## Maximize Volume

Did you know it is possible to cha nge a two-dimensional object (a 5 inch x 8 inch index card) into a three-dimensional object (open box)? Your task is to find the maximum volume of that box. First, you need to find a formula for your box. Then you can explore the volume.

## Exercise 1

Start with a 5 inch $x 8$ inch index card. In order to move from a two-dimensional figure to a three-dimensional figure, you must cut out a small square from each corner of the index card. Now fold up the sides so that the dimension of your corner is the height of the box.


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a. The figure below shows how the squares are cut from the index card. Once the corners are cut, you get three dimensions for an open box, length, width, and heigh $t$. If you choose to have a height of 1 inch, label the three dimensions on the picture below.

b. If you choose to have a height of 2 inches, label the three dimensions on the picture below.

c. Could you choose to have a height of 3 inches? Explain your answer.
d. What is the largest corner you can cut? How do you know?
e. As the height of the box increases, how are the length and width affected?

## Exercise 2

In Exercise 1, you labeled two figures, one with a height of 1 inch and another with a height of 2 inches. Instead of labeling figures, create a table of values for length and width using the given height. You will also compute the volume of each box and record it in the table.
a. Fill in the table below.

| Height | Width | Length | Volume |
| :---: | :--- | :--- | :--- |
| 0 |  |  |  |
| 0.5 |  |  |  |
| 1 |  |  |  |
| 1.5 |  |  |  |
| 2 |  |  |  |
| 2.5 |  |  |  |
| 3 |  |  |  |
| $x$ |  |  |  |

b. For computing the volume, what values of height are possible?
c. You have already discovered what happens to length and width as the height increases. What happens to the volume as the height increases?
d. Based on the values in your table, estimate the height value that will give the largest volume.

## Exercise 3

Rather than estimating the height that will give the box with the largest volume, we can use the HP 39gs to find a more exact value. Transfer the VOLUME Aplet from your teacher or another student.

Press the PLOT key to see a graph of the volume function you discovered in Exercise 1. Keep in mind that there are some values for $x$ and $y$ that do not make sense, but are helpful for seeing a good graph.
a. Use the TRACE feature of the HP 39 gs to verify some of the values in your table from Exercise 1. You may not see all the values in your table, but you should also see some that are not in your table. Describe what you see as you trace from $x=0$ to $x=3$.
b. Trace to a value that you think is close to the largest volume. What is that value?
c. You can verify your estimate from exercise b using a different feature of the HP 39gs. Press the FCN menu key and then choose Extremum. After you press the ENTER key, you should see the answer at the bottom of your screen.

