## Tower of Hanoi

You explored the Tower of Hanoi problem in Inquiry Investigation 4 on page 114 . One strategy for solving the problem is to solve a simpler problem. This investigation will lead you through the process of solving simpler problems to make a prediction for solving the Tower problem.

## Exercises

1) Use numbered blocks or pieces of paper to find the number of moves nee ded to move 1,2 , and 3 disks. Record your answers in the table.

| Number of Disks | Number of Moves Needed |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 | 15 |
| 4 | 31 |
| 5 | 63 |
| 6 |  |

2) Describe the relationship between the number of disks and the number of moves needed.
3) In Exercise 2, you discovered a relationship between the number of disks and the number of moves needed. There are two ways to state this relationship. One way is to base your answer on the previous number of moves in the table. Use your answers from Exercise 1 and fill in the extra column in the table below. Notice that with 1 disk, there is no previous number of moves to double.

| Number of Disks | Number of Moves <br> Needed | Twice the Previous <br> Number of Moves |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 | 31 |  |
| 5 | 63 |  |
| 6 |  |  |

4) Describe in words how the number of moves needed is based on the previous number of moves.
5) Based on your answer in Exercise 4, how many moves would be needed to move 10 disks?
6) In Exercise 4, you developed a way to determine the number of moves needed to move disks based on the previous number of moves. Now, you will look at the number of moves needed from a different perspective, powers of 2. Use your answers from Exercise 1 and fill in the extra column in the table below.

| Number of Disks <br> $(\boldsymbol{N})$ | Number of Moves <br> Needed | Power of 2 <br> $\left(\mathbf{2}^{\boldsymbol{N}}\right)$ |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 | 15 |  |
| 5 | 31 |  |
| 6 | 63 |  |

7) Describe in words how the number of moves needed can be based on powers of 2 .
8) Based on your answer in Exercise 7, how many moves would be needed to move 10 disks?
9) How does your answer to Exercise 8 compare to your answer from Exercise 5 ?
10) To complete this exercise, you will need to get the TOWER Aplet from your teacher. You can use this HP Aplet to show that the methods used in Exercises 2 and 3 give the same result and to find the number of moves needed for 64 disks. Once you have the HP Aplet, press the SYMB key to see a symbolic view of the descriptions from Exercises 2 and 3. Now press the NUM key to see the table of values. This will show you that both methods used give the same values.
