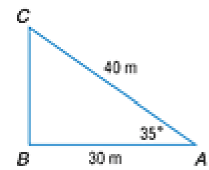
Lesson 13–4

Example 1 Find the Area of a Triangle

Find the area of ΔABC to the nearest tenth.



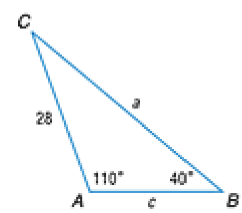
In this triangle, $b = 40$, $c = 30$, and $A = 35^\circ$. Chose the first formula because you know the value of its variables.

$\text{Area} = \frac{1}{2}bc \sin A$	Area formula
$= \frac{1}{2}(40)(30) \sin 35^\circ$	Replace b with 40, c with 30, and A with 35°.
≈ 344.1	Use a calculator.

To the nearest tenth, the area is 344.1 square meters.

Example 2 Solve a Triangle Given Two Angles and a Side

Solve ΔABC.



You are given the measures of two angles and a side. First, find the measure of the third angle.

$110^\circ + 40^\circ + C = 180^\circ$	The sum of the angle measures of a triangle is 180°.
$C = 30^\circ$	180 – (110 + 40) = 30

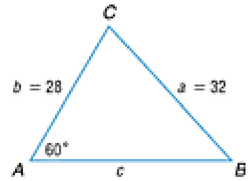
Now use the Law of Sines to find a and c. Write two equations, each with one variable.

$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines	$\frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin 110^\circ}{a} = \frac{\sin 40^\circ}{28}$	Replace A with 110°, B with 40°, C with 30°, and b with 28.	$\frac{\sin 40^\circ}{28} = \frac{\sin 30^\circ}{c}$
$a = \frac{28 \sin 110^\circ}{\sin 40^\circ}$	Solve for the variable.	$c = \frac{28 \sin 30^\circ}{\sin 40^\circ}$
$a \approx 40.9$	Use a calculator.	$c \approx 21.8$

Therefore, $C = 30^\circ$, $a \approx 40.9$, and $c \approx 21.8$.

Example 3 One Solution

Determine whether ΔABC has *no* solution, *one* solution, or *two* solutions. Then solve ΔABC for $A = 60^\circ$, $b = 28$, and $a = 32$.



Because angle A is acute and $a > b$, you know that one solution exists.

Make a sketch and then use the Law of Sines to find B.

$$\frac{\sin B}{28} = \frac{\sin 60^\circ}{32}$$ Law of Sines

$$\sin B = \frac{28 \sin 60^\circ}{32}$$ Multiply each side by 28.

$\sin B \approx 0.7578$ Use a calculator.

$B \approx 49.3^\circ$ Use the $\sin^{-1}$ function.

The measure of angle C is approximately $180 - (60 + 49.3)$ or 70.7°.
Use the Law of Sines again to find c.

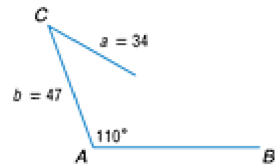
$$\frac{\sin 70.7^\circ}{c} = \frac{\sin 60^\circ}{32}$$ Law of Sines

$$c = \frac{32 \sin 70.7^\circ}{\sin 60^\circ}$$ or about 34.9 Use a calculator.

Therefore, $B \approx 49.3^\circ$, $C \approx 70.7^\circ$, and $c \approx 34.9$.

Example 4 No Solution

Determine whether ΔABC has *no* solution, *one* solution, or *two* solutions. Then solve ΔABC for $A = 110^\circ$, $a = 34$, and $b = 47$.



Since angle A is obtuse, compare the lengths of a and b. Since $a < b$, there is no solution.

Since $34 < 47$, there is no solution.

Example 5 Two Solutions

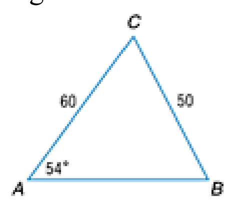
Determine whether $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve $\triangle ABC$ for $A = 54^\circ$, $a = 50$, and $b = 60$.

Since angle A is acute, find $b \sin A$ and compare it with a.

$$\begin{aligned} b \sin A &= 60 \sin 54^\circ && \text{Replace } b \text{ with 60 and } A \text{ with } 54^\circ. \\ &\approx 48.54 && \text{Use a calculator.} \end{aligned}$$

Since 60 > 50 > 48.54, there are two solutions. Thus, there are two possible triangles to be solved.

Case 1 Acute Angle *B*



First, use the Law of Sines to find *B*.

$$\frac{\sin B}{60} = \frac{\sin 54^\circ}{50}$$

$$\sin B = \frac{60 \sin 54^\circ}{50}$$

$$\sin B \approx 0.9708$$

$$B \approx 76.1^\circ$$

The measure of angle *C* is approximately 180 – (54 + 76.1) or 49.9°.

Use the Law of Sines again to find *c*.

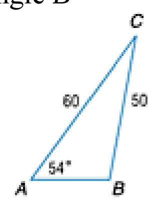
$$\frac{\sin 49.9^\circ}{c} = \frac{\sin 54^\circ}{50}$$

$$c = \frac{50 \sin 49.9^\circ}{\sin 54^\circ}$$

$$c \approx 47.3$$

Therefore, $B \approx 76.1°$, $C \approx 49.9°$, and $c \approx 47.3$.

Case 2 Obtuse Angle *B*



To find *B*, you need to find an obtuse angle whose sine is also 0.9708. To do this, subtract the angle given by your calculator, 76.1°, from 180°. So *B* is approximately 180 – 76.1 or 103.9°.

The measure of angle *C* is approximately 180 – (54 + 103.9) or 22.1°.

Use the Law of Sines to find *c*.

$$\frac{\sin 22.1^\circ}{c} = \frac{\sin 54^\circ}{50}$$

$$c = \frac{50 \sin 22.1^\circ}{\sin 54^\circ}$$

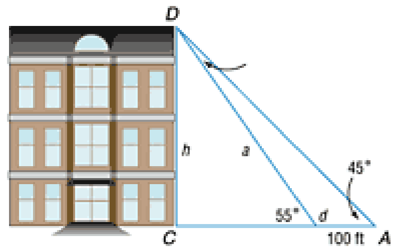
$$c \approx 23.3$$

Therefore, $B \approx 103.9°$, $C \approx 22.1°$, and $c \approx 23.3$.

Example 6 Use the Law of Sines to Solve a Problem

Two people are standing at different distances from the base of a building and sighting in the top. The person at *A* looks up at the top of the building with an angle of elevation of 45°. The person at *B* is 100 feet closer to the base of the building and looks up at the top of the building with an angle of elevation of 55°.

a. What is the height of the building?



Draw and label a diagram of the situation.

First you need to use ΔDBA to find the measure of a. Since angle α and the angle measuring 55° are supplementary, $\alpha = 125^\circ$. Then angle θ measures 10°. Use the Law of Sines to find a.

$$\frac{\sin 45^\circ}{a} = \frac{\sin 10^\circ}{100}$$ Law of Sines

$$a = \frac{100 \sin 45^\circ}{\sin 10^\circ}$$ Solve for a.

$$a \approx 407.2$$ Use a calculator.

ΔDBC is a right triangle with hypotenuse of length a or about 407.2 feet. You know the measure of the angle at B, 55°, so use sine of $\angle DBC$ to find the height of the building, h.

$$\sin \angle DBC = \frac{h}{a}$$ Sine ratio

$$\sin 55^\circ = \frac{h}{407.2}$$ $B = 55^\circ$ and $a = 407.2$

$$h = 407.2 \sin 55^\circ$$ Solve for h.

$$h \approx 333.6.$$ Use a calculator.

The height of the building is about 333.6 feet.

b. How far is the person at *B* from the base of the building?

ΔDBC is a right triangle and the length of one leg and the hypotenuse is known, so use the Pythagorean Theorem to find BC.

$$h^2 + BC^2 = a^2$$ Pythagorean Theorem

$$(333.6)^2 + BC^2 = (407.2)^2$$ Replace h with 333.6 and a with 407.2.

$$BC^2 = (407.2)^2 - (333.6)^2$$ Subtract $(333.6)^2$ from each side.

$$BC = \sqrt{(407.2)^2 - (333.6)^2}$$ Take the square root of each side.

$$BC \approx 233.5$$ Simplify.

The person at B is about 233.5 feet from the base of the building.