Algebraic Fractions

Introduction

It is easy to see that $\dfrac{3}{27} = \dfrac{1}{9}$ because $\dfrac{3}{27} = \dfrac{3 \cdot 1}{3 \cdot 9} = \dfrac{3}{3} \cdot \dfrac{1}{9} = 1 \cdot \dfrac{1}{9} = \dfrac{1}{9}$. It is probably less easy

to see that $\dfrac{5}{7} = \dfrac{7.285}{10.199}$, but those two fractions are also equal (check this yourself!). In

this activity, we will be working with *algebraic fractions* like $\dfrac{x+1}{x+2}$ and $\dfrac{x^2-1}{x^2+x-2}$. Are

these two fractions equal? It can be difficult to tell but this activity will give you a
method to check if two algebraic fractions are equal.

First, it is very important to understand the difference between an equation and an
identity. Both equations and identities have equal signs in them, but they use those equal
signs in different ways.

Equations

A statement like $x + 3 = 2x + 1$ is an *equation*. It is true for some values of the variable
like $x = 2$ and not true for other values of the variable like $x = 1$. This is easy to see by
substituting and evaluating. We can also see this using the table on the HP 39gs.

Turn the HP 39gs on and press the APLET key, highlight the **Function** aplet (if it isn't
already) and press the RESET menu key. Type the expression $x + 3$ into F1(X) and type
$2x + 1$ into F2(X) (see Figure 1). Press SHIFT NUM to bring up the NUMERIC SETUP
screen and enter the values as shown in Figure 2. Finally, press the NUM key. Do you see
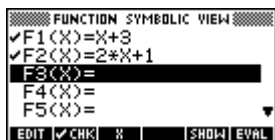that $x = 2$ makes $x + 3 = 2x + 1$ true and that $x = 1$ does not?
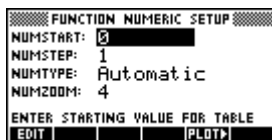

Figure 1

Figure 2

Figure 3

Identities

Let us explore the statement $\dfrac{x+1}{x+2} = \dfrac{(x+1)^2}{x^2+3x+1}$ in a similar way.

Press SYMB and type $\dfrac{x+1}{x+2}$ into F1(X) and $\dfrac{(x+1)^2}{x^2+3x+1}$ into F2(X) (see Figure 4 below).
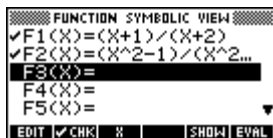Press NUM.

Figure 4

Exercises

1) The table for $\dfrac{x+1}{x+2} = \dfrac{(x+1)^2}{x^2+3x+2}$ is different from the table for $x + 3 = 2x + 1$. Explain the difference.

Use the up arrow to scroll the table into negative values of x.

2) Something interesting is happening between $x = -5$ and $x = 0$. What do you see in the table?

Equations that are true for **all** values of x (except for values that make either side undefined) are called *identities*.

However, two statements that look so similar, such as

$$\frac{x+1}{x+2} = \frac{x^2-1}{x^2+x-2} \qquad \text{and} \qquad \frac{x-2}{x-1} = \frac{x^2-16}{x^2-x-12} \ ,$$

can mean very different things. As we have seen, the first statement is an identity. The second statement is only true for one value of x.

3) Using a table, find the value of x that makes the equation $\dfrac{x-2}{x-1} = \dfrac{x^2-16}{x^2-x-12}$ true.

Hint: the solution is negative.

4) Is $\dfrac{x-2}{x-1} = \dfrac{x^2-16}{x^2-x-12}$ an identity? Explain your reasoning.

5) Which one of the equations below is an identity? Explain your reasoning.

$$\frac{x+1}{x+2} = \frac{x^2-1}{x^2+x-2} \qquad\qquad \frac{2x+4}{x+2} = \frac{3}{x+1}$$

6) Use a table to show that $x = 0.5$ makes $\dfrac{2x+4}{x+2} = \dfrac{3}{x+1}$ true. Explain how you got the table to display a line for $x = 0.5$.

Another way to solve this equation is by graphing $y = \dfrac{2x+4}{x+2}$ and $y = \dfrac{3}{x+1}$ then find the points where the graphs intersect. The point where the graphs intersect is an x-value and a corresponding y-value that make both $y = \dfrac{2x+4}{x+2}$ and $y = \dfrac{3}{x+1}$ true. If both of those equations are true, then $\dfrac{2x+4}{x+2} = \dfrac{3}{x+1}$ must be true.

Turn on the HP 39gs. Press SYMB and define F1(X) to be $\dfrac{2x+4}{x+2}$. Press PLOT.

7) Describe the graph of $y = \dfrac{2x+4}{x+2}$. Does the shape of the graph surprise you? Can you find a hole in the graph? If so, where is the hole?

Press SYMB again and define F2(X) to be $\dfrac{3}{x+1}$. Press PLOT and both graphs should be plotted. While looking at the graphs, press the MENU and then the FCN menu keys. Select **Intersection** and choose **Using F1(X) and F2(X)** (the first option). Press OK and the coordinates of the point of intersection should be displayed on the plot screen

8) What are the coordinates of the point where the graphs of $y = \dfrac{3}{x+1}$ and $y = \dfrac{2x+4}{x+2}$ intersect?

The value of x you just found (the x-coordinate of the intersection point) is where $\dfrac{2x+4}{x+2} = \dfrac{3}{x+1}$. In other words, you have found a value of x that makes $\dfrac{2x+4}{x+2} = \dfrac{3}{x+1}$ true. You have found a solution to the equation by graphing.

Try to solve $\dfrac{x+1}{x+2} = \dfrac{x^2-1}{x^2+x-2}$ by graphing just as you did in Exercise 8.

Put $\dfrac{x+1}{x+2}$ in for F1(X) and $\dfrac{x^2-1}{x^2+x-2}$ in for F2(X). Press PLOT. Something strange is happening. Press SYMB, uncheck F1(X) and press PLOT. This shows the graph of $y = \dfrac{x+1}{x+2}$ alone. Try this again with only F2(X) checked.

9) How do the graphs of $y = \dfrac{x+1}{x+2}$ and $y = \dfrac{x^2-1}{x^2+x-2}$ compare? Is $\dfrac{x+1}{x+2} = \dfrac{x^2-1}{x^2+x-2}$ an identity? Explain how you know from the graphs.

You can also use the Solve aplet on the HP 39gs to check for identities. Is $\dfrac{x-2}{x^2-4} = \dfrac{x^2+x-2}{x-1}$ an identity? If not, find a value of x that makes the equation true.

Press the APLET key and select **Solve** from the list. This brings up the symbolic view where you should type (X-2)/(X^2-4)=(X^2+X-2)/(X-1) into E1. Press NUM to go to the numeric view, where you can solve for x numerically. You will see an input box for x that will already have a value in it. This value is called the seed. The calculator will start looking for a solution near this value. Set this value to zero and press the SOLVE menu key. You should see that one of the solutions is $x=-1$.

10) Press PLOT. Do you think $x=-1$ is the only solution to the equation? Explain. If not, find the other solution (try a different seed).

11) Do you think $x=0$ is a solution? Explain.

12) Do you think $\dfrac{x-2}{x^2-4} = \dfrac{x^2+x-2}{x-1}$ is an identity? Explain.

Is $\dfrac{x^2-1}{x-1} = \dfrac{x^2+3x+2}{x+2}$ an identity? Press SYMB, uncheck E1 and type (X^2-1)/(X-1) = (X^2+3X+2)/(X+2) in for E2. Press NUM and try a few different seeds.

13) Is $\dfrac{x^2-1}{x-1} = \dfrac{x^2+3x+2}{x+2}$ an identity? What behavior in the numeric view leads you to believe that the equation is or is not an identity?

Press PLOT.

14) What about this screen suggests that $\dfrac{x^2-1}{x-1} = \dfrac{x^2+3x+2}{x+2}$ is or is not an identity?