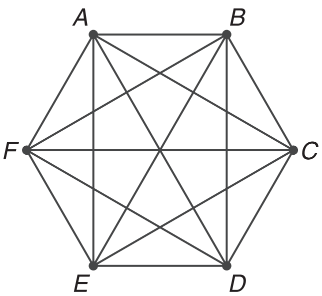
Postulate 2.1
Through any two points, there is exactly one line.

Postulate 2.2
Through any three points not on the same line, there is exactly one plane.

EXAMPLE Points and Lines

1 SNOW CRYSTALS Some snow crystals are shaped like regular hexagons. How many lines must be drawn to interconnect all vertices of a hexagonal snow crystal?

Draw a diagram of a hexagon to illustrate the solution. Connect each point with every other point. Then, count the number of segments.



Between every two points there is exactly ________ segment.

Be sure to include the sides of the hexagon. For the six points, ________ segments can be drawn.

Check Your Progress Jodi is making a string art design. She has positioned ten nails, similar to the vertices of a decagon, onto a board. How many strings will she need to interconnect all vertices of the design?

REVIEW IT

Collinear means lying on the same line, so noncollinear means not lying on the same line. *(Lesson 1-1)*

Postulate 2.3
A line contains at least two points.

Postulate 2.4
A plane contains at least three points not on the same line.

Postulate 2.5
If two points lie on a plane, then the entire line containing those points lies in that plane.

Postulate 2.6
If two lines intersect, then their intersection is exactly one point.

Postulate 2.7
If two planes intersect, then their intersection is a line.

EXAMPLE **Use Postulates**

2 **Determine whether the statement is *always*, *sometimes*, or *never* true. Explain.**

a. If plane $\mathcal{T}$ contains $\overleftrightarrow{EF}$ and $\overleftrightarrow{EF}$ contains point G, then plane $\mathcal{T}$ contains point G.

________; Postulate 2.5 states that if two ________ lie in a plane, then the entire ________ containing those points lies in the plane.

b. For $\overleftrightarrow{XY}$, if X lies in plane $\mathcal{Q}$ and Y lies in plane $\mathcal{R}$, then plane $\mathcal{Q}$ intersects plane $\mathcal{R}$.

________; planes $\mathcal{Q}$ and $\mathcal{R}$ can both be ________, and can intersect both planes.

c. $\overleftrightarrow{GH}$ contains three noncollinear points.

________; noncollinear points do not lie on the same ________ by definition.

Check Your Progress Determine whether the statement is *always*, *sometimes*, or *never* true. Explain.

Plane $\mathcal{A}$ and plane $\mathcal{B}$ intersect in one point.

Theorem 2.1 Midpoint Theorem
If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

EXAMPLE Write a Paragraph Proof

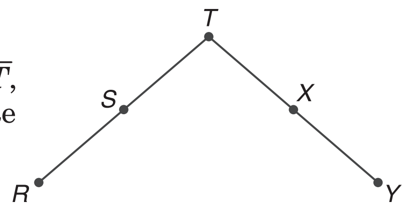
1 Given $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{CD}$, write a paragraph proof to show that A, C, and D determine a plane.

Given: $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{CD}$.

Prove: A, C, and D determine a plane.

$\overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$ must intersect at C because if ______ lines intersect, then their intersection is exactly ______ point. Point A is on $\overleftrightarrow{AC}$ and point D is on $\overleftrightarrow{CD}$. Therefore, points A and D are ______. Therefore, A, C, and D determine a plane.

Check Your Progress Given $\overline{RT} \cong \overline{TY}$, S is the midpoint of $\overline{RT}$, and X is the midpoint of $\overline{TY}$, write a paragraph proof to show that $\overline{ST} \cong \overline{TX}$.



KEY CONCEPT

Proofs Five essential parts of a good proof:

- State the theorem or conjecture to be proven.
- List the given information.
- If possible, draw a diagram to illustrate the given information.
- State what is to be proved.
- Develop a system of deductive reasoning.

FOLDABLES™ Copy Example 3 under the tab for Lesson 2-5 as an example of a paragraph proof.

HOMEWORK ASSIGNMENT

Page(s):
Exercises:

2–6 Algebraic Proof

Standard 2.0 Students write geometric proofs, including proofs by contradiction. (Key) **Standard 3.0 Students construct and judge the validity of a logical argument** and give counterexamples to disprove a statement. (Key)

MAIN IDEAS

- Use algebra to write two-column proofs.
- Use properties of equality in geometry proofs.

EXAMPLE Verify Algebraic Relationships

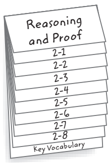
1 **Solve $2(5 - 3a) - 4(a + 7) = 92$.**

Algebraic Steps	Properties
$2(5 - 3a) - 4(a + 7) = 92$	Original equation
$10 - 6a - 4a - 28 = 92$	______ Property
$-18 - 10a = 92$	______ Property
______ = ______	Addition Property
$a =$ ______	______ Property

Check Your Progress Solve $-3(a + 3) + 5(3 - a) = -50$.

FOLDABLES™ ORGANIZE IT

Write a two-column proof for the statement *if $5(2x - 4) = 20$, then $x = 4$* under the tab for Lesson 2-6.



BUILD YOUR VOCABULARY (pages 32–33)

A **two-column proof**, or **formal proof**, contains ______ and ______ organized in two columns.

EXAMPLE Write a Two-Column Proof

Write It

What are the five essential parts of a good proof? *(Lesson 2-5)*

____

____

____

____

____

2 **Write a two-column proof to show that if $\frac{7d + 3}{4} = 6$, then $d = 3$.**

Statements	Reasons
1. $\frac{7d+3}{4} = 6$	**1.** Given
2. $4\left(\frac{7d+3}{4}\right) = 4(6)$	**2.**
3. $7d + 3 = 24$	**3.** Substitution
4. $7d = 21$	**4.**
5.	**5.**

Check Your Progress Write a two-column proof for the following.

Given: $\frac{10 - 8n}{3} = -2$

Proof: $n = 2$

EXAMPLE Geometric Proof

3 **SEA LIFE** **A starfish has five arms. If the length of arm 1 is 22 centimeters, and arm 1 is congruent to arm 2, and arm 2 is congruent to arm 3, prove that arm 3 has a length of 22 centimeters.**

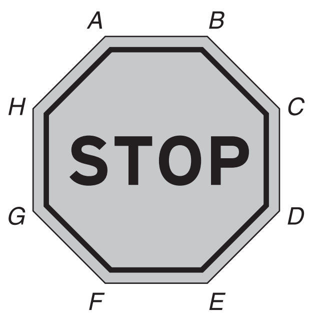
Given: arm 1 ≅ arm 2
arm 2 ≅ arm 3
m arm 1 = 22 cm

Prove: m arm 3 = 22 cm

Proof:

Statements	Reasons
1. arm 1 ≅ arm 2; arm 2 ≅ arm 3	**1.** Given
2. arm 1 ≅ arm 3	**2.**
3. m arm 1 = m arm 3	**3.** Definition of congruence
4.	**4.** Given
5. m arm 3 = 22 cm	**5.** Transitive Property

Check Your Progress A stop sign as shown at right is a regular octagon. If the measure of angle A is 135 and angle A is congruent to angle G, prove that the measure of angle G is 135.



HOMEWORK ASSIGNMENT
Page(s):
Exercises:

2–7 Proving Segment Relationships

Standard 4.0 Students prove basic theorems involving congruence and similarity. (Key)

MAIN IDEAS

- Write proofs involving segment addition.
- Write proofs involving segment congruence.

Postulate 2.8 Ruler Postulate
The points on any line or line segment can be paired with real numbers so that, given any two points *A* and *B* on a line, *A* corresponds to zero, and *B* corresponds to a positive real number.

Postulate 2.9 Segment Addition Postulate
If *B* is between *A* and *C*, then $AB + BC = AC$.
If $AB + BC = AC$, then *B* is between *A* and *C*.

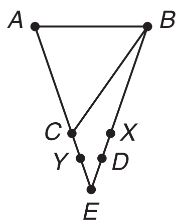
EXAMPLE Proof with Segment Addition

1 Prove the following.

Given: $PR = QS$
Prove: $PQ = RS$

Statements	Reasons
1. $PR = QS$	**1.** Given
2. $PR - QR = QS - QR$	**2.** Subtraction Property
3. $PR - QR = PQ$; $QS - QR = RS$	**3.** Segment Addition Postulate
4. $PQ = RS$	**4.** Substitution

Check Your Progress Prove the following.

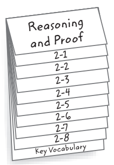


Given: $AC = AB$
$AB = BX$
$CY = XD$
Prove: $AY = BD$

FOLDABLES™

ORGANIZE IT

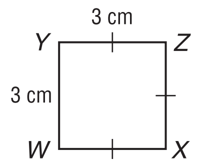
Under the tab for Lesson 2-7, write the Segment Addition Postulate, draw an example, and write an equation for your example.



Theorem 2.2
Congruence of segments is reflexive, symmetric, and transitive.

EXAMPLE Transitive Property of Congruence

2 **Prove the following.**

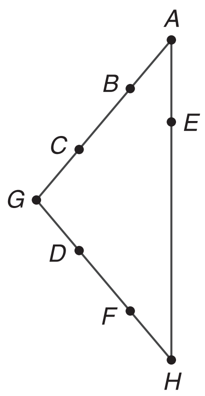


Given: $WY = YZ$, $\overline{YZ} \cong \overline{XZ}$
$\overline{XZ} \cong \overline{WY}$

Prove: $\overline{WX} \cong \overline{WY}$

Statements	Reasons
1. $WY = YZ$	1. Given
2. $\overline{WY} \cong \overline{YZ}$	2. Definition of ______ segments
3. $\overline{YZ} \cong \overline{XZ}$; $\overline{XZ} \cong \overline{WX}$	3. Given
4. ______	4. Transitive Property
5. $\overline{WX} \cong \overline{WY}$	5. ______

Check Your Progress Prove the following.



Given: $\overline{GD} \cong \overline{BC}$, $\overline{BC} \cong \overline{FH}$,
$\overline{FH} \cong \overline{AE}$

Prove: $\overline{AE} \cong \overline{GD}$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

2–8 Proving Angle Relationships

Standard 4.0 Standard 4.0 Students prove basic theorems involving congruence and similarity. (Key)

MAIN IDEAS

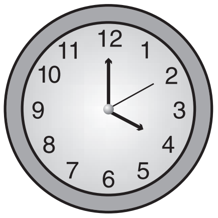
- Write proofs involving supplementary and complementary angles.
- Write proofs involving congruent and right angles.

Postulate 2.10 Protractor Postulate
Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

Postulate 2.11 Angle Addition Postulate
If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$. If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.

EXAMPLE Angle Addition

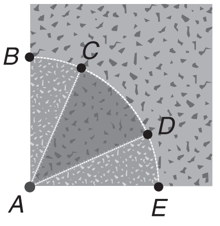
1 TIME **At 4 o'clock, the angle between the hour and minute hands of a clock is 120°. If the second hand stops where it bisects the angle between the hour and minute hands, what are the measures of the angles between the minute and second hands and between the second and hour hands?**



If the second hand stops where the angle is bisected, then the angle between the minute and second hands is ______ the measure of the angle formed by the hour and minute hands, or ______ (120) = ______.

By the Angle Addition Postulate, the sum of the two angles is ______, so the angle between the second and hour hands is also ______.

Check Your Progress The diagram shows one square for a particular quilt pattern. If $m\angle BAC = m\angle DAE = 20°$, and $\angle BAE$ is a right angle, find $m\angle CAD$.



Theorem 2.3 Supplement Theorem
If two angles form a linear pair, then they are supplementary angles.

Theorem 2.4 Complement Theorem
If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

REVIEW IT
The angles of a linear pair are always supplementary, but supplementary angles need not form a linear pair. *(Lesson 1-5)*

EXAMPLE Supplementary Angles

2 **If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 166$, find $m\angle 1$.**

$m\angle 1 + m\angle 2 = 180$ ____________

$m\angle 1 +$ ______ $= 180$ $m\angle 2 =$ ______

$m\angle 1 = 14$ Subtraction Property

Check Your Progress If $\angle 1$ and $\angle 2$ are complementary angles and $m\angle 1 = 62$, find $m\angle 2$.

Theorem 2.5
Congruence of angles is reflexive, symmetric, and transitive.

Theorem 2.6
Angles supplementary to the same angle or to congruent angles are congruent.

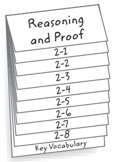
Theorem 2.7
Angles complementary to the same angle or to congruent angles are congruent.

EXAMPLE Use Supplementary Angles

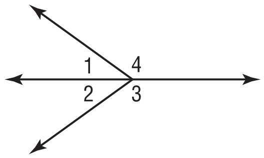
FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 2-8, copy Theorem 2.12: *If two angles are congruent and supplementary, then each angle is a right angle.* Illustrate this theorem with a diagram.



3 **In the figure, ∠1 and ∠4 form a linear pair, and $m\angle 3 + m\angle 1 = 180$. Prove that ∠3 and ∠4 are congruent.**



Given: ∠1 and ∠4 form a linear pair.
$m\angle 3 + m\angle 1 = 180$

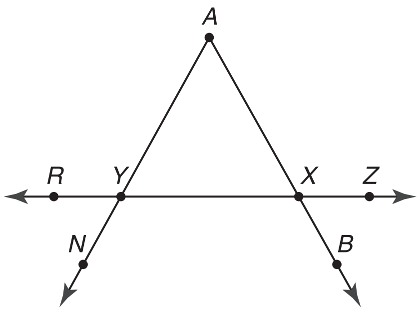
Prove: $\angle 3 \cong \angle 4$

Proof:

Statements	Reasons
1. $m\angle 3 + m\angle 1 = 180$; ∠1 and ∠4 form a linear pair.	1. Given
2. ∠1 and ∠4 are supplementary.	2.
3. ∠3 and ∠1 are supplementary.	3. Definition of supplementary angles.
4. $\angle 3 \cong \angle 4$	4.

Check Your Progress

In the figure, ∠*NYR* and ∠*RYA* form a linear pair, ∠*AXY* and ∠*AXZ* form a linear pair, and ∠*RYA* and ∠*AXZ* are congruent. Prove that ∠*RYN* and ∠*AXY* are congruent.



Theorem 2.8 Vertical Angle Theorem
If two angles are vertical angles, then they are congruent.

EXAMPLE Vertical Angles

REMEMBER IT Be sure to read problems carefully in order to provide the information requested. Often the value of the variable is used to find the answer.

4 **If $\angle 1$ and $\angle 2$ are vertical angles and $m\angle 1 = d - 32$ and $m\angle 2 = 175 - 2d$, find $m\angle 1$ and $m\angle 2$.**

$\angle 1 \cong \angle 2$	[ ] Theorem
$m\angle 1 = m\angle 2$	Definition of congruent angles
[ ] = [ ]	Substitution
$3d = 207$	Addition Property
$d = 69$	Divide each side by 3.

$m\angle 1 =$ [ ]
$=$ [ ] $- 32$
$=$ [ ]

$m\angle 2 = 175 - 2d$
$= 175 - 2($ [ ] $)$
$=$ [ ]

Check Your Progress If $\angle A$ and $\angle Z$ are vertical angles and $m\angle A = 3b - 23$ and $m\angle Z = 152 - 4b$, find $m\angle A$ and $m\angle Z$.

[ ]

Theorem 2.9
Perpendicular lines intersect to form four right angles.

Theorem 2.10
All right angles are congruent.

Theorem 2.11
Perpendicular lines form congruent adjacent angles.

Theorem 2.12
If two angles are congruent and supplementary, then each angle is a right angle.

Theorem 2.13
If two congruent angles form a linear pair, then they are right angles.

HOMEWORK ASSIGNMENT
Page(s):
Exercises:

CHAPTER 2

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	Build your Vocabulary
Use your **Chapter 2 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 2, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 32–33)* to help you solve the puzzle.

2-1 Inductive Reasoning and Conjecture

Make a conjecture about the next number in the pattern.

1. $-6, -3, 0, 3, 6$

2. $4, -2, 1, -\frac{1}{2}, \frac{1}{4}$

3. Make a conjecture based on the given information. Points A, B, and C are collinear. D is between A and B.

2-2 Logic

Use the statements p: $-1 + 4 = 3$, q: A pentagon has 5 sides, and r: $5 + 3 > 8$ to find the truth value of each statement.

4. $p \vee r$

5. $q \wedge r$

6. $(p \vee q) \wedge r$

7. Construct a truth table for the compound statement $p \vee (\sim p \wedge q)$.

2-3 Conditional Statements

8. Identify the hypothesis and conclusion of the statement.
If $x - 7 = 4$, then $x = 11$.

9. Write the statement in if-then form.
Supplementary angles have measures with a sum of 180°.

2-4 Deductive Reasoning

10. Use the Law of Detachment to tell whether the following reasoning is valid. Write *valid* or *not valid*.

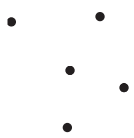
If a car has air bags it is safe. Maria's car has air bags. Therefore, Maria's car is safe.

11. Use the Law of Syllogism to write a valid conclusion from this set of statements.

(1) All squares are rectangles.
(2) All rectangles are parallelograms.

2-5 Postulates and Paragraph Proofs

12. Determine the number of line segments that can be drawn connecting each pair of points.



Name the definition, property, postulate, or theorem that justifies each statement.

13. $\overline{CD} \cong \overline{CD}$

14. If A is the midpoint of $\overline{CD}$, then $\overline{CA} \cong \overline{AD}$.

2-6

Algebraic Proof

Name the definition, property, postulate, or theorem that justifies each statement.

15. If $x - 7 = 9$, then $x = 16$

16. $5(x + 8) = 5x + 40$

17. If $\overline{CD} \cong \overline{EF}$ and $\overline{EF} \cong \overline{GH}$, then $\overline{CD} \cong \overline{GH}$.

2-7

Proving Segment Relationships

Name the definition, property, postulate, or theorem that justifies each statement.

18. If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

19. If $AB + BD = AD$, then B is between A and D.

2-8

Proving Angle Relationships

Name the definition, property, postulate, or theorem that justifies each statement.

20. If $m\angle 1 + m\angle 2 = 90$, and $m\angle 2 + m\angle 3 = 90$, then $m\angle 1 = m\angle 3$.

21. If $\angle A$ and $\angle B$ are vertical angles, then $\angle A \cong \angle B$.

CHAPTER 2

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 2.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 2 Practice Test on page 137 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 2 Study Guide and Review on pages 132–136 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 137 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 2 Foldable.
- Then complete the Chapter 2 Study Guide and Review on pages 132–136 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 2 Practice Test on page 137.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 3

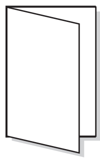
Parallel and Perpendicular Lines

FOLDABLES™

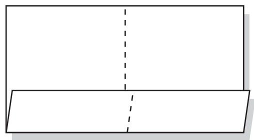
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of $8\frac{1}{2}$" × 11" paper.

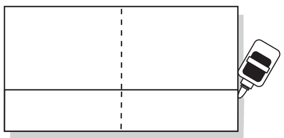
STEP 1 **Fold** in half matching the short sides.



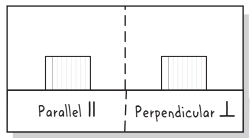
STEP 2 **Unfold** and fold the long side up 2 inches to form a pocket.



STEP 3 **Staple** or glue the outer edges to complete the pocket.



STEP 4 **Label** each side as shown. Use index cards to record examples.



NOTE-TAKING TIP: When taking notes, writing a paragraph that describes the concepts, the computational skills, and the graphics will help you to understand the math in the lesson.

CHAPTER 3

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
alternate exterior angles			
alternate interior angles			
consecutive interior angles			
corresponding angles			
equidistant [ee-kwuh-DIS-tuhnt]			
non-Euclidean geometry [yoo-KLID-ee-yuhn]			
parallel lines			

Vocabulary Term	Found on Page	Definition	Description or Example
parallel planes			
point-slope form			
rate of change			
skew lines			
slope			
slope-intercept form			
transversal			

3–1 Parallel Lines and Transversals

Preparation for Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Key)

MAIN IDEAS

- Identify the relationships between two lines or two planes.
- Name angles formed by a pair of lines and a transversal.

BUILD YOUR VOCABULARY (pages 62–63)

Coplanar lines that do not ______ are called **parallel lines.**

Two ______ that do not ______ are called **parallel planes.**

Lines that do not intersect and are not ______ are called **skew lines.**

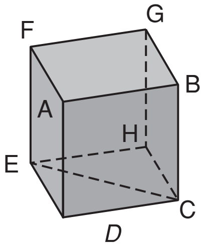
A line that intersects ______ or more lines in a plane at different points is called a **transversal.**

EXAMPLE Identify Relationships

1 **Use the figure shown at the right.**

a. Name all planes that are parallel to plane *AEF*.

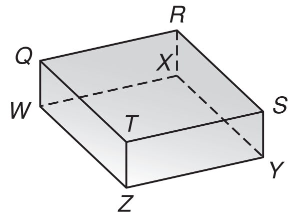
b. Name all segments that intersect $\overline{AF}$.



c. Name all segments that are skew to $\overline{AD}$.

Check Your Progress **Use the figure shown.**

a. Name all planes that are parallel to plane *RST*.

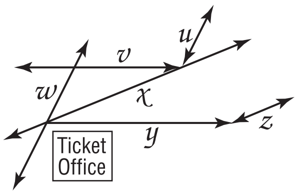


b. Name all segments that intersect $\overline{YZ}$.

c. Name all segments that are skew to $\overline{TZ}$.

EXAMPLE Identify Transversals

2 BUS STATION **Some of a bus station's driveways are shown. Identify the sets of lines to which each given line is a transversal.**

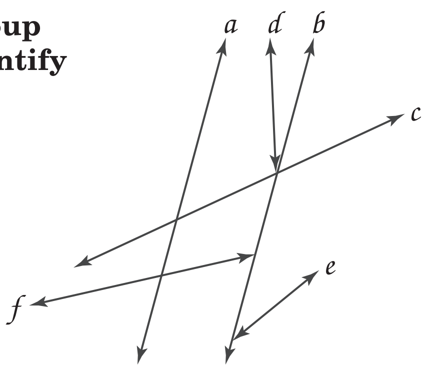


a. line v If the lines are extended, line v intersects lines ______.

b. line y

c. line u

Check Your Progress **A group of nature trails is shown. Identify the set of lines to which each given line is a transversal.**



a. line a

b. line b

c. line c

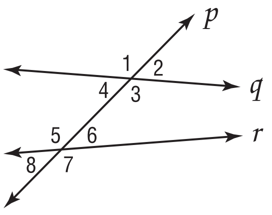
d. line d

BUILD YOUR VOCABULARY (page 62)

consecutive interior angles:
∠4 and ∠5, ∠3 and ∠6

alternate interior angles:
∠4 and ∠6, ∠3 and ∠5

alternate exterior angles:
∠1 and ∠7, ∠2 and ∠8

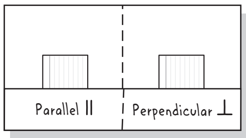


corresponding angles: ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8

FOLDABLES™

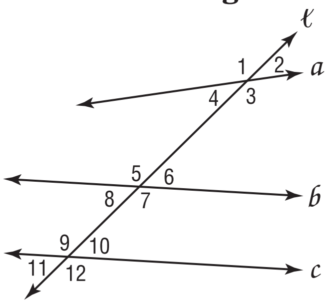
ORGANIZE IT

Using a separate card for each type of angle pair, draw two parallel lines cut by a transversal and identify consecutive interior angles, alternate exterior angles, alternate interior angles, and corresponding angles. Place your cards in the Parallel Lines pocket.



EXAMPLE Identify Angle Relationships

3 **Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.**



a. **∠7 and ∠3**

b. **∠8 and ∠2**

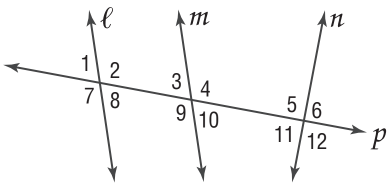
c. **∠4 and ∠11**

d. **∠7 and ∠1**

e. **∠3 and ∠9**

f. **∠7 and ∠10**

Check Your Progress **Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.**



a. ∠4 and ∠5

b. ∠7 and ∠9

c. ∠4 and ∠7

d. ∠2 and ∠11

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–2 Angles and Parallel Lines

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Key)

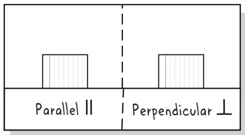
MAIN IDEAS

- Use the properties of parallel lines to determine congruent angles.
- Use algebra to find angle measures.

FOLDABLES™

ORGANIZE IT

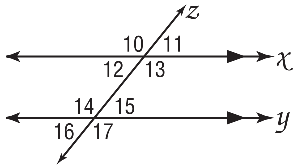
Sketch two parallel lines and a transversal on a card. Below the sketch, write a brief note to explain how knowing one of the angle measures allows you to find the measures of all the other angles. Place the card in the Parallel Lines pocket.



Postulate 3-1 Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

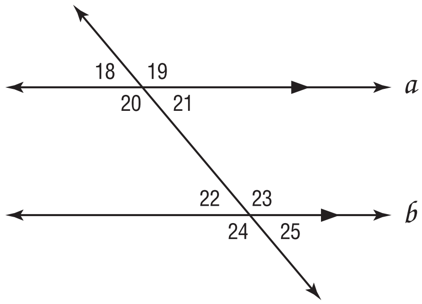
EXAMPLE Determine Angle Measures

1 **In the figure $x \parallel y$ and $m\angle 11 = 51$. Find $m\angle 16$.**



$\angle 11 \cong \angle 15$	Corresponding Angles Postulate
______ $\cong$ ______	Vertical Angles Theorem
$\angle 11 \cong \angle 16$	Transitive Property
$m\angle 11 = m\angle 16$	Definition of congruent angles
______ $= m\angle 16$	Substitution

Check Your Progress In the figure, $m\angle 18 = 42$. Find $m\angle 25$.



Theorem 3.1 Alternate Interior Angles Theorem
If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

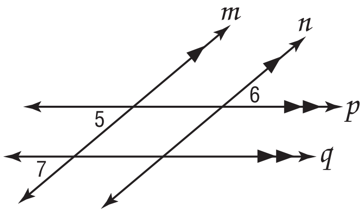
Theorem 3.2 Consecutive Interior Angles Theorem
If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Theorem 3.3 Alternate Exterior Angles Theorem
If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Theorem 3.4 Perpendicular Transversal Theorem
In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

EXAMPLE Find Values of Variables

2 ALGEBRA **If $m\angle 5 = 2x - 10$, $m\angle 6 = 4(y - 25)$, and $m\angle 7 = x + 15$, find x and y.**



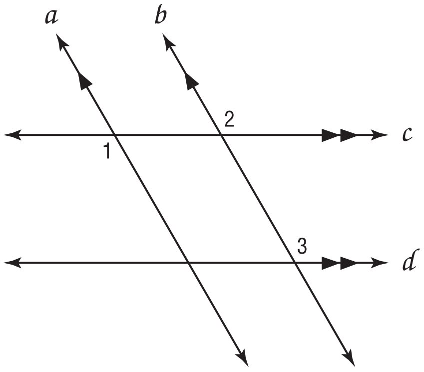
Find x. Since $p \parallel q$, $m\angle 5 \cong m\angle 7$ by the Corresponding Angles Postulate.

$m\angle 5 = m\angle 7$	Definition of congruent angles
______ = ______	Substitution
$x =$ ______	Subtract x from each side and add 10 to each side.

Find y. Since $m \parallel n$, $m\angle 5 \cong m\angle 6$ by the Alternate Exterior Angles Theorem.

$m\angle 5 = m\angle 6$	Definition of congruent angles
______ = ______	Substitution
______ = ______	$x = 25$
______ $= y$	Add 100 to each side and divide each side by 4.

Check Your Progress
If $m\angle 1 = 9x + 6$, $m\angle 2 = 2(5x - 3)$, and $m\angle 3 = 5y + 14$, find x and y.



REVIEW IT

When two lines intersect, what types of angle pairs are congruent? (*Lesson 2-8*)

______
______
______
______

HOMEWORK ASSIGNMENT

Page(s):
Exercises:

3–3 Slope of Lines

Reinforcement of Algebra I Standard 8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

MAIN IDEAS

- Find slopes of lines.
- Use slope to identify parallel and perpendicular lines.

KEY CONCEPT

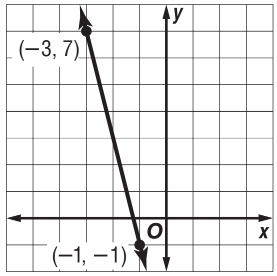
Slope The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

BUILD YOUR VOCABULARY (page 63)

The **slope** of a line is the ratio of the vertical rise to its horizontal run.

EXAMPLE Find the Slope of a Line

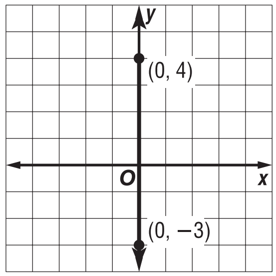
1 **a. Find the slope of the line.**



Use the $\frac{\text{rise}}{\text{run}}$ method. From (−3, 7) to (−1, −1), go down ____ units and ____ 2 units.

$\frac{\text{rise}}{\text{run}} = \frac{\text{____}}{\text{____}}$ or ____

b. Find the slope of the line.



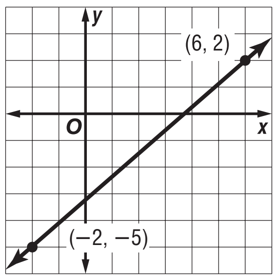
Use the slope formula.

Let (0, 4) be (x_1, y_1) and (0, −3) be (x_2, y_2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\text{____} - \text{____}}{0 - 0}$ or $\frac{-7}{0}$, which is ____.

c. Find the slope of the line.

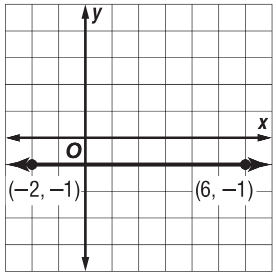


$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\text{____} - \text{____}}{6 - (-2)}$

$=$ ____

d. Find the slope of the line.



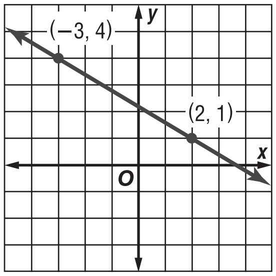
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

= ______

= ______ or ______

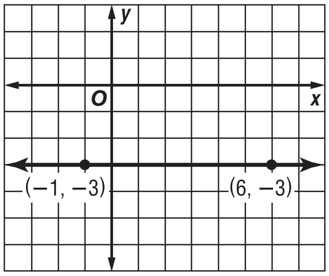
Check Your Progress **Find the slope of each line.**

a.



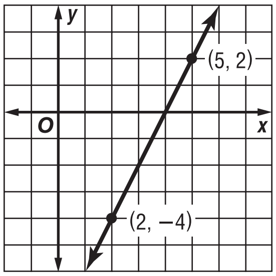
______

b.



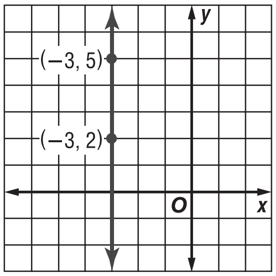
______

c.



______

d.



______

REMEMBER IT

Lines with positive slopes *rise* as you move from left to right, while lines with negative slopes *fall* as you move from left to right.

BUILD YOUR (page 63)

The **rate of change** describes how a quantity is ______ over ______.

Postulate 3.2
Two nonvertical lines have the same slope if and only if they are parallel.

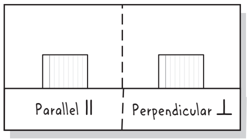
Postulate 3.3
Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

EXAMPLE Determine Line Relationships

FOLDABLES™

ORGANIZE IT

On a study card, write the equation of two lines that are parallel and explain what is true about their slopes. Place this card in the Parallel Lines pocket.



2 **Determine whether $\overleftrightarrow{FG}$ and $\overleftrightarrow{HJ}$ are *parallel*, *perpendicular*, or *neither*.**

a. ***F*(1, –3), *G*(–2, –1), *H*(5, 0), *J*(6, 3)**

Find the slopes of $\overleftrightarrow{FG}$ and $\overleftrightarrow{HJ}$.

slope of $\overleftrightarrow{FG} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square}$ or $-\frac{2}{3}$

slope of $\overleftrightarrow{HJ} = \frac{\square - \square}{\square - \square} = \frac{3}{1}$ or $\square$

The slopes are not the same, so $\overleftrightarrow{FG}$ and $\overleftrightarrow{HJ}$ are ________. The product of the slopes is ________. So $\overleftrightarrow{FG}$ and $\overleftrightarrow{HJ}$ are neither ________ nor ________.

b. ***F*(4, 2), *G*(6, –3), *H*(–1, 5), *J*(–3, 10)**

slope of $\overleftrightarrow{FG} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square}$ or $-\frac{5}{2}$

slope of $\overleftrightarrow{HJ} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square}$ or $-\frac{5}{2}$

The slopes are the same, so $\overleftrightarrow{FG}$ and $\overleftrightarrow{HJ}$ are ________.

Check Your Progress **Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.**

a. *A*(–2, –1), *B*(4, 5), *C*(6, 1), *D*(9, –2)

____

b. *A*(7, –3), *B*(1, –2), *C*(4, 0), *D*(–3, 1)

____

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–4 Equations of Lines

Preparation for Standard 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

MAIN IDEAS

- Write an equation of a line given information about its graph.
- Solve problems by writing equations.

BUILD YOUR VOCABULARY (page 63)

The **slope-intercept form** of a linear equation is ________, where *m* is the ________ of the line and *b* is the *y*-intercept.

The **point-slope form** is ________, where (x_1, y_1) are the coordinates of any point on the line and *m* is the slope of the line.

EXAMPLE Slope and *y*-intercept

1 **Write an equation in slope-intercept form of the line with slope of 6 and *y*-intercept of –3.**

$y = mx + b$ Slope-intercept form

$y =$ ____ $x +$ ____ $m = 6, b = -3$

The slope-intercept form of the equation is ________.

Check Your Progress Write an equation in slope-intercept form of the line with slope of –1 and *y*-intercept of 4.

____

REMEMBER IT

Note that the point-slope form of an equation is different for each point used. However, the slope-intercept form of an equation is unique.

EXAMPLE Slope and a Point

2 **Write an equation in point-slope form of the line whose slope is $-\frac{3}{5}$ and contains (–10, 8).**

$y - y_1 = m(x - x_1)$ Point-slope form

$y -$ ____ $= -\frac{3}{5}[x -$ ____ $]$ $m = -\frac{3}{5}$, $(x_1, y_1) = (-10, 8)$

$y - 8 = -\frac{3}{5}(x + 10)$ Simplify.

Check Your Progress Write an equation in point-slope form of the line whose slope is $\frac{1}{3}$ and contains (6, –3).

EXAMPLE **Two Points**

3 **Write an equation in slope-intercept form for a line containing (4, 9) and (–2, 0).**

Find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{0 - 9}{-2 - 4}$ $x_1 = 4$, $x_2 = -2$, $y_1 = 9$, $y_2 = 0$

$= \frac{\square}{\square}$ or $\square$ Simplify.

Use the point-slope form to write an equation.

Using (4, 9):

$y - y_1 = m(x - x_1)$ Point-slope form

$y - \square = \square(x - 4)$ $m = \square$, $(x_1, y_1) = (4, 9)$

$y = \frac{3}{2}x + 3$ Distributive Property and add 9 to both sides.

Using (–2, 0):

$y - y_1 = m(x - x_1)$ Point-slope form

$y - \square = \square[x - (-2)]$ $m = \square$, $(x_1, y_1) = (-2, 0)$

$y = \square$ Distributive Property

Check Your Progress Write an equation in slope-intercept form for a line containing (3, 2) and (6, 8).

EXAMPLE One Point and an Equation

4 **Write an equation in slope-intercept form for a line containing (1, 7) that is perpendicular to the line $y = -\frac{1}{2}x + 1$.**

Since the slope of the line is ____, the slope of a line perpendicular to it is ____.

$y - y_1 = m(x - x_1)$	Point-slope form
$y -$ ____ $=$ ____ $(x -$ ____ $)$	$m = 2, (x_1, y_1) = (1, 7)$
$y =$ ____	Distributive Property and add 7 to each side.

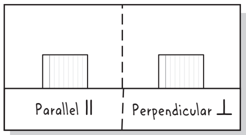
Check Your Progress Write an equation in slope-intercept form for a line containing (–3, 4) that is perpendicular to the line $y = \frac{3}{5}x - 4$.

____

FOLDABLES™

ORGANIZE IT

On a study card, write the slope-intercept equations of two lines that are perpendicular, and explain what is true about their slopes. Place this card in the Perpendicular Lines pocket.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–5 Proving Lines Parallel

MAIN IDEAS

- Recognize angle conditions that occur with parallel lines.
- Prove that two lines are parallel based on given angle relationships.

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Key)
Standard 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and **the line parallel to a given line through a point off the line.** (Key)

Postulate 3.4
If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Postulate 3.5 Parallel Postulate
If there is a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Theorem 3.5
If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.

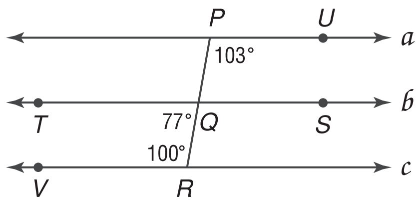
Theorem 3.6
If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.

Theorem 3.7
If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.

Theorem 3.8
In a plane, if two lines are perpendicular to the same line, then they are parallel.

EXAMPLE Identify Parallel Lines

1 Determine which lines, if any, are parallel.



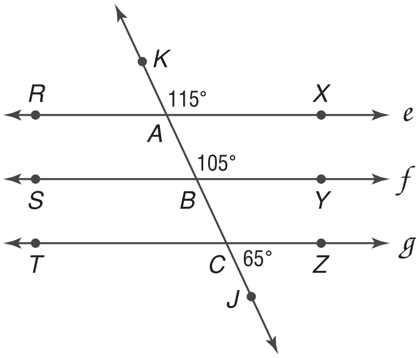
Since $\angle RQT$ and $\angle SQP$ are ______ angles,

$m\angle SQP =$ ______.

Since $m\angle UPQ + m\angle SQP =$ ______ $+$ ______ or ______, consecutive interior angles are supplementary. So, $a \parallel b$.

Since $m\angle TQR + m\angle VRQ = 77 + 100$ or 177, consecutive interior angles are ______. So, c is not parallel to a or b.

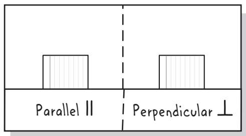
Check Your Progress Determine which lines, if any, are parallel.



FOLDABLES

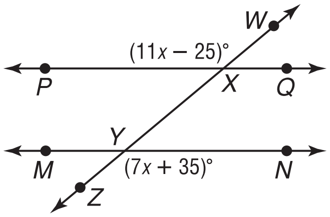
ORGANIZE IT

On a card, write the biconditional statement that you obtain by combining Postulates 3.1 and 3.4. Place the card in the Parallel Lines pocket.



EXAMPLE Solve Problems with Parallel Lines

2 ALGEBRA **Find x and $m\angle ZYN$ so that $\overleftrightarrow{PQ} \parallel \overleftrightarrow{MN}$.**



$m\angle WXP = m\angle ZYN$	Alternate exterior angles
______ = ______	Substitution
$4x - 25 = 35$	Subract $7x$ from each side.
$4x = 60$	Add 25 to each side.
$x = 15$	Divide each side by 4.

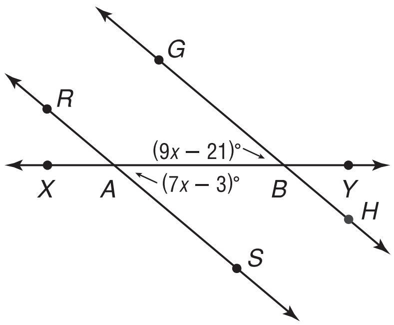
Now use the value of x to find $m\angle ZYN$.

$m\angle ZYN = 7x + 35$	Original equation
$= 7(\_\_\_\_) + 35$	$x = 15$
$= 140$	Simplify.

So, $x = 15$ and $m\angle ZYN = 140$.

Check Your Progress

Find x and $m\angle GBA$ so that $\overleftrightarrow{GH} \parallel \overleftrightarrow{RS}$.



EXAMPLE Prove Lines Parallel

Write It

Write how you can use angles formed by two lines and a transversal to decide whether the two lines are parallel or not parallel.

____

____

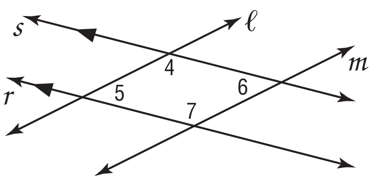
____

____

3 **Given:** $\ell \parallel m$

$\angle 4 \cong \angle 7$

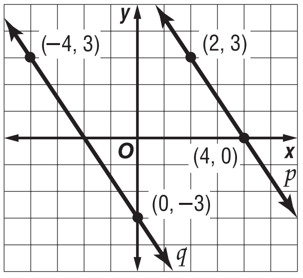
Prove: $r \parallel s$



Statements	Reasons
1. $\ell \parallel m$, $\angle 4 \cong \angle 7$	**1.** Given
2. $\angle 4$ and $\angle 6$ are suppl.	**2.** Consecutive Interior $\angle$Thm.
3. $\angle 4 + \angle 6 = 180$	**3.** ______
4. ______ $= m\angle 7$	**4.** Def. of congruent $\angle$s
5. $m\angle 7 +$ ______ $= 180$	**5.** ______
6. $\angle 7$ and $\angle 6$ are suppl.	**6.** Def. of suppl. $\angle$s
7. $r \parallel s$	**7.** If cons. int. $\angle$s are suppl., then lines are parallel.

EXAMPLE Slope and Parallel Lines

4 **Determine whether $p \parallel q$.**



slope of p: $m = \dfrac{\square - \square}{\square - \square}$

$= \square$

slope of q: $m = \dfrac{\square - (\square)}{\square - \square}$

$= \square$

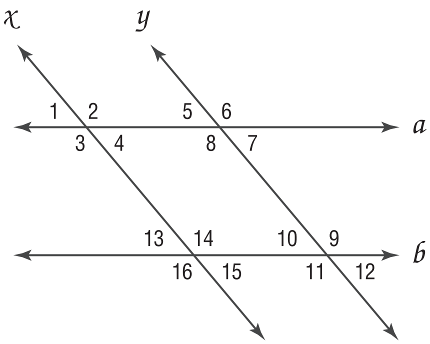
Since the slopes are ______, $p \parallel q$.

Check Your Progress

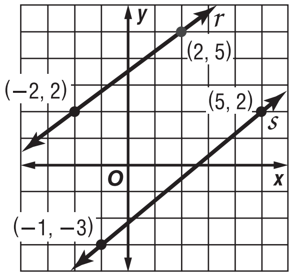
a. Prove the following lines parallel.

Given: $x \parallel y$
$\angle 1 \cong \angle 4$

Prove: $a \parallel b$



b. Determine whether $r \parallel s$.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

3–6 Perpendiculars and Distance

Standard 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. (Key)

MAIN IDEAS

- Find the distance between a point and a line.
- Find the distance between parallel lines.

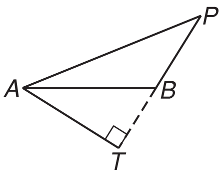
KEY CONCEPT

Distance Between a Point and a Line The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.

FOLDABLES™ On a card, describe how to find the distance from a point in the coordinate plane to a line that does not pass through the point. Place the card in the Perpendicular Lines pocket.

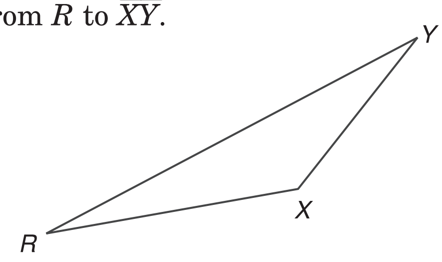
EXAMPLE Distance from a Point to a Line

1 **Draw the segment that represents the distance from A to $\overleftrightarrow{BP}$.**



Since the distance from a line to a point not on the line is the ________ of the segment ________ to the line from the point, extend ________ and draw ________ so that ________.

Check Your Progress Draw the segment that represents the distance from R to $\overline{XY}$.



BUILD YOUR VOCABULARY (page 62)

Equidistant means that the ________ between two lines measured along a ________ to the lines is always the same.

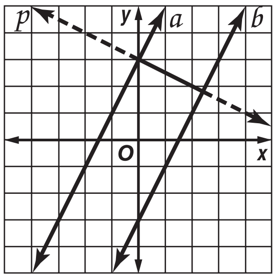
Theorem 3.9
In a plane, if two lines are each equidistant from a third line, then the two lines are parallel to each other.

EXAMPLE Distance Between Lines

KEY CONCEPT

Distance Between Parallel Lines The distance between two parallel lines is the distance between one of the lines and any point on the other line.

2 **Find the distance between parallel lines *a* and *b* whose equations are $y = 2x + 3$ and $y = 2x - 3$, respectively.**



- First, write an equation of a line *p* perpendicular to *a* and *b*. The slope of *p* is the opposite _____ of 2, or _____. Use the *y*-intercept of line *a*, (0, 3), as one of the endpoints of the perpendicular segment.

$y - y_1 = m(x - x_1)$	Point-slope form
$y -$ _____ $=$ _____ $(x -$ _____ $)$	$x_1 =$ _____, $y_1 =$ _____, $m =$ _____
$y = -\frac{1}{2}x + 3$	Simplify and add 3 to each side.

- Next, use a system of equations to determine the point of intersection of line *b* and *p*.

_____ $=$ _____	Substitute _____ for *y* in the second equation.
$2x + \frac{1}{2}x = 3 + 3$	Group like terms on each side.
$x = 2.4$	Divide each side $\frac{5}{2}$.
$y =$ _____ $=$ _____	Substitute 2.4 for *x* in the equation for *p*.

The point of intersection is _____.

Then, use the Distance Formula to determine the distance between (0, 3) and (2.4, 1.8).

$d =$ ______ Distance Formula

$=$ ______ $x_2 = 2.4,\ x_1 = 0,$ $y_2 = 1.8,\ y_1 = 3$

$=$ ______

The distance between the lines is ______ or about ______ units.

WRITE IT

What is the point-slope form of a linear equation?

______

______

______

______

Check Your Progress Find the distance between the parallel lines a and b whose equations are $y = \frac{1}{3}x + 1$ and $y = \frac{1}{3}x - 2$, respectively.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 3

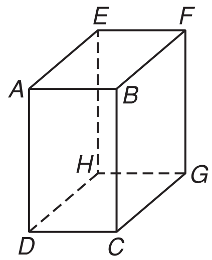
BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 3 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 3, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 62–63)* to help you solve the puzzle.

3-1 Parallel Lines and Transversals

Refer to the figure at the right.

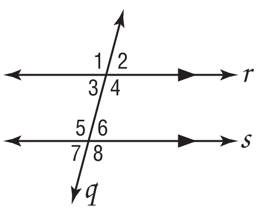


1. Name all planes that are parallel to plane *ABC*.

2. Name all segments that are parallel to $\overline{FG}$.

3-2 Angles and Parallel Lines

In the figure, $m\angle 5 = 100$. Find the measure of each angle.



3. $\angle 1$

4. $\angle 3$

5. $\angle 4$

6. $\angle 7$

7. $\angle 6$

8. $\angle 2$

3-3 Slope of Lines

Determine the slope of the line that contains the given points.

9. $F(-6, 2), M(7, 9)$

10. $Z(1, 10), L(5, -3)$

11. Determine whether $\overline{EF}$ and $\overline{PQ}$ are parallel, perpendicular, or neither. $E(0, 4)$, $F(2, 3)$, $P(-3, 5)$, $Q(1, 3)$

3-4 Equations of lines

12. Write an equation in slope-intercept form of the line with slope -2 that contains $(2, 5)$.

13. Write an equation in slope-intercept form of the line that contains $(-4, -2)$ and $(-1, 7)$.

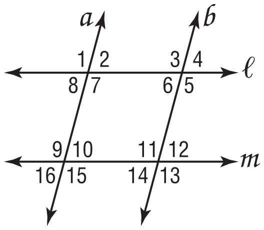
3-5 Proving Lines Parallel

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

14. $\angle 1 \cong \angle 15$

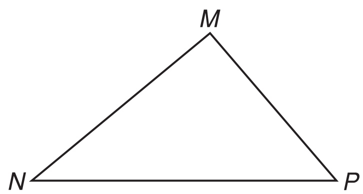
15. $\angle 9 \cong \angle 11$

16. $\angle 2 \cong \angle 6$



3-6 Perpendiculars and Distance

17. Draw the segment that represents the distance from m to $\overleftrightarrow{NP}$.



Find the distance between each pair of parallel lines.

18. $x = 3$
$x = -5$

19. $y = x + 5$
$y = x - 5$

CHAPTER 3

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 3.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 3 Practice Test on page 195 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 3 Study Guide and Review on pages 191–194 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 195.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 3 Foldable.
- Then complete the Chapter 3 Study Guide and Review on pages 191–194 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 3 Practice Test on page 195.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 4

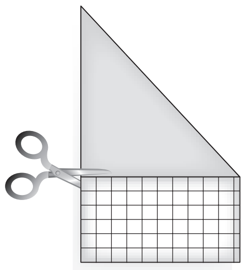
Congruent Triangles

FOLDABLES™

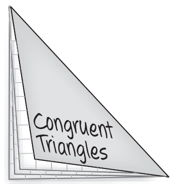
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with two sheets of grid paper and one sheet of construction paper.

STEP 1 **Stack** the grid paper on the construction paper. Fold diagonally to form a triangle and cut off the excess.



STEP 2 **Staple** the edge to form a booklet. Write the chapter title on the front and label each page with a lesson number and title.



NOTE-TAKING TIP: Before each lesson, skim through the lesson and write any questions that come to mind in your notes. As you work through the lesson, record the answer to your question.

CHAPTER 4

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
acute triangle			
base angles			
congruence transformation [kuhn-GROO-uhns]			
congruent triangles			
coordinate proof			
corollary			
equiangular triangle			
equilateral triangle			
exterior angle			

Vocabulary Term	Found on Page	Definition	Description or Example
flow proof			
included angle			
included side			
isosceles triangle			
obtuse triangle			
remote interior angles			
right triangle			
scalene triangle [SKAY-leen]			
vertex angle			

4–1 Classifying Triangles

Standard 12.0 Students find and use measures of sides and of interior and exterior **angles of triangles** and polygons to **classify figures and solve problems.** (Key)

MAIN IDEAS

- Identify and classify triangles by angles.
- Identify and classify triangles by sides.

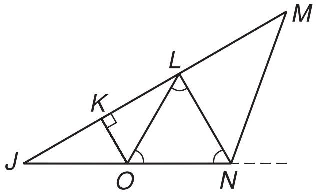
KEY CONCEPTS

Classifying Triangles by Angles In an acute triangle, all of the angles are acute. In an obtuse triangle, one angle is obtuse. In a right triangle, one angle is right.

Classifying Triangles by Sides No two sides of a scalene triangle are congruent. At least two sides of an isosceles triangle are congruent. All of the sides of an equilateral triangle are congruent.

EXAMPLE Classify Triangles by Angles

1 **ARCHITECTURE The triangular truss below is modeled for steel construction. Classify $\triangle JMN$, $\triangle JKO$, and $\triangle OLN$ as *acute, equiangular, obtuse,* or *right.***

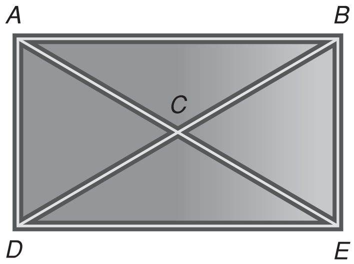


$\triangle JMN$ has one angle with measure greater than ____, so it is an ____ triangle.

$\triangle JKO$ has one angle with measure equal to ____, so it is a ____ triangle.

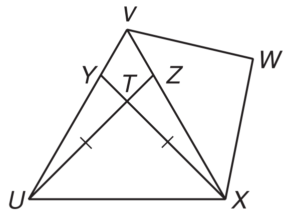
$\triangle OLN$ is an ____ triangle with all angles congruent, so it is an ____ triangle.

Check Your Progress The frame of this window design is made up of many triangles. Classify $\triangle ABC$, $\triangle ACD$, and $\triangle ADE$ as *acute, equiangular, obtuse,* or *right*.



EXAMPLE Classify Triangles by Sides

2 **Identify the indicated triangles in the figure if $\overline{UV} \cong \overline{VX} \cong \overline{UX}$.**



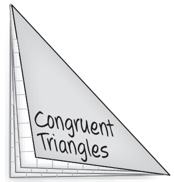
a. isosceles triangles

Isosceles triangles have at least two sides congruent. Triangle ____ and ____ are ____.

FOLDABLES™

ORGANIZE IT

On the page for Lesson 4-1, draw the 6 different types of triangles and describe each one.

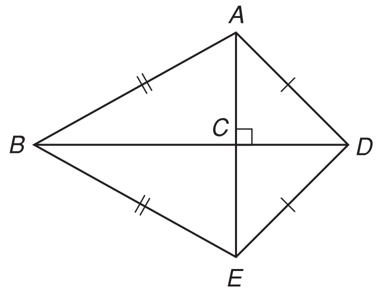


b. scalene triangles

Scalene triangles have no ____ sides.

The scalene triangles are ____.

Check Your Progress **Identify the indicated triangles in the figure.**



a. isosceles triangles

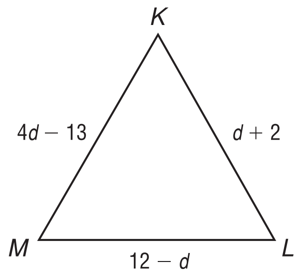
____

b. scalene triangles

____

EXAMPLE Find Missing Values

1 ALGEBRA **Find d and the measure of each side of equilateral triangle KLM if $KL = d + 2$, $LM = 12 - d$, and $KM = 4d - 13$.**



Since $\triangle KLM$ is equilateral, each side has the same length.

So ____ = ____.

____ = ____ Substitution

$2 = 3d - 13$ Subtract d from each side.

$15 = 3d$ ____.

____ = ____ Divide each side by 3.

Next, substitute to find the length of each side.

$KL = d + 2$ $LM = 12 - d$ $KM = 4d - 13$

$=$ ____ $+ 2$ $= 2 -$ ____ $= 4($ ____ $) - 13$

$=$ ____ $=$ ____ $=$ ____

For $\triangle KLM$, $d =$ ____ and the measure of each side is ____.

WRITE IT

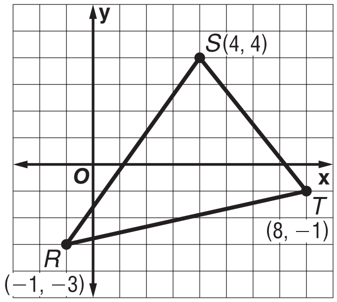
Explain why an equilateral triangle is a special kind of isosceles triangle.

Check Your Progress Find x and the measure of each side of equilateral triangle ABC if $AB = 6x - 8$, $BC = 7 + x$, and $AC = 13 - x$.

EXAMPLE Use the Distance Formula

4 COORDINATE GEOMETRY **Find the measures of the sides of $\triangle RST$. Classify the triangle by sides.**

Use the Distance Formula to find the length of each side.



$RS =$ ______

$= \sqrt{25 + 49}$

$=$ ______

$ST =$ ______

$= \sqrt{(16 + 25)}$

$=$ ______

$RT =$ ______

$= \sqrt{81 + 4}$

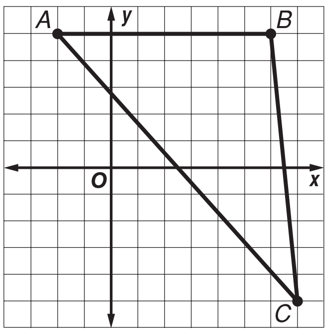
$=$ ______

$RS =$ ______, $ST =$ ______, $RT =$ ______. Since all 3 sides have different lengths, $\triangle RST$ is ______.

REVIEW IT

Write the Distance Formula. (*Lesson 1-3*)

Check Your Progress Find the measures of the sides of $\triangle ABC$. Classify the triangle by its sides.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–2 Angles of Triangles

Standard 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

MAIN IDEAS

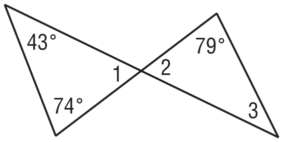
- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

Theorem 4.1 Angle Sum Theorem
The sum of the measures of the angles of a triangle is 180.

EXAMPLE Interior Angles

1 Find the missing angle measures.

Find $m\angle 1$ first because the measure of two angles of the triangle are known.



$m\angle 1 + 43 + 74 = 180$	Angle Sum Theorem
$m\angle 1 +$ ______ $= 180$	Simplify.
$m\angle 1 =$ ______	Subtract 117 from each side.

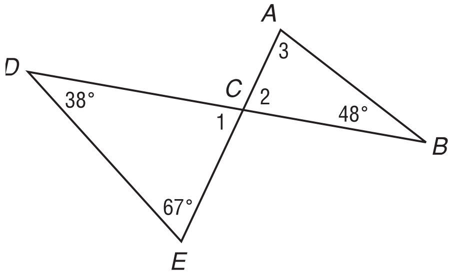
$\angle 1$ and $\angle 2$ are congruent ______ angles.

So, $m\angle 2 =$ ______.

$m\angle 3 +$ ______ $+$ ______ $= 180$	Angle Sum Theorem
$m\angle 3 + 142 = 180$	Simplify.
$m\angle 3 =$ ______	Subtract 142 from each side.

So, $m\angle 1 =$ ______, $m\angle 2 =$ ______, and $m\angle 3 =$ ______.

Check Your Progress Find the missing angle measures.



BUILD YOUR VOCABULARY (pages 86–87)

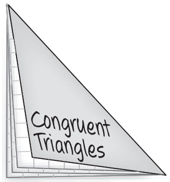
An **exterior angle** of a triangle is formed by one side of a ____ and the ____ of another side.

The ____ angles of the triangle not ____ to a given exterior angle are called **remote interior angles** of the exterior angle.

FOLDABLES

ORGANIZE IT

On the page for Lesson 4-2, draw an acute triangle and identify an exterior angle and its two remote interior angles.



Theorem 4.2 Third Angle Theorem
If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

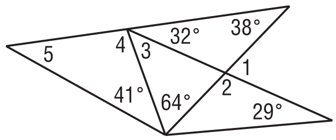
Theorem 4.3 Exterior Angle Theorem
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

EXAMPLE Exterior Angles

REMEMBER IT

Two angles are supplementary if the sum of the angles is 180.

2 **Find the measure of each numbered angle in the figure.**

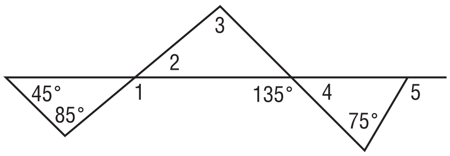


$m\angle 1 =$ ____ $+$ ____	Exterior Angle Theorem
$=$ ____	Simplify.
$m\angle 1 + m\angle 2 = 180$	Linear pairs are supplementary.
____ $+ m\angle 2 = 180$	Substitution
$m\angle 2 =$ ____	Subtract 70 from each side.
$m\angle 2 = m\angle 3 + 64$	Exterior Angle Theorem
____ $= m\angle 3 + 64$	Substitution
____ $= m\angle 3$	Subtract 64 from each side.

$m\angle 4 + (m\angle 3 + 32) = 180$	Linear pairs are supplementary.
$m\angle 4 + (\_\_\_\_ + 32) = 180$	Substitution
$m\angle 4 = \_\_\_\_$	Simplify and subtract 78 from each side.
$m\angle 5 + m\angle 4 + 41 = 180$	Angle Sum Theorem
$m\angle 5 + 102 + 41 = 180$	Substitution
$m\angle 5 = \_\_\_\_$	Simplify and subtract 143 from each side.

So, $m\angle 1 = \_\_\_\_$, $m\angle 2 = \_\_\_\_$, $m\angle 3 = \_\_\_\_$, $m\angle 4 = \_\_\_\_$, and $m\angle 5 = \_\_\_\_$.

Check Your Progress Find the measure of each numbered angle in the figure.



BUILD YOUR VOCABULARY (page 86)

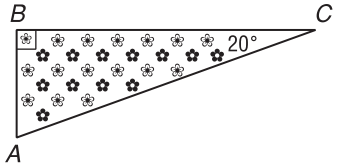
A ______ that can be easily proved using a ______ is often called a **corollary**.

Corollary 4.1
The acute angles of a right triangle are complementary.

Corollary 4.1
There can be at most one right or obtuse angle in a triangle.

EXAMPLE Right Angles

3 GARDENING **The flower bed shown is in the shape of a right triangle. Find $m\angle A$ if $m\angle C$ is 20.**

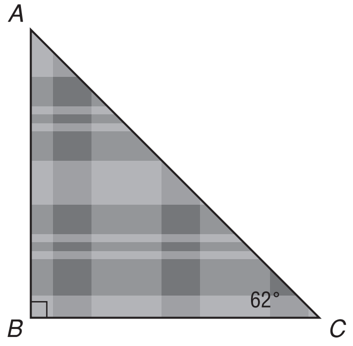


[ ] + [ ] = [ ] Corollary 4.1

$m\angle A +$ [ ] = [ ] Substitution

$m\angle A =$ [ ] Subtract [ ] from each side.

Check Your Progress The piece of quilt fabric is in the shape of a right triangle. Find $m\angle A$ if $m\angle C$ is 62.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–3 Congruent Triangles

Standard 5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

MAIN IDEAS

- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

BUILD YOUR VOCABULARY (page 86)

Triangles that are the same ______ and ______ are **congruent triangles**.

If you slide, flip, or turn a triangle, the size and ______ do not change. These three transformations are called **congruence transformations**.

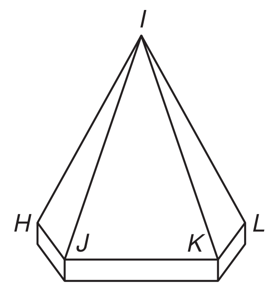
KEY CONCEPT

Definition of Congruent Triangles (CPCTC) Two triangles are congruent if and only if their corresponding parts are congruent.

FOLDABLES™ Write this definition in your notes. Be sure to include a diagram.

EXAMPLE Corresponding Congruent Parts

1 ARCHITECTURE **A drawing of a tower's roof is composed of congruent triangles all converging at a point at the top.**



a. Name the corresponding congruent angles and sides of $\triangle HIJ$ and $\triangle LIK$.

Since corresponding parts of congruent triangles are congruent, $\angle HJI \cong \angle LKI$, $\angle ILK \cong \angle IHJ$,

$\angle HIJ \cong$ ______, $\overline{HI} \cong$ ______, $\overline{HJ} \cong$ ______

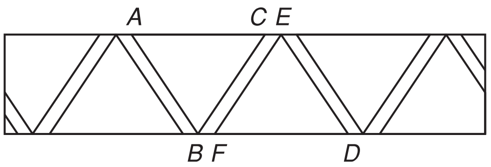
and $\overline{JI} \cong \overline{KI}$.

b. Name the congruent triangles.

Name the triangles in the order of their corresponding congruent parts. So, $\triangle HIJ \cong$ ______.

Check Your Progress

The support beams on the fence form congruent triangles.



a. Name the corresponding congruent angles and sides of $\triangle ABC$ and $\triangle DEF$.

b. Name the congruent triangles.

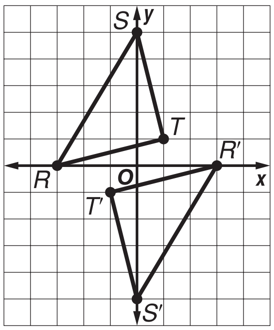
REMEMBER IT

When naming congruent triangles, always list the name in the order of congruent parts.

Theorem 4.4
Congruence of triangles is reflexive, symmetric, and transitive.

EXAMPLE Transformations in the Coordinate Plan

2 **a. Verify that $\triangle RST \cong \triangle R'S'T'$.**



Use the Distance Formula to find the length of each side of the triangles.

$RS =$

$= \sqrt{9 + 25}$ or

$R'S' =$

$= \sqrt{9 + 25}$ or

$ST = \sqrt{(0 - 1)^2 + (5 - 1)^2}$

$= \sqrt{1 + 16}$ or

$S'T' = \sqrt{(0 - (-1))^2 + (5 - (-1))^2}$

$= \sqrt{1 + 16}$ or

$$TR = \sqrt{(1 - (-3))^2 + (1 - 0)^2}$$

$$= \sqrt{16 + 1} \text{ or } \sqrt{17}$$

$$T'R' = \sqrt{(-1 - 3)^2 + (-1 - 0)^2}$$

$$= \sqrt{16 + 1} \text{ or } \sqrt{17}$$

The lengths of the corresponding sides of two triangles are equal. Therefore, by the definition of congruence,

$\overline{RS} \cong$, $\overline{ST} \cong$, $\overline{TR} \cong$.

In conclusion, because $\overline{RS} \cong \overline{R'S'}$, $\overline{ST} \cong \overline{S'T'}$, $\overline{TR} \cong \overline{T'R'}$,

$\angle R \cong$, $\angle S \cong$, and $\cong \angle T'$,

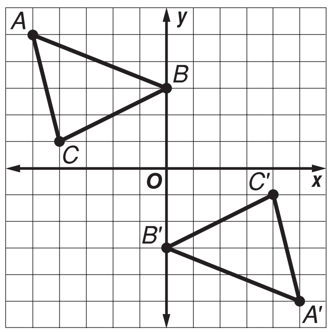
$\triangle RST \cong$.

b. Name the congruence transformation for $\triangle RST$ and $\triangle R'S'T'$.

$\triangle R'S'T'$ is a of .

Check Your Progress **The vertices of $\triangle ABC$ are $A(-5, 5)$, $B(0, 3)$, and $C(-4, 1)$. The vertices of $\triangle A'B'C'$ are $A'(5, -5)$, $B'(0, -3)$, and $C'(4, -1)$.**

a. Verify that $\triangle ABC \cong \triangle A'B'C'$.



b. Name the congruence transformation for $\triangle ABC$ and $\triangle A'B'C'$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–4 Proving Congruence – SSS, SAS

Standard 5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

Main Ideas

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

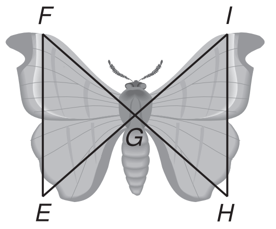
Postulate 4.1 Side-Side-Side Congruence (SSS)
If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

EXAMPLE Use SSS in Proofs

1 ENTOMOLOGY The wings of one type of moth form two triangles. Write a two-column proof to prove that $\triangle FEG \cong \triangle HIG$ if $\overline{EI} \cong \overline{FH}$, $\overline{FE} \cong \overline{HI}$, and G is the midpoint of both $\overline{EI}$ and $\overline{FH}$.

Given: $\overline{EI} \cong \overline{FH}$; $\overline{FE} \cong \overline{HI}$; G is the midpoint of both $\overline{EI}$ and $\overline{FH}$.

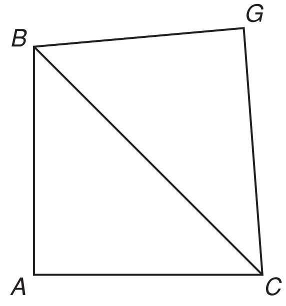
Prove: $\triangle FEG \cong \triangle HIG$



Proof:

Statements	Reasons
1. $\overline{FE} \cong \overline{HI}$; G is the midpoint of $\overline{EI}$; G is the midpoint of $\overline{FH}$.	1.
2.	2. Midpoint Theorem
3.	3.

Check Your Progress Write a two-column proof to prove that $\triangle ABC \cong \triangle GBC$ if $\overline{GB} \cong \overline{AB}$ and $\overline{AC} \cong \overline{GC}$.



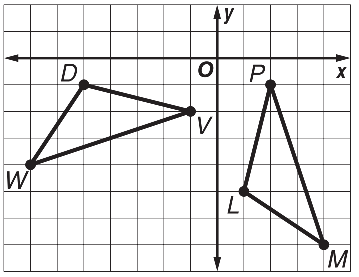
Proof:

Statements	Reasons
1.	1.
2.	2.
3.	3.

EXAMPLE **SSS on the Coordinate Plane**

2 **COORDINATE GEOMETRY** **Determine whether $\triangle WDV \cong \triangle MLP$ for $D(-5, -1)$, $V(-1, -2)$, $W(-7, -4)$, $L(1, -5)$, $P(2, -1)$, and $M(4, -7)$. Explain.**

Use the Distance Formula to show that the corresponding sides are congruent.



$WD = \sqrt{(-7 - (-5))^2 + (-4 - (-1))^2}$

$= \underline{\qquad}$ or $\sqrt{13}$

$ML = \sqrt{(4 - 1)^2 + (-7 - (-5))^2}$

$= \underline{\qquad}$ or $\sqrt{13}$

$DV = \underline{\qquad}$

$= \sqrt{16 + 1}$ or $\sqrt{17}$

$LP = \sqrt{(1 - 2)^2 + (-5 - (-1))^2}$

$= \underline{\qquad}$ or $\sqrt{17}$

$VW = \sqrt{(-1 - (-7))^2 + (-2 - (-4))^2}$

$= \sqrt{36 + 4}$ or $\underline{\qquad}$

$PM = \sqrt{(2 - 4)^2 + (-1 - (-7))^2}$

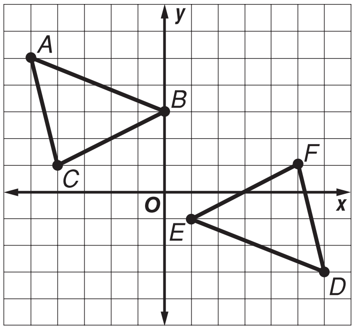
$= \underline{\qquad}$ or $\underline{\qquad}$

$WD = \underline{\qquad}$, $DV = \underline{\qquad}$, and $VW = \underline{\qquad}$.

By definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle WDV \cong \underline{\qquad}$ by $\underline{\qquad}$.

Check Your Progress

Determine whether $\triangle ABC \cong \triangle DEF$ for $A(-5, 5)$, $B(0, 3)$, $C(-4, 1)$, $D(6, -3)$, $E(1, -1)$, and $F(5, 1)$. Explain.



BUILD YOUR VOCABULARY (page 87)

In a triangle, the ______ formed by ______ is the **included angle** for those two sides.

Postulate 4.2 Side-Angle-Side Congruence (SAS)
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

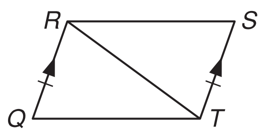
EXAMPLE Use SAS in Proofs

1 **Write a flow proof.**

Given: $\overline{RQ} \parallel \overline{TS}$

$\overline{RQ} \cong \overline{TS}$

Prove: $\triangle QRT \cong \triangle STR$

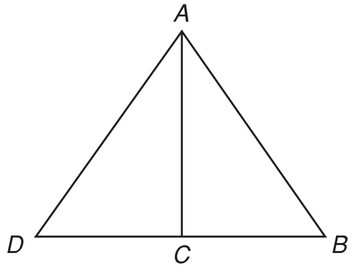


Proof:

- $\overline{RQ} \parallel \overline{TS}$ (Given) → $\angle QRT \cong \angle STR$ (Alt. int. $\angle$s are $\cong$.) → $\triangle QRT \cong \triangle STR$
- $\overline{RQ} \cong \overline{TS}$ (Given) → $\triangle QRT \cong \triangle STR$
- $\overline{RT} \cong \overline{TR}$ (Reflexive Property) → $\triangle QRT \cong \triangle STR$

$\triangle QRT \cong \triangle STR$ (SAS)

Check Your Progress Write a flow proof.



Given: C is the midpoint of $\overline{DB}$; $\angle ACB \cong \angle ACD$.

Prove: $\triangle ABC \cong \triangle ADC$

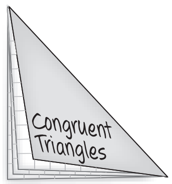
Proof:

______

FOLDABLES™

ORGANIZE IT

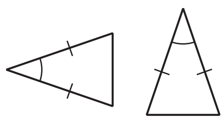
On the page for Lesson 4-4, copy two sample flow proofs to show how to use SSS and SAS to prove triangles congruent.



EXAMPLE Identify Congruent Triangles

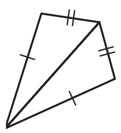
4 **Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.**

a.



Two sides of each triangle are ______. The included angles are congruent. Therefore, the triangles are congruent by ______.

b.

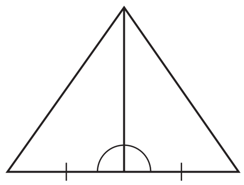


Each pair of ______ sides are congruent. Two are given and the third is congruent by the ______ Property.

So the triangles are congruent by ______.

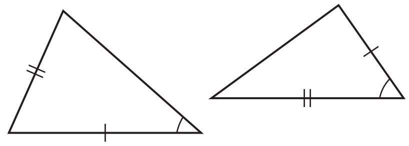
Check Your Progress **Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.**

a.



______

b.



______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–5 Proving Congruence – ASA, AAS

Standard 5.0 **Students prove that triangles are congruent** or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

Main Ideas

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.

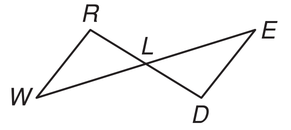
Postulate 4.3 Angle-Side-Angle Congruence (ASA)
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.

Build Your Vocabulary (page 87)

The side of a triangle that is contained in each of two angles is called the **included side** for the two angles.

EXAMPLE Use ASA in Proofs

1 **Write a paragraph proof.**



Given: L is the midpoint of $\overline{WE}$; $\overline{WR} \parallel \overline{ED}$.

Prove: $\triangle WRL \cong \triangle EDL$

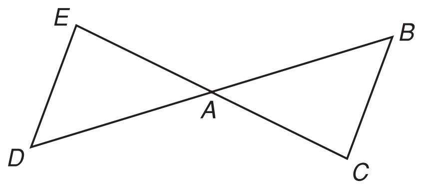
Proof:

____ $\cong$ ____ because ____ angles are congruent. By the Midpoint Theorem, ____ $\cong$ ____.

Since vertical angles are congruent, ____ $\cong$ ____.

$\triangle WRL \cong$ ____ by ASA.

Check Your Progress Write a paragraph proof.



Given: $\overline{DE} \parallel \overline{CB}$; A is the midpoint of $\overline{EC}$.

Prove: $\triangle DEA \cong \triangle CBA$

Proof:

Theorem 4.5 Angle-Angle-Side Congruence (AAS)
If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

REMEMBER IT

When triangles overlap, it may be helpful to draw each triangle separately.

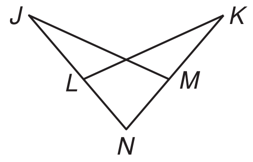
EXAMPLE Use AAS in Proofs

1 Write a flow proof.

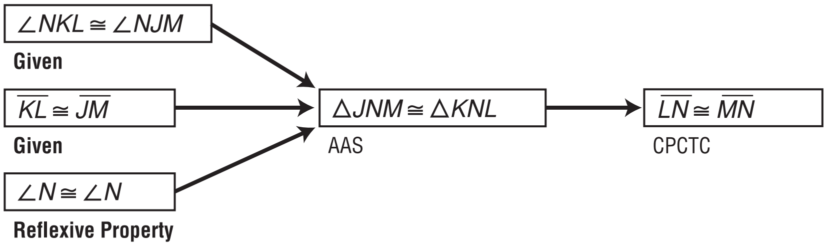
Given: $\angle NKL \cong \angle NJM$

$\overline{KL} \cong \overline{JM}$

Prove: $\overline{LN} \cong \overline{MN}$



Proof:



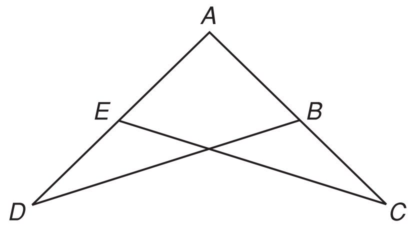
Check Your Progress

Write a flow proof.

Given: $\angle ADB \cong \angle ACE$

$\overline{EC} \cong \overline{BD}$

Prove: $\angle AEC \cong \angle ABD$



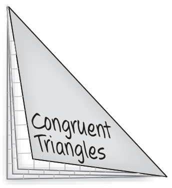
Proof:

EXAMPLE Determine if Triangles are Congruent

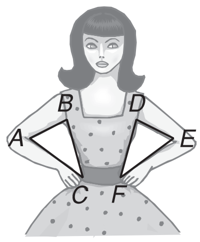
FOLDABLES™

ORGANIZE IT

On the page for Lesson 4-5, draw a picture to show how ASA is different from AAS.



3 STANCES **When Ms. Gomez puts her hands on her hips, she forms two triangles with her upper body and arms. Suppose her arm lengths *AB* and *DE* measure 9 inches, and *AC* and *EF* measure 11 inches. Also suppose that you are given that $\overline{BC} \cong \overline{DF}$. Determine whether $\triangle ABC \cong \triangle EDF$. Justify your answer.**



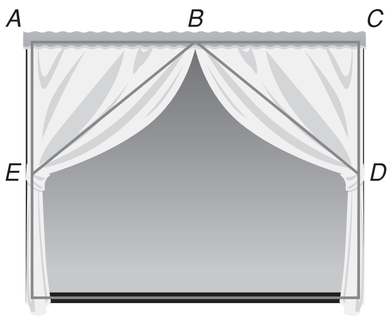
Since ______ $= DE =$ ______, $\overline{AB} \cong$ ______. Likewise,

$AC = EF =$ ______, $\overline{AC} \cong$ ______. We are given $\overline{BC} \cong \overline{DF}$.

Check each possibility using the five methods you know.

We are given information about ______. Since all three pairs of corresponding sides of the triangles are congruent, $\triangle ABC \cong \triangle EDF$ by ______.

Check Your Progress The curtain decorating the window forms 2 triangles at the top. *B* is the midpoint of $\overline{AC}$. $AE = 13$ inches and $CD = 13$ inches. *BE* and *BD* each use the same amount of material, 17 inches. Determine whether $\triangle ABE \cong \triangle CBD$. Justify your answer.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–6 Isosceles Triangles

Standard 4.0 Students prove basic theorems involving congruence and similarity. (Key)

MAIN IDEAS

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

BUILD YOUR VOCABULARY (pages 86–87)

(pages 86–87)

In an ______ triangle, the angle formed by the congruent sides is called the **vertex angle**.

The two angles formed by the base and one of the ______ sides are called **base angles**.

Theorem 4.9 Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

WRITE IT

State the four ways to prove triangles are congruent.

______

______

______

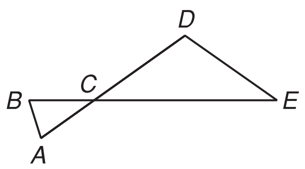
______

EXAMPLE Find the Measure of a Missing Angle

1 TEST EXAMPLE **If $\overline{DE} \cong \overline{CD}$, $\overline{BC} \cong \overline{AC}$, and $m\angle CDE = 120$, what is the measure of $\angle BAC$?**

A 45.5 **C** 68.5

B 57.5 **D** 75



Read the Test Item

$\triangle CDE$ is isosceles with base $\overline{CE}$. Likewise, $\triangle CBA$ is isosceles with base $\overline{BA}$. The base angles of $\triangle CDE$ are congruent.

Solve the Test Item

Let $x = m\angle DEC = m\angle DCE$.

$m\angle DEC + m\angle DCE + m\angle CDE = 180$	Angle Sum Theorem
____ + ____ + ____ $= 180$	Substitution
____ + ____ $= 180$	Add.

$\square = \square$		Subtract 120 from each side.
$x = \square$		Divide each side by 2.

So, $m\angle DEC = m\angle DCE = \square$.

$\angle DCE$ and $\angle BCA$ are vertical angles, so they have equal measures.

$m\angle DCE = m\angle BCA$	Definition of vertical angles
$\square = m\angle BCA$	Substitution

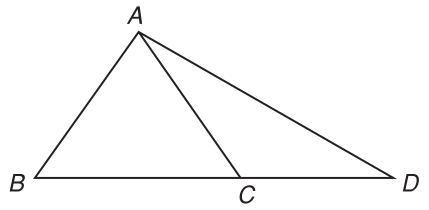
The base angles of $\triangle CBA$ are congruent.
Let $y = m\angle CBA = m\angle BAC$.

$m\angle CBA + m\angle BAC + m\angle BCA = 180$	Angle Sum Theorem
$y + y + 30 = 180$	Substitution
$\square + 30 = 180$	Add.
$2y = \square$	Subtract 30 from each side.
$y = \square$	Divide each side by 2.

The measure of $\angle BAC$ is $\square$. The answer is D.

Check Your Progress

If $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $m\angle ABC = 80$, what is the measure of $\angle ADC$?



Theorem 4.10
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

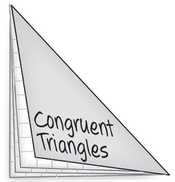
Corollary 4.3
A triangle is equilateral if and only if it is equiangular.

Corollary 4.4
Each angle of an equilateral triangle measures 60°.

FOLDABLES™

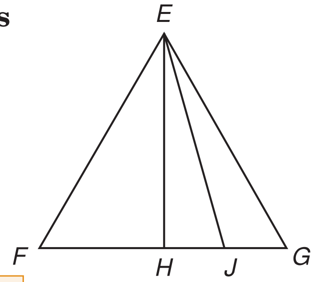
ORGANIZE IT

On the page for Lesson 4-6, list the properties and theorems you learned about each type of triangle.



EXAMPLE Use Properties of Equilateral Triangles

2 **$\triangle EFG$ is equilateral, and $\overline{EH}$ bisects $\angle E$. $\overline{EJ}$ bisects $\angle HEG$.**



a. Find $m\angle HEJ$ and $m\angle EJH$.

Each angle of an equilateral triangle measures ____.

So, $m\angle FEH$ + ____ = ____. Since the angle was ____, $m\angle FEH = m\angle HEG$ = ____.

EJ bisects ____, so $m\angle HEJ = m\angle JEG$ = ____.

$m\angle EJH = m\angle GEJ + m\angle EGJ$	Exterior Angle Theorem
= ____ + ____	Substitution
= ____	Add.

So, $m\angle HEJ$ = ____, $m\angle EJH$ = ____.

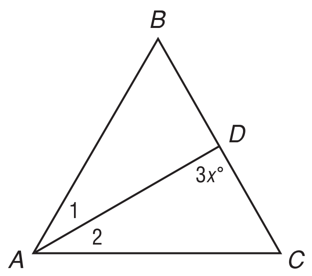
b. Find $m\angle EJG$.

$\angle EJH$ and $\angle EJG$ form a ____.

$\angle EJH$ and $\angle EJG$ = ____	Def. of linear pairs
____ $+ m\angle EJG = 180$	Substitution.
$m\angle EJG$ = ____	Subtract.

Check Your Progress **$\triangle ABC$ is an equilateral triangle. $\overline{AD}$ bisects $\angle BAC$.**

a. Find x.



b. Find $m\angle ADB$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

4–7 Triangles and Coordinate Proof

Standard 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles. (Key)

MAIN IDEAS

- Position and label triangles for use in coordinate proofs.
- Write coordinate proofs.

KEY CONCEPT

Placing Figures on the Coordinate Plane

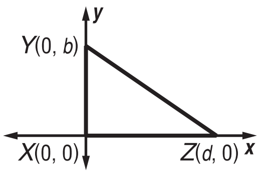
1. Use the origin as a vertex or center of the figure.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

BUILD YOUR VOCABULARY (page 86)

Coordinate proof uses figures in the ______ and ______ to prove geometric concepts.

EXAMPLE Position and Label a Triangle

1 Position and label right triangle *XYZ* with leg $\overline{XZ}$ *d* units long on the coordinate plane.



Use the ______ as vertex *X* of the triangle. Place the base of the triangle along the positive ______.

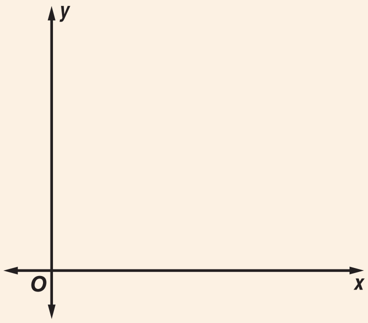
Position the triangle in the ______ quadrant.

Since *Z* is on the *x*-axis, its *y*-coordinate is ______.

Its *x*-coordinate is ______ because the base is *d* units long.

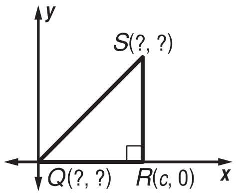
Since triangle *XYZ* is a right triangle the *x*-coordinate of *Y* is ______. We cannot determine the *y*-coordinate so call it ______.

Check Your Progress Position and label equilateral triangle *ABC* with side $\overline{BC}$ *w* units long on the coordinate plane.



EXAMPLE **Find the Missing Coordinates**

2 **Name the missing coordinates of isosceles right triangle QRS.**

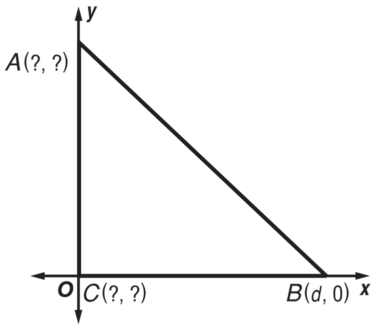


Q is on the origin, so its coordinates are ______. The x-coordinate of S is ______.

The y-coordinate for S is the distance from R to S. Since $\triangle QRS$ is an isosceles right triangle, ______.

The distance from Q to R is ______ units. The distance from R to S must be the same. So, the coordinates of S are ______.

Check Your Progress Name the missing coordinates of isosceles right $\triangle ABC$.



EXAMPLE **Coordinate Proof**

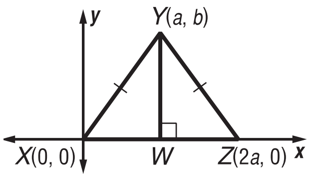
3 **Write a coordinate proof to prove that the segment that joins the vertex angle of an isosceles triangle to the midpoint of its base is perpendicular to the base.**

REVIEW IT

The vertex angle of an isosceles triangle is the angle formed by the congruent sides. (*Lesson 4-6*)

Given: $\triangle XYZ$ is isosceles.
$\overline{XW} \cong \overline{WZ}$

Prove: $\overline{YW} \perp \overline{XZ}$



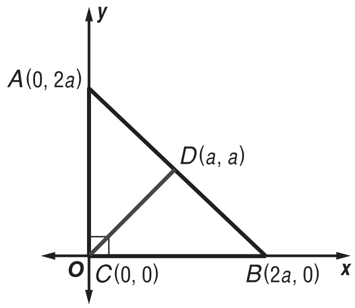
Proof:

By the ______ Formula, the coordinates of W, the midpoint of $\overline{XZ}$, are $\left(\frac{0 + 2a}{2}, \frac{0 + 0}{2}\right)$ or ______.

The slope of $\overline{YW}$ is $\left(\frac{0 - b}{a - a}\right)$ or ______.

The slope of $\overline{XZ}$ is $\left(\frac{0 - 0}{0 - 2a}\right)$ or ______, therefore, ______.

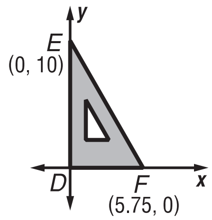
Check Your Progress Write a coordinate proof to prove that the segment drawn from the right angle to the midpoint of the hypotenuse of an isosceles right triangle is perpendicular to the hypotenuse.



Proof:

EXAMPLE Classify Triangles

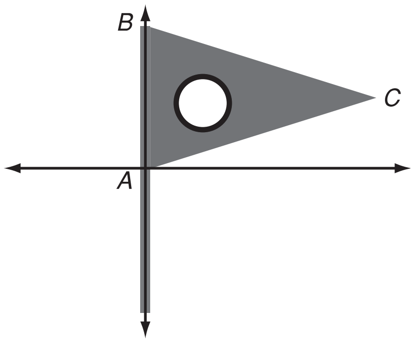
4 DRAFTING **Write a coordinate proof to prove that the outside of this drafter's tool is shaped like a right triangle. The length of one side is 10 inches and the length of another side is 5.75 inches.**



Proof:

The slope of $\overline{ED}$ is $\left(\frac{10 - 0}{0 - 0}\right)$ or ______. The slope of $\overline{DF}$ is $\left(\frac{0 - 0}{0 - 5.75}\right)$ or ______, therefore ______. $\triangle DEF$ is a ______ triangle. The drafter's tool is shaped like a right triangle.

Check Your Progress Write a coordinate proof to prove this flag is shaped like an isosceles triangle. The length is 16 inches and the height is 10 inches.

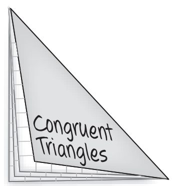


Proof:

FOLDABLES™

ORGANIZE IT

Suppose a triangle is in the coordinate plane. Explain how you can classify the triangle by its sides if you know the coordinates of all three vertices. Include the explanation on the page for Lesson 4-7.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 4

Bringing It All Together

Study Guide

FOLDABLES™	Vocabulary PuzzleMaker	Build your Vocabulary
Use your **Chapter 4 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 4, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 86–87)* to help you solve the puzzle.

4-1

Classifying Triangles

Find x and the measure of each side of the triangle.

1. $\triangle ABC$ is equilateral with $AB = 3x - 15$, $BC = 2x - 4$, and $CA = x + 7$.

2. $\triangle DEF$ is isosceles, $\angle D$ is the vertex angle, $DE = x + 5$, $DF = 5x - 7$ and $EF = 2x - 1$.

3. Find the measures of the sides of $\triangle RST$ and classify the triangle by its sides. Triangle RST has vertices $R(2, -2)$, $S(0, 1)$, and $T(2, 4)$.

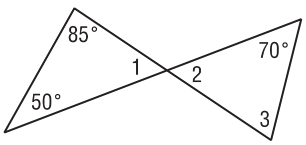
4-2

Angles of Triangles

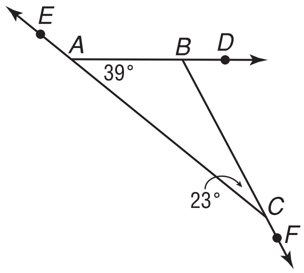
Find the measure of each angle.

4. $m\angle 1$

5. $m\angle 3$



Find the measure of each angle without using a protractor.



6. $\angle DBC$

7. $\angle ABC$

8. $\angle ACF$

9. $\angle EAB$

4-3

Congruent Triangles

Complete each congruence statement if $\triangle TSR \cong \triangle WVU$.

10. $\angle R \cong$

11. ____ $\cong W$

12. $\angle S \cong$

13. $\overline{RT} \cong$

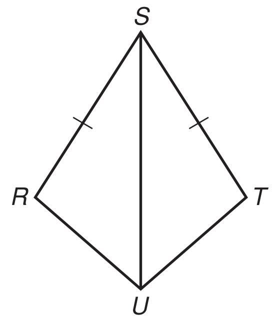
14. ____ $\cong \overline{VU}$

15. ____ $\cong \overline{WV}$

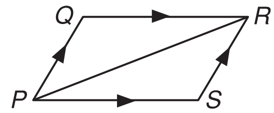
4-4

Proving Congruence – SSS, SAS

16. In quadrilateral $RSTU$, $\overline{RS} \cong \overline{TS}$ and $\overline{SU}$ bisects $\angle RST$. Name the postulate that could be used to prove $\triangle RSU \cong \triangle TSU$.



17. In quadrilateral $PQRS$, $\overline{QR} \parallel \overline{SP}$ and $\overline{PQ} \parallel \overline{RS}$.
Name the postulate that could be used to prove $\triangle PQR \cong \triangle RSP$.

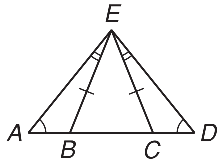


4-5

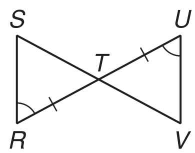
Proving Congruence – ASA, AAS

Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write *not possible*.

18.

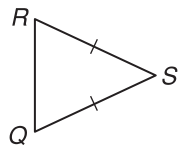


19. T is the midpoint of $\overline{RU}$.



4-6 Isosceles Triangle

Refer to the figure.



20. What kind of triangle is $\triangle QRS$?

21. Name the legs of $\triangle QRS$.

22. Name the base of $\triangle QRS$.

23. Name the vertex angle of $\triangle QRS$.

24. Name the base angles of $\triangle QRS$.

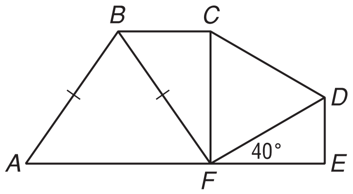
$\triangle ABF$ is equilateral, and $\overline{AE} \perp \overline{CF}$. Find each measure.

25. $m\angle CFD$

26. $m\angle BFC$

27. $m\angle ABF$

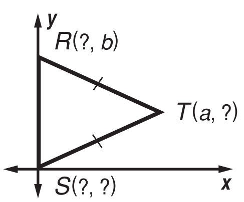
28. $m\angle A$



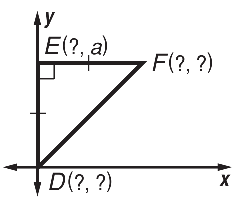
4-7 Triangles and Coordinate Proof

Find the missing coordinates of each triangle.

29.



30.



CHAPTER 4

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 4.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 4 Practice Test on page 261 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 4 Study Guide and Review on pages 256–260 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 261 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 4 Foldable.
- Then complete the Chapter 4 Study Guide and Review on pages 256–260 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 4 Practice Test on page 261 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 5

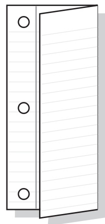
Relationships in Triangles

FOLDABLES™

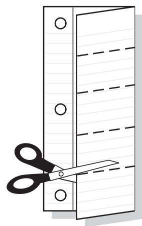
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of notebook paper.

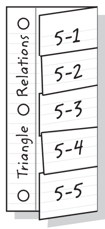
STEP 1 **Fold** lengthwise to the holes.



STEP 2 **Cut** five tabs.



STEP 3 **Label** the edge. Then label the tabs using lesson numbers.



NOTE-TAKING TIP: When you take notes, write concise definitions in your own words. Add examples that illustrate the concepts.

CHAPTER 5

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
altitude			
centroid			
circumcenter [SUHR-kuhm-SEN-tuhr]			
concurrent lines			
incenter			
indirect proof			

Vocabulary Term	Found on Page	Definition	Description or Example
indirect reasoning			
median			
orthocenter [OHR-thoh-CEN-tuhr]			
perpendicular bisector			
point of concurrency			
proof by contradiction			

5–1 Bisectors, Medians, and Altitudes

Standard 12.0 Students find and use measures of sides and of interior and exterior angles **of triangles** and polygons **to** classify figures and **solve problems.** (Key)

MAIN IDEAS

- Identify and use perpendicular bisectors and angle bisectors in triangles.
- Identify and use medians and altitudes in triangles.

BUILD YOUR VOCABULARY (pages 116–117)

A **perpendicular bisector** of a side of a triangle is a line, segment, or ray that passes through the midpoint of the side and is ________ to that side.

When three or more lines intersect at a common point, the lines are called **concurrent lines**, and their point of ________ is called the **point of concurrency**. The point of concurrency of the ________ bisectors of a triangle is called the **circumcenter**.

Theorem 5.1
Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Theorem 5.2
Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Theorem 5.3 Circumcenter Theorem The circumcenter of a triangle is equidistant from the vertices of the triangle.

WRITE IT

State the Angle Sum Theorem. *(Lesson 4-2)*

____

____

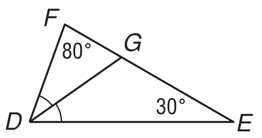
____

____

EXAMPLE Use Angle Bisectors

1 **Given:** $m\angle F = 80$ and $m\angle E = 30$
$\overline{DG}$ bisects $\angle EDF$

Prove: $m\angle DGE = 115$



Proof:

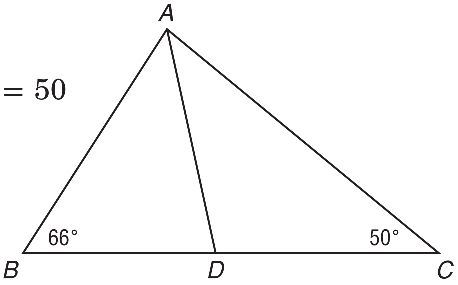
Statements	Reasons
1. $m\angle F = 80$, $m\angle E = 30$, and $\overline{DG}$ bisects $\angle EDF$.	**1.** Given
2. $m\angle EDF + m\angle E + m\angle F = 180$	**2.** ________

Statements	Reasons
3.	**3.** Substitution
4. $m\angle EDF = 180 - 110 = 70$	**4.** Subtraction Property
5. $m\angle GDE = 35$	**5.** Definition of angle bisector
6. $m\angle GDE + m\angle E + m\angle DGE = 180$	**6.** Angle Sum Theorem
7. $35 + 30 + m\angle DGE = 180$	**7.** Substitution
8.	**8.** Subtraction Property

Check Your Progress

Given: $m\angle B = 66$ and $m\angle C = 50$
$\overline{AD}$ bisects $\angle BAC$.

Prove: $m\angle ADC = 98$



Proof:

Statements	Reasons
1. $m\angle B = 66$, $m\angle C = 50$, and $\overline{AD}$ bisects $\angle BAC$.	**1.** Given
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**
7.	**7.**
8.	**8.**

BUILD YOUR VOCABULARY (pages 116–117)

The angle bisectors of a triangle are concurrent, and their point of concurrency is called the **incenter** of a triangle.

A **median** is a segment whose endpoints are a vertex of a triangle and the ________ of the side opposite the ________. The point of concurrency for the medians of a triangle is called a **centroid**.

Theorem 5.4
Any point on the angle bisector is equidistant from the sides of the angle.

Theorem 5.5
Any point equidistant from the sides of an angle lies on the angle bisector.

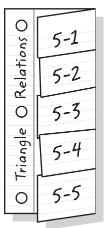
Theorem 5.6 Incenter Theorem
The incenter of a triangle is equidistant from each side of the triangle.

Theorem 5.7 Centroid Theorem
The centroid of a triangle is located two-thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

FOLDABLES™

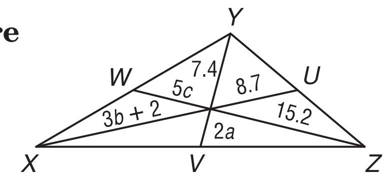
ORGANIZE IT

Under the tab for Lesson 5-1, draw separate pictures that show the centroid, circumcenter, incenter, and orthocenter. Write a description for each.



EXAMPLE Segment Measures

1 **ALGEBRA** **Points U, V, and W are midpoints of $\overline{YZ}$, $\overline{ZX}$, and $\overline{XY}$, respectively. Find a, b, and c.**



Find a.

$VY = 2a + 7.4$	Segment Addition Postulate
________ $= \frac{2}{3}VY$	Centroid Theorem
$7.4 = \frac{2}{3}(2a + 7.4)$	Substitution
________ $= 4a +$ ________	Multiply each side by 3 and simplify.
________ $= a$	Subtract 14.8 from each side and divide by 4.

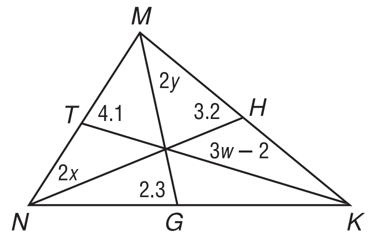
Find b.

$XU = 3b + 2 + 8.7$	Segment Addition Postulate
$3b + 2 = \frac{2}{3}XU$	______
$3b + 2 = \frac{2}{3}(3b + 2 + 8.7)$	Substitution
______ $=$ ______	Multiply each side by 3 and simplify.
$b \approx$ ______	Solve for b.

Find c.

$WZ =$ ______ $+$ ______	Segment Addition Postulate
$15.2 = \frac{2}{3}WZ$	Centroid Theorem
$15.2 = \frac{2}{3}(5c + 15.2)$	Substitution
______ $= c$	Simplify and solve for c.

Check Your Progress Points T, H, and G are the midpoints of $\overline{NM}$, $\overline{MK}$, and $\overline{NK}$, respectively. Find w, x, and y.



BUILD YOUR VOCABULARY (pages 116–117)

An **altitude** of a triangle is a segment from a ______ to the line containing the opposite side and ______ to the line containing that side.

The intersection point of the ______ of a triangle is called the **orthocenter**.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–2 Inequalities and Triangles

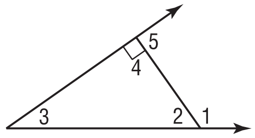
MAIN IDEAS

- Recognize and apply properties of inequalities to the measures of angles of a triangle.
- Recognize and apply properties of inequalities to the relationships between angles and sides of a triangle.

Standard 12.0 Students find and use measures of sides and of interior and **exterior angles of triangles** and polygons **to** classify figures and **solve problems**. (Key)
Standard 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and **exterior angles.**

EXAMPLE Compare Angle Measures

1 Determine which angle has the greatest measure.



Compare $m\angle 3$ to $m\angle 1$.

$m\angle 1 =$ ______ Exterior Angle Theorem

$m\angle 1$ ______ $m\angle 3$ Definition of Inequality

Compare $m\angle 4$ to $m\angle 1$.

$m\angle 1 =$ ______ Exterior Angle Theorem

$m\angle 1$ ______ $m\angle 4$ Definition of Inequality

$\angle 4 \cong \angle 5$ ______

$m\angle 4 = m\angle 5$ Definition of $\cong$ $\angle s$

$m\angle 1 > m\angle 5$ ______

Compare ______ to ______.

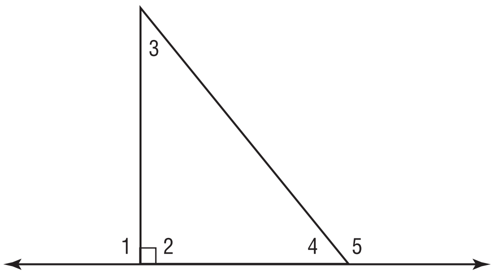
By the Exterior Angle Theorem, $m\angle 5 = m\angle 2 + m\angle 3$.

By the definition of inequality, $m\angle 5$ ______ $m\angle 2$.

Since we know that $m\angle 1 > m\angle 5$, by the Transitive Property, $m\angle 1$ ______ $m\angle 2$. Therefore, ______ has the greatest measure.

Check Your Progress

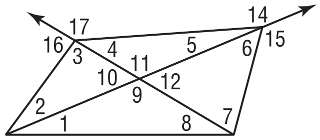
Determine which angle has the greatest measure.



Theorem 5.8 Exterior Angle Inequality Theorem
If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.

EXAMPLE Exterior Angles

2 **Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.**



a. measures less than $m\angle 14$

By the Exterior Angle Inequality Theorem, $m\angle 14 > m\angle 4$, $m\angle 14 > m\angle 11$, $m\angle 14 > m\angle 2$, and $m\angle 14 >$ ____ + ____ .

Since $\angle 11$ and $\angle 9$ are ____ angles, they have equal measures, so $m\angle 14 > m\angle 9$. $> m\angle 9 > m\angle 6$ and $m\angle 9 > m\angle 7$, so $m\angle 14 > m\angle 6$ and $m\angle 14 > m\angle 7$.

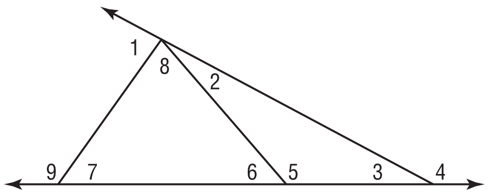
Thus, the measures of ____ are all less than $m\angle 14$.

b. measures greater than $m\angle 5$

By the Exterior Angle Inequality Theorem, $m\angle 5 < m\angle 10$, $m\angle 5 < m\angle 16$, $m\angle 5 < m\angle 12$, $m\angle 5 < m\angle 15$, and $m\angle 5 < m\angle 17$.

Thus, the measures of ____ are each greater than $m\angle 5$.

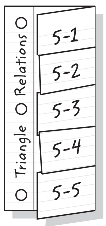
Check Your Progress Use the Exterior Angle Inequality Theorem to list all angles whose measures are greater than $m\angle 8$.



FOLDABLES™

ORGANIZE IT

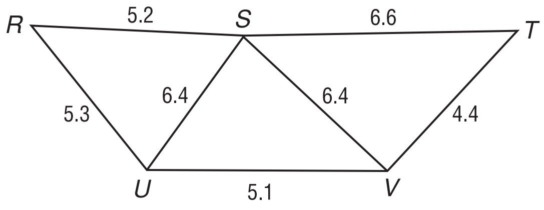
Under the tab for Lesson 5-2, summarize the proof of Theorem 5.9 using your own words in paragraph form.



Theorem 5.9
If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

EXAMPLE Side-Angle Relationships

3 **Determine the relationship between the measures of the given angles.**



a. ∠*RSU* and ∠*SUR*

The side opposite ∠*RSU* is ________ than the side opposite ∠*SUR*, so *m*∠*RSU* > ________.

b. ∠*TSV* and ∠*STV*

The side opposite ∠*TSV* is shorter than the side opposite ________, so *m*∠*TSV* ________ *m*∠*STV*.

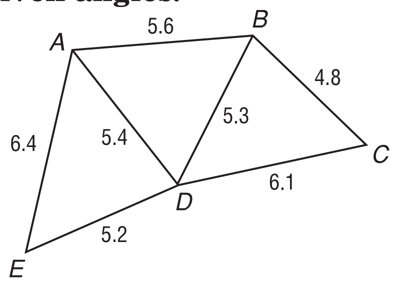
Check Your Progress **Determine the relationship between the measures of the given angles.**

a. ∠*ABD*, ∠*DAB*

____

b. ∠*AED*, ∠*EAD*

____



Theorem 5.10
If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

REMEMBER IT

The longer and shorter sides must be opposite the larger and smaller angles respectively, not adjacent to them.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–3 Indirect Proof

Standard 2.0 Students write geometric proofs, including proofs by contradiction. (Key)

MAIN IDEAS

- Use indirect proof with algebra.
- Use indirect proof with geometry.

BUILD YOUR VOCABULARY (pages 116–117)

When using **indirect reasoning**, you assume that the ______ is false and then show that this assumption leads to a contradiction of the ______, or some other accepted fact, such as a definition, postulate, theorem, or corollary. A proof of this type is called an **indirect proof** or **proof by contradiction**.

KEY CONCEPT

Steps for Indirect Proof:

(1) Assume that the conclusion is false.

(2) Show that this assumption leads to a contradiction of the hypothesis or of some other fact.

(3) Point out that the original conclusion must be true, since this is the only way to avoid the contradiction.

EXAMPLE Stating Conclusions

1 State the assumption you would make to start an indirect proof of each statement.

a. $\overline{EF}$ is not a perpendicular bisector.

______

b. $3x = 4y + 1$

______

c. If B is the midpoint of $\overline{LH}$ and $\overline{LH} = 26$, then $\overline{BH}$ is congruent to $\overline{LB}$.

______

Check Your Progress **State the assumption you would make to start an indirect proof of each statement.**

a. $\overline{AB}$ is not an altitude.

______

b. $a = \frac{1}{2}b - 4$

______

EXAMPLE Algebraic Proof

2 Write an indirect proof.

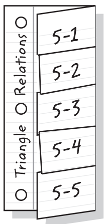
Given: $\frac{1}{2y + 4} = 20$

Prove: $y \neq -2$

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 5-3, list the steps for writing an indirect proof. Then, give an example of an indirect proof.



Indirect Proof:

STEP 1 Assume that ______.

STEP 2 Substitute -2 for y in the equation $\frac{1}{2y + 4} = 20$.

$\frac{1}{2y + 4} = 20$		
______ $= 20$	Substitution	
$\frac{1}{-4 + 4} = 20$	Multiply.	
$\frac{1}{0} = 20$	Add.	

This is a contradiction because the ______ cannot be 0.

STEP 3 The assumption leads to a contradiction. Therefore, the assumption that $y = -2$ must be ______, which means that $y \neq -2$ must be ______.

Check Your Progress Write an indirect proof.

Given: $\frac{1}{2a + 6} \leq 12$

Prove: $a \neq -3$

Indirect Proof:

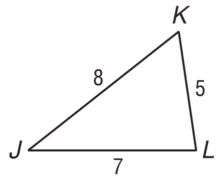
STEP 1 ______

STEP 2 ______

STEP 3

EXAMPLE **Geometry Proof**

3 **Write an indirect proof.**



Given: $\triangle JKL$ with side lengths 5, 7, and 8 as shown.

Prove: $m\angle K < m\angle L$

Indirect Proof:

STEP 1 Assume that ______.

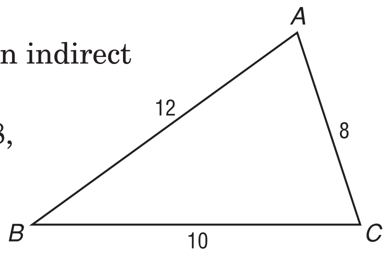
STEP 2 By angle-side relationships, $JL \geq JK$.

By substitution, ______.

This inequality is a false statement.

STEP 3 This contradicts the given side lengths, so the assumption must be false. Therefore, ______.

Check Your Progress Write an indirect proof.



Given: $\triangle ABC$ with side lengths 8, 10, and 12 as shown.

Prove: $m\angle C > m\angle A$

Indirect Proof:

STEP 1

STEP 2

STEP 3

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

5–4 The Triangle Inequality

Standard 6.0 Students know and are able to use the triangle inequality theorem.

Theorem 5.11 Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

MAIN IDEAS

- Apply the Triangle Inequality Theorem.
- Determine the shortest distance between a point and a line.

EXAMPLE Identify Sides of a Triangle

1 **Determine whether the given measures can be lengths of the sides of a triangle.**

a. $6\frac{1}{2}$, $6\frac{1}{2}$, **and** $14\frac{1}{2}$

Because the ______ of two measures is not greater than the length of the ______ side, the sides cannot form a triangle.

$6\frac{1}{2} + 6\frac{1}{2} \overset{?}{>} 14\frac{1}{2}$

______ $\not>$ ______

b. 6.8, 7.2, and 5.1

Check each inequality.

$6.8 + 7.2 \overset{?}{>} 5.1$	$6.8 + 5.1 \overset{?}{>} 7.2$	$5.1 + 7.2 \overset{?}{>} 6.8$
______ > 5.1	______ > 7.2	______ > 6.8

All of the inequalities are ______, so 6.8, 7.2, and 5.1 can be the lengths of the sides of a triangle.

Check Your Progress **Determine whether the given measures can be lengths of the sides of a triangle.**

a. 6, 9, 16

______

b. 14, 16, 27

______

EXAMPLE Determine Possible Side Length

1 **TEST EXAMPLE** **In $\triangle PQR$, $PQ = 7.2$ and $QR = 5.2$. Which measure cannot be PR?**

A 7 **B** 9 **C** 11 **D** 13

Read the Test Item

Let $PR = n$. Solve each inequality to determine the range of values for PR.

Solve the Test Item

$PQ + QR > PR$

$7.2 +$ ______ $> n$

______ $> n$ or $n <$ ______

$PQ + PR > QR$

$7.2 +$ ______ > 5.2

$n >$ ______

$PR + QR > PQ$

$n +$ ______ > 7.2

$n >$ ______

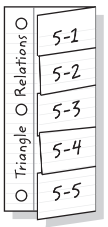
The range of values that fits all three inequalities is ______ $< n <$ ______. The answer is ______.

Check Your Progress In $\triangle XYZ$, $XY = 6$ and $YZ = 9$. Find the range of possible values for XZ.

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 5-4, write the Triangle Inequality Theorem in your own words. Draw and label a triangle to show why the theorem makes sense.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Theorem 5.12
The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Corollary 5.1
The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

5–5 Inequalities Involving Two Triangles

Standard 6.0 Students know and **are able to use the triangle inequality theorem.**

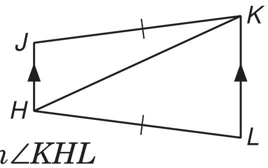
Main Ideas

- Apply the SAS Inequality.
- Apply the SSS Inequality.

Theorem 5.13 SAS Inequality/Hinge Theorem
If two sides of a triangle are congruent to two sides of another triangle and the included angle in one triangle has a greater measure than the included angle in the other, then the third side of the first triangle is longer than the third side of the second triangle.

EXAMPLE Use SAS Inequality in a Proof

1 **Write a two-column proof.**



Given: $\overline{KL} \parallel \overline{JH}$, $JK = HL$

$m\angle JKH + m\angle HKL < m\angle JHK + m\angle KHL$

Prove: $JH < KL$

Proof:

Statements	Reasons
1. $m\angle JKH + m\angle HKL < m\angle JHK + m\angle KHL$	1. Given
2. $m\angle HKL = m\angle JHK$	2.
3. $m\angle JKH + m\angle JHK < m\angle JHK + m\angle KHL$	3. Substitution
4. $m\angle JKH < m\angle KHL$	4. Subtraction Property
5.	5. Given
6. $HK = HK$	6. Reflexive Property
7.	7.

Write It

Explain why the SAS Inequality Theorem is also called the Hinge Theorem.

____

____

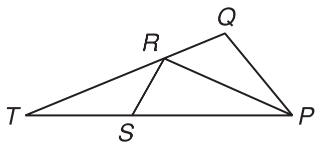
____

____

Theorem 5.14 SSS Inequality
If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.

EXAMPLE Prove Triangle Relationships

2 **Given:** $ST = PQ$
$SR = QR$
$ST = \frac{2}{3}SP$



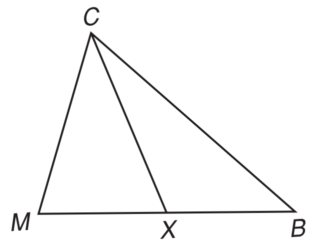
Prove: $m\angle SRP > m\angle PRQ$

Proof:

Statements	Reasons
1. $SR = QR$	**1.** Given
2. $PR = PR$	**2.**
3. $ST = PQ$	**3.** Given
4. $ST = \frac{2}{3}SP$; $SP > ST$	**4.** Given
5. $SP > PQ$	**5.** Substitution
6.	**6.**

Check Your Progress

Given: X is the midpoint of $\overline{MB}$.
$\triangle MCX$ is isosceles.
$CB > CM$



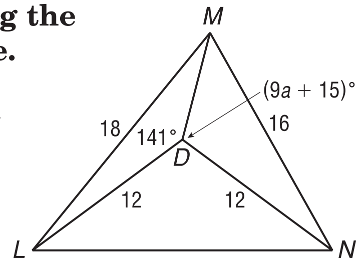
Prove: $m\angle CXB > m\angle CMX$

Proof:

Statements	Reasons
1. X is the midpoint of $\overline{MB}$.	**1.** Given
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**
7.	**7.**

EXAMPLE Relationships Between Two Triangles

3 **Write an inequality using the information in the figure.**



a. Compare $m\angle LDM$ and $m\angle MDN$.

In $\triangle MDL$ and $\triangle MDN$, $\overline{LD} \cong \overline{ND}$, $\overline{MD} \cong \overline{MD}$, and $ML > MN$.

The ______ allows us to conclude that

______ > ______.

b. Find the range of values containing a.

By the SSS Inequality, $m\angle LDM > m\angle MDN$ or $m\angle MDN < m\angle LDM$.

$m\angle MDN < m\angle LDM$ ______

$9a + 15 < 141$ Substitution

______ < ______ Subtract 15 from each side and divide each side by 9.

Also, recall that the measure of any angle is always greater than 0.

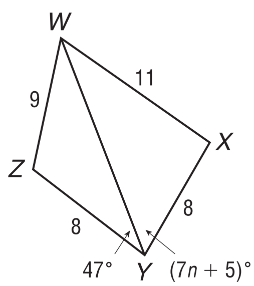
$9a + 15 > 0$

$9a > -15$ Subtract 15 from each side.

______ > ______ or $-\frac{5}{3}$ Divide each side by 9.

The two inequalities can be written as the compound inequality ______.

Check Your Progress **Write an inequality using the information in the figure.**



a. Compare $m\angle WYX$ and $m\angle ZYW$.

______

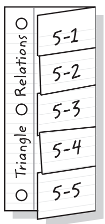
b. Find the range of values containing n.

______

FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 5-5, draw two sets of two triangles each with two pairs of congruent sides. Measure and mark the sides and angles to illustrate the SAS Inequality Theorem and the SSS Inequality Theorem.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 5

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 5 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 116–117)* to help you solve the puzzle.

5-1

Bisectors, Medians, and Altitudes

Fill in the correct word or phrase to complete each sentence.

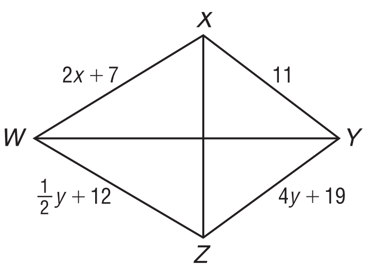
1. A(n) ________ of a triangle is a segment drawn from a vertex of the triangle perpendicular to the line containing the opposite side.

2. The point of concurrency of the three perpendicular bisectors of a triangle is called the ________.

3. Any point in the interior of an angle that is equidistant from the sides of that angle lies on the ________.

4. The vertices of $\triangle PQR$ are $P(0, 0)$, $Q(2, 6)$, and $R(6, 4)$. Find the coordinates of the orthocenter of $\triangle PQR$.

5. If $\overline{XZ}$ is the perpendicular bisector of $\overline{WY}$ and $\overline{WY}$ is the perpendicular bisector of $\overline{XZ}$, find x and y.

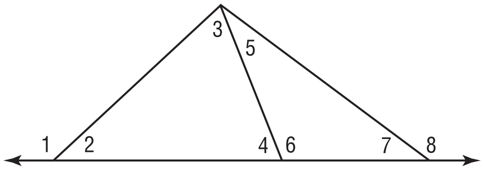


5-2 Inequalities and Triangles

Determine which angle has the greatest measure.

6. $\angle 1, \angle 3, \angle 4$

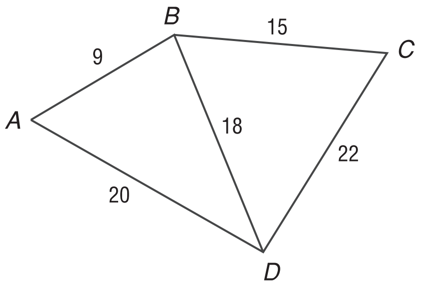
7. $\angle 5, \angle 6, \angle 8$



Determine the relationship between the measures of the given angles.

8. $m\angle ABD, m\angle BDA$

9. $m\angle DBC, m\angle BCD$



5-3 Indirect Proof

Write the assumption you would make to start an indirect proof of each statement.

10. If $3x - 8 = 10$, then $x = 6$.

11. If $m\angle 1 = 100$ and $m\angle 2 = 80$, then $\angle 1$ and $\angle 2$ are supplementary.

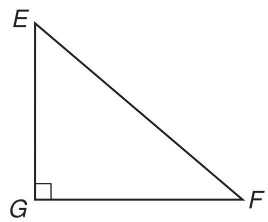
Write the assumption you would make to start an indirect proof.

12. Given: $5x + 4 < 14$

Prove: $x < 2$

13. Given: $\triangle EFG$ is a right triangle. $\angle G$ is a right angle.

Prove: $m\angle E < 90$



5-4

The Triangle Inequality

Determine whether the given measures can be the lengths of a triangle. Write *yes* or *no*.

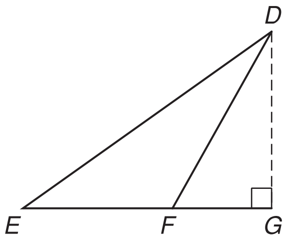
14. 5, 6, 7

15. 6, 8, 10

16. 10, 10, 21

17. 12, 12, 12

Refer to the figure. Determine whether each statement is *true* or *false*.



18. $DE > EF + FD$

19. $EG = EF + FG$

20. The shortest distance from D to $\overleftrightarrow{EG}$ is DF.

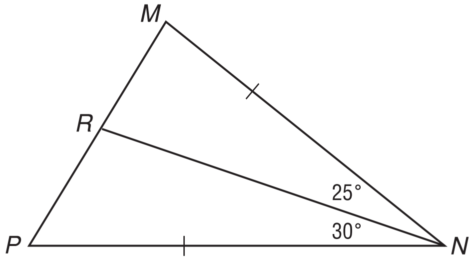
21. The shortest distance from D to $\overleftrightarrow{EG}$ is DG.

5-5

Inequalities Involving Two Triangles

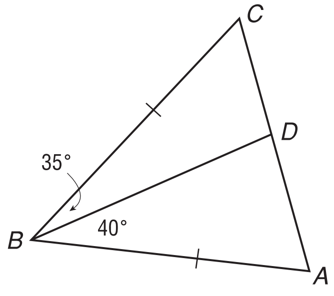
Write an inequality relating the given pair of segment measures.

22.



MR ☐ RP

23.



AD ☐ CD

CHAPTER 5

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 5.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 5 Practice Test on page 313 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 5 Study Guide and Review on pages 310–312 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 313.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 5 Foldable.
- Then complete the Chapter 5 Study Guide and Review on pages 310–312 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 5 Practice Test on page 313.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 6

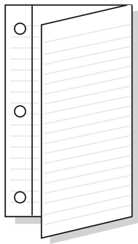
Quadrilaterals

FOLDABLES™

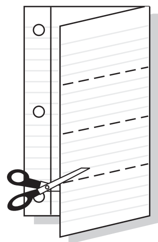
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of notebook paper.

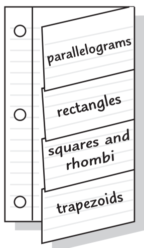
STEP 1 **Fold** lengthwise to the left margin.



STEP 2 **Cut** four tabs.



STEP 3 **Label** the tabs using the lesson concepts.



NOTE-TAKING TIP: When taking notes, always write clear and concise notes so they can be easily read when studying for a quiz or exam.

CHAPTER 6

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 6. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
diagonal			
isosceles trapezoid			
kite			
median			
parallelogram			
rectangle			
rhombus			
square			
trapezoid			

6–1 Angles of Polygons

Standard 12.0 Students find and use measures of sides and **of interior and exterior angles of** triangles and **polygons to classify figures and solve problems.** (Key)

BUILD YOUR VOCABULARY (page 138)

The **diagonals** of a polygon are segments that connect any two nonconsecutive ______.

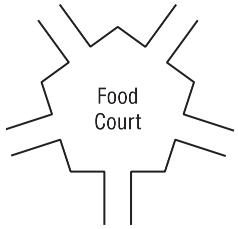
MAIN IDEAS

- Find the sum of the measures of the interior angles of a polygon.
- Find the sum of the measures of the exterior angles of a polygon.

Theorem 6.1 Interior Angle Sum Theorem
If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

EXAMPLE Interior Angles of Regular Polygons

1 ARCHITECTURE **A mall is designed so that five walkways meet at a food court that is in the shape of a regular pentagon. Find the sum of measures of the interior angles of the pentagon.**

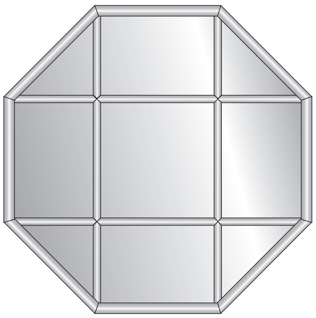


A pentagon is a convex polygon. Use the Angle Sum Theorem.

$S = 180(n - 2)$	Interior Angle Sum Theorem
$= 180(\_\_\_\_ - 2)$	$n =$ ____
$= 180(\_\_\_\_)$ or ____	Simplify.

The sum of the measures of the angles is ______.

Check Your Progress A decorative window is designed to have the shape of a regular octagon. Find the sum of the measures of the interior angles of the octagon.



EXAMPLE Sides of a Polygon

2 The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.

Use the Interior Angle Sum Theorem.

$S = 180(n - 2)$	Interior Angle Sum Theorem
$(135)n = 180(n - 2)$	$S = 135n$
______ $= 180n -$ ______	Distributive Property
$0 = 45n - 360$	Subtract.
______ $=$ ______	Add.
______ $= n$	The polygon has ______ sides.

Write It

Can you use the method in Example 2 if the polygon is not regular? Explain why or why not.

____

____

____

____

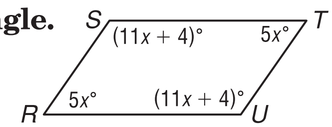
____

____

____

EXAMPLE Interior Angles

3 Find the measure of each interior angle.



Since $n = 4$, the sum of the measures of the interior angles is $180(4 - 2)$ or 360. Write an equation to express the sum of the measures of the interior angles of the polygon.

$360 = m\angle R + m\angle S + m\angle T + m\angle U$

$360 =$ ______ $+$ ______ $+$ ______ $+$ ______

$360 =$ ______

______ $=$ ______

______ $=$ ______

Use the value of x to find the measure of each angle.

$m\angle R =$ ______ or ______

$m\angle S =$ ______ or ______

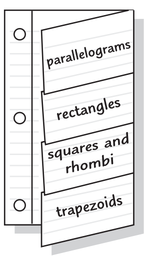
$m\angle T =$ ______ or ______

$m\angle U =$ ______ or ______

FOLDABLES™

Organize It

Sketch a quadrilateral with two pairs of parallel sides. Write the sum of the angle measures below the figure. Include the sketch under the parallelograms tab of your Foldable.



REVIEW IT

What is an exterior angle? (*Lesson 4-2*)

____

____

____

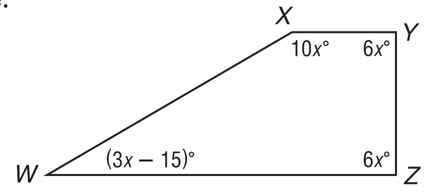
____

____

Check Your Progress

a. The measure of an interior angle of a regular polygon is 144. Find the number of sides in the polygon.

b. Refer to the figure shown. Find the measure of each interior angle.

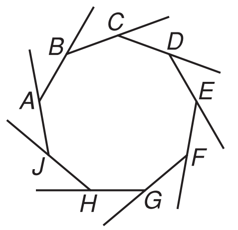


Theorem 6.2 Exterior Angle Sum Theorem
If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

EXAMPLE Exterior Angles

4 **Find the measures of an exterior angle and an interior angle of convex regular nonagon *ABCDEFGHJ*.**

At each vertex, extend a side to form one exterior angle. The sum of the measures of the exterior angles is 360. A convex regular nonagon has 9 congruent exterior angles.



$9n = 360$

$n =$ ______ Divide each side by 9.

Since each exterior angle and its corresponding interior angle form a linear pair, the measure of the interior angle is

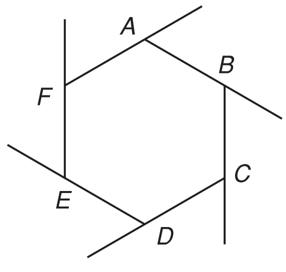
$180 -$ ______ or ______.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress Find the measures of an exterior angle and an interior angle of convex regular hexagon *ABCDEF*.



6–2 Parallelograms

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, **the properties of quadrilaterals,** and the properties of circles. (Key)

MAIN IDEAS

- Recognize and apply properties of the sides and angles of parallelograms.
- Recognize and apply properties of the diagonals of parallelograms.

BUILD YOUR VOCABULARY (page 138)

A quadrilateral with ______ opposite sides is called a **parallelogram.**

Theorem 6.3
Opposite sides of a parallelogram are congruent.

Theorem 6.4
Opposite angles in a parallelogram are congruent.

Theorem 6.5
Consecutive angles in a parallelogram are supplementary.

Theorem 6.6
If a parallelogram has one right angle, it has four right angles.

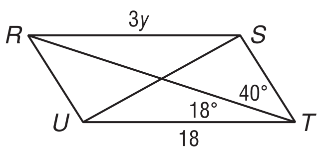
KEY CONCEPT

Parallelogram A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

FOLDABLES™ Write the properties of parallelograms under the parallelograms tab.

EXAMPLE Properties of Parallelograms

1 Quadrilateral *RSTU* is a parallelogram. Find $m\angle URT$, $m\angle RST$, and *y*.



First, find $m\angle URT$.

$\angle URT \cong \angle STR$ — You know that if parallel lines are cut by a transversal, alternate interior ∠s are congruent.

$m\angle URT = m\angle STR$ — Congruent angles

$m\angle URT =$ ______ — Substitution

Now, find $m\angle RST$.

$m\angle STU =$ ______ $+$ ______ or ______ — Angle Addition Postulate

$m\angle RST + m\angle STU =$ ______ — Consecutive ∠s in ▱ are supplementary.

$m\angle RST + 58 =$ ______ — Substitution

$m\angle RST =$ ______ — Subtract.

$\overline{RS} \cong$ ______ — Opposite sides of ▱ are ≅.

$RS = TU$ Congruent segments

$3y =$ ___, so $y =$ ___ Substitution; Divide.

$m\angle URT =$ ___, $m\angle RST =$ ___, $y =$ ___

Write It

Write what it means for diagonals to bisect each other.

____

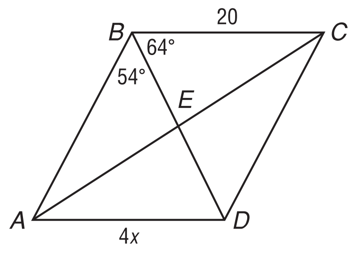
____

____

____

Check Your Progress

ABCD is a parallelogram. Find $m\angle BDC$, $m\angle BCD$, and x.



Theorem 6.7
The diagonals of a parallelogram bisect each other.

Theorem 6.8
The diagonal of a parallelogram separates the parallelogram into two congruent triangles.

EXAMPLE Diagonals of a Parallelogram

2 TEST EXAMPLE What are the coordinates of the intersection of the diagonals of parallelogram *MNPR*, with vertices *M*(–3, 0), *N*(–1, 3), *P*(5, 4), and *R*(3, 1)?

A (2, 4) **B** $\left(\frac{9}{2}, \frac{5}{2}\right)$ **C** (1, 2) **D** $\left(-2, \frac{3}{2}\right)$

Read the Test Item Since the diagonals of a parallelogram bisect each other, the intersection point is the midpoint of $\overline{MP}$ and $\overline{NR}$.

Solve the Test Item Find the midpoint of $\overline{MP}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\quad}{2}, \frac{\quad}{2}\right) = \underline{\quad}$$

The answer is ___.

Homework Assignment

Page(s):

Exercises:

Check Your Progress What are the coordinates of the intersection of the diagonals of parallelogram *LMNO*, with vertices *L*(0, –3), *M*(–2, 1), *N*(1, 5), *O*(3, 1)?

6–3 Tests for Parallelograms

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, **the properties of quadrilaterals,** and the properties of circles. (Key)

MAIN IDEAS

- Recognize the conditions that ensure a quadrilateral is a parallelogram.
- Prove that a set of points forms a parallelogram in the coordinate plane.

Theorem 6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

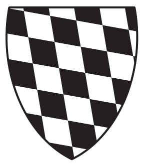
Theorem 6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

EXAMPLE Properties of Parallelograms

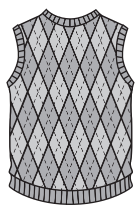
(parallels Example 2 in text)

1 **Some of the shapes in this Bavarian crest appear to be parallelograms. Describe the information needed to determine whether the shapes are parallelograms.**



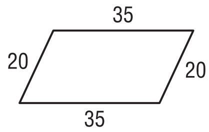
If both pairs of opposite sides are the same length or if one pair of opposite sides is ________________, the quadrilateral is a parallelogram. If both pairs of opposite angles are ____________ or if the ____________ bisect each other, the quadrilateral is a parallelogram.

Check Your Progress The shapes in the vest pictured here appear to be parallelograms. Describe the information needed to determine whether the shapes are parallelograms.



EXAMPLE **Properties of Parallelograms**

2 **Determine whether the quadrilateral is a parallelogram. Justify your answer.**



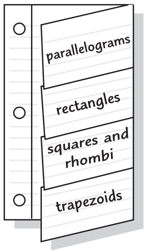
Each pair of opposite sides have the same measure. Therefore, they are ______.

If both pairs of opposite sides of a quadrilateral are ______, the quadrilateral is a parallelogram.

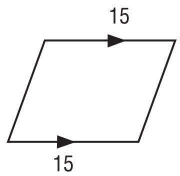
FOLDABLES

ORGANIZE IT

Record the tests for parallelograms under the parallelograms tab.

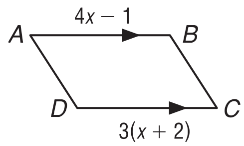


Check Your Progress Determine whether the quadrilateral is a parallelogram. Justify your answer.



EXAMPLE **Find Measures**

3 **Find x so that the quadrilateral is a parallelogram.**



Opposite sides of a parallelogram are congruent.

$\overline{AB} \cong \overline{DC}$		Opposite sides of a ▱ are ≅.
$AB = DC$		Definition of ≅ segments.
______	= ______	Substitution
______	= ______	Distributive Property
______	= ______	Subtract $3x$ from each side.
$x =$	______	Add 1 to each side.

When x is ______, $ABCD$ is a parallelogram.

REVIEW IT

What does it mean for two segments to be congruent? (*Lesson 1-2*)

______

______

______

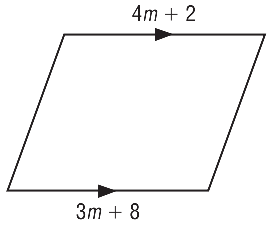
______

______

______

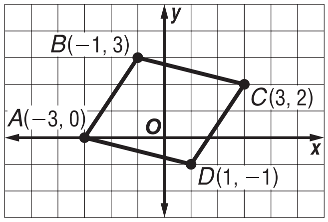
______

Check Your Progress Find m so that the quadrilateral is a parallelogram.



EXAMPLE **Use Slope and Distance**

1 **COORDINATE GEOMETRY** **Determine whether *ABCD* with vertices *A*(–3, 0), *B*(–1, 3), *C*(3, 2), and *D*(1, –1) is a parallelogram. Use the Slope Formula.**



If the opposite sides of a quadrilateral are parallel, then it is a parallelogram.

slope of $\overline{AB} = \frac{3 - 0}{-1 - (-3)}$ or

slope of $\overline{CD} = \frac{-1 - 2}{1 - 3}$ or

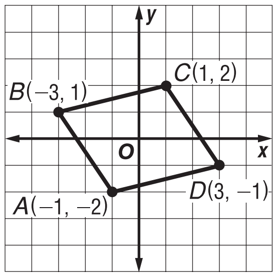
slope of $\overline{AD} = \frac{-1 - 0}{1 - (-3)}$ or

slope of $\overline{BC} = \frac{3 - 2}{-1 - 3}$ or

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, *ABCD* is a parallelogram by definition.

Check Your Progress Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.

A(–1, –2), *B*(–3, 1), *C*(1, 2), *D*(3, –1); Slope Formula



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–4 Rectangles

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, **the properties of quadrilaterals,** and the properties of circles. (Key)

MAIN IDEAS

- Recognize and apply properties of rectangles.
- Determine whether parallelograms are rectangles.

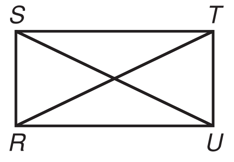
BUILD YOUR VOCABULARY (page 138)

A **rectangle** is a quadrilateral with four ________.

Theorem 6.13 If a parallelogram is a rectangle, then the diagonals are congruent.

EXAMPLE Diagonals of a Rectangle

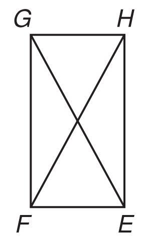
1 **Quadrilateral *RSTU* is a rectangle. If $RT = 6x + 4$ and $SU = 7x - 4$, find x.**



The diagonals of a rectangle are congruent, so $\overline{RT} \cong \overline{SU}$.

$\overline{RT} \cong$ ________	Diagonals of a rectangle are $\cong$.
$RT =$ ________	Definition of congruent segments
________ $=$ ________	Substitution
________ $=$ ________	Subtract $6x$ from each side.
________ $= x$	Add 4 to each side.

Check Your Progress Quadrilateral *EFGH* is a rectangle. If $FH = 5x + 4$ and $GE = 7x - 6$, find x.



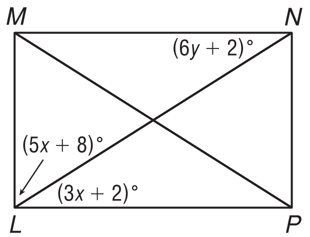
KEY CONCEPT

Properties of a Rectangle

1. Opposite sides are congruent and parallel.
2. Opposite angles are congruent.
3. Consecutive angles are supplementary.
4. Diagonals are congruent and bisect each other.
5. All four angles are right angles.

EXAMPLE Angles of a Rectangle

2 **Quadrilateral *LMNP* is a rectangle. Find *x*.**



$\angle MLP$ is a right angle, so $m\angle MLP = 90$.

$m\angle MLN + m\angle NLP = m\angle MLP$	Angle Addition Postulate
$5x + 8 + 3x + 2 =$ ______	Substitution.
______ $=$ ______	Simplify.
______ $=$ ______	Subtract.
$x =$ ______	Divide each side by 8.

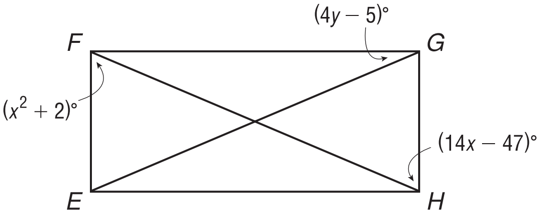
Check Your Progress **Quadrilateral *EFGH* is a rectangle.**

a. Find *x*.

______

b. Find *y*.

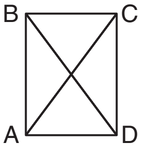
______



Theorem 6.14
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

EXAMPLE Diagonals of a Parallelogram

3 **Kyle is building a barn for his horse. He measures the diagonals of the door opening to make sure that they bisect each other and they are congruent. How does he know that the measure of each corner is 90?**

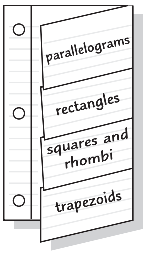


We know that $\overline{AC} \cong \overline{BD}$. A parallelogram with ______ diagonals is a rectangle. Therefore, the corners are ______ angles.

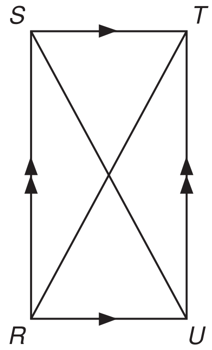
FOLDABLES™

ORGANIZE IT

Record similarities and differences between rectangles and other types of parallelograms under the rectangles tab.

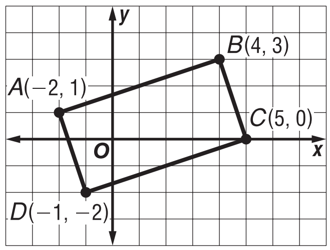


Check Your Progress Max is building a swimming pool in his backyard. He measures the length and width of the pool so that opposite sides are parallel. He also measures the diagonals of the pool to make sure that they are congruent. How does he know that the measure of each corner is 90?



EXAMPLE Rectangle on a Coordinate Plane

4 **Quadrilateral *ABCD* has vertices *A*(–2, 1), *B*(4, 3), *C*(5, 0), and *D*(–1, –2). Determine whether *ABCD* is a rectangle.**



Method 1

Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to see if consecutive sides are perpendicular.

slope of $\overline{AB} = \frac{3 - 1}{4 - (-2)}$ or

slope of $\overline{CD} = \frac{-2 - 0}{-1 - 5}$ or

slope of $\overline{BC} = \frac{0 - 3}{5 - 4}$ or

slope of $\overline{AD} = \frac{1 - (-2)}{-2 - (-1)}$ or

REVIEW IT

What do you know about the slopes of two perpendicular lines? (*Lesson 3-3*)

____

____

____

____

____

____

Because $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$, quadrilateral *ABCD* is a ______. The product of the slopes of consecutive sides is ______. This means that $\overline{AB} \perp \overline{BC}$, $\overline{AB} \perp \overline{AD}$, $\overline{AD} \perp \overline{CD}$, and $\overline{BC} \perp \overline{CD}$.

The perpendicular segments create four right angles.

Therefore, by definition *ABCD* is a ______.

Method 2

Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to determine whether opposite sides are congruent.

AB

$= \sqrt{[4 - (-2)]^2 + (3 - 1)^2}$

= ______

= ______

DC

$= \sqrt{[5 - (-1)]^2 + [0 - (-2)]^2}$

= ______

= ______

AD

$= \sqrt{[-1 - (-2)]^2 + (-2 - 1)^2}$

= ______

= ______

BC

$= \sqrt{(5 - 4)^2 + (0 - 3)^2}$

= ______

= ______

Since each pair of opposite sides of the quadrilateral have the same measure, they are congruent. Quadrilateral *ABCD* is a parallelogram.

Find the length of the diagonals.

$AC = \sqrt{[-5 - (-2)]^2 + (0 - 1)^2}$

$=$ ____

$=$ ____

$DB = \sqrt{[4 - (-1)]^2 + [3 - (-2)]^2}$

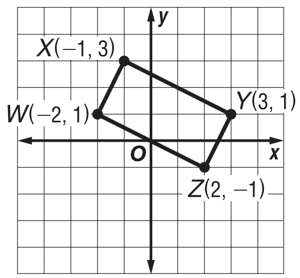
$=$ ____

$=$ ____

The length of each diagonal is ____.

Since the diagonals are congruent, *ABCD* is a rectangle.

Check Your Progress Quadrilateral *WXYZ* has vertices $W(-2, 1)$, $X(-1, 3)$, $Y(3, 1)$, and $Z(2, -1)$. Determine whether *WXYZ* is a rectangle using the Distance Formula.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–5 Rhombi and Squares

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, **the properties of quadrilaterals,** and the properties of circles. (Key)

MAIN IDEAS

- Recognize and apply the properties of rhombi.
- Recognize and apply the properties of squares.

BUILD YOUR VOCABULARY (page 138)

A **rhombus** is a quadrilateral with all four sides congruent.

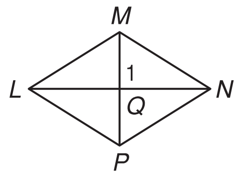
Theorem 6.15 The diagonals of a rhombus are perpendicular.

Theorem 6.16 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Theorem 6.17 Each diagonal of a rhombus bisects a pair of opposite angles.

EXAMPLE Measures of a Rhombus

1 **Use rhombus *LMNP* and the given information to find the value of each variable.**



a. Find y if $m\angle 1 = y^2 - 54$.

$m\angle 1 =$		The diagonals of a rhombus are perpendicular.
$=$		Substitution
$y^2 = 144$		Add 54 to each side.
$y =$		Take the square root of each side.

The value of y can be .

b. Find $m\angle PNL$ if $m\angle MLP = 64$.

$m\angle PNM =$		Opposite angles are congruent.
$m\angle PNM =$		Substitution

The diagonals of a rhombus bisect the angles.

So, $m\angle PNL =$ or .

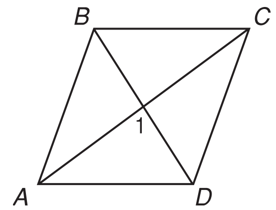
REMEMBER IT

A square is a rhombus, but a rhombus is not necessarily a square.

Check Your Progress **Use rhombus *ABCD* and the given information to find the value of each variable.**

a. Find x if $m\angle 1 = 2x^2 - 38$.

b. Find $m\angle CDB$ if $m\angle ABC = 126$.

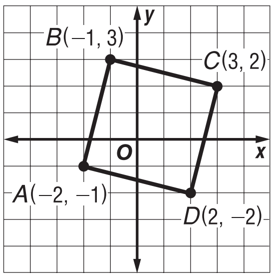


BUILD YOUR VOCABULARY (page 138)

If a quadrilateral is both a ________ and a rectangle, then it is a **square**.

EXAMPLE Squares

2 **Determine whether parallelogram *ABCD* is a *rhombus*, a *rectangle*, or a *square* for *A*(–2, –1), *B*(–1, 3), *C*(3, 2), and *D*(2, –2). List all that apply. Explain.**



Use the Distance Formula to compare the lengths of the diagonals.

$DB = \sqrt{(-1 - 2)^2 + [3 - (-2)]^2}$

= ________

= ________

$AC = \sqrt{[3 - (-2)]^2 + [2 - (-1)]^2}$

= ________

= ________

Use slope to determine whether the diagonals are perpendicular.

slope of $\overline{DB} = \frac{3 - (-2)}{-1 - 2}$ or ______

slope of $\overline{AC} = \frac{2 - (-1)}{3 - (-2)}$ or ______

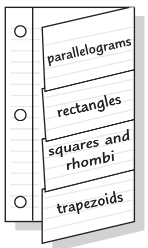
Since the slope of $\overline{AC}$ is the negative ______ of the slope of $\overline{DB}$ the diagonals are ______. The lengths of $\overline{DB}$ and $\overline{AC}$ are the same so the diagonals are congruent.

ABCD is a ______.

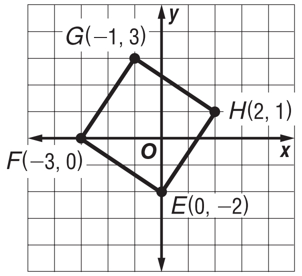
FOLDABLES™

ORGANIZE IT

Record the concepts about squares and rhombi, including their similarities and differences, under the squares and rhombi tab.



Check Your Progress Determine whether parallelogram *EFGH* is a rhombus, a rectangle, or a square for *E*(0, −2), *F*(−3, 0), *G*(−1, 3), and *H*(2, 1). List all that apply. Explain.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–6 Trapezoids

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, **the properties of quadrilaterals,** and the properties of circles. (Key)

MAIN IDEAS

- Recognize and apply the properties of trapezoids.
- Solve problems involving the medians of trapezoids.

BUILD YOUR VOCABULARY (page 138)

A **trapezoid** is a quadrilateral with exactly one pair of ______ sides.

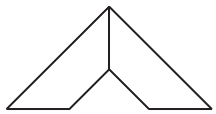
If the legs are ______, then the trapezoid is an **isosceles trapezoid**.

Theorem 6.18 Both pairs of base angles of an isosceles trapezoid are congruent.

Theorem 6.19 The diagonals of an isosceles trapezoid are congruent.

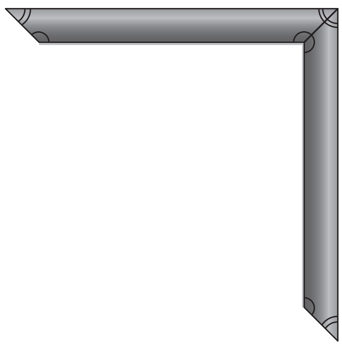
EXAMPLE Identify Isosceles Trapezoids

1 The top of this work station appears to be two adjacent trapezoids. Determine if they are isosceles trapezoids.



Each pair of base angles is ______, so the legs are the same length. Both trapezoids are ______.

Check Your Progress The sides of a picture frame appear to be two adjacent trapezoids. Determine if they are isosceles trapezoids.

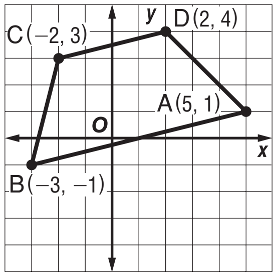


EXAMPLE Identify Trapezoids

2 ***ABCD* is a quadrilateral with vertices *A*(5, 1), *B*(–3, –1), *C*(–2, 3), and *D*(2, 4).**

a. Verify that *ABCD* is a trapezoid.

A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel. Use the Slope Formula.



slope of $\overline{AB} = \frac{-1-1}{-3-5}$

$=$

slope of $\overline{CD} = \frac{4-3}{2-(-2)}$

$=$

slope of $\overline{DA} = \frac{1-4}{5-2}$

$=$

slope of $\overline{BC} = \frac{3-(-1)}{-2-(-3)}$

$=$

Exactly one pair of opposite sides are parallel, ______ and ______. So, *ABCD* is a trapezoid.

b. Determine whether *ABCD* is an isosceles trapezoid. Explain.

First use the Distance Formula to determine whether the legs are congruent.

$DA = \sqrt{(2-5)^2 + (4-1)^2}$

$=$

$=$

$BC = \sqrt{[-2-(-3)]^2 + [3-(-1)]^2}$

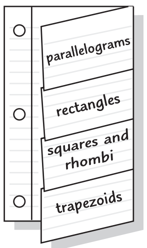
$=$

$=$

Since the legs are not ______, *ABCD* is not an isosceles trapezoid.

FOLDABLES™ ORGANIZE IT

Draw an isosceles trapezoid under the trapezoids tab. Include labels on the drawing to show congruent angles, congruent diagonals, and parallel sides.



Check Your Progress ***QRST*** **is a quadrilateral with vertices** $Q(-3, -2)$, $R(-2, 2)$, $S(1, 4)$, **and** $T(6, 4)$.

a. Verify that *QRST* is a trapezoid.

b. Determine whether *QRST* is an isosceles trapezoid. Explain.

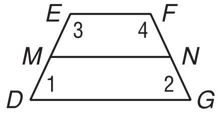
BUILD YOUR VOCABULARY (page 138)

The segment that joins ______ of the ______ of a trapezoid is the **median**.

Theorem 6.20
The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

EXAMPLE Median of a Trapezoid

3 ***DEFG*** **is an isosceles trapezoid with median** $\overline{MN}$.



a. Find DG **if** $EF = 20$ **and** $MN = 30$.

$MN = \frac{1}{2}(EF + DG)$ Theorem 6.20

$\square = \frac{1}{2}(\square + DG)$ Substitution

$\square = \square + DG$ Multiply each side by 2.

$\square = \square$ Subtract 20 from each side.

b. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$ if $m\angle 1 = 3x + 5$ and $m\angle 3 = 6x - 5$.

Since $\overline{EF} \parallel \overline{DG}$, $\angle 1$ and $\angle 3$ are supplementary. Because this is an isosceles trapezoid, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

$m\angle 1 + m\angle 3 =$ ______ Consecutive Interior Angles Theorem

______ $+$ ______ $=$ ______ Substitution

______ $=$ ______ Combine like terms.

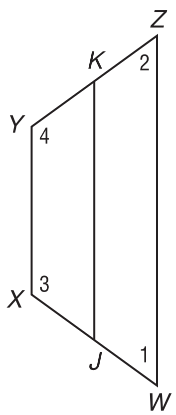
$x =$ ______ Divide each side by 9.

If $x =$ ______, then $m\angle 1 =$ ______ and $m\angle 3 =$ ______.

Because $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, $m\angle 2 =$ ______ and $m\angle 4 =$ ______.

Check Your Progress ***WXYZ* is an isosceles trapezoid with median $\overline{JK}$.**

a. Find XY if $JK = 18$ and $WZ = 25$.



b. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$ if $m\angle 2 = 2x - 25$ and $m\angle 4 = 3x + 35$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

6–7 Coordinate Proof With Quadrilaterals

MAIN IDEAS

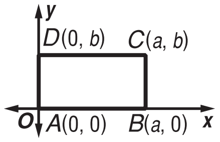
- Position and label quadrilaterals for use in coordinate proofs.
- Prove theorems using coordinate proofs.

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, **the properties of quadrilaterals,** and the properties of circles. (Key)
Standard 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles. (Key)

EXAMPLE Positioning a Rectangle

1 **Position and label a rectangle with sides *a* and *b* units long on the coordinate plane.**

- Let A, B, C, and D be vertices of a rectangle with sides $\overline{AB}$ and $\overline{CD}$ a units long, and sides $\overline{BC}$ and $\overline{AD}$ b units long.
- Place the rectangle with vertex A at the ______, $\overline{AB}$ along the positive ______, and $\overline{AD}$ along the ______. Label the vertices A, B, C, and D.
- The y-coordinate of B is ______ because the vertex is on the x-axis. Since the side length is a, the x-coordinate is ______.
- D is on the y-axis so the x-coordinate is ______. Since the side length is b, the y-coordinate is ______.
- The x-coordinate of C is also ______. The y-coordinate is $0 + b$ or b because the side $\overline{BC}$ is b units long.

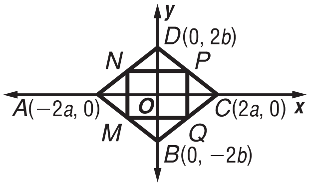


Check Your Progress Position and label a parallelogram with sides a and c units long on the coordinate plane.

EXAMPLE Coordinate Proof

2 **Place a rhombus on the coordinate plane. Label the midpoints of the sides *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a rectangle.**

The first step is to position a rhombus on the coordinate plane so that the origin is the midpoint of the diagonals and the diagonals are on the axes, as shown. Label the vertices to make computations as simple as possible.



Given: *ABCD* is a rhombus as labeled. *M*, *N*, *P*, *Q* are midpoints.

Prove: *MNPQ* is a rectangle.

Proof: By the Midpoint Formula, the coordinates of *M*, *N*, *P*, and *Q* are as follows.

$$M\left(\frac{-2a+0}{2}, \frac{0-2b}{2}\right) = \square$$

$$N\left(\frac{0-2a}{2}, \frac{2b+0}{2}\right) = \square$$

$$P\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right) = \square$$

$$Q\left(\frac{0+2a}{2}, \frac{-2b+0}{2}\right) = \square$$

Find the slopes of $\overline{QP}$, $\overline{MN}$, $\overline{QM}$, and $\overline{PN}$.

$$\text{slope of } \overline{QP} = \frac{\square - \square}{a - a} \text{ or } \square$$

$$\text{slope of } \overline{MN} = \frac{b-(-b)}{\square - \square} \text{ or } \square$$

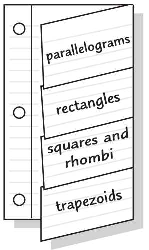
$$\text{slope of } \overline{QM} = \frac{\square - (-b)}{-a - a} \text{ or } \square$$

$$\text{slope of } \overline{PN} = \frac{b - \square}{-a - a} \text{ or } \square$$

FOLDABLES™

ORGANIZE IT

Make sketches to show how each type of quadrilateral in this chapter can be placed in the coordinate plane to have the simplest coordinates for the vertices. Label the vertices with their coordinates. Include each sketch under the appropriate tab.



WRITE IT

When proving theorems about quadrilaterals, why is it convenient to place a figure on the coordinate plane with one side parallel to an axis and one vertex at (0, 0)?

____

____

____

____

____

____

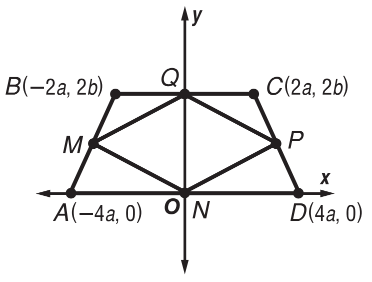
____

A segment with slope 0 is perpendicular to a segment with [ ] slope. Therefore, consecutive sides of this quadrilateral are [ ]. Since consecutive sides are perpendicular, *MNPQ* is, by definition, a [ ].

Check Your Progress Write a coordinate proof.

Given: *ABCD* is an isosceles trapezoid. *M*, *N*, *P*, and *Q* are midpoints.

Prove: *MNPQ* is a rhombus.



Proof:

[ ]

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 6

Bringing It All Together

Study Guide

FOLDABLES™	Vocabulary Puzzlemaker	Build your Vocabulary
Use your **Chapter 6 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 6, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(page 138)* to help you solve the puzzle.

6-1

Angles of Polygons

Give the measure of an interior angle and the measure of an exterior angle of each polygon.

1. equilateral triangle

2. regular hexagon

3. Find the sum of the measures of the interior angles of a convex 20-gon.

6-2

Parallelograms

For Exercises 4–7, let *ABCD* be a parallelogram with $AB \neq BC$ and with no right angles.

4. Sketch a parallelogram that matches the description above and draw diagonal $\overline{BD}$.

Complete each sentence.

5. $\overline{AB} \parallel$ ______ and $\overline{AD} \parallel$ ______ .

6. $\overline{AB} \cong$ ______ and $\overline{BC} \cong$ ______ .

7. $\angle A \cong$ ______ and $\angle ABC \cong$ ______ .

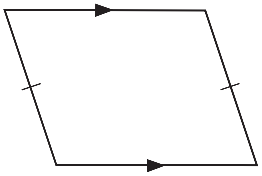
6-3 Tests for Parallelograms

8. Which of the following conditions guarantee that a quadrilateral is a parallelogram?

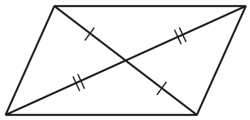
a. Two sides are parallel.
b. Both pairs of opposite sides are congruent.
c. A pair of opposite sides is both parallel and congruent.
d. There are two right angles.
e. All four sides are congruent.
f. Both pairs of opposite angles are congruent.
g. The diagonals bisect each other.
h. All four angles are right angles.

Determine whether there is enough given information to know that each figure is a parallelogram. If so, state the definition or theorem that justifies your conclusion.

9.

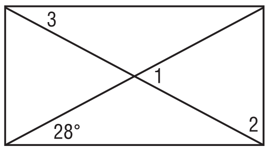


10.

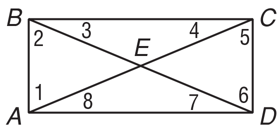


6-4 Rectangles

11. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the rectangle shown.



***ABCD* is a rectangle with *AD* > *AB*. Name each of the following in this figure.**



12. all segments that are congruent to $\overline{BE}$

13. all angles congruent to $\angle 1$

14. two pairs of congruent triangles

6-5 Rhombi and Squares

Sketch each of the following.

15. a quadrilateral with perpendicular diagonals that is not a rhombus

16. a quadrilateral with congruent diagonals that is not a rectangle

List all of the special quadrilaterals that have each listed property: *parallelogram*, *rectangle*, *rhombus*, *square*.

17. Opposite sides are congruent.

18. The diagonals are perpendicular.

19. The quadrilateral is equilateral.

20. The quadrilateral is equiangular.

21. The diagonals are perpendicular and congruent.

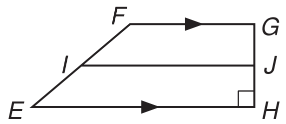
6-6 Trapezoids

Complete each sentence.

22. A quadrilateral with only one pair of opposite sides parallel and the other pair of opposite sides congruent is a(n) ______.

23. The segment joining the midpoints of the nonparallel sides of a trapezoid is called the ______.

24. A quadrilateral with only one pair of opposite sides parallel is a(n) ______.



***EFGH* is a trapezoid, *I* is the midpoint of $\overline{FE}$, and *J* is the midpoint of $\overline{GH}$. Identify each of the following segments or angles.**

25. the bases of trapezoid *EFGH*

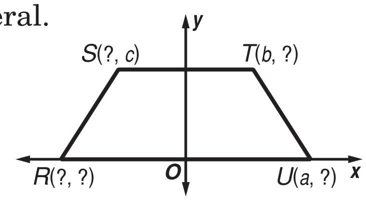
26. the legs of trapezoid *EFGH*

27. the median of trapezoid *EFGH*

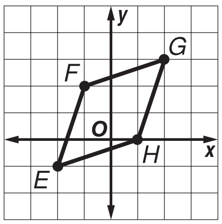
6-7

Coordinate Proof with Quadrilaterals

28. Find the missing coordinates in the figure. Then write the coordinates of the four vertices of the quadrilateral.



Refer to quadrilateral *EFGH*.



29. Find the slope of each side.

30. Find the length of each side.

31. Find the slope of each diagonal.

32. Find the length of each diagonal.

33. What can you conclude about the sides of *EFGH*?

34. What can you conclude about the diagonals of *EFGH*?

CHAPTER 6

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 6.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 6 Practice Test on page 373 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 6 Study Guide and Review on pages 369–372 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 373.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 6 Foldable.
- Then complete the Chapter 6 Study Guide and Review on pages 369–372 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 6 Practice Test on page 373.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 7

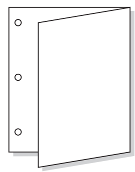
Proportions and Similarity

FOLDABLES™

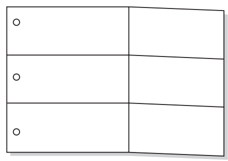
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" × 17" paper.

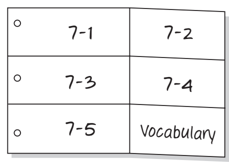
STEP 1 **Fold** widthwise. Leave space to punch holes so it can be placed in your binder.



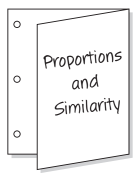
STEP 2 **Open** the flap and draw lines to divide the inside into six equal parts.



STEP 3 **Label** each part using the lesson numbers.



STEP 4 **Write** the name of the chapter on the front flap.



NOTE-TAKING TIP: A visual study guide like the Foldable shown above helps you organize what you know and remember what you have learned. You can use them to review main ideas or keywords.

CHAPTER 7

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 7. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
cross products			
extremes			
means			

Vocabulary Term	Found on Page	Definition	Description or Example
midsegment			
proportion			
ratio			
scale factor			
similar polygons			

7–1 Proportions

BUILD YOUR VOCABULARY (pages 168–169)

A **ratio** is a comparison of two quantities. The ratio of a to b can be expressed ______, where b is not zero.

An equation stating that two ratios are ______ is called a **proportion**.

MAIN IDEAS

- Write ratios.
- Use properties of proportions.

Reinforcement of Standard 6NS1.3 Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse. (Key)

EXAMPLE Write a Ratio

1 **The total number of students who participate in sports programs at Central High School is 520. The total number of students in the school is 1850. Find the athlete-to-student ratio to the nearest tenth.**

To find this ratio, divide the number of athletes by the total number of students.

$$\frac{\text{number of athletes}}{\text{total number of students}} = \frac{\square}{\square} \text{ or about } \square$$

The athlete-to-student ratio is ______ athletes for each student in the school.

Check Your Progress The country with the longest school year is China with 251 days. Find the ratio of school days to total days in a year for China to the nearest tenth. (Use 365 as the number of days in a year.)

______

EXAMPLE Solve Proportions by Using Cross Products

2 **Solve each proportion.**

a. $\frac{6}{18.2} = \frac{9}{y}$.

$\frac{6}{18.2} = \frac{9}{y}$	Original proportion
$\square = \square$	Cross products
$y = \square$	Divide each side by 6.

KEY CONCEPT

Property of Proportions For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

b. $\frac{4x - 5}{3} = \frac{-26}{6}$

$\frac{4x - 5}{3} = \frac{-26}{6}$	Original proportion
$6\,\square = 3\,\square$	Cross multiply.
$24x - 30 = -78$	Multiply.
$24x = -48$	Add 30 to each side.
$x = \square$	Divide.

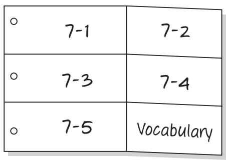
Check Your Progress **Solve each proportion.**

a. $\frac{13.5}{42} = \frac{b}{14}$

b. $\frac{7n - 1}{8} = \frac{15.5}{2}$

FOLDABLES™ ORGANIZE IT

As you skim the lesson, write down questions that you have on the section for Lesson 7-1. Then write the answers next to each question.



EXAMPLE Solve Problems Using Proportions

3 TRAINS **A boxcar on a train has a length of 40 feet and a width of 9 feet. A scale model is made with a length of 16 inches. Find the width of the model.**

Write and solve a proportion.

$\frac{\text{boxcar's length (ft)}}{\text{model's length (in.)}} = \frac{\text{boxcar's width (ft)}}{\text{model's width (in.)}}$

$\square = \square$	Substitution
$40x = 16(9)$	Cross products
$40x = 144$	Multiply.
$x = \square$	Divide each side by 40.

The width of the model is ☐ inches.

Check Your Progress Two large cylindrical containers are in proportion. The height of the larger container is 25 meters with a diameter of 8 meters. The height of the smaller container is 7 meters. Find the diameter of the smaller container.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–2 Similar Polygons

Standard 11.0 Students determine how changes in dimensions affect the perimeter, area, and volume **of common geometric figures** and solids.

Main Ideas

- Identify similar figures.
- Solve problems involving scale factors.

Build Your Vocabulary (page 169)

When polygons have the same shape but may be different in ___, they are called **similar polygons**.

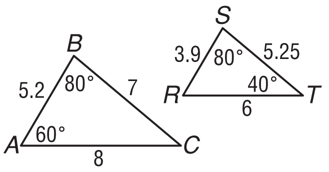
When you compare the lengths of ___ sides of similar figures, you usually get a numerical ratio. This ratio is called the **scale factor** for the two figures.

Key Concept

Similar polygons Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Example: Similar Polygons

1 **Determine whether the pair of figures is similar. Justify your answer.**



Since $m\angle B = m\angle S$, ___.

The $m\angle C = 40$ and $m\angle R = 60$.

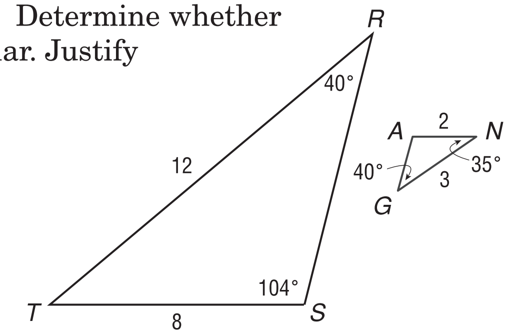
So, $\angle C \cong \angle T$ and ___.

Thus, all the corresponding angles are congruent.
Now determine whether corresponding sides are proportional.

$\frac{AC}{RT} = \frac{8}{6}$ or $1.\overline{3}$ $\quad \frac{AB}{RS} = \frac{\_\_\_}{\_\_\_}$ or $1.\overline{3}$ $\quad \frac{BC}{ST} = \frac{\_\_\_}{\_\_\_}$ or $1.\overline{3}$

The ratio of the measures of the corresponding sides are equal and the corresponding angles are ___, so $\triangle ABC \sim \triangle RST$.

Check Your Progress Determine whether the pair of figures is similar. Justify your answer.



___

EXAMPLE Scale Factor

2 ARCHITECTURE **An architect prepared a 12-inch model of a skyscraper to look like an actual 1100-foot building. What is the scale factor of the model compared to the actual building?**

Before finding the scale factor you must make sure that both measurements use the same unit of measure.

1100(12) = 13,200 inches

$$\frac{\text{height of model}}{\text{height of actual building}} = \frac{\square}{\square}$$

$$= \square$$

The ratio comparing the two heights is ______ or ______ : ______. The scale factor is ______, which means that the model is ______ the height of the actual skyscraper.

WRITE IT

Explain why two congruent polygons must also be similar.

____

____

____

____

____

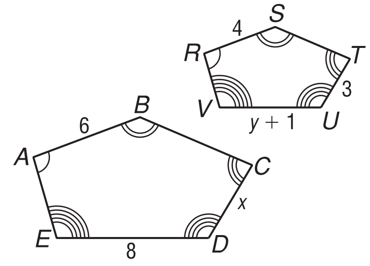
Check Your Progress A space shuttle is about 122 feet in length. The Science Club plans to make a model of the space shuttle with a length of 24 inches. What is the scale factor of the model compared to the real space shuttle?

____

EXAMPLE Proportional Parts and Scale Factor

3 **The two polygons are similar.**

a. Write a similarity statement. Then find *x*, *y*, and *UV*.

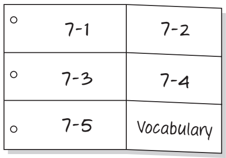


Use the congruent angles to write the corresponding vertices in order.

polygon ______ ~ polygon ______

ORGANIZE IT

Write a description of the information you would include in a diagram of two polygons to enable a friend to decide that the polygons are similar. Record your description on the section for Lesson 7-2.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Now write propotions to find x and y.

To find x:

$\frac{AB}{RS} = \frac{CD}{TU}$	Similarity proportion
$\frac{\square}{4} = \frac{\square}{3}$	$AB = 6$, $RS = 4$, $CD = x$, $TU = 3$
$18 = 4x$	Cross products
$\square = x$	Divide each side by 4.

To find y:

$\frac{AB}{RS} = \frac{DE}{UV}$	Similarity proportion
$\frac{6}{\square} = \frac{8}{\square}$	$AB = 6$, $RS = 4$, $DE = 8$, $UV = y + 1$
$6y + 6 = 32$	Cross products
$6y = 26$	Subtract 6 from each side.
$y = \square$	Divide each side by 6 and simplify.

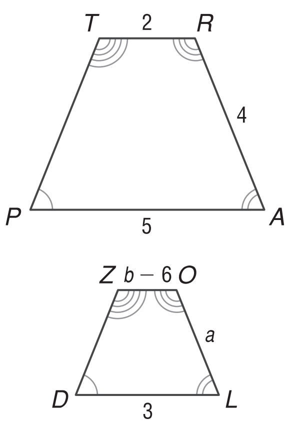
$UV = y + 1$, so $UV = \square + 1$ or $\square$.

b. Find the scale factor of polygon *ABCDE* to polygon *RSTUV*.

The scale factor is the ratio of the lengths of any two corresponding sides.

$$\frac{AB}{RS} = \frac{\square}{4} \text{ or } \frac{3}{2}$$

Check Your Progress **The two polygons are similar.**



a. Write a similarity statement. Then find a, b, and ZO.

b. Find the scale factor of polygon *TRAP* to polygon *ZOLD*.

7–3 Similar Triangles

Standard 4.0 Students prove basic theorems involving congruence and **similarity.** (Key) **Standard 5.0 Students prove that triangles are** congruent or **similar,** and they are able to use the concept of corresponding parts of congruent triangles.

MAIN IDEAS

- Identify similar triangles.
- Use similar triangles to solve problems.

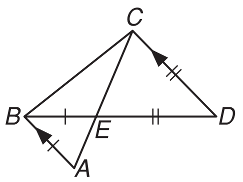
Postulate 7.1 Angle-Angle (AA) Similarity
If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Theorem 7.1 Side-Side-Side (SSS) Similarity
If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Theorem 7.2 Side-Angle-Side (SAS) Similarity
If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

EXAMPLE Determine Whether Triangles are Similar

1 In the figure, $\overline{AB} \parallel \overline{DC}$, $BE = 27$, $DE = 45$, $AE = 21$, and $CE = 35$. Determine which triangles in the figure are similar.

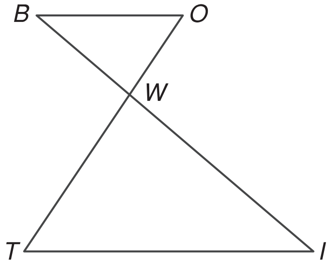


Since $\overline{AB} \parallel \overline{DC}$, $\angle BAC \cong$ ______ by the Alternate Interior Angles Theorem.

Vertical angles are congruent, so ______ $\cong$ ______.

Therefore, by the AA Similarity Theorem, ______.

Check Your Progress In the figure, $OW = 7$, $BW = 9$, $WT = 17.5$, and $WI = 22.5$. Determine which triangles in the figure are similar.



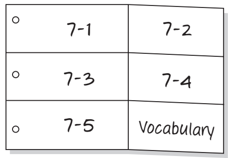
Theorem 7.3
Similarity of triangles is reflexive, symmeteric, and transitive.

EXAMPLE Parts of Similar Triangles

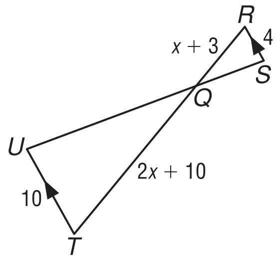
FOLDABLES™

ORGANIZE IT

Write a short paragraph to describe how you could apply the postulate and theorems in this lesson to help you construct similar triangles. Include your paragraph on the section for Lesson 7-3.



2 ALGEBRA **Given $\overline{RS} \parallel \overline{UT}$, $RS = 4$, $RQ = x + 3$, $QT = 2x + 10$, $UT = 10$, find RQ and QT.**



Since $\overline{RS} \parallel \overline{UT}$, $\angle SRQ \cong$ ____ and ____ $\cong TUQ$ because they are alternate interior angles. By AA Similarity, ____ $\cong$ ____. Using the definition of similar polygons, $\dfrac{\_\_\_\_}{\_\_\_\_} = \dfrac{\_\_\_\_}{\_\_\_\_}$.

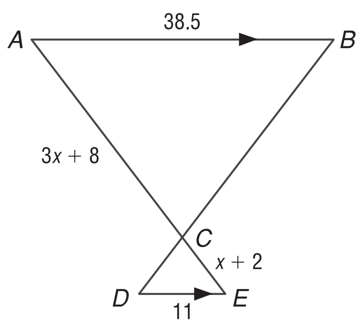
$\dfrac{4}{10} = \dfrac{x+3}{2x+10}$	Substitution
$4(2x + 10) = 10(x + 3)$	Cross products
$8x + 40 = 10x + 30$	Distributive Property
____ $= x$	Simplify.

Now find RQ and QT.

$RQ =$ ____ $\qquad QT = 2x + 10$

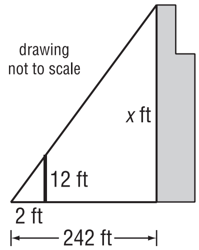
$=$ ____ $+$ ____ or ____ $\qquad = 2($____$) + 10$ or ____

Check Your Progress Given $\overline{AB} \parallel \overline{DE}$, $AB = 38.5$, $DE = 11$, $AC = 3x + 8$, and $CE = x + 2$, find AC and CE.



EXAMPLE Find a Measurement

3 INDIRECT MEASUREMENT Josh wanted to measure the height of the Sears Tower in Chicago. He used a 12-foot light pole and measured its shadow at 1 P.M. The length of the shadow was 2 feet. Then he measured the length of the Sears Tower's shadow and it was 242 feet at the time. What is the height of the Sears Tower?



Assuming that the sun's rays form similar triangles, the following proportion can be written.

$$\frac{\text{height of Sears Tower (ft)}}{\underline{\qquad}} = \frac{\underline{\qquad}}{\text{light pole shadow length (ft)}}$$

Now substitute the known values and let x be the height of the Sears Tower.

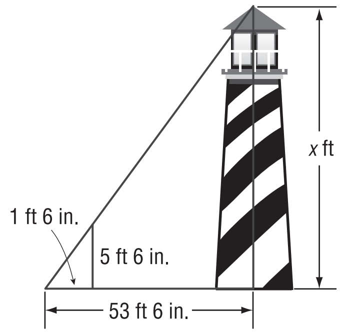
$\frac{\underline{\quad}}{\underline{\quad}} = \frac{\underline{\quad}}{\underline{\quad}}$ Substitution

$x \cdot 2 = 242(12)$ Cross products

$x = \underline{\qquad}$ Simplify and divide each side by 2.

The Sears Tower is ______ feet tall.

Check Your Progress On her trip along the East coast, Jennie stops to look at the tallest lighthouse in the U.S. located at Cape Hatteras, North Carolina. Jennie measures her shadow to be 1 feet 6 inches in length and the length of the shadow of the lighthouse to be 53 feet 6 inches. Jennie's height is 5 feet 6 inches. What is the height of the Cape Hatteras lighthouse to the nearest foot?



REMEMBER IT

Shadows and similar triangles are commonly used for indirectly measuring the heights of objects that are otherwise too tall to measure.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–4 Parallel Lines and Proportional Parts

Standard 12.0 Students find and use measures of sides and of interior and exterior angles **of triangles** and polygons **to** classify figures and **solve problems.** (Key)

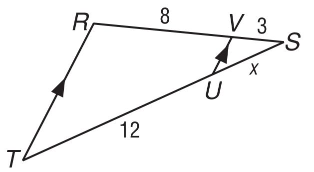
MAIN IDEAS

- Use proportional parts of triangles.
- Divide a segment into parts.

Theorem 7.4 Triangle Proportionality Theorem
If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

EXAMPLE Find the Length of a Side

1 **In $\triangle RST$, $\overline{RT} \parallel \overline{VU}$, $SV = 3$, $VR = 8$, and $UT = 12$. Find SU.**

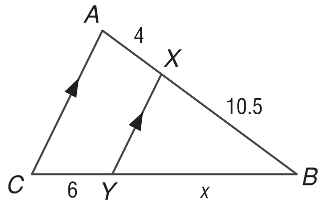


From the Triangle Proportionality Theorem, $\frac{SV}{VR} = \frac{\square}{\square}$.
Substitute the known measures.

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$3(12) = 8x$	Cross products
$36 = 8x$	Multiply.
$\frac{36}{8} = x$	Divide each side by 8.
$\square = x$	Simplify.

Check Your Progress In $\triangle ABC$, $\overline{AC} \parallel \overline{XY}$, $AX = 4$, $XB = 10.5$, and $CY = 6$. Find BY.



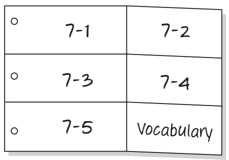
Theorem 7.5 Converse of the Triangle Proportionality If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

EXAMPLE Determine Parallel Lines

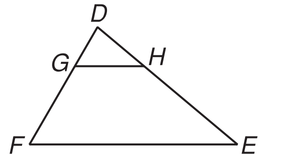
FOLDABLES™

ORGANIZE IT

Sketch a pentagon that is not regular. Write a brief explanation of how you could use the theorems in this lesson to construct a pentagon similar to the given pentagon. Include the sketch and explanation on the section for Lesson 7-4.



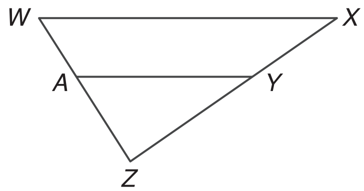
2 **In △*DEF*, *DH* = 18, *HE* = 36, and $DG = \frac{1}{2}GF$. Determine whether $\overline{GH} \parallel \overline{FE}$. Explain.**



In order to show that $\overline{GH} \parallel \overline{FE}$, we must show that $\frac{DG}{GF} = \frac{DH}{HE}$. $DG = \frac{1}{2}GF$, so $\frac{DG}{GF} = \frac{1}{2}$.

Since, $\frac{DG}{GF} =$ ______ and $\frac{DH}{HE} =$ ______ or ______, the sides have ______ lengths. Since the segments have proportional lengths, ______.

Check Your Progress In △*WXZ*, *XY* = 15, *YZ* = 25, *WA* = 18, and *AZ* = 32. Determine whether $\overline{WX} \parallel \overline{AY}$. Explain.



______

BUILD YOUR VOCABULARY (page 135)

A **midsegment** of a triangle is a segment whose endpoints are the ______ of the two sides of the triangle.

WRITE IT

Write the Midpoint Formula.

______

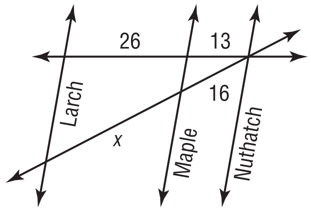
______

______

Theorem 7.6 Triangle Midsegment Theorem
A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half of that side.

EXAMPLE Proportional Segments

3 MAPS In the figure, Larch, Maple, and Nuthatch Streets are all parallel. The figure shows the distances in between city blocks. Find x.

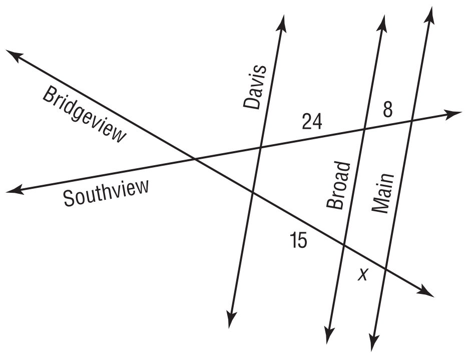


From Corollary 7.1, if three or more parallel lines intersect two transversals, then they cut off the transversals

__________. Write a proportion.

$\frac{26}{13} = \frac{x}{16}$	Triangle Proportionality Theorem
$26(16) = 13x$	__________.
$\_\_\_ = 13x$	Multiply.
$\_\_\_ = x$	Divide.

Check Your Progress In the figure, Davis, Broad, and Main Streets are all parallel. The figure shows the distances in city blocks that the streets are apart. Find x.



A 4 **B** 5 **C** 6 **D** 7

____

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

7–5 Parts of Similar Triangles

Standard 4.0 Students prove basic theorems involving congruence and **similarity.** (Key)

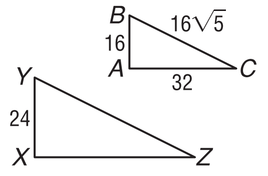
Main Ideas

- Recognize and use proportional relationships of corresponding perimeters of similar triangles.
- Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.

Theorem 7.7 Proportional Perimeters Theorem
If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

EXAMPLE Perimeters of Similar Triangles

1 **If $\triangle ABC \sim \triangle XYZ$, $AC = 32$, $AB = 16$, $BC = 16\sqrt{5}$, and $XY = 24$, find the perimeter of $\triangle XYZ$.**



Let x represent the perimeter of $\triangle XYZ$. The perimeter of

$\triangle ABC = 16 + 16\sqrt{5} + 32$ or ____ $+ 16\sqrt{5}$.

$\frac{XY}{AB} = \frac{\text{perimeter of ____}}{\text{perimeter of ____}}$ Proportional Perimeter Theorem

____ = ____ Substitution

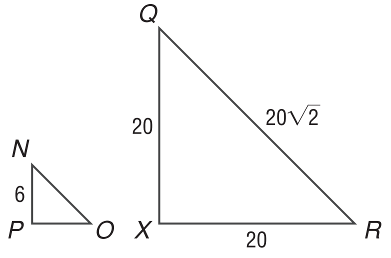
$24(48 + 16\sqrt{5}) = 16x$ Cross products

$1152 + 384\sqrt{5} = 16x$ Multiply.

____ $+ 24\sqrt{5} = x$ Divide.

The perimeter of $\triangle XYZ$ is ____ units.

Check Your Progress If $\triangle PNO \sim \triangle XQR$, $PN = 6$, $XQ = 20$, $QR = 20\sqrt{2}$, and $RX = 20$, find the perimeter of $\triangle PNO$.



Theorem 7.8
If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

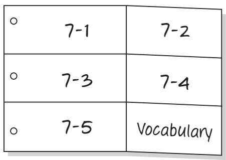
Theorem 7.9
If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides.

Theorem 7.10
If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

FOLDABLES™

ORGANIZE IT

Use a pair of similar isosceles triangles to illustrate all of the theorems in this lesson. Include a sketch and explanation on the section for Lesson 7-5.



EXAMPLE **Write a Proof**

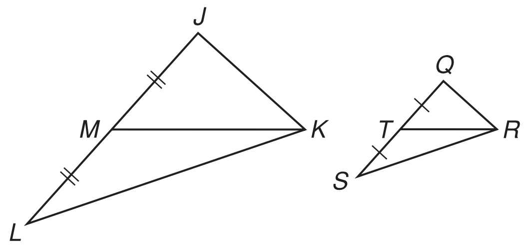
2 **Write a paragraph proof.**

Given: $\triangle JKL \sim \triangle QRS$

$\overline{MK}$ is a median of $\triangle JKL$.

$\overline{TR}$ is a median of $\triangle QRS$.

Prove: $\triangle JKM \sim \triangle QRT$

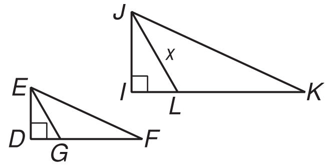


Proof: By ______, $\frac{JK}{QR} = \frac{JL}{QS}$.

Since similar triangles have corresponding ______ proportional to the corresponding sides, $\frac{JL}{QS} = \frac{KM}{RT}$. By substitution, $\frac{JK}{QR} = \frac{KM}{RT}$. Since $\overline{MK}$ and $\overline{TR}$ are ______ of $\triangle JKL$ and $\triangle QRS$, respectively, M and T are midpoints of $\overline{JL}$ and $\overline{QS}$. By ______, $2JM = JL$ and $2QT = QS$. Substitute in the equation $\frac{JK}{QR} = \frac{JL}{QS}$ to get $\frac{JK}{QR} = \frac{2JM}{2QT}$. Simplify to find $\frac{JK}{QR} = \frac{JM}{QT}$. Therefore, $\triangle JKM \sim \triangle QRT$ by ______.

EXAMPLE Medians of Similar Triangles

3 **In the figure, $\triangle EFD \sim \triangle JKI$. $\overline{EG}$ is a median of $\triangle EFD$, and $\overline{JL}$ is a median of $\triangle JKI$. Find JL if $EF = 36$, $EG = 18$, and $JK = 56$.**



$\frac{EG}{JL} = \frac{EF}{JK}$ Write a proportion.

$\frac{18}{x} = \frac{36}{56}$ $EG = 18$, $JL = x$, $EF = 36$, and $JK = 56$

$1008 = 36x$ Cross products

______ $= x$ Divide each side by 36.

Thus, ______.

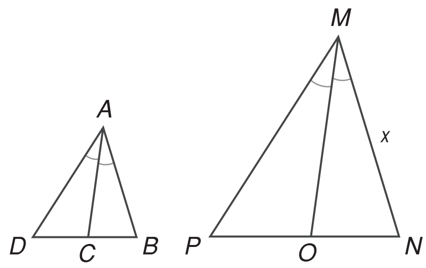
REVIEW IT

An altitude of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing the side. *(Lesson 5-1)*

Check Your Progress

a. $\triangle EFG \sim \triangle MSY$ and $EF = \frac{5}{4}MS$. Find the ratio of the length of an altitude of $\triangle EFG$ to the length of an altitude of $\triangle MSY$.

b. In the figure, $\triangle ABD \sim \triangle MNP$. $\overline{AC}$ is a median of $\triangle ABD$ and $\overline{MO}$ is a median of $\triangle MNP$. Find x if $AC = 5$, $AB = 7$, and $MO = 12.5$.



Theorem 7.11 Angle Bisector Theorem
An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 7

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 7 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 7, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 168–169)* to help you solve the puzzle.

7-1 Proportions

1. **ADVERTISEMENT** A poster measures 10 inches by 14 inches. If it is enlarged to have a width of 60 inches, how tall will the new poster be?

Solve each proportion.

2. $\frac{3}{8} = \frac{x}{40}$

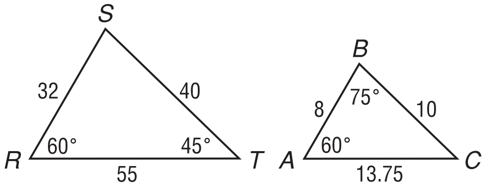
3. $\frac{9}{11} = \frac{15}{x}$

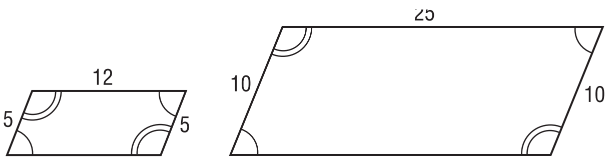
4. $\frac{x+2}{5} = \frac{4}{3}$

7-2 Similar Polygons

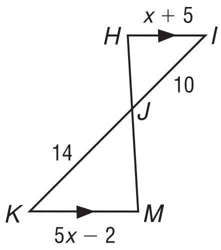
Determine whether each pair of figures is similar. If so, write the appropriate similarity statement.

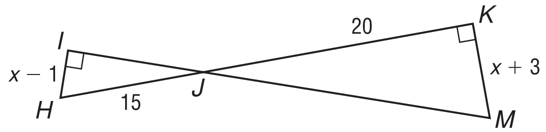
5.



6. 

If $\triangle HIJ \sim \triangle MKJ$, find x and the scale factor of $\triangle HIJ$ to $\triangle MKJ$.

7. 

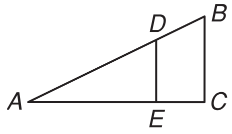
8. 

7-3 Similar Triangles

9. **SHADOWS** A tree casts a 60 foot shadow. At the same time, a 6-foot tall man casts a shadow that is 2 feet long. How tall is the tree?

7-4 Parallel Lines and Proportional Parts

Determine whether $\overline{BC} \parallel \overline{DE}$.

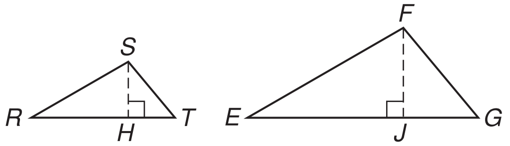


10. $AD = 16$, $DB = 12$, $AE = 14$, and $EC = 10.5$

11. $BD = 5$, $BA = 20$, and CE is one third of EA

7-5 Parts of Similar Triangles

12. Find FG if $\triangle RST \sim \triangle EFG$, $\overline{SH}$ is an altitude of $\triangle RST$, $\overline{FJ}$ is an altitude of $\triangle EFG$, $ST = 10$, $SH = 8$, and $FJ = 10$.



CHAPTER 7

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 7.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 7 Practice Test on page 427 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 7 Study Guide and Review on pages 424–426 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 427 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 7 Foldable.
- Then complete the Chapter 7 Study Guide and Review on pages 424–426 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 7 Practice Test on page 427 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 8

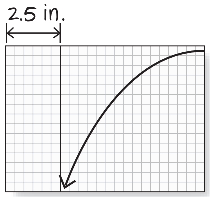
Right Triangles and Trigonometry

FOLDABLES™

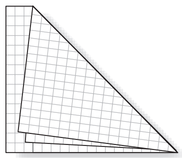
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with seven sheets of grid paper.

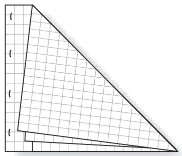
STEP 1 **Stack** the sheets. Fold the top right corner to the bottom edge to form a square.



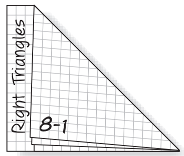
STEP 2 **Fold** the rectangular part in half.



STEP 3 **Staple** the sheets along the fold in four places.



STEP 4 **Label** each sheet with a lesson number and the rectangular part with the chapter title.



NOTE-TAKING TIP: When you take notes, draw a visual (graph, diagram, picture, chart) that presents the information introduced in the lesson in a concise, easy-to-study format.

CHAPTER 8

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 8. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
angle of depression			
angle of elevation			
cosine			
geometric mean			
Law of Cosines			
Law of Sines			

Vocabulary Term	Found on Page	Definition	Description or Example
Pythagorean triple			
sine			
solving a triangle			
tangent			
trigonometric ratio			
trigonometry			

8–1 Geometric Mean

Standard 4.0 Students prove basic theorems involving congruence and **similarity.** (Key)

MAIN IDEAS

- Find the geometric mean between two numbers.
- Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.

KEY CONCEPT

Geometric Mean For two positive numbers *a* and *b*, the geometric mean is the positive number *x* where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

BUILD YOUR VOCABULARY (page 188)

The **geometric mean** between two numbers is the ________ square root of their ________.

EXAMPLE Geometric Mean

1 **Find the geometric mean between 25 and 7.**

$\frac{\square}{x} = \frac{\square}{16}$ Definition of geometric mean

$x^2 = 175$ Cross multiply

$x =$ ________ Take the positive square root of each side.

$x \approx$ ________ Use a calculator.

Check Your Progress **Find the geometric mean between each pair of numbers.**

a. 3 and 12 ________

b. 25 and 7 ________

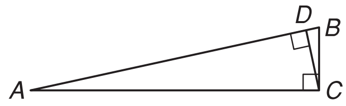
Theorem 8.1
If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.

Theorem 8.2
The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

EXAMPLE **Altitude and Segments of the Hypotenuse**

2 **In △*ABC*, *BD* = 6 and *AD* = 27. Find *CD*.**

Let $x = CD$.



$$\frac{\square}{CD} = \frac{CD}{\square}$$

$$\frac{\square}{x} = \frac{x}{\square}$$

$BD =$ ____ , $AD =$ ____ , and $CD = x$.

$x^2 =$ ____ Cross products

$x = 9\sqrt{2}$ Take the positive square root of each side.

$x \approx$ ____ Use a calculator.

CD is about ____ .

WRITE IT

Why do you automatically discard the negative root when finding the altitude or geometric mean?

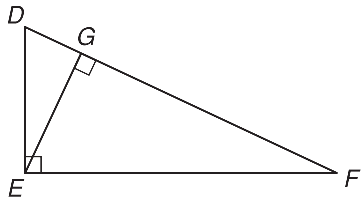
____

____

____

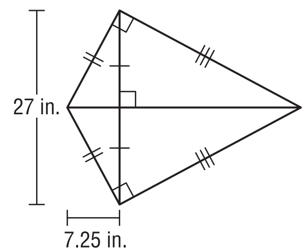
____

Check Your Progress In △*DEF*, $FG = 18$, and $DG = 4$. Find *EG*.

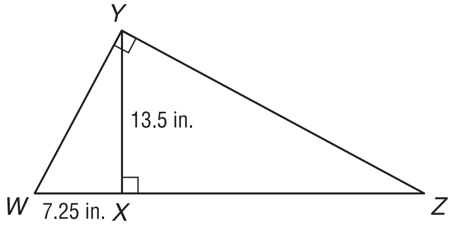


EXAMPLE **Altitude and Length of the Hypotenuse**

3 KITES **Ms. Alspach constructed a kite for her son. She had to arrange perpendicularly two support rods, the shorter of which was 27 inches long. If she had to place the short rod 7.25 inches from one end of the long rod in order to form two right triangles with the kite fabric, what was the length of the long rod?**



Draw a diagram of one of the right triangles formed.



Let XY be the altitude drawn from the right angle of $\triangle WYZ$.

$$\frac{WX}{YX} = \frac{YX}{ZX}$$

$$\frac{\text{____}}{\text{____}} = \frac{\text{____}}{ZX}$$ $WX =$ ____ and $YX =$ ____

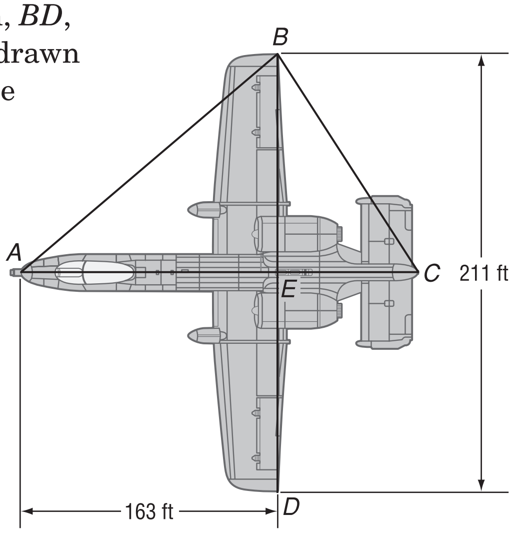
$7.25ZX =$ ____ Cross products

$ZX \approx$ ____ Divide each side by ____.

The length of the long rod is 7.25 + ____, or about ____ inches long.

Check Your Progress

A jetliner has a wingspan, BD, of 211 feet. The segment drawn from the front of the plane to the tail, $\overline{AC}$, intersects $\overline{BD}$ at point E. If AE is 163 feet, what is the length of the aircraft?



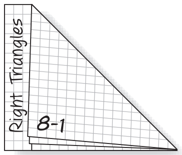
Theorem 8.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

FOLDABLES™

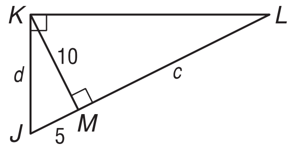
ORGANIZE IT

On the Lesson 8-1 Foldable, include drawings to show how to solve problems involving the relationships between parts of a right triangle and the altitude to its hypotenuse.



EXAMPLE Hypotenuse and Segment of Hypotenuse

4 **Find c and d in JKL.**



$\overline{KM}$ is the altitude of right triangle JKL. Use Theorem 8.2 to write a proportion.

$$\frac{JM}{KM} = \frac{KM}{LM}$$

$$\frac{\_\_\_}{\_\_\_} = \frac{\_\_\_}{\_\_\_}$$ $JM =$ ___, $KM =$ ___, and $LM =$ ___

$100 = 5c$ Cross products

___ $= c$ Divide each side by ___.

$\overline{JK}$ is a leg of right triangle JKL. Use Theorem 8.3 to write a proportion.

$$\frac{JL}{JK} = \frac{JK}{JM}$$

$$\frac{\_\_\_}{d} = \frac{d}{\_\_\_}$$ $JL =$ ___, $JM = 5$, and $JK = d$

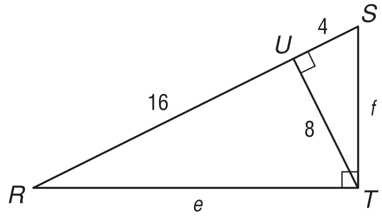
___ $=$ ___ Cross products

$d = \sqrt{125}$ Take the ___.

$d =$ ___ Simplify.

$d \approx 11.2$ Use a calculator.

Check Your Progress Find e and f.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–2 The Pythagorean Theorem and Its Converse

Main Ideas

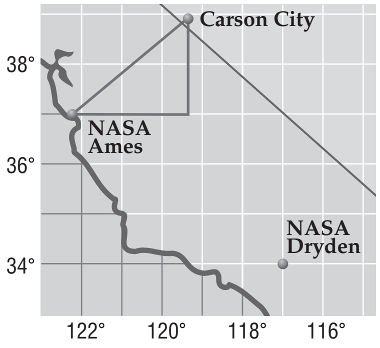
- Use the Pythagorean Theorem.
- Use the converse of the Pythagorean Theorem.

Standard 12.0 Students find and use measures of sides and of interior and exterior angles **of triangles** and polygons **to classify figures and solve problems.** (Key)
Standard 14.0 Students prove the Pythagorean theorem. (Key)
Standard 15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.

Theorem 8.4 Pythagorean Theorem
In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

EXAMPLE Find the Length of the Hypotenuse

1 LONGITUDE AND LATITUDE **Carson City, Nevada, is located at about 120 degrees longitude and 39 degrees latitude; NASA Ames is located at about 122 degrees longitude and 37 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth degree if you were to travel directly from NASA Ames to Carson City.**



The change in longitude between NASA Ames and Carson City is $|119 - 122|$ or 3 degrees. Let this distance be a.
The change in latitude is $|39 - 37|$ or 2 degrees.
Let this distance be b.
Use the Pythagorean Theorem to find the distance from NASA Ames to Carson City.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$3^2 + 2^2 = c^2$	$a = 3$, $b = 2$
____ + ____ $= c^2$	Simplify.
$13 = c^2$	Add.
____ $= c$	Take the square root of each side.
____ $\approx c$	Use a calculator.

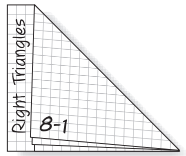
The degree distance between NASA Ames and Carson City is about ____ degrees.

Check Your Progress Carson City, Nevada, is located at about 120 degrees longitude and 39 degrees latitude. NASA Dryden is located about 117 degrees longitude and 34 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth degree if you were to travel directly from NASA Dryden to Carson City.

FOLDABLES™

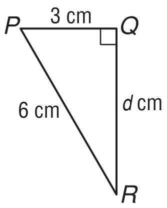
ORGANIZE IT

On the Lesson 8-2 Foldable, write the Pythagorean Theorem and its converse.



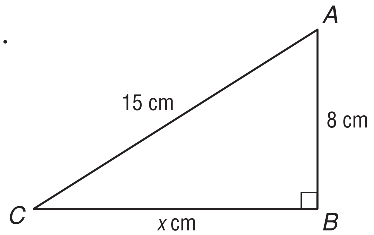
EXAMPLE Find the Length of a Leg

2 **Find d.**



$(PQ)^2 + (QR)^2 = (PR)^2$	Pythagorean Theorem
$3^2 + d^2 = 6^2$	$PQ = 3$, $PR = 6$
$9 + d^2 = 36$	Simplify.
$d^2 =$ ______	Subtract ______ from each side.
$d =$ ______	Take the square root of each side.
$d \approx$ ______	Use a calculator.

Check Your Progress Find x.

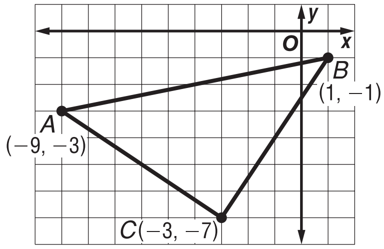


Theorem 8.5 Converse of the Pythagorean Theorem
If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

EXAMPLE Verify a Triangle is a Right Triangle

3 COORDINATE GEOMETRY **Verify that $\triangle ABC$ is a right triangle.**

Use the Distance Formula to determine the lengths of the sides.



$AB = \sqrt{[1 - (-9)]^2 + [-1 - (-3)]^2}$ — $x_1 = -9, y_1 = -3,$ $x_2 = 1, y_2 = -1$

$= \sqrt{\square^2 + \square^2}$ — Subtract.

$= \square$ — Simplify.

$BC = \sqrt{(-3 - 1)^2 + [-7 - (-1)]^2}$ — $x_1 = 1, y_1 = -1,$ $x_2 = -3, y_2 = -7$

$= \sqrt{(-4)^2 + (-6)^2}$ — Subtract.

$= \square$ — Simplify.

$AC = \sqrt{[-3 - (-9)]^2 + [-7 - (-3)]^2}$ — $x_1 = -9, y_1 = -3,$ $x_2 = -3, y_2 = -7$

$= \sqrt{6^2 + (-4)^2}$ — Subtract.

$= \square$ — Simplify.

By the converse of the Pythagorean Theorem, if the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

$(BC)^2 + (AC)^2 = (AB)^2$ — Converse of the Pythagorean Theorem

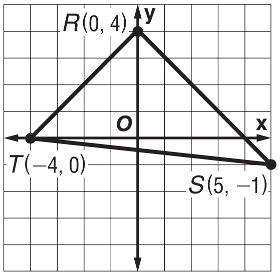
$(\square)^2 + (\square)^2 = (\square)^2$

$\square + \square = 104$ — Simplify.

$\square = 104$ — Add.

Since the sum of the squares of two sides equals the square of the ________, $\triangle ABC$ is a right triangle.

Check Your Progress Verify that $\triangle RST$ is a right triangle.



BUILD YOUR VOCABULARY (page 189)

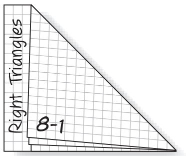
A **Pythagorean triple** is three whole numbers that satisfy the equation ______, where c is the greatest number.

EXAMPLE Pythagorean Triples

FOLDABLES™

ORGANIZE IT

Write examples of Pythagorean triples under the Lesson 8-2 tab.



4 **Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.**

a. 9, 12, 15

Since the measure of the longest side is 15, 15 must be c. Let a and b be 9 and 12.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$9^2 + 12^2 \stackrel{?}{=} 15^2$	$a = 9$, $b = 12$, $c = 15$
$\square + \square \stackrel{?}{=} \square$	Simplify.
$\square = \square$	Add.

These segments form the sides of a right triangle since they satisfy the Pythagorean Theorem. The measures are whole numbers and form a Pythagorean triple.

b. $4\sqrt{3}$, 4, and 8

Since the measure of the longest side is 8, let $c = 8$.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$(4\sqrt{3})^2 + 4^2 \stackrel{?}{=} 8^2$	Substitution
$\square + \square \stackrel{?}{=} \square$	Simplify.
$\square = \square$	Add.

Since these measures satisfy the Pythagorean Theorem, they form a right triangle. Since the measures are not all whole numbers, they do not form a Pythagorean triple.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress **Determine whether each set of measures are the sides of a right triangle. Then state whether they form a Pythagorean triple.**

a. 5, 8, 9

b. $3, \sqrt{5}, \sqrt{14}$

8–3 Special Right Triangles

CA Standard 20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.

MAIN IDEAS

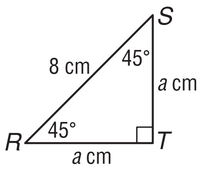
- Use properties of 45°-45°-90° triangles.
- Use properties of 30°-60°-90° triangles.

Theorem 8.6

In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

EXAMPLE **Find the Measure of the Legs**

1 **Find *a*.**



The length of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times as long as a leg of the triangle.

$RS = (ST)\sqrt{2}$

$\_\_\_\_ = \_\_\_\_\sqrt{2}$ $\quad$ $RS = \_\_\_\_$, $ST = \_\_\_\_$

$\frac{8}{\sqrt{2}} = a$ $\quad$ Divide each side by $\sqrt{2}$.

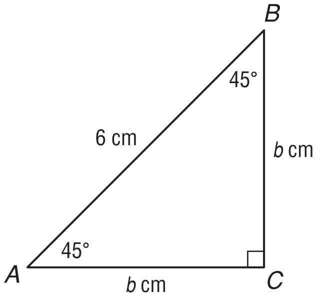
$\frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = a$ $\quad$ Rationalize the denominator.

$\_\_\_\_ = a$ $\quad$ Divide.

REMEMBER IT

To rationalize a denominator, multiply the fraction by 1 in the form of a radical over itself so that the product in the denominator is a rational number.

Check Your Progress Find *b*.

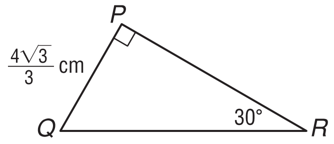


Theorem 8.7

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

EXAMPLE 30°-60°-90° Triangles

2 **Find QR.**



$\overline{PR}$ is the longer leg, $\overline{PQ}$ is the shorter leg, and $\overline{QR}$ is the hypotenuse.

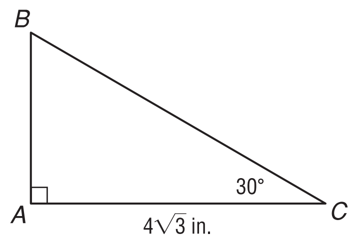
$$PQ = \frac{1}{2}(QR)$$

$\square = \frac{1}{2}(QR)$ $\qquad$ $PQ = \square$

$\square = QR$ $\qquad$ Multiply each side by 2.

The length of QR is $\square$ centimeters.

Check Your Progress Find BC.



REVIEW IT

Explain what it means for a triangle to be classified as a right triangle. (*Lesson 4-1*)

____

____

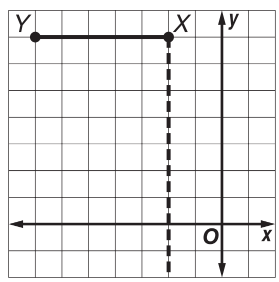
____

____

EXAMPLE Special Triangles in a Coordinate Plane

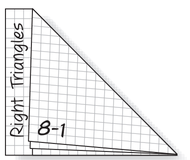
3 **COORDINATE GEOMETRY $\triangle WXY$ is a 30°-60°-90° triangle with right angle X and $\overline{WX}$ as the longer leg. Graph points $X(-2, 7)$ and $Y(-7, 7)$ and locate point W in Quadrant III.**

Graph X and Y. $\overline{XY}$ lies on a horizontal gridline of the coordinate plane. Since $\overline{WX}$ will be perpendicular to $\overline{XY}$, it lies on a vertical gridline. Find the length of $\overline{XY}$.

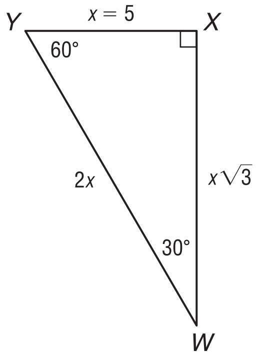


FOLDABLES™ ORGANIZE IT

On the Lesson 8-3 Foldable, include drawings of a 45°-45°-90° triangle and of a 30°-60°-90° triangle. Show how to use the Pythagorean Theorem and properties of these special triangles to find missing parts.



$XY = |-7 - (-2)| = |-7 + 2| =$ ______



$\overline{XY}$ is the shorter leg. $\overline{WX}$ is the longer leg. So, $WX = \sqrt{3}(XY)$. Use XY to find WX.

$WX = \sqrt{3}(XY)$

$= \sqrt{3}($ ______ $)$ $XY =$ ______

$=$ ______

Point W has the same x-coordinate as X. W is located

______ units below X. So, the coordinates of W are

$(-2, 7 - 5\sqrt{3})$ or about ______.

Check Your Progress $\triangle RST$ is a 30°-60°-90° triangle with right angle R and $\overline{RT}$ as the shorter leg. Graph points $T(3, -3)$ and $R(3, -6)$ and locate point S in Quadrant III.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–4 Trigonometry

Standard 18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$. (Key)
Standard 19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side. (Key)

Main Ideas

- Find trigonometric ratios using right triangles.
- Solve problems using trigonometric ratios.

Build Your Vocabulary (page 189)

A ratio of the ________ of sides of a right triangle is called a **trigonometric ratio**.

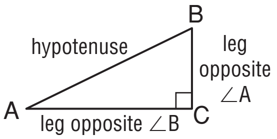
The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**.

Key Concept

Trigonometric Ratios

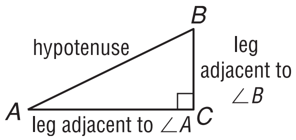
$\sin A = \frac{BC}{AB}$

$\sin B = \frac{AC}{AB}$



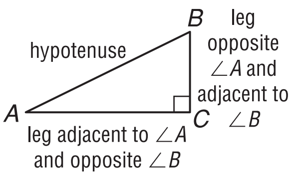
$\cos A = \frac{AC}{AB}$

$\cos B = \frac{BC}{AB}$



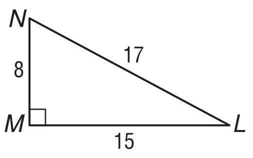
$\tan A = \frac{BC}{AC}$

$\tan B = \frac{AC}{BC}$



EXAMPLE Find Sine, Cosine, and Tangent Ratios

1 Find sin L, cos L, tan L, sin N, cos N, and tan N. Express each ratio as a fraction and as a decimal.



$\sin L = \frac{\text{opposite leg}}{\text{hypotenuse}}$

$= \frac{MN}{LN}$

$= \frac{\square}{\square}$ or 0.47

$\sin N = \frac{\text{opposite leg}}{\text{hypotenuse}}$

$= \frac{LM}{LN}$

$= \frac{\square}{\square}$ or 0.88

$\cos L = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

$= \frac{LM}{LN}$

$= \frac{\square}{\square}$ or 0.88

$\cos N = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

$= \frac{MN}{LN}$

$= \frac{\square}{\square}$ or 0.47

$\tan L = \frac{\text{opposite leg}}{\text{adjacent leg}}$

$= \frac{MN}{LM}$

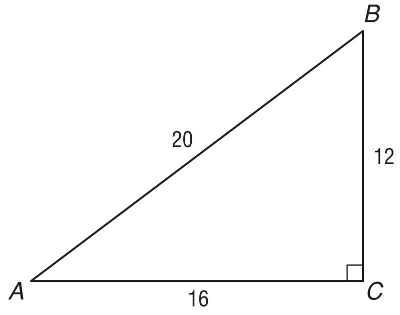
$= \frac{\square}{\square}$ or 0.53

$\tan N = \frac{\text{opposite leg}}{\text{adjacent leg}}$

$= \frac{LM}{MN}$

$= \frac{\square}{\square}$ or 1.88

Check Your Progress Find sin A, cos A, tan A, sin B, cos B, and tan B. Express each ratio as a fraction and as a decimal.



EXAMPLE Evaluate Expressions

2 **Use a calculator to find each value to the nearest ten-thousandth.**

a. tan 56°

TAN [ ] ENTER 1.482560969

tan 56° ≈ [ ]

b. cos 90°

COS [ ] ENTER 0

cos 90° ≈ [ ]

Check Your Progress **Use a calculator to find each value to the nearest ten-thousandth.**

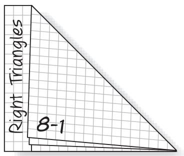
a. sin 48°

b. cos 85°

FOLDABLES™

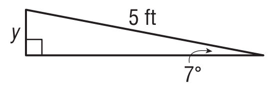
ORGANIZE IT

On your Lesson 8-4 Foldable, include steps to follow when finding trigonometric ratios on a calculator. Be sure to note that the calculator should be in degree mode rather than radian mode.



EXAMPLE Use Trigonometric Ratios to Find a Length

3 EXERCISING A fitness trainer sets the incline on a treadmill to 7°. The walking surface is 5 feet long. Approximately how many inches did the trainer raise the end of the treadmill from the floor?



Let y be the height of the treadmill from the floor in inches.

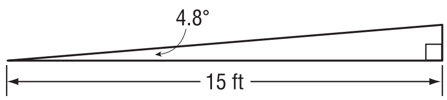
The length of the treadmill is ______ feet, or ______ inches.

$\sin 7° = \frac{\quad}{\quad}$ $\qquad$ $\sin = \frac{\text{leg opposite}}{\text{hypotenuse}}$

______ $\sin 7° = y$ $\qquad$ Multiply each side by ______.

The treadmill is about ______ inches high.

Check Your Progress The bottom of a handicap ramp is 15 feet from the entrance of a building. If the angle of the ramp is about 4.8°, how high does the ramp rise off the ground to the nearest inch?



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–5 Angles of Elevation and Depression

Standard 19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side. (Key)

MAIN IDEAS

- Solve problems involving angles of elevation.
- Solve problems involving angles of depression.

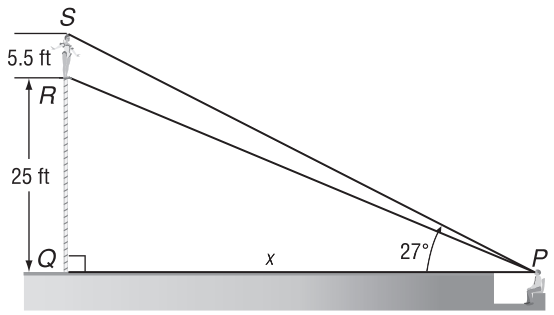
BUILD YOUR VOCABULARY (page 188)

An **angle of elevation** is the angle between the line of sight and the horizontal when an observer looks ________.

EXAMPLE Angle of Elevation

1 CIRCUS ACTS **At the circus, a person in the audience watches the high-wire routine. A 5-foot-6-inch tall acrobat is standing on a platform that is 25 feet off the ground. How far is the audience member from the base of the platform, if the angle of elevation from the audience member's line of sight to the top of the acrobat is 27°?**

Make a drawing.



Since QR is 25 feet and RS is 5 feet 6 inches or ________ feet, QS is 30.5 feet. Let x represent PQ.

$\tan 27° = \dfrac{\_\_\_\_}{\_\_\_\_}$	$\tan = \dfrac{\text{opposite}}{\text{adjacent}}$
$\tan 27° = \dfrac{30.5}{x}$	$QS = 30.5$, $PQ = x$
$x \tan 27° = 30.5$	Multiply each side by x.
$x = \dfrac{30.5}{\tan 27°}$	Divide each side by tan 27°.
$x \approx$ ________	Simplify.

The audience member is about ________ feet from the base of the platform.

REVIEW IT

What is true about the sum of the measures of the acute angles of a right triangle? *(Lesson 4-2)*

____

____

____

____

____

Check Your Progress At a diving competition, a 6-foot-tall diver stands atop the 32-foot platform. The front edge of the platform projects 5 feet beyond the end of the pool. The pool itself is 50 feet in length. A camera is set up at the opposite end of the pool even with the pool's edge. If the camera is angled so that its line of sight extends to the top of the diver's head, what is the camera's angle of elevation to the nearest degree?

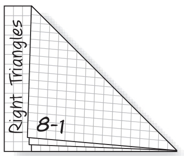
BUILD YOUR VOCABULARY (page 188)

An **angle of depression** is the angle between the line of sight when an observer looks ________, and the horizontal.

FOLDABLES™

ORGANIZE IT

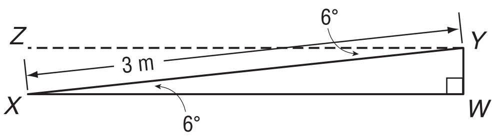
On the Lesson 8-5 Foldable, include a drawing that illustrates angle of elevation and one that illustrates angle of depression.



EXAMPLE Angle of Depression

2 TEST EXAMPLE A wheelchair ramp is 3 meters long and inclines at 6°. Find the height of the ramp to the nearest tenth centimeter.

A 0.3 cm **B** 31.4 cm **C** 31.5 cm **D** 298.4 cm



Read the Test Item

The angle of depression between the ramp and the horizontal is ________. Use trigonometry to find the height of the ramp.

Solve the Test Item

The ground and the horizontal level with the platform to which the ramp extends are ________. Therefore, $m\angle ZYX = m\angle WXY$ since they are ________ angles.

REMEMBER IT

There may be more than one way to solve a problem. Refer to page 465 of your textbook for another method you could use to solve Example 2.

$\sin 6° = \frac{WY}{\square}$ — $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin 6° = \frac{WY}{\square}$ — $XY = \square$

$\square \sin 6° = WY$ — Multiply each side by 3.

$\square \approx WY$ — Simplify.

The height of the ramp is about ☐ meters, or 0.314(100) = 31.4 centimeters. The answer is ☐.

Check Your Progress A roller coaster car is at one of its highest points. It drops at a 63° angle for 320 feet. How high was the roller coaster car to the nearest foot before it began its fall?

Homework Assignment

Page(s):

Exercises:

8–6 The Law of Sines

Preparation for Trigonometry Standard 13.0 Students know the law of sines and the law of cosines **and apply those laws to solve problems.** (Key)

BUILD YOUR VOCABULARY (page 188)

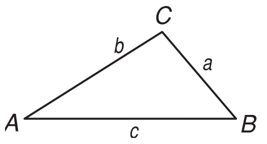
In trigonometry, the **Law of Sines** can be used to find missing parts of triangles that are not right triangles.

MAIN IDEAS

- Use the Law of Sines to solve triangles.
- Solve problems by using the Law of Sines.

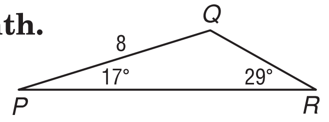
KEY CONCEPT

Law of Sines Let $\triangle ABC$ be any triangle with *a*, *b*, and *c* representing the measures of the sides opposite the angles with measures *A*, *B*, and *C*, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Use the Law of Sines

a. Find *p*. Round to the nearest tenth.



$\frac{\sin P}{p} = \frac{\sin R}{r}$ Law of Sines

$\frac{\sin 17°}{p} = \frac{\sin 29°}{8}$ $m\angle P = 17$, $m\angle R = 29$, $r = 8$

$8 \sin \underline{\hspace{1cm}} = p \sin \underline{\hspace{1cm}}$ Cross products

$\frac{8 \sin \underline{\hspace{1cm}}}{\sin \underline{\hspace{1cm}}} = p$ Divide.

$4.8 \approx p$ Use a calculator.

b. Find $m\angle L$ to the nearest degree in $\triangle LMN$ if $n = 7$, $\ell = 9$, and $m\angle N = 43$.

$\frac{\sin L}{\ell} = \frac{\sin N}{n}$ Law of Sines

$\frac{\sin L}{\underline{\hspace{1cm}}} = \frac{9 \sin 43}{\underline{\hspace{1cm}}}$ $\ell = \underline{\hspace{1cm}}$, $m\angle N = 43$, $n = \underline{\hspace{1cm}}$

$\underline{\hspace{1cm}} \sin L = \underline{\hspace{1cm}} \sin 43$ Cross Products

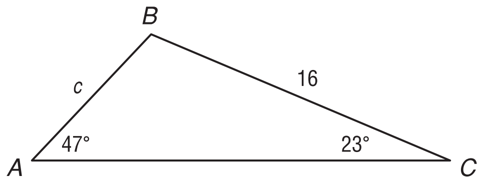
$\sin L = \frac{9 \sin 43}{7}$ Divide.

$L = \sin^{-1}\left(\frac{9 \sin 43}{7}\right)$ Solve for *L*.

$L \approx \underline{\hspace{1cm}}$ Use a calculator.

Check Your Progress

a. Find c.



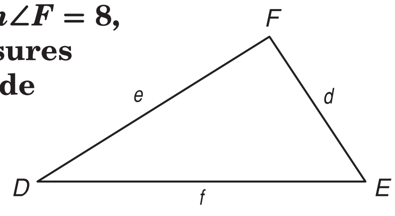
b. Find $m\angle T$ to the nearest degree in $\triangle RST$ if $r = 12$, $t = 7$, and $m\angle R = 76$.

BUILD YOUR VOCABULARY (page 189)

Solving a triangle means finding the measures of all of the ______ and ______ of a triangle.

EXAMPLE Solve Triangles

2 a. Solve $\triangle DEF$ if $m\angle D = 112$, $m\angle F = 8$, and $f = 2$. Round angle measures to the nearest degree and side measures to the nearest tenth.



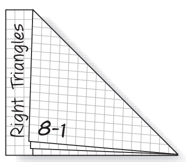
We know the measures of two angles of the triangle. Use the Angle Sum Theorem to find $m\angle E$.

$m\angle D + m\angle E + m\angle F = 180$	Angle Sum Theorem
______ $+ m\angle E +$ ______ $= 180$	$m\angle D =$ ______, $m\angle F =$ ______
______ $+ m\angle E = 180$	Add.
$m\angle E =$ ______	Subtract 120 from each side.

Since we know $m\angle F$ and f, use proportions involving $\frac{\sin F}{f}$.

FOLDABLES ORGANIZE IT

On the Lesson 8-6 Foldable, write an example of how the Law of Sines is used.



To find *d*:

$\frac{\sin F}{f} = \frac{\sin d}{d}$ — Law of Sines

$\frac{\sin 8°}{2} = \frac{\sin \square}{d}$ — Substitute.

$d \sin 8° = 2 \sin \square$ — Cross products.

$d = \frac{2 \sin \square}{\sin 8°}$ — Divide each side by sin 8°.

$d \approx \square$ — Use a calculator.

To find *e*:

$\frac{\sin F}{f} = \frac{\sin E}{e}$ — Law of Sines

$\frac{\sin 8°}{\square} = \frac{\sin 60°}{e}$ — Substitute.

$e \sin 8° = \square \sin 60°$ — Cross products.

$e = \frac{\square \sin 60°}{\sin 8°}$ — Divide each side by sin 8°.

$e \approx \square$ — Use a calculator.

Therefore, $m\angle E = \square$, $d \approx \square$, and $e \approx \square$.

Check Your Progress Solve $\triangle RST$ if $m\angle R = 43$, $m\angle T = 103$, and $r = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.

b. Solve $\triangle HJK$ if $m\angle J = 32$, $h = 30$, and $j = 16$. Round angle measures to the nearest degree and side measures to the nearest tenth.

We know the measure of two sides and an angle opposite one of the sides. Use the Law of Sines.

$\frac{\sin J}{j} = \frac{\sin H}{h}$	Law of Sines
$\frac{\sin 32°}{16} = \frac{\sin H}{30}$	$m\angle J = 32$, $j = 16$, $h = 30$
____ $\sin 32° =$ ____ $\sin H$	Cross products
$\frac{30 \sin 32°}{16} = \sin H$	Divide.
$\sin^{-1}\left(\frac{30 \sin 32°}{16}\right) = H$	Solve for H.
____ $\approx H$	Use a calculator.
$m\angle H + m\angle J = m\angle K = 180$	Angle Sum Theorem
____ $+$ ____ $+ m\angle K = 180$	Substitute.
$m\angle K \approx$ ____	Subtract 116 from each side.
$\frac{\sin J}{j} = \frac{\sin K}{k}$	Law of Sines
$\frac{\sin 32°}{16} = \frac{\sin 64°}{k}$	$m\angle J = 32$, $m\angle K = 64$, $j = 16$
$k \sin 32° = 16 \sin 64°$	Cross products
$k = \frac{16 \sin 64°}{\sin 32°}$	Divide.
$k \approx$ ____	Use a calculator.

So, $m\angle H \approx$ ____, $m\angle K \approx$ ____, and $k \approx$ ____ .

REMEMBER IT If you round before the final answer, your results may differ from results in which rounding was not done until the final answer.

Check Your Progress Solve $\triangle TUV$ if $m\angle T = 54$, $t = 12$, and $v = 9$. Round angle measures to the nearest degree and side measures to the nearest tenth.

____

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

8–7 The Law of Cosines

Preparation for Trigonometry Standard 13.0 Students know the law of sines **and the law of cosines and apply those laws to solve problems.** (Key)

MAIN IDEAS

- Use the Law of Cosines to solve triangles.
- Solve problems by using the Law of Cosines.

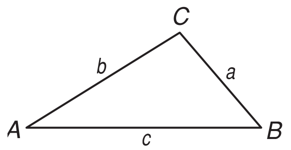
KEY CONCEPT

Law of Cosines Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then the following equations are true.

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

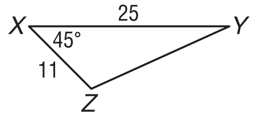


BUILD YOUR VOCABULARY (page 188)

The **Law of Cosines** allows us to solve a triangle when the ____ cannot be used.

EXAMPLE Two Sides and the Included Angle

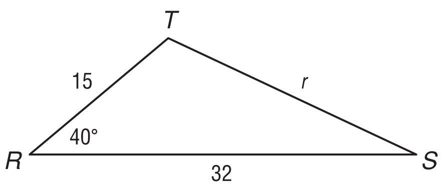
1 **Find x if $y = 11$, $z = 25$, and $m\angle X = 45$.**



Use the Law of Cosines since the measures of two sides and the included angle are known.

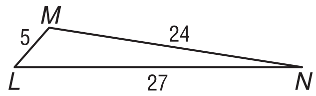
$x^2 = y^2 + z^2 - 2yz \cos$ ____	Law of Cosines
$x^2 = 11^2 + 25^2 - 2(11)(25) \cos$ ____	$y = 11$, $z = 25$, $m\angle$ ____ $=$ ____
$x^2 = 746 - 550 \cos$ ____	Simplify.
$x = \sqrt{746 - 550 \cos \text{____}}$	Take the square root of each side.
$x \approx$ ____	Use a calculator.

Check Your Progress Find r if $s = 15$, $t = 32$, and $m\angle R = 40$.



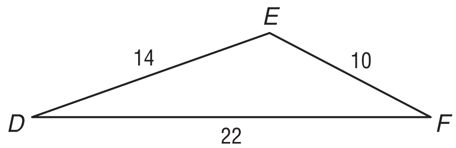
EXAMPLE Three Sides

2 **Find $m\angle L$.**



$\ell^2 = m^2 + n^2 - 2mn \cos L$	Law of Cosines
$\underline{\qquad} = 27^2 + 5^2 - 2(27)(5) \cos L$	Replace ℓ, m, and n.
$\underline{\qquad} = 754 - 270 \cos L$	Simplify.
$-178 = -270 \cos L$	Subtract.
$\frac{-178}{-270} = L$	Divide.
$\cos^{-1}\left(\frac{178}{270}\right) = L$	Solve for L.
$\underline{\qquad} \approx L$	Use a calculator.

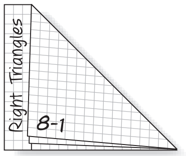
Check Your Progress Find $m\angle F$.



FOLDABLES

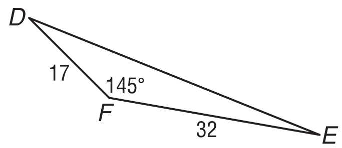
ORGANIZE IT

On the Lesson 8-7 Foldable, try to include your own example of a problem that can be solved using the Law of Cosines. Show how you solved your problem.



EXAMPLE Select a Strategy

3 **Solve $\triangle DEF$. Round angle measures to the nearest degree and side measures to the nearest tenth.**



Since we know the measures of two sides and the included angle, use the Law of Cosines.

$f^2 = d^2 + e^2 - 2de \cos F$	Law of Cosines
$f^2 = 32^2 + 17^2 - 2(32)(17) \cos 145°$	$d = 32$, $e = 17$, $m\angle F = 145$
$f = \sqrt{\underline{\qquad} - 1088 \cos 145°}$	Take the square root of each side.
$f \approx \underline{\qquad}$	Use a calculator.

WRITE IT

Name two cases when you would use the Law of Cosines to solve a triangle and two cases when you would use the Law of Sines to solve a triangle.

Next, we can find $m\angle D$ or $m\angle E$. If we decide to find $m\angle D$, we can use either the Law of Sines or the Law of Cosines to find this value.

$$\frac{\sin F}{f} = \frac{\sin D}{d}$$ Law of Sines

$$\frac{\sin F\ \_\_\_\_}{\_\_\_\_} = \frac{\sin D}{\_\_\_\_}$$ $f =$ ____, $d =$ ____, $m\angle F =$ ____

$$32 \sin 145° = 46.9 \sin D$$ Cross products

$$\frac{32 \sin 145°}{46.9} = \sin D$$ Divide each side by 46.9.

$$\sin^{-1}\left(\frac{32 \sin 145°}{46.9}\right) = D$$ Take the inverse of each side.

$$\_\_\_\_ \approx D$$ Use a calculator.

Use the Angle Sum Theorem to find $m\angle E$.

$$m\angle D + m\angle E + m\angle F = 180$$ Angle Sum Theorem

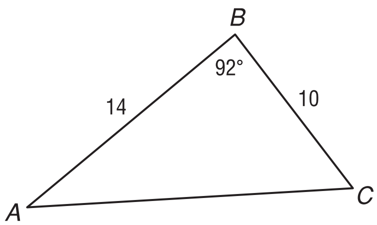
$$\_\_\_\_ + m\angle E + \_\_\_\_ \approx 180$$ $m\angle D \approx 23$, $m\angle F = 145$

$$m\angle E + 168 \approx 180$$

$$m\angle E \approx \_\_\_\_$$ Subtract ____ from each side.

Therefore, $f \approx$ ____, and $m\angle D \approx$ ____, $m\angle E \approx$ ____.

Check Your Progress Determine whether the Law of Sines or the Law of Cosines should be used first to solve $\triangle ABC$. Then solve $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 8

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 8 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 8, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 188–189)* to help you solve the puzzle.

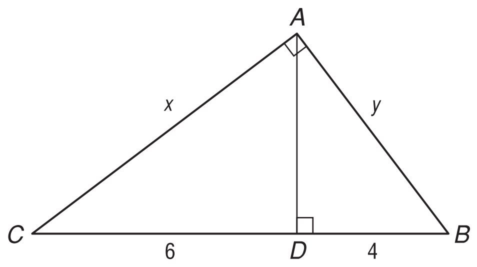
8-1 Geometric Mean

Find the geometric mean between each pair of numbers.

1. 4 and 9

2. 20 and 30

3. Find x and y.



8-2 The Pythagorean Theorem and Its Converse

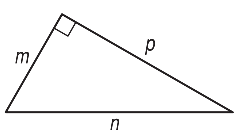
4. For the figure shown, which statements are true?

a. $m^2 + n^2 = p^2$ **b.** $n^2 = m^2 + p^2$

c. $m^2 = n^2 + p^2$ **d.** $m^2 = p^2 - n^2$

e. $p^2 = n^2 - m^2$ **f.** $n^2 - p^2 = m^2$

g. $n = \sqrt{m^2 + p^2}$ **h.** $p = \sqrt{m^2 - n^2}$



Which of the following are Pythagorean triples? Write *yes* or *no*.

5. 10, 24, 26

6. $\sqrt{2}, \sqrt{2}, 2$

7. 10, 6, 8

8-3 Special Right Triangles

Complete each statement.

8. In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a leg by ______.

9. In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by ______.

10. In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by ______.

Indicate whether each statement is *always*, *sometimes*, or *never* true.

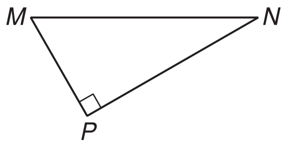
11. The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem. ______

12. The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple. ______

13. The lengths of all three sides of a 30°-60°-90° triangle are positive integers. ______

8-4 Trigonometry

Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.



14. sin N ______

15. cos N ______

16. tan N ______

17. tan M ______

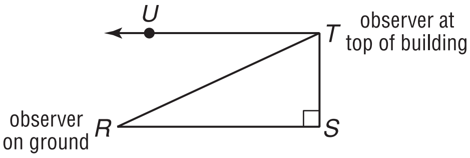
18. sin M ______

19. cos M ______

8-5

Angles of Elevation and Depression

Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.



20. What is the line of sight?

a. line *RS* **b.** line *ST* **c.** line *RT* **d.** line *TU*

21. What is the angle of elevation?

a. $\angle RST$ **b.** $\angle SRT$ **c.** $\angle RTS$ **d.** $\angle UTR$

22. What is the angle of depression?

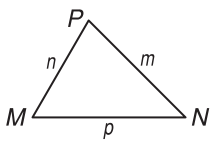
a. $\angle RST$ **b.** $\angle SRT$ **c.** $\angle RTS$ **d.** $\angle UTR$

23. A tree that is 12 meters tall casts a shadow that is 15 meters long. What is the angle of elevation of the sun?

8-6

The Law of Sines

24. Refer to the figure. According to the Law of Sines, which of the following are correct statements?



a. $\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$ **b.** $\frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$

c. $\frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$ **d.** $\frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$

e. $(\sin M)^2 + (\sin N)^2 = (\sin P)^2$ **f.** $\frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$

25. Solve $\triangle ABC$ if $m\angle A = 50$, $m\angle B = 65$, and $a = 12$. Round angle measures to the nearest degree and side measures to the nearest tenth.

8-7

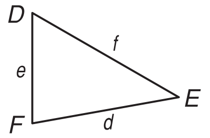
The Law of Cosines

Write *true* or *false* for each statement. If the statement is false, explain why.

26. The Law of Cosines applies to right triangles.

27. The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle.

28. Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$?



a. $d^2 = e^2 + f^2 - ef \cos D$

b. $e^2 = d^2 + f^2 - 2df \cos E$

c. $d^2 = e^2 + f^2 + 2ef \cos D$

d. $f^2 = d^2 + e^2 - 2ef \cos F$

e. $f^2 = d^2 + e^2 - 2de \cos F$

f. $d^2 = e^2 + f^2$

g. $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$

h. $d^2 = e^2 + f^2 - 2ef \cos D$

29. Solve $\triangle DEF$ if $m\angle F = 37$, $d = 3$, and $e = 7$. Round angle measures to the nearest degree and side measures to the nearest tenth.

CHAPTER 8

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 8.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 8 Practice Test on page 491 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 8 Study Guide and Review on pages 486–490 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 491 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 8 Foldable.
- Then complete the Chapter 8 Study Guide and Review on pages 486–490 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 8 Practice Test on page 491 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 9

Transformations

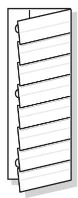
FOLDABLES™

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

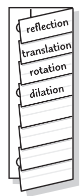
Begin with one sheet of notebook paper.

STEP 1 **Fold** a sheet of notebook paper in half lengthwise.

STEP 2 **Cut** on every third line to create 8 tabs.



STEP 3 **Label** each tab with a vocabulary word from this chapter.



NOTE-TAKING TIP: In addition to writing important definitions in your notes, be sure to include your own examples of the concepts presented.

CHAPTER 9

Build Your Vocabulary

This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
angle of rotation			
center of rotation			
component form			
composition			
dilation			
direction			
invariant points			
isometry			
line of reflection			
line of symmetry			
magnitude			
point of symmetry			
reflection			

Vocabulary Term	Found on Page	Definition	Description or Example
regular tessellation			
resultant			
rotation			
rotational symmetry			
scalar			
scalar multiplication			
semi-regular tessellation			
similarity transformation			
standard position			
tessellation			
translation			
uniform			
vector			

9–1 Reflections

Standard 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including** rotations, translations, and **reflections.** (Key)

MAIN IDEAS

- Draw reflected images.
- Recognize and draw lines of symmetry and points of symmetry.

BUILD YOUR VOCABULARY (pages 220–221)

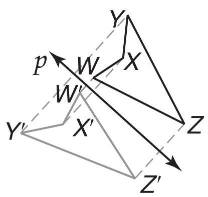
A **reflection** is a transformation representing a ______ of a figure.

The segment connecting a point and its image is ______ to a line m, and is ______ by line m. Line m is called the **line of reflection**.

A reflection is a congruence transformation, or an **isometry**.

EXAMPLE Reflecting a Figure in a Line

1 **Draw the reflected image of quadrilateral *WXYZ* in line *p*.**

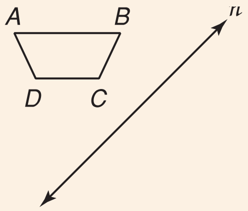


STEP 1 Draw segments perpendicular to line p from each point W, X, Y, and Z.

STEP 2 Locate W', X', Y', and Z' so that line p is the ______ bisector of $\overline{WW'}$, $\overline{XX'}$, $\overline{YY'}$, and $\overline{ZZ'}$. Points W', X', Y', and Z' are the respective images of ______.

STEP 3 Connect vertices W', X', Y', and Z'.

Check Your Progress Draw the reflected image of quadrilateral $ABCD$ in line n.



EXAMPLE Relection in the *x*-axis

1 COORDINATE GEOMETRY **Quadrilateral *ABCD* has vertices *A*(1, 1), *B*(3, 2), *C*(4, –1), and *D*(2, –3).**

a. Graph *ABCD* and its image under reflection in the *x*-axis. Compare the coordinates of each vertex with the coordinates of its image.

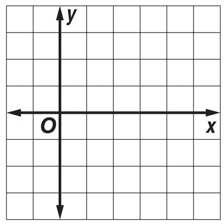
Use the vertical grid lines to find the corresponding point for each vertex so that the *x*-axis is equidistant from each vertex and its image.

$A(1, 1) \longrightarrow A'$

$B(3, 2) \longrightarrow B'$

$C(4, -1) \longrightarrow C'$

$D(2, -3) \longrightarrow D'$



Plot the reflected vertices and connect to form the image $A'B'C'D'$. The *x*-coordinates stay the same, but the *y*-coordinates are opposite. That is, $(a, b) \longrightarrow (a, -b)$.

b. Graph *ABCD* and its image under reflection in the origin. Compare the coordinates of each vertex with the coordinates of its image.

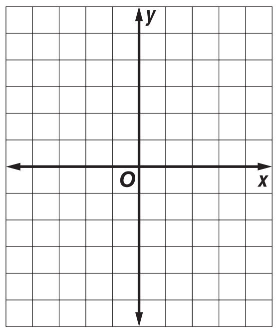
Use the horizontal and vertical distances. From *A* to the origin is 2 units down and 1 unit left. So, A' is located by repeating that pattern from the origin.

$A(1, 1) \longrightarrow A'$

$B(3, 2) \longrightarrow B'$

$C(4, -1) \longrightarrow C'$

$D(2, -3) \longrightarrow D'$

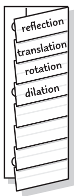


Plot the reflected vertices and connect to form the image $A'B'C'D'$. Both the *x*-coordinates and *y*-coordinates are opposite. That is, $(a, b) \longrightarrow (-a, -b)$.

FOLDABLES™

ORGANIZE IT

Write the definition of reflection under the reflection tab. Include a sketch to illustrate a reflection.



c. Graph *ABCD* and its image under reflection in the line $y = x$. Compare the coordinates of each vertex with the coordinates of its image.

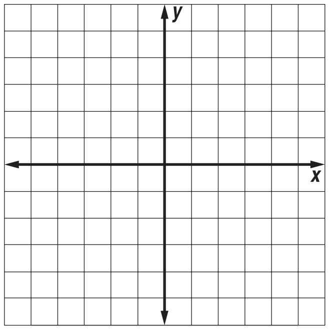
The slope of $y = x$ is 1. $\overline{C'C}$ is perpendicular to $y = x$, so its slope is -1. From C, move up 5 units and to the left 5 units to C'.

$A(1, 1) \longrightarrow A'$

$B(3, 2) \longrightarrow B'$

$C(4, -1) \longrightarrow C'$

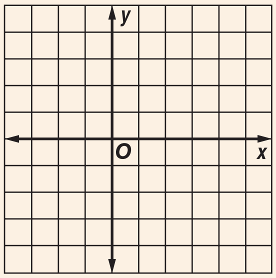
$D(2, -3) \longrightarrow D'$



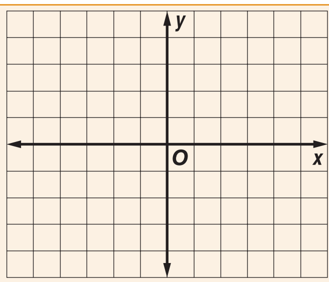
Plot the reflected vertices and connect. Comparing coordinates shows that $(a, b) \longrightarrow (b, a)$.

Check Your Progress

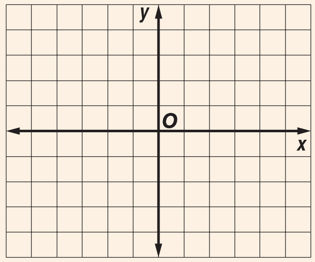
a. Quadrilateral *LMNP* has vertices $L(-1, 1)$, $M(5, 1)$, $N(4, -1)$, and $P(0, -1)$. Graph *LMNP* and its image under reflection in the x-axis.



b. Quadrilateral *LMNP* has vertices $L(-1, 1)$, $M(5, 1)$, $N(4, -1)$, and $P(0, -1)$. Graph *LMNP* and its image under reflection in the origin.



c. Quadrilateral *LMNP* has vertices $L(-1, 1)$, $M(5, 1)$, $N(4, -1)$, and $P(0, -1)$. Graph *LMNP* and its image under reflection in the line $y = x$.



BUILD YOUR VOCABULARY (pages 220–221)

Some figures can be folded so that the two halves ________. The fold is a line of reflection called a **line of symmetry**.

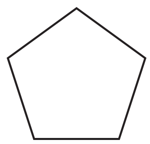
For some figures, a point can be found that is a common point of reflection for all points on a figure. This common point of reflection is called a **point of symmetry**.

EXAMPLE Draw Lines of Summetry

1 **Determine how many lines of symmetry a regular pentagon has. Then determine whether a regular pentagon has a point of symmetry.**

A regular pentagon has ________ lines of symmetry.

A point of symmetry is a point that is a common point of reflection for all points on the figure. There is not one point of symmetry in a regular pentagon.



Check Your Progress Determine how many lines of symmetry an equilateral triangle has. Then determine whether an equilateral triangle has a point of symmetry.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–2 Translations

Standard 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, **translations,** and reflections. (Key)

MAIN IDEAS

- Draw translated images using coordinates.
- Draw translated images by using repeated reflections.

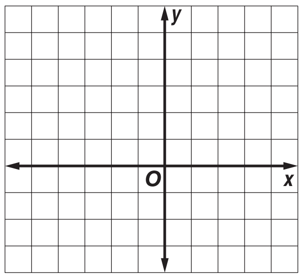
BUILD YOUR VOCABULARY (pages 220–221)

A **translation** is a transformation that moves all points of a figure the same distance in the same ______.

EXAMPLE Translations in the Coordinate Plane

1 COORDINATE GEOMETRY

Parallelogram *TUVW* has vertices *T*(–1, 4), *U*(2, 5), *V*(4, 3), and *W*(1, 2). Graph *TUVW* and its image for the translation $(x, y) \rightarrow (x - 4, y - 5)$.



This translation moved every point of the preimage 4 units left and 5 units down.

$T(-1, 4) \rightarrow T'$ ______ or T' ______

$U(2, 5) \rightarrow U'$ ______ or U' ______

$V(4, 3) \rightarrow V'$ ______ or V' ______

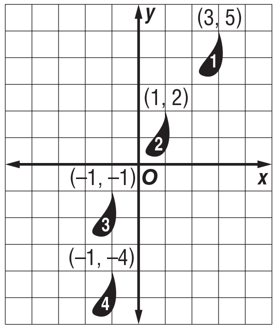
$W(1, 2) \rightarrow W'$ ______ or W' ______

Plot and then connect the translated vertices $T'U'V'$ and W'.

Check Your Progress Parallelogram *LMNP* has vertices *L*(–1, 2), *M*(1, 4), *N*(3, 2), and *P*(1, 0). Graph *LMNP* and its image for the translation $(x, y) \rightarrow (x + 3, y - 4)$.

EXAMPLE Repeated Translations

2 ANIMATION The graph shows repeated translations that result in the animation of a raindrop. Find the translation that moves raindrop 2 to raindrop 3, and then the translation that moves raindrop 3 to raindrop 4.



To find the translation from raindrop 2 to raindrop 3, use the coordinates at the top of each raindrop. Use the coordinates (1, 2) and (−1, −1) in the formula.

$(x, y) \longrightarrow (x + a, y + b)$.

$(1, 2) \longrightarrow (-1, -1)$

$x + a =$ ____

____ $+ a =$ ____ $x = 1$

$a =$ ____ Subtract 1 from each side.

$y + b =$ ____

____ $+ b =$ ____ $y = 2$

$b =$ ____ Subtract 2 from each side.

The translation is (____ , ____) from raindrop 2 to raindrop 3. Use the coordinates (−1, −1) and (−1, −4) to find the translation from raindrop 3 to raindrop 4.

$(x, y) \longrightarrow (x + a, y + b)$

$(-1, -1) \longrightarrow (-1, -4)$

$x + a = -1$ $y + b = -4$

____ $+ a =$ ____ ____ $+ b =$ ____

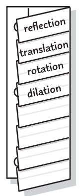
$a =$ ____ $b =$ ____

The translation is ____ from raindrop 3 to raindrop 4.

FOLDABLES™

ORGANIZE IT

Write the definition of *translation* under the translation tab. Draw a figure and its image for a translation on a coordinate plane.



REMEMBER IT

When you translate the point at (*x*, *y*) to the point at (*x* + *a*, *y* + *b*), the number *a* tells how far to move left or right. The number *b* tells how far to move up or down.

9–3 Rotations

Standard 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including rotations,** translations, and reflections. (Key)

Main Ideas

- Draw rotated images using the angle of rotation.
- Identify figures with rotational symmetry.

Build Your Vocabulary (pages 220–221)

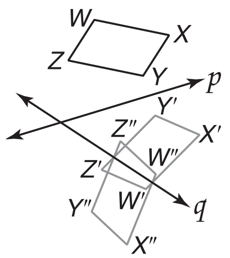
A **rotation** is a transformation that turns every point of a preimage through a specified ______ and ______ about a fixed point.

Postulate 9.1
In a given rotation, if *A* is the preimage, *A′* is the image, and *P* is the center of rotation, then the measure of the angle of rotation, $\angle APA'$ is twice the measure of the acute or right angle formed by the intersecting lines of reflection.

Corollary 9.1
Reflecting an image successively in two perpendicular lines results in a 180° rotation.

EXAMPLE Reflections in Intersection Lines

1 **Find the image of parallelogram *WXYZ* under reflections in line *p* and then line *q*.**



First reflect parallelogram *WXYZ* in line ______. Then label the image *W′X′Y′Z′*.

Next, reflect the image in line ______. Then label the image *W″X″Y″Z″*.

Parallelogram *W″X″Y″Z″* is the image of parallelogram ______ under reflections in lines *p* and *q*.

9–3 Rotations

Standard 22.0 **Students know the effect of rigid motions on figures in the coordinate plane** and space, **including rotations,** translations, and reflections. (Key)

MAIN IDEAS

- Draw rotated images using the angle of rotation.
- Identify figures with rotational symmetry.

BUILD YOUR VOCABULARY (pages 220–221)

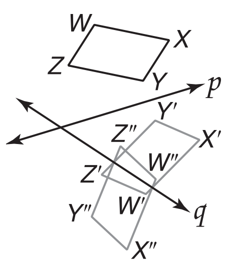
A **rotation** is a transformation that turns every point of a preimage through a specified ______ and ______ about a fixed point.

Postulate 9.1
In a given rotation, if A is the preimage, A' is the image, and P is the center of rotation, then the measure of the angle of rotation, $\angle APA'$ is twice the measure of the acute or right angle formed by the intersecting lines of reflection.

Corollary 9.1
Reflecting an image successively in two perpendicular lines results in a 180° rotation.

EXAMPLE Reflections in Intersection Lines

1 **Find the image of parallelogram *WXYZ* under reflections in line *p* and then line *q*.**

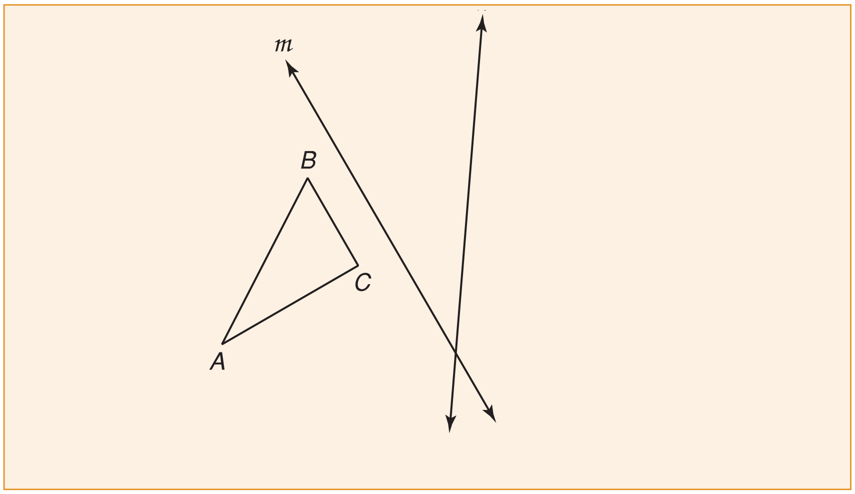


First reflect parallelogram *WXYZ* in line ______. Then label the image *W′X′Y′Z′*.

Next, reflect the image in line ______. Then label the image *W″X″Y″Z″*.

Parallelogram *W″X″Y″Z″* is the image of parallelogram ______ under reflections in lines *p* and *q*.

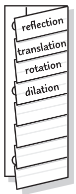
Check Your Progress Find the image of $\triangle ABC$ under reflections in line m and then line n.



FOLDABLES™

ORGANIZE IT

Write the definition of *rotation* under the rotation tab. Include a sketch that indicates the angle, direction, and center of rotation.



EXAMPLE Identifying Rotational Symmetry

2 QUILTS **Using the quilt in Example 3 in the Student Edition, identify the order and magnitude of the symmetry in each part of the quilt.**

a. medium star directly to the left of the large star in the center of the quilt

The medium-sized star has ______ points. So it has a rotational symmetry of order 16. To find the magnitude, divide ______ by 16 to find that the magnitude is ______.

b. tiny star above the medium sized star in part a

The tiny star has 8 points, so the order is ______. Divide 360° by 8 to find that the magnitude is ______.

WRITE IT

How many degrees are in a half-turn rotation? a full-turn rotation?

______

______

______

______

______

______

Check Your Progress **Identify the order and magnitude of the rotational symmetry for each regular polygon.**

a. nonagon ______

b. 18-gon ______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–4 Tessellations

Standard 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections. (Key)

MAIN IDEAS

- Identify regular tessellations.
- Create tessellations with specific attributes.

BUILD YOUR VOCABULARY (pages 220–221)

A pattern that ________ a plane by ________ the same figure or set of figures so that there are no overlapping or empty spaces is called a **tessellation.**

A **regular tessellation** is a tessellation formed by only one type of regular polygon.

Tessellations containing the same arrangement of shapes and ________ at each vertex are called **uniform.**

A uniform tessellation formed using two or more regular ________ is called a **semi-regular tessellation.**

EXAMPLE Regular Polygons

1 Determine whether a regular 16-gon tessellates the plane. Explain.

Use the Interior Angle Theorem. Let ∠1 represent one interior angle of a regular 16-gon.

$$m\angle 1 = \frac{180(n-2)}{n} = \frac{180(16-2)}{16} \text{ or } \_\_\_\_\_\_$$

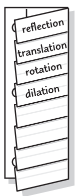
Since ________ is not a factor of 360, a 16-gon will not tessellate the plane.

Check Your Progress Determine whether a regular 20-gon tessellates the plane. Explain.

____

FOLDABLES™ ORGANIZE IT

Write the definitions of *tessellation*, *regular tessellation*, and *uniform tessellation* under the appropriate tabs. In each case, include a sketch that illustrates the definition.



EXAMPLE Semi-Regular Tessellation

2 Determine whether a semi-regular tessellation can be created from regular nonagons and squares, all having sides 1 unit long.

Each interior angle of a regular nonagon measures 140°. Each angle of a square measures 90°. Find whole-number values for

REVIEW IT

What is the sum of the interior angles of a regular pentagon? (*Lesson 8-1*)

____

____

n and s such that $140n + 90s = 360$. All whole numbers greater than 3 will result in a negative value for s.

Let $n = 1$.Let $n = 2$.

$140(1) + 90s = 360$ $140(2) + 90s = 360$

[ ] $+ 90s = 360$ [ ] $+ 90s = 360$

[ ] $= 220$ [ ] $= 80$

$s =$ [ ] $s =$ [ ]

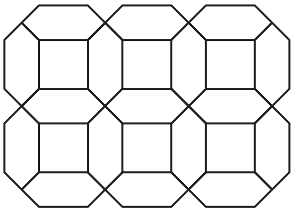
There are no whole number values for n and s so that $140n + 90s = 360$.

Check Your Progress Determine whether a semi-regular tessellation can be created from regular hexagon and squares, all having sides 1 unit long. Explain.

[ ]

EXAMPLE Classify Tessellations

3 STAINED GLASS **Determine whether the pattern is a tessellation. If so, describe it as *uniform*, *regular*, *semi-regular*, or *not uniform*.**



The pattern is a tessellation because at the different vertices

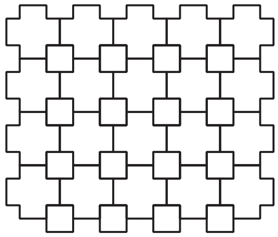
the sum of the angles is [ ]. The tessellation is not

uniform because each vertex does not have the same

arrangement of shapes and [ ].

Check Your Progress Determine whether the pattern is a tessellation. If so, describe it as *uniform*, *regular*, *semi-regular*, or *not uniform*.

[ ]



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–5 Dilations

Standard 11.0 **Students determine how changes in dimensions affect the perimeter, area,** and volume of common geometric figures and solids.

Main Ideas

- Determine whether a dilation is an enlargement, a reduction, or a congruence transformation.
- Determine the scale factor for a given dilation.

Build Your Vocabulary (pages 220–221)

A **dilation** is a transformation that changes the ________ of a figure.

A dilation is a **similarity transformation;** that is, dilations produce ________ figures.

Theorem 9.1
If a dilation with center C and a scale factor of r transforms A to E and B to D, then $ED = |r|(AB)$.

Theorem 9.2
If $P(x, y)$ is the preimage of a dilation centered at the origin with a scale factor r, then the image is $P'(rx, ry)$.

Key Concept

Dilation

- If $|r| > 1$, the dilation is an enlargement.
- If $0 < |r| < 1$, the dilation is a reduction.
- If $|r| = 1$, the dilation is a congruence transformation.
- If $r > 0$, P' lies on $\overrightarrow{CP}$ and $CP = r \cdot CP$. If $r < 0$, P' lies on $\overrightarrow{CP'}$ the ray opposite $\overrightarrow{CP}$, and $CP' = |r| \cdot CP$. The center of a dilation is always its own image.

EXAMPLE Determine Measures Under Dilations

1 **Find the measure of the dilation image or the preimage of $\overline{CD}$ using the given scale factor.**

a. $CD = 15, r = 3$

Since $|r| > 1$, the dilation is an enlargement.

$C'D' = |r|(CD)$	Dilation Theorem
$=$ ________	$|r| = 3$, $CD = 15$
$=$ ________	Multiply.

b. $C'D' = 7, r = -\frac{2}{3}$

Since $0 < |r| < 1$, the dilation is a reduction.

$C'D' = |r|(CD)$	Dilation Theorem
________ $=$ ________ (CD)	$|r| = \frac{2}{3}$, $C'D' = 7$
________ $= CD$	Multiply each side by $\frac{3}{2}$.

Check Your Progress **Find the measure of the dilation image or the preimage of $\overline{AB}$ using the given scale factor.**

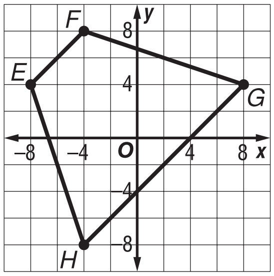
a. $AB = 16$, $r = 22$

b. $A'B' = 24$, $r = \frac{2}{3}$

EXAMPLE Dilations in the Coordinate Plane

2 COORDINATE GEOMETRY **Trapezoid *EFGH* has vertices *E*(–8, 4), *F*(–4, 8), *G*(8, 4) and *H*(–4, –8). Find the image of trapezoid *EFGH* after a dilation centered at the origin with a scale factor of $\frac{1}{4}$. Sketch the preimage and the image. Name the vertices of the image.**

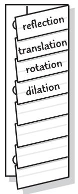
Preimage (*x*, *y*)	Image $\left(\frac{1}{4}x, \frac{1}{4}y\right)$
E(–8, 4)	*E*′
F(–4, 8)	*F*′
G(8, 4)	*G*′
H(–4, –8)	*H*′



FOLDABLES™

ORGANIZE IT

Write the definition of a dilation under the dilation tab. Then show with figures how dilations can result in a larger figure and a smaller figure than the original.



Check Your Progress Triangle *ABC* has vertices *A*(–1, 1), *B*(2, –2), and *C*(–1, –2). Find the image of △*ABC* after a dilation centered at the origin with a scale factor of 2. Sketch the preimage and the image.

WRITE IT

What image is produced when a dilation has a scale factor of $r = 1$?

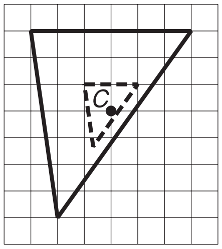
____

____

EXAMPLE Identify Scale Factor

3 **Determine the scale factor used for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.**

a.



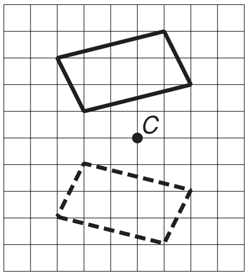
scale factor $= \dfrac{\text{image length}}{\text{preimage length}}$

$=$ ______ ← image length ← preimage length

$=$ ______ Simplify.

Since the scale factor is less than 1, the dilation is a ______.

b.



scale factor $= \dfrac{\text{image length}}{\text{preimage length}}$

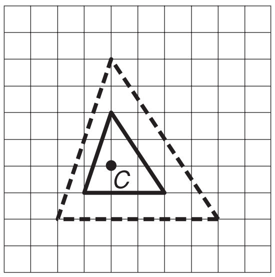
$=$ ______ ← image length ← preimage length

$=$ ______ Simplify.

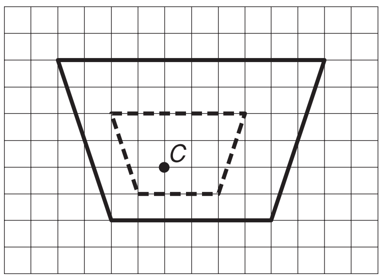
Since the image falls on the opposite side of the center, *C*, than the preimage, the scale factor is ______. So the scale factor is $|-1|$. The absolute value of the scale factor equals 1, so the dilation is a ______ transformation.

Check Your Progress **Determine the scale factor used for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.**

a.



b.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

9–6 Vectors

Standard 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, **translations,** and reflections. (Key)

MAIN IDEAS

- Find magnitudes and directions of vectors.
- Perform translations with vectors.

KEY CONCEPT

Vectors A **vector** is a quantity that has both **magnitude**, or length, and **direction**, and is represented by a directed segment.

BUILD YOUR VOCABULARY (pages 220–221)

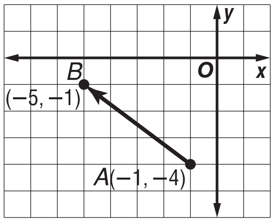
A vector in **standard position** has its initial point at the ______.

The ______ representation of a vector is called the **component form** of the vector.

EXAMPLE Write Vectors in Component Form

1 Write the component form of $\overrightarrow{AB}$.

Find the change of x-values and the corresponding change in y-values.



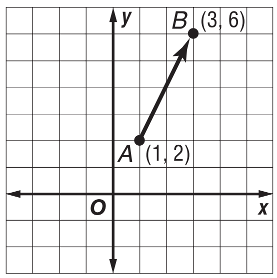
$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$ Component form of vector

$= \langle \_\_\_ - (\_\_\_), \_\_\_ - (\_\_\_) \rangle$ $x_1 =$ ___ , $y_1 =$ ___ , $x_2 =$ ___ , $y_2 =$ ___

$= \langle \_\_\_, \_\_\_ \rangle$ Simplify.

Because the magnitude and direction of a vector are not changed by translation, the vector ______ represents the same vector as $\overrightarrow{AB}$.

Check Your Progress Write the component form of $\overrightarrow{AB}$.



EXAMPLE Magnitude and Direction of a Vector

KEY CONCEPTS

Equal Vectors Two vectors are equal if and only if they have the same magnitude and direction.

Parallel Vectors Two vectors are parallel if and only if they have the same or opposite direction.

1 **Find the magnitude and direction of $\overrightarrow{ST}$ for $S(-3, -2)$ and $T(4, -7)$.**

Find the magnitude.

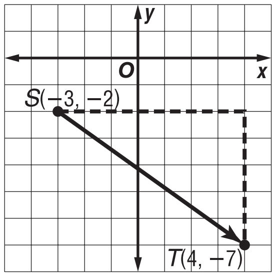
$|\overrightarrow{ST}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \underline{\qquad\qquad\qquad}$ $x_1 = -3, y_1 = -2,$ $x_2 = -4, y_2 = -7$

$= \underline{\qquad}$ Simplify.

$\approx \underline{\qquad}$ Use a calculator.

Graph $\overrightarrow{ST}$ to determine how to find the direction. Draw a right triangle that has $\overrightarrow{ST}$ as its hypotenuse and an acute angle at S.



$\tan S = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\tan = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

$= \dfrac{\underline{\qquad} - \underline{\qquad}}{\underline{\qquad} - \underline{\qquad}}$ Substitution.

$= \dfrac{\underline{\qquad}}{\underline{\qquad}}$

So, $m\angle S = \tan^{-1}\left(\underline{\qquad}\right)$ Simplify.

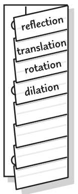
$\approx \underline{\qquad}$ Use a calculator.

A vector in standard position that is equal to $\overrightarrow{ST}$ forms a ______ angle with the positive x-axis in the fourth quadrant. So it forms a 360 + (−35.5) or ______ angle with the positive x-axis.

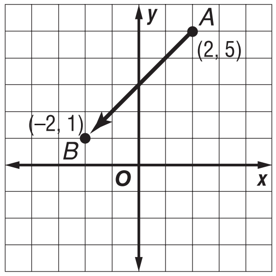
FOLDABLES™

ORGANIZE IT

Write the definition of vector under the vector tab. Include a sketch to illustrate the definition.



Check Your Progress Find the magnitude and direction of $\overrightarrow{AB}$ for $A(2, 5)$ and $B(-2, 1)$.



EXAMPLE Translations with Vectors

3 **Graph the image of quadrilateral *HJLK* with vertices *H*(–4, 4), *J*(–2, 4), *L*(–1, 2) and *K*(–3, 1) under the translation of $\vec{v} = \langle 5, -5 \rangle$.**

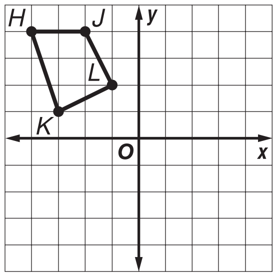
First graph quadrilateral *HJLK*.

Next translate each vertex by $\vec{v}$,

_____ units _____ and

_____ units _____.

Connect the vertices for quadrilateral *H′J′L′K′*.



Check Your Progress Graph the image of triangle *ABC* with vertices *A*(7, 6), *B*(6, 2), and *C*(2, 3) under the translation of $\vec{v}$ $\langle -3, -4 \rangle$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 9

BRINGING IT ALL TOGETHER

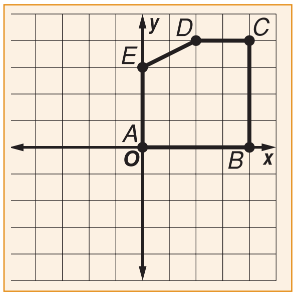
STUDY GUIDE

FOLDABLES	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 9 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 9, go to: **glencoe.com**	You can use your completed **Vocabulary Builder** *(pages 220–221)* to help you solve the puzzle.

9-1

Reflections

1. Draw the reflected image for a reflection of pentagon *ABCDE* in the origin. Label the image of *ABCDE* as *A′B′C′D′E′*.



Determine the number of lines of symmetry for each figure described below. Then determine whether the figure has point symmetry and indicate this by writing *yes* or *no*.

2. a square

3. an isosceles trapezoid

4. the letter E

5. a regular hexagon

9-2

Translations

Find the image of each preimage under the indicated translation.

6. (x, y); 5 units right and 3 units up

7. (x, y); 2 units left and 4 units down

8. $(-7, 5)$; 7 units right and 5 units down

9. $\triangle RST$ has vertices $R(-3, 3)$, $S(0, -2)$, and $T(2, 1)$. Graph $\triangle RST$ and its image $\triangle R'S'T'$ under the translation $(x, y) \longrightarrow (x + 3, y - 2)$. List the coordinates of the vertices of the image.

9-3 Rotations

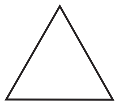
List all of the following types of transformations that satisfy each description: *reflection*, *translation*, *rotation*.

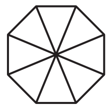
10. The transformation is also called a slide.

11. The transformation is also called a flip.

12. The transformation is also called a turn.

Determine the order and magnitude of the rotational symmetry for each figure.

13. 

14. 

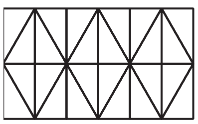
9-4 Tessellations

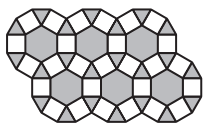
Underline the correct word, phrase, or number to form a true statement.

15. A tessellation that uses only one type of regular polygon is called a (uniform/regular/semi-regular) tessellation.

16. The sum of the measures of the angles at any vertex in any tessellation is (90/180/360).

Write all of the following words that describe each tessellation: *uniform*, *non-uniform*, *regular*, *semi-regular*.

17. 

18. 

9-5 Dilations

19. $\triangle XYZ$ has vertices $X(-4, 3)$, $Y(6, 2)$, and $Z(8, -3)$. Find the coordinates of the image of $\triangle XYZ$ after a dilation centered at the origin with a scale factor of 2.

Each value of *r* represents the scale factor for a dilation. In each case, determine whether the dilation is an *enlargement*, a *reduction*, or a *congruence transformation*.

20. $r = 3$

21. $r = 0.5$

22. $r = -1$

9-6 Vectors

23. Find the magnitude and direction to the nearest degree of $\vec{x} = \langle 2, -5 \rangle$.

Write each vector described below in component form.

24. a vector with initial point (a, b) and endpoint (c, d)

25. a vector in standard position with endpoint $(-3, 5)$

26. a vector with initial point $(2, -3)$ and endpoint $(6, -8)$

CHAPTER 9

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 9.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 9 Practice Test on page 547 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 9 Study Guide and Review on pages 543–546 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 547.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 9 Foldable.
- Then complete the Chapter 9 Study Guide and Review on pages 543–546 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 547.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 10

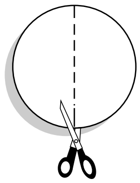
Circles

FOLDABLES™

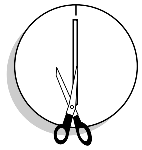
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of plain $8\frac{1}{2}" \times 11"$ paper, and cut out five large circles that are the same size.

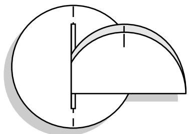
STEP 1 **Fold** two of the circles in half and cut one-inch slits at each end of the folds.



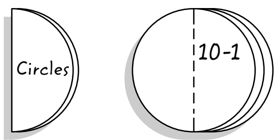
STEP 2 **Fold** the remaining three circles in half and cut a slit in the middle of the fold.



STEP 3 **Slide** the two circles with slits on the ends through the large slit of the other circles.



STEP 4 **Fold** to make a booklet. Label the cover with the title of the chapter and each sheet with a lesson number.



NOTE-TAKING TIP: Take notes in such a manner that someone who did not understand the topic will understand after reading what you have written.

Chapter 10

CHAPTER 10

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 10. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
arc			
center			
central angle			
chord			
circle			
circumference			
circumscribed			
diameter			
inscribed			

Vocabulary Term	Found on Page	Definition	Description or Example
intercepted			
major arc			
minor arc			
pi (π)			
point of tangency			
radius			
secant			
semicircle			
tangent			

10–1 Circles and Circumference

Standard 8.0 Students know, derive, and solve problems involving the perimeter, **circumference,** area, volume, lateral area, and surface area **of common geometric figures.** (Key)

MAIN IDEAS

- Identify and use parts of circles.
- Solve problems involving the circumference of a circle.

BUILD YOUR VOCABULARY (pages 244–245)

A **circle** is the locus of all points in a plane ______ from a given point called the **center** of the circle.

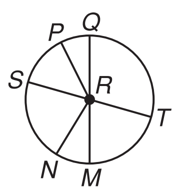
Any segment with ______ that are on the circle is a **chord** of the circle.

A chord that passes through the ______ is a **diameter** of the circle.

Any segment with endpoints that are the ______ and a point on the circle is a **radius.**

EXAMPLE Find Radius and Diameter

1 **Circle R has diameters $\overline{ST}$ and $\overline{QM}$.**



a. If $ST = 18$, find RS.

$r = \frac{1}{2}d$

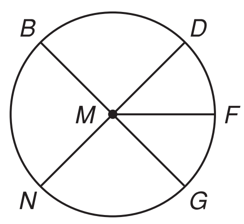
$r = \frac{1}{2}$ ______ or ______ Substitute and simplify.

b. If $RN = 2$, find RP.

Since all radii are congruent, $RN = RP$.

So, $RP =$ ______.

Check Your Progress **Circle M has diameters $\overline{BG}$ and $\overline{DN}$.**



a. If $BG = 25$, find MG.

______

b. If $MF = 8.5$, find MG.

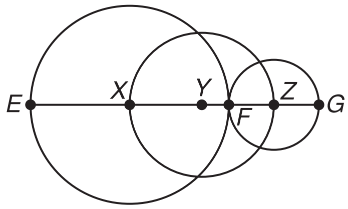
______

REMEMBER IT

The letters d and r are usually used to represent diameter and radius. So, $d = 2r$ and $r = \frac{d}{2}$ or $r = \frac{1}{2}d$.

EXAMPLE Find Measures in Intersecting Circles

2 **The diameters of ⊙X, ⊙Y, and ⊙Z are 22 millimeters, 16 millimeters, and 10 millimeters, respectively. Find EZ.**



Since the diameter of ⊙X is ______, $EF =$ ______.

Since the diameter of ⊙Z is ______, $FZ =$ ______.

$\overline{FZ}$ is part of $\overline{EZ}$.

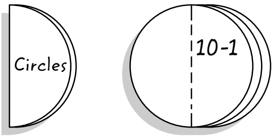
$EF + FZ = EZ$	Segment Addition Postulate
______ + ______ $= EZ$	Substitution
______ $= EZ$	Simplify.

So, the measure of $\overline{EZ}$ is ______ millimeters.

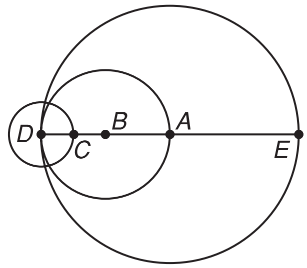
FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 10-1, sketch a circle and label its parts. List the parts of the circle and their definitions.



Check Your Progress **The diameters of ⊙D, ⊙B, and ⊙A are 5 inches, 9 inches, and 18 inches respectively.**



a. Find AC.

______

b. Find EB.

______

BUILD YOUR VOCABULARY (pages 244–245)

The **circumference** of a circle is the ______ around a circle.

The ratio ______ is an irrational number called **pi (π)**.

KEY CONCEPT

Circumference For a circumference of C units and a diameter of d units or a radius of r units, $C = \pi d$ or $C = 2\pi r$.

EXAMPLE Find Circumference, Diameter, and Radius

3 **Find the indicated measure for each circle.**

a. Find C if $r = 13$ inches.

$C = 2\pi r$ Circumference formula

$= 2\pi$ ______ Substitution.

$=$ ______ π or about ______

b. Find C if $d = 6$ millimeters.

$C = \pi d$

$= \pi$ ______ or about ______

c. Find d and r to the nearest hundredth if $C = 65.4$ feet.

$C = \pi d$ Circumference formula

______ $= \pi d$ Substitution

$\frac{65.4}{\pi} = d$ Divide each side by π.

$r = \frac{1}{2}d$ Radius formula

$r = \frac{1}{2}$(______) Substitution.

$r \approx$ ______ Use a calculator.

The diameter is approximately ______ feet and the radius is approximately ______ feet.

Check Your Progress **Find the indicated measure for each circle.**

a. Find C if $d = 3$ feet.

______

b. Find d and r to the nearest hundredth if $C = 16.8$ meters.

______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–2 Angles and Arcs

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and **the properties of circles.** (Key)

BUILD YOUR VOCABULARY (pages 244–245)

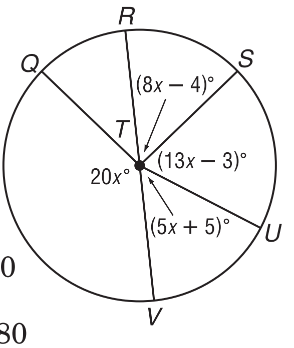
A **central angle** has the ____ of the circle as its vertex, and its sides contain two radii of the circle.

MAIN IDEAS

- Recognize major arcs, minor arcs, semicircles, and central angles and their measures.
- Find arc length.

EXAMPLE Measures of Central Angles

1 ALGEBRA $\overline{RV}$ **is a diameter of** $\odot T$**. Find** $m\angle RTS$**.**



The sum of the measures of $\angle RTS$, $\angle STU$, and $\angle UTV$ is 180.

$$m\angle RTS + m\angle STU + m\angle UTV = 180$$

$(8x - 4) +$ ____ $+$ ____ $= 180$

____ $= 180$ Simplify.

____ $= 182$ Add 2.

$x =$ ____ Divide.

Use the value of x to find $m\angle RTS$.

$m\angle RTS = 8x - 4$ Given

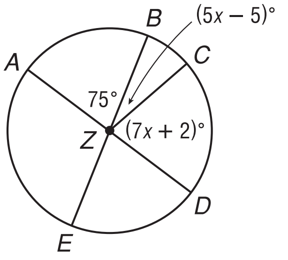
$= 8$ ____ $- 4$ or ____ Substitution

KEY CONCEPT

Sum of Central Angles The sum of the measures of the central angles of a circle with no interior points in common is 360.

FOLDABLES™ Include this concept in your notes.

Check Your Progress Find $m\angle CZD$.



BUILD YOUR VOCABULARY (pages 244–245)

A central angle separates the circle into two parts, each of which is an **arc**.

Theorem 10.1
Two arcs are congruent if and only if their corresponding central angles are congruent.

Postulate 10.1 Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of two arcs.

KEY CONCEPTS

Arcs of a Circle

A **minor arc** can be named by its endpoints and has a measure less than 180.

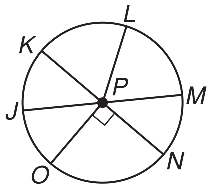
A **major arc** can be named by its endpoints and another point on the arc, and its measure is 360 minus the measure of the related minor arc.

A **semicircle** can be named by its endpoints and another point on the arc, and its measure is 180.

EXAMPLE Measures of Arcs

2 **In $\odot P$, $m\angle NPM = 46$, $\overline{PL}$ bisects $\angle KPM$, and $\overline{OP} \perp \overline{KN}$.**

a. Find $m\overset{\frown}{OK}$.



$\overset{\frown}{OK}$ is a minor arc, so $m\overset{\frown}{OK} = m\angle KPO$.

$\overset{\frown}{KON}$ is a semicircle.

$m\overset{\frown}{ON} = m\angle NPO$	
$= \_\_\_\_\_$	$\angle NPO$ is a right angle.
$m\overset{\frown}{KON} = m\overset{\frown}{OK} + m\overset{\frown}{ON}$	Arc Addition Postulate
$\_\_\_\_\_ = m\overset{\frown}{OK} + \_\_\_\_\_$	Substitution
$\_\_\_\_\_ = m\overset{\frown}{OK}$	Subtract.

b. Find $m\overset{\frown}{LM}$.

$m\overset{\frown}{LM} = \frac{1}{2}\overset{\frown}{KM}$ since $\overline{PL}$ bisects $\angle KPM$.

$\overset{\frown}{KMN}$ is a semicircle.

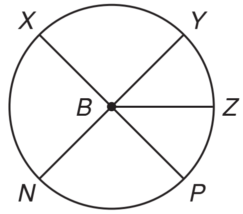
$m\overset{\frown}{KM} + m\overset{\frown}{MN} = m\overset{\frown}{KMN}$	Arc Addition Postulate
$m\overset{\frown}{KM} + \_\_\_\_\_ = \_\_\_\_\_$	$m\overset{\frown}{MN} = m\angle NPM = 46$
$m\overset{\frown}{KM} = \_\_\_\_\_$	Subtract.
$m\overset{\frown}{LM} = \frac{1}{2}\_\_\_\_\_$ or 67	

c. $m\overset{\frown}{JKO}$

$\overset{\frown}{JKO}$ is a major arc.

$m\overset{\frown}{JKO} = m\overset{\frown}{JLM} + m\overset{\frown}{MN} + m\overset{\frown}{NO}$	Arc Addition Postulate
$m\overset{\frown}{JKO} = 180 + \_\_\_\_\_ + \_\_\_\_\_$	Substitution
$m\overset{\frown}{JKO} = \_\_\_\_\_$	

Check Your Progress **In $\odot B$, $\overline{XP}$ and $\overline{YN}$ are diameters, $m\angle XBN = 108$, and $\overline{BZ}$ bisects $\angle YBP$. Find each measure.**



a. $m\widehat{YZ}$

b. $m\widehat{XY}$

c. $m\widehat{XNZ}$

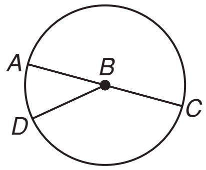
EXAMPLE **Arc Length**

Key Concept

Arc Length Suppose a circle has radius r and circumference C. If an arc of the circle has degree measure A and length ℓ, then $\frac{A}{360} = \frac{\ell}{2\pi r}$ and $\frac{A}{360} \cdot C = \ell$.

3 **In $\odot B$, $AC = 9$ and $m\angle ABD = 40$. Find the length of $\widehat{AD}$.**

In $\odot B$, $AC = 9$ so $C = \pi(9)$ or 9π and $m\widehat{AD} = m\angle ABD$ or 40. Write a proportion to compare each part to its whole.



degree measure of arc → $\frac{\square}{\square} = \frac{\square}{\square}$ ← arc length

degree measure of circumference → ← whole circle

Now solve the proportion for ℓ.

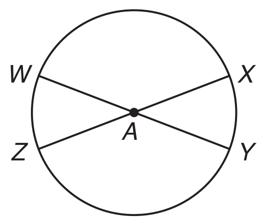
$\frac{\square}{\square} = \frac{\square}{\square}$

$\frac{40}{360}\square = \ell$ Multiply each side by 9π.

$\square = \ell$ Simplify.

The length of $\widehat{AD}$ is ☐ units or about ☐ units.

Check Your Progress In $\odot A$, $AY = 21$ and $m\angle XAY = 45$. Find the length of $\widehat{WX}$.



Homework Assignment

Page(s):

Exercises:

10–3 Arcs and Chords

Main Ideas

- Recognize and use relationships between arcs and chords.
- Recognize and use relationships between chords and diameters.

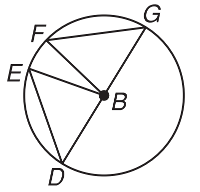
Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and **the properties of circles.** (Key)

Standard 21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons **of circles.** (Key)

Theorem 10.2
In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

EXAMPLE Prove Theorems

1 **PROOF** **Write a proof.**



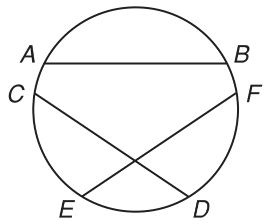
Given: $\overline{DE} \cong \overline{FG}$,
$m\angle EBF = 24$
$\widehat{DFG}$ is a semicircle.

Prove: $m\angle FBG = 78$

Proof:

Statements	Reasons
1. $\overline{DE} \cong \overline{FG}$; $m\angle EBF = 24$; $\widehat{DFG}$ is a semicircle.	**1.** Given
2. $m\widehat{DFG} =$ ______	**2.** Def. of semicircle
3. ______ = ______	**3.** In a circle, if 2 chords are ≅, corr. minor arcs are ≅.
4. $m\widehat{DE} = m\widehat{FG}$	**4.** Def. of ≅ arcs
5. $m\widehat{EF} =$ ______	**5.** Def. of arc measure
6. $m\widehat{ED} + m\widehat{EF} + m\widehat{FG} = m\widehat{DFG}$	**6.** Arc Addition Postulate
7. $m\widehat{FG} +$ ______ $+ m\widehat{FG} = 180$	**7.** ______
8. $2(m\widehat{FG}) = 156$	**8.** Subtraction Property and simplify
9. $m\widehat{FG} =$ ______	**9.** Division Property
10. $m\widehat{FG} = \angle mFBG$	**10.** ______
11. $m\angle FBG = 78$	**11.** ______

Check Your Progress Write a proof.



Given: $\overset{\frown}{AB} \cong \overset{\frown}{EF}$
$\overline{AB} \cong \overline{CD}$

Prove: $\overset{\frown}{CD} \cong \overset{\frown}{EF}$

Proof:

REVIEW IT

What is a biconditional statement? Include an example in your explanation. (*Lesson 2-3*)

____

____

____

____

____

____

BUILD YOUR VOCABULARY (pages 244–245)

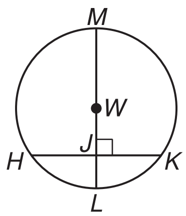
A figure is considered **inscribed** if all of its vertices lie on the circle.

A circle is considered **circumscribed** about a polygon if it contains all the vertices of the polygon.

Theorem 10.3
In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

EXAMPLE Radius Perpendicular to a Chord

2 **Circle W has a radius of 10 centimeters. Radius $\overline{WL}$ is perpendicular to chord $\overline{HK}$, which is 16 centimeters long.**



a. If $m\overset{\frown}{HL} = 53$, find $m\overset{\frown}{MK}$.

You know that $\overset{\frown}{HL} \cong \overset{\frown}{LK}$.

$m\overset{\frown}{MK} + m\overset{\frown}{KL} = m\overset{\frown}{ML}$

$m\overset{\frown}{MK} +$ ______ $= m\overset{\frown}{ML}$ Substitution

$m\overset{\frown}{MK} +$ ______ $=$ ______ Substitution

$m\overset{\frown}{MK} =$ ______ Subtract.

b. Find *JL*.

Draw radius $\overline{WK}$. $\triangle WJK$ is a right triangle.

$WK =$ ☐ $r =$ ☐

$\overline{HL}$ bisects HK. A radius perpendicular to a chord bisects it.

$JK = \frac{1}{2}(HK)$ Definition of segment bisector.

$=$ ☐ or ☐ $HK =$ ☐

Use the Pythagorean Theorem to find *WJ*.

$(WJ)^2 + (JK)^2 = (WK)^2$ Pythagorean Theorem

$(WJ)^2 +$ ☐ $=$ ☐ $JK = 8$, $WK = 10$

$(WJ)^2 +$ ☐ $=$ ☐ Simplify.

$(WJ)^2 =$ ☐ Subtract 64 from each side.

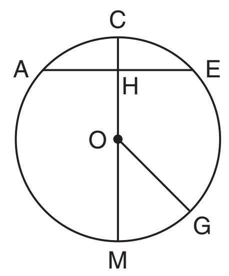
$WJ =$ ☐ Take the square root of each side.

$WJ + JL = WL$ Segment addition

☐ $+ JL =$ ☐ $WJ =$ ☐, $WL =$ ☐

$JL =$ ☐ Subtract 6 from each side.

Check Your Progress **Circle *O* has a radius of 25 units. Radius $\overline{OC}$ is perpendicular to chord $\overline{AE}$, which is 40 units long.**



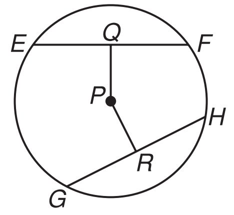
a. If $m\widehat{MG} = 35$, find $m\widehat{CG}$.

b. Find *CH*.

Theorem 10.4
In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

EXAMPLE Chords Equidistant from Center

3 **Chords $\overline{EF}$ and $\overline{GH}$ are equidistant from the center. If the radius of $\odot P$ is 15 and $EF = 24$, find PR and RH.**



$\overline{EF}$ and $\overline{GH}$ are equidistant from P, so $\overline{EF} \cong \overline{GH}$.

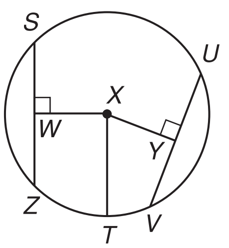
$QF = \frac{1}{2}EF$, so $QF =$ ______ or 12

$RH = \frac{1}{2}GH$, so $RH =$ ______ or 12

Draw $\overline{PH}$ to form a right triangle. Use the Pythagorean Theorem.

$(PR)^2 + (RH)^2 = (PH)^2$	Pythagorean Theorem
$(PR)^2 +$ ______ $=$ ______	$RH =$ ______, $PH =$ ______
$(PR)^2 +$ ______ $=$ ______	Simplify.
$(PR)^2 =$ ______	Subtract.
$PR =$ ______	Take the square root of each side.

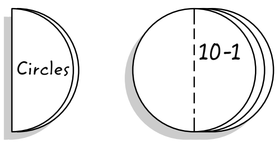
Check Your Progress Chords $\overline{SZ}$ and $\overline{UV}$ are equidistant from the center of $\odot X$. If TX is 39 and XY is 15, find WZ and UV.



FOLDABLES™

ORGANIZE IT

Summarize what you have learned in this lesson about arcs and chords of a circle. Include sketches that illustrate the important facts. Include this summary under the tab for Lesson 10-3.



HOMEWORK ASSIGNMENT

Page(s):
Exercises:

10–4 Inscribed Angles

MAIN IDEAS

- Find measures of inscribed angles.
- Find measures of angles of inscribed polygons.

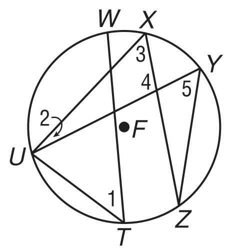
Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and **the properties of circles.** (Key)

Standard 21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, **inscribed angles,** and inscribed and circumscribed polygons **of circles.** (Key)

Theorem 10.5
If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

EXAMPLE Measures of Inscribed Angles

1 **In $\odot F$, $m\widehat{WX} = 20$, $m\widehat{XY} = 40$, $m\widehat{UZ} = 108$, and $m\widehat{UW} = m\widehat{YZ}$. Find the measures of the numbered angles.**



First determine $m\widehat{YZ}$ and $m\widehat{UW}$.
Use the Arc Addition Thereom.

$$m\widehat{WX} + m\widehat{XY} + m\widehat{YZ} + m\widehat{UZ} + m\widehat{UW} = 360$$
$$20 + 40 + m\widehat{YZ} + 108 + m\widehat{YZ} = 360$$
$$\square + 2(m\widehat{YZ}) = \square \quad \text{Simplify.}$$
$$2(m\widehat{YZ}) = \square \quad \text{Subtract.}$$
$$m\widehat{YZ} = \square \quad \text{Divide.}$$

So, $m\widehat{YZ} = \square$ and $m\widehat{UW} = \square$. Now find the measures of the numbered angles.

$m\angle 1 = \frac{1}{2}m\widehat{UW}$
$= \frac{1}{2}\square$ or $\square$

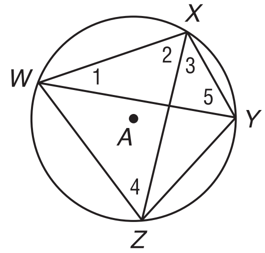
$m\angle 2 = \frac{1}{2}m\widehat{XY}$
$= \frac{1}{2}\square$ or $\square$

$m\angle 3 = \frac{1}{2}m\widehat{UZ}$
$= \frac{1}{2}\square$ or $\square$

$m\angle 5 = \frac{1}{2}m\widehat{UZ}$
$= \frac{1}{2}\square$ or $\square$

$$m\angle 2 + m\angle 3 + m\angle 4 = 180$$
$$\square + \square + m\angle 4 = 180$$
$$m\angle 4 = \square$$

Check Your Progress In $\odot A$, $m\widehat{XY} = 60$, $m\widehat{YZ} = 80$, and $m\widehat{WX} = m\widehat{WZ}$. Find the measures of the numbered angles.



Theorem 10.6
If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

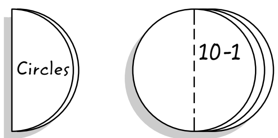
Theorem 10.7
If an inscribed angle intercepts a semicircle, the angle is a right angle.

EXAMPLE Angles of an Inscribed Triangle

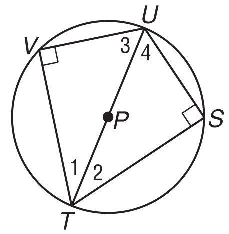
FOLDABLES™

ORGANIZE IT

Explain how to find the measure of an inscribed angle in a circle if you know the measure of the intercepted arc. Include your explanation under the tab for Lesson 10-4.



2 ALGEBRA **Triangles *TVU* and *TSU* are inscribed in $\odot P$ with $\widehat{VU} \cong \widehat{SU}$. Find the measure of each numbered angle if $m\angle 2 = x + 9$ and $m\angle 4 = 2x + 6$.**



$\triangle UVT$ and $\triangle UST$ are right triangles. $m\angle 1 = m\angle 2$ since they intercept congruent arcs. Then the third angles of the triangles are also congruent, so $m\angle 3 = m\angle 4$.

$m\angle 2 + m\angle 4 + m\angle S = 180$

$\_\_\_ + \_\_\_ + \_\_\_ = 180$

$\_\_\_ = 180$ Simplify.

$\_\_\_ = \_\_\_$ Subtract.

$x = \_\_\_$ Divide.

Use the value of x to find the measures of $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.

$m\angle 2 = x + 9$

$= \_\_\_ + 9$ or 34

$m\angle 4 = 2x + 6$

$= 2\_\_\_ + 6$ or 56

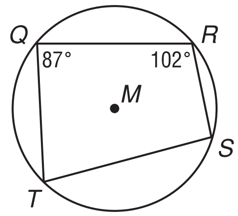
$m\angle 1 = m\angle 2 = \_\_\_$

$m\angle 3 = m\angle 4 = \_\_\_$

EXAMPLE Angles of an Inscribed Quadrilateral

3 Quadrilateral *QRST* is inscribed in ⊙*M*. If $m\angle Q = 87$ and $m\angle R = 102$, find $m\angle S$ and $m\angle T$.

Draw a sketch of this situation.
To find $m\angle S$, we need to know $m\widehat{RQT}$.
To find $m\widehat{RQT}$, first find $m\widehat{RST}$.



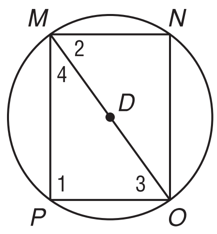
$m\widehat{RST} = 2(m\angle Q)$	Inscribed Angle Theorem
$= 2(\ \_\_\_\ )$ or 174	$m\angle Q =$ ___
$m\widehat{RST} + m\widehat{RQT} = 360$	Sum of angles $= 360$
___ $+ m\widehat{RQT} = 360$	$m\widehat{RST} =$ ___
$m\widehat{RQT} =$ ___	Subtract.
$m\widehat{RQT} = 2(m\angle S)$	Inscribed Angle Theorem
___ $= 2(m\angle S)$	Substitution
___ $= (m\angle S)$	Divide.

To find $m\angle T$, we need to know $m\widehat{QRS}$, but first find $m\widehat{STQ}$.

$m\widehat{STQ} = 2(m\angle R)$	Inscribed Angle Theorem
$= 2(\ \_\_\_\ )$ or 204	$m\angle R =$ ___
$m\widehat{STQ} + m\widehat{QRS} = 360$	Sum of angles in circle $= 360$
___ $+ m\widehat{QRS} = 360$	$m\widehat{STQ} =$ ___
$m\widehat{QRS} =$ ___	Subtract.
$m\widehat{QRS} = 2(m\angle T)$	Inscribed Angle Theorem
___ $= 2(m\angle T)$	$m\widehat{QRS} =$ ___
___ $= m\angle T$	Divide.

Check Your Progress

a. Triangles *MNO* and *MPO* are inscribed in ⊙*D* with $\widehat{MN} \cong \widehat{OP}$. Find the measure of each numbered angle if $m\angle 2 = 4x - 8$ and $m\angle 3 = 3x + 9$.



b. *BCDE* is inscribed in ⊙*X*. If $m\angle B = 99$ and $m\angle C = 76$, find $m\angle D$ and $m\angle E$.

Theorem 10.8
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–5 Tangents

BUILD YOUR VOCABULARY (pages 244–245)

A ray is **tangent** to a circle if the line containing the ray intersects the circle in exactly one point. This point is called the **point of tangency**.

MAIN IDEAS

- Use properties of tangents.
- Solve problems involving circumscribed polygons.

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and **the properties of circles.** (Key)

Standard 21.0 Students prove and solve problems regarding relationships among chords, secants, **tangents,** inscribed angles, and inscribed and circumscribed polygons **of circles.** (Key)

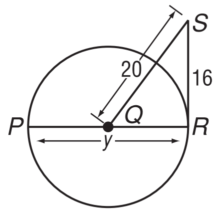
Theorem 10.9
If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Theorem 10.10
If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.

Theorem 10.11
If two segments from the same exterior point are tangent to a circle, then they are congruent.

EXAMPLE Find Lengths

1 ALGEBRA $\overline{RS}$ is tangent to $\odot Q$ at point R. Find y.

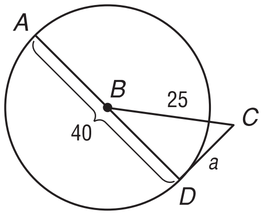


Use the Pythagorean Theorem to find QR, which is one-half the length y.

$(SR)^2 + (QR)^2 = (SQ)^2$	Pythagorean Theorem
____ $+ (QR)^2 =$ ____	$SR =$ ____ , $SQ =$ ____
____ $+ (QR)^2 =$ ____	Simplify.
$(QR)^2 = 144$	Subtract from each side.
$QR =$ ____	Take the square root of each side.

Because y is the length of the diameter, ignore the negative result. Thus, y is twice QR or $y =$ ____.

Check Your Progress $\overline{CD}$ is a tangent to $\odot B$ at point D. Find a.

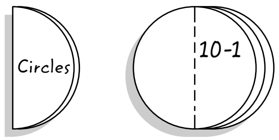


____

FOLDABLES™

ORGANIZE IT

Explain what a tangent to a circle is. Provide a sketch to illustrate the explanation. Include your explanation and sketch under the tab for Lesson 10-5.



REVIEW IT

Write the Pythagorean Theorem in words and in symbols. Include a diagram. (*Lesson 8-2*)

____

____

____

____

____

____

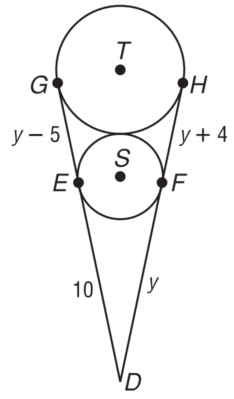
HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Congruent Tangents

2 ALGEBRA **Find y. Assume that segments that appear tangent to circles are tangent.**

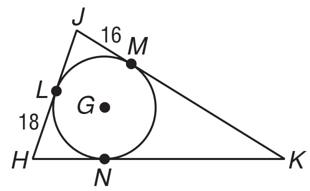


$\overline{DE}$ and $\overline{DF}$ are each drawn from the same exterior point and are both tangent to $\odot S$, so $\overline{DE} \cong \overline{DF}$. Likewise, $\overline{DG}$ and $\overline{DH}$ are both drawn from D and are tangent to $\odot T$, so $\overline{DG} \cong \overline{DH}$. From the Segment Addition Postulate, $DG = DE + EG$ and $DH = DF + FH$.

$DG = DH$	Definition of congruent segments
$DE + EG = DF + FH$	Substitution
____ $+ y - 5 = y +$ ____	$DE =$ ____ , $EG = y - 5$, $DF = y$, $FH =$ ____
$y + 5 = 2y + 4$	Simplify.
$5 = y + 4$	Subtract y from each side.
____ $= y$	Subtract 4 from each side.

EXAMPLE Triangles Circumscribed About a Circle

3 **Triangle HJK is circumscribed about $\odot G$. Find the perimeter of $\triangle HJK$ if $NK = JL + 29$.**



Use Theorem 10.10. $JM = JL =$ ____ ,

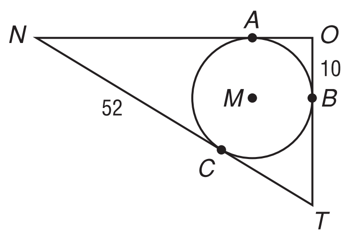
$LH = HN =$ ____ , and $NK =$ ____ .

Since $NK = JL + 29$, $NK = 16 + 29$ or 45. Then $MK = 45$.

$P = JM + MK + HN + NK + JL + LH$

$= 16 + 45 + 18 + 45 + 16 + 18$ or ____ units

Check Your Progress Triangle NOT is circumscribed about $\odot M$. Find the perimeter of $\triangle NOT$ if $CT = NC - 28$.



10–6 Secants, Tangents, and Angle Measures

Main Ideas

- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and **the properties of circles.** (Key)

Standard 21.0 Students prove and solve problems regarding relationships among chords, **secants, tangents,** inscribed angles, and inscribed and circumscribed polygons **of circles.** (Key)

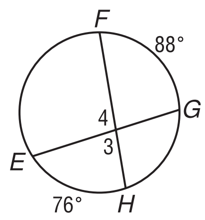
Build Your Vocabulary (pages 244–245)

A line that intersects a circle in exactly two points is called a **secant**.

Theorem 10.12 If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

EXAMPLE Secant-Secant Angle

1 Find $m\angle 4$ if $m\widehat{FG} = 88$ and $m\widehat{EH} = 76$.



Method 1

$m\angle 3 = \frac{1}{2}(m\widehat{FG} + m\widehat{EH})$

$= \frac{1}{2}(\_\_\_\_ + \_\_\_\_)$ or $\_\_\_\_$

$m\angle 4 = 180 - m\angle 3$

$= \_\_\_\_ - \_\_\_\_$ or $\_\_\_\_$

Method 2

Find $m\widehat{EF} + m\widehat{GH}$.

$m\widehat{EF} + m\widehat{GH} = 360 - (m\widehat{FG} + m\widehat{EH})$

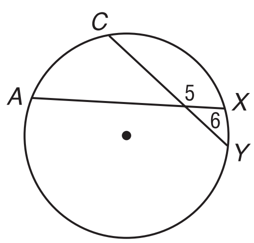
$= 360 - (\_\_\_\_ + \_\_\_\_)$

$= 360 - \_\_\_\_$ or $\_\_\_\_$

$m\angle 4 = \frac{1}{2}(m\widehat{EF} + m\widehat{GH})$

$= \frac{1}{2}(\_\_\_\_)$ or $\_\_\_\_$

Check Your Progress Find $m\angle 5$ if $m\widehat{AC} = 63$ and $m\widehat{XY} = 21$.



Theorem 10.13
If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

Theorem 10.14
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

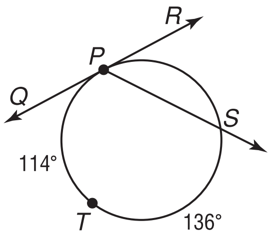
EXAMPLE Secant-Tangent Angle

2 **Find $m\angle RPS$ if $m\widehat{PT} = 114$ and $m\widehat{TS} = 136$.**

$m\widehat{PS} = 360 - m\widehat{PTS}$

$= 360 - ($ _____ $+$ _____ $)$

$=$ _____

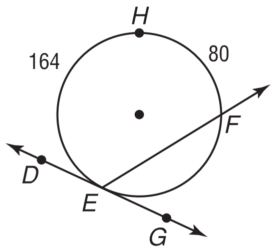


$m\angle RPS = \frac{1}{2}m\widehat{PS}$

$= \frac{1}{2}($ _____ $)$ or _____

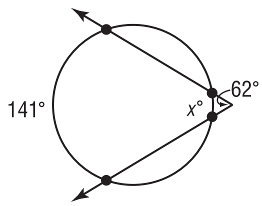
Check Your Progress Find $m\angle FEG$ if $m\widehat{HF} = 80$ and $m\widehat{HE} = 164$.

_____



EXAMPLE Secant-Secant Angle

3 **Find x.**



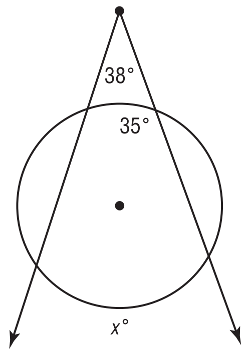
_____ $= \frac{1}{2}(141 - x)$ — Theorem 10.14

_____ $= 141 - x$ — Multiply each side by 2.

_____ $= 141$ — Add x to each side.

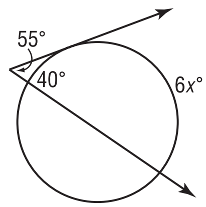
$x =$ _____ — Subtract 124 from each side.

Check Your Progress Find x.



EXAMPLE Secant-Tangent Angle

4 **Find x.**



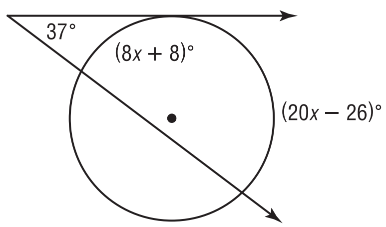
$$55 = \frac{1}{2}(6x - 40)$$

______ = ______ − ______ Multiply each side by 2.

______ = ______ Add ______ to each side.

______ $= x$ Divide each side by ______.

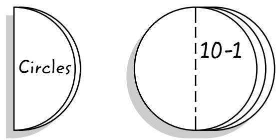
Check Your Progress Find x.



FOLDABLES™

ORGANIZE IT

Devise a chart that summarizes how the measure of an angle formed by chords, secants, or tangents is related to the measures of intercepted arcs. Include your chart under the tab for Lesson 10-6.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–7 Special Segments in a Circle

Main Ideas

- Find measures of segments that intersect in the interior of a circle.
- Find measures of segments that intersect in the exterior of a circle.

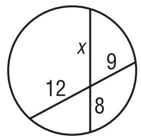
Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and **the properties of circles.** (Key)
Standard 21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons **of circles.** (Key)

Theorem 10.15
If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

EXAMPLE Intersection of Two Chords

1 **Find x.**

To find x, use Theorem 10.15.



____ $\cdot$ ____ $=$ ____ $\cdot$ ____

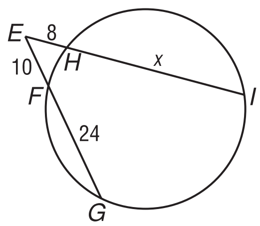
$8x = 108$ Multiply.

$x =$ ____ Divide each side by 8.

Theorem 10.16
If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

EXAMPLE Intersection of Two Secants

2 **Find x if $EF = 10$, $EH = 8$, and $FG = 24$.**



$EH \cdot EI = EF \cdot EG$ Secant Segment Products

____ $\cdot (8 + x) =$ ____ $\cdot (10 + 24)$ Substitution

$64 + 8x =$ ____ Distributive Property

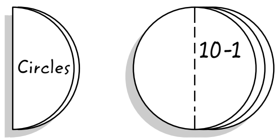
$8x = 276$ Subtract 64 from each side.

$x =$ ____ Divide each side by ____.

FOLDABLES™

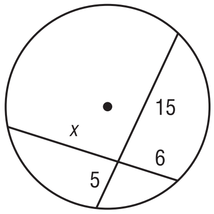
ORGANIZE IT

Write a paragraph to describe what you have learned about the relationships between the lengths of special segments in a circle. Include your paragraph under the tab for Lesson 10-7.

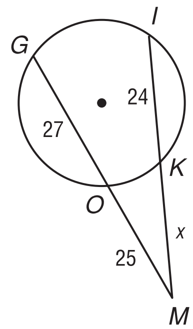


Check Your Progress

a. Find x.



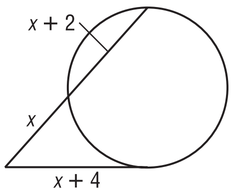
b. Find x if $GO = 27$, $OM = 25$, and $KI = 24$.



Theorem 10.17 If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

EXAMPLE Intersection of Secant and a Tangent

1 **Find x. Assume that segments that appear to be tangent are tangent.**



$(x + 4)^2 = x(x + x + 2)$

$x^2 + 8x + 16 = x(2x + 2)$

$x^2 + 8x + 16 = 2x^2 + 2x$

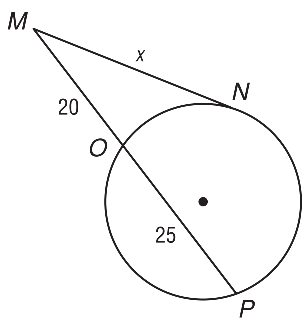
$0 =$ ____

$0 =$ ____

$x - 8 = 0$ $\quad$ $x + 2 = 0$

$x =$ ____ $\quad$ $x =$ ____ Disregard.

Check Your Progress Find x. Assume that segments that appear to be tangent are tangent.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

10–8 Equations of Circles

Standard 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of **equations of** lines and **circles.**

MAIN IDEAS

- Write the equation of a circle.
- Graph a circle on the coordinate plane.

KEY CONCEPT

Standard Equation of a Circle An equation for a circle with center at (h, k) and radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.

EXAMPLE Equation of a Circle

1 **Write an equation for the circle with center at (3, –3), $d = 12$.**

If $d = 12$, $r = 6$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(\underline{\qquad})^2 + [y - (-3)]^2 = \underline{\qquad}$ $(h, k) = (3, -3)$, $r = 6$

$\underline{\qquad\qquad} = \underline{\qquad}$ Simplify.

Check Your Progress **Write an equation for each circle.**

a. center at $(0, -5)$, $d = 18$

b. center at $(7, 0)$, $r = 20$

EXAMPLE Graph a Circle

2 **Graph $(x - 2)^2 + (y + 3)^2 = 4$.**

Compare each expression in the equation to the standard form.

$(x - h)^2 =$

$x - h =$

$-h =$

$h =$

$(y - k)^2 =$

$y - k =$

$-k =$

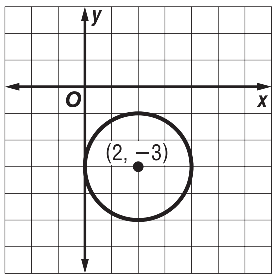
$k =$

$r^2 =$, so $r =$.

The center is at , and

the radius is .

Graph the center. Use a compass set to a width of grid squares to draw the circle.



Check Your Progress Graph $x^2 + (y - 5)^2 = 25$.

HOMEWORK ASSIGNMENT

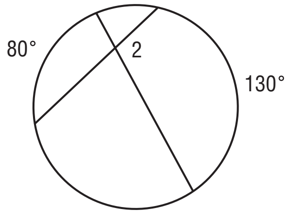
Page(s):

Exercises:

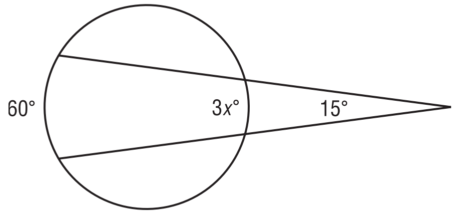
10-6

Secants, Tangents, and Angle Measures

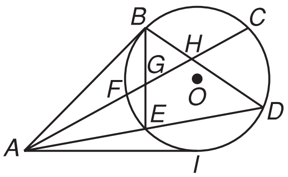
22. Find $m\angle 2$.



23. Find x.



Supply the missing length to complete each equation.



24. $BH \cdot HD = FH \cdot$ ______

25. $AD \cdot AE = AB \cdot$ ______

26. $AF \cdot AC = ($ ______ $)^2$

10-7

Special Segments in a Circle

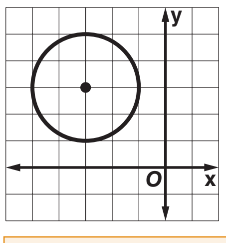
Write an equation for each circle.

27. center at origin, $r = 8$

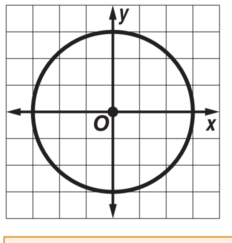
28. center at $(3, 9)$, $r = 1$

Write an equation for each circle.

29.



30.



CHAPTER 10

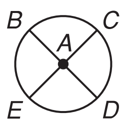
BRINGING IT ALL TOGETHER

STUDY GUIDE

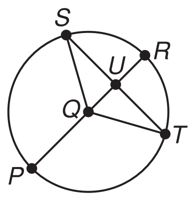
FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 10 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 10, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 244–245)* to help you solve the puzzle.

10-1 Circles and Circumference

1. In ⊙*A*, if *BD* = 18, find *AE*.



Refer to the figure.

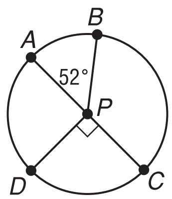


2. Name four radii of the circle.

3. Name two chords of the circle.

10-2 Angles and Arcs

Refer to ⊙*P*. Indicate whether each statement is *true* or *false*.



4. $\widehat{DAB}$ is a major arc.

5. $\widehat{ADC}$ is a semicircle.

6. $\widehat{AD} \cong \widehat{CD}$

7. $\widehat{DA}$ and $\widehat{AB}$ are adjacent arcs.

Refer to ⊙*P*. Give each of the following arc measures.

8. $m\widehat{AB}$

9. $m\widehat{BC}$

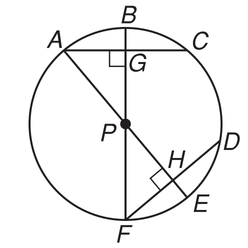
10. $m\widehat{DAB}$

11. $m\widehat{DAC}$

10-3

Arcs and Chords

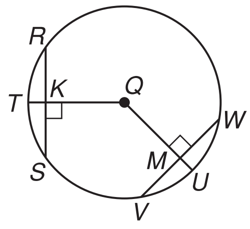
If ⊙*P* has a diameter 40 centimeters long, and *AC* = *FD* = 24 centimeters, find each measure.



12. *PA*

13. *HE*

In ⊙*Q*, *RS* = *VW* and $m\widehat{RS} = 70$. Find each measure.



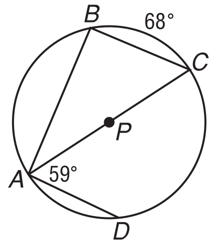
14. $m\widehat{RT}$

15. $m\widehat{VW}$

10-4

Inscribed Angles

Refer to the figure. Find each measure.



16. $m\angle ABC$

17. $m\angle\widehat{AD}$

18. $m\angle BCA$

19. $m\angle\widehat{BCD}$

10-5

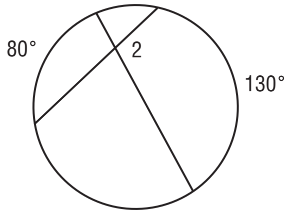
Tangents

20. Two segments from *P* are tangent to ⊙*O*. If $m\angle P = 120$ and the radius of ⊙*O* is 8 feet, find the length of each tangent segment.

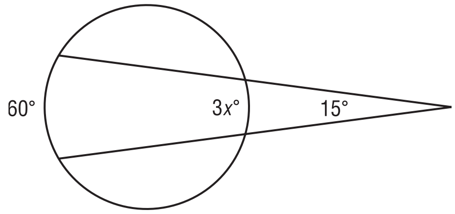
21. Each side of a circumscribed equilateral triangle is 10 meters. Find the radius of the circle.

10-6 Secants, Tangents, and Angle Measures

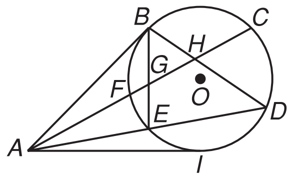
22. Find $m\angle 2$.



23. Find x.



Supply the missing length to complete each equation.



24. $BH \cdot HD = FH \cdot$ ______

25. $AD \cdot AE = AB \cdot$ ______

26. $AF \cdot AC = (\ \_\_\_\_\_\_\ )^2$

10-7 Special Segments in a Circle

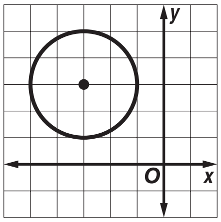
Write an equation for each circle.

27. center at origin, $r = 8$

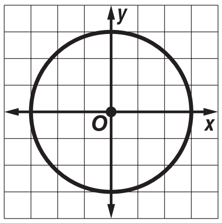
28. center at $(3, 9)$, $r = 1$

Write an equation for each circle.

29.



30.



CHAPTER 10

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 10.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 10 Practice Test on page 625 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 10 Study Guide and Review on pages 620–624 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 625.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 10 Foldable.
- Then complete the Chapter 10 Study Guide and Review on pages 620–624 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 10 Practice Test on page 625.

Student Signature

Parent/Guardian Signature

Teacher Signature

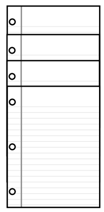
CHAPTER 11

Areas of Polygons and Circles

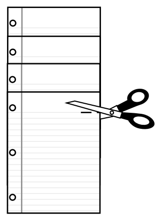
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of notebook paper.

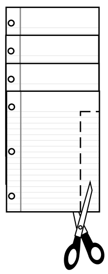
STEP 1 **Stack** 4 of the 5 sheets of notebook paper as illustrated.



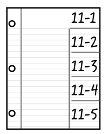
STEP 2 **Cut** in about 1 inch along the heading line on the top sheet of paper.



STEP 3 **Cut** the margins off along the right edge.



STEP 4 **Stack** in order of cuts, placing the uncut fifth sheet at the back. Label the tabs as shown. Staple edge to form a book.



NOTE-TAKING TIP: When you take notes, write a summary of the lesson, or write in your own words what the lesson was about.

Chapter 11

CHAPTER 11

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 11. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
apothem			
composite figure			
geometric probability			
height of a parallelogram			
sector			
segment			

11–1 Areas of Parallelograms

MAIN IDEAS

- Find perimeters and areas of parallelograms.
- Determine whether points on a coordinate plane define a parallelogram.

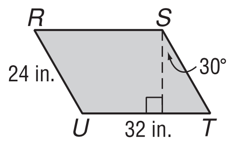
KEY CONCEPT

Area of a Parallelogram
If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.** (Key)
Standard 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, **parallelograms,** and trapezoids. (Key)

EXAMPLE Perimeter and Area of a Parallelogram

1 **Find the perimeter and area of $\square RSTU$.**



Base and Side:

Each base is ______ inches long, and each side is ______ inches long.

Perimeter:

The perimeter of $\square RSTU$ is $2(\_\_\_\_) + 2(\_\_\_\_)$ or ______ inches.

Height:

Use a 30°-60°-90° triangle to find the height. Recall that if the measure of the leg opposite the 30° angle is x, then the length of the hypotenuse is ______, and the length of the leg opposite the 60° angle is $x\sqrt{3}$.

$24 = 2x$ Substitute 24 for the hypotenuse.

$12 = x$ Divide each side by 2.

So, the height is $x\sqrt{3}$ or ______ inches.

Area:

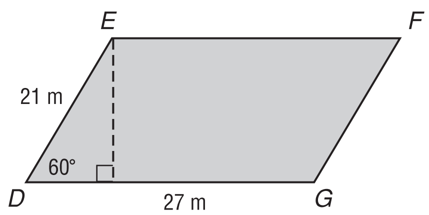
$A = bh$

$=$ ______

$=$ ______ or about ______ square inches.

Check Your Progress

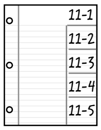
Find the perimeter and area of $\square DEFG$.



FOLDABLES™

ORGANIZE IT

Under the tab for Lesson 11-1, make a sketch to show how a parallelogram can be cut apart and be reassembled to form a rectangle. Write the formula for the area of the parallelogram.



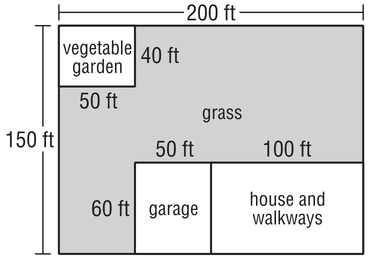
HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Area of a Parallelogram

2 **The Kanes want to sod a portion of their yard. Find the number of square yards of grass needed to sod the shaded region in the diagram.**



The area of the shaded region is the sum of two rectangles. The dimensions of the first rectangle are 50 feet by 150 − 40 or 110 feet. The dimensions of the second rectangle are 150 − 60 or 90 feet and 50 + 100 or 150 feet.

Area of shaded region = Area of Rectangle 1 + Area of Rectangle 2

= ______ + ______

= ______ + ______

= ______ square feet

Next, change square feet to square yards.

$19{,}000 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2} \approx$ ______

The Kanes need approximately ______ square yards of sod.

11–2 Areas of Triangles, Trapezoids, and Rhombi

MAIN IDEAS

- Find areas of triangles.
- Find areas of trapezoids and rhombi.

KEY CONCEPTS

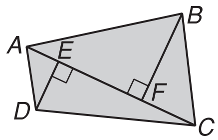
Area of a Triangle If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.

Area of a Trapezoid If a trapezoid has an area of A square units, bases of b_1 units and b_2 units and a height of h units, then $A = \frac{1}{2}h(b_1 + b_2)$.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.** (Key)
Standard 10.0 Students compute areas of polygons, including rectangles, **scalene triangles, equilateral triangles, rhombi,** parallelograms, and **trapezoids.** (Key)

EXAMPLE Area of Triangles

1 Find the area of quadrilateral $ABCD$ if $AC = 35$, $BF = 18$, and $DE = 10$.



area of $ABCD$

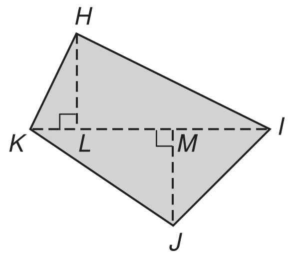
$= \text{area of } \triangle ABC + \text{area of } \triangle ADC$

$= \frac{1}{2}bh + \frac{1}{2}bh$ — Area formula

= ____ + ____ — Substitution

= ____ square units — Simplify.

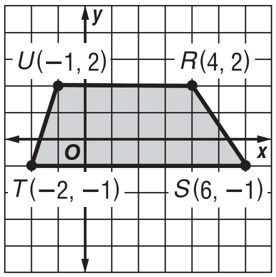
Check Your Progress Find the area of quadrilateral $HIJK$ if $IK = 16$, $HL = 5$, and $JM = 9$.



____

EXAMPLE Area of a Trapezoid on the Coordinate Plane

2 Find the area of trapezoid $RSTU$ with vertices $R(4, 2)$, $S(6, -1)$, $T(-2, -1)$ and $U(-1, 2)$.



Bases:
Since $\overline{UR}$ and $\overline{TS}$ are horizontal, find their length by subtracting the x-coordinates of their endpoints.

$UR = |\ \_\_\_\_ - \_\_\_\_\ |$ $\qquad$ $TS = |\ \_\_\_\_ - \_\_\_\_\ |$

$= |\ \_\_\_\_\ |$ or ____ $\qquad$ $= |\ \_\_\_\_\ |$ or ____

Height:
Because the bases are horizontal segments, the distance between them can be measured on a vertical line. So, subtract the y-coordinates to find the trapezoid's height.

$h = \left|2 - (\square)\right|$ or $\square$

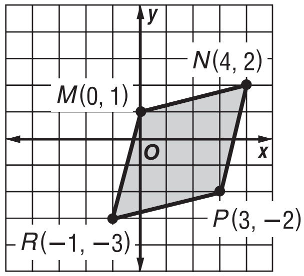
Area:

$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(\square)[(\square) + (\square)]$ or $\square$ square units

EXAMPLE Area of a Rhombus on the Coordinate Plane

3 Find the area of rhombus *MNPR* with vertices at *M*(0, 1), *N*(4, 2), *P*(3, –2), and *R*(–1, –3).



Let $\overline{MP}$ be d_1 and $\overline{NR}$ be d_2.
Use the Distance Formula to find *MP*.

$d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 3)^2 + [\square]^2} = \sqrt{18}$ or $\square$

Use the Distance Formula to find *NR*.

$d^2 = \sqrt{[\square]^2 + [\square]^2}$

$= \square$ or $\square$

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$= \frac{1}{2}(\square)(\square)$ or $\square$ square units

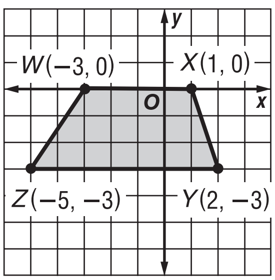
KEY CONCEPT

Area of a Rhombus If a rhombus has an area of *A* square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$.

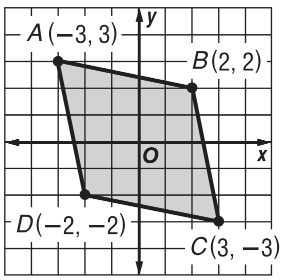
FOLDABLES™ Solve three problems under the tab for Lesson 11-2: find the area of a triangle, find the area of a trapezoid, and find the area of a rhombus.

Check Your Progress **Find the area of each figure.**

a.

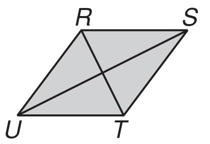


b.



EXAMPLE Algebra: Find Missing Measures

4 **Rhombus *RSTU* has an area of 64 square inches. Find *US* if *RT* = 8 inches.**

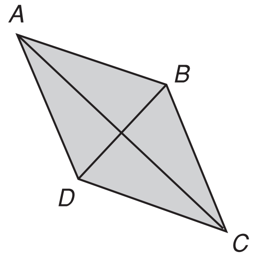


Use the formula for the area of a rhombus and solve for d_2.

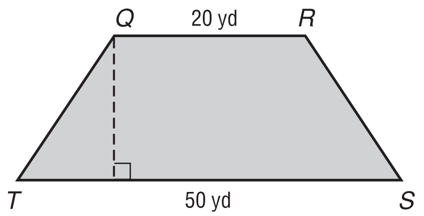
$A = \frac{1}{2}d_1d_2$	Area of rhombus
$\square = \frac{1}{2}(\square)(d_2)$	$A = \square$, $d_1 = \square$
$64 = 4(d_2)$	Multiply.
$\square = d_2$	$\overline{US}$ is $\square$ inches long.

Check Your Progress

a. Rhombus *ABCD* has an area of 81 square centimeters. Find *BD* if *AC* = 6 centimeters.



b. Trapezoid *QRST* has an area of 210 square yards. Find the height of *QRST*.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–3 Areas of Regular Polygons and Circles

MAIN IDEAS

- Find areas of regular polygons.
- Find areas of circles.

BUILD YOUR VOCABULARY (page 278)

An **apothem** is a segment that is drawn from the ______ of a regular polygon ______ to a side of the polygon.

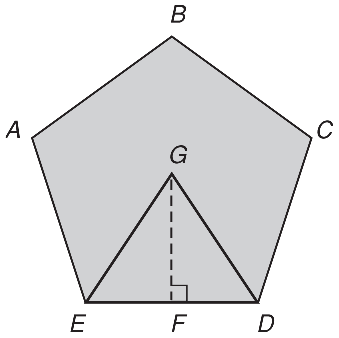
KEY CONCEPT

Area of a Regular Polygon If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

FOLDABLES Write the formula for the area of a regular polygon under the tab for Lesson 11-3.

EXAMPLE Area of a Regular Polygon

1 **Find the area of a regular pentagon with a perimeter of 90 meters.**



Apothem:
The central angles of a regular pentagon are all congruent. Therefore, the measure of each angle is $\frac{360}{5}$ or 72. $\overline{GF}$ is an apothem of pentagon $ABCDE$. It bisects $\angle EGD$ and is a perpendicular bisector of $\overline{ED}$. So, $m\angle DGF = \frac{1}{2}(72)$ or 36. Since the perimeter is 90 meters, each side is 18 meters and $FD = 9$ meters.

Write a trigonometric ratio to find the length of $\overline{GF}$.

$\tan\angle DGF = \frac{DF}{GF}$ — $\tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$

$\tan$ ______ $= \frac{\text{______}}{GF}$ — $m\angle DGF =$ ______, $DF =$ ______

$(GF)\tan$ ______ $=$ ______ — Multiply each side by GF.

$GF = \frac{\text{______}}{\tan \text{______}}$ — Divide each side by tan ______.

$GF \approx$ ______ — Use a calculator.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.** (Key)
Standard 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids. (Key)

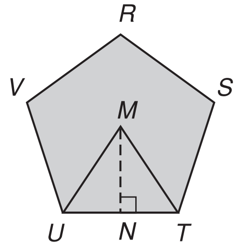
Area: $A = \frac{1}{2}Pa$ — Area of a regular polygon

$\approx$ [ ] — $P =$ [ ], $a \approx$ [ ]

≈ 558 — Simplify.

The area of the pentagon is about [ ] square meters.

Check Your Progress Find the area of a regular pentagon with a perimeter of 120 inches.



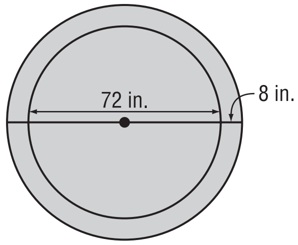
[ ]

EXAMPLE Use Area of a Circle to Solve a Real-World Problem

KEY CONCEPT

Area of a Circle If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$.

2 MANUFACTURING **An outdoor accessories company manufactures circular covers for outdoor umbrellas. If the cover is 8 inches longer than the umbrella on each side, find the area of the cover in square yards.**



The diameter of the umbrella is 72 inches, and the cover must extend 8 inches in each direction. So the diameter of the cover is [ ] + [ ] + [ ] or [ ] inches. Divide by 2 to find

that the radius is 44 inches.

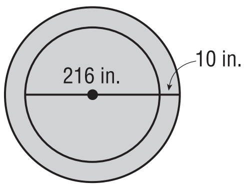
$A = \pi r^2$ — Area of a circle

$= \pi$ [ ] — Substitution

$\approx$ [ ] — Use a calculator.

The area of the cover is [ ] square inches. To convert to square yards, divide by 1296. The area of the cover is [ ] square yards to the nearest tenth.

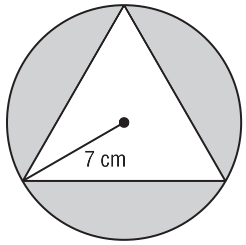
Check Your Progress A swimming pool company manufactures circular covers for above-ground pools. If the cover is 10 inches longer than the pool on each side, find the area of the cover in square yards.



[ ]

EXAMPLE Area of an Inscribed Polygon

3 **Find the area of the shaded region. Assume that the triangle is equilateral.**



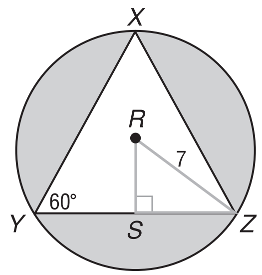
The area of the shaded region is the difference between the area of the circle and the area of the triangle. First, find the area of the circle.

$A = \pi r^2$ Area of a circle

$= \pi$ ☐ Substitution

$\approx$ ☐ Use a calculator.

To find the area of the triangle, use properties of 30°-60°-90° triangles. First, find the length of the base. The hypotenuse of $\triangle RSZ$ is 7, so RS is 3.5 and SZ is $3.5\sqrt{3}$. Since $YZ = 2(SZ)$, $YZ = 7\sqrt{3}$.



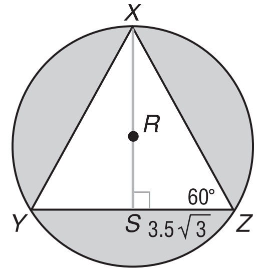
Next, find the height of the triangle, XS. Since $m\angle XZY$ is 60, $XS = 3.5\sqrt{3}(\sqrt{3})$ or 10.5.

Use the formula to find the area of the triangle.

$A = \frac{1}{2}bh$

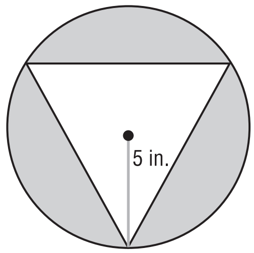
$= \frac{1}{2}($☐$)($☐$)$

$\approx$ ☐



The area of the shaded region is ☐ or approximately ☐ square centimeters to the nearest tenth.

Check Your Progress Find the area of the shaded region. Assume that the triangle is equilateral. Round to the nearest tenth.



REVIEW IT

Draw a 30°-60°-90° triangle with the shorter leg labeled 5 meters long. Label the angles and the remaining sides. *(Lesson 8-3)*

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–4 Areas of Composite Figures

Standard 8.0 Students know, derive, and **solve problems involving the** perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.** (Key)

Main Ideas

- Find areas of composite figures.
- Find areas of composite figures on the coordinate plane.

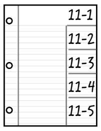
Build Your Vocabulary (page 274)

A **composite figure** is a figure that ______ be classified into the specific shapes that we have studied.

Postulate 11.2
The area of a region is the sum of the areas of all of its nonoverlapping parts.

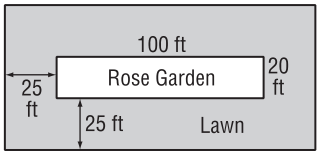
Foldables™ Organize It

Explain how to find the area of a composite figure. Include the explanation under the tab for Lesson 11-4. Also include an example to show how to find the area of such a figure.



EXAMPLE Find the Area of a Composite Figure

1 **A rectangular rose garden is centered in a border of lawn. Find the area of the lawn around the garden in square feet.**



One method to find the area of the lawn around the garden is to find the total area and then subtract the area of the garden.

The overall length of the lawn and garden is 25 + 100 + 25 or ______ feet. The overall width of the lawn and garden is 25 + 20 + 25 or ______ feet.

Area of lawn = Area of lawn and garden − Area of garden

$= \ell_1 w_1 - \ell_2 w_2$	Area formulas
$= (150)(70) - (100)(20)$	$\ell_1 = 150$, $w_1 = 70$, $\ell_2 = 100$, $w_2 = 20$
$= 10{,}500 - 2000$	Multiply.
= ______	Subtract.

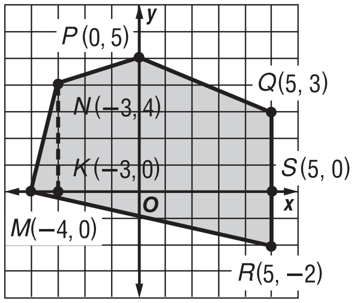
The area of the lawn around the garden is ______ square feet.

EXAMPLE Coordinate Plane

REMEMBER IT

Estimate the area of the figure by counting the unit squares. Use the estimate to determine if your answer is reasonable.

2 **Find the area of polygon *MNPQR*.**



First, separate the figure into regions. Draw an auxiliary line perpendicular to $\overline{QR}$ from M (we will call this point of intersection S) and an auxiliary line from N to the x-axis (we will call this point of intersection K).

This divides the figure into triangle *MRS*, triangle *NKM*, trapezoid *POKN*, and trapezoid *PQSO*.

Now, find the area of each of the figures. Find the difference between x-coordinates to find the lengths of the bases of the triangles and the lengths of the bases of the trapezoids. Find the difference between y-coordinates to find the heights of the triangles and trapezoids.

area of *MNPQR*

= area of $\triangle MRS$ + area of $\triangle NKM$ + area of trapezoid *POKN*
+ area of trapezoid *PQSO*

$= \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2) + \frac{1}{2}h(b_1 + b_2)$

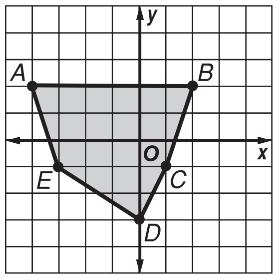
= ______ + ______ + ______

+ ______

= ______

The area of polygon *MNPQR* is ______ square units.

Check Your Progress Find the area of polygon *ABCDE*.



______

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

11–5 Geometric Probability

Standard 8.0 Students know, derive, and **solve problems involving the** perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.** (Key)

Main Ideas

- Solve problems involving geometric probability.
- Solve problems involving sectors and segments of circles.

Build Your Vocabulary (page 274)

Probability that involves a geometric measure such as length or area is called **geometric probability**.

A **sector** of a circle is a region of a circle bounded by a ____ and its ____.

Key Concepts

Probability and Area
If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is

$P(B) = \dfrac{\text{area of region } B}{\text{area of region } A}$.

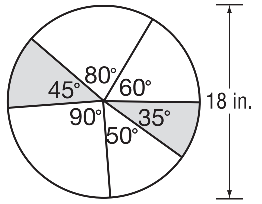
Area of a Sector
If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then

$A = \dfrac{N}{360}\pi r^2$.

EXAMPLE Probability with Sectors

1 Refer to the figure.

a. Find the total area of the shaded sectors.



The shaded sectors have degree measures of 45 and 35 or 80° total. Use the formula to find the total area of the shaded sectors.

$A = \dfrac{N}{360}\pi r^2$ Area of a sector

$= \dfrac{\text{____}}{360}\pi(\text{____})^2$ $N =$ ____ , $r =$ ____

$=$ ____ Simplify.

b. Find the probability that a point chosen at random lies in the shaded region.

To find the probability, divide the area of the shaded sectors by the area of the circle. The area of the circle is πr^2 with a radius of 9.

$P(\text{shaded}) = \dfrac{\text{area of sectors}}{\text{area of circle}}$ Geometric probability formula

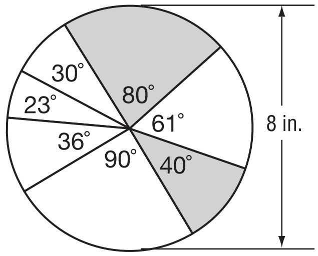
$= \dfrac{\text{____}}{\pi \cdot \text{____}}$ area of circle = ____

area of circle = ____

$\approx$ ____ Use a calculator.

The probability that a random point is in the shaded sectors is about ____.

Check Your Progress Find the area of the shaded sectors. Then find the probability that a point chosen at random lies in the shaded regions.

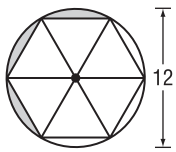


BUILD YOUR VOCABULARY (page 274)

The region of a circle bounded by an arc and a ______ is called a **segment** of a circle.

EXAMPLE Probability with Segments

2 **A regular hexagon is inscribed in a circle with a diameter of 12.**



a. Find the area of the shaded regions.

Area of a sector:

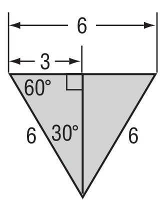
$A = \frac{N}{360}\pi r^2$ — Area of a sector

$= \frac{\square}{360}\pi(\square)^2$ — $N =$ ______, $r =$ ______

$= \square$ or about ______ — Simplify. Then use a calculator.

Area of a triangle:

Since the hexagon was inscribed in the circle, the triangle is equilateral, with each side 6 units long. Use properties of 30°-60°-90° triangles to find the apothem. The value of x is 3 and the apothem is $x\sqrt{3}$ or $3\sqrt{3}$, which is approximately 5.20.



Next, use the formula for the area of a triangle.

$A = \frac{1}{2}bh$ — Area of a triangle

$\approx \frac{1}{2}$ ____ ____ — $b =$ ____ , $h \approx$ ____

$\approx$ ____ — Simplify.

Area of segment:

area of one segment = area of sector − area of triangle

$\approx$ ____ − ____ — Substitution

$\approx$ ____ — Simplify.

Since three segments are shaded, we will multiply this by 3.

3(____) = ____

The area of the shaded regions about ____ square units.

b. Find the probability that a point chosen at random lies in the shaded regions.

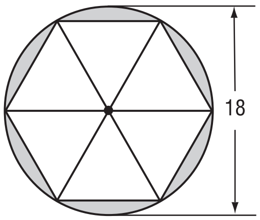
Divide the area of the shaded regions by the area of the circle to find the probability. First, find the area of the circle. The radius is 6, so the area is or about 113.10 square units.

$$P(\text{shaded}) = \frac{\text{area of shaded region}}{\text{area of circle}}$$

$\approx$ ____ or about ____

The probability that a random point is on the shaded region is about ____ or ____ %.

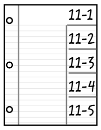
Check Your Progress A regular hexagon is inscribed in a circle with a diameter of 18. Find the area of the shaded regions. Find the probability that a point chosen at random lies in the shaded regions.



FOLDABLES™

ORGANIZE IT

Write a paragraph to summarize what you have learned about finding geometric probabilities. Illustrate your remarks with sketches. Include your paragraph and sketches under the tab for Lesson 11-5.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 11

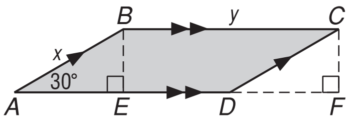
BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 11 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 11, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(page 274)* to help you solve the puzzle.

11-1 Areas of Parallelograms

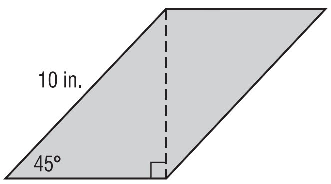
Refer to the figure. Determine whether each statement is *true* or *false*. If the statement is false, explain why.



1. $\overline{AB}$ is an altitude of the parallelogram.

2. $\overline{CD}$ is a base of parallelogram *ABCD*.

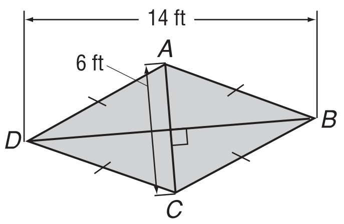
3. Find the perimeter and area of the parallelogram. Round to the nearest tenth if necessary.



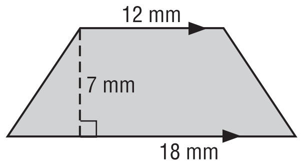
11-2 Area of Triangles, Trapezoids, and Rhombi

Find the area of each quadrilateral.

4.



5.

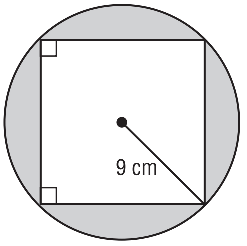


11-3

Areas of Regular Polygons and Circles

6. Find the area of a regular pentagon with a perimeter of 100 meters.

7. Find the area of the shaded region to the nearest tenth. Assume that the polygon is regular.

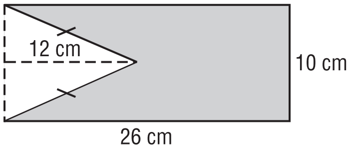


11-4

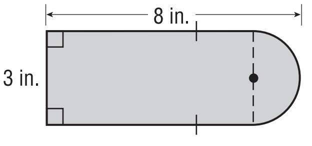
Areas of Composite Figures

Find the area of the shaded region of each figure to the nearest tenth.

8.



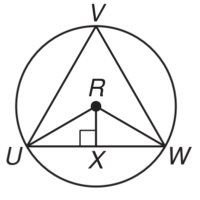
9.



11-5

Geometric Probability

10. Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Find the probability of getting a point. Round to the nearest thousandth.



CHAPTER 11

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 11.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 11 Practice Test on page 675 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 11 Study Guide and Review on pages 672–674 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 675.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 11 Foldable.
- Then complete the Chapter 11 Study Guide and Review on pages 672–674 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 11 Practice Test on page 675.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 12

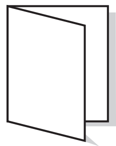
Extending Surface Area

FOLDABLES™

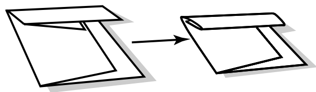
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11" × 17" paper.

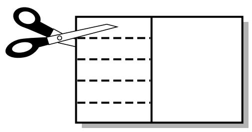
STEP 1 **Fold** lengthwise leaving a two-inch tab.



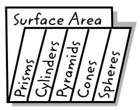
STEP 2 **Fold** the paper into five sections.



STEP 3 **Open.** Cut along each fold to make five tabs.



STEP 4 **Label** as shown.



NOTE-TAKING TIP: When taking notes, place a question mark next to anything you do not understand. Then be sure to ask questions before any quizzes or tests.

CHAPTER 12

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 12. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
axis			
great circle			
hemisphere			
lateral area			
lateral edges			
lateral faces			

Vocabulary Term	Found on Page	Definition	Description or Example
oblique cone			
reflection symmetry			
regular pyramid			
right cone			
right cylinder			
right prism [PRIZ-uhm]			
slant height			

12–1 Representations of Three-Dimensional Figures

Reinforcement of Standard 7MG3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect). (Key)

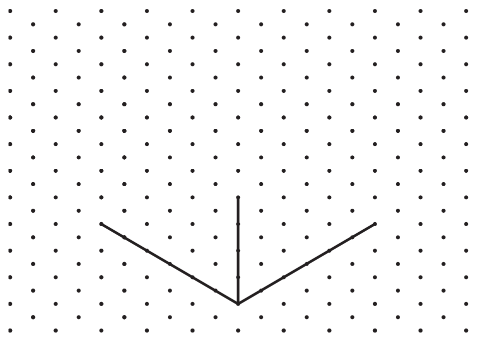
MAIN IDEAS

- Draw isometric views of three-dimensional figures.
- Investigate cross sections of three-dimensional figures.

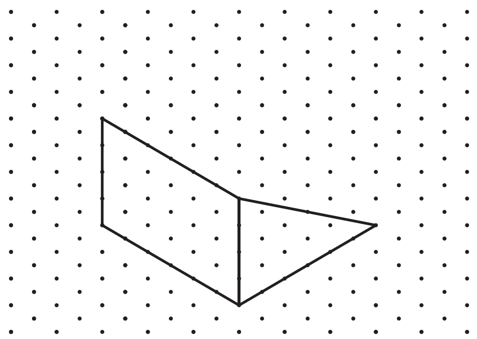
EXAMPLE Draw a Solid

1 **Sketch a triangular prism 6 units high with bases that are right triangles with legs 6 units and 4 units long.**

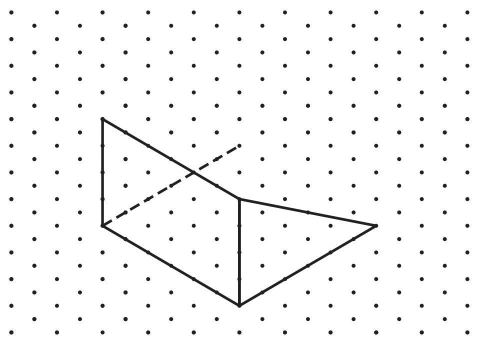
STEP 1 Draw the corner of the solid: 4 units up, 6 units to the left, and 6 units to the right.



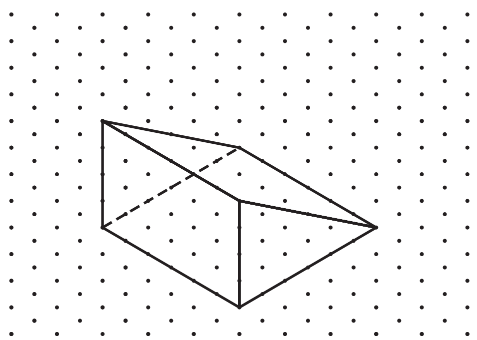
STEP 2 Draw a parallelogram for the back of the solid. Draw the hypotenuse of one base.



STEP 3 Draw a dashed line 6 units long.



STEP 4 Connect the corresponding vertices. Use dashed lines for the hidden edges.

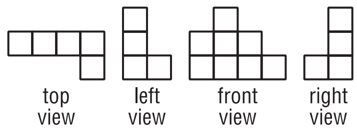


Check Your Progress

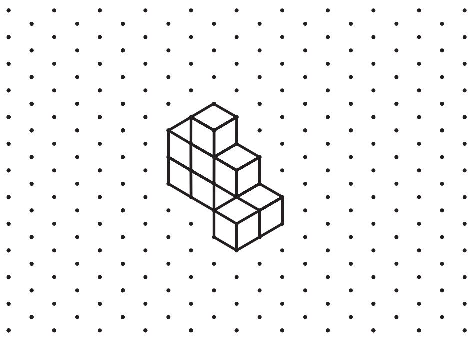
Sketch a rectangular prism 1 unit high, 5 units long, and 2 units wide.

EXAMPLE Use Orthographic Drawings

2 **Draw the corner view of the figure given its orthographic drawing.**



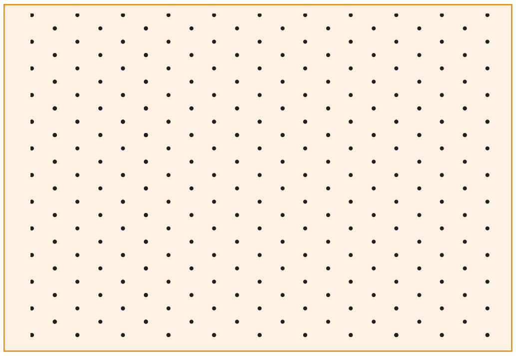
- The top view indicates one row of different heights and one column in the front right.
- The front view indicates that there are ______ standing columns. The first column to the left is ______ blocks high, the second column is ______ blocks high, the third column is ______ blocks high, and the fourth column to the far right is ______ block high. The dark segments indicate breaks in the surface.
- The right view indicates that the front right column is only ______ block high. The dark segments indicate breaks in the surface.
- The ______ column should be visible. Connect the dots on the isometric dot paper to represent the solid.

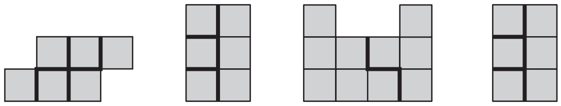


WRITE IT

What views are included in an orthographic drawing? Is each view the same shape?

Check Your Progress Draw the corner view of the figure given its orthogonal drawing.



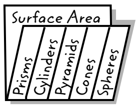


top view left view front view right view

FOLDABLES™

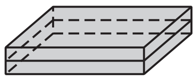
ORGANIZE IT

Sketch a prism, pyramid, cylinder, cone, and sphere. Write the name of each figure below the sketch. Include each sketch under the appropriate tab.



EXAMPLE

3 BAKERY **A customer ordered a two-layer sheet cake. Describe and draw the possible cross sections of the cake.**



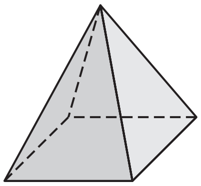
If the cake is cut ________, the cross section will be a ________.

If the cake is cut ________, the cross section will be a ________. Sketch the cross sections.

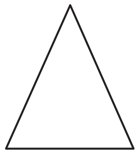
Horizontal: Vertical:

Check Your Progress ARCHITECTURE

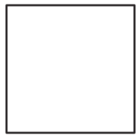
An architect is building a scale model of the Great Pyramids of Egypt. Describe the possible cross sections of the model.



If the pyramid is cut vertically the cross section is a triangle.



If the pyramid is cut horizontally the cross section is a square.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–2 Surface Areas of Prisms

BUILD YOUR VOCABULARY (pages 292–293)

In a prism, the faces that are not ________ are called **lateral faces**.

The lateral faces intersect at the **lateral edges**. Lateral edges are ________ segments.

A prism with lateral edges that are also ________ is called a **right prism**.

The **lateral area** L is the sum of the ________ of the lateral faces.

MAIN IDEAS

- Find lateral areas of prisms.
- Find surface areas of prisms.

KEY CONCEPT

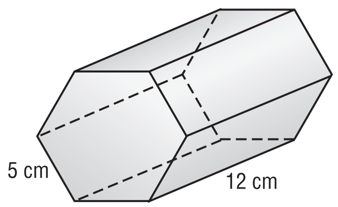
Lateral Area of a Prism If a right prism has a lateral area of L square units, a height of h units, and each base has a perimeter of P units, then $L = Ph$.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, **lateral area, and surface area of common geometric figures.** (Key)

Standard 9.0 Students compute the volumes and **surface areas of prisms,** pyramids, cylinders, cones, and spheres; **and students commit to memory the formulas for prisms,** pyramids, and cylinders.

EXAMPLE Lateral Area of a Hexagonal Prism

1 Find the lateral area of the regular hexagonal prism.



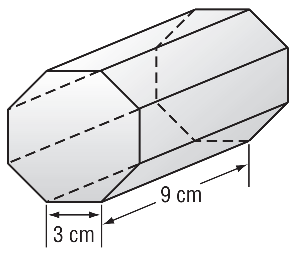
The bases are regular hexagons. So the perimeter of one base is 6(5) or 30 centimeters.

$L = Ph$

$= (\quad)(\quad)$

$= \quad$ square centimeters

Check Your Progress Find the lateral area of the regular octagonal prism.



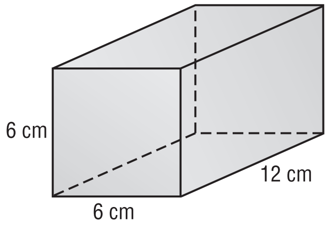
EXAMPLE Surface Area of a Square Prism

KEY CONCEPT

Surface Area of a Prism
If the surface area of a right prism is T square units, its height is h units, and each base has an area of B square units and a perimeter of P units, then $T = L + 2B$.

FOLDABLES™ Sketch a right prism. Below the sketch, explain how finding the perimeter of the base can help you calculate the surface area of the prism. Include the sketch and explanation under the tab for Prisms.

2 **Find the surface area of the square prism.**



$T = L + 2B$ Surface area of a prism

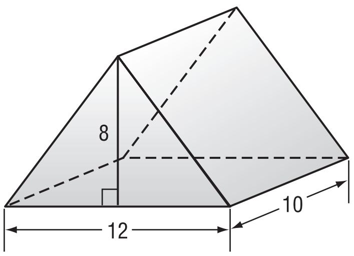
$=$ ☐ $+$ ☐ $L = Ph$

$=$ ☐ $+$ ☐ Substitution

$=$ ☐ Simplify.

The surface area is ☐ square centimeters.

Check Your Progress Find the surface area of the triangular prism.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–3 Surface Areas of Cylinders

Main Ideas

- Find lateral areas of cylinders.
- Find surface areas of cylinders.

Key Concepts

Lateral Area of a Cylinder If a right cylinder has a lateral area of L square units, a height of h units, and the bases have radii of r units, then $L = 2\pi rh$.

Surface Area of a Cylinder If a right cylinder has a surface area of T square units, a height of h units, and the bases have radii of r units, then $T = 2\pi rh + 2\pi r^2$.

FOLDABLES™ Take notes about cylinders under the Cylinders tab.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, **lateral area, and surface area of common geometric figures.** (Key)
Standard 9.0 Students compute the volumes and **surface areas of** prisms, pyramids, **cylinders,** cones, and spheres; **and students commit to memory the formulas for** prisms, pyramids, and **cylinders.**

Build Your Vocabulary (pages 292–293)

The **axis** of a cylinder is the segment with endpoints that are ______ of the circular bases.

An **altitude** of a cylinder is a segment that is ______ to the bases of the cylinder and has its endpoints on the bases.

If the axis is also the ______, then the cylinder is called a **right cylinder**. Otherwise, the cylinder is an **oblique cylinder**.

EXAMPLE Lateral Area of a Cylinder

1 MANUFACTURING **A fruit juice can is cylindrical with aluminum sides and bases. The can is 12 centimeters tall, and the diameter of the can is 6.3 centimeters. How many square centimeters of aluminum are used to make the sides of the can?**

The aluminum sides of the can represent the ______ area of the cylinder. If the diameter of the can is 6.3 centimeters, then the radius is ______ centimeters.
The height is 12 centimeters. Use the formula to find the lateral area.

$L = 2\pi rh$	Lateral area of a cylinder
$= 2\pi(\_\_\_\_)(\_\_\_\_)$	$r =$ ____, $h =$ ____
$\approx$ ____	Use a calculator.

About ______ square centimeters of aluminum are used to make the sides of the can.

Check Your Progress A set of toy blocks are sold in a cylindrical shape container. A product label wraps around all sides of the container without any overlaps or gaps. How much paper is used to make the label the appropriate size if the diameter of the container is 12 inches and the height is 18 inches?

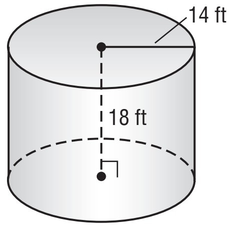
REVIEW IT

What is the area of a circle with a diameter of 7 centimeters? (*Lesson 11-3*)

EXAMPLE Surface Area of a Cylinder

2 **Find the surface area of the cylinder.**

The radius of the base and the height of the cylinder are given. Substitute these values in the formula to find the surface area.



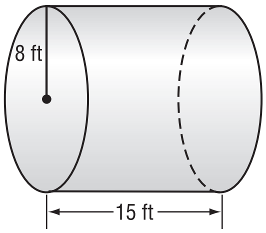
$T = 2\pi rh + 2\pi r^2$ Surface area of a cylinder

$= 2\pi(\_\_\_)(\_\_\_) + 2\pi(\_\_\_)^2$ $r = \_\_\_$, $h = \_\_\_$

$\approx \_\_\_$ Use a calculator.

The surface area is approximately ______ square feet.

Check Your Progress Find the surface area of the cylinder.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

12–4

Surface Areas of Pyramids

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, **lateral area, and surface area of common geometric figures.** (Key) **Standard 9.0 Students compute the** volumes and **surface areas of** prisms, **pyramids,** cylinders, cones, and spheres; and **students commit to memory the formulas for** prisms, **pyramids,** and cylinders.

MAIN IDEAS

- Find lateral areas of regular pyramids.
- Find surface areas of regular pyramids.

KEY CONCEPTS

Lateral Area of a Regular Pyramid If a regular pyramid has a lateral area of L square units, a slant height of ℓ units, and its base has a perimeter of P units,

then $L = \frac{1}{2}P\ell$.

Surface Area of a Regular Pyramid If a regular pyramid has a surface area of T square units, a slant height of ℓ units, and its base has a perimeter of P units, and an area of B square units, then

$T = \frac{1}{2}P\ell + B$.

FOLDABLES™ Define the basic properties of pyramids under the Pyramids tab. Include a sketch of a pyramid with the parts of a pyramid labeled.

EXAMPLE Use Lateral Area to Solve a Problem

1 CANDLES **A candle store offers a pyramidal candle that burns for 20 hours. The square base is 6 centimeters on a side and the slant height of the candle is 22 centimeters. Find the lateral area of the candle.**

The sides of the base measure 6 centimeters, so the perimeter is 4(6) or 24 centimeters.

$L = \frac{1}{2}P\ell$ Lateral area of a regular pyramid

= ______ P = ______, ℓ = ______

= ______ Multiply.

The lateral area of the candle is ______ square centimeters.

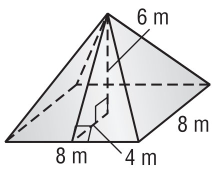
Check Your Progress A pyramidal shaped tent is put up by two campers. The square base is 7 feet on a side and the slant height of the tent is 7.4 feet. Find the lateral area of the tent.

______

EXAMPLE Surface Area of a Square Pyramid

2 **Find the surface area of the square pyramid. Round to the nearest tenth if necessary.**

To find the surface area, first find the slant height of the pyramid. The slant height is the hypotenuse of a right triangle with legs that are the altitude and a segment with a length that is one-half the side measure of the base.



$c^2 = a^2 + b^2$ Pythagorean Theorem

______ = ______ Replace a, b, and ℓ.

$\ell \approx$ ______ Use a calculator.

Now find the surface area of the regular pyramid. The perimeter of the base is 4(8) or 32 meters, and the area of the base is 8^2 or 64 square meters.

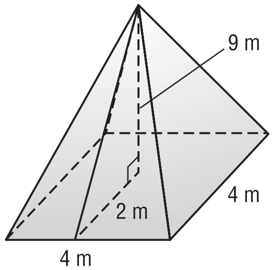
$T = \frac{1}{2}P\ell + B$ Surface area of a regular pyramid

$T \approx \frac{1}{2}(\_\_\_)(\_\_\_) + \_\_\_$ Replace P, ℓ, and B.

$T \approx \_\_\_$ Use a calculator.

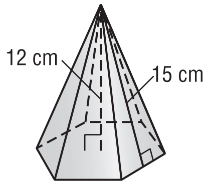
The surface area is ___ square meters to the nearest tenth.

Check Your Progress Find the surface area of the regular pyramid to the nearest tenth.



EXAMPLE Surface Area of a Pentagonal Pyramid

3 Find the surface area of the regular pyramid. Round to the nearest tenth.



The altitude, slant height, and apothem form a right triangle. Use the Pythagorean Theorem to find the apothem. Let x represent the length of the apothem.

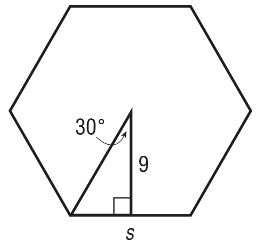
$c^2 = a^2 + b^2$ Pythagorean Theorem

___ $=$ ___ $+$ ___ $b =$ ___, $c =$ ___

___ $= a$ Simplify.

Now find the length of the sides of the base. The central angle of the hexagon measures $\frac{360°}{6}$ or 60°. Let a represent the measure of the angle formed by a radius and the apothem. Then $a = \frac{60}{2}$ or 30.

Use trigonometry to find the length of the sides.



$\tan 30° = \dfrac{\square}{\square}$ — $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

$9(\tan 30°) = \square$ — Multiply each side by 9.

$\square(\tan 30°) = s$ — Multiply each side by 2.

$\square \approx s$ — Use a calculator.

Next, find the perimeter and area of the base.

$P = 6s$

$\approx 6(\square)$ or $\square$

$B = \frac{1}{2}Pa$

$\approx \frac{1}{2}(\square)(\square)$ or $\square$

Finally, find the surface area.

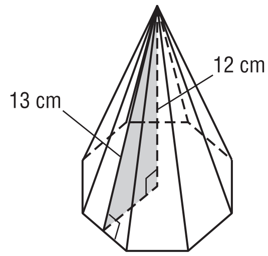
$T = \frac{1}{2}P\ell + B$ — Surface area of a regular pyramid

$\approx \frac{1}{2}(\square)(\square) + \square$ — Replace P, ℓ, and B.

$\approx \square$ — Simplify.

The surface area is approximately $\square$ square centimeters.

Check Your Progress Find the surface area of the regular pyramid.



Homework Assignment

Page(s):

Exercises:

12–5 Surface Areas of Cones

Main Ideas

- Find lateral areas of cones.
- Find surface areas of cones.

BUILD YOUR VOCABULARY (pages 292–293)

The shape of a tepee suggests a **circular cone**.

KEY CONCEPT

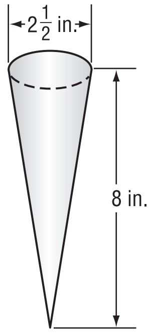
Lateral Area of a Cone If a right circular cone has a lateral area of L square units, a slant height of ℓ units, and the radius of the base is r units, then $L = \pi r \ell$.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, **lateral area, and surface area of common geometric figures.** (Key)

Standard 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, **cones,** and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

EXAMPLE Lateral Area of a Cone

1 ICE CREAM **A sugar cone has an altitude of 8 inches and a diameter of $2\frac{1}{2}$ inches. Find the lateral area of the sugar cone.**



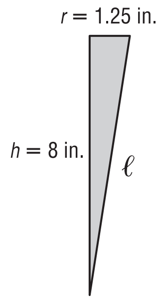
Use the Pythagorean Theorem. Write an equation and solve for ℓ.

$\ell^2 = \boxed{}^2 + \boxed{}^2$

$\ell^2 = \boxed{}$

$\ell \approx \boxed{}$

Next, use the formula for the lateral area of a right circular cone.



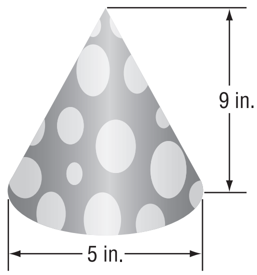
$L = \pi r \ell$

$\approx \pi(\boxed{})(\boxed{})$

$\approx \boxed{}$

The lateral area is approximately $\boxed{}$ square inches.

Check Your Progress A hat for a child's birthday party has a conical shape with an altitude of 9 inches and a diameter of 5 inches. Find the lateral area of the birthday hat.



KEY CONCEPT

Surface Area of a Cone
If a right circular cone has a surface area of T square units, a slant height of ℓ units, and the radius of the base is r units, then $T = \pi r\ell + \pi r^2$.

FOLDABLES™ Sketch a right circular cone. Use the letters r, h, and ℓ to indicate the radius, height and slant height, respectively. Write the formula for the surface area. Include all this under the tab for Cones.

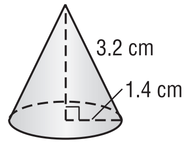
HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Surface Area of a Cone

2 **Find the surface area of the cone. Round to the nearest tenth.**



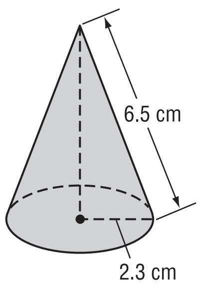
$T = \pi r\ell + \pi r^2$ Surface area of a cone

$= \pi(\_\_\_\_)(\_\_\_\_) + \pi(\_\_\_\_)^2$ $r =$ ____, $\ell =$ ____

$\approx$ ____ Use a calculator.

The surface area is approximately ____ square centimeters.

Check Your Progress Find the surface area of the cone. Round to the nearest tenth.



12–6 Surface Areas of Spheres

Build Your Vocabulary (pages 292–293)

When a plane intersects a sphere so that it contains the ______ of the sphere, the intersection is called a **great circle**.

Each great circle separates a sphere into ______, each called a **hemisphere**.

Main Ideas

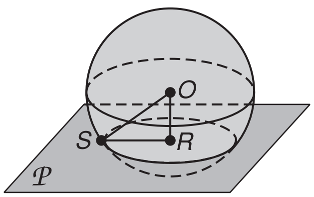
- Recognize and define basic properties of spheres.
- Find surface areas of spheres.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and **surface area of common geometric figures.** (Key)

Standard 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and **spheres;** and students commit to memory the formulas for prisms, pyramids, and cylinders.

EXAMPLE Spheres and Circles

1 In the figure, *O* is the center of the sphere, and plane $\mathcal{P}$ intersects the sphere in the circle *R*. If $OR = 6$ centimeters and $OS = 14$ centimeters, find *RS*.

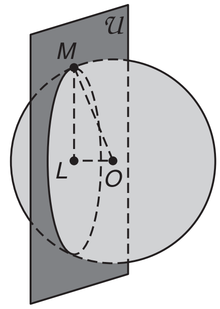


The radius of circle R is segment ______, and S is a point on circle R and on sphere O. Use the Pythagorean Theorem for right triangle ORS to solve for RS.

$OS^2 = RS^2 + OR^2$	Pythagorean Theorem
______ $= RS^2 +$ ______	$OS =$ ______, $OR =$ ______
______ $= RS^2 +$ ______	Simplify.
______ $= RS^2$	Subtract ______ from each side.
______ $\approx RS$	Use a calculator.

RS is approximately ______ centimeters.

Check Your Progress In the figure, O is the center of the sphere, and plane $\mathcal{U}$ intersects the sphere in circle L. If $OL = 3$ inches and $LM = 8$ inches, find OM.



EXAMPLE Surface Area

2 a. Find the surface area of the sphere, given a great circle with an area of approximately 907.9 square centimeters.

The surface area of a sphere is four times the area of the great circle.

$T = 4\pi r^2$	Surface area of a sphere
$= 4(\_\_\_\_)$	$\pi r^2 \approx$ ____
$=$ ____	Multiply.

The surface area is approximately ____ square centimeters.

b. Find the surface area of a hemisphere with a radius of 3.8 inches.

A hemisphere is half of a sphere. To find the surface area, find half of the surface area of the sphere and add the area of the great circle.

$T = \frac{1}{2}(4\pi r^2) + \pi r^2$	Surface area of a hemisphere
$= \frac{1}{2}[4\pi(\_\_\_\_)^2] + \pi(\_\_\_\_)^2$	Substitution
$\approx$ ____	Use a calculator.

The surface area is approximately ____ square inches.

Key Concept

Surface Area of a Sphere If a sphere has a surface area of T square units and a radius of r units, then $T = 4\pi r^2$.

FOLDABLES Define the terms great circle and hemisphere under the Spheres tab. Also, include the formula for finding the surface area of a sphere.

Check Your Progress

a. Find the surface area of the sphere, given a great circle with an area of approximately 91.6 square centimeters.

b. Find the surface area of a hemisphere with a radius of 6.4 inches.

REVIEW IT

What is the circumference of a circle with a radius of 6 centimeters? (*Lesson 10-1*)

EXAMPLE

3 **A ball is a sphere with a circumference of 24 inches. Find the approximate surface area of the ball to the nearest tenth of a square inch.**

To find the surface area, first find the radius of the sphere.

$C = 2\pi r$	Circumference of a circle
___ $= 2\pi r$	$C =$ ___
$\frac{\_\_\_}{\_\_\_} = r$	Divide each side by ___.
___ $= r$	Simplify.

Next, find the surface area of the sphere.

$T = 4\pi r^2$	Surface area of a sphere.
$\approx 4\pi(\_\_\_)^2$	$r =$ ___
$\approx$ ___	Use a calculator.

The surface area is approximately ___ square inches.

Check Your Progress Find the approximate surface area of a ball with a circumference of 18 inches to the nearest tenth of a square inch.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

CHAPTER 12

BRINGING IT ALL TOGETHER

STUDY GUIDE

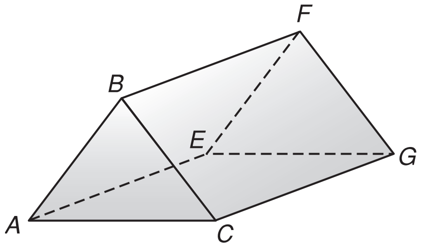
FOLDABLES	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 12 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 12, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(pages 292–293)* to help you solve the puzzle.

12-1

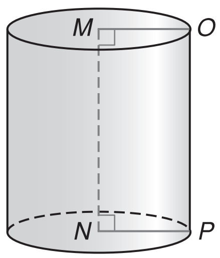
Representations of Three-Dimensional Figures

Identify each solid. Name the bases, faces, edges, and vertices.

1.



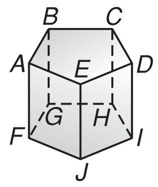
2.



12-2

Surface Areas of Prisms

Refer to the figure.



3. Name this solid with as specific a name as possible.

4. Name the bases of the solid.

5. Name the lateral faces.

6. Name the edges.

7. Name an altitude of the solid.

8. The lateral area of a prism is 90 square inches and the perimeters of its base is 15 inches. Find the height.

12-3

Surface Areas of Cylinders

Underline the correct word or phrase to form a true statement.

9. The bases of a cylinder are (rectangles/regular polygons/circles).

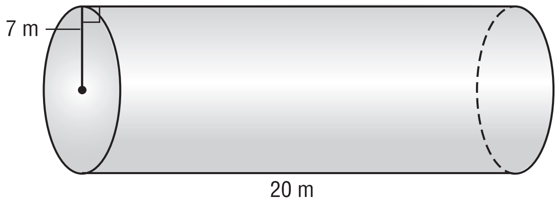
10. The (axis/radius/diameter) of a cylinder is the segment with endpoints that are the centers of the bases.

11. The net of a cylinder is composed of two congruent (rectangles/circles) and one (rectangle/semicircle).

12. In a right cylinder, the axis of the cylinders is also a(n) (base/lateral edge/altitude).

13. A cylinder that is not a right cylinder is called an (acute/obtuse/oblique) cylinder.

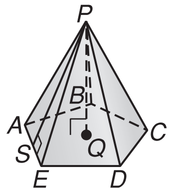
14. Find the lateral area and surface area of the cylinder. Round to the nearest tenth.



12-4

Surface Areas of Pyramids

In the figure, *ABCDE* has congruent sides and congruent angles.



15. Use the figure to name the base of this pyramid.

16. Describe the base of the pyramid.

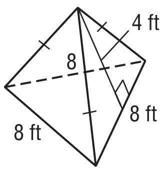
17. Name the vertex of the pyramid.

18. Name the altitude of the pyramid.

19. Write an expression for the height of the pyramid.

20. Write an expression for the slant height of the pyramid.

21. Find the lateral area and surface area of the regular figure. Round to the nearest tenth.



12-5

Surface Areas of Cones

A right circular cone has a radius of 7 meters and a slant height of 13 meters.

22. Find the lateral area to the nearest tenth.

23. Find the surface area to the nearest tenth.

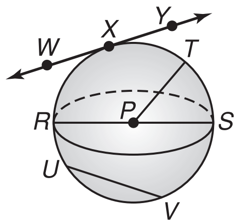
24. Suppose you have a right cone with radius r, diameter d, height h, and slant height ℓ. Which of the following relationships involving these lengths are correct?

a. $r = 2d$	**b.** $r + h = \ell$	**c.** $r^2 + h^2 = \ell^2$
d. $r^2 + \ell^2 = h^2$	**e.** $r = \sqrt{\ell^2 - h^2}$	**f.** $h = \pm\sqrt{\ell^2 - r^2}$

12-6

Surface Areas of Spheres

In the figure, *P* is the center of the sphere. Name each of the following in the figure.



25. two chords of the sphere

26. a great circle of the sphere

27. a tangent to the sphere

28. Complete: A sphere is separated by a great circle into two congruent halves, each called a(n) ________.

Determine whether each sentence is *sometimes*, *always*, or *never* true.

29. If a sphere and a plane intersect in more than one point, their intersection will be a great circle.

30. A great circle has the same center as the sphere.

31. A chord of a sphere is a diameter of the sphere.

32. $T = \pi r\ell + \pi r^2$

33. $T = PH + 2B$

34. $T = 4\pi r^2$

35. $T = \frac{1}{2}P\ell + B$

36. $T = 2\pi rh + 2\pi r^2$

37. $T = 3\pi r^2$

a. regular pyramid

b. hemisphere

c. cylinder

d. prism

e. sphere

f. cone

38. A sphere has a radius that is 28 inches long. Find the surface area to the nearest tenth.

39. The radius of a sphere is doubled. How is the surface area changed?

CHAPTER 12

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 12.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 12 Practice Test on page 723 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 12 Study Guide and Review on pages 719–722 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 723 of your textbook.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 12 Foldable.
- Then complete the Chapter 12 Study Guide and Review on pages 719–722 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 723 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

CHAPTER 13

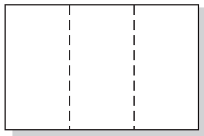
Extending Volume

FOLDABLES™

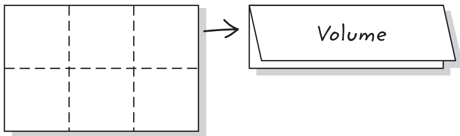
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with one sheet of 11" × 17" paper.

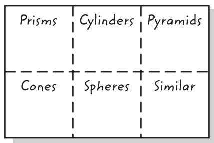
STEP 1 **Fold** in thirds.



STEP 2 **Fold** in half lengthwise. Label as shown.



STEP 3 **Unfold** book. Draw lines along folds and label as shown.



NOTE-TAKING TIP: When you take notes in geometry, be sure to make comparisons among the different formulas and concepts. For example, how are pyramids and cones similar? different? This will help you learn the material.

CHAPTER 13

BUILD YOUR VOCABULARY

This is an alphabetical list of new vocabulary terms you will learn in Chapter 13. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

Vocabulary Term	Found on Page	Definition	Description or Example
congruent solids			
ordered triple			
similar solids			

13–1 Volumes of Prisms and Cylinders

MAIN IDEAS

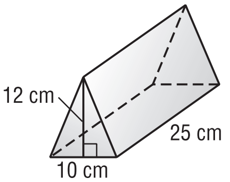
- Find volumes of prisms.
- Find volumes of cylinders.

KEY CONCEPT

Volume of a Prism If a prism has a volume of V cubic units, a height of h units, and each base has an area of B square units, then $V = Bh$.

EXAMPLE Volume of a Triangular Prism

1 **Find the volume of the triangular prism.**



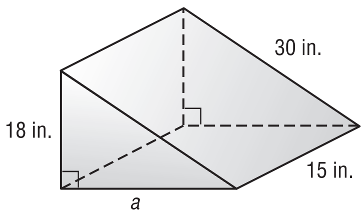
$V = Bh$

$= \frac{1}{2}(10)(\_\_\_\_)(\_\_\_\_)$

$= \_\_\_\_$

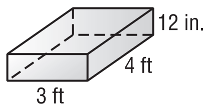
The volume of the prism is ____ cubic centimeters.

Check Your Progress Find the volume of the triangular prism.



EXAMPLE Volume of a Rectangular Prism

2 **The weight of water is 0.036 pound times the volume of water in cubic inches. How many pounds of water would fit into a rectangular child's pool that is 12 inches deep, 3 feet wide, and 4 feet long?**



First, convert feet to inches.

3 feet = 3 × ____ or ____ inches

4 feet = 4 × ____ or ____ inches

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, **volume**, lateral area, and surface area **of common geometric figures.** (Key)
Standard 9.0 Students compute the volumes and surface areas **of prisms,** pyramids, **cylinders,** cones, and spheres; **and students commit to memory the formulas for prisms,** pyramids, and **cylinders.**

To find the pounds of water that would fit into the child's pool, find the volume of the pool.

$V = Bh$ — Volume of a prism

$= 36(48)(\quad)$ — $B = 36(48)$, $h = \quad$

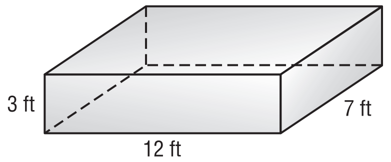
$= \quad$

Now multiply the volume by 0.036.

$\quad \times 0.036 \approx \quad$ — Simplify.

A rectangular child's pool that is 12 inches deep, 3 feet wide, and 4 feet long, will hold about ______ pounds of water.

Check Your Progress The weight of water is 62.4 pounds per cubic foot. How many pounds of water would fit into a back yard pond that is rectangular prism 3 feet deep, 7 feet wide, and 12 feet long?



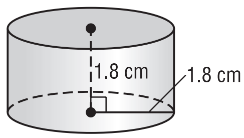
KEY CONCEPT

Volume of a Cylinder
If a cylinder has a volume of V cubic units, a height of h units, and the bases have radii of r units, then $V = Bh$ or $V = \pi r^2 h$.

FOLDABLES Explain how to find the volume of a prism and a cylinder. Put the explanations under their respective tabs in the Foldable.

EXAMPLE Volume of a Cylinder

3 **Find the volume of the cylinder.**



The height h is ______ centimeters,

and the radius r is ______ centimeters.

$V = \pi r^2 h$ — Volume of a cylinder

$= \pi(\quad)^2(\quad)$ — $r = \quad$, $h = \quad$

$\approx \quad$ — Use a calculator.

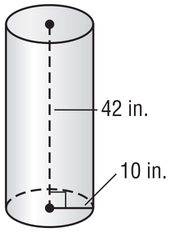
The volume is approximately ______ cubic centimeters.

Remember It

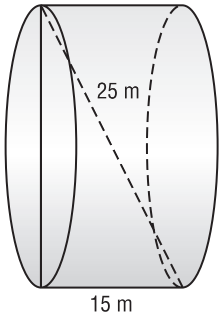
The diameter of the base, the diagonal, and the lateral edge of the cylinder form a right triangle.

Check Your Progress **Find the volume of each cylinder to the nearest tenth.**

a.



b.

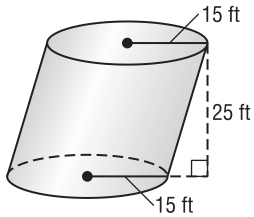


Key Concept

Cavalieri's Principle
If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE Volume of an Oblique Cylinder

4 **Find the volume of the oblique cylinder to the nearest tenth.**



To find the volume, use the formula for a right cylinder.

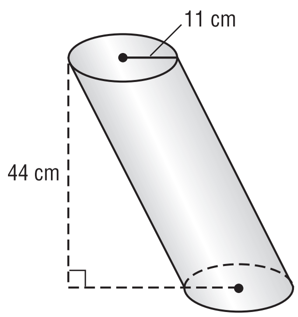
$V = \pi r^2 h$ Volume of a cylinder

$= \pi(\quad)^2(\quad)$ $r =$ ____ , $h =$ ____

$\approx$ ____ Use a calculator.

The volume is approximately ____ cubic feet.

Check Your Progress Find the volume of the oblique cylinder to the nearest tenth.



Homework Assignment

Page(s):

Exercises:

13–2 Volumes of Pyramids and Cones

MAIN IDEAS

- Find volumes of pyramids.
- Find volumes of cones.

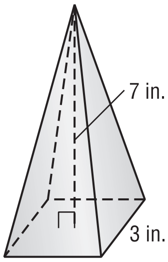
KEY CONCEPT

Volume of a Pyramid
If a pyramid has a volume of V cubic units, a height of h units, and a base with an area of B square units, then $V = \frac{1}{3}Bh$.

Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, **volume**, lateral area, and surface area **of common geometric figures.** (Key)
Standard 9.0 Students compute the volumes and surface areas **of** prisms, **pyramids,** cylinders, **cones, and spheres; and students commit to memory the formulas for** prisms, **pyramids,** and cylinders.

EXAMPLE Volume of a Pyramid

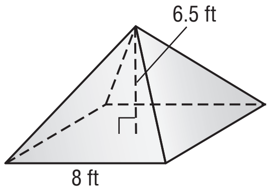
1 CLOCKS **Teofilo has a solid clock that is in the shape of a square pyramid. The clock has a base of 3 inches and a height of 7 inches. Find the volume of the clock.**



$V = \frac{1}{3}Bh$	Volume of a pyramid
$= \frac{1}{3}s^2h$	$B = s^2$
$= \frac{1}{3}(\square)^2(\square)$	$s = \square$, $h = \square$
$= \square$	Multiply.

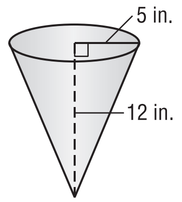
The volume of the clock is $\square$ cubic inches.

Check Your Progress Brad is building a model pyramid for a social studies project. The model is a square pyramid with a base edge of 8 feet and a height of 6.5 feet. Find the volume of the pyramid.



EXAMPLE Volumes of Cones

2 Find the volume of the cone to the nearest tenth.



$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$= \frac{1}{3}\pi(\quad)^2(\quad)$ $r = \quad$, $h = \quad$

$\approx \quad$ Use a calculator.

The volume is approximately ______ cubic inches.

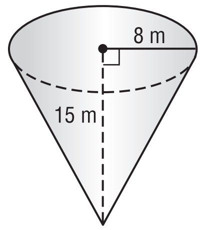
KEY CONCEPT

Volume of a Cone If a right circular cone has a volume of V cubic units, a height of h units, and the base has a radius of r units, then

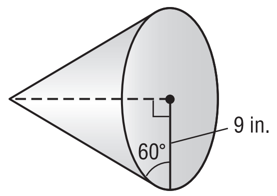
$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$.

Check Your Progress **Find the volume of each cone to the nearest tenth.**

a.

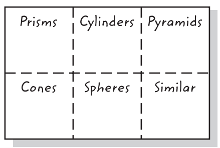


b.



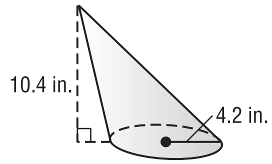
FOLDABLES™ ORGANIZE IT

Explain how to find the volume of a pyramid and a cone. Put the explanations under their respective tabs in the Foldable.



EXAMPLE Volume of an Oblique Cone

3 Find the volume of the oblique cone to the nearest tenth.



$V = \frac{1}{3}Bh$ Volume of a cone

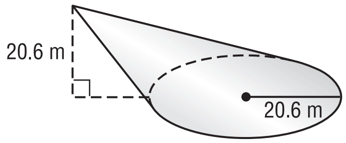
$= \frac{1}{3}(\quad)h$ $B = \quad$

$= \frac{1}{3}\pi(\quad)^2(\quad)$ $r \approx \quad$, $h = \quad$

$\approx \quad$ Use a calculator.

The volume is approximately ______ cubic inches.

Check Your Progress Find the volume of the oblique cone to the nearest tenth.



Homework Assignment

Page(s):

Exercises:

13–3 Volumes of Spheres

MAIN IDEAS

- Find volumes of spheres.
- Solve problems involving volumes of spheres.

KEY CONCEPT

Volume of a Sphere If a sphere has a volume of V cubic units and a radius of r units, then $V = \frac{4}{3}\pi r^3$.

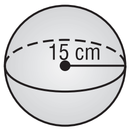
Standard 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, **volume**, lateral area, and surface area **of common geometric figures.** (Key)

Standard 9.0 Students compute the volumes and surface areas **of** prisms, pyramids, cylinders, cones, and **spheres**; and students commit to memory the formulas for prisms, pyramids, and cylinders.

EXAMPLE Volumes of Spheres

1 **Find the volume of each sphere. Round to the nearest tenth.**

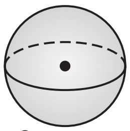
a.



$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$= \frac{4}{3}\pi(\square)^3$	$r = \square$
$\approx \square$	Use a calculator.

The volume is approximately $\square$ cubic centimeters.

b.



$C = 25$ cm

First find the radius of the sphere.

$C = 2\pi r$	Circumference of a circle
$\square = 2\pi r$	$C = \square$
$\square = r$	Solve for r.

Now find the volume.

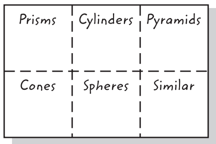
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$= \frac{4}{3}\pi(\square)^3$	$r = \square$
$\approx \square$	Use a calculator.

The volume is approximately $\square$ cubic centimeters.

FOLDABLES™

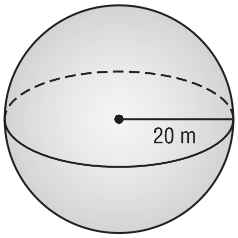
ORGANIZE IT

Write a paragraph that includes formulas for the volume of a sphere, cylinder, and cone. Then describe how the volumes of these three solids are related. Put your paragraph under the Spheres tab.

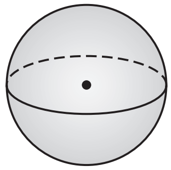


Check Your Progress **Find the volume of each sphere to the nearest tenth.**

a.

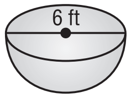


b.



EXAMPLE Volume of a Hemisphere

2 **Find the volume of the hemisphere.**



The volume of a hemisphere is one-half the volume of the sphere.

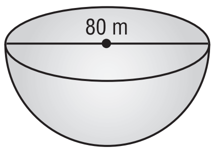
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Volume of a hemisphere

$= \square\,\pi(3)^3$ $r = 3$

$\approx \square$ Use a calculator.

The volume of the hemisphere is approximately ______ cubic feet.

Check Your Progress Find the volume of the hemisphere.



HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–4 Congruent and Similar Solids

Standard 11.0 Students determine how changes in dimensions affect the perimeter, area, and **volume of common geometric figures and solids.**

Main Ideas

- Identify congruent or similar solids.
- State the properties of similar solids.

Key Concept

Congruent Solids Two solids are congruent if the corresponding angles are congruent, the corresponding edges are congruent, the corresponding faces are congruent, and the volumes are equal.

Build Your Vocabulary (page 316)

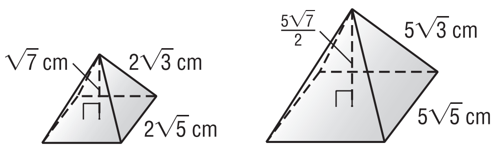
Similar solids are solids that have exactly the same ____ but not necessarily the same ____.

Congruent solids are exactly the same ____ and exactly the same ____.

Example: Similar and Congruent Solids

1 **Determine whether each pair of solids is *similar*, *congruent*, or *neither*.**

a.



Find the ratios between the corresponding parts of the square pyramids.

$$\frac{\text{base edge of larger pyramid}}{\text{base edge of smaller pyramid}} = \frac{5\sqrt{5}}{2\sqrt{5}}$$ Substitution

$$= \square$$ Simplify.

$$\frac{\text{height of larger pyramid}}{\text{height of smaller pyramid}} = \frac{\frac{5\sqrt{7}}{2}}{\sqrt{7}}$$ Substitution

$$= \square$$ Simplify.

$$\frac{\text{lateral edge of larger pyramid}}{\text{lateral edge of smaller pyramid}} = \frac{5\sqrt{3}}{2\sqrt{3}}$$ Substitution

$$= \square$$ Simplify.

The ratios of the measures are equal, so we can conclude that the pyramids are similar. Since the scale factor is not 1, the solids are not congruent.

REVIEW IT

An architect drew blueprints of a house, where the 44-foot length of the house was represented by 20 inches. What is the scale factor of the blueprints compared to the real house? *(Lesson 7-2)*

____

____

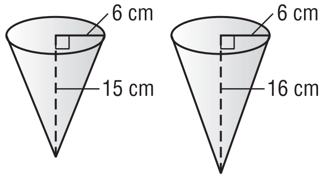
____

____

____

____

b.



Compare the ratios between the corresponding parts of the cones.

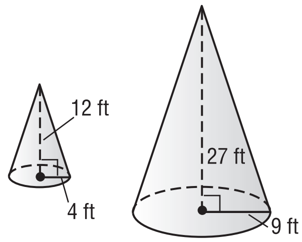
$\frac{\text{radius of larger cone}}{\text{radius of smaller cone}} = \frac{\square}{\square}$ or $\square$ — Use substitution. Then simplify.

$\frac{\text{height of larger cone}}{\text{height of smaller cone}} = \frac{\square}{\square}$ — Substitution

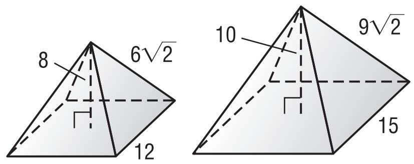
Since the ratios are not the same, the cones are neither similar nor congruent.

Check Your Progress **Determine whether each pair of solids is *similar*, *congruent*, or *neither*.**

a.



b.



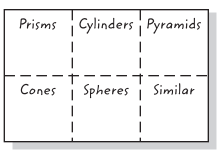
REMEMBER IT

All spheres are similar, just as all circles are similar.

Theorem 13.1
If two solids are similar with a scale factor of a:b, then the surface areas have a ratio of a^2:b^2, and the volumes have a ratio of a^3:b^3.

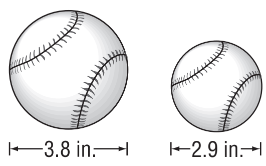
FOLDABLES ORGANIZE IT

Describe the differences between congruent and similar solids under the Similar tab in the Foldables.



EXAMPLE Sports

2 SOFTBALL **Softballs and baseballs are both used to play a game with a bat. A softball has a diameter of 3.8 inches, while a baseball has a diameter of 2.9 inches.**



a. Find the scale factor of the two balls.

Write the ratio of the radii. The scale factor of the two balls is 3.8 : 2.9 or about ______ : ______.

b. Find the ratio of the surface areas of the two balls.

If the scale factor is $a:b$, then the ratio of the surface areas is $a^2:b^2$.

$$\frac{\text{surface area of the larger ball}}{\text{surface area of the smaller ball}} = \frac{a^2}{b^2}$$ Theorem 13.1

$$\approx \frac{1.3^2}{\square^2} \approx \square$$

The ratio of the surface areas is about ______.

c. Find the ratio of the volumes of the two balls.

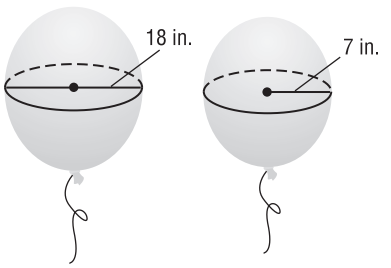
If the scale factor is $a:b$, then the ratio of the volumes is $a^3:b^3$.

$$\frac{\text{volume of the larger ball}}{\text{volume of the smaller ball}} = \frac{a^3}{b^3}$$ Theorem 13.1

$$\approx \frac{\square^3}{1^3} \approx \square$$

The ratio of the volume of the two balls is about ______.

Check Your Progress **Two sizes of balloons are being used for decorating at a party. When fully inflated, the balloons are spheres. The first balloon has a diameter of 18 inches while the second balloon has a radius of 7 inches.**



a. Find the scale factor of the two balloons.

b. Find the ratio of the surface areas of the two balloons.

c. Find the ratio of the volumes of the two balloons.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

13–5 Coordinates in Space

Standard 22.0 Students know the effect of rigid motions on figures in the coordinate plane and **space, including rotations, translations, and reflections.** (Key)

MAIN IDEAS

- Graph solids in space.
- Use the Distance and Midpoint Formulas for points in space.

REMEMBER IT

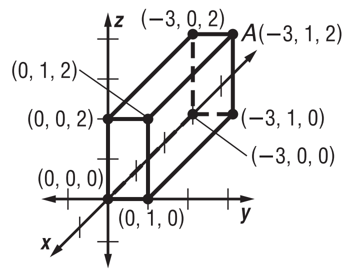
The three planes determined by the axes of a three-dimensional coordinate system separate space into eight regions. These regions are called *octants*.

BUILD YOUR VOCABULARY (page 316)

A point in space is represented by an **ordered triple** of real numbers (*x*, *y*, *z*).

EXAMPLE Graph a Rectangular Solid

1 **Graph the rectangular solid that contains the ordered triple *A*(–3, 1, 2) and the origin as vertices. Label the coordinates of each vertex.**



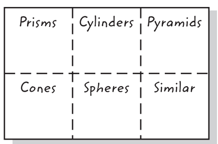
- Plot the *x*-coordinate first. Draw a segment from the origin ______ units in the negative direction.
- To plot the *y*-coordinate, draw a segment ______ unit in the positive direction.
- Next, to plot the *z*-coordinate, draw a segment ______ units long in the positive direction.
- Label the coordinate *A*.
- Draw the rectangular prism and label each vertex.

Check Your Progress Graph the rectangular solid that contains the ordered triple *N*(1, 2, –3) and the origin. Label the coordinates of each vertex.

FOLDABLES™

ORGANIZE IT

Write a short paragraph to explain how to graph a rectangular solid. Record your paragraph under the Prisms tab.



KEY CONCEPTS

Distance Formula in Space Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space, the distance between A and B is given by the following equation.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Midpoint Formula in Space Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space, the midpoint of $\overline{AB}$ is at

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

EXAMPLE Distance and Midpoint Formulas in Space

2 **a. Determine the distance between $F(4, 0, 0)$ and $G(-2, 3, -1)$.**

$$FG = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$ Distance Formula in Space

$= \sqrt{\_\_\_\_ + \_\_\_\_ + \_\_\_\_}$ Substitution

$= \_\_\_\_$ Simplify.

b. Determine the midpoint M of $\overline{FG}$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$ Midpoint Formula in Space

$= (\_\_\_\_, \_\_\_\_, \_\_\_\_)$ Substitution

$= (\_\_\_\_)$ Simpliify.

Check Your Progress

a. Determine the distance between $A(0, -5, 0)$ and $B(1, -2, -3)$.

b. Determine the midpoint M of $\overline{AB}$.

CHAPTER 13

BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES™	VOCABULARY PUZZLEMAKER	BUILD YOUR VOCABULARY
Use your **Chapter 13 Foldable** to help you study for your chapter test.	To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 13, go to: glencoe.com	You can use your completed **Vocabulary Builder** *(page 316)* to help you solve the puzzle.

13-1

Volumes of Prisms and Cylinders

In each case, write a formula for the volume V of the solid in terms of the given variables.

1. a rectangular box with length a, width b, and height c

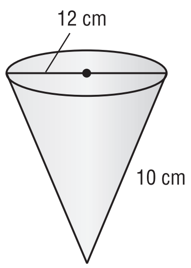
2. a cylinder with height h whose bases each have diameter d

3. The volume of a rectangular prism is 224 cubic centimeters, the length is 7 centimeters, and the height is 8 centimeters. Find the width.

13-2

Volumes of Pyramids and Cones

4. Find the volume to the nearest tenth.



13-3 Volumes of Spheres

Let *r* represent the radius and *d* represent the diameter of a sphere. Determine whether each formula below can be used to find the volume of a *sphere*, a *hemisphere*, or *neither*.

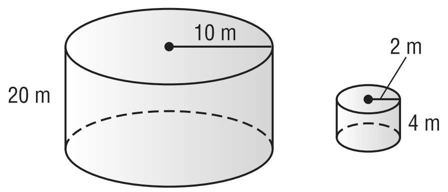
5. $V = \frac{2\pi r^3}{3}$

6. $V = \frac{1}{3}\pi r^3$

7. $V = \frac{1}{6}\pi d^3$

13-4 Congruent and Similar Solids

8. Determine whether the cylinders are *congruent*, *similar*, or *neither*.



9. If the ratio of the diameters of two spheres is 3:1, then the ratio of their surface areas is ______, and the ratio of their volumes is ______.

13-5 Coordinates in Space

For the following questions, *A*(−4, 3, 12) and *B*(7, 2, 8).

10. Find the coordinates of the midpoint of $\overline{AB}$.

11. Find the distance between *A* and *B*.

12. If *B* is the midpoint of $\overline{AC}$, find the coordinates of *C*.

CHAPTER 13

Checklist

ARE YOU READY FOR THE CHAPTER TEST?

Math Online

Visit **glencoe.com** to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 13.

Check the one that applies. Suggestions to help you study are given with each item.

☐ **I completed the review of all or most lessons without using my notes or asking for help.**

- You are probably ready for the Chapter Test.
- You may want to take the Chapter 13 Practice Test on page 769 of your textbook as a final check.

☐ **I used my Foldable or Study Notebook to complete the review of all or most lessons.**

- You should complete the Chapter 13 Study Guide and Review on pages 765–768 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 13 Practice Test on page 769.

☐ **I asked for help from someone else to complete the review of all or most lessons.**

- You should review the examples and concepts in your Study Notebook and Chapter 13 Foldable.
- Then complete the Chapter 13 Study Guide and Review on pages 765–768 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 13 Practice Test on page 769.

Student Signature

Parent/Guardian Signature

Teacher Signature