

Lesson 13-7**Example 1**

Graph $-y^2 + x^2 > 9$.

Solution

Graph the hyperbola $-y^2 + x^2 = 9$. Use the intercepts $(3, 0)$, $(-3, 0)$, $(0, 3)$, and $(0, -3)$ to draw the rectangle. Then draw the asymptotes and sketch the hyperbola.

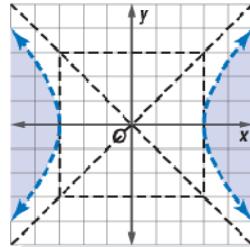
Because $-y^2 + x^2 = 9$ is not part of the solution, the hyperbola is drawn with a dashed line.

To decide which points are part of the solution set, select points on the graph and substitute their coordinates into the equation.

$$\begin{array}{ll} \text{Select } (0, 0): & -(0)^2 + (0)^2 > 9 \\ & 0 + 0 > 9 \end{array} \quad \begin{array}{l} \text{The point } (0, 0) \text{ and the region that} \\ \text{contains it are not in the solution set.} \end{array}$$

$$\begin{array}{ll} \text{Select } (-4, 1): & -(1)^2 + (-4)^2 > 9 \\ & (-1) + 16 > 9 \\ & 15 > 9 \end{array} \quad \begin{array}{ll} \text{Select } (4, 0): & -(0)^2 + (4)^2 > 9 \\ & 0 + 16 > 9 \\ & 16 > 9 \end{array}$$

These points and the regions they contain are in the solution set. The solution set is the shaded region shown on the graph.



Example 2

ASTRONOMY Radio commands may be sent to a space probe during a specific portion of its flight. If commands are sent too soon or too late they will not be received by the probe. The solution set of the following system of inequalities is used to determine when commands may be sent. Solve the system of inequalities by graphing.

$$\begin{aligned}4x^2 + 25y^2 &< 100 \\y &\geq x - 3\end{aligned}$$

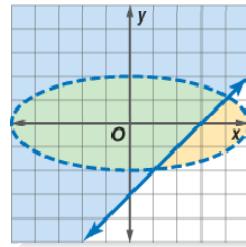
Solution

Graph the ellipse $4x^2 + 25y^2 = 100$. The center is at the origin. The x -intercepts are $(5, 0)$ and $(-5, 0)$. The y -intercepts are $(0, 2)$ and $(0, -2)$.

The points on the ellipse are not in the solution set, so the ellipse is drawn with a dashed line. Point $(0, 0)$ is in the solution set so points inside the ellipse are part of the solution set.

Graph $y \geq x - 3$. The solution set includes the line, so the line is solid. The area above the line, $y \geq x - 3$, is part of the solution set.

The intersection of the two equations is shown by the area of the graph shaded green.



Example 3

Solve this system of inequalities by graphing.

$$\begin{aligned}x^2 + y^2 &\leq 16 \\y &> 2x^2 + 1\end{aligned}$$

Solution

Graph the circle with radius 6. The circle is in the solution set; draw the circle with a solid line. Point (0,0) is in the solution set. The area inside the circle is in the solution set.

Graph parabola $y > 2x^2 + 1$. The parabola is not in the solution set; graph the parabola with a dashed line. Point (0,0) is not in the solution set, so the area inside the parabola is the solution set.

The area shaded orange shows the intersection of the two equations.

