**Lesson 12–9**

## Example 1 Binomial Theorem

**Suppose you take a multiple–choice test with six questions for which each question has five answer choices. What is the probability of getting exactly three questions right?**

There are two possible outcomes for each question: right or wrong. The probability of a right answer $r$ is $\frac{1}{5}$ and the probability of a wrong answer $w$ is $\frac{4}{5}$.

$(r + w)^6 = r^6 + 6r^5w + 15r^4w^2 + 20r^3w^3 + 15r^2w^4 + 6rw^5 + w^6$

The term $20r^3w^3$ represents 3 right and 3 wrong answers.

$$\begin{aligned} P(\text{3 right, 3 wrong}) &= 20r^3w^3 \\ &= 20\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3 \qquad r = \frac{1}{5},\ w = \frac{4}{5} \\ &= \frac{256}{3125} \end{aligned}$$

The probability of getting exactly three questions right is $\frac{256}{3125}$ or about 8%.

**Example 2** **Binomial Experiment**

**QUALITY CONTROL A company makes fragile glass ornaments. At the end of the manufacturing process, 15% of the ornaments are defective. Five ornaments are selected at random during a particular shift.**

**a. What is the probability that exactly 3 ornaments selected are defective?**

The probability of a defective ornament is $\frac{15}{100}$ or $\frac{3}{20}$. The probability that an ornament is not defective is $\frac{17}{20}$. There are $C(5, 3)$ ways to choose the defective ornaments.

$$P(3 \text{ defective ornaments}) = C(5, 3)\left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^2$$

If 3 ornaments are defective, 2 ornaments are not defective.

$$= \frac{5 \cdot 4}{2}\left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^2$$

$C(5, 3) = \frac{5!}{3!2!}$

$$= \frac{7803}{320,000}$$

Simplify.

The probability that exactly 3 ornaments out of the 5 selected are defective is $\frac{7803}{320,000}$ or about 2.4%.

**b. What is the probability that no more than 2 ornaments of the five chosen are defective?**

No more than 2 defective means 0, 1, or 2 defective. Add the probabilities of 0 defective, 1 defective, and 2 defective.

$$P(\text{no more than 2 defective}) = P(0 \text{ defective}) + P(1 \text{ defective}) + P(2 \text{ defective})$$

$$= C(5, 0)\left(\frac{3}{20}\right)^0\left(\frac{17}{20}\right)^5 + C(5, 1)\left(\frac{3}{20}\right)^1\left(\frac{17}{20}\right)^4 + C(5, 2)\left(\frac{3}{20}\right)^2\left(\frac{17}{20}\right)^3$$

$$= \frac{1,557,421}{1,600,000}$$

Simplify.

The probability that no more than 2 out of the five selected will be defective is $\frac{1,557,421}{1,600,000}$ or about 97%.