Teacher Edition

Glencoe Secondary Mathematics

ALIGNED TO THE

COMMON CORE STATE STANDARDS

Algebra 2

McGraw Hill Education

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STEM

McGraw-Hill is committed to providing instructional materials in Science, Technology, Engineering, and Mathematics (STEM) that give students a solid foundation, one that prepares them for college and careers in the 21st Century.

Common Core State Standards

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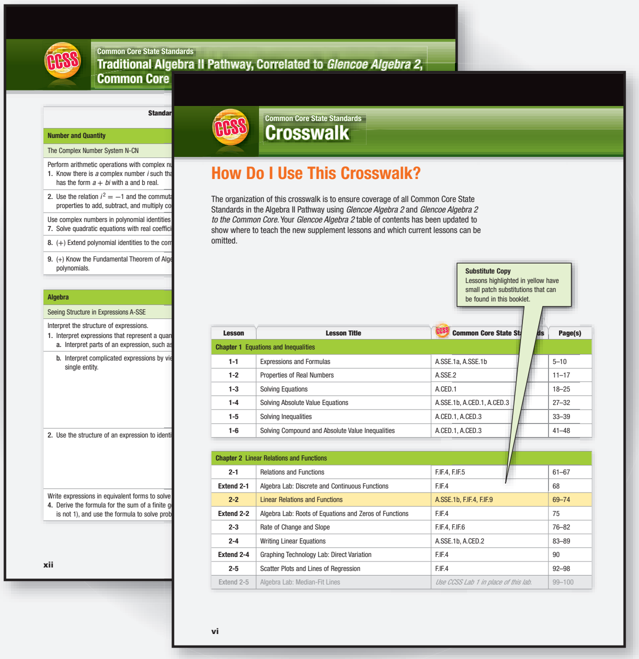
Welcome to Glencoe Secondary Mathematics to the Common Core

How to Use This Supplement

This supplement is your tour guide to understanding how Glencoe Secondary Mathematics programs teach the new Common Core State Standards. Its purpose is to help make a smooth transition from your state standards to the new Common Core State Standards.

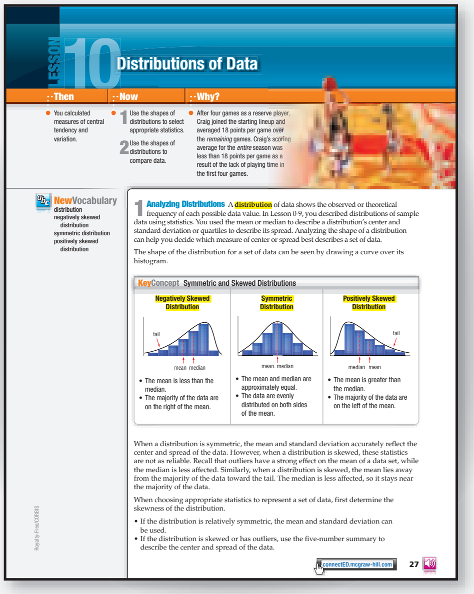
Crosswalk

The crosswalk is your guide to understanding how to use your current *Glencoe Algebra 2* program with this supplement to create a Common Core State Standards curriculum. Pages vi–xii show you which lessons in your textbook should be kept, which can be considered optional, which lessons have additional available content, and how the new material fits into the flow of the chapters you already use.



Correlations

Glencoe Algebra 2 and *Glencoe Secondary Mathematics to the Common Core* align your curriculum with the Common Core State Standards and the Traditional Algebra II Pathway. You can use pages xii–xviii to map each standard to the lesson(s) that address each standard.

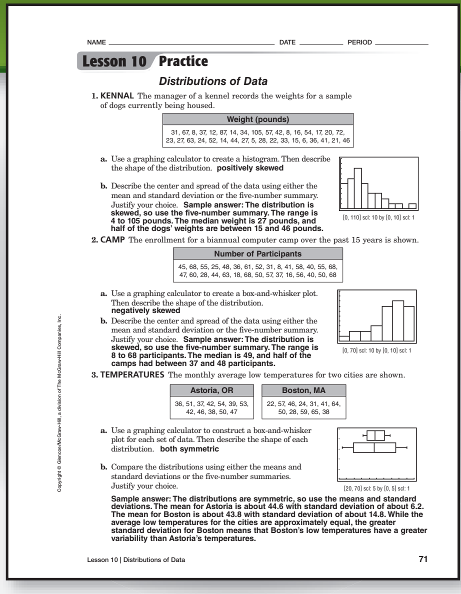


In This Booklet

Glencoe Secondary Mathematics to the Common Core contains additional lessons and labs to address the Common Core State Standards and the Traditional Algebra II Pathway. (See pages 1–64.) You can also find copy for patch substitutions that can help you better meet the Common Core State Standards using your existing program. (See pages 65–68.) Refer to the Crosswalk on pages v–xi for appropriate placement of this content in your *Glencoe Algebra 2* textbook.

Homework Practice

Pages 69–75 of the Student Edition of *Glencoe Secondary Mathematics to the Common Core* contain homework practice pages for the lessons added to meet the Common Core State Standards.



NAME ________________ DATE ________ PERIOD ________

Lesson 10 Practice

Distributions of Data

1. **KENNAL** The manager of a kennel records the weights for a sample of dogs currently being housed.

Weight (pounds)
31, 67, 8, 37, 12, 87, 14, 34, 105, 57, 42, 8, 16, 54, 17, 20, 72, 23, 27, 63, 24, 52, 14, 44, 27, 5, 28, 22, 33, 15, 6, 36, 41, 21, 46

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution. **positively skewed**

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice. **Sample answer: The distribution is skewed, so use the five-number summary. The range is 4 to 105 pounds. The median weight is 27 pounds, and half of the dogs' weights are between 15 and 46 pounds.**

[0, 110] scl: 10 by [0, 10] scl: 1

2. **CAMP** The enrollment for a biannual computer camp over the past 15 years is shown.

Number of Participants
45, 68, 55, 25, 48, 36, 61, 52, 31, 8, 41, 58, 40, 55, 68, 47, 60, 28, 44, 63, 18, 68, 50, 57, 37, 16, 56, 40, 50, 68

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution. **negatively skewed**

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice. **Sample answer: The distribution is skewed, so use the five-number summary. The range is 8 to 68 participants. The median is 49, and half of the camps had between 37 and 48 participants.**

[0, 70] scl: 10 by [0, 10] scl: 1

3. **TEMPERATURES** The monthly average low temperatures for two cities are shown.

Astoria, OR	Boston, MA
36, 51, 37, 42, 54, 39, 53, 42, 46, 38, 50, 47	22, 57, 46, 24, 31, 41, 64, 50, 28, 59, 65, 38

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution. **both symmetric**

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

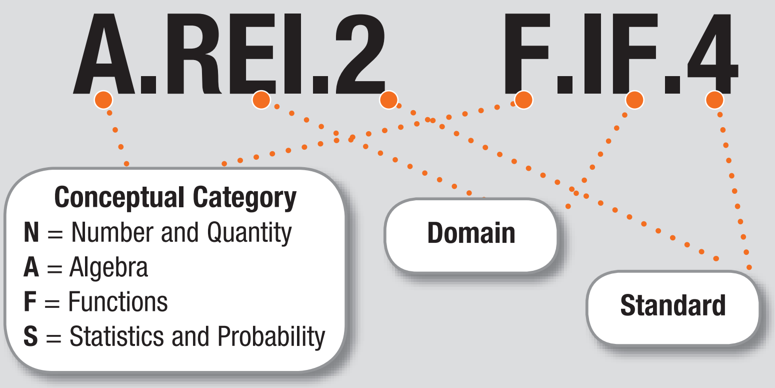
[20, 70] scl: 5 by [0, 5] scl: 1

Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean for Astoria is about 44.6 with standard deviation of about 6.2. The mean for Boston is about 43.8 with standard deviation of about 14.8. While the average low temperatures for the cities are approximately equal, the greater standard deviation for Boston means that Boston's low temperatures have a greater variability than Astoria's temperatures.

Lesson 10 | Distributions of Data 71

Decoding the Common Core State Standards

This diagram provides clarity for decoding the standard identifiers.



Domain Names	Abbreviations
The **C**omplex **N**umber System	**CN**
Seeing **S**tructure in **E**xpressions	**SSE**
Arithmetic with **P**olynomials and **R**ational Expressions	**APR**
Creating **E**quations	**CED**
Reasoning with Equations and **I**nequalities	**REI**
Interpreting **F**unctions	**IF**
Building **F**unctions	**BF**
Linear, Quadratic, and **E**xponential Models	**LE**
Trigonometric **F**unctions	**TF**
Interpreting Categorical and Quantitative **D**ata	**ID**
Making **I**nferences and Justifying **C**onclusions	**IC**
Using Probability to **M**ake **D**ecisions	**MD**

You can choose how to use *Glencoe Secondary Mathematics to the Common Core* in your classroom.

- To print, go to connectED.mcgraw-hill.com.
- To display for whole class instruction, go to connectED.mcgraw-hill.com.
- To order print copies of the Student Edition, contact your local sales representative.

Common Core State Standards

Crosswalk

How Do I Use This Crosswalk?

The organization of this crosswalk is to ensure coverage of all Common Core State Standards in the Algebra II Pathway using *Glencoe Algebra 2* and *Glencoe Algebra 2 to the Common Core*. Your *Glencoe Algebra 2* table of contents has been updated to show where to teach the new supplement lessons and which current lessons can be omitted.

Substitute Copy
Lessons highlighted in yellow have small patch substitutions that can be found in this booklet.

Lesson	Lesson Title	Common Core State Standards	Page(s)
Chapter 1 Equations and Inequalities			
1-1	Expressions and Formulas	A.SSE.1a, A.SSE.1b	5–10
1-2	Properties of Real Numbers	A.SSE.2	11–17
1-3	Solving Equations	A.CED.1	18–25
1-4	Solving Absolute Value Equations	A.SSE.1b, A.CED.1, A.CED.3	27–32
1-5	Solving Inequalities	A.CED.1, A.CED.3	33–39
1-6	Solving Compound and Absolute Value Inequalities	A.CED.1, A.CED.3	41–48
Chapter 2 Linear Relations and Functions			
2-1	Relations and Functions	F.IF.4, F.IF.5	61–67
Extend 2-1	Algebra Lab: Discrete and Continuous Functions	F.IF.4	68
2-2	Linear Relations and Functions	A.SSE.1b, F.IF.4, F.IF.9	69–74
Extend 2-2	Algebra Lab: Roots of Equations and Zeros of Functions	F.IF.4	75
2-3	Rate of Change and Slope	F.IF.4, F.IF.6	76–82
2-4	Writing Linear Equations	A.SSE.1b, A.CED.2	83–89
Extend 2-4	Graphing Technology Lab: Direct Variation	F.IF.4	90
2-5	Scatter Plots and Lines of Regression	F.IF.4	92–98
Extend 2-5	Algebra Lab: Median-Fit Lines	*Use CCSS Lab 1 in place of this lab.*	99–100

Lesson	Lesson Title	CCSS Common Core State Standards	Page(s)
CCSS Lab 1	Algebra Lab: Correlation and Causation	F.IF.4	CCSS 1–2
2-6	Special Functions	A.SSE.1b, A.CED.2, A.CED.3, F.IF.4, F.IF.5, F.IF.7b	101–107
Explore 2-7	Graphing Technology Lab: Families of Lines	F.BF.3	108
2-7	Parent Functions and Transformations	A.SSE.1b, A.CED.2, A.CED.3, F.IF.4, F.IF.9, F.BF.3	109–116
2-8	Graphing Linear and Absolute Value Inequalities	A.CED.3	117–121
Chapter 3 Systems of Equations and Inequalities			
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3-2	Solving Systems of Equations Algebraically	A.CED.3	143–150
3-3	Solving Systems of Inequalities by Graphing	A.CED.3	151–157
3-4	Optimization with Linear Programming	A.CED.3	160–166
3-5	Systems of Equations in Three Variables	A.CED.3	167–173

Add This In
Lessons and labs highlighted in blue can be found in this booklet.

Lesson	Lesson Title	CCSS Common Core State Standards	Page(s)
Chapter 4 Matrices			
4-1	Introduction to Matrices		185–191
4-2	Operations with Matrices		193–199
4-3	Multiplying Matrices		200–207
4-4	Transformations with Matrices		209–217
4-5	Determinants and Cramer's Rule	A.CED.3	220–228
4-6	Inverse Matrices and Systems of Equations	A.CED.3	229–235
Chapter 5 Quadratic Functions and Relations			
5-1	Graphing Quadratic Functions	A.SSE.1a, A.SSE.1b, A.CED.2, F.IF.4, F.IF.5, F.IF.9	249–257
Extend 5-1	Graphing Technology Lab: Modeling Real-World Data	F.IF.4	258
5-2	Solving Quadratic Equations by Graphing	A.CED.2, F.IF.4	259–266
Extend 5-2	Graphing Technology Lab: Solving Quadratic Equations by Graphing	*Use CCSS Lab 2 in place of this lab.*	267
CCSS Lab 2	Graphing Technology Lab: Solving Quadratic Equations by Graphing	A.REI.11	CCSS 3

Leave This Outt
Lessons highlighted in gray should be considered optional because these concepts are not included in the Geometry Algebra II Pathway.

Lesson	Lesson Title	Common Core State Standards	Page(s)
5-3	Solving Quadratic Equations by Factoring	*Use CCSS Lesson 3 in place of this lesson.*	268–275
CCSS Lesson 3	Solving Quadratic Equations by Factoring	A.SSE.2, A.APR.1, A.APR.4, A.CED.1, F.IF.8a	CCSS 4–10
5-4	Complex Numbers	N.CN.1, N.CN.2, N.CN.8	276–282
5-5	Completing the Square	N.CN.7, A.SSE.2, A.APR.4, A.CED.1, F.IF.8a	284–290
Extend 5-5	Graphing Technology Lab: Solving Quadratic Equations	N.CN.7	291
5-6	The Quadratic Formula and the Discriminant	N.CN.7, N.CN.8, A.SSE.1b, A.APR.4, A.CED.1, A.CED.4	292–300
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5-7	Transformations with Quadratic Functions	A.SSE.1b, A.CED.2, F.IF.4, F.IF.8a, F.BF.3	305–310
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CCSS Lab 5	Algebra Lab: Power Functions	F.IF.4	CCSS 13–14
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6-4	Analyzing Graphs of Polynomial Functions	A.SSE.1b, A.CED.2, F.IF.4, F.IF.7c	357–364
Extend 6-4	Graphing Technology Lab: Modeling Data Using Polynomial Functions	F.IF.4, F.IF.7c	365–366
6-5	Solving Polynomial Equations	A.CED.1	368–375
Extend 6-5	Graphing Technology Lab: Solving Polynomial Equations	*Use CCSS Lab 6 in place of this lab.*	376
CCSS Lab 6	Graphing Technology Lab: Polynomial Identities	A.REI.11	CCSS 15–16
6-6	The Remainder and Factor Theorems	A.APR.2, A.CED.1, F.IF.7c	377–382
6-7	Roots and Zeros	N.CN.9, A.APR.3, A.CED.1, F.IF.4, F.IF.7c	383–390
CCSS Lab 7	Graphing Technology Lab: Analyzing Polynomial Functions	A.APR.4	CCSS 17
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Lesson	Lesson Title	CCSS Common Core State Standards	Page(s)
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7-2	Inverse Functions and Relations	F.IF.4, F.IF.5, F.BF.4a	417–422
7-3	Square Root Functions and Inequalities	A.CED.2, F.IF.4, F.IF.5, F.IF.7b, F.IF.9, F.BF.3	424–430
7-4	*n*th Roots	A.SSE.2	431–436
Extend 7-4	Graphing Technology Lab: Graphing nth Root Functions	F.IF.7b, F.BF.3	437
7-5	Operations with Radical Expressions	A.SSE.2	439–445
7-6	Rational Exponents		446–452
7-7	Solving Radical Equations and Inequalities	A.REI.2	453–459
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Lesson	Lesson Title	Common Core State Standards	Page(s)
Chapter 8 Exponential and Logarithmic Functions and Relations			
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Explore 8-2	Graphing Technology Lab: Solving Exponential Equations and Inequalities	A.REI.11	483–484
8-2	Solving Exponential Equations and Inequalities	A.SSE.2, A.CED.1, F.LE.4	485–491
8-3	Logarithms and Logarithmic Functions	A.SSE.2, A.CED.2, F.IF.4, F.IF.5, F.IF.7e, F.BF.3	492–499
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8-4	Solving Logarithmic Equations and Inequalities	A.SSE.2, A.CED.1	502–507
8-5	Properties of Logarithms	A.CED.1	509–515
8-6	Common Logarithms	A.CED.1	516–522
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8-7	Base e and Natural Logarithms	A.SSE.2	525–531
8-8	Using Exponential and Logarithmic Functions	A.SSE.2, A.CED.1, A.CED.2, A.CED.3, F.IF.4, F.IF.8b, F.LE.4	533–539
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11-7	Proof by Mathematical Induction	A.SSE.1b, A.APR.5	727–731
Chapter 12 Statistics and Probability *Use the CCSS lessons/labs below in place of the existing content.*			
CCSS Lesson 8	Designing a Study	S.IC.2, S.IC.3, S.IC.5	CCSS 18–24

Lesson	Lesson Title	Common Core State Standards	Page(s)
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CCSS Lesson 11	Probability Distributions	S.SMD.7	CCSS 35–42
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14-2	Verifying Trigonometric Identities	F.TF.8	898–903
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14-4	Double-Angle and Half-Angle Identities	F.TF.8	911–917
14-5	Solving Trigonometric Equations	F.TF.8	919–925

Traditional Algebra II Pathway, Correlated to *Glencoe Algebra 2*, Common Core Edition

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Number and Quantity		
The Complex Number System N-CN		
Perform arithmetic operations with complex numbers. **1.** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	5-4	276–282
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	5-4	276–282
Use complex numbers in polynomial identities and equations. **7.** Solve quadratic equations with real coefficients that have complex solutions.	5-5, Extend 5-5, 5-6	284–290, 291, 292–300
8. (+) Extend polynomial identities to the complex numbers.	5-4, 5-6	276–282, 292–300
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	6-7	383–390

Algebra		
Seeing Structure in Expressions A-SSE		
Interpret the structure of expressions. **1.** Interpret expressions that represent a quantity in terms of its context.★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients.	1-1, 5-1	5–10, 249–257
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	1-1, 1-4, 2-2, 2-4, 2-6, 2-7, 5-1, 5-6, 5-7, 6-4, 10-2, 10-3, 10-4, 10-5, 10-6, 11-7	5–10, 27–32, 69–74, 83–89, 101–107, 109–116, 249–257, 292–300, 305–310, 357–364, 623–629, 631–637, 639–646, 648–655, 656–660, 727–731
2. Use the structure of an expression to identify ways to rewrite it.	1-2, 5-5, 7-4, 7-5, 8-2, 8-3, 8-4, 8-7, 8-8 CCSS Lesson 3	11–17, 284–290, 431–436, 439–445, 485–491, 492–499, 502–507, 525–531, 533–539 CCSS 4–10
Write expressions in equivalent forms to solve problems. **4.** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.★	11-3	696–702

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Arithmetic with Polynomials and Rational Expressions A-APR		
Perform arithmetic operations on polynomials. **1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	6-1 CCSS Lesson 3	333–339 CCSS 4–10
Understand the relationship between zeros and factors of polynomials. **2.** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	6-6	377–382
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	6-7	383–390
Use polynomial identities to solve problems. **4.** Prove polynomial identities and use them to describe numerical relationships.	5-5, 5-6 CCSS Lesson 3, CCSS Lab 7	284–290, 292–300 CCSS 4-10, CCSS 17
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	11-6, Extend 11-6, 11-7 CCSS Lesson 12	721–725, 726, 727–731 CCSS 43–49
Rewrite rational expressions. **6.** Rewrite simple rational expressions in different forms; write in the form $q(x) +$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	6-2 CCSS Lab 4	341–347 CCSS 11-12
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	9-1, 9-2, 9-4, 9-6	553–561, 562–568, 577–584, 594–602
Creating Equations★ A-CED		
Create equations that describe numbers or relationships. **1.** Create equations and inequalities in one variable and use them to solve problems.	1-3, 1-4, 1-5, 1-6, 5-5, 5-6, 5-8, 6-5, 6-6, 6-7, 8-2, 8-4, 8-5, 8-6, 8-8, 9-6 CCSS Lesson 3	18–25, 27–32, 33–39, 41–48, 284–290, 292–300, 312–318, 368–375, 377–382, 383–390, 485–491, 502–507, 509–515, 516–522, 533–539, 594–602 CCSS 4-10
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Throughout the text; for example, 2-4, 3-1, 3-2, 5-2, 7-3, 10-3, 13-7	Throughout the text; for example, 83-89, 135-141, 143-150, 259-266, 424-430, 631-637, 855-861

Standards	Student Edition Lesson(s)	Student Edition Page(s)
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	1-4, 1-5, 1-6, 2-6, 2-7, 2-8, 3-1, 3-2, 3-3, 3-4, 3-5, 4-5, 4-6, 5-8, 8-8, 9-6	27–32, 33–39, 41–48, 101–107, 109–116, 117–121, 135–141, 143–150, 151–157, 160–166, 167–173, 220–228, 229–235, 312–318, 533–539, 594–602
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	5-6, 10-1, 10-3, 11-2	292–300, 617–622, 631–637, 688–695
Reasoning with Equations and Inequalities A-REI		
Understand solving equations as a process of reasoning and explain the reasoning. **2.** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	7-7, Extend 7-7, 9-6, Extend 9-6	453–459, 460–461, 594–602, 603–604
Represent and solve equations and inequalities graphically. **11.** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	3-1, Extend 7-7, Explore 8-2, Extend 8-6, Extend 9-6, 10-7 CCSS Lab 2, CCSS Lab 6	135–141, 460–461, 483–484, 523–524, 603–604, 662–667 CCSS 3, CCSS 15-16

Functions		
Interpreting Functions F-IF		
Interpret functions that arise in applications in terms of the context. **4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.★	Throughout the text; for example, 2-1, Extend 2-1, Extend 2-2, 2-6, 6-3, 9-4, 11-1, 13-6	Throughout the text; for example, 61–67, 68, 75, 101–107, 348–355, 577–584, 681–687, 848–854
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	2-1, 2-6, 5-1, 6-3, 7-2, 7-3, 8-1, 8-3, 9-3, 9-4, 13-7	61–67, 101–107, 249–257, 348–355, 417–422, 424–430, 475–482, 492–499, 569–575, 577–584, 855–861
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	2-3, Extend 5-7	76–82, 311

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Analyze functions using different representations. **7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	2-6, 7-3, Extend 7-4	101–107, 424–430, 437
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	6-3, 6-4, Extend 6-4, 6-6, 6-7	348–355, 357–364, 365–366, 377–382, 383–390
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	8-1, 8-3, 13-7, 13-8	475–482, 492–499, 855–861, 863–870
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	5-5, 5-7 CCSS Lesson 3	284–290, 305–310 CCSS 4–10
b. Use the properties of exponents to interpret expressions for exponential functions.	8-1, 8-8	475–482, 533–539
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	2-2, 2-7, 5-1, 6-3, 7-1, 7-3, 8-1, 9-4, 10-6	69–74, 109–116, 249–257, 348–355, 409–416, 424–430, 475–482, 577–584, 656–660
Building Functions F-BF		
1. Build a function that models a relationship between two quantities. **b.** Combine standard function types using arithmetic operations.	7-1, Extend 8-8, 13-8	409–416, 540, 863–870
Build new functions from existing functions. **3.** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	Explore 2-7, 2-7, Explore 5-7, 5-7, 7-3, Extend 7-4, 8-1, 8-3, 9-3, Explore 13-8, 13-8	108, 109–116, 303–304, 305-310, 424–430, 437, 475–482, 492–499, 569–575, 862, 863–870
4. Find inverse functions. **a.** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.	7-2	417–422
Linear, Quadratic, and Exponential Models F-LE		
Construct and compare linear and exponential models and solve problems. **4.** For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	8-2, 8-8	485–491, 533–539

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Trigonometric Functions F-TF		
Extend the domain of trigonometric functions using the unit circle. **1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	13-2, 13-6	817–823, 848–854
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	13-6	848–854
Model periodic phenomena with trigonometric functions. **5.** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	13-7, 13-8	855–861, 863–870
Prove and apply trigonometric identities. **8.** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.	14-1, 14-2, 14-3, 14-4, 14-5	891–897, 898–903, 904–909, 911–917, 919–925

Statistics and Probability		
Interpreting Categorical and Quantitative Data S-ID		
Summarize, represent, and interpret data on a single count or measurement variable. **4.** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	CCSS Lesson 13, CCSS Lab 14	CCSS 50-55, CCSS 56-57
Making Inferences and Justifying Conclusions S-IC		
Understand and evaluate random processes underlying statistical experiments **1.** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	CCSS Lesson 10, CCSS Lesson 13	CCSS 27-34, CCSS 50-55
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	CCSS Lesson 8, CCSS Lab 9	CCSS 18-24, CCSS 25-26
Make inferences and justify conclusions from sample surveys, experiments, and observational studies **3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	CCSS Lesson 8	CCSS 18-24
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	CCSS Lab 9, CCSS Lesson 15	CCSS 25-26, CCSS 58-64
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	CCSS Lesson 8	CCSS 18-24
6. Evaluate reports based on data.	CCSS Lab 9	CCSS 25-26

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Using Probability to Make Decisions S-MD		
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	CCSS Lesson 12	CCSS 43-49
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	CCSS Lesson 11, CCSS Lesson 12, CCSS Lesson 15	CCSS 35-42, CCSS 43-49, CCSS 58-64

LAB 1

Algebra Lab
Correlation and Causation

You have learned that the correlation coefficient measures how well an equation fits a set of data. A correlation coefficient close to 1 or −1 indicates a high correlation. However, this does not imply causation. When there is **causation**, one data set is the direct cause of the other data set. When there is a correlation between two sets of data, the data sets are related.

The news clip to the right uses correlation to imply causation. While a study may have found a high correlation between these two variables, this does not mean that cell phone use causes brain tumors. Other variables, such as an individual's family history, diet, and environment could also have an effect on the formation of brain tumors.

A **lurking variable** affects the relationship between two other variables, but is not included in the study that compares them.

NEWS

Cell Phones Cause Brain Tumors!

Studies have shown that brain tumors are linked to cell phone use. One particular study showed a very strong correlation between the two events.

EXAMPLE 1

Determine whether the following correlations possibly show causation. Write *yes* or *no*. If not, identify other lurking variables.

a. Studies have shown that students are less energetic after they eat lunch.

No; students could have had gym class before lunch, or the class after lunch could be one that they do not find interesting.

b. If Kevin lifts weights, he will make the football team.

No; Kevin may need to maintain a certain grade-point average to make the team. He also needs to be talented enough to make the team.

c. When the Sun is visible, we have daylight.

Yes; no other variables cause daylight.

Exercises

Determine whether the following correlations possibly show causation. Write *yes* or *no*. If not, identify other lurking variables. **1–3. See margin.**

1. If Allison studies, she will get an A.
2. When Lisa exercises, she is in a better mood.
3. The gun control laws have reduced violent crime.
4. Smoking causes lung cancer. **yes**
5. If we have a Level 2 snow emergency, we do not have school. **yes**
6. Reading more increases one's intelligence.

6. No; a person's experiences in school, at the workplace, and in everyday life could improve his or her intelligence.

7. Sample answer: A large number of unbiased studies need to be completed that show a direct link between the variables and all of the lurking variables need to be eliminated from the studies.

Think About It

7. What do you think must be done to show that a correlation between two variables actually shows causation?

(continued on the next page)

1 Focus

Objective Explore the difference between correlation and causation.

Materials for Each Group
- graphing calculator

Teaching Tip
This activity requires students to calculate and analyze the correlation coefficient. You may wish to review this concept as a class before you begin the Examples.

2 Teach

Working in Cooperative Groups
For each Example, each student should work with a partner.

Have the pairs discuss lurking variables and their effect on other variables. Have them think of three situations where lurking variables are influencing a relationship between two other variables that show a high correlation. Then have them try to think of two variables in which there is causation.

Practice Have students complete Exercises 1–7.

Additional Answers
1. No; she will have to complete her assignments with high quality work, participate in class, understand the material, score high on her quizzes and tests, and meet any other requirements designated by her instructor.
2. No; she could have an argument with her family, her friends could do something to upset her, or something could happen at work.
3. No; more violent criminals could already be in prison, rehabilitation could be working, unemployment could be falling, or the economy could be improving.

3 Assess

Formative Assessment
Use Exercise 7 to assess whether students understand the difference between correlation and causation.

From Concrete to Abstract
Ask students why they think there is a causation in Exercise 4. Discuss why this is accepted as fact in today's society.

Extending the Concept
Ask students to identify another situation in which causation is possible. Have them collect the data and calculate the correlation coefficient. Then ask them to say why they think the relationship shows causation.

Practice Have students complete Exercises 8–11.

Additional Answers

8. −0.73, very weak negative correlation; no causation is possible when the correlation is weak.

9. 0.89, strong positive correlation; no causation is possible because humidity, rain, time of day, and price of the ice cream are also factors.

10. 0.99, very strong positive correlation; no causation is possible because the popularity of the company, the products they sell, the economy, and the accessibility and user-friendliness of the company's Web site are also factors.

11. 1.00, very strong correlation, yes, causation exists because the age of a tree is a direct cause for the number of rings from pith to bark.

Algebra Lab
Correlation and Causation *Continued*

Statistics are often provided that show a correlation between two events and causation is implied, but not confirmed. The best method to find evidence that *x* causes *y* is to actually perform multiple experiments with large sample sizes. When an experiment is not available, the only option is to determine if causation is *possible.*

EXAMPLE 2

The table shows IQ scores for ten high school students and their corresponding grade-point averages.

IQ	GPA	IQ	GPA
106	3.4	118	3.9
110	3.6	107	3.3
109	3.5	111	3.6
98	2.9	109	3.7
115	3.8	102	3.1

a. Calculate the correlation coefficient.
The value of r is about 0.96.

b. Describe the correlation.
There is a strong positive correlation.

c. Is there causation? Explain your reasoning.
There is no causation. Many other factors determine grade-point averages, including effort, study habits, knowledge of the material in the particular classes, and extracurricular activities.

EXAMPLE 3

The table shows the approximate boiling temperatures of water at different altitudes.

Altitude (ft)	Temp. (°F)
0	212
984	210
2000	208
3000	206
5000	203
7500	198
10,000	194
20,000	178
26,000	168

a. Calculate the correlation coefficient.
The value of r is about −0.99.

b. Describe the correlation.
There is a strong negative correlation.

c. Is there causation? Explain your reasoning.
There appears to be causation. Altitude is a direct cause for the boiling temperature of water to decrease.

Exercises

Calculate the correlation coefficient and describe the correlation. Determine if causation is possible. Explain your reasoning. **8–11. See margin.**

8.

Car Value ($)	Miles per Gallon	Car Value ($)	Miles per Gallon
14,000	55	25,000	33
16,000	48	29,000	29
18,500	37	32,000	19
20,000	35	35,000	26
22,500	29	40,000	33

9.

Temp. (°F)	Ice Cream Sales ($)	Temp. (°F)	Ice Cream Sales ($)
76	850	85	905
78	885	86	1005
80	875	87	1060
82	920	88	1105
84	955	89	1165

10.

Year	Sales via Internet ($)	Year	Sales via Internet ($)
2001	1923	2006	7067
2002	2806	2007	8568
2003	3467	2008	9991
2004	4848	2009	10,587
2005	5943	2010	12,695

11.

Number of Rings from Pith to Bark	Age of Tree (yr.)	Number of Rings from Pith to Bark	Age of Tree (yr.)
5	5	30	30
10	10	40	40
15	15	50	50
20	20	60	60

LAB 2 Graphing Technology Lab
Solving Quadratic Equations by Graphing

You can use a TI-83/84 Plus graphing calculator to solve quadratic equations.

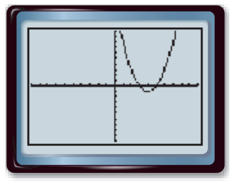
Activity Solving Quadratic Equations

Solve $x^2 - 8x + 15 = 0$.

Step 1 Let **Y1** $= x^2 - 8x + 15$ and **Y2** $= 0$.

Step 2 Graph **Y1** and **Y2** in the standard viewing window.

KEYSTROKES: [Y=] [X,T,θ,*n*] [x^2] [(−)] 8 [X,T,θ,*n*] [+] 15 [ENTER] 0 [ZOOM] 6



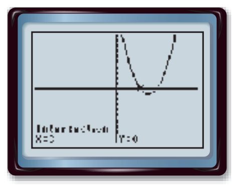
[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 3 To find the x-intercepts, determine the points where **Y1** = **Y2**.

KEYSTROKES: [2nd] [CALC] 5

Press [ENTER] for the first equation. Press [ENTER] for the second equation.

Move the cursor as close to the left x-intercept and press [ENTER].



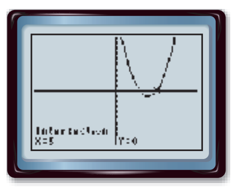
[−10, 10] scl: 1 by [−10, 10] scl: 1

Find the second x-intercept.

KEYSTROKES: [2nd] [CALC] 5

Press [ENTER] for the first equation. Press [ENTER] for the second equation.

Move the cursor as close to the right x-intercept and press [ENTER].



[−10, 10] scl: 1 by [−10, 10] scl: 1

The x-intercepts are 3 and 5, so $x = 3$ and $x = 5$.

Exercises

Solve each equation. Round to the nearest tenth if necessary.

1. $x^2 - 7x + 12 = 0$ **3, 4**
2. $x^2 + 5x + 6 = 0$ **−3, −2**
3. $x^2 - 3 = 2x$ **−1, 3**
4. $x^2 + 5x + 6 = 12$ **−6, 1**
5. $x^2 + 5x = 0$ **0, −5**
6. $x^2 - 4 = 0$ **−2, 2**
7. $x^2 + 8x + 16 = 0$ **−4**
8. $x^2 - 10x = -25$ **5**
9. $9x^2 + 48x + 64 = 0$ **−2.7**
10. $2x^2 + 3x - 1 = 0$ **−1.8, 0.3**
11. $5x^2 - 7x = -2$ **0.4, 1**
12. $6x^2 + 2x + 1 = 0$ **no real solution**

From Concrete to Abstract

Have students explain why the intercepts of the graph of a function can be used to identify the roots of an equation. The x-intercepts of a graph occur where $y = 0$. At these points, the equation of the function reduces to a single-variable equation in x.

Extending the Concept

Ask:

- The x-intercepts in the Activity are 3 and 5. Have students multiply $(x - 3)(x - 5)$ and ask them to compare the result to the equation they have solved.

1 Focus

Objective Use a graphing calculator to solve quadratic equations.

Materials for Each Group

- TI-83/84 Plus or other graphing calculator

Teaching Tips

In order to use a graph to solve the quadratic equation $ax^2 + bx + c = 0$, students must graph the related function $y = ax^2 + bx + c$.

2 Teach

Working in Cooperative Groups

Put students in groups of two or three, mixing abilities. Have each group work through the Activity, comparing screens with each other as they go.

- Steps 3 and 4 can only be done while the graph is showing on the screen. If students try to do these steps from the home screen, remind them to display the graph first.
- When using [TRACE], the cursor's initial position is at the y-intercept, so it may not appear on the screen.

Practice Have students complete Exercises 1–12.

3 Assess

Formative Assessment

Use Exercise 3 to assess whether students understand that a quadratic equation should first be written in the form $ax^2 + bx + c = 0$ before solving.

1 Focus

Vertical Alignment

Before Lesson 3 Find the greatest common factors of sets of numbers.

Lesson 3 Write quadratic equations in intercept form. Solve quadratic equations by factoring.

After Lesson 3 Solve quadratic equations using the Quadratic Formula.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- Does $x^2 - 8x + 12$ have a maximum or minimum value? minimum
- Solve $y = x^2 - 8x + 12 = 0$ by graphing. 2, 6
- Compare the soultions to $x^2 - 8x + 12 = 0$ and $(x - 6)(x - 2) = 0$. The solutions are the same because the equations are equivalent.

LESSON 3 Solving Quadratic Equations by Factoring

Then

- You found the greatest common factors of sets of numbers.

Now

1. Write quadratic equations in intercept form.
2. Solve quadratic equations by factoring.

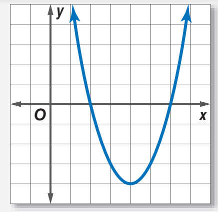
Why?

- The **factored form** of a quadratic equation is $0 = a(x - p)(x - q)$. In the equation, p and q represent the x-intercepts of the graph of the equation.

The x-intercepts of the graph at the right are 2 and 6. In this lesson, you will learn how to change a quadratic equation in factored form into standard form and vice versa.

Standard Form	Factored Form
$0 = x^2 - 8x + 12$	$0 = (x - 6)(x - 2)$

Factors



Related Graph 2 and 6 are x-intercepts.

New Vocabulary
factored form
FOIL method

1 Factored Form

You can use the FOIL method to write a quadratic equation that is in factored form in standard form. The **FOIL method** uses the Distributive Property to multiply binomials.

Key Concept FOIL Method for Multiplying Binomials

Words To multiply two binomials, find the sum of the products of **F** the *First terms*, **O** the *Outer terms*, **I** the *Inner terms*, and **L** the *Last terms*.

Examples

	Product of First Terms		Product of Outer Terms		Product of Inner Terms		Product of Last Terms
$(x - 6)(x - 2) =$	$(x)(x)$	+	$(x)(-2)$	+	$(-6)(x)$	+	$(-6)(-2)$

$= x^2 - 2x - 6x + 12$ or $x^2 - 8x + 12$

Example 1 Translate Sentences into Equations

Write a quadratic equation in standard form with $-\frac{1}{3}$ and 6 as its roots.

$(x - p)(x - q) = 0$ Write the pattern.

$\left[x - \left(-\frac{1}{3}\right)\right](x - 6) = 0$ Replace p with $-\frac{1}{3}$ and q with 6.

$\left(x + \frac{1}{3}\right)(x - 6) = 0$ Simplify.

$x^2 - \frac{17}{3}x - 2 = 0$ Multiply.

$3x^2 - 17x - 6 = 0$ Multiply each side by 3 so that b and c are integers.

Guided Practice

$4x^2 + 17x - 15 = 0$

1. Write a quadratic equation in standard form with $\frac{3}{4}$ and -5 as its roots.

2 Solve Equations by Factoring

Solving quadratic equations by factoring is an application of the Zero Product Property.

KeyConcept Zero Product Property

Words	For any real numbers *a* and *b*, if $ab = 0$, then either $a = 0$, $b = 0$, or both *a* and *b* equal zero.
Example	If $(x + 3)(x - 5) = 0$, then $x + 3 = 0$ or $x - 5 = 0$.

Example 2 Factor the GCF

Solve $16x^2 + 8x = 0$.

$16x^2 + 8x = 0$	Original equation.
$8x(2x) + 8x^2(1) = 0$	Factor the GCF.
$8x(2x + 1) = 0$	Distributive Property
$8x = 0$ or $2x + 1 = 0$	Zero Product Property
$x = 0 \qquad 2x = -1$ $x = -\frac{1}{2}$	Solve both equations.

GuidedPractice Solve each equation.

2A. $20x^2 + 15x = 0$ **$0, -\frac{3}{4}$** **2B.** $4y^2 + 16y = 0$ **$0, -4$** **2C.** $6a^5 + 18a^4 = 0$ **$0, -3$**

ReviewVocabulary
perfect square a number with a positive square root that is a whole number

Trinomials and binomials that are perfect squares have special factoring rules. In order to use these rules, the first and last terms need to be perfect squares and the middle term needs to be twice the product of the square roots of the first and last terms.

Example 3 Perfect Squares and Differences of Squares

Solve each equation.

a. $x^2 + 16x + 64 = 0$

$x^2 = (x)^2;\ 64 = (8)^2$	First and last terms are perfect squares.
$16x = 2(x)(8)$	Middle term equals $2ab$.

$x^2 + 16x + 64$ is a perfect square trinomial.

$x^2 + 16x + 64 = 0$	Original equation
$(x + 8)^2 = 0$	Factor using the pattern.
$x + 8 = 0$	Take the square root of each side.
$x = -8$	Solve.

b. $x^2 = 64$

$x^2 = 64$	Original equation
$x^2 - 64 = 0$	Subtract 64 from each side.
$x^2 - (8)^2 = 0$	Write in the form $a^2 - b^2$.
$(x + 8)(x - 8) = 0$	Factor the difference of squares.
$x + 8 = 0$ or $x - 8 = 0$	Zero Product Property
$x = -8 \qquad x = 8$	Solve.

StudyTip
Square Roots By inspection, notice that the square root of 64 is −8 and 8. Also, for $x^2 = 4$, the solutions would be −2 and 2.

GuidedPractice

3A. $4x^2 - 12x + 9 = 0$ **$\frac{2}{3}$** **3B.** $81x^2 - 9x = 0$ **$0, \frac{1}{9}$** **3C.** $6a^2 - 3a = 0$ **$0, \frac{1}{2}$**

DifferentiatedInstruction AL OL BL

If students think that the steps in Example 1 provide the only possible equation for the given roots,

Then provide each student with a sheet of grid paper. Have students begin by drawing a coordinate grid with two points on the *x*-axis plotted as the roots of a quadratic equation. Ask students to draw several parabolas that might be the graphs of different equations having those two points as their solutions. Point out that this demonstrates that the steps shown in Example 1 yield just *one* of the possible equations having the given roots.

1 Factored Form

Example 1 shows how to write a quadratic equation for a given pair of roots.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Example

1 Write a quadratic equation in standard form with $\frac{1}{2}$ and −5 as its roots. Sample answer: $2x^2 + 9x - 5 = 0$

Additional Examples also in Interactive Classroom PowerPoint® Presentations

Interactive White Board READY

2 Solve Equations by Factoring

Examples 2 and 3 show how to solve quadratic equations by factoring and by inspection. **Example 4** shows how to solve quadratic equations using a pattern. **Example 5** shows how to solve a real-world problem using factoring to solve a quadratic equation.

Additional Examples

2 Solve each equation.

a. $9y^2 + 3y = 0$ $-\frac{1}{3}, 0$

b. $5a^2 - 20a = 0$ $0, 4$

3 Solve each equation.

a. $x^2 - 6x + 9 = 0$ 3

b. $y^2 = 36$ $-6, 6$

Additional Example

4 Solve each equation.

a. $x^2 - 2x - 15 = 0$

$-3, 5$

b. $5x^2 + 34x + 24 = 0$

$-\frac{4}{5}, -6$

StudyTip

Trinomials If values for m and p exist, then the trinomial can always be factored.

A special pattern is used when factoring trinomials of the form $ax^2 + bx + c$. First, multiply the values of a and c. Then, find two values, m and p, such that their product equals ac and their sum equals b.

Consider $6x^2 + 13x - 5$: $ac = 6(-5) = -30$.

Factors of −30	Sum	Factors of −30	Sum
1, −30	−29	−1, 30	29
2, −15	−13	**−2, 15**	**13**
3, −10	−7	−3, 10	7
5, −6	−1	−5, 6	1

Now the middle term, $13x$, can be rewritten as $-2x + 15x$.

This polynomial can now be factored by grouping.

$6x^2 + 13x - 5 = 6x^2 + mx + px - 5$ Write the pattern.
$= 6x^2 - 2x + 15x - 5$ $m = -2$ and $p = 15$
$= (6x^2 - 2x) + (15x - 5)$ Group terms.
$= 2x(3x - 1) + 5(3x - 1)$ Factor the GCF.
$= (2x + 5)(3x - 1)$ Distributive Property

PT

Example 4 Factor Trinomials

Solve each equation.

a. $x^2 + 9x + 20 = 0$

$ac = 20$ $a = 1, c = 20$

Factors of 20	Sum	Factors of 20	Sum
1, 20	21	−1, −20	−21
2, 10	12	−2, −10	−12
4, 5	**9**	−4, −5	−9

StudyTip

Trinomials It does not matter if the values of m and p are switched when grouping.

$x^2 + 9x + 20 = 0$ Original expression
$x^2 + mx + px + 20 = 0$ Write the pattern.
$x^2 + 4x + 5x + 20 = 0$ $m = 4$ and $p = 5$
$(x^2 + 4x) + (5x + 20) = 0$ Group terms with common factors.
$x(x + 4) + 5(x + 4) = 0$ Factor the GCF from each grouping.
$(x + 5)(x + 4) = 0$ Distributive Property
$x + 5 = 0$ or $x + 4 = 0$ Zero Product Property
$x = -5$ $x = -4$ Solve each equation.

b. $6y^2 - 23y + 20 = 0$

$ac = 120$ $a = 6, c = 20$
$m = -8, p = -15$ $-8(-15) = 120; -8 + (-15) = -23$
$6y^2 - 23y + 20 = 0$ Original equation
$6y^2 + my + py + 20 = 0$ Write the pattern.
$6y^2 - 8y - 15y + 20 = 0$ $m = -8$ and $p = -15$
$(6y^2 - 8y) + (-15y + 20) = 0$ Group terms with common factors.
$2y(3y - 4) - 5(3y - 4) = 0$ Factor the GCF from each grouping.
$(2y - 5)(3y - 4) = 0$ Distributive Property
$2y - 5 = 0$ or $3y - 4 = 0$ Zero Product Property
$2y = 5$ $3y = 4$ Solve both equations.
$y = \frac{5}{2}$ $y = \frac{4}{3}$

Focus on Mathematical Content

Solving Quadratics by Factoring Quadratic equations can be solved using several different methods. Factoring can be a quick method. Once a polynomial has been factored, the Zero Product Property may be used to find the roots of the equation. If the polynomial is difficult to factor or is not factorable, then other methods must be used.

GuidedPractice

4A. $x^2 - 11x + 30 = 0$ **5, 6**

4B. $x^2 - 4x - 21 = 0$ **−3, 7**

4C. $15x^2 - 8x + 1 = 0$ $\frac{1}{5}, \frac{1}{3}$

4D. $-12x^2 + 8x + 15 = 0$ $-\frac{5}{6}, \frac{3}{2}$

Real-WorldLink

Cuba's Osleidys Menendez broke the javelin world record in 2002 with a distance of 234 feet 8 inches.

Source: *New York Times*

Stu Forster/Getty Images Sport/Getty Images

Real-World Example 5 Solve Equations by Factoring

TRACK AND FIELD **The height of a javelin in feet is modeled by $h(t) = -16t^2 + 79t + 5$, where t is the time in seconds after the javelin is thrown. How long is it in the air?**

To determine how long the javelin is in the air, we need to find when the height equals 0. We can do this by solving $-16t^2 + 79t + 5 = 0$.

$-16t^2 + 79t + 5 = 0$	Original equation
$m = 80;\ p = -1$	$-16(5) = -80,\ 80 \cdot (-1) = -80,\ 80 + (-1) = 79$
$-16t^2 + 80t - t + 5 = 0$	Write the pattern.
$(-16t^2 + 80t) + (-t + 5) = 0$	Group terms with common factors.
$16t(-t + 5) + 1(-t + 5) = 0$	Factor GCF from each group.
$(16t + 1)(-t + 5) = 0$	Distributive Property
$16t + 1 = 0$ or $-t + 5 = 0$	Zero Product Property
$16t = -1$ $\quad -t = -5$	Solve both equations.
$t = -\frac{1}{16}$ $\quad t = 5$	Solve.

CHECK We have two solutions.

- The first solution is negative and since time cannot be negative, this solution can be eliminated.
- The second solution of 5 seconds seems reasonable for the time a javelin spends in the air.
- The answer can be confirmed by substituting back into the original equation.

$$-16t^2 + 79t + 5 = 0$$
$$-16(5)^2 + 79(5) + 5 \stackrel{?}{=} 0$$
$$-400 + 395 + 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

The javelin is in the air for 5 seconds.

GuidedPractice

5. **BUNGEE JUMPING** Juan recorded his brother bungee jumping from a height of 1100 feet. At the time the cord lifted his brother back up, he was 76 feet above the ground. If Juan started recording as soon as his brother fell, how much time elapsed when the cord snapped back? Use $f(t) = -16t^2 + c$, where c is the height in feet. **8 seconds**

Additional Example

5 **ARCHITECTURE** The entrance to an office building is an arch in the shape of a parabola whose vertex is the height of the arch. The height of the arch is given by $h = 9 - x^2$, where x is the horizontal distance from the center of the arch. Both h and x are measured in feet. How wide is the arch at ground level? **6 ft**

WatchOut!

Common Misconceptions In Example 5, some students may suggest solving the equation by dividing both sides by t. Point out that this cannot be done because the value of t could be zero, and division by zero is undefined.

Teach with Tech

Document Camera Choose several students to share their work with the class and explain their answers. Have students check their work by substituting their solutions into the original equation.

3 Practice

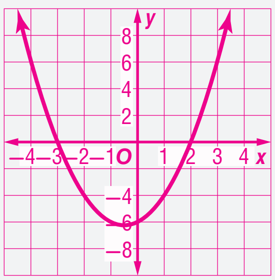
Formative Assessment

Use Exercises 1–16 to check for understanding.

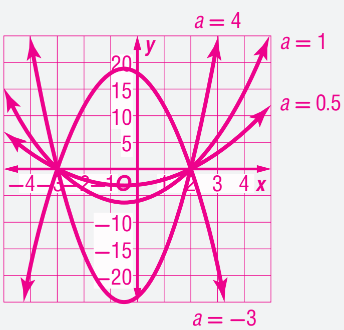
Use the chart at the bottom of this page to customize assignments for your students.

Additional Answers

68a.



68c.



Check Your Understanding

Example 1 Write a quadratic equation in standard form with the given root(s).

1. $-8, 5$ **$x^2 + 3x - 40 = 0$**
2. $\frac{3}{2}, \frac{1}{4}$ **$8x^2 - 14x + 3 = 0$**
3. $-\frac{2}{3}, \frac{5}{2}$ **$6x^2 - 11x - 10 = 0$**

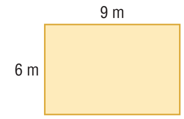
Examples 2–4 Factor each polynomial. **5. $(6x - 1)(3x + 4)$ 7. $(x - 7)(x + 3)$ 8. $(2x - 5)(x + 6)$**

4. $35x^2 - 15x$ **$5x(7x - 3)$**
5. $18x^2 - 3x + 24x - 4$
6. $x^2 - 12x + 32$ **$(x - 8)(x - 4)$**
7. $x^2 - 4x - 21$
8. $2x^2 + 7x - 30$
9. $16x^2 - 16x + 3$ **$(4x - 3)(4x - 1)$**

Example 5 Solve each equation.

10. $x^2 - 36 = 0$ **$-6, 6$**
11. $12x^2 - 18x = 0$ **$0, \frac{3}{2}$**
12. $12x^2 - 2x - 2 = 0$ **$-\frac{1}{3}, \frac{1}{2}$**
13. $x^2 - 9x = 0$ **$0, 9$**
14. $x^2 - 3x - 28 = 0$ **$-4, 7$**
15. $2x^2 - 24x = -72$ **6**

16. **GARDENING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then? **9 m by 12 m**



Practice and Problem Solving

Example 1 Write a quadratic equation in standard form with the given root(s).

17. 7 **$x^2 - 14x + 49 = 0$**
18. $-5, \frac{1}{2}$ **$2x^2 + 9x - 5 = 0$**
19. $\frac{1}{5}, 6$ **$5x^2 - 31x + 6 = 0$**

Examples 2–4 Factor each polynomial.

20. $40a^2 - 32a$ **$8a(5a - 4)$**
21. $51c^3 - 34c$ **$17c(3c^2 - 2)$**
22. $32xy + 40bx - 12ay - 15ab$
23. $3x^2 - 12$ **$3(x + 2)(x - 2)$**
24. $15y^2 - 240$
25. $48cg + 36cf - 4dg - 3df$
26. $x^2 + 13x + 40$
27. $x^2 - 9x - 22$
28. $3x^2 + 12x - 36$
29. $15x^2 + 7x - 2$
30. $4x^2 + 29x + 30$
31. $18x^2 + 15x - 12$
32. $8x^2z^2 - 4xz^2 - 12z^2$ **$4z^2(2x - 3)(x + 1)$**
33. $9x^2 - 25$ **$(3x + 5)(3x - 5)$**
34. $18x^2y^2 - 24xy^2 + 36y^2$ **$6y^2(3x^2 - 4x + 6)$**

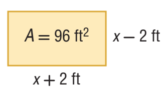
22. $(8x - 3a)\bullet(4y + 5b)$
24. $15(y + 4)\bullet(y - 4)$
25. $(12c - d)\bullet(4g + 3f)$
26. $(x + 8)\bullet(x + 5)$
27. $(x - 11)\bullet(x + 2)$
28. $3(x + 6)\bullet(x - 2)$
29. $(5x - 1)\bullet(3x + 2)$
30. $(4x + 5)\bullet(x + 6)$
31. $3(2x - 1)\bullet(3x + 4)$

Example 3 Solve each equation.

35. $15x^2 - 84x - 36 = 0$ **$-\frac{2}{5}, 6$**
36. $12x^2 + 13x - 14 = 0$ **$-\frac{7}{4}, \frac{2}{3}$**
37. $12x^2 - 108x = 0$ **$0, 9$**
38. $x^2 + 4x - 45 = 0$ **$5, -9$**
39. $x^2 - 5x - 24 = 0$ **$8, -3$**
40. $x^2 = 121$ **$11, -11$**
41. $x^2 + 13 = 17$ **$2, -2$**
42. $-3x^2 - 10x + 8 = 0$ **$-4, \frac{2}{3}$**
43. $-8x^2 + 46x - 30 = 0$ **$5, \frac{3}{4}$**
44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle. **7 cm, 24 cm, 25 cm**
45. **NUMBER THEORY** Find two consecutive even integers with a product of 624. **24 and 26 or −24 and −26**

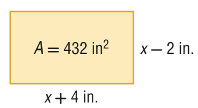
GEOMETRY Find x and the dimensions of each rectangle.

46.



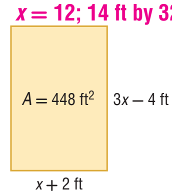
$x = 10$; 8 ft by 12 ft

47.



$x = 20$; 24 in. by 18 in.

48. **$x = 12$; 14 ft by 32 ft**



Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	17–48, 79, 82, 84–86	17–47 odd	18–48 even, 79, 82, 84–86
OL Core	17–47 odd, 49–63 odd, 65–70, 71–77 odd, 79, 82, 84–86	17–48	49–79, 82, 84–86
BL Advanced	49–86		

B **Solve each equation by factoring.**

49. $12x^2 - 4x = 5$ $-\frac{1}{2}, \frac{5}{6}$ **50.** $5x^2 = 15x$ **0, 3** **51.** $16x^2 + 36 = -48x$ $-\frac{3}{2}$

52. $75x^2 - 60x = -12$ $\frac{2}{5}$ **53.** $4x^2 - 144 = 0$ **6, −6** **54.** $-7x + 6 = 20x^2$ $\frac{2}{5}, -\frac{3}{4}$

55. MOVIE THEATER A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where x is the number of movie screens, and $P(x)$ is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money. **16 to 32 screens**

Write a quadratic equation in standard form with the given root(s).

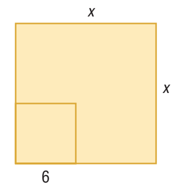
56. $-\frac{4}{7}, \frac{3}{8}$ $56x^2 + 11x - 12 = 0$ **57.** 3.4, 0.6 $25x^2 - 100x + 51 = 0$ **58.** $\frac{2}{11}, \frac{5}{9}$ $99x^2 - 73x + 10 = 0$

Solve each equation by factoring.

59. $10x^2 + 25x = 15$ $-3, \frac{1}{2}$ **60.** $27x^2 + 5 = 48x$ $\frac{5}{3}, \frac{1}{9}$ **61.** $x^2 + 0.25x = 1.25$ $1, -\frac{5}{4}$

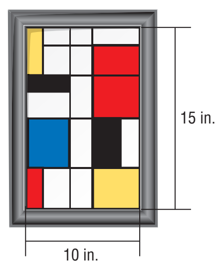
62. $48x^2 - 15 = -22x$ $\frac{3}{8}, -\frac{5}{6}$ **63.** $3x^2 + 2x = 3.75$ $-\frac{3}{2}, \frac{5}{6}$ **64.** $-32x^2 + 56x = 12$ $\frac{1}{4}, \frac{3}{2}$

65. DESIGN A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression. $x^2 - 6^2$; $(x + 6)(x - 6)$



66. FINANCIAL LITERACY After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by $P(x) = -16x^2 + 368x - 2035$, where x is the price of each Web site and $P(x)$ is the company's profit. Determine the price range of the Web sites that will be profitable for the company. **\$9.25 to \$13.75**

67. PAINTINGS Enola wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included? **20 in. by 15 in.**



68. MULTIPLE REPRESENTATIONS In this problem, you will consider $a(x - p)(x - q) = 0$.

a. Graphical Graph the related function for $a = 1$, $p = 2$, and $q = -3$. **See margin.**

b. Analytical What are the solutions of the equation? **2 and −3**

c. Graphical Graph the related functions for $a = 4, -3$, and $\frac{1}{2}$ on the same graph. **See margin.**

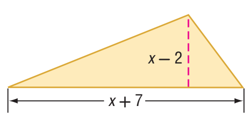
d. Verbal What are the similarities and differences between the graphs?

e. Verbal What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

68d. Sample answer: They all have the same roots, p and q. Therefore, they all have the same solutions. The graphs are shaped differently due to the value of a. The graph with $a = -3$ is flipped due to the negative.

68e. When quadratic equations have the same factors, they will have the same solutions, regardless of the value of a, which only affects the shape of the graphs.

69. GEOMETRY The area of the triangle is 26 square centimeters. Find the length of the base. **13 cm**



Multiple Representations

In Exercise 68, students use algebra and a graph in the coordinate plane to relate the factors of a quadratic equation to its solutions.

WatchOut!

Error Analysis For Exercise 79 remind students that when a polynomial is subtracted, every term in the polynomial is subtracted. Always check answers by substitution in the original equation.

4 Assess

Name the Math Have students explain the Zero Product Property. Have them discuss why it is true and how it is used in finding the roots of a quadratic equation.

Additional Answers

83. Sample answer:

$(x - p)(x - q) = 0$ (Original equation)

$x^2 - px - qx + pq = 0$ (Multiply)

$x^2 - (p + q)x + pq = 0$ (Simplify)

$x = -\frac{b}{2a}$ (Formula for axis of symmetry)

$x = -\frac{-(p+q)}{2(1)}$

$a = 1$ and $b = -(p + q)$

$x = \frac{p+q}{2}$ (Simplify)

x is midway between p and q. (Definition of midpoint)

86. Sample answer: In standard form, we have $ax^2 + bx + c$. Multiply a and c. Then find a pair of integers, g and h, that multiply to equal ac and add to equal b. Then write out the quadratic, turning the middle term, bx, into $gx + hx$. We now have $ax^2 + gx + hx + c$. Now factor the GCF from the first two terms and then factor the GCF from the second two terms. So we now have $\text{GCF}(x - q) + \text{GCF}_2(x - q)$. Simplifying, we get $(\text{GCF} + \text{GCF}_2)(x - q)$ or $(x - p)(x - q)$.

70. SOCCER When a ball is kicked in the air, its height in meters above the ground can be modeled by $h(t) = -4.9t^2 + 14.7t$ and the distance it travels can be modeled by $d(t) = 16t$, where t is the time in seconds.

a. How long is the ball in the air? **3 seconds**

b. How far does it travel before it hits the ground? (*Hint*: Ignore air resistance.) **48 m**

c. What is the maximum height of the ball? **11.025 m**

73. $6b^2(a^2 - 2a - 3b)$
75. $2(2x - 3y)(8a + 3b)$

C Factor each polynomial.

71. $18a - 24ay + 48b - 64by$ **$2(3 - 4y)(3a + 8b)$**

72. $3x^2 + 2xy + 10y + 15x$ **$(3x + 2y)(x + 5)$**

73. $6a^2b^2 - 12ab^2 - 18b^3$

74. $12a^2 - 18ab + 30ab^3$ **$6a(2a - 3b + 5b^3)$**

75. $32ax + 12bx - 48ay - 18by$

76. $30ac + 80bd + 40ad + 60bc$ **$10(a + 2b)(3c + 4d)$**

77. $5ax^2 - 2by^2 - 5ay^2 + 2bx^2$ **$(x + y)(x - y)(5a + 2b)$**

78. $12c^2x + 4d^2y - 3d^2x - 16c^2y$ **$(2c + d)(2c - d)(3x - 4y)$**

H.O.T. Problems Use Higher-Order Thinking Skills

79. Sample answer: Morgan; Gwen did not have like terms in the parentheses in the third line.

85. Sample answer: Always; in order to factor using perfect square trinomials, the coefficient of the linear term, bx, must be a multiple of 2, or even.

79. ERROR ANALYSIS Gwen and Morgan are solving $-12x^2 + 5x + 2 = 0$. Is either of them correct? Explain your reasoning.

Gwen

$-12x^2 + 5x + 2 = 0$
$-12x^2 + 8x - 3x + 2 = 0$
$4x(-3x + 2) - (3x + 2) = 0$
$(4x - 1)(3x + 2) = 0$
$x = \frac{1}{4}$ or $-\frac{2}{3}$

Morgan

$-12x^2 + 5x + 2 = 0$
$-12x^2 + 8x - 3x + 2 = 0$
$4x(-3x + 2) + (-3x + 2) = 0$
$(4x + 1)(-3x + 2) = 0$
$x = -\frac{1}{4}$ or $\frac{2}{3}$

82. Sample answer: 3 and 6 → $x^2 - 9x + 18 = 0$. −3 and −6 → $x^2 + 9x + 18 = 0$. The linear term changes sign.

84. Sample answer: What do you know about $a \cdot c$ to use guess-and-check to factor trinomials of the form $ax^2 + bx + c$?

80. CHALLENGE Solve $3x^6 - 39x^4 + 108x^2 = 0$ by factoring. **0, 3, −3, 2, or −2**

81. CHALLENGE The rule for factoring a difference of cubes is shown below. Use this rule to factor $40x^5 - 135x^2y^3$. **$5x^2(2x - 3y)(4x^2 + 6xy + 9y^2)$**

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

82. OPEN ENDED Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

83. CHALLENGE For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the x-intercepts p and q. **See margin.**

84. WRITE A QUESTION A classmate is using the guess-and-check strategy to factor trinomials of the form $x^2 + bx + c$. Write a question to help him think of a way to use that strategy for $ax^2 + bx + c$.

85. REASONING Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

In a quadratic equation in standard form where a, b, and c are integers, if b is odd, then the quadratic cannot be a perfect square trinomial.

86. WRITING IN MATH Explain how to factor a trinomial in standard form with $a > 1$. **See margin.**

Differentiated Instruction OL BL

Extension Pose the following question to students:

If the roots of a quadratic equation are 6 and −3, what is the equation of the axis of symmetry? $x = \frac{3}{2}$

LAB 4

Graphing Technology Lab
Dividing Polynomials

Long division and synthetic division are two alternatives for dividing polynomials with linear divisors. You can use a graphing calculator with a computer algebra system (CAS) to divide polynomials with any divisor.

Activity 1 Divide Polynomials Without Remainders

Use CAS to find $(x^4 + 3x^3 - x^2 - 5x + 2) \div (x^2 + 2x - 1)$.

Step 1 Add a new **Calculator** page on the TI-Nspire.

Step 2 From the menu, select **Algebra**, then **Polynomial Tools** and **Quotient of Polynomial**.

Step 3 Type the dividend, a comma, and the divisor.

The CAS indicates that $(x^4 + 3x^3 - x^2 - 5x + 2) \div (x^2 + 2x - 1)$ is $x^2 + x - 2$.

Step 4 To verify that there is no remainder, select **Remainder of a Polynomial** from the **Algebra, Polynomial Tools** menu then type the dividend, a comma, and the divisor.

In Activity 1, there was no remainder. But in many cases, there will be a remainder.

Activity 2 Divide Polynomials With Remainders

Use CAS to find $(4x^5 - 12x^4 - 7x^3 + 32x^2 + 3x + 20) \div (x^2 - 2x + 4)$.

Step 1 Add a new **Calculator** page on the TI-Nspire.

Step 2 From the menu, select **Algebra, Polynomial Tools,** and **Quotient of Polynomial**.

Step 3 Type the dividend, a comma, and the divisor.

The CAS indicates that $(4x^5 - 12x^4 - 7x^3 + 32x^2 + 3x + 20) \div (x^2 - 2x + 4)$ is $4x^3 - 4x^2 - 31x + 14$.

We need to determine whether there is a remainder.

Step 4 Use the **Remainder of a Polynomial** option from the **Algebra, Polynomial Tools** menu to determine the remainder. Then type the dividend, a comma, and the divisor.

The remainder is $99x + 76$.
Therefore, $(4x^5 - 12x^4 - 7x^3 + 32x^2 + 3x + 20) \div (x^2 - 2x + 4)$ is $4x^3 - 4x^2 - 31x - 14 + \frac{99x + 76}{x^2 - 2x + 4}$.

(continued on the next page)

1 Focus

Objective Use a graphing calculator with CAS to divide polynomials.

Materials

- TI-Nspire CAS technology

Teaching Tip

- Sometimes long equations cannot be displayed on one screen. Use the right arrow to scroll the display to the right.
- Remind students that they cannot edit a line once they press **enter**. However, they can use **ctrl C** and **ctrl V** to copy and paste an equation, and then make edits.
- In Step 2 of Activity 3, students can use long division or the CAS to divide.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activities 1 and 2 as a class. Then ask students to work with their partners to complete Activity 3.

Practice Have students complete Exercises 1–5.

3 Assess

Formative Assessment

Use Exercises 6 and 7 to assess each student's ability to divide polynomials.

From Concrete to Abstract

Ask students to summarize what they learned about dividing polynomials.

Graphing Technology Lab
Dividing Polynomials *Continued*

You can also use a graphing calculator to determine roots of a polynomial so you can divide with synthetic division.

Activity 3 Divide with Synthetic Division

Use synthetic division to find $(x^6 - 28x^4 + 14x^3 + 147x^2 - 14x - 120) \div (x^3 + 3x^2 - 18x - 40)$.

Step 1 Graph the dividend as $f1(x)$ and the divisor as $f2(x)$ on the same calculator page. Use the **intersection points** tool from **Points and Lines** menu to find where the graphs have the same x-intercepts.

[−10, 10] scl: 1 by [−100, 100] scl: 10

Step 2 Use the roots from Step 1 as the divisors for synthetic division.

−5	1	0	−28	14	147	−14	−120
		−5	25	15	−145	−10	120
	1	−5	−3	29	2	−24	0
−2	1	−5	−3	29	2	−24	
		−2	14	−22	−14	24	
	1	−7	11	7	−12	0	
4	1	−7	11	7	−12		
		4	−12	−4	12		
	1	−3	−1	3	0		

Step 3 Use the **Expand** function to verify that $x^3 - 3x^2 - x + 3$ is the quotient when −5, −2, and 4 are roots.

Thus, $(x^6 - 28x^4 + 14x^3 + 147x^2 - 14x - 120) \div (x^3 + 3x^2 - 18x - 40)$ is $x^3 - 3x^2 - x + 3$.

Exercises

Find each quotient.

1. $(2x^4 + x^3 - 8x^2 + 17x - 12) \div (x^2 + 2x - 3)$ **$2x^2 - 3x + 4$**
2. $(x^4 + 7x^3 + 8x^2 + x - 12) \div (x^2 + 3x - 4)$ **$x^2 + 4x + \frac{7x - 12}{x^2 + 3x - 4}$**
3. $(9x^5 - 9x^3 - 5x^2 + 5) \div (9x^3 - 5)$ **$x^2 - 1$**
4. $(x^5 - 8x^4 + 10x^3 + 14x^2 + 61x - 30) \div (x^2 - 5x + 3)$ **$x^3 - 3x^2 - 8x - 17 + \frac{21}{x^2 - 5x + 3}$**
5. $(2x^6 + 2x^5 - 4x^4 - 18x^3 - 16x^2 + 8x + 16) \div (2x^3 + 2x^2 - 4x - 2)$ **$x^3 - 8 - \frac{24x}{2x^3 + 2x^2 - 4x - 2}$**
6. $(6x^6 - 2x^5 - 14x^4 + 10x^3 - 4x^2 - 28x - 5) \div (3x^3 - x^2 - 7x - 1)$ **$2x^3 + 4 - \frac{1}{3x^3 - x^2 - 7x - 1}$**
7. Use synthetic division to find $(x^6 - 7x^5 - 21x^4 + 175x^3 + 56x^2 - 924x + 720) \div (x^3 - 5x^2 - 12x + 36)$. **$x^3 - 2x^2 - 19x + 20$**

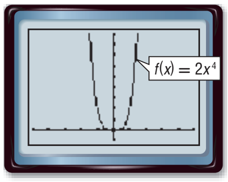
Graphing Technology Lab
Power Functions

A **power function** is any function of the form $f(x) = ax^n$, where a and n are nonzero constant real numbers. A power function in which n is a positive integer is called a **monomial function**.

Activity 1 $y = ax^4$

Graph $f(x) = 2x^4$. Analyze the graph.

Step 1 Graph the function.



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Step 2 Analyze the graph.

The graph resembles the graph of $g(x) = x^2$, but it flattens out more at its vertex.

Analyze the Results

1. What assumptions can you make about the graph of $f(x) = 2x^4$ as x becomes more positive or more negative? Use the TABLE feature to evaluate the function for values of x to confirm this assumption, if necessary. **It appears that the value of $f(x)$ will continue to increase.**

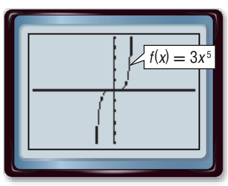
2. State the domain and range of $f(x) = 2x^4$. Compare these values with the domain and range of $g(x) = x^2$. **D = {all real numbers}, R = {$f(x)$| $f(x) > 0$}; they are the same.**

3. What other characteristics does the graph of $f(x) = 2x^4$ share with the graph of $g(x) = x^2$? **The graphs have the same vertex, (0, 0); axis of symmetry, the line $x = 0$ (the y-axis); and y-intercept, 0.**

Activity 2 $y = ax^5$

Graph $f(x) = 3x^5$. Analyze the graph.

Step 1 Graph the function.



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Step 2 Analyze the graph.

The graph resembles the graph of $f(x) = x^3$, but it flattens out more as the graph approaches the origin.

(continued on the next page)

1 Focus

Objective Use a graphing calculator to explore power functions.

Materials

- TI-83/84 Plus or other graphing calculator

Teaching Tip

- Students may need to adjust the windows of their calculators to explore the graphs. Remind them that they can zoom in and out as well as use **zoomfit.**
- Remind students that they can use **zoomstandard** to restore the standard window.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activities 1 and 2 as a class. Then ask students to work with their partners to complete Activity 3.

Practice Have students complete Exercises 1–9.

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3 Assess

Formative Assessment

Use Exercises 10–15 to assess each student's understanding of power functions.

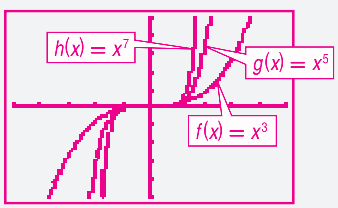
From Concrete to Abstract

Ask students to summarize what they learned about how the graphs of power functions behave as x increases or decreases.

Additional Answers

7. The graphs are reflections in the origin. The graph of $f(x)$ increases as x increases while the graph of $h(x)$ decreases as x increases. When a is positive, the graph will increase from left to right, appearing to approach negative infinity as x approaches negative infinity, and appearing to approach positive infinity as x approaches positive infinity. When a is negative, the opposite is true.

9. The graphs will appear to decrease to the origin as x goes from negative infinity to zero, then cross the origin, and increase as x goes from 0 to infinity. The graphs will get narrower as the exponent increases to greater even numbers.

10. 

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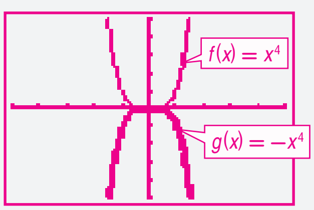
13. 

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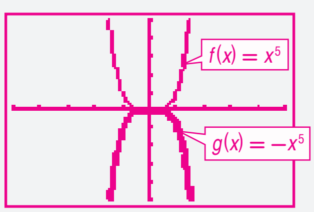
14. 

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Graphing Technology Lab
Power Functions *Continued*

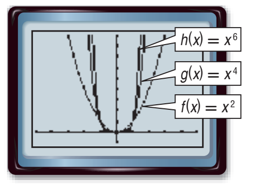
Analyze the Results

4. What assumptions can you make about the graph of $f(x) = 3x^5$ as x becomes more positive or more negative? Use the **TABLE** feature to evaluate the function for values of x to confirm this assumption, if necessary. **It appears that the value of $f(x)$ will continue to increase as x becomes more positive and decrease as x becomes more negative.**
5. State the domain and range of $f(x) = 3x^5$. Compare these values with the domain and range of $g(x) = x^3$. **D = {all real numbers}, R = {all real numbers}; they are the same.**
6. What other characteristics does the graph of $f(x) = 3x^5$ share with the graph of $g(x) = x^3$? **The graphs have the same y-intercept, 0, and the same shape.**
7. Compare the characteristics of the graph $h(x) = -3x^5$ with the graph of $f(x) = 3x^5$. What conclusions can you make about $f(x) = ax^5$ when a is positive and when a is negative? **See margin.**

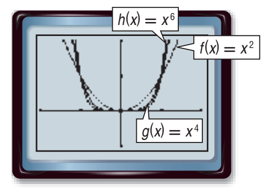
Activity 3 $y = x^n$, where n is even

Graph $f(x) = x^2$, $g(x) = x^4$, and $h(x) = x^6$ on the same screen. Analyze the graphs.

Step 1 Graph the function.



[−5, 5] scl: 1 by [−1, 9] scl: 1



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Step 2 The graphs are all U-shaped, but the widths are different. The graphs also differ around the origin. If you zoom in around the origin, you can see this more clearly. You can do this by using the **ZOOM** feature or by adjusting the window manually.

Analyze the Results

8. What assumptions can you make about the graphs of $a(x) = x^8$, $b(x) = x^{10}$, and so on? **8. The graphs will have the same appearance as $f(x) = x^2$, but they will get narrower and narrower as the exponent increases to greater even numbers.**
9. Identify the common characteristics of the graphs of power functions in which the power is an even number. **See margin.**
10. Graph $f(x) = x^3$, $g(x) = x^5$, and $h(x) = x^7$ on the same graph. **See margin.**
11. What assumptions can you make about the graphs of $a(x) = x^9$, $b(x) = x^{11}$, and so on? **The graphs will have the same appearance as $f(x) = x^3$, but they will get narrower and narrower as the exponent increases to greater odd numbers.**
12. Identify the common characteristics of the graphs of power functions in which the power is an odd number. **12. The graphs will appear to increase to the origin as x goes from negative infinity to zero, then cross the origin, then increase as x goes from 0 to infinity. The graphs will get narrower and narrower as the exponent increases to greater odd numbers.**
13. Graph $f(x) = x^4$ and $g(x) = -x^4$ on the same graph. **See margin.**
14. Graph $f(x) = x^5$ and $g(x) = -x^5$ on the same graph. **See margin.**
15. What assumptions can you make about the effects of a negative value of a in $f(x) = ax^n$ when n is even? when n is odd? **When x is odd, changing a from positive to negative will reflect the graph in the origin. When x is even, changing a from positive to negative will reflect the graph in the x-axis.**

LAB 6

Graphing Technology Lab
Polynomial Identities

An **identity** is an equation that is satisfied by any numbers that replace the variables. Thus, a **polynomial identity** is a polynomial equation that is true for any values that are substituted for the variables.

Common Core State Standards
A.REI.11

You can use a table or a spreadsheet on your graphing calculator to determine whether a polynomial equation may be an identity.

Activity 1 Use a Table

Determine whether $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ may be an identity.

Step 1 Add a new **List/Spreadsheet** page on the TI-Nspire. Label column A x and column B y. Type any values in columns A and B.

Step 2 Move the cursor to the formula row in column C and type $= x^3 - y^3$. In column D, type $= (x - y)(x^2 + xy + y^2)$.

No matter what values are entered for x and y in column A and B, the values columns C and D are the same. Thus, the equation may be an identity.

If you want to prove that an equation is an identity, you need to show that it is true for all values of the variables.

KeyConcept Verifying Identities by Transforming One Side

Step 1 Simplify one side of an equation until the two sides of the equation are the same. It is often easier to work with the more complicated side of the equation.

Step 2 Transform that expression into the form of the simpler side.

Activity 2 Transform One Side

Prove that $(x + y)^2 = x^2 + 2xy + y^2$ is an identity.

$(x + y)^2 \stackrel{?}{=} x^2 + 2xy + y^2$	Original equation
$(x + y)(x + y) \stackrel{?}{=} x^2 + 2xy + y^2$	Write $(x + y)^2$ as two factors
$x^2 + xy + xy + y^2 \stackrel{?}{=} x^2 + 2xy + y^2$	FOIL Method
$x^2 + 2xy + y^2 = x^2 + 2xy + y^2$ ✓	Simplify.

Thus, the identity $(x + y)^2 = x^2 + 2xy + y^2$ is verified.

1 Focus

Objective Prove polynomial identities.

Materials

- TI-Nspire CAS technology

Teaching Tip

- A spreadsheet program can also be used to complete Activity 1.
- When formulas are entered into the TI-Nspire technology spreadsheet, it will prompt you to indicate whether x and y are column or variable references. Indicate that they are variable references.
- Sometimes the entire equation cannot be displayed on one CAS screen. Use the right arrow to scroll the display to the right.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activities 1 and 2 as a class. Then ask students to work with their partners to complete Activities 3 and 4 and Exercise 1.

Ask:

- After Activity 2, ask students if it matters which side of the equation they simplify. no

Practice Have students complete Exercises 2–5.

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3 Assess

Formative Assessment

Use Exercises 6–8 to assess each student's ability to prove identities.

From Concrete to Abstract

Have students prove the identity $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ without the use of technology.

You can also use a TI-Nspire with a computer algebra system (CAS) to prove an identity.

Activity 3 Use CAS

Prove that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ is an identity.

Step 1 Add a new **Calculator** page on the TI-Nspire CAS. Simplify the right side of the equation one step at a time.

Step 2 Enter the right side of the equation and then distribute.

Step 3 Multiply next. The CAS system will do the final simplification step.

The final step shown on the CAS screen is the results in $x^3 - y^3$. Thus, the identity has been proved.

You can also prove identities by transforming each side of the equation.

Activity 4 Use CAS to Transform Each Side

Prove that $(x + 2)^3(x - 1)^3 = (x^2 + x - 2)(x^4 + 2x^3 - 3x^2 - 4x + 4)$ is an identity.

Add a new **Calculator** page on the TI-Nspire. Simplify the left and the right sides of the equation simultaneously.

The CAS will indicate if the changes are true, otherwise it will simplify for you.

The CAS system will do the final simplification step. The identity $(x + 2)^3 (x - 1)^3 = (x^2 + x - 2)(x^4 + 2x^3 - 3x^2 - 4x + 4)$ has been proved.

Analyze

1. Use CAS to prove $a^2 - b^2 = (a + b)(a - b)$.

1–8. See Answer Appendix.

Exercises

Use CAS to prove each identity.

2. $x^3 + y^3 = (x + y)(x^2 - xy + y^3)$
3. $p^4 - q^4 = (p - q)(p + q)(p^2 + q^2)$
4. $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$
5. $g^6 + h^6 = (g^2 + h^2)(g^4 - g^2h^2 + h^4)$
6. $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$
7. $u^6 - w^6 = (u + w)(u - w)(u^2 + vw + w^2)(u^2 - vw + w^2)$
8. $(x + 1)^2(x - 4)^3 = (x^2 - 3x - 4)(x^3 - 7x^2 + 8x + 16)$

16 | Lab 6 | Graphing Technology Lab: Polynomial Identities

LAB 7

Graphing Technology Lab

Analyzing Polynomial Functions

You can use graphing technology to help you identify zeros, maximum and minimum points, multiplicity of zeros, *y*-intercepts, and symmetry of polynomial functions.

Common Core State Standards
A.APR.4

Activity Identify Polynomial Characteristics

Graph each function. Identify the zeros, maximum and minimum points, multiplicity of zeros, *y*-intercepts, and symmetry.

a. $g(x) = 3x^4 - 15x^3 + 87x^2 - 375x + 300$

Step 1 Graph the equation.

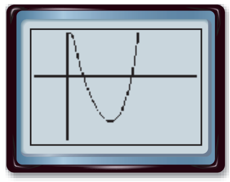
Step 2 Use 2nd [CALC] **zero** to find the zeros at $x = 1$ and $x = 4$.

Step 3 Use 2nd [CALC] **minimum** to find the relative minimum at (2.68, −214.11). There is no relative maximum point.

Step 4 $g(x)$ has degree 4 and can have at most 4 zeros. Two were found through graphing. The other two roots are either multiple roots or imaginary roots.

Step 5 Use 2nd [CALC] **value** 0 to find the *y*-intercept, 300.

Step 6 The line of symmetry passes through the vertex. Its equation is $x = 2.68$.



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b. $f(x) = 2x^5 - 5x^4 - 3x^3 + 8x^2 + 4x$

Step 1 Graph the equation.

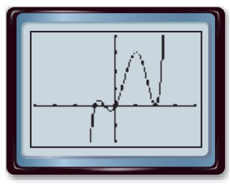
Step 2 Locate the zeros at $x = -1$, $x = 0$, $x = \frac{1}{2}$ and $x = 2$.

Step 3 Find the relative maxima at (−0.81, 0.75) and (1.04, 6.02) and the relative minima at (−0.24, −0.48) and (2, 0).

Step 4 $f(x)$ has degree 5 and can have at most 5 zeros. Four roots were found through graphing. The other root is either a multiple root or an imaginary root. In this case, there is a double root at $x = 2$.

Step 5 The *y*-intercept is 0 because the graph goes through the origin.

Step 6 There is no symmetry.



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Exercises

Graph each function. Identify the zeros, maximum and minimum points, multiplicity of zeros, *y*-intercepts, and symmetry. **1–8. See Answer Appendix.**

1. $f(x) = x^3 - 5x^2 + 6x$

2. $g(x) = x^4 - 3x^2 - 4$

3. $k(x) = -x^4 - x^3 + 2x^2$

4. $f(x) = -2x^3 - 4x^2 + 16x$

5. $g(x) = 3x^5 - 18x^4 + 27x^3$

6. $k(x) = x^4 - 8x^2 + 15$

7. $f(x) = -x^3 + 2x^2 + 8x$

8. $g(x) = x^5 + 3x^4 - 10x^2$

1 Focus

Objective Use a graphing calculator to analyze polynomial functions.

Materials

- TI-83/84 Plus or other graphing calculator

Teaching Tip

- When finding zeros, students will have to perform Step 2 twice to find two zeros. In part **b,** they will need to perform Step 2 three times.
- Encourage students to set the window setting to match the samples in the Activity.
- Numbers have been rounded to two decimal places in this lab.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Ask students to work with their partners to complete the Activity.

Practice Have students complete Exercises 1–7.

Formative Assessment

Use Exercise 8 to assess each student's ability to analyze polynomials.

From Concrete to Abstract

Ask students to summarize what they learned about analyzing polynomial functions.

1 Focus

Vertical Alignment

Before Lesson 8 Identify sampling techniques.

Lesson 8 Classify study types. Design statistical studies.

After Lesson 8 Use the shapes of distributions to select appropriate statistics and compare data.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- What do you think the purpose of the study was? Sample answer: to determine the number of teen cell phone users that send texts and the amount of texts that these users send per day
- How do you think the data could have been collected for this study? Sample answer: Teen cell phone users could have been asked if they send texts and, if so, how many texts they send per day.
- According to the study, if 50 teen cell phone users were randomly selected, how many would be texters? 44

LESSON 8 Designing a Study

Then

- You identified various sampling techniques.

Now

1. Classify study types.
2. Design statistical studies.

Why?

- According to a recent study, 88% of teen cell phone users in the U.S. send text messages, and one in three teens sends more than 100 texts per day.

Radius Images/Jupiterimages

New Vocabulary
parameter
statistic
bias
random sample
survey
experiment
observational study

Common Core State Standards
S.IC.2, S.IC.3, S.IC.5

1 Classifying Studies In a statistical study, data are collected and used to answer questions about a population characteristic or **parameter**. Due to time and money constraints, it may be impractical or impossible to collect data from each member of a population. Therefore, in many studies, a sample of the population is taken, and a measure called a **statistic** is calculated using the data. The sample statistic, such as the sample mean or sample standard deviation, is then used to make inferences about the population parameter.

The steps in a typical statistical study are shown below.

Identify the objective of the study. → Choose a sample, and collect data. → Organize the data, and calculate sample statistics. → Make inferences, and draw conclusions about the population.

To obtain good information and draw accurate conclusions about a population, it is important to select an *unbiased* sample. A **bias** is an error that results in a misrepresentation of members of a population. A poorly chosen sample can cause biased results. To reduce the possibility of selecting a biased sample, a **random sample** can be taken, in which members of the population are selected entirely by chance.
You will review other sampling methods in Exercise 32.

The following study types can be used to collect sample information.

Key Concept Study Types

Definition	Example
In a **survey**, data are collected from responses given by members of a population regarding their characteristics, behaviors, or opinions.	To determine whether the student body likes the new cafeteria menu, the student council asks a random sample of students for their opinion.
In an **experiment**, the sample is divided into two groups: • an *experimental group* that undergoes a change, and • a *control group* that does not undergo the change. The effect on the experimental group is then compared to the control group.	A restaurant is considering creating meals with chicken instead of beef. They randomly give half of a group of participants meals with chicken and the other half meals with beef. Then they ask how they like the meals.
In an **observational study**, members of a sample are measured or observed without being affected by the study.	Researchers at an electronics company observe a group of teenagers using different laptops and note their reactions.

Example 1 Classify Study Types

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

a. MUSIC **A record label wants to test three designs for an album cover. They randomly select 50 teenagers from local high schools to view the covers while they watch and record their reactions.**


This is an observational study, because the company is going to observe the teens without them being affected by the study. The sample is the 50 teenagers selected, and the population is all potential purchasers of this album.

b. RECYCLING **The city council wants to start a recycling program. They send out a questionnaire to 200 random citizens asking what items they would recycle.**

This is a survey, because the data are collected from participants' responses in the questionnaire. The sample is the 200 people who received the questionnaire, and the population is all of the citizens of the city.

GuidedPractice

1A. experiment; sample: dogs given the heartworm medication; population: all dogs

1A. RESEARCH Scientists study the behavior of one group of dogs given a new heartworm treatment and another group of dogs given a false treatment or *placebo*.

1B. YEARBOOKS The yearbook committee conducts a study to determine whether students would prefer to have a print yearbook or both print and digital yearbooks.

StudyTip

Census
A *census* is a survey in which each member of a population is questioned. Therefore, when a census is conducted, there is no sample.

1B. survey; sample: students participating in the survey; population: the student body

To determine when to use a survey, experiment, or observational study, think about how the data will be obtained and whether or not the participants will be affected by the study.

Example 2 Choose a Study Type

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

a. MEDICINE **A pharmaceutical company wants to test whether a new medicine is effective.**

The treatment will need to be tested on a sample group, which means that the members of the sample will be affected by the study. Therefore, this situation calls for an experiment.

b. ELECTIONS **A news organization wants to randomly call citizens to gauge opinions on a presidential election.**

This situation calls for a survey because members of the sample population are asked for their opinion.

GuidedPractice

2A. RESEARCH A research company wants to study smokers and nonsmokers to determine whether 10 years of smoking affects lung capacity.

2B. PETS A national pet chain wants to know whether customers would pay a small annual fee to participate in a rewards program. They randomly select 200 customers and send them questionnaires.

2A. Observational study; sample answer: The lung capacities of the participants are observed and compared without them being affected by the study.
2B. Survey; sample answer: The data is obtained from opinions given by members of the sample population.

Classifying Studies

Examples 1 shows how to classify study types. **Example 2** shows how to choose a study type.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Example

1 Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

a. MOVIES A retro movie theater wants to determine what genre of movies to play during the next year. They plan to poll 50 random area residents and ask them what their favorite movies are. survey; sample: the 50 residents that were polled; population: all potential movie-goers

b. DRIVING A driving school wants to determine the main issue drivers face while taking the driving test. They watch and record 30 random people taking the test. observational study; sample: the people observed while taking the test; population: potential driving school students

Additional Examples also in Interactive Classroom PowerPoint® Presentations

IWB **Interactive White Board READY**

Focus on Mathematical Content

Survey and Census When a census is conducted, data are collected from every member of a population. Therefore, the results are known to be correct. Since a survey investigates only part of a population, the results always involve some uncertainty.

Designing Studies

Example 3 shows how to identify biased survey questions. **Example 4** shows how to design a survey. **Example 5** shows how to identify flaws in experiments. **Example 6** shows how to design an experiment.

Additional Examples

2 Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

a. VIDEO GAMES A gaming company plans to test whether a new controller is preferable to the old one. A group of teens will be observed while using the controllers to see which one they use the most. Observational study; sample answer: The teens will be observed without being affected by the study.

b. RESTAURANTS A restaurant wants to conduct an online study in which they will ask customers whether they were satisfied with their dining experience. Survey; sample answer: The members of the sample will be asked for their opinion.

3 Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

a. What is your favorite type of music? This question is unbiased because it is clearly stated and does not encourage a certain response.

b. Do you think that poisons, such as pesticides, should be sprayed on crops? This question is biased because the term "poison" could cause a strong reaction from the respondent.

2 Designing Studies

The questions chosen for a survey or procedures used in an experiment can also introduce bias, and thus, affect the results of the study.

A survey question that is poorly written may result in a response that does not accurately reflect the opinion of the participant. Therefore, it is important to write questions that are clear and precise. Avoid survey questions that:

- are confusing or wordy
- cause a strong reaction
- encourage a certain response
- address more than one issue

Questions can also introduce bias if there is not enough information given for the participant to give an accurate response.

Example 3 Identify Bias in Survey Questions

PT

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

a. Don't you agree that the cafeteria should serve healthier food?

This question is biased because it encourages a certain response. The phrase "don't you agree" encourages you to agree that the cafeteria should serve healthier food.

b. How often do you exercise?

This question is unbiased because it is clearly stated and does not encourage a certain response.

GuidedPractice

3A. How many glasses of water do you drink a day? **unbiased**

3B. Do you prefer watching exciting action movies or boring documentaries?

3B. Biased; sample answer: The adjectives "exciting" and "dull" suggest that the respondent should agree that action movies are preferable to documentaries.

4. objective: to determine teachers' interest in teaching an online course; population: teachers at the school with at least five years of experience; sample survey questions: How long have you been teaching? If offered, would you be interested in teaching an online course?

When designing a survey, clearly state the objective, identify the population, and carefully choose unbiased survey questions.

Real-World Example 4 Design a Survey

PT

TECHNOLOGY Jim is writing an article for his school newspaper about online courses. He wants to conduct a survey to determine how many students at his school would be interested in taking an online course from home. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Step 1 **State the objective of the survey.**

The objective of the survey is to determine students' interest in taking an online course from home.

Step 2 **Identify the population.**

The population is the student body.

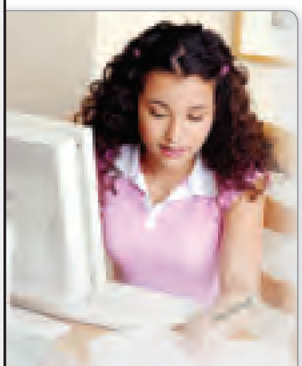
Step 3 **Write unbiased survey questions.**

Possible survey questions:

- "Do you have Internet access at home?"
- "If offered, would you take an online course?"

GuidedPractice

4. **TECHNOLOGY** In a follow-up article, Jim decides to conduct a survey to determine how many teachers from his school with at least five years of experience would be interested in teaching an online course. State the objective of the survey, suggest a population, and write two unbiased survey questions.



Real-WorldLink

Online Courses In 2009, about 1.2 million students took at least one online course.

Source: International Association for K-12 Online Learning

Ephraim Ben-Shimon/Corbis

DifferentiatedInstruction OL ELL

Verbal/Linguistic Learners Divide students into small groups. Have each group design a survey question and practice asking it in such a way that there is bias built into the tone of voice and facial expression of the questioner. Then have them ask other groups the question and record their answers. As a class, discuss whether the answers corresponded to the bias that the question was designed to elicit.

To avoid introducing bias in experiments, the experimental and control groups should be randomly selected and the experiment should be designed so that everything about the two groups is alike (except for the treatment or procedure).

StudyTip

Bias in Experiments An experiment is biased when the participants know which group they are in.

Example 5 Identify Flaws in Experiments

Identify any flaws in the design of the experiment, and describe how they could be corrected.

Experiment: An electronics company wants to test whether using a new graphing calculator increases students' test scores. A random sample is taken. Calculus students in the experimental group are given the new calculator to use, and Algebra 2 students in the control group are asked to use their own calculator.

Results: When given the same test, the experimental group scored higher than the control group. The company concludes that the use of this calculator increases test scores.

Calculus students are more likely to score higher when given the same test as Algebra 2 students. Therefore, the flaw is that the experimental group consists of Calculus students and the control group consists of Algebra 2 students. This flaw could be corrected by selecting a random sample of all Calculus or all Algebra 2 students.

GuidedPractice

5. Experiment: A research firm tests the effectiveness of a de-icer on car locks. They use a random sample of drivers in California and Minnesota for the control and experimental groups.

Results: They concluded that the de-icer is effective.

5. Sample answer: The flaw is that the control group is drivers in California while the experimental group is drivers in Minnesota. This flaw can be corrected by selecting both groups from a cold area like Minnesota where the de-icer is needed more.

When designing an experiment, clearly state the objective, identify the population, determine the experimental and control groups, and define the procedure.

Real-World Example 6 Design an Experiment

PLANTS **A research company wants to test the claim of the advertisement shown at the right. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.**

Step 1 **State the objective, and identify the population.**

The objective of the experiment is to determine whether tomato plants given the plant food grow taller in three weeks than tomato plants not given the food. The population is all tomato plants.

Step 2 **Determine the experimental and control groups.**

The experimental group is the tomato plants given the food, and the control group is the tomato plants not given the food.

Step 3 **Describe a sample procedure.**

Measure the heights of the plants in each group, and give the experimental group the plant food. Then, wait three weeks, measure the heights of the plants again, and compare the heights for each group to see if the claim was valid.

GuidedPractice **See margin.**

6. **SPORTS** A company wants to determine whether wearing a new tennis shoe improves jogging time. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Additional Examples

4 **COLLEGE** A community college wants to determine whether college-bound students from local high schools would be interested in taking a class at the college. State the objective of the survey, suggest a population, and write two unbiased survey questions. objective: to determine whether students that are planning on going to college would be interested in taking a class at the community college; population: all local high school students; sample survey questions: Are you planning on attending college after high school? Would you be interested in taking a course at the community college during high school?

5 Identify any flaws in the design of the experiment, and describe how they could be corrected.

Experiment: A research company wants to conduct a study to determine whether a new fishing reel is more effective than the old reel. The experimental procedure consists of using the new reel to catch fish in one lake, and using the old reel to catch fish in another nearby lake that is randomly chosen.

Results: The company concludes that the new reel is twice as effective as the old reel. Sample answer: The flaw is that the fishing reels were used in two different lakes. The amount or type of fish in the two lakes may not be similar enough to perform a controlled experiment.

Additional Answer (Guided Practice)

6. objective: To determine whether wearing the tennis shoe improves jogging time; experimental group: participants that wear the new shoe; control group: participants that wear another type of shoe; sample procedure: The company could select participants that jog on a regular basis, are all healthy, and are close to the same age. The company could then randomly divide the sample into two groups: an experimental group of participants that wear the new tennis shoe and a control group of participants that wear another type of shoe. The company could record the jogging times of each group over a certain period of time and then determine if there were larger improvements in the experimental group compared to the control group.

Additional Example

6 **PETS** A research company wants to test a new food for overweight cats that promotes weight loss. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure. objective: to determine whether overweight cats given a new weight loss food will lose weight; population: all overweight cats; experimental group: a group of overweight cats given the food; control group: a group of overweight cats given their regular food; sample procedure: The two groups of cats could be weighed at the start of the experiment and then the experimental group could be given the new food and the control group could be given their regular food. Then, after a certain period of time, the two groups could be weighed again, and the weights of the cats could be compared to determine any effect.

3 Practice

Formative Assessment

Use Exercises 1–9 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Additional Answer

2. observational study; sample: the participants in the study; population: potential customers

Check Your Understanding

Example 1 **Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.**

1. **SCHOOL** A group of high school students is randomly selected and asked to complete the form shown. survey; sample: the students in the study; population: the student body

Do you agree with the new lunch rules?
- ☐ agree
- ☐ disagree
- ☐ don't care

2. **DESIGN** An advertising company wants to test a new logo design. They randomly select 20 participants and watch them discuss the logo. See margin.

Example 2 **Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.** 3–4. See Answer Appendix.

3. **LITERACY** A literacy group wants to determine whether high school students that participated in a recent national reading program had higher standardized test scores than high school students that did not participate in the program.

4. **RETAIL** The research department of a retail company plans to conduct a study to determine whether a dye used on a new T-shirt will begin fading before 50 washes.

Example 3 **Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.**

5. Which student council candidate's platform do you support? unbiased

6. How long have you lived at your current address? unbiased

Example 4

7. **HYBRIDS** A car manufacturer wants to determine what the demand in the U.S. is for hybrid vehicles. State the objective of the survey, suggest a population, and write two unbiased survey questions. 7–9. See Answer Appendix.

Example 5

8. Identify any flaws in the experiment design, and describe how they could be corrected.

   Experiment: A research company wants to determine whether a new vitamin boosts energy levels and decides to test the vitamin at a college campus. A random sample is taken. The experimental group consists of students who are given the vitamin, and the control group consists of instructors who are given a placebo.

   Results: When given a physical test, the experimental group outperformed the control group. The company concludes that the vitamin is effective.

Example 6

9. **SPORTS** A research company wants to conduct an experiment to test the claim of the protein shake shown. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Practice and Problem Solving

Example 1 **Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.** 10–13. See Answer Appendix.

10. **FOOD** A grocery store conducts an online study in which customers are randomly selected and asked to provide feedback on their shopping experience.

11. **GRADES** A research group randomly selects 80 college students, half of whom took a physics course in high school, and compares their grades in a college physics course.

12. **HEALTH** A research group randomly chooses 100 people to participate in a study to determine whether eating blueberries reduces the risk of heart disease for adults.

13. **TELEVISION** A television network mails a questionnaire to randomly selected people across the country to determine whether they prefer watching sitcoms or dramas.

Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	10–24, 28–30, 32	11–23 odd	10–24 even, 28–30, 32
OL Core	11–23 odd, 25–30, 32	10–24	25–30, 32
BL Advanced	25–32		

Example 2 **Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.** **14–17. See margin.**

14. **FASHION** A fashion magazine plans to poll 100 people in the U.S. to determine whether they would be more likely to buy a subscription if given a free issue.

15. **TRAVEL** A travel agency randomly calls 250 U.S. citizens and asks them what their favorite vacation destination is.

16. **FOOD** Chee wants to examine the eating habits of 100 random students at lunch to determine how many students eat in the cafeteria.

17. **ENGINEERING** An engineer is planning to test 50 metal samples to determine whether a new titanium alloy has a higher strength than a different alloy.

Example 3 **Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.** **18–21. See margin.**

18. Do you think that the school needs a new gym and football field?

19. Which is your favorite football team, the Dallas Cowboys or the Pittsburgh Steelers?

20. Do you play any extracurricular sports?

21. Don't you agree that students should carpool to school?

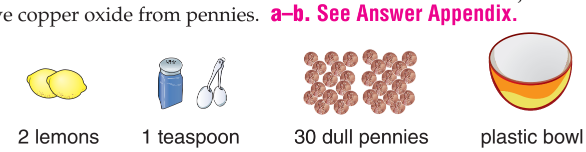
Example 4 22. **COLLEGE** A school district wants to conduct a survey to determine the number of juniors in the district who are planning to attend college after high school. State the objective of the survey, suggest a population, and write two unbiased survey questions. **See margin.**

Example 5 23. Identify any flaws in the experiment design, and describe how they could be corrected.

Experiment: A supermarket chain wants to determine whether shoppers are more likely to buy sunscreen if it is located near the checkout line. The experimental group consists of a group of stores in the midwest in which the sunscreen was moved next to the checkout line, and the control group consists of stores in Arizona in which the sunscreen was not moved. **See margin.**

Results: The Arizona stores sold more sunscreen than the midwest stores. The company concluded that moving the sunscreen closer to the checkout line did not increase sales.

Example 6 24. **CHEMISTRY** In chemistry class, Pedro learned that copper objects become dull over time because the copper reacts with air to form a layer of copper oxide. He plans to use the supplies shown below to determine whether a mixture of lemon juice and salt will remove copper oxide from pennies. **a–b. See Answer Appendix.**



a. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

b. What factors do you think should be considered when selecting pennies for the experiment? Explain your reasoning.

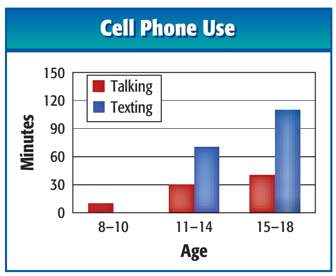
B 25. **REPORTS** The graph shown is from a report on the average number of minutes 8- to 18-year-olds in the U.S. spend on cell phones each day. **a, c–d. See margin.**

a. Describe the sample and suggest a population.

b. What type of sample statistic do you think was calculated for this report? **average time**

c. Describe the results of the study for each age group.

d. Who do you think would be interested in this type of report? Explain your reasoning.



Additional Answers

14. Survey; sample answer: The data will be obtained from opinions given by members of the sample population.

15. Survey; sample answer: The data will be obtained from opinions given by members of the sample population.

16. Observational study; sample answer: The eating habits of the participants will be observed and compared without them being affected by the study.

17. Experiment; sample answer: Metal samples will need to be tested, which means that the members of the sample will be affected by the study.

18. Biased; sample answer: The question is asking about two issues: whether the school needs a new gym *and* whether the school needs a new football field.

19. Biased; sample answer: The question only gives two options, and thus encourages a certain response.

20. unbiased

21. Biased; sample answer: The question encourages a certain response. The phrase "don't you agree" suggests that the people surveyed should agree.

22. objective: to determine the number of juniors in the district planning to attend college after high school; population: all juniors in the district; sample survey questions: What grade are you in? Do you plan on attending college after graduation?

23. Sample answer: The flaw is that the experimental group consists of stores in the Midwest, and the control group consists of stores in Arizona. On average, the temperature is higher in Arizona than in the Midwest, and people use more sunscreen. Therefore, the sunscreen sales in stores located in those regions would likely be different and should not be compared in an experiment.

25a. sample: the 8- to 18-year-olds surveyed; population: all 8- to 18-year-olds in the U.S.

25c. Sample answer: The 8- to 10-year-old group talked for about 10 minutes a day and did not text at all. The 11- to 14-year-old group talked for about 30 minutes a day and texted for about 70 minutes a day. The 15- to 18-year-old group talked for about 40 minutes a day and texted for about 110 minutes a day.

25d. Sample answer: A cell phone company might use a report like this to determine which age group to target in their ads.

4 Assess

Ticket Out the Door Have each student research an experiment of their choosing, and identify the objective of the experiment, the population, the experimental and control groups, and the procedure.

Additional Answers

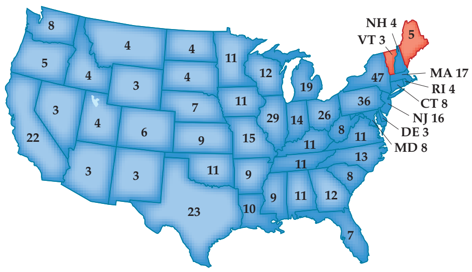
26a. survey; sample: the 2.4 million people polled, population: all U.S. citizens of voting age in 1936

26b. According to the predicted results, Landon should have won 57% of the popular vote. However, in the actual election, Roosevelt won 60.8% of the popular vote.

26c. Yes; sample answer: The people sampled could afford magazine subscriptions, automobiles, and telephones, suggesting that they were wealthier than the average American citizen. The sampling method did not represent citizens that could not afford these things, and therefore, was not representative of the entire population.

26. **HISTORY** In 1936, the *Literary Digest* reported the results of a statistical study used to predict whether Alf Landon or Franklin D. Roosevelt would win the presidential election that year. The sample consisted of 2.4 million Americans, including subscribers to the magazine, registered automobile owners, and telephone users. The results concluded that Landon would win 57% of the popular vote. The actual election results are shown.

ELECTORAL VOTE



POPULAR VOTE

Roosevelt	60.8%
Landon	36.5%

a–c. See margin.

a. Describe the type of study performed, the sample taken, and the population.

b. How do the predicted and actual results compare?

c. Do you think that the survey was biased? Explain your reasoning.

27. **MULTIPLE REPRESENTATIONS** The results of two experiments concluded that Product A is 70% effective and Product B is 80% effective. **a–d. See Answer Appendix.**

a. **NUMERICAL** To simulate the experiment for Product A, use the random number generator on a graphing calculator to generate 30 integers between 0 and 9. Let 0–6 represent an effective outcome and 7–9 represent an ineffective outcome.

b. **TABULAR** Copy and complete the frequency table shown using the results. from part **a.** Then use the data to calculate the probability that Product A was effective. Repeat to find the probability for Product B.

Product A	
Number	**Frequency**
0–6	
7–9	

c. **ANALYTICAL** Compare the probabilities that you found in part **b.** Do you think that the difference in the effectiveness of each product is significant enough to justify selecting one product over the other? Explain.

d. **LOGICAL** Suppose Product B costs twice as much as Product A. Do you think the probability of the product's effectiveness justifies the price difference to a consumer? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

REASONING **Determine whether each statement is *true* or *false*. If false, explain.** **28, 30–32. See Answer Appendix.**

28. To save time and money, population parameters are used to estimate sample statistics.

29. Observational studies and experiments can both be used to study cause-and-effect relationships. **true**

30. **OPEN ENDED** Design an observational study. Identify the objective of the study, define the population and sample, collect and organize the data, and calculate a sample statistic.

31. **CHALLENGE** What factors should be considered when determining whether a given statistical study is reliable?

32. **WRITING IN MATH** Research each of the following sampling methods. Then describe each method and discuss whether using the method could result in bias.

a. convenience sample
b. self-selected sample
c. stratified sample
d. systematic sample

Differentiated Instruction OL BL

Extension Have students investigate techniques that could be used to select random samples. For example, a random number table from the appendix of a statistics text or the random number generator of a spreadsheet or calculator could be used.

LAB 9 Graphing Calculator Lab
Simulations and Margin of Error

The Pew Research Center conducted a survey of a random sample of teens and concluded that 43% of all teens who take their cell phones to school text in class on a daily basis. How accurately did their random sample represent all teens?

Common Core State Standards
S.IC.2, S.IC.4, S.IC.6

As you learned in the previous lesson, a survey of a random sample is a valuable tool for generalizing information about a larger population. The program in the following activity makes use of a random number generator (randInt(a, b)) to simulate the results of a random sampling survey.

Activity 1 Random Sampling Simulation

Use the following program that simulates the texting survey to measure the percent of teens who text in class for random sample sizes of 20, 50, and 100 students.

Step 1 Input the following program into a graphing calculator.

```
Program:SIMTEST
:Input "SAMPLE SIZE ",S
:0→A
:0→B
:Lbl Z
:A + 1→A
:randInt(1, 100) →C
:If C ≤ 43
:B + 1→B
:If A<S
:Goto Z
:100B/S→P
:Disp "PERCENT WHO TEXT",P
:Stop
```

Step 2 Run 10 trials of the program for each sample size of 20, 50, and 100. Press ENTER to run the program again each time.

Step 3 Record the percent who text for each trial in the table below.

Sample Size	1	2	3	4	5	6	7	8	9	10
20										
50										
100										

Analyze the Results

1. Discard the percent that is farthest from the Pew survey result of 43% for each sample size. What is the range of the remaining nine percents for each sample size? **See students' work.**

2. What is the farthest any of these remaining trials is from the 43% for each sample size? **See students' work.**

3. The positive or negative of the result found in Exercise 2 is known as the **margin of error**. For your results, which sample size had the smallest margin of error? **samples of size 100**

4. What would you expect to happen to the margin of error if we used a sample size of 500?

(continued on the next page)

Sample answer: The margin of error would decrease.

1 Focus

Objective Use a simulation to develop margins of error for various sizes of random samples.

Materials for Each Student

- TI-83/84 Plus or other graphing calculator

Teaching Tip

If students have limited programming experience on the calculator, review the list of commands by pressing PRGM when the calculator is in program editing mode. To save classroom time, the program can be downloaded from one calculator to the others.

2 Teach

Working in Cooperative Groups

Have students work in pairs, mixing abilities, to complete Activities 1–2 and Exercises 1–4.

Ask:

- What does the program line: randInt(1,100)→*C* do? It assigns a random integer from 1 to 100 to the variable *C*.
- What happens if *C* is not less than or equal to 43? The program will skip the next line (it will not add 1 to the value of *B*).
- What happens if a sample size of 500 or 1000 is entered into the program? The calculator will take several seconds longer to run the program.

3 Assess

Formative Assessment

Use Exercises 5 and 6 to assess whether students can correctly apply the margin of error formula.

From Concrete to Abstract

Ask:

- In Activity 1, what percent of the trials for each sample size did you use? 90%
- Why were all the trials not used to develop the margin of error? Sample answer: As seen in Exercise 9, a wide range of percents is possible, but not likely. By using 90% of the trials, a range with a smaller margin of error is obtained that more closely resembles our results.

Additional Answers

9. Yes; sample answer: The margin of error is a prediction of what is most likely to happen. It is possible, but not likely, that a random sample could give results anywhere between 0% and 100%.
10. Sample answer: The margin of error predicts that using a sample size of 2500 should produce a percent that falls within the range 41%–45%. While it is possible that a random sample could give results anywhere between 0% and 100%, it is not likely that 10 trials all consisting of a sample size of 2500 would produce percents with a range of 19%–23%. Therefore, the validity of the model should be questioned.

Graphing Calculator Lab
Simulations and Margin of Error *Continued*

Statisticians have found that for large populations, the margin of error for a random sample of size n can be approximated by the following formula.

KeyConcept Margin of Error Formula

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}}(100)$$

Since n is in the denominator, the margin of error will decrease as the size of the random sample increases. This expression can also be used to determine the size of a random sample necessary to achieve a desired level of reliability.

Activity 2 Margin of Error and Sample Size

You are a member of a research team.

a. You need to decide whether to conduct a survey with a margin of error of ±3% or ±2%. What sample size would be needed to achieve each goal?

Set each percent equal to the margin of error formula and solve for n.

$\pm 3\% = \pm\frac{1}{\sqrt{n}}(100)$	Margin of error formula	$\pm 2\% = \pm\frac{1}{\sqrt{n}}(100)$
$0.03\sqrt{n} = 1$	Multiply by $\frac{\sqrt{n}}{100}$.	$0.02\sqrt{n} = 1$
$\sqrt{n} = 33.333$	Divide.	$\sqrt{n} = 50$
$n = 1111.11$	Square each side.	$n = 2500$

A random sample of about 1100 would have a margin of error of about ±3%, while a random sample of 2500 would have a margin of error of ±2%.

b. Suppose the finance director would like to reduce the cost of the survey by using a random sample of 100. What would be the margin of error for this sample size?

Substitute 100 for n in the margin of error formula.

$$\text{margin of error} = \pm\frac{1}{\sqrt{n}}(100) \quad \text{Margin of error formula}$$

$$= \pm\frac{1}{\sqrt{100}}(100) \text{ or } \pm 10\% \quad n = 100$$

A random sample of 100 would have a margin of error of ±10%.

Exercises

5. What random sample size would produce a margin of error of ±1%? **10,000**
6. What margin of error can be expected when using a sample size of 500? **about ±4.5%**
7. What are some reasons that a research center might decide that a survey with a margin of error of ±3% would be more desirable than one with a margin of error of ±2%? **Sample answer: The cost would be less, the survey would take less time, and it would require fewer people to administer.**
8. What is the range for the percent of students that text in class that the research center can expect from any random survey they conduct with a sample size of 2500? **41%–45%**
9. If a survey with a random sample of 2500 students is conducted, is it possible that only 20% of the students could respond that they text during class? If so, how could this be possible? **See margin.**
10. If Step 2 from Activity 1 is repeated using a sample size of 2500 and the range for the percents is found to be 19%–23%, would this result cause you to question the model? **See margin.**

LESSON 10 Distributions of Data

Then

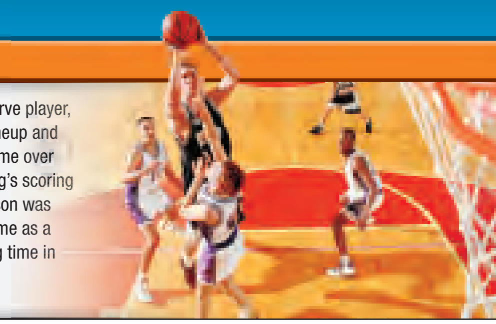
- You calculated measures of central tendency and variation.

Now

1. Use the shapes of distributions to select appropriate statistics.
2. Use the shapes of distributions to compare data.

Why?

- After four games as a reserve player, Craig joined the starting lineup and averaged 18 points per game over the *remaining* games. Craig's scoring average for the *entire* season was less than 18 points per game as a result of the lack of playing time in the first four games.



NewVocabulary
distribution
negatively skewed distribution
symmetric distribution
positively skewed distribution

Common Core State Standards
S.IC.1

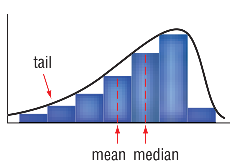
1 Analyzing Distributions

A **distribution** of data shows the observed or theoretical frequency of each possible data value. In Lesson 0-9, you described distributions of sample data using statistics. You used the mean or median to describe a distribution's center and standard deviation or quartiles to describe its spread. Analyzing the shape of a distribution can help you decide which measure of center or spread best describes a set of data.

The shape of the distribution for a set of data can be seen by drawing a curve over its histogram.

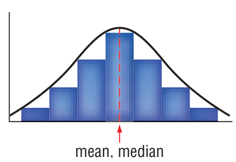
KeyConcept Symmetric and Skewed Distributions

Negatively Skewed Distribution



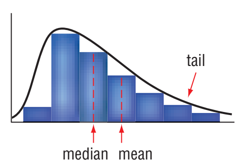
- The mean is less than the median.
- The majority of the data are on the right of the mean.

Symmetric Distribution



- The mean and median are approximately equal.
- The data are evenly distributed on both sides of the mean.

Positively Skewed Distribution



- The mean is greater than the median.
- The majority of the data are on the left of the mean.

When a distribution is symmetric, the mean and standard deviation accurately reflect the center and spread of the data. However, when a distribution is skewed, these statistics are not as reliable. Recall that outliers have a strong effect on the mean of a data set, while the median is less affected. Similarly, when a distribution is skewed, the mean lies away from the majority of the data toward the tail. The median is less affected, so it stays near the majority of the data.

When choosing appropriate statistics to represent a set of data, first determine the skewness of the distribution.

- If the distribution is relatively symmetric, the mean and standard deviation can be used.
- If the distribution is skewed or has outliers, use the five-number summary to describe the center and spread of the data.

1 Focus

VerticalAlignment

Before Lesson 10 Calculate measures of central tendency and variation.

Lesson 10 Use the shapes of distributions to select appropriate statistics. Use the shapes of distributions to compare data.

After Lesson 10 Learn about probability distributions.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- Why is Craig's scoring average for the entire season less than 18 points per game? Sample answer: If Craig did not receive much playing time as a reserve player, he probably did not score many points during the first four games. As a result, these scores brought down his average.
- Is Craig's scoring average for the entire season a good representation of his scoring average? Explain. Sample answer: No; his scoring average for the entire season is lower than it would have been had Craig been in the starting lineup for the entire season.

1 Analyzing Distributions

Example 1 shows how to choose appropriate statistics to describe a set of data using a histogram. **Example 2** shows how to choose appropriate statistics to describe a set of data using a box-and-whisker plot.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Example

1 **PRESENTATIONS** Ms. Shroyer's students each gave a presentation as part of their class project. The length of each presentation is shown in the table below.

Time (minutes)				
17	13	11	17	20
15	23	7	16	10
13	20	12	21	14
17	19	20	18	19

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

See bottom margin for graph; negatively skewed

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice. Sample answer: The distribution is skewed, so use the five-number summary. The range is 7 to 23 minutes. The median is 17 minutes, and half of the times are between 13 and 19.5 minutes.

Additional Examples also in Interactive Classroom PowerPoint® Presentations

Interactive White Board READY

Real-World Link

The first portable computer, the Osborne I, was available for sale in 1981 for $1795. The computer weighed 24 pounds and included a 5-inch display. Laptops can now be purchased for as little as $250 and can weigh as little as 3 pounds.

Source: Computer History Museum

Real-World Example 1 Describe a Distribution Using a Histogram

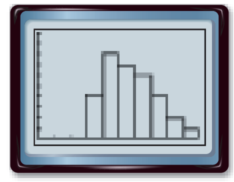
COMPUTERS The prices for a random sample of personal computers are shown.

Price (dollars)							
723	605	847	410	440	386	572	523
374	915	734	472	420	508	613	659
706	463	470	752	671	618	538	425
811	502	490	552	390	512	389	621

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

First, press STAT ENTER and enter each data value. Then, press 2nd [STAT PLOT] ENTER ENTER and choose the histogram icon. Finally, adjust the window to the dimensions shown.

The majority of the computers cost between $400 and $700. Some of the computers are priced significantly higher, forming a tail for the distribution on the right. Therefore, the distribution is positively skewed.



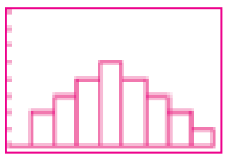
[0, 1000] scl: 100 by [0, 10] scl: 1

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is skewed, so use the five-number summary to describe the center and spread. Press STAT ▶ ENTER ENTER and scroll down to view the five-number summary.

The prices for this sample range from $374 to $915. The median price is $530.50, and half of the computers are priced between $451.50 and $665.

1A.



[15, 42] scl: 3 by [0, 8] scl: 1

symmetric

1B. Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean rainfall was 29 inches with standard deviation of 5.4 inches.

Guided Practice

1. RAINFALL The annual rainfall for a region over a 24-year period is shown below.

A. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

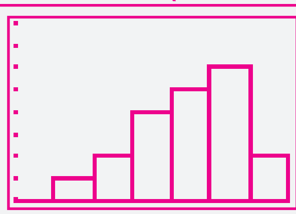
B. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Annual Rainfall (in.)					
27.2	30.2	35.8	26.1	39.3	20.6
28.9	23.0	32.7	26.8	22.7	25.4
29.6	36.8	33.4	28.4	21.9	20.8
24.7	30.6	27.7	31.4	34.9	37.1

A box-and-whisker plot can also be used to identify the shape of a distribution. The position of the line representing the median indicates the center of the data. The "whiskers" show the spread of the data. If one whisker is considerably longer than the other and the median is closer to the shorter whisker, then the distribution is skewed.

Additional Answer (Additional Example)

1a.

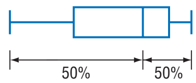
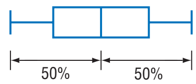
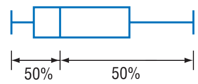


[3, 24] scl: 3 by [0, 8] scl: 1

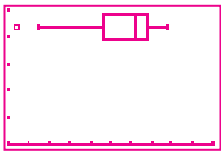
Tips for New Teachers

Skewed Distributions Students may confuse negatively and positively skewed distributions. Remind them that when the tail is on the left of a distribution, the data appear to be going uphill, and since it is harder to go uphill, the distribution is negatively skewed. When the tail is on the right of a distribution, the data appear to be going downhill, and since it is easier to go downhill, the distribution is positively skewed.

KeyConcept Box-and-Whisker Plots as Distributions

Negatively Skewed	Symmetric	Positively Skewed
		
The data to the left of the median are distributed over a wider range than the data to the right. The data have a tail to the left.	The data are equally distributed to the left and right of the median.	The data to the right of the median are distributed over a wider range than the data to the left. The data have a tail to the right.

2A.



[500, 750] scl: 25 by [0, 5] scl: 1

negatively skewed

2B. Sample answer: The distribution is skewed, so use the five-number summary. Janet's used minutes range from 511 minutes to 695 minutes. The median is 655.5 minutes, and half of the data are between 616.5 and 670.5 minutes.

WatchOut!

Standard Deviation Recall from Lesson 0-9 that the formulas for standard deviation for a population σ and for a sample s are slightly different. In Example 2, times for all of the students in Mr. Fejis' class are being analyzed, so use the population standard deviation.

Example 2 Describe a Distribution Using a Box-and-Whisker Plot

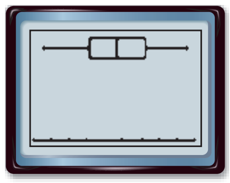
HOMEWORK The students in Mr. Fejis' language arts class found the average number of minutes that they each spent on homework each night.

Minutes per Night					
62	53	46	66	38	45
52	46	73	39	42	56
64	54	48	59	70	60
49	54	48	57	70	33

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

Enter the data as L1. Press 2nd [STAT PLOT] ENTER ENTER and choose the box plot icon. Adjust the window to the dimensions shown.

The lengths of the whiskers are approximately equal, and the median is in the middle of the data. This indicates that the data are equally distributed to the left and right of the median. Thus, the distribution is symmetric.



[30, 75] scl: 5 by [0, 5] scl: 1

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is symmetric, so use the mean and standard deviation to describe the center and spread. The average number of minutes that a student spent on homework each night was 53.5 with standard deviation of about 10.5.

GuidedPractice

2. CELL PHONE Janet's parents have given her a prepaid cell phone. The number of minutes she used each month for the last two years are shown in the table.

Minutes Used per Month			
582	608	670	620
667	598	671	613
537	511	674	627
638	661	642	641
668	673	680	695
658	653	670	688

A. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

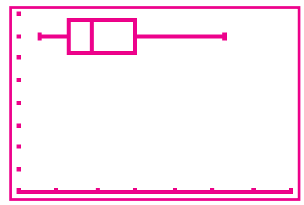
B. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Additional Example

2 WAGES The hourly wages for a random sample of employees of a restaurant are shown in the table.

Wages ($)				
10.50	7.75	8.25	9.50	6.50
7.50	11.25	7.25	6.50	8.25
8.00	7.50	6.75	7.25	9.00
7.50	10.00	7.25	8.00	9.00

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.



[6, 13] scl: 1 by [0, 8] scl: 1

positively skewed

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice. Sample answer: The distribution is skewed, so use the five-number summary. The range is $6.50 to $11.25. The median is about $7.88, and half of the data are between $7.25 and $9.00.

DifferentiatedInstruction AL OL BL ELL

Interpersonal Learners Have students work in pairs to think of examples of data that may have distributions that are symmetric, negatively skewed, or positively skewed.

2 Comparing Distributions

Example 3 shows how to use histograms to compare the distributions of two data sets. **Example 4** shows how to use box-and-whisker plots to compare the distributions of two data sets.

Additional Example

3 **GAMES** Tyler and Jordan are working through several brainteasers on the computer. The time in minutes that it took to complete each game is shown.

Tyler (minutes)				
5.1	2.9	3.5	6.0	2.8
3.4	4.1	3.8	4.3	6.1
5.8	5.1	6.0	3.4	4.6
4.4	3.6	4.8	3.1	5.2

Jordan (minutes)				
4.1	5.4	3.8	2.9	6.4
3.1	3.7	5.3	4.5	2.7
3.4	4.3	4.2	5.8	4.7
6.1	5.9	5.4	4.4	3.9

a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution. See bottom margin for graphs; Tyler, positively skewed; Jordan, symmetric

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice. Sample answer: One distribution is symmetric and the other is skewed, so use the five-number summaries. The median for both sets is 4.35 but 50% of Tyler's times occur between 3.45 and 5.15, while 50% of Jordan's times occur between 3.75 and 5.4. The smaller interquartile range for Jordan may suggest that she was slightly more consistent than Tyler.

2 Comparing Distributions

To compare two sets of data, first analyze the shape of each distribution. Use the mean and standard deviation to compare two symmetric distributions. Use the five-number summaries to compare two skewed distributions or a symmetric distribution and a skewed distribution.

Example 3 Compare Data Using Histograms

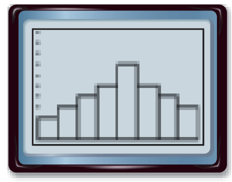
TEST SCORES **Test scores from Mrs. Morash's class are shown below.**

Chapter 3 Test Scores
81, 81, 92, 99, 61, 67, 86, 82, 76, 73, 62, 97, 97, 72, 72, 84, 77, 88, 92, 93, 76, 74, 66, 78, 76, 69, 84, 87, 83, 87, 92, 87, 82

Chapter 4 Test Scores
87, 73, 69, 83, 74, 86, 74, 69, 79, 84, 79, 74, 83, 74, 86, 69, 91, 73, 79, 83, 69, 79, 83, 74, 86, 79, 79, 78, 83, 79, 86, 79, 84

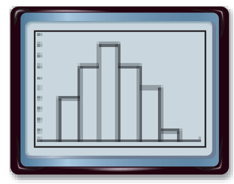
a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

Chapter 3 Test Scores



[60, 100] scl: 5 by [0, 10] scl: 1

Chapter 4 Test Scores



[60, 100] scl: 5 by [0, 10] scl: 1

Both distributions are symmetric.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

The distributions are symmetric, so use the means and standard deviations.

Chapter 3 Test Scores

Chapter 4 Test Scores

The Chapter 4 test scores, while lower in average, have a much smaller standard deviation, indicating that the scores are more closely grouped about the mean. Therefore, the mean for the Chapter 4 test scores is a better representation of the data than the mean for the Chapter 3 test scores.

Guided Practice

3. **TYPING** The typing speeds of the students in two classes are shown below.

A. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

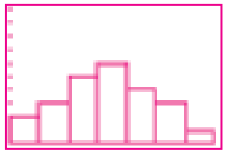
B. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

3rd Period (wpm)
23, 38, 27, 28, 40, 45, 32, 33, 34, 27, 40, 22, 26, 34, 29, 31, 35, 33, 37, 38, 28, 29, 39, 42

6th Period (wpm)
38, 26, 43, 46, 23, 24, 27, 36, 22, 21, 26, 27, 31, 32, 27, 25, 23, 22, 28, 29, 28, 33, 23, 24

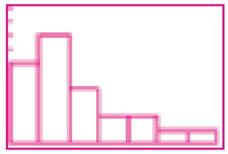
3A.

3rd Period



[20, 50] scl: 4 by [0, 10] scl: 1

6th Period



[20, 50] scl: 4 by [0, 10] scl: 1

3rd period, symmetric; 6th period, positively skewed

StudyTip

Multiple Data Sets To compare two sets of data, enter one set as **L1** and the other as **L2**. In order to calculate statistics for a set of data in **L2**, press STAT ▶ ENTER 2nd [L2] ENTER.

3B. Sample answer: One distribution is symmetric and the other is skewed, so use the five-number summaries. The range for 3rd period is 23, and the range for 6th period is 25. However, the median for 3rd period is 33, and the median for 6th period is 27. The lower quartile for 3rd period is 28. Since this is greater than the median for 6th period, this means that 75% of the speeds for 3rd period are greater than 50% of the speeds for 6th period. Therefore, we can conclude that 3rd period had slightly better typing speeds overall.

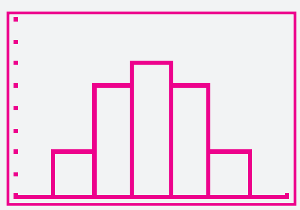
Additional Answer (Additional Example)

3a.

Tyler

[1, 8] scl: 1 by [0, 8] scl: 1

Jordan



[1, 8] scl: 1 by [0, 8] scl: 1

Box-and-whisker plots can be displayed alongside one another, making them useful for side-by-side comparisons of data.

Example 4 Compare Data Using Box-and-Whisker Plots

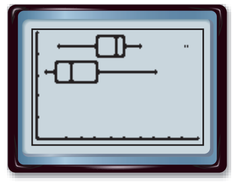
POINTS **The points scored per game by a professional football team for the 2008 and 2009 football seasons are shown.**

2008							
7	51	24	27	17	35	27	33
28	30	27	21	24	30	14	20

2009							
20	9	3	10	6	14	3	10
3	37	7	21	13	41	20	23

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

Enter the 2008 scores as L1. Graph these data as Plot1 by pressing [2nd] [STAT PLOT] [ENTER] [ENTER] and choosing ⊏⊐··. Enter the 2009 scores as L2. Graph these data as Plot2 by pressing [2nd] [STAT PLOT] [▼] [ENTER] [ENTER] and choosing ⊏⊐··. For Xlist, enter L2. Adjust the window to the dimensions shown.



[0, 55] scl: 5 by [0, 5] scl: 1

For the 2008 scores, the left whisker is longer than the right and the median is closer to the right whisker. The distribution is negatively skewed.

For the 2009 scores, the right whisker is longer than the left and the median is closer to the left whisker. The distribution is positively skewed.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

The distributions are skewed, so use the five-number summaries to compare the data.

The lower quartile for the 2008 season and the upper quartile for the 2009 season are both 20.5. This means that 75% of the scores from the 2008 season were greater than 20.5 and 75% of the scores from the 2009 season were less than 20.5.

The minimum of the 2008 season is approximately equal to the lower quartile for the 2009 season. This means that 25% of the scores from the 2009 season are lower than any score achieved in the 2008 season. Therefore, we can conclude that the team scored a significantly higher amount of points during the 2008 season than the 2009 season.

GuidedPractice

4. **GOLF** Robert recorded his golf scores for his sophomore and junior seasons.

A. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

B. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

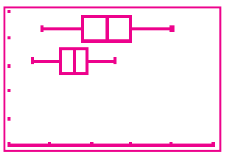
Sophomore Season
42, 47, 43, 46, 50, 47, 52, 45, 53, 55, 48, 39, 40, 49, 47, 50

Junior Season
44, 38, 46, 48, 42, 41, 42, 46, 43, 40, 43, 43, 44, 45, 39, 44

StudyTip

Outliers Recall from Lesson 0-9 that outliers are data that are more than 1.5 times the interquartile range beyond the upper or lower quartile. All outliers should be plotted, but the whiskers should be drawn to the least and greatest values that are not outliers.

4A.



[35, 60] scl: 5 by [0, 5] scl: 1

both symmetric

4B. Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean score for Robert's sophomore season is about 47.1 with standard deviation of about 4.4. The mean score for Robert's junior season is 43 with standard deviation of about 2.6. The lower mean and standard deviation from Robert's junior season indicates that he not only improved, but was also more consistent.

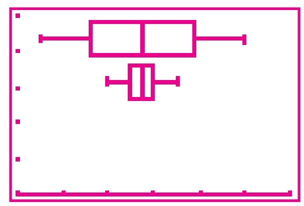
Additional Example

4 **TEMPERATURES** The daily high temperatures over a 20-day period for two cities are shown.

Clintonville (°F)				
60	55	68	63	70
59	63	61	54	68
66	57	72	65	67
65	62	58	72	58

Stockton (°F)				
61	63	62	61	64
63	64	60	65	63
63	65	62	60	64
64	63	66	62	66

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.



[52, 76] scl: 4 by [0, 5] scl: 1

both symmetric

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice. Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean temperature for Clintonville is about 63.15° with standard deviation of about 5.40°. The mean temperature for Stockton is about 63.05° with standard deviation of about 1.76°. The average temperatures for both cities are about the same, but the lower standard deviation for Stockton means that the temperatures there are more consistently near 63° than at Clintonville.

Teach with Tech

Graphing Calculator Have students enter the data points from Example 2 in a graphing calculator. Have students calculate the mean and median of the data set. Have students add outliers to the data set and then have them recalculate the mean and median. After they have reviewed their results, reiterate the concept of the median being less resistant to the effect of outliers.

3 Practice

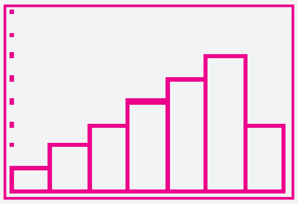
Formative Assessment

Use Exercises 1–4 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Additional Answers

1a.

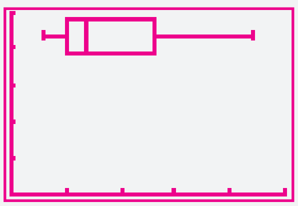


[4, 32] scl: 4 by [0, 8] scl: 1

negatively skewed

1b. Sample answer: The distribution is skewed, so use the five-number summary. The times range from 7 to 30 minutes. The median is 22.5 minutes, and half of the data are between 15.5 and 26 minutes.

2a.

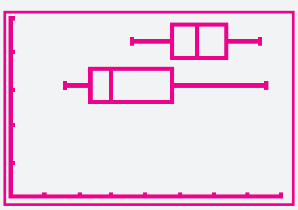


[0, 25] scl: 5 by [0, 5] scl: 1

positively skewed

2b. Sample answer: The distribution is skewed, so use the five-number summary. The data range from 3 to 22 times. The median number is 7 times, and half of the data are between 5 and 13 times.

4a.



[0, 40] scl: 5 by [0, 5] scl: 1

junior class, positively skewed; senior class, symmetric

Check Your Understanding

Example 1

1. **EXERCISE** The amount of time that James ran on a treadmill for the first 24 days of his workout is shown. **a–b. See margin.**

Time (minutes)											
23	10	18	24	13	27	19	7	25	30	15	22
10	28	23	16	29	26	26	22	12	23	16	27

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Example 2

2. **RESTAURANTS** The total number of times that 20 random people either ate at a restaurant or bought fast food in a month are shown. **a–b. See margin.**

Restaurants or Fast Food									
4	7	5	13	3	22	13	6	5	10
7	18	4	16	8	5	15	3	12	6

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Example 3

3. **SALES** The total fundraiser sales for the students in two classes at Cantonville High School are shown. **a–b. See Answer Appendix.**

Mrs. Johnson's Class (dollars)					
6	14	17	12	38	15
11	12	23	6	14	28
16	13	27	34	25	32
21	24	21	17	16	

Mr. Edmunds' Class (dollars)					
29	38	21	28	24	33
14	19	28	15	30	6
31	23	33	12	38	28
18	34	26	34	24	37

a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Example 4

4. **RECYCLING** The weekly totals of recycled paper for the junior and senior classes are shown. **a–b. See margin.**

Junior Class (pounds)					
14	24	8	26	19	38
12	15	12	18	9	24
12	21	9	15	13	28

Senior Class (pounds)					
25	31	35	20	37	27
22	32	24	28	18	32
25	32	22	29	26	35

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	5–10, 14–16	5–9 odd	6–10 even, 14–16
OL Core	5–12, 14–16	5–10	11, 12, 14–16
BL Advanced	11–16		

32 | Lesson 10 | Distributions of Data

Practice and Problem Solving

Examples 1–2 For Exercises 5 and 6, complete each step.

a. Use a graphing calculator to create a histogram and a box-and-whisker plot. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

5. **FANTASY** The weekly total points of Kevin's fantasy football team are shown. **a–b. See margin.**

Total Points							
165	140	88	158	101	137	112	127
53	151	120	156	142	179	162	79

6. **MOVIES** The students in one of Mr. Peterson's classes recorded the number of movies they saw over the past month. **a–b. See Answer Appendix.**

Movies Seen											
14	11	17	9	6	11	7	8	12	13	10	9
5	11	7	13	9	12	10	9	15	11	13	15

Example 3 For Exercises 7 and 8, complete each step.

a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice. **7–8. See Answer Appendix.**

7. **SAT** A group of students took the SAT their sophomore year and again their junior year. Their scores are shown.

Sophomore Year Scores					
1327	1663	1708	1583	1406	1563
1637	1521	1282	1752	1628	1453
1368	1681	1506	1843	1472	1560

Junior Year Scores					
1728	1523	1857	1789	1668	1913
1834	1769	1655	1432	1885	1955
1569	1704	1833	2093	1608	1753

8. **INCOME** The total incomes for 18 households in two neighboring cities are shown.

Yorkshire (thousands of dollars)					
68	59	61	78	58	66
56	72	86	58	63	53
68	58	74	60	103	64

Applewood (thousands of dollars)					
52	55	60	61	55	65
65	60	45	37	41	71
50	61	65	66	87	55

Example 4 **9.** **TUITION** The annual tuitions for a sample of public colleges and a sample of private colleges are shown. Complete each step. **a–b. See Answer Appendix.**

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Public Colleges (dollars)					
3773	3992	3004	4223	4821	3880
3163	4416	5063	4937	3321	4308
4006	3508	4498	3471	4679	3612

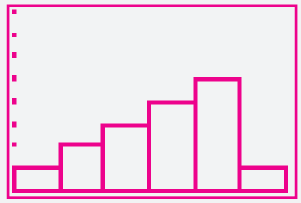
Private Colleges (dollars)					
10,766	13,322	12,995	15,377	16,792	9147
15,976	11,084	17,868	7909	12,824	10,377
14,304	10,055	12,930	16,920	10,004	11,806

WatchOut!

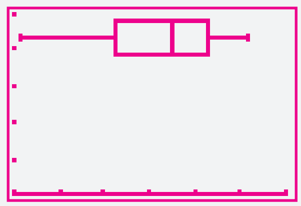
Standard Deviations For Exercises 6 and 7, students should calculate the population standard deviation. Every student in Mr. Peterson's class is included, and every student that was in the group of sophomores was included in the group of juniors. For Exercise 9, students should calculate the sample standard deviation since the tuitions for only a sample of colleges were collected.

Additional Answers

5a.



[50, 200] scl: 25 by [0, 8] scl: 1



[50, 200] scl: 25 by [0, 5] scl: 1

negatively skewed

5b. Sample answer: The distribution is skewed, so use the five-number summary. The points range from 53 to 179. The median is 138.5 points, and half of the data are between 106.5 and 157 points.

Additional Answer

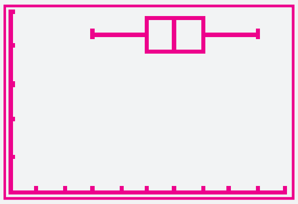
4b. Sample answer: One distribution is symmetric and the other is skewed, so use the five-number summaries. The median for the junior class is 15, and the median for the senior class is 27.5. The minimum value for the senior class is 18. This means that all of the weekly totals for the senior class are greater than 50% of the weekly totals for the junior class. Therefore, we can conclude that the senior class's weekly totals were far greater than the junior class's weekly totals.

4 Assess

Ticket Out the Door Have students generate a set of data, create a histogram using the data, and describe the shape of the distribution. Then have the students describe the center and spread of the data using either the mean and standard deviation or the five-number summary.

Additional Answers

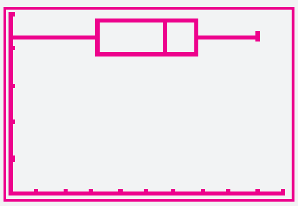
11a.



[0, 30] sc: 3 by [0, 5] scl: 1

Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean of the data is 18 with sample standard deviation of about 5.2 points.

11b.



[0, 30] sc: 3 by [0, 5] scl: 1

mean: 14.6; median: 17

11c. Sample answer: Adding the scores from the first four games causes the shape of the distribution to go from being symmetric to being negatively skewed. Therefore, the center and spread should be described using the five-number summary.

13a. Sample answer: mean = 14; median = 10

13b. Sample answer: mean = 20; median = 24

13c. Sample answer: mean = 17; median = 17

10. DANCE The total amount of money that a random sample of seniors spent on prom is shown. Complete each step. **a–b. See Answer Appendix.**

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Boys (dollars)					
253	288	304	283	348	276
322	368	247	404	450	341
291	260	394	302	297	272

Girls (dollars)					
682	533	602	504	635	541
489	703	453	521	472	368
562	426	382	668	352	587

11a–c. See margin.

B **11. BASKETBALL** Refer to the beginning of the lesson. The points that Craig scored in the remaining games are shown.

Points Scored			
18	10	18	21
9	25	13	17
17	12	24	19
20	17	27	21

a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.

b. Craig scored 0, 2, 1, and 0 points in the first four games. Use a graphing calculator to create a box-and-whisker plot that includes the new data. Then find the mean and median of the new data set.

c. What effect does adding the scores from the first four games have on the shape of the distribution and on how you should describe the center and spread?

12a–b. See Answer Appendix.

12. SCORES Allison's quiz scores are shown.

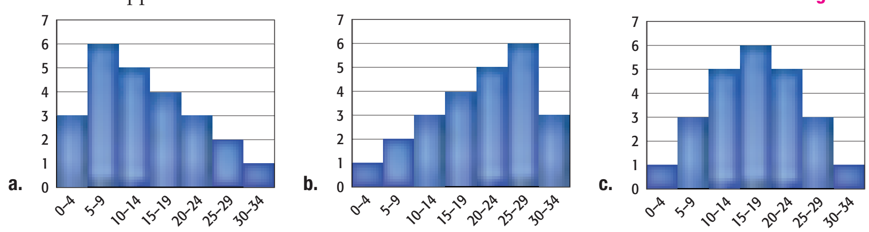
Math Quiz Scores					
83	76	86	82	84	57
86	62	90	96	76	89
76	88	86	86	92	94

a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread.

b. Allison's teacher allows students to drop their two lowest quiz scores. Use a graphing calculator to create a box-and-whisker plot that reflects this change. Then describe the center and spread of the new data set.

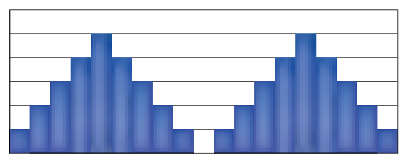
H.O.T. Problems Use Higher-Order Thinking Skills

C **13. CHALLENGE** Approximate the mean and median for each distribution of data. **a–c. See margin.**



14. REASONING Distributions of data are not always symmetric or skewed. If a distribution has a gap in the middle, like the one shown, two separate clusters of data may result, forming a *bimodal distribution*. How can the center and spread of a bimodal distribution be described?

14. See Answer Appendix.



15. OPEN ENDED Find a real-world data set that appears to represent a symmetric distribution and one that does not. Describe each distribution. Create a visual representation of each set of data. **See Answer Appendix.**

16. WRITING IN MATH Explain the difference between positively skewed, negatively skewed, and symmetric sets of data, and give an example of each. **See Answer Appendix.**

Differentiated Instruction OL BL

Extension Have students complete Exercise 14. Examples of data that may have bimodal distributions are the annual tuitions for private and public schools, the selling prices for suburban and urban houses, and the heights of 6th grade and 12th grade students. Have students think of other examples that may result in bimodal distributions.

LESSON 11 Probability Distributions

Then

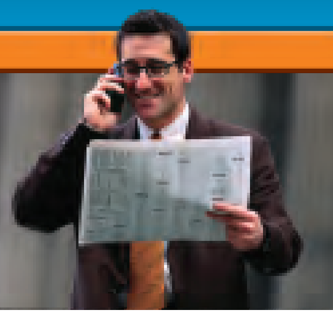
- You used statistics to describe symmetrical and skewed distributions of data.

Now

1. Construct a probability distribution.
2. Analyze a probability distribution and its summary statistics.

Why?

- Mutual funds are professionally managed investments that offer diversity to investors. An accurate analysis of the fund's current and expected performance can help an investor determine if the fund suits their needs.



NewVocabulary

random variable
discrete random variable
continuous random variable
probability distribution
theoretical probability distribution
experimental probability distribution
Law of Large Numbers
expected value

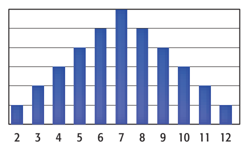
Jack Hollingsworth/CORBIS

Common Core State Standards
S.SMD.7

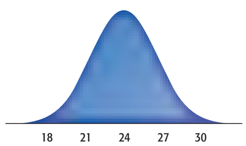
1 Construct a Probability Distribution

A sample space is the set of all possible outcomes in a distribution. Consider a distribution of values represented by the sum of the values on two dice and a distribution of the miles per gallon for a sample of cars.

Sum of Two Dice



Miles Per Gallon



The sum of the values on the dice can be any integer from 2 to 12. So, the sample space is [2, 3, …, 11, 12]. This distribution is *discrete* because the number of possible values in the sample space can be counted.

The distribution of miles per gallon is *continuous.* While the sample space includes any positive value less than a certain maximum (around 100), the data can take on an infinite number of values within this range.

The value of a **random variable** is the numerical outcome of a random event. A random variable can be discrete or continuous. **Discrete random variables** represent countable values. **Continuous random variables** can take on any value.

Example 1 Identify and Classify Random Variables

PT

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

a. the number of songs found on a random selection of mp3 players

The random variable X is the number of songs on any mp3 player in the random selection of players. The number of songs is countable, so X is discrete.

b. the weights of football helmets sent by a manufacturer

The random variable X is the weight of any particular helmet. The weight of any particular helmet can be anywhere within a certain range, typically 6 to 8 pounds. Therefore, X is continuous.

GuidedPractice **1A–1B. See Answer Appendix.**

1A. the exact distances of a sample of discus throws

1B. the ages of counselors at a summer camp

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1 Focus

VerticalAlignment

Before Lesson 11 Describe symmetrical and skewed distributions of data.

Lesson 11 Construct a probability distribution. Analyze a probability distribution and its summary statistics.

After Lesson 11 Understand the difference between continuous and discrete probability distributions.

Tips for New Teachers

While both discrete and continuous variables can represent infinitely many values, it is important to distinguish between the two.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- What do mutual funds offer investors? diversity
- Why is it beneficial to have an accurate analysis of the fund's performance? Sample answer: to determine if it is the right fund for the investor.

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Construct a Probability Distribution

Example 1 shows how to identify and classify random variables. **Example 2** shows how to use a relative frequency table to construct a probability distribution. **Example 3** shows how to use simulations to construct an experimental probability distribution.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Examples

1 Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

a. the number of hits for the players of a baseball team The random variable *X* is the number of hits, which is finite and countable, so *X* is discrete.

b. the distances traveled by the tee shots in a golf tournament The random variable *X* is the distance traveled, which can take on any value in a certain range, so *X* is continuous.

2 *X* represents the sum of two cards drawn from a stack of cards numbered 1 through 8 with replacement.

a. Construct a relative-frequency table.

b. Graph the theoretical probability distribution.

StudyTip

Discrete vs. Continuous Variables representing height, weight, and capacity will always be continuous variables because they can take on any positive value.

A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. Probability distributions can be represented by tables, equations, or graphs. In this lesson, we will focus on discrete probability distributions.

A probability distribution has the following properties.

KeyConcept Probability Distribution

- A probability distribution can be determined theoretically or experimentally.
- A probability distribution can be discrete or continuous.
- The probability of each value of *X* must be at least 0 and not greater than 1.
- The sum of all the probabilities for all of the possible values of *X* must equal 1. That is, $\Sigma P(X) = 1$.

ReviewVocabulary

Theoretical and Experimental Probability Theoretical probability is based on assumptions, and experimental probability is based on experiments. (Lesson 0-5)

A **theoretical probability distribution** is based on what is expected to happen. For example, the distribution for flipping a fair coin is $P(\text{heads}) = 0.5$, $P(\text{tails}) = 0.5$.

Example 2 Construct a Theoretical Probability Distribution

X represents the sum of the values on two dice.

a. Construct a relative-frequency table.

The theoretical probabilities associated with rolling two dice can be described using a relative-frequency table. When two dice are rolled, 36 total outcomes are possible. To determine the relative frequency, or theoretical probability, of each outcome, divide the frequency by 36.

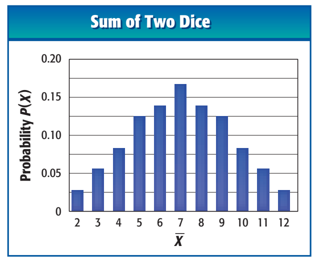
Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	2	3	4	5	6	5	4	3	2	1
Relative Frequency	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Sum: 36

b. Graph the theoretical probability distribution.

The graph shows the probability distribution for the sum of the values on two dice *X*. The bars are separated on the graph because the distribution is discrete (no other values of *X* are possible).

Each unique outcome of *X* is indicated on the horizontal axis, and the probability of each outcome occurring *P*(*X*) is indicated on the vertical axis.



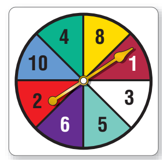
GuidedPractice

2. *X* represents the sum of the values of two spins of the wheel.

A. Construct a relative-frequency table.

B. Graph the theoretical probability distribution.

2A–2B. See Answer Appendix.

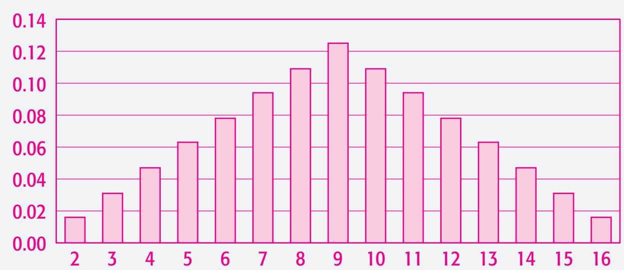


Additional Answers (Additional Examples)

2a.

Sum	2	3	4	5	6	7	8	9
Relative Frequency	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{3}{64}$	$\frac{1}{16}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{7}{64}$	$\frac{1}{8}$
Sum	10	11	12	13	14	15	16	
Relative Frequency	$\frac{7}{64}$	$\frac{3}{32}$	$\frac{5}{64}$	$\frac{1}{16}$	$\frac{3}{64}$	$\frac{1}{32}$	$\frac{1}{64}$	

2b.



An **experimental probability distribution** is a distribution of probabilities estimated from experiments. Simulations can be used to construct an experimental probability distribution. When constructing this type of distribution, use the frequency of occurrences of each observed value to compute its probability.

Example 3 Construct an Experimental Probability Distribution

X represents the sum of the values found by rolling two dice.

a. Construct a relative-frequency table.

Roll two dice 100 times or use a random number generator to complete the simulation and create a simulation tally sheet.

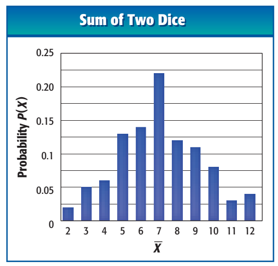
Sum	Tally	Frequency	Sum	Tally	Frequency
2	||	2	8	卌 卌 ||	12
3	卌	5	9	卌 卌 |	11
4	卌 |	6	10	卌 |||	8
5	卌 卌 |||	13	11	|||	3
6	卌 卌 ||||	14	12	||||	4
7	卌 卌 卌 卌 ||	22			

Calculate the experimental probability of each value by dividing its frequency by the total number of trials, 100.

Sum	2	3	4	5	6	7	8	9	10	11	12
Relative Frequency	0.02	0.05	0.06	0.13	0.14	0.22	0.12	0.11	0.08	0.03	0.04

b. Graph the experimental probability distribution.

The graph shows the discrete probability distribution for the sum of the values shown on two dice X.



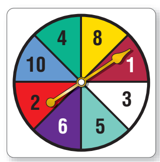
GuidedPractice

3. X represents the sum of the values of two spins of the wheel.

A. Construct a relative-frequency table for 100 trials.

B. Graph the experimental probability distribution.

3A–3B. See Answer Appendix.



Notice that this graph is different from the theoretical graph in Example 2. With small sample sizes, experimental distributions may vary greatly from their associated theoretical distributions. However, as the sample size increases, experimental probabilities will more closely resemble their associated theoretical probabilities. This is due to the **Law of Large Numbers**, which states that the variation in a data set decreases as the sample size increases.

ReviewVocabulary

Simulations and Random Number Generators For more practice on simulations and random number generators, see Extend 12-1.

StudyTip

Random Number Generators and Proportions When using a random number generator to simulate events with different probabilities, set up a proportion. For example, suppose there are 3 possible outcomes with probabilities of A: 0.25, B:0.35, and C:0.40. Random numbers 1–25 can represent A, 26–60 represent B, and 61–100 represent C.

Additional Example

3 X represents the sum of two cards drawn from a stack of cards numbered 1 through 8 with replacement.

a. Construct a relative frequency table for 100 trials.

b. Graph the experimental probability distribution.

Teach with Tech

Interactive Whiteboard Choose two students to work through an example in front of the class. Have one student explain how to calculate a frequency distribution, and have the other student explain how to create a histogram from that distribution.

Tips for New Teachers

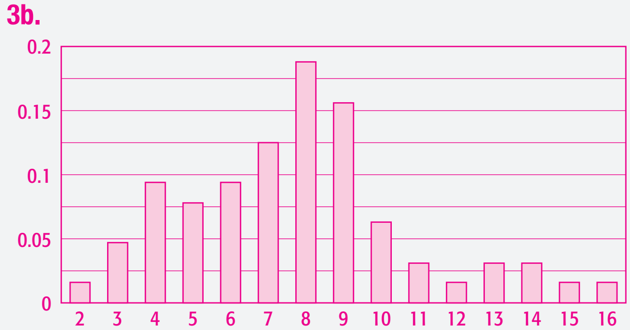
Law of Large Numbers Have students demonstrate the Law of Large Numbers by simulating the flipping of a coin for 10, 25, 50, 100, and 200 trials, and comparing the distributions. Show how the experimental probability more closely resembles the theoretical probability as the number of trials increases.

Additional Answers (Additional Examples)

3a.

Sum	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	3	6	5	6	8	12	10	4	2	1	2	2	1	1
Relative Frequency	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{3}{32}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{32}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$

3b.



Focus on Mathematical Content

Symbols The expected value $E(X)$ of a discrete random variable is equal to the mean μ of the variable. Therefore, the symbols are interchangeable.

Analyze a Probability Distribution

Example 4 shows how to calculate the expected value. **Example 5** shows how to calculate and analyze the standard deviation.

Additional Example

4 A tetrahedral die has four sides numbered 1, 2, 3, and 4. Find the expected value of one roll of this die. 2.5

Math HistoryLink

Christian Huygens
(1629–1695) This Dutchman was the first to discuss games of chance. "Although in a pure game of chance the results are uncertain, the chance that one player has to win or to lose depends on a determined value." This became known as the *expected value.*

akg-images

2 Analyze a Probability Distribution

Probability distributions are often used to analyze financial data. The two most common statistics used to analyze a discrete probability distribution are the mean, or expected value, and the standard deviation. The **expected value** $E(X)$ of a discrete random variable of a probability distribution is the weighted average of the variable.

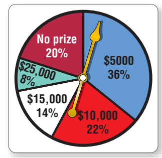
KeyConcept Expected Value of a Discrete Random Variable

Words The expected value of a discrete random variable is the weighted average of the values of the variable. It is calculated by finding the sum of the products of every possible value of X and its associated probability $P(X)$.

Symbols $E(X) = \Sigma\,[X \cdot P(X)]$

Real-World Example 4 Expected Value

GAME SHOWS A game-show contestant has won one spin of the wheel at the right. Find the expected value of his winnings.



Each prize value represents a value of X and each percent represents the corresponding probability $P(X)$. Find $E(X)$.

$$E(X) = \Sigma[X \cdot P(X)]$$
$$= 0(0.20) + 25{,}000(0.08) + 15{,}000(0.14) + 10{,}000(0.22) + 5000(0.36)$$
$$= 0 + 2000 + 2100 + 2200 + 1800$$
$$= 8100$$

The expected value of the contestant's winnings is $8100.

WatchOut!

Expected Value The expected value is what you *expect* to happen in the long run, not necessarily what you *will* happen.

GuidedPractice

4. PRIZES Curt won a ticket for a prize. The distribution of the values of the tickets and their relative frequencies are shown. Find the expected value of his winnings. **about $8.48**

Value ($)	1	10	100	1000	5000	25,000
Frequency	5000	100	25	5	1	1

Sometimes the expected value does not provide enough information to fully analyze a probability distribution. For example, suppose two wheels had roughly the same expected value. Which one would you choose? Which one is *riskier*? The standard deviation can provide more insight into the expected value of a probability distribution.

The formula for calculating the standard deviation of a probability distribution is similar to the one used for a set of data.

KeyConcept Standard Deviation of a Probability Distribution

Words For each value of X, subtract the mean from X and square the difference. Then multiply by the probability of X. The sum of each of these products is the variance. The standard deviation is the square root of the variance.

Symbols Variance: $\sigma^2 = \Sigma[[X - E(X)]^2 \cdot P(X)]$

Standard Deviation: $\sigma = \sqrt{\sigma^2}$

DifferentiatedInstruction AL OL BL ELL

Social Learners It is important to be aware that some students may have cultural or familial prohibitions against cards, dice, or gambling of any kind. Explain that, historically, the laws of probability were actually developed in the context of gambling but are now used in many other ways, including medicine and meteorology.

Real-World Example 5 Standard Deviation of a Distribution

DECISION MAKING **Jimmy is thinking about investing $10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below.**

Fund A	Fund B
50% chance of an $800 profit	30% chance of a $2400 profit
20% chance of a $1200 profit	10% chance of a $1900 profit
20% chance of a $600 profit	40% chance of a $200 loss
10% chance of a $100 loss	20% chance of a $400 loss

a. Find the expected value of each investment.

Fund A: $E(X) = 0.50(800) + 0.20(1200) + 0.20(600) + 0.10(-100)$ or 750

Fund B: $E(X) = 0.30(2400) + 0.10(1900) + 0.40(-200) + 0.20(-400)$ or 750

An investment of $10,000 in Fund A or Fund B will expect to yield $750.

b. Find each standard deviation.

Fund A:

Profit, X	$P(X)$	$[X - E(X)]^2$	$[X - E(X)]^2 \cdot P(X)$
800	0.50	$(800 - 750)^2 = 2500$	$2500 \cdot 0.50 = 1250$
1200	0.20	$(1200 - 750)^2 = 202{,}500$	$202{,}500 \cdot 0.20 = 40{,}500$
600	0.20	$(600 - 750)^2 = 22{,}500$	$22{,}500 \cdot 0.20 = 4500$
−100	0.10	$(-100 - 750)^2 = 722{,}500$	$722{,}500 \cdot 0.10 = 72{,}250$
			$\Sigma[[(X - E(X)]^2 \cdot P(X)] = 118{,}500$
			$\sqrt{118{,}500} \approx 344.2$

Fund B:

Profit, X	$P(X)$	$[X - E(X)]^2$	$[X - E(X)]^2 \cdot P(X)$
2400	0.30	$(2400 - 750)^2 = 2{,}722{,}500$	$2{,}722{,}500 \cdot 0.30 = 816{,}750$
1900	0.10	$(1900 - 750)^2 = 1{,}322{,}500$	$1{,}322{,}500 \cdot 0.10 = 132{,}250$
−200	0.40	$(-200 - 750)^2 = 902{,}500$	$902{,}500 \cdot 0.40 = 361{,}000$
−400	0.20	$(-400 - 750)^2 = 1{,}322{,}500$	$1{,}322{,}500 \cdot 0.20 = 264{,}500$
			$\Sigma[[(X - E(X)]^2 \cdot P(X)] = 1{,}574{,}500$
			$\sqrt{1{,}574{,}500} \approx 1254.8$

c. Which investment would you advise Jimmy to choose, and why?

Jimmy should choose Fund A. While the funds have identical expected values, the standard deviation of Fund B is almost four times the standard deviation for Fund A. This means that the expected value for Fund B will have about four times the variability than Fund A and will be riskier with a greater chance for gains and losses.

GuidedPractice

5. **DECISION MAKING** Compare a $10,000 investment in the two funds. Which investment would you recommend, and why?

Fund C	Fund D
30% chance of a $1000 profit	40% chance of a $1000 profit
40% chance of a $500 profit	30% chance of a $600 profit
20% chance of a $100 loss	15% chance of a $100 profit
10% chance of a $300 loss	15% chance of a $200 loss

Tetra Images/Getty Images

Real-WorldCareer

Mutual Fund Manager

Mutual fund managers buy and sell investments according to the investment objective of the fund. Investment management includes financial statement analysis, asset and stock selection, and monitoring of investments. Certification beyond a bachelor's degree is required.

Source: International Financial Services, London

5. Fund C: $E(X) = \$450$; $\sigma \approx 456.6$; Fund D: $E(X) = \$565$, $\sigma \approx 443$; sample answer: Funds C and D have roughly the same standard deviation, so they will have about the same risk. Therefore, Fund D will be the better investment because its expected value of $565 is greater than Fund C's expected value of $450.

StudyTip

Return on Investment When investing $1000 in a product that has a 6% expected return, the investor can expect a 0.06(1000) or $60 profit.

Additional Example

5 **RAFFLE** At a raffle, 400 tickets are sold for $1 each. One ticket wins $100, five tickets win $10, and ten tickets win $5. Calculate the expected value and standard deviation of the distribution of winnings for a $1 ticket.
−$0.50; 5.26

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3 Practice

Formative Assessment

Use Exercises 1–5 to check for understanding.

Use the chart on the bottom of this page to customize assignments for your students.

Additional Answers

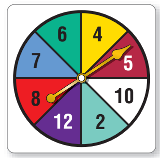
1. The random variable X is the number of pages linked to a Web page. The pages are countable, so X is discrete.
2. The random variable X is the number of stations in a cable package. The stations are countable, so X is discrete.
3. The random variable X is the amount of precipitation in a city per month. Precipitation can be anywhere within a certain range. Therefore, X is continuous.
4. The random variable X is the number of cars passing through an intersection. The cars are countable, so X is discrete.
6. The random variable X is the number of texts per week. The texts are countable, so X is discrete.
7. The random variable X is the number of diggs for a web page. The diggs are countable, so X is discrete.
8. The random variable X is the height of a plant. Height can be anywhere within a certain range. Therefore, X is continuous.
9. The random variable X is the number of files infected by a computer virus. The files are countable, so X is discrete.

Check Your Understanding

Example 1 **Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.** 1–4. See margin.

1. the number of pages linked to a Web page
2. the number of stations in a cable package
3. the amount of precipitation in a city per month
4. the number of cars passing through an intersection in a given time interval

Examples 2–5 5. X represents the sum of the values of two spins of the wheel.



a. Construct a relative-frequency table showing the theoretical probabilities.
b. Graph the theoretical probability distribution.
c. Construct a relative-frequency table for 100 trials.
d. Graph the experimental probability distribution.
e. Find the expected value for the sum of two spins of the wheel. 13.5
f. Find the standard deviation for the sum of two spins of the wheel. 4.29

5a–d. See Answer Appendix.

Practice and Problem Solving

Example 1 **Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.** 6–9. See margin.

6. the number of texts received per week
7. the number of diggs (or "likes") for a Web page
8. the height of a plant after a specific amount of time
9. the number of files infected by a computer virus

10a–d. See Answer Appendix.

Examples 2–5 10. **GAME SHOWS** A contestant has won a prize on a game show. The frequency table at the right shows the number of winners for 3200 hypothetical players.

a. Construct a relative-frequency table showing the theoretical probability.
b. Graph the theoretical probability distribution.
c. Construct a relative-frequency table for 50 trials.
d. Graph the experimental probability distribution.
e. Find the expected value. \$922.50
f. Find the standard deviation. 1711.91

Prize, X	Winners
\$100	1120
\$250	800
\$500	480
\$1000	320
\$2500	256
\$5000	128
\$7500	64
\$10,000	32

B 11. **SNOW DAYS** The following probability distribution lists the probable number of snow days per school year at North High School. Use this information to determine the expected number of snow days per year. 3.34

Number of Snow Days Per Year									
Days	0	1	2	3	4	5	6	7	8
Probability	0.1	0.1	0.15	0.15	0.25	0.1	0.08	0.05	0.02

12. **CARDS** In a standard deck of 52 cards, there are 4 different suits.
    a. If jacks = 11, queens = 12, kings = 13, and aces = 1, what is the expected value of a card that is drawn from a standard deck? 7
    b. If you are dealt 7 cards, what is the expected number of spades? 1.75

Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	6–10, 19–24	7, 9	6–10 even
OL Core	7, 9, 11–24	6–10	11–24
BL Advanced	11–24		

13. **RAFFLES** The table shows the probability distribution for a raffle if 100 tickets are sold for $1 each. There is 1 prize for $20, 5 prizes for $10, and 10 prizes for $5.

Distribution of Prizes				
Prize	no prize	$20	$10	$5
Probability	0.84	0.01	0.05	0.10

a. Construct a relative frequency table. **See Answer Appendix.**

b. Find the expected value. **$1.20**

c. Interpret the results you found in part **b**. What can you conclude about the raffle? **See margin.**

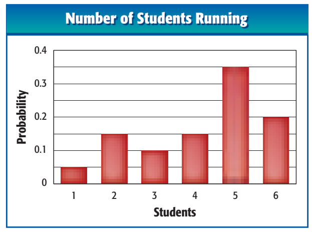
14a–c. See Answer Appendix.

14. **STUDENT GOVERNMENT** Based on previous data, the probability distribution of the number of students running for class president is shown.

a. Determine the expected number of students who will run. Interpret your results.

b. Construct a relative-frequency table for 50 trials.

c. Graph the experimental probability distribution.



15c–d. See Answer Appendix.

15. **BASKETBALL** The distribution below lists the probability of the number of major upsets in the first round of a basketball tournament each year.

Number of Upsets Per Year									
Upsets	0	1	2	3	4	5	6	7	8
Probability	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

a. Determine the expected number of upsets. Interpret your results.

b. Find the standard deviation. **1.90**

c. Construct a relative-frequency table for 50 trials.

d. Graph the experimental probability distribution.

15a. 4.34; Sample answer: The expected number is 4.34, so we can expect there to be 4 upsets. We cannot have 0.34 upsets, so we round to the nearest whole number.

16. **RAFFLES** The French Club sold 500 raffle tickets for $1 each. The first prize ticket will win $100, 2 second prize tickets will each win $10, and 5 third prize tickets each win $5.

a. What is the expected value of a single ticket? **−$0.71**

b. Calculate the standard deviation of the probability distribution. **4.53**

c. **DECISION MAKING** The Glee Club is offering a raffle with a similar expected value and a standard deviation of 2.2. In which raffle should you participate? Explain your reasoning. **See Answer Appendix.**

17. **DECISION MAKING** Carmen is thinking about investing $10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below. Compare the two investments using the expected value and standard deviation. Which investment would you advise Carmen to choose, and why? **See Answer Appendix.**

Fund A	Fund B
30% chance of a $1900 profit	40% chance of a $1600 profit
30% chance of a $600 profit	10% chance of a $900 profit
15% chance of a $200 loss	10% chance of a $300 loss
25% chance of a $500 loss	40% chance of a $400 loss

Additional Answer

13c. Sample answer: The expected value is positive, so a person buying a ticket can expect to win $0.20 even after the cost of the ticket is considered. Thus, a person would want to participate in this raffle. On the other hand, this raffle is guaranteed to lose money for the organizers and they should change the distribution of prizes or not do the raffle.

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4 Assess

Crystal Ball Ask students how they think today's study of probability distributions will help them with their study of the binomial distribution in the next lesson.

Multiple Representations

In Exercise 18, students use a diagram, information organized in a table, symbolic formulas, and verbal analysis to determine geometric probabilities.

WatchOut!

Exercise 14 For Exercise 14, let 1–5 represent *1 student*, 6–20 represent *2 students*, 21–30 *3 students*, and so on. This way, when you run **rand** (100) and get "28", you know that "28" represents *3 students* for that trial.

WatchOut!

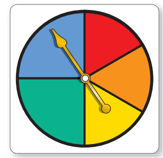
Error Analysis For Exercise 19, one common error would be considering spinning a "2" and a "3", but not considering a "3" and a "2".

Follow-up Students have explored probability.

Ask:

- How can probability be used in decision making? Sample answer: You can use probability to predict the most likely outcomes, and then make a decision based on those findings.

18. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric probability.



a. **Tabular** The spinner shown has a radius of 2.5 inches. Copy and complete the table below.

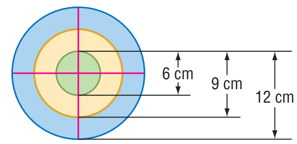
Color	Probability	Sector Area	Total Area	Sector Area / Total Area
red	$\frac{1}{6}$	3.27 in^2	19.63 in^2	0.166
orange	$\frac{1}{6}$	3.27 in^2	19.63 in^2	0.166
yellow	$\frac{1}{6}$	3.27 in^2	19.63 in^2	0.166
green	$\frac{1}{4}$	4.91 in^2	19.63 in^2	0.25
blue	$\frac{1}{4}$	4.91 in^2	19.63 in^2	0.25

18b. Sample answer: The probability is equal to the ratio of the sector area to the total area.

18c. green: $\frac{1}{9}$; yellow: $\frac{1}{3}$; blue: $\frac{5}{9}$

b. **Verbal** Make a conjecture about the relationship between the ratio of the area of the sector to the total area and the probability of the spinner landing on each color.

c. **Analytical** Consider the dartboard shown. Predict the probability of a dart landing in each area of the board. Assume that any dart thrown will land on the board and is equally likely to land at any point on the board.



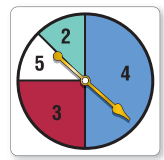
d. **Tabular** Construct a relative-frequency table for throwing 100 darts.

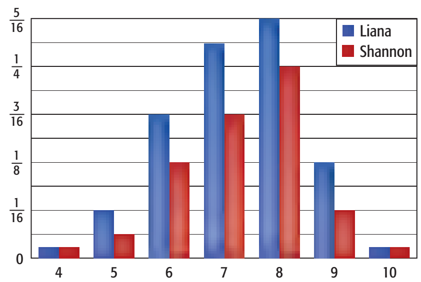
e. **Graphical** Graph the experimental probability distribution.

18d–e. See Answer Appendix.

H.O.T. Problems Use Higher-Order Thinking Skills

19. **ERROR ANALYSIS** Liana and Shannon each created a probability distribution for the sum of two spins on the spinner at the right. Is either of them correct? Explain your reasoning.





19. Sample answer: Liana; Shannon didn't consider every scenario in determining the total probability. For example, in calculating the probability of a sum of 5, she considered spinning a 3 then a 2, but not a 2 then a 3.

20. Sample answer: False; the expected value is 3.5 which is not a possible outcome of a single roll.

20. **REASONING** Determine whether the following statement is *true* or *false*. Explain.

If you roll a die 10 times, you will roll the expected value at least twice.

21. **OPEN ENDED** Create a discrete probability distribution that shows five different outcomes and their associated probabilities. **See Answer Appendix.**

22. **REASONING** Determine whether the following statement is *true* or *false*. Explain.

Random variables that can take on an infinite number of values are continuous.

22. See Answer Appendix.

23. **OPEN ENDED** Provide examples of a discrete probability distribution and a continuous probability distribution. Describe the differences between them. **See Answer Appendix.**

24. **WRITING IN MATH** Compare and contrast two investments that have identical expected values and significantly different standard deviations. **See Answer Appendix**

Differentiated Instruction BL

Extension Write the numbers 1, 1, 1, 2, 2, 3, 4, 4 on eight slips of paper and mix them up in a paper bag. Have students construct a probability distribution for the number that results when one slip is drawn at random from the bag. Then have them take turns drawing (with replacement) and compare the relative frequencies with the theoretical probability distribution.

The theoretical probabilities are $P(1) = \frac{3}{8}$, $P(2) = \frac{1}{4}$, $P(3) = \frac{1}{8}$, and $P(4) = \frac{1}{4}$.

LESSON 12 The Binomial Distribution

Then

- You used the Binomial Theorem.

Now

1. Identify and conduct a binomial experiment.
2. Find probabilities using binomial distributions.

Why?

- Jessica forgot to study for her civics quiz. The quiz consists of five multiple-choice questions with each question having four answer choices. Jessica randomly circles an answer for each question. In order to pass, she needs to answer at least four questions correctly.

Fancy Photography/Veer

NewVocabulary
binomial experiment
binomial distribution

Common Core State Standards
A.APR.5, S.MD.6, S.MD.7

1 Binomial Experiments

Each question on a multiple-choice quiz, like the one described above, can be thought of as a trial with two possible outcomes, correct or incorrect. If Jessica guesses on each question, the probability that she answers a question correctly is the same for all five questions.

Jessica's guessing on each question is an example of a binomial experiment. A **binomial experiment** is a probability experiment that satisfies the following conditions.

KeyConcept Binomial Experiments

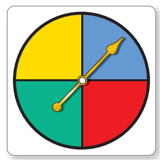
- There is a fixed number of independent trials n.
- Each trial has only two possible outcomes, success or failure.
- The probability of success p is the same in every trial. The probability of failure q is $1 - p$.
- The random variable X is the number of successes in n trials.

Many probability experiments are or can be reduced to binomial experiments.

Example 1 Identify a Binomial Experiment

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

a. The spinner at the right is spun 20 times to see how many times it lands on red.



This experiment can be reduced to a binomial experiment with success being that the spinner lands on red and failure being any other outcome. Thus, a trial is a spin, and the random variable X represents the number of reds spun. The number of trials n is 20, the probability of success p is $\frac{1}{4}$ or 0.25, and the probability of failure q is $1 - 0.25$ or 0.75.

b. One hundred students are randomly asked their favorite food.

This is not a binomial experiment because there are many possible outcomes.

GuidedPractice 1A–1B. See Answer Appendix.

1A. Seventy-five students are randomly asked if they own a car.

1B. Four cards are removed from a deck to see how many aces are selected.

1 Focus

VerticalAlignment

Before Lesson 12 Use the Binomial Theorem.

Lesson 12 Identify and conduct a binomial experiment. Find probabilities using binomial distribution.

After Lesson 12 Use the normal distribution to find probabilities.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- What is the probability that Jessica answers a question correctly? incorrectly? $\frac{1}{4}$; $\frac{3}{4}$
- Are answering two successive questions independent events? yes
- If answering successive questions are independent events, what is the probability that Jessica correctly answers the first two questions? $\frac{1}{16}$ or 0.0625

1 Binomial Experiments

Example 1 shows how identify a binomial experiment. **Example 2** shows how to design a binomial experiment.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Example

1 Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

a. In a class that consists of 12 male and 13 female students, a group of 5 students is randomly chosen to see how many males are selected. This experiment cannot be reduced to a binomial experiment because the events are not independent. The probability of choosing a male student changes after each selection.

b. A school study found that 65% of students have seen an episode of a specific show. Seven students are randomly asked if they have seen an episode of the show. This experiment can be reduced to a binomial experiment. Success is a yes, failure is a no, a trial is asking a student, and the random variable is the number of yeses; $n = 7$, $p = 0.65$, $q = 0.35$.

Additional Examples also in Interactive Classroom PowerPoint® Presentations

Interactive White Board READY

Use the following guidelines when conducting a binomial experiment.

KeyConcept Conducting Binomial Experiments

Step 1 Describe a trial for the situation, and determine the number of trials to be conducted.

Step 2 Define a success, and calculate the theoretical probabilities of success and failure.

Step 3 Describe the random variable X.

Step 4 Design and conduct a simulation to determine the experimental probability.

A binomial experiment can be conducted to compare experimental and theoretical probabilities.

Example 2 Design a Binomial Experiment

Conduct a binomial experiment to determine the probability of drawing an odd-numbered card from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Step 1 A trial is drawing a card from a deck. The number of trials conducted can be any number greater than 0. We will use 52.

Step 2 A success is drawing an odd-numbered card. The odd-numbered cards in a deck are 3, 5, 7, and 9, and they occur once in each of the four suits. Therefore, there are $4 \cdot 4$ or 16 odd-numbered cards in the deck. The probability of drawing an odd-numbered card, or the probability of success, is $\frac{16}{52}$ or $\frac{4}{13}$. The probability of failure is $1 - \frac{4}{13}$ or $\frac{9}{13}$.

Step 3 The random variable X represents the number of odd-numbered cards drawn in 52 trials.

Step 4 Use the random number generator on a calculator to create a simulation. Assign the integers 0–12 to accurately represent the probability data.

Odd-numbered cards	0, 1, 2, 3
Other cards	4, 5, 6, 7, 8, 9, 10, 11, 12

Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Odd-Numbered Card	𝍸 𝍸 ||	12
Other Cards	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40

An odd-numbered card was drawn 12 times, so the experimental probability is $\frac{12}{52}$ or about 23.1%. This is less than the theoretical probability of $\frac{16}{52}$ or about 30.8%.

StudyTip

Random Number Generator To generate all 52 random numbers using a graphing calculator, press MATH ◀ 5 and then enter the desired range followed by the number of trials. For example, type (0, 12, 52) for Example 2.

GuidedPractice

2. Conduct a binomial experiment to determine the probability of drawing an even-numbered card from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment. **See Answer Appendix.**

44 | Lesson 12 | The Binomial Distribution

2 Binomial Distribution

In the binomial experiment in Example 2, there were 12 successes in 52 trials. If you conducted that same experiment again, there may be any number of successes from 0 to 52. This situation can be represented by a binomial distribution. A **binomial distribution** is a frequency distribution of the probability of each value of X, where the random variable X represents the number of successes in n trials. Because X is a discrete random variable, a binomial distribution is a *discrete probability distribution.*

The probabilities in a binomial distribution can be calculated using the following formula.

StudyTip

Binomial Probability Formula In the Binomial Probability Formula, X represents the number of successes in n trials. Thus, the exponent for q, $n - X$, represents the number of failures in n trials.

KeyConcept Binomial Probability Formula

The probability of X successes in n independent trials is

$$P(X) = {}_nC_X\, p^X q^{n-X},$$

where p is the probability of success of an individual trial and q is the probability of failure on that same individual trial ($q = 1 - p$).

Notice that the Binomial Probability Formula is an adaptation of the Binomial Theorem you have already studied. The expression ${}_nC_X\, p^X q^{n-X}$ represents the $p^X q^{n-X}$ term in the binomial expansion of $(p + q)^n$.

Standardized Test Example 3 Find a Probability

PT

Garrett is selling items from a catalog to raise money for school. He has a 40% chance of making a sale each time he solicits a potential customer. Garrett asks 10 people to purchase an item. Find the probability that 6 people make a purchase.

A 8.6% **B** 11.2% **C** 24% **D** 40%

Read the Test Item

We need to find the probability that 6 people purchase an item. A success is making a sale, so $p = 0.4$, $q = 1 - 0.4$ or 0.6, and $n = 10$.

Solve the Test Item

$P(X) = {}_nC_X\, p^X q^{n-X}$ — Binomial Probability Formula

$P(6) = {}_{10}C_6\,(0.4)^6\,(0.6)^{10-6}$ — $n = 10$, $X = 6$, $p = 0.4$, and $q = 0.6$

≈ 0.111 — Simplify.

The probability of Garrett making six sales is about 0.111 or 11.1%. So, the correct answer is B.

Test-TakingTip

Failures A common error when using the Binomial Probability Formula is to focus on only the successes and to forget the failures. Notice that $P(6) \neq (0.4)^6$ and $P(6) \neq {}_{10}C_6(0.4)^6$.

GuidedPractice

3. **TELEMARKETING** At Jenny's telemarketing job, 15% of the calls that she makes to potential customers result in a sale. She makes 20 calls in a given hour. What is the probability that 5 calls result in a sale? **H**

F 6.7% **G** 8.3% **H** 10.3% **J** 11.9%

If, on average, 40% of the people Garrett solicits make a purchase and he solicits 10 people, he can probably expect to make 10(0.40) or 4 sales. This value represents the mean of the binomial distribution. In general, the mean of a binomial distribution can be calculated by the following formula.

KeyConcept Mean of a Binomial Distribution

The mean μ of a binomial distribution is given by $\mu = np$, where n is the number of trials and p is the probability of success.

2 Binomial Distribution

Example 3 shows how to find a probability by using the Binomial Probability Formula. **Example 4** shows how to create a probability distribution and find expected value.

Additional Examples

2 Conduct a binomial experiment to determine the probability of two fair coins both landing on heads. Then compare the experimental and theoretical probabilities of the experiment. See margin.

3 **CANDY** A candy company produces strawberry and vanilla flavored candies, 65% of which are strawberry on average. The production line mixes the candies randomly and packages 15 per bag. What is the probability that 7 candies in a bag are strawberry? 0.071 or 7.1%

Additional Answer (Additional Example)

2. Sample answer:

Step 1 A trial is tossing two coins. The simulation will consist of 20 trials.

Step 2 A success is both coins landing on heads. The probability of success is $\frac{1}{4}$ and the probability of failure is $\frac{3}{4}$.

Step 3 The random variable X represents the number of times both coins land on heads in 20 trials.

Step 4 Use a random number generator. Let 0 represent both coins landing on heads. Let 1–3 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Two Heads	𝍸 I	6
Other Outcomes	𝍸 𝍸 IIII	14

The experimental probability is $\frac{6}{20}$ or 30%. This is greater than the theoretical probability of $\frac{1}{4}$ or 25%.

Additional Example

4 **FAMILY** A family has 4 children. Assume that when a child is born, there is a 50% chance that the child is female.

a. Determine the probabilities associated with the number of daughters in the family by calculating the probability distribution. 0 daughters, 0.063 or 6.3%; 1 daughter, 0.25 or 25%; 2 daughters, 0.375 or 37.5%; 3 daughters, 0.25 or 25%; 4 daughters, 0.063 or 6.3%

b. What is the probability that the family has at least 3 daughters? 0.313 or 31.3%

c. How many daughters should the family expect to have? 2

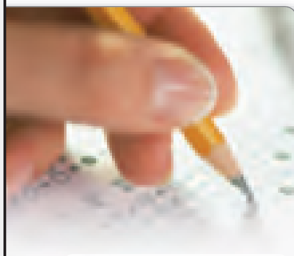
Focus on Mathematical Content

Binomial Distributions A binomial distribution is called a *two-parameter* distribution, with the parameters being the number of trials n and the probability of one of the outcomes p. Probabilities for a binomial distribution are determined once the parameters are specified.

Teach with Tech

Web Search Have students search the Web for binomial distribution applets. Allow students to create and explore binomial probability histograms, and search for additional examples of binomial experiments.

You can find the probability distribution for a binomial experiment by fully expanding the binomial $(p + q)^n$. A probability distribution can be helpful when solving for problems that allow multiple numbers of successes.



Real-WorldLink

ACT The math portion of the ACT college entrance exam includes 60 multiple-choice questions that each have five answer choices.

Source: ACT

Real-World Example 4 Full Probability Distribution

TEST TAKING **Refer to the beginning of the lesson.**

a. Determine the probabilities associated with the number of questions Jessica answered correctly by calculating the probability distribution.

If there are four answer choices for each question, then the probability that Jessica guesses and answers a question correctly is $\frac{1}{4}$ or 0.25. In this binomial experiment, $n = 5$, $p = 0.25$, and $q = 1 - 0.25$ or 0.75. Expand the binomial $(p + q)^n$.

$$(p + q)^n$$

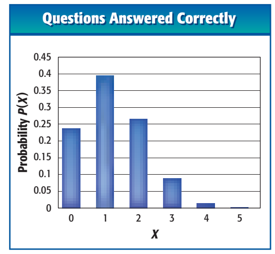
$$= 1p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + 1q^5$$

$$= (0.25)^5 + 5(0.25)^4(0.75) + 10(0.25)^3(0.75)^2 + 10(0.25)^2(0.75)^3 + 5(0.25)(0.75)^4 + (0.75)^5$$

$$\approx 0.001 + 0.015 + 0.089 + 0.264 + 0.396 + 0.237$$

0.1%	1.5%	8.9%	26.4%	39.6%	23.7%
5 correct	4 correct	3 correct	2 correct	1 correct	0 correct

The graph shows the binomial probability distribution for the number of questions that Jessica answered correctly.



b. What is the probability that Jessica passes the quiz?

Jessica must answer at least four questions correctly to pass the quiz. The probability that Jessica answers *at least* four correct is the sum of the probabilities that she answers four or five correct is about 1.5% + 0.1% or 1.6%. So, Jessica has about a 1.6% chance of passing, which is not likely.

StudyTip

Mean and Expected Value The mean of a binomial distribution can be any positive rational number. The expected value of a binomial distribution, however, should be rounded to the nearest whole number since a fraction of a success is not possible.

c. How many questions should Jessica expect to answer correctly?

Find the mean.

$\mu = np$ — Mean of a Binomial Distribution

$= 5(0.25)$ or 1.25 — $n = 5$ and $p = 0.25$

The mean of the distribution is 1.25. On average, Jessica should expect to answer one question correctly when she guesses on five.

GuidedPractice

4A. 0 questions, 0.031 or 3.1%; 1 question, 0.156 or 15.6%; 2 questions, 0.313 or 31.3%; 3 questions, 0.313 or 31.3%; 4 questions, 0.156 or 15.6%; 5 questions, 0.031 or 3.1%

4. **TEST TAKING** Suppose Jessica's civics quiz consisted of five true-or-false questions instead of multiple-choice questions.

A. Determine the probabilities associated with the number of answers Jessica answered correctly by calculating the probability distribution.

B. What is the probability that Jessica passes the quiz? **0.187 or 18.7%**

C. How many questions should Jessica expect to answer correctly? **3**

Stockbyte/Getty Images

DifferentiatedInstruction AL OL BL ELL

Kinesthetic Learners Have students in small groups do a binomial experiment by tossing a ball into a wastebasket about 20 times to establish the probability of scoring a goal. Then have them find the probability that they will score exactly 4 goals in 8 tries.

46 | Lesson 12 | The Binomial Distribution

Check Your Understanding

Example 1 **1–4. See margin.**

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

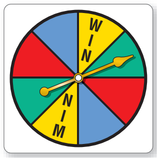
1. A study finds that 58% of people have pets. You ask 100 people how many pets they have.
2. You roll a die 15 times and find the sum of all of the rolls.
3. A poll found that 72% of students plan on going to the homecoming dance. You ask 30 students if they are going to the homecoming dance.

Example 2

4. Conduct a binomial experiment to determine the probability of drawing an ace or a king from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Example 3

5. **GAMES** Aiden has earned five spins of the wheel on the right. He will receive a prize each time the spinner lands on WIN. What is the probability that he receives three prizes? **D**



A 4.2% **C** 7.1%
B 5.8% **D** 8.8%

Example 4

6. **PARKING** A poll at Steve's high school was taken to see if students are in favor of spending class money to expand the junior-senior parking lot. Steve surveyed 6 random students from the population.

Expand the Parking Lot	
favor	85%
oppose	15%

6a. See Answer Appendix.

a. Determine the probabilities associated with the number of students that Steve asked who are in favor of expanding the parking lot by calculating the probability distribution.
b. What is the probability that no more than 2 people are in favor of expanding the parking lot? **0.00589 or 0.589%**
c. How many students should Steve expect to find who are in favor of expanding the parking lot? **5**

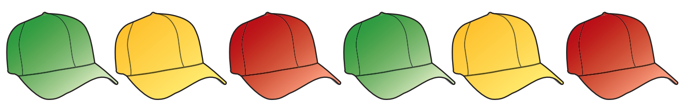
Practice and Problem Solving

Example 1

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

7–9. See Answer Appendix.

7. There is a 35% chance that it rains each day in a given month. You record the number of days that it rains for that month.
8. A survey found that on a scale of 1 to 10, a movie received a 7.8 rating. A movie theater employee asks 200 patrons to rate the movie on a scale of 1 to 10.
9. A ball is hidden under one of the hats shown below. A hat is chosen, one at a time, until the ball is found.



Example 2

10. **DICE** Conduct a binomial experiment to determine the probability of rolling a 7 with two dice. Then compare the experimental and theoretical probabilities of the experiment.

10–11. See Answer Appendix.

11. **MARBLES** Conduct a binomial experiment to determine the probability of pulling a red marble from the bag. Then compare the experimental and theoretical probabilities of the experiment.

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Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	7–23, 43–46	7–23 odd	8–22 even, 43–46
OL Core	7–41 odd, 43–46	7–23	24–41, 43–46
BL Advanced	24–46		

3 Practice

Formative Assessment

Use Exercises 1–6 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Additional Answers

1. This experiment cannot be reduced to a binomial experiment because there are more than two possible outcomes.
2. This experiment cannot be reduced to a binomial experiment because there are more than two possible outcomes.
3. This experiment can be reduced to a binomial experiment. Success is yes, failure is no, a trial is asking a student, and the random variable is the number of yeses; $n = 30$, $p = 0.72$, $q = 0.28$.
4. Sample answer:

Step 1 A trial is drawing a card from a deck. The simulation will consist of 20 trials.

Step 2 A success is drawing an ace or a king. The probability of success is $\frac{2}{13}$ and the probability of failure is $\frac{11}{13}$.

Step 3 The random variable X represents the number of aces or kings drawn in 20 trials.

Step 4 Use a random number generator. Let 0–1 represent drawing an ace or a king. Let 2–12 represent drawing all other cards. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Ace or King	𝍸 I	6
Other Outcomes	𝍸 𝍸 IIII	14

The experimental probability is $\frac{6}{20}$ or 30%. This is greater than the theoretical probability of $\frac{2}{13}$ or about 15.4%.

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WatchOut!

Preventing Errors Encourage students to think about the reasonableness of their solution. Emphasize the fact that a randomly selected item will most likely be clustered around the mean. The further that a value is from the mean, the lower the percentage should be.

Additional Answers

21a. 0 own a laptop, 0.0006 or 0.06%; 1 owns a laptop, 0.007 or 0.7%, 2 own a laptop, 0.0343 or 3.43%; 3 own a laptop, 0.0991 or 9.91%; 4 own a laptop, 0.1878 or 18.78%; 5 own a laptop, 0.2441 or 24.41%; 6 own a laptop, 0.2204 or 22.04%; 7 own a laptop, 0.1364 or 13.64%; 8 own a laptop, 0.0554 or 5.54%; 9 own a laptop, 0.0133 or 1.33%; 10 own a laptop, 0.0014 or 0.14%

22a. 0 play at least 1 sport, 0.00006 or 0.006%; 1 plays at least 1 sport, 0.00154 or 0.154%; 2 play at least 1 sport, 0.01536 or 1.536%; 3 play at least 1 sport, 0.08192 or 8.192%; 4 play at least 1 sport, 0.24576 or 24.576%; 5 play at least 1 sport, 0.39322 or 39.322%; 6 play at least 1 sport, 0.26214 or 26.214%

12. SPINNER Conduct a binomial experiment to determine the probability of the spinner stopping on an even number. Then compare the experimental and theoretical probabilities of the experiment.

12–13. See Answer Appendix.

13. CARDS Conduct a binomial experiment to determine the probability of drawing a face card out of a standard deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Example 3

14. MP3 PLAYERS According to a recent survey, 85% of high school students own an MP3 player. What is the probability that 6 out of 10 random high school students own an MP3 player? **0.04 or 4%**

15. CARS According to a recent survey, 92% of high school seniors own their own car. What is the probability that 10 out of 12 random high school students own their own car? **0.183 or 18.3%**

16. SENIOR PROM According to a recent survey, 25% of high school upperclassmen think that the junior-senior prom is the most important event of the school year. What is the probability that 3 out of 15 random high school upperclassmen think this way? **0.225 or 22.5%**

17. FOOTBALL A certain football team has won 75.7% of their games. Find the probability that they win 7 of their next 12 games. **0.096 or 9.6%**

18. GARDENING Peter is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Peter knows that the probability of having a blue flower is 75%. What is the probability that 20 of the flowers will be blue? **0.132 or 13.2%**

19. FOOTBALL A field goal kicker is accurate 75% of the time from within 35 yards. What is the probability that he makes exactly 7 of his next 10 kicks from within 35 yards? **0.25 or 25%**

Range (yd)	Accuracy (%)
0–35	75
35–45	62
45+	20

20. BABIES Mr. and Mrs. Davis are planning to have 3 children. The probability of each child being a boy is 50%. What is the probability that they will have 2 boys? **0.375 or 37.5%**

Example 4

21. LAPTOPS According to a recent survey, 52% of high school students own a laptop. Ten random students are chosen.

a. Determine the probabilities associated with the number of students that own a laptop by calculating the probability distribution. **See margin.**

b. What is the probability that at least 8 of the 10 students own a laptop? **about 7%**

c. How many students should you expect to own a laptop? **5**

22. ATHLETICS A survey was taken to see the percent of students that participate in sports for their school. Six random students are chosen.

Student Athletics	
0 sports	20%
1 sport	55%
2 sports	20%
3+ sports	5%

a. Determine the probabilities associated with the number of students playing in at least one sport by calculating the probability distribution. **See margin.**

b. What is the probability that no more than 2 of the students participated in a sport? **0.01696 or 1.696%**

c. How many students should you expect to have participated in at least one sport? **5**

23. **MUSIC** An online poll showed that 57% of adults still own vinyl records. Moe surveyed 8 random adults from the population.

a. Determine the probabilities associated with the number of adults that still own vinyl records by calculating the probability distribution. **See margin.**

b. What is the probability that no less than 6 of the people surveyed still own vinyl records? **0.256 or 25.6%**

c. How many people should Moe expect to still own vinyl records? **5**

A binomial distribution has a 60% rate of success. There are 18 trials.

B **24.** What is the probability that there will be at least 12 successes? **0.374 or 37.4%**

25. What is the probability that there will be 12 failures? **0.015 or 1.5%**

26. What is the expected number of successes? **11**

27. **DECISION MAKING** Six roommates randomly select someone to wash the dishes each day.

a. What is the probability that the same person has to wash the dishes 3 times in a given week? **about 7.8%**

b. What method can the roommates use to select who washes the dishes each day? **b. Sample answer: They can roll a six-sided die.**

28. **DECISION MAKING** A committee of five people randomly selects someone to take the notes of each meeting.

a. What is the probability that a person takes notes less than twice in 10 meetings? **about 37.6%**

b. What method can the committee use to select the notetaker each meeting? **b. Sample answer: They can roll a six-sided die. If a six is rolled, they roll again.**

c. If the method described in part **b** results in the same person being notetaker for nine straight meetings, would this result cause you to question the method? **See margin.**

Each binomial distribution has n trials and p probability of success. Determine the most likely number of successes.

29. $n = 8, p = 0.6$ **5**

30. $n = 10, p = 0.4$ **4**

31. $n = 6, p = 0.8$ **5**

32. $n = 12, p = 0.55$ **7**

33. $n = 9, p = 0.75$ **7**

34. $n = 11, p = 0.35$ **4**

35. **SWEEPSTAKES** A beverage company is having a sweepstakes. The probabilities of winning selected prizes are shown at the right. If Ernesto purchases 8 beverages, what is the probability that he wins at least one prize? **0.603 or 60.3%**

Odds of Winning	
beverage	1 in 10
CD	1 in 200
hat	1 in 250
MP3 player	1 in 20,000
car	1 in 25,000,000

Each binomial distribution has n trials and p probability of success. Determine the probability of s successes.

36. $n = 8, p = 0.3, s \geq 2$ **0.745 or 74.5%**

37. $n = 10, p = 0.2, s > 2$ **0.322 or 32.2%**

38. $n = 6, p = 0.6, s \leq 4$ **0.767 or 76.7%**

39. $n = 9, p = 0.25, s \leq 5$ **0.99 or 99%**

40. $n = 10, p = 0.75, s \geq 8$ **0.526 or 52.6%**

41. $n = 12, p = 0.1, s < 3$ **0.889 or 88.9%**

H.O.T. Problems Use Higher-Order Thinking Skills

C **42.** **CHALLENGE** A poll of students determined that 88% wanted to go to college. Eight random students are chosen. The probability that at least x students want to go to college is about 0.752 or 75.2%. Solve for x. **7**

43–46. See margin.

43. **WRITING IN MATH** What should you consider when using a binomial distribution to make a decision?

44. **OPEN ENDED** Describe a real-world setting within your school or community activities that seems to fit a binomial distribution. Identify the key components of your setting that connect to binomial distributions.

45. **WRITING IN MATH** Describe how binomial distributions are connected to Pascal's triangle.

46. **WRITING IN MATH** Explain the relationship between a binomial experiment and a binomial distribution.

DifferentiatedInstruction OL BL

Extension Have students look at a histogram of a binomial distribution created on a graphing calculator for $n = 20$ and $p = 0.5$. Point out that they can use the histogram to answer a question like "What is the probability that x is at least 12?" by adding the heights of all the bars beginning with $x = 12$. Then have them find the probability that x is at least 12. 0.252 or 25.2%

4 Assess

Name the Math Have each student use his or her own family or the family of a friend (for example, 2 boys and a girl), and find the probabilities for that particular group of siblings. Then have students explain the steps they used.

Additional Answers

23a. 0 own vinyl records, 0.001 or 0.1%; 1 owns vinyl records, 0.012 or 1.2%; 2 own vinyl records, 0.058 or 5.8%; 3 own vinyl records, 0.152 or 15.2%; 4 own vinyl records, 0.253 or 25.3%; 5 own vinyl records, 0.268 or 26.8%; 6 own vinyl records, 0.178 or 17.8%; 7 own vinyl records, 0.067 or 6.7%; 8 own vinyl records, 0.011 or 1.1%

28c. Sample answer: While it is possible for the same person to be chosen nine straight times, it is rather unlikely. The fairness of the die should be questioned.

43. Sample answer: You should consider the type of situation for which the binomial distribution is being used. For example, if a binomial distribution is being used to predict outcomes regarding an athletic event, the probabilities of success and failure could change due to other variables such as weather conditions or player health. So, binomial distributions should be used cautiously when making decisions invoiving events that are not completely random.

44. Sample answer: During May and June, lunches are held outside, weather permitting. Also during this time, there has historically been a 15% chance of rain. So, to determine the probability of not having rain for at least 24 of these 28 days, the binomial distribution would use $p = 0.85$, $q = 0.15$, and $n = 28$.

45. Sample answer: A full binomial distribution can be determined by expanding the binomial, which itself utilizes Pascal's triangle.

46. Sample answer: A binomial distribution shows the probabilities of the outcomes of a binomial experiment.

1 Focus

Vertical Alignment

Before Lesson 13 Analyze probability distributions of data.

Lesson 13 Use the Empirical Rule to analyze normally distributed variables.

After Lesson 13 Understand how to apply the standard normal distribution.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- What dates are one standard deviation from the mean? April 11 and May 1
- What do you think the likelihood is of a Swiss cherry tree flowering before April 1? about 2.5%

LESSON 13 The Normal Distribution

Then

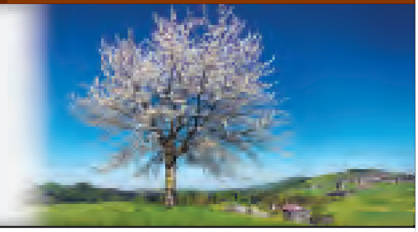
- You constructed and analyzed discrete probability distributions.

Now

1. Use the Empirical Rule to analyze normally distributed variables.
2. Apply the standard normal distribution and z-values.

Why?

- Extensive observations of Swiss cherry trees found that the mean flowering date is April 21 with a standard deviation of about 10 days. Therefore, 95% of the time, a Swiss cherry tree will have a flowering date between April 1 and May 3.

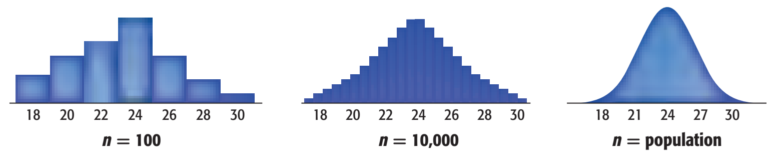


Frank Krahmer/Photographer's Choice RF/Getty Images

New Vocabulary
normal distribution
Empirical Rule
z-value
standard normal distribution

Common Core State Standards
S.ID.4

1 The Normal Distribution Distributions of mileages of different sample sizes of cars are shown below. As the sample size increases, the distributions become more and more symmetrical and resemble the curve at the right, due to the Law of Large Numbers.



The curve at the right is a **normal distribution**, a continuous, symmetric, bell-shaped distribution of a random variable. It is the most common *continuous probability distribution*. The characteristics of the normal distribution are as follows.

Key Concept The Normal Distribution

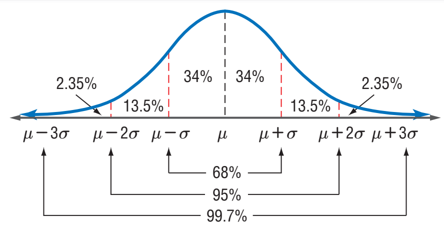
- The graph of the curve is continuous, bell-shaped, and symmetric with respect to the mean.
- The mean, median, and mode are equal and located at the center.
- The curve approaches, but never touches, the x-axis.
- The total area under the curve is equal to 1 or 100%.

The area under the normal curve represents the amount of data within a certain interval or the probability that a random data value falls within that interval. The **Empirical Rule** can be used to determine the area under the normal curve at specific intervals.

Key Concept The Empirical Rule

In a normal distribution with mean μ and standard deviation σ,

- approximately 68% of the data fall within 1σ of the mean,
- approximately 95% of the data fall within 2σ of the mean, and
- approximately 99.7% of the data fall within 3σ of the mean.



Study Tip

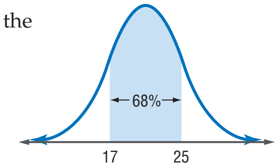
Normal Distributions In all of these cases, the number of data values must be large for the distribution to be approximately normal.

Example 1 Use the Empirical Rule to Analyze Data

A normal distribution has a mean of 21 and a standard deviation of 4.

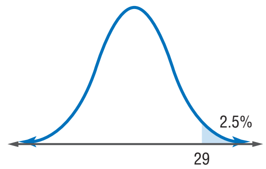
a. Find the range of values that represent the middle 68% of the distribution.

The middle 68% of data in a normal distribution is the range from $\mu - \sigma$ to $\mu + \sigma$. Therefore, the range of values in the middle 68% is $17 < X < 25$.



b. What percent of the data will be greater than 29?

29 is 2σ more than μ. 95% of the data fall between $\mu - 2\sigma$ and $\mu + 2\sigma$, so the remaining data values represented by the two tails covers 5% of the distribution. We are only concerned with the upper tail, so 2.5% of the data will be greater than 29.



Guided Practice

1. A normal distribution has a mean of 8.2 and a standard deviation of 1.3.
   - A. Find the range of values that represent the middle 95% of the distribution. $5.6 < X < 10.8$
   - B. What percent of the data will be less than 4.3? 0.15%

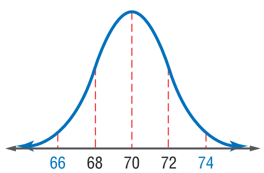
Real-World Example 2 Use the Empirical Rule to Analyze a Distribution

HEIGHTS The heights of 1800 adults are normally distributed with a mean of 70 inches and a standard deviation of 2 inches.

a. About how many adults are between 66 and 74 inches?

66 and 74 are 2σ away from the mean. Therefore, about 95% of the data are between 66 and 74.

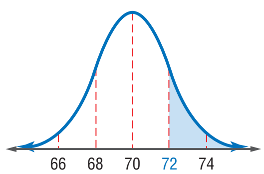
Since $1800 \times 95\% = 1710$, we know that about 1710 of the adults are between 66 and 74 inches tall.



b. What is the probability that a random adult is more than 72 inches tall?

From the curve, values greater than 72 are more than 1σ from the mean. 13.5% are between 1σ and 2σ, 2.35% are between 2σ and 3σ, and 0.15% are greater than 3σ.

So, the probability that an adult selected at random has a height greater than 72 inches is $13.5 + 2.35 + 0.15$ or 16%.



Real-World Link

While the average adult American male is 5 feet 10 inches, the average height of adult males in the Netherlands is the highest worldwide, at almost 6 feet 1 inch.

Source: Eurostats Statistical Yearbook

Guided Practice

2. **NETWORKING SITES** The number of friends per member in a sample of 820 members is normally distributed with a mean of 38 and a standard deviation of 12.
   - A. About how many members have between 26 and 50 friends? 558
   - B. What is the probability that a random member will have more than 14 friends? 97.5%

The Empirical Rule

Example 1 shows how to use the Empirical Rule to analyze data. **Example 2** shows how to use the Empirical Rule to analyze a distribution.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Examples

1 A normal distribution has a mean of 45.1 and a standard deviation of 9.6.

- **a.** Find the values that represent the middle 99.7% of the distribution. $16.3 < X < 73.9$
- **b.** What percent of the data will be greater than 54.7? 16%

2 **PACKAGING** Students counted the number of candies in 100 small packages. They found that the number of candies per package was normally distributed with a mean of 23 candies per package and a standard deviation of 1 piece of candy.

- **a.** About how many packages have between 22 and 24 candies? about 68 packages
- **b.** What is the probability that a package selected at random has more than 25 candies? about 2.5%

Teach with Tech

Instant Messaging Have students work in pairs and send each other questions about a normal distribution (for example: "What percent of the values are within two standard deviations above the mean?") Students should reply with the answers and check each others' work.

Focus on Mathematical Content

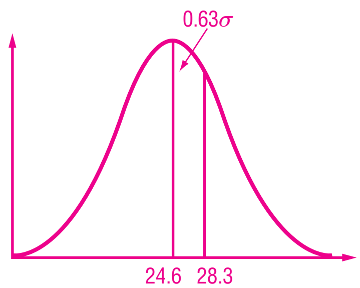
Normal Distributions The graphs of all normally distributed variables have essentially the same shape. With appropriate labeling of the mean and the points that are one standard deviation from the mean, the same normal curve can represent any normal distribution.

The Standard Normal Distribution

Example 3 shows how to use *z*-values to locate position. **Example 4** shows how to use *z*-values and the standard normal distribution to find probabilities.

Additional Example

3 Find σ if $X = 28.3$, $\mu = 24.6$, and $z = 0.63$. Indicate the position of X in the distribution. 5.87; 0.63 standard deviations greater than the mean



Tips for New Teachers

Relative Position Like percentiles, *z*-values can be used to compare the relative positions of two values in two different sets of data.

2 Standard Normal Distribution

The Empirical Rule is only useful for evaluating specific values, such as $\mu + \sigma$. Once the data set is *standardized*, however, any data value can be evaluated. Data are standardized by converting them to *z*-values, also known as *z*-scores. The **z-value** represents the number of standard deviations that a given data value is from the mean. Therefore, *z*-values can be used to determine the position of any data value within a set of data.

KeyConcept Formula for *z*-Values

The *z*-value for a data value X in a set of normally distributed data is given by $z = \frac{X - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation.

StudyTip

Symmetry The normal distribution is symmetrical, so when you are asked for the middle or outside set of data, the *z*-values will be opposites.

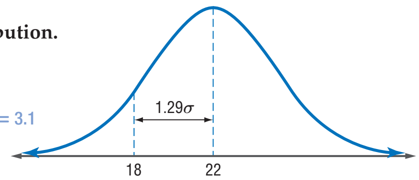
Example 3 Use *z*-Values to Locate Position

Find z if $X = 18$, $\mu = 22$, and $\sigma = 3.1$. Indicate the position of X in the distribution.

$z = \frac{X - \mu}{\sigma}$ — Formula for *z*-values

$= \frac{18 - 22}{3.1}$ — $X = 18$, $\mu = 22$, $\sigma = 3.1$

≈ -1.29 — Simplify.



The *z*-value that corresponds to $X = 18$ is approximately -1.29. Therefore, 18 is about 1.29 standard deviations less than the mean of the distribution.

GuidedPractice

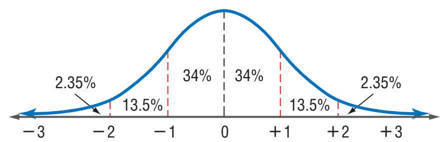
3. Find X if $\mu = 39$, $\sigma = 8.2$, and $z = 0.73$. Indicate the position of X in the distribution. **See margin.**

Any combination of mean and standard deviation is possible for a normally distributed set of data. As a result, there are infinitely many normal probability distributions. This makes comparing two individual distributions difficult. Different distributions *can* be compared, however, once they are standardized using *z*-values. The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

StudyTip

Standard Normal Distribution The standard normal distribution is the set of all *z*-values.

KeyConcept Characteristics of the Standard Normal Distribution



- The total area under the curve is equal to 1 or 100%.
- Almost all of the area is between $z = -3$ and $z = 3$.
- The distribution is symmetric.
- The mean is 0, and the standard deviation is 1.

The standard normal distribution allows us to assign actual areas to the intervals created by *z*-values. The area under the normal curve corresponds to the proportion of data values in an interval as well as the probability of a random data value falling within the interval. For example, the area between $z = 0$ and $z = 1$ is 0.34. Therefore, the probability of a *z*-value being in this interval is 34%.

DifferentiatedInstruction AL OL

If students need an aid in drawing a normal curve,

Then it may be useful to know that the concave side of the curve switches from facing downward to facing upward at points that are one standard deviation from the mean. Drawing a normal curve for each problem can help students with their estimates.

Real-WorldLink

Video Uploading According to a recent study, 52% of people who said they upload videos to the Web do it through sites such as Facebook and MySpace. The rest use video-sharing sites like YouTube and Google Video.

Source: *Pew Internet and American Life Project*

Real-World Example 4 Find Probabilities

VIDEOS The number of videos uploaded daily to a video sharing site is normally distributed with $\mu = 181{,}099$ videos and $\sigma = 35{,}644$ videos. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

a. $P(180{,}000 < X < 200{,}000)$

The question is asking for the percent of days when between 180,000 and 200,000 videos are uploaded. First, find the corresponding z-values for $X = 180{,}000$ and $X = 200{,}000$.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$

$$= \frac{180{,}000 - 181{,}099}{35{,}644} \text{ or about } -0.03 \quad X = 180{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

Use 200,000 to find the other z-value.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$

$$= \frac{200{,}000 - 181{,}099}{35{,}644} \text{ or about } 0.53 \quad X = 200{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

The range of z-values that c s to $180{,}000 < X < 200{,}000$ is $-0.03 < z < 0.53$. Find the area under the normal curve within this interval.

You can use a graphing calculator to display the area that corresponds to any z-value by selecting [2nd] **[DISTR]**. Then, under the **DRAW** menu, select **ShadeNorm(lower *z* value, upper *z* value)**. The area between $z = -0.03$ and $z = 0.53$ is about 0.21 as shown in the graph.

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Therefore, about 21% of the time, there will be between 180,000 and 200,000 video uploads on a given day.

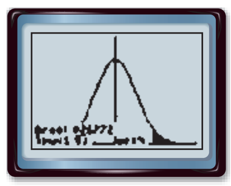
b. $P(X > 250{,}000)$

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$

$$= \frac{250{,}000 - 181{,}099}{35{,}644} \text{ or about } 1.93 \quad X = 250{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

Using a graphing calculator, you can find the area between $z = 1.93$ and $z = 4$ to be about 0.027.

Therefore, the probability that more than 250,000 videos will be uploaded is about 2.7%.



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StudyTip

Range of *z*-values The majority of data values are within $\pm 4\sigma$ of the mean, so setting the maximum value of *z* equal to 4 is sufficient in part **b**. Use the window [−4, 4] by [0, 0.5] when using **ShadeNorm**.

GuidedPractice

4. TIRES The life spans of a certain tread of tire are normally distributed with $\mu = 31{,}066$ miles and $\sigma = 1644$ miles. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

A. $P(30{,}000 < X < 32{,}000)$ **B.** $P(X > 35{,}000)$ **4A–B. See margin.**

Another method for calculating the area between two z-values is [2nd] **[DISTR]** **normalcdf**(*lower z value, upper z value*).

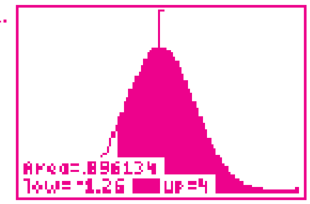
Additional Example

4 HEALTH The cholesterol levels for adult males of a specific racial group are normally distributed with a mean of 158.3 and a standard deviation of 6.6. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

a. $P(X > 150)$ 0.90

b. $P(145 < X < 165)$ 0.82

a.



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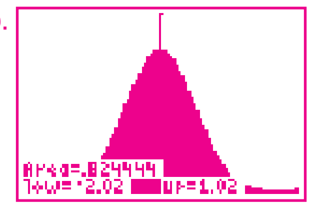
b.



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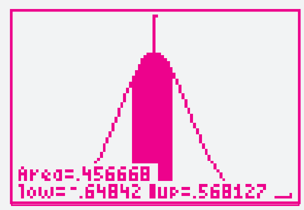
Additional Answers (Guided Practice)

4A. 0.46



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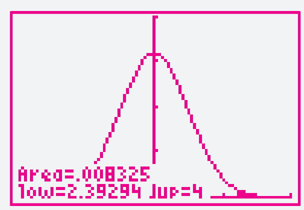
4B. 0.008



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Additional Answer (Guided Practice)

3. $X \approx 45.0$

Tips for New Teachers

ClrDraw Have students select [2nd] **[DRAW]** 1 to clear any previous drawings.

Tips for New Teachers

Sign Usage In a continuous distribution, there is no difference between $P(X \geq c)$ and $P(X > c)$ because the probability that x is equal to c is zero.

3 Practice

Formative Assessment

Use Exercises 1–8 to check for understanding.

Use the chart on the bottom of this page to customize assignments for your students.

Tips for New Teachers

Tell students that when data are normally distributed, they have the characteristics of a normal distribution and can be analyzed as such.

> **WatchOut!**
>
> **Rounding** For Exercise 17a, if students round to 35.1% before multiplying by 20,000, they will get an answer of 7020 batteries.

Additional Answers

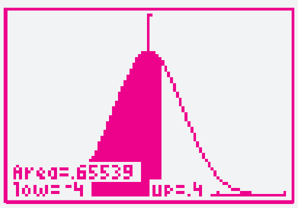
4. −0.70; 0.70 standard deviations less than the mean

5. 2.05; 2.05 standard deviations greater than the mean

6. 59.8; 1.38 standard deviations less than the mean

7. 3.08; 2.40 standard deviations less than the mean

8a.



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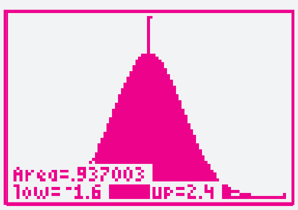
8b.



[−4, 4] scl: 1 by [0, .5] scl: 0.125

12. 2.5; 2.5 standard deviations greater than the mean

13. −1.33; 1.33 standard deviations less than the mean

14. 51.8; 0.92 standard deviations less than the mean

15. 177.7; 1.73 standard deviations greater than the mean

Check Your Understanding

Example 1 **A normal distribution has a mean of 416 and a standard deviation of 55.**

1. Find the range of values that represent the middle 99.7% of the distribution. **$251 < X < 581$**
2. What percent of the data will be less than 361? **16%**

Example 2

3. **TEXTING** The number of texts sent per day by a sample of 811 teens is normally distributed with a mean of 38 and a standard deviation of 7.
   a. About how many teens sent between 24 and 38 texts? **about 386 teens**
   b. What is the probability that a teen selected at random sent less than 818 texts? **84%**

Example 3 **Find the missing variable. Indicate the position of X in the distribution.** **4–7. See margin.**

4. z if $\mu = 89$, $X = 81$, and $\sigma = 11.5$
5. z if $\mu = 13.3$, $X = 17.2$, and $\sigma = 1.9$
6. X if $z = -1.38$, $\mu = 68.9$, and $\sigma = 6.6$
7. σ if $\mu = 21.1$, $X = 13.7$, and $z = -2.40$

Example 4

8. **CONCERTS** The number of concerts attended per year by a sample of 925 teens is normally distributed with a mean of 1.8 and a standard deviation of 0.5. Find each probability. Then use a graphing calculator to sketch the area under each curve. **a–b. See margin for graphs.**
   a. $P(X < 2)$ **65.5%**
   b. $P(1 < X < 3)$ **93.7%**

Practice and Problem Solving

Example 1 **A normal distribution has a mean of 29.3 and a standard deviation of 6.7.**

9. Find the range of values that represent the outside 5% of the distribution. **$X < 15.9$ or $X > 42.7$**
10. What percent of the data will be between 22.6 and 42.7? **81.5%**

Example 2

11. **GYMS** The number of visits to a gym per year by a sample of 522 members is normally distributed with a mean of 88 and a standard deviation of 19.
    a. About how many members went to the gym at least 50 times? **about 509 members**
    b. What is the probability that a member selected at random went to the gym more than 145 times? **0.15%**

Example 3 **Find the missing variable. Indicate the position of X in the distribution.** **12–15. See margin.**

12. z if $\mu = 3.3$, $X = 3.8$, and $\sigma = 0.2$
13. z if $\mu = 19.9$, $X = 18.7$, and $\sigma = 0.9$
14. μ if $z = -0.92$, $X = 44.2$, and $\sigma = 8.3$
15. X if $\mu = 138.8$, $\sigma = 22.5$, and $z = 1.73$

Example 4

16. **VENDING** A vending machine dispenses about 8.2 ounces of coffee. The amount varies and is normally distributed with a standard deviation of 0.3 ounce. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve. **16a–b. See Answer Appendix for graphs.**
    a. $P(X < 8)$ **25.2%**
    b. $P(X > 7.5)$ **99.0%**

B

17. **CAR BATTERIES** The useful life of a certain car battery is normally distributed with a mean of 113,627 miles and a standard deviation of 14,266 miles. The company makes 20,000 batteries a month.
    a. About how many batteries will last between 90,000 and 110,000 miles? **about 7000 batteries**
    b. About how many batteries will last more than 125,000 miles? **about 4200 batteries**
    c. What is the probability that if you buy a car battery at random, it will last less than 100,000 miles? **17.0%**
18. **FOOD** The shelf life of a particular snack chip is normally distributed with a mean of 173.3 days and a standard deviation of 23.6 days.
    a. About what percent of the product lasts between 150 and 200 days? **70.9%**
    b. About what percent of the product lasts more than 225 days? **1.4%**
    c. What range of values represents the outside 5% of the distribution? **$X > 220.5$ or $X < 126.1$**

Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	9–16, 21, 23–26	9–15 odd	10–16 even, 21, 23–26
OL Core	9–15 odd, 17–21, 23–26	9–16	17–21, 23–26
BL Advanced	17–26		

19. FINANCIAL LITERACY The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual's insurance rate. Suppose insurance rates for a sample population are normally distributed.

a. If the mean annual cost per person is $829 and the standard deviation is $115, what is the range of rates you would expect the middle 68% of the population to pay annually? **between $714 and $944**

b. If 900 people were sampled, how many would you expect to pay more than $1000 annually? **62**

c. Where on the distribution would you expect a person with several traffic citations to lie? Explain your reasoning. **c–d. See Answer Appendix.**

d. How do you think auto insurance companies use each factor to calculate an individual's insurance rate?

20. STANDARDIZED TESTS Nikki took three national standardized tests and scored an 86 on all three. The table shows the mean and standard deviation of each test.

	Math	Science	Social Studies
μ	76	81	72
σ	9.7	6.2	11.6

a. Calculate the *z*-values that correspond to her score on each test.

b. What is the probability of a student scoring an 86 or *lower* on each test?

c. On which test was Nikki's standardized score the highest? Explain your reasoning.
20a–c. See Answer Appendix.

H.O.T. Problems Use Higher-Order Thinking Skills

21. ERROR ANALYSIS A set of normally distributed tree diameters have mean 11.5 centimeters, standard deviation 2.5, and range from 3.6 to 19.8. Monica and Hiroko are to find the range that represents the middle 68% of the data. Is either of them correct? Explain.
Sample answer: Hiroko; Monica's solution would work with a uniform distribution.

C

Monica	Hiroko
The data span 16.2 cm. 68% of 16.2 is about 11 cm. Center this 11-cm range around the mean of 11.5 cm. This 68% group will range from about 6 cm to about 17 cm.	The middle 68% span from $\mu + \sigma$ to $\mu - \sigma$. So we move 2.5 cm below 11.5 and then 2.5 cm above 11.5. The 68% group will range from 9 cm to 14 cm.

22. CHALLENGE A case of MP3 players has an average battery life of 8.2 hours with a standard deviation of 0.7 hour. Eight of the players have a battery life greater than 9.3 hours. If the sample is normally distributed, how many MP3 players are in the case? **about 138**

23. REASONING The term *six sigma process* comes from the notion that if one has six standard deviations between the mean of a process and the nearest specification limit, there will be practically no items that fail to meet the specifications. Is this a true assumption? Explain. **See margin.**

24. REASONING *True* or *false*: According to the Empirical Rule, in a normal distribution, most of the data will fall within one standard deviation of the mean. Explain. **See margin.**

25. OPEN ENDED Find a set of real-world data that appears to be normally distributed. Calculate the range of values that represent the middle 68%, the middle 95%, and the middle 99.7% of the distribution. **See Answer Appendix.**

26. WRITING IN MATH Describe the relationship between the *z*-value, the position of an interval of *X* in the normal distribution, the area under the normal curve, and the probability of the interval occurring. Use an example to explain your reasoning. **See Answer Appendix.**

4 Assess

Name the Math Ask students to explain the difference between a normal distribution and the standard normal distribution.

WatchOut!

Error Analysis For Exercise 21, remind students that the middle 68% of any normal distribution is within one standard deviation of the mean.

Follow-up

Students have explored binomial distributions.

Ask:

- Can statistics lie? Sample answer: Statistics can "lie" when they are manipulated and then used to influence the intended audiences' beliefs and behaviors.

Additional Answers

23. Sample answer: True; according to the Empirical Rule, 99% of the data lie within 3 standard deviations of the mean. Therefore, only 1% will fall outside of three sigma. An infinitesimally small amount will fall outside of six-sigma.

24. Sample answer: True; according to the Empirical Rule, 68% of the data lie within
1 standard deviation of the mean.

DifferentiatedInstruction BL

Extension Explain to students that they can use a graphing calculator to identify the range of *z*-values that corresponds with any area under the normal curve. For example, to find the range of *z*-values that corresponds to the middle 50% of data, they need to find the *z*-values that correspond to the data between 25% and 75% in the distribution. Select 2nd **[DISTR] 3** for the **InvNorm(** feature. Then enter 0.25). Do the same for 0.75).

1 Focus

Objective Use a normal distribution to approximate a binomial distribution.

Materials for Each Student

- computer with spreadsheet program

Teaching Tip

Have students review the Binomial Probability Formula and Example 4 in Lesson 12 before completing Activity 1.

Before Activity 2, remind students that in Lesson 13, they used *z*-values and graphing calculators to find probabilities associated with normally distributed data.

From Concrete to Abstract

Have students refer to Activity 1.

Ask:

- Find the probability that Tom selects no more than 30 hearts using the normal approximation if he chooses 100 cards. 87.5%
- Find the sum of the probabilities for 0 to 30 hearts by highlighting cells D1:D31 on the spreadsheet from Step 4 and applying the AutoSum function. 89.6%
- Explain why the probabilities found using these two methods are not the same. Sample answer: Using the normal distribution is an approximation. The larger the number of trials, the closer the answers should get to each other.

LAB 14 Spreadsheet Lab: Normal Approximation of Binomial Distributions

In Lesson 11-4, you used a binomial expansion to find a full probability distribution. A spreadsheet can be used to quickly find and graph a full distribution for any number of trials.

Common Core State Standards
S.ID.4

Activity 1 Full Probability Distribution

PLAYING CARDS Tom randomly selects a card from a deck of 52 playing cards, records its suit, and replaces it. Use a spreadsheet to construct and graph a full probability distribution for *X*, the number of hearts that Tom selects if he chooses 4, 20, or 100 cards.

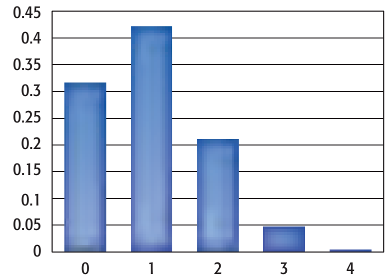
Since one fourth of the cards in a standard deck are hearts, the probability of success is 25% or 0.25, and the probability of failure is 75% or 0.75.

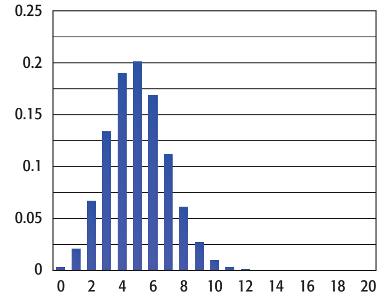
Step 1 Enter the numbers 0 to 4 in column A. In B1, enter the binomial probability formula as =COMBIN(4,A1)*(0.25)^A1*(0.75)^(4–A1). Copy and paste this formula in cells B2:B5.

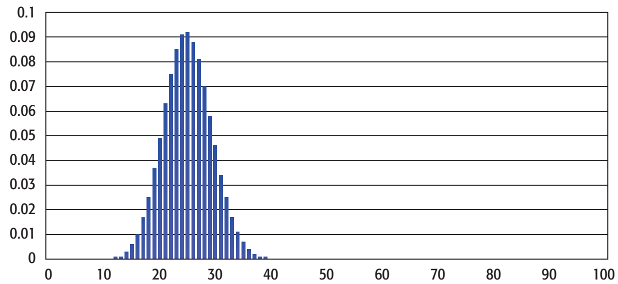
Step 2 Select cells B1:B5, and insert a clustered column bar graph. Use the values in column A for Category *(X)* axis labels.

Step 3 Select cells A1:A5, and autofill the through A21. In C1, enter the formula =COMBIN(20,A1)*(0.25)^A1* (0.75)^(20–A1). Copy this formula in cells C2:C21. Repeat Step 2 to graph this distribution.

Step 4 Autofill column A through A101. In D1, enter the formula =COMBIN(100,A1)*(0.25)^A1* (0.75)^(100–A1). Copy this formula in cells D2:C1011. Repeat Step 2 to graph this distribution.







Notice that the number of trials increases, the shape of the graph of the distribution becomes more symmetric with its center at the mean, $\mu = np$. If the number of trials becomes large enough, the shape of the distribution approaches a normal curve. Therefore, a normal distribution can be used to approximate the binomial distribution.

KeyConcept Normal Approximation of a Binomial Distribution

In a binomial distribution with *n* trials, a probability of success *p*, and a probability of failure *q*, such that $np \geq 5$ and $nq \geq 5$, a binomial distribution can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$.

56 | Lab 14 | Spreadsheet Lab: Normal Approximation of Binomial Distributions

Once the mean and standard deviation are calculated, a z-value can be determined, and the corresponding probability can be found as demonstrated in the next activity.

Activity 2 Normal Approximation of a Binomial Distribution

JURY DUTY According to a poll, 60% of the registered voters in a city have never been called for jury duty. Mariah conducts a random survey of 300 registered voters. What is the probability that at least 170 of those voters have never been called for jury duty?

This is a binomial experiment with $n = 300$, $p = 0.6$, and $q = 0.4$. Since $np = 300(0.6)$ or 180 and $nq = 300(0.4)$ or 120 are both greater than 5, the normal distribution can be used to approximate the binomial distribution.

Step 1 The mean μ is np or 180. Find the standard deviation σ.

$\sigma = \sqrt{npq}$ — Standard deviation of a binomial distribution

$= \sqrt{300(0.6)(0.4)}$ — $n = 300$, $p = 0.6$, and $q = 0.4$

≈ 8.49 — Simplify.

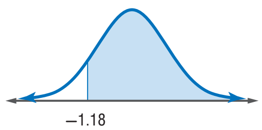
Step 2 Write the problem in probability notation using X. The probability that at least 170 people have never been called for jury duty is $P(X \geq 170)$.

Step 3 Find the corresponding z-value for X.

$z = \frac{X - \mu}{\sigma}$ — Formula for z-values

$= \frac{170 - 180}{8.49}$ — $X = 170$, $\mu = 180$, and $\sigma = 8.49$

≈ -1.18 — Simplify.



Step 4 Use a calculator to find the area under the normal curve to the right of z.

KEYSTROKES: 2nd [DISTR] 2 (−) 1 . 18 , 4) ENTER

The approximate area to the right of z is 0.881. Therefore, the probability that at least 170 of the registered voters have never been called for jury duty is about 88.1%.

Exercises

1. Use a spreadsheet to construct the graph of the full probability distribution for X, the number of times a 3 is rolled from rolling a die 25 times. **See margin.**
2. **SUMMER JOBS** According to an online poll, 80% of high school upperclassmen have summer jobs. Tadeo thinks the number should be lower so he conducts a survey of 480 random upperclassmen. What is the probability that no more than 380 of the surveyed upperclassmen have summer jobs? **about 32.3%**
3. **WORK** According to an online poll, 28% of adults feel that the standard 40-hour work week should be increased. Sheila interviews 300 adults who work at the mall. What is the probability that more than 80 but fewer than 100 of those surveyed will say that the work week should be increased? **about 67.7%**

2 Teach

Working in Cooperative Groups

Have students work in pairs to complete Activities 1 and 2 and Exercise 1. Mix abilities so that a student with more knowledge of spreadsheets is partnered with one who has less experience.

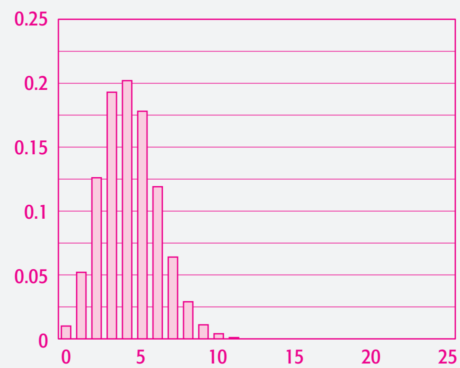
3 Assess

Formative Assessment

Use Exercise 2 to assess whether students can use a normal distribution to approximate a binomial distribution.

Additional Answer

1.



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1 Focus

VerticalAlignment

Before Lesson 15 Apply the standard normal distribution.

Lesson 15 Find confidence intervals and perform hypothesis tests on normally distributed data.

After Lesson 15 Understand how to draw conclusions about a population using a sample.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- To what do you think the margin of sampling error refers? Sample answer: The average obtained using the sample is within 31 songs of the population average.
- Why do you think they are only 95% confident? Sample answer: The sample is only an estimate of the population. The entire population would need to be used in order to be 100% confident.

LESSON 15 Confidence Intervals and Hypothesis Testing

Then

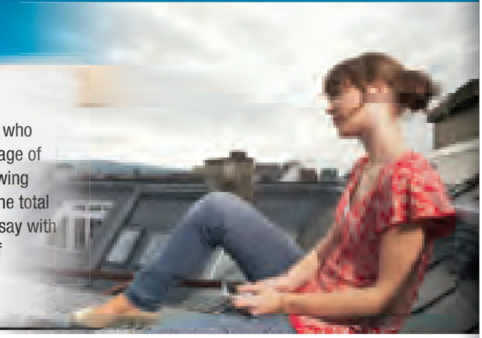
- You applied the standard normal distribution and z-values.

Now

1. Find confidence intervals for normally distributed data.
2. Perform hypothesis tests on normally distributed data.

Why?

- In a recent Gallup Poll, 1514 teens who owned an mp3 player had an average of 1033 songs. The poll had the following disclaimer: "For results based on the total sample of national teens, one can say with 95% confidence that the margin of sampling error is ± 31 songs."



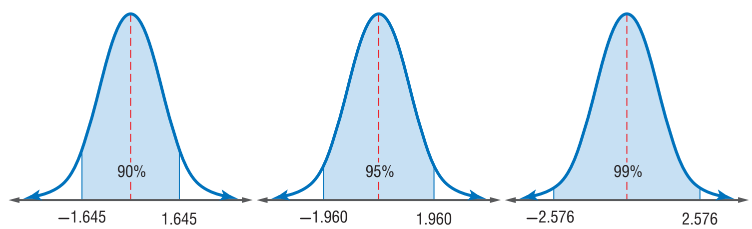
NewVocabulary
inferential statistics
statistical inference
confidence interval
maximum error of estimate
hypothesis test
null hypothesis
alternative hypothesis
critical region
left-tailed test
two-tailed test
right-tailed test

1 Confidence Intervals

Inferential statistics are used to draw conclusions or **statistical inferences** about a population using a sample. For example, the sample mean of 1033 songs per mp3 player can be used to estimate the population mean.

A **confidence interval** is an estimate of a population parameter stated as a range with a specific degree of certainty. Typically, statisticians use 90%, 95%, and 99% confidence intervals, but any percentage can be considered. In the opening example, we are 95% confident that the population mean is within 31 songs of 1033.

The confidence interval for a normal distribution is equivalent to the area under the standard normal curve between $-z$ and z, as shown. A 95% confidence interval for a population mean implies that we are 95% sure that the mean will fall within the range of z-values.



Suppose you want 95% confidence when conducting an experiment. The corresponding z-value is 1.960, where 2.5% of the area lies to the left of $-z$ and 2.5% lies to the right of z.

To find the confidence interval, use the maximum error of estimate and the sample mean. The **maximum error of estimate** E is the maximum difference between the estimate of the population mean μ and its actual value.

KeyConcept **Maximum Error of Estimate**

The maximum error of estimate E for a population mean is given by

$$E = z \cdot \frac{s}{\sqrt{n}},$$

where z is the z-value that corresponds to a particular confidence level, s is the standard deviation of the sample, and n is the sample size; $n \geq 30$.

Fancy Photography/Veer

Common Core State Standards
S.IC.1, S.IC.4, S.MD.7

Example 1 Maximum Error of Estimate

SOCIAL NETWORKING A poll of 218 randomly selected members of a social networking Web site showed that they spent an average of 14 minutes per day on the site with a standard deviation of 3.1 minutes. Use a 95% confidence interval to find the maximum error of estimate for the time spent on the site.

In a 95% confidence interval, 2.5% of the area lies in each tail. The corresponding z-value is 1.960.

$E = z \cdot \frac{s}{\sqrt{n}}$	Maximum Error of Estimate
$= 1.960 \cdot \frac{3.1}{\sqrt{218}}$	$z = 1.960$, $s = 3.1$, and $n = 218$
≈ 0.41	Simplify.

This means that you can be 95% confident that the population mean time spent on the site will be within 0.41 minute of the sample mean of 14 minutes.

StudyTip

Critical Values The z-value that corresponds to a particular confidence level is known as the *critical value.* The most commonly used levels and their corresponding z-value are shown below.

Confidence Level	z-Value
90%	1.645
95%	1.960
99%	2.576

GuidedPractice

1. **TEACHING** A poll of 184 randomly selected high school teachers showed that they spend an average of 16.8 hours per week grading, planning, and preparing for class. The standard deviation is 2.9 hours. Use a 90% confidence interval to find the maximum error of estimate for the amount of time spent per week. **0.35**

Once the maximum error of estimate E is found, a confidence interval CI for the population mean can be determined by adding $\pm E$ to the sample mean.

KeyConcept Confidence Interval for the Population Mean

A confidence interval for a population mean is given by

$$CI = \overline{x} \pm E \text{ or } \overline{x} \pm z \cdot \frac{s}{\sqrt{n}},$$

where $\overline{x}$ is the sample mean and E is the maximum error of estimate.

Real-WorldLink

On average, students sleep 8.1 hours per weekday and spend 7.5 hours on educational activities, such as attending class or doing homework.

Source: Bureau of Labor Statistics

BananaStock/Jupiterimages

Real-World Example 2 Confidence Interval

SCHOOL WORK A sample of 200 students was asked the average time they spent on homework each weeknight. The mean time was 52.5 minutes with a standard deviation of 5.1 minutes. Determine a 99% confidence interval for the population mean.

$CI = \overline{x} \pm z \cdot \frac{s}{\sqrt{n}}$	Confidence Interval for Population Mean
$= 52.5 \pm 2.576 \cdot \frac{5.1}{\sqrt{200}}$	$\overline{x} = 52.5$, $z = 2.576$, $s = 5.1$, and $n = 200$
$\approx 52.5 \pm 0.93$	Use a calculator.

The 99% confidence interval is $51.57 \leq \mu \leq 53.43$. Therefore, we are 99% confident that the population mean time is between 51.57 and 53.43 minutes.

GuidedPractice

2. **SCHOOL ATHLETICS** A sample of 224 students showed that they attend an average of 2.6 school athletic events per year with a standard deviation of 0.8. Determine a 90% confidence interval for the population mean. **$2.51 \leq \mu \leq 2.69$**

Confidence Intervals

Example 1 shows how to find a maximum error of estimate. **Example 2** shows how to find a confidence interval.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Examples

1. **ORAL HYGIENE** A poll of 422 randomly selected adults showed that they brushed their teeth an average of 11.4 times per week with a standard deviation of 1.6. Use a 99% confidence interval to find the maximum error of estimate for the number of times per week adults brush their teeth. 0.20

2. **PACKAGING** A poll of 156 randomly selected members of a golf course showed that they play an average of 4.6 times every summer with a standard deviation of 1.1. Determine a 95% confidence interval for the population mean. $4.43 \leq \mu \leq 4.77$

Additional Examples also in Interactive Classroom PowerPoint® Presentations

IWB Interactive White Board READY

Hypothesis Testing

Example 3 shows how to identify claims and hypotheses. **Example 4** shows how to perform a one-sided hypothesis test. **Example 5** shows how to perform a two-sided hypothesis test.

Additional Example

3 Identify the null and alternative hypotheses for each statement. Then identify which statement represents the claim.

a. Cindy thinks her employees spend over 60 minutes per day surfing the Internet at work. $H_0: \mu \leq 60$; $H_a: \mu > 60$ (claim)

b. Mr. Bacon thinks that he tells at least five jokes every class period. $H_0: \mu \geq 5$ (claim); $H_a: \mu < 5$

2 Hypothesis Testing While a confidence interval provides an estimate of the population mean, a **hypothesis test** is used to assess a specific claim about the mean. Typical claims are that the mean is equal to, is greater than, or is less than a specific value.

There are two parts to a hypothesis test: the null hypothesis and the alternative hypothesis. The **null hypothesis** H_0 is a statement of equality to be tested. The **alternative hypothesis** H_a is a statement of inequality that is the complement of the null hypothesis. A *claim* can be part of the null or alternative hypothesis.

Example 3 Claims and Hypotheses

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

a. A school administrator thinks it takes less than 3 minutes to evacuate the entire building for a fire drill.

less than 3 minutes	not less than three minutes
$\mu < 3$	$\mu \geq 3$

The claim is $\mu < 3$, and it is the alternative hypothesis because it does not include equality. The null hypothesis is $\mu \geq 3$, which is the complement of $\mu < 3$.

$H_0: \mu \geq 3$ $\qquad$ $H_a: \mu < 3$ (claim)

b. The owner of a deli says that there are 2 ounces of ham in a sandwich.

$H_0: \mu = 2$ (claim); $H_a: \mu \neq 2$

The claim is $\mu = 2$, and it is the null hypothesis because it is an equality. The alternative hypothesis is $\mu \neq 2$, which is the complement of $\mu = 2$.

Guided Practice

3A. Jill thinks it takes longer than 10 minutes for the school bus to reach school from her stop. $H_0: \mu \leq 10$; $H_a: \mu > 10$ (claim)

3B. Mindy thinks there aren't 35 peanuts in every package. $H_0: \mu = 35$; $H_a: \mu \neq 35$ (claim)

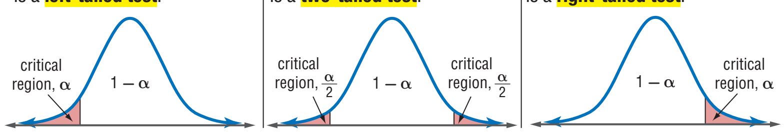
Reading Math

Significance Level A significance level of 5% means that we need the data to provide evidence against H_0 so strong that the data would occur no more than 5% of the time when H_0 is true.

Once a claim is made, a sample is collected and analyzed, and the null hypothesis is either *rejected* or *not rejected*. From this information, the claim is then rejected or not rejected. This is determined by whether the sample mean falls within the critical region. The **critical region** is the range of values that suggests a significant enough difference to reject the null hypothesis. The critical region is determined by the significance level α, most commonly 1%, 5%, or 10%, and the inequality sign of the alternative hypothesis.

Concept Summary Tests of Significance

If H_a: $\mu < k$, the hypothesis test is a **left-tailed test**.	If H_a: $\mu \neq k$, the hypothesis test is a **two-tailed test**.	If H_a: $\mu > k$, the hypothesis test is a **right-tailed test**.



The value k represents the claim about the population mean μ.

Study Tip

***z*-Values and *z*-Statistics** In hypothesis testing, a *z*-value is used to determine the critical region. A *z*-statistic is calculated to see if the sample mean falls within that critical region, and thus determines whether or not to reject the null hypothesis.

The type of hypothesis test that we are using in this lesson is known as a *z*-test. Once the area corresponding to the significance level is determined, a *z*-statistic for the sample mean is calculated. The *z*-statistic is the *z*-value for the sample. When testing at 5% significance, it is only 5% likely for the *z*-statistic to fall within the critical region *and* for H_0 to be true. Therefore, when the *z*-statistic falls within the critical region, H_0 is rejected.

Differentiated Instruction OL ELL

Linguistic Learners The formal conclusion of a hypothesis test is stated as "Reject the null hypothesis" or "Do not reject the null hypothesis." Students should always restate conclusions in their own words.

60 | Lesson 15 | Confidence Intervals and Hypothesis Testing

Use these steps to test a hypothesis.

KeyConcept Steps for Hypothesis Testing

Step 1 State the hypotheses and identify the claim.

Step 2 Determine the critical value(s) and critical region.

Step 3 Calculate the *z*-statistic using $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Step 4 Reject or fail to reject the null hypothesis.

Step 5 Make a conclusion about the claim.

Real-WorldCareer

Politician Politicians are elected officials in the federal, state, and local governments. They influence public decision making and make laws that affect everyone. Many politicians have business, teaching, or legal experience prior to getting involved in politics.

Real-World Example 4 One-Sided Hypothesis Test

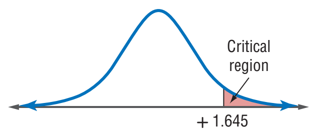
STUDENT COUNCIL **Lindsey, the president of the student council, has heard complaints that the cafeteria lunch line moves too slowly. The dining services coordinator assures her that the average wait time is 6 minutes or less. Using a sample of 65 customers, Lindsey calculated a mean wait time of 6.3 minutes and a standard deviation of 1.1 minutes. Test the hypothesis at 5% significance.**

Step 1 State the claim and hypotheses. H_0: $\mu \leq 6$ (claim) and H_a: $\mu > 6$.

Step 2 Determine the critical region.

The alternative hypothesis is $\mu > 6$, so this is a right-tailed test. We are testing at 5% significance, so we need to identify the *z*-value that corresponds with the upper 5% of the distribution.

KEYSTROKES: 2nd [DISTR] 3 .95 ENTER



Step 3 Calculate the *z*-statistic for the sample data.

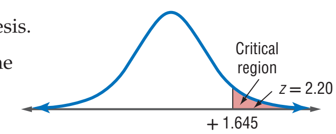
$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$ Formula for *z*-statistic

$$= \frac{6.3 - 6}{\frac{1.1}{\sqrt{65}}}$$ $\bar{x} = 6.3$, $\mu = 6$, $s = 1.1$, and $n = 65$

$$\approx 2.20$$ Simplify.

Step 4 Decide whether to reject the null hypothesis.

H_0 is rejected because the *z*-statistic for the sample falls within the critical region.



Step 5 Make a conclusion about the claim.

Therefore, there is enough evidence to reject the claim that the average wait time is 6 minutes or less.

GuidedPractice

4. **EXERCISE** Julian thinks it takes less than 60 minutes for people to get in a full workout at the gym. Using a sample of 128 members, he calculated a mean workout time of 58.4 minutes with a standard deviation of 8.9 minutes. Test the hypothesis at 10% significance.

StudyTip

Rejecting the Null Hypothesis When the null hypothesis is rejected, it is implied that the alternative hypothesis is accepted.

4. H_0: $\mu \geq 60$; H_a: $\mu < 60$ (claim); H_0 is rejected. Therefore, there is not enough evidence to reject Julian's claim that it takes less than 60 minutes for people to get in a full workout.

Additional Example

4 MAGAZINES Jenny thinks there are more than 10 pages of advertisements in every issue of a particular magazine. Using a sample of 42 magazines, she calculated a mean of 10.5 pages with a standard deviation of 1.6. Test the hypothesis at 1% significance.
H_0 is not rejected. There is not enough evidence to support Jenny's claim that there are more than 10 pages of advertisements in every issue.

Tip for New Teachers

Failing to Reject Versus Accepting the Null Hypothesis When the null hypothesis is not rejected, it cannot be accepted as true. There is merely not enough evidence to say that it is false.

Teach with Tech

Blog Have students write a blog entry to describe hypothesis testing in their own words. Check students' entries to be sure they understand the concept and importance of hypothesis testing.

Focus on Mathematical Content

Tests and Confidence Intervals While a confidence interval is used to estimate the value of a quantity, a hypothesis test is performed in order to answer a *yes* or *no* question about whether a null hypothesis is true. A confidence interval can be used to help answer that question.

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Additional Example

5 **CALORIES** A label on a package of food states that there are 120 Calories per serving. Using a sample of 38 servings, Libby calculated a mean of 120.6 Calories with a standard deviation of 2.2. Test the hypothesis at 5% significance. H_0 is not rejected. There is not enough evidence to reject the label's claim that there are 120 Calories per serving.

Tip for New Teachers

***t*-Test** When σ is unknown and $n < 30$, the *z*-test is no longer accurate. A different hypothesis test called the *t*-test needs to be performed.

For a two-sided test, the significance level α must be divided by 2 in order to determine the critical value at each tail.

Example 5 Two-Sided Hypothesis Test

ADVERTISEMENTS **Jerrod wants to determine if the advertisement shown is accurate. Using a sample of 42 pizzas, Jerrod calculates a mean of 99.8 pieces and a standard deviation of 0.8. Test the hypothesis at 5% significance.**

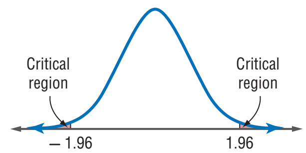
Step 1 State the claim and hypotheses.

H_0: $\mu = 100$ (claim) and H_a: $\mu \neq 100$.

Step 2 Determine the critical region.

The alternative hypothesis is $\mu \neq 100$, so this is a two-tailed test. We are testing at 5% significance, so we need to identify the *z*-value that corresponds with upper and lower 2.5% of the distribution.

KEYSTROKES: [2nd] [DISTR] 3 0.25 [ENTER] and [2nd] [DISTR] 3 .975 [ENTER].



StudyTip

***z*-Test** The *z*-test can be used when σ is known or when $n \geq 30$.

Step 3 Calculate the *z*-statistic for the sample data.

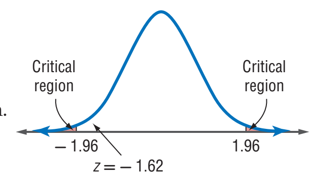
$$z = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$ Formula for *z*-statistic

$$= \frac{99.8 - 100}{\frac{0.8}{\sqrt{42}}}$$ $\overline{x} = 99.8$, $\mu = 100$, $s = 0.8$, and $n = 42$

$$\approx -1.62$$ Simplify.

Step 4 Reject or fail to reject the null hypothesis.

H_0 is not rejected because the *z*-value for the sample does not fall within the critical region.



Step 5 Make a conclusion about the claim.

Therefore, there is not enough evidence to reject the claim that there are 100 pieces of pepperoni on every large pizza.

WatchOut!

Inferential Statistics Inferential statistics are not designed to definitely prove a hypothesis. They depend on probability statements and therefore, may not be correct.

5. H_0: $\mu = 12$ (claim); H_a: $\mu \neq 12$; H_0 is rejected. Therefore, the claim that there are 12 ounces in every can is rejected.

GuidedPractice

5. **JUICE** A manufacturer claims there are 12 ounces of juice in every can. Using a sample of 68 cans, Susan calculated a mean of 11.95 ounces with a standard deviation of 0.15 ounce. Test the hypothesis at 1% significance.

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Check Your Understanding

Example 1

1. **LUNCH** A sample of 145 high school seniors was asked how many times they go out for lunch per week. The mean number of times was 2.4 with a standard deviation of 0.7. Use a 90% confidence level to calculate the maximum error of estimate. **0.096**

Example 2

2. **PRACTICE** A poll of 233 randomly chosen high school athletes showed that they spend an average of 1.6 hours practicing their sport during the off-season. The standard deviation is 0.5 hour. Determine a 99% confidence interval for the population mean. **$1.52 \le \mu \le 1.68$**

Example 3

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim. **5. H_0: $\mu \ge 20$ (claim) ; H_a: $\mu < 20$**

3. Lori thinks it takes a fast food restaurant less than 2 minutes to serve her meal after she orders it. **H_0: $\mu \ge 2$; H_a: $\mu < 2$ (claim)**
4. A snack label states that one serving contains one gram of fat. **H_0: $\mu = 1$ (claim); H_a: $\mu \ne 1$**
5. Mrs. Hart's review game takes at least 20 minutes to complete.
6. The tellers at a bank can complete no more than 18 transactions per hour. **H_0: $\mu \le 18$ (claim); H_a: $\mu > 18$**

7. H_0: $\mu \ge 84$ (claim), H_a: $\mu < 84$; Do not reject H_0; The manufacturer's claim that the discs can hold at least 84 minutes cannot be rejected.

Identify the hypotheses and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

Example 4

7. **COMPACT DISCS** A manufacturer of blank compact discs claims that each disc can hold at least 84 minutes of music. Using a sample of 219 compact discs, Cayla calculated a mean time of 84.1 minutes per disc with a standard deviation of 1.9 minutes. Test the hypothesis at 5% significance.

Example 5

8. **GOLF TEES** A company claims that each golf tee they produce is 5 centimeters in length. Using a sample of 168 tees, Angelene calculated a mean of 5.1 centimeters with a standard deviation of 0.3. Test the hypothesis at 10% significance. **H_0: $\mu = 5$ (claim), H_a: $\mu \ne 5$; Reject H_0; The company's claim that each tee is 5 centimeters is rejected.**

Practice and Problem Solving

Example 1

9. **MUSIC** A sample of 76 albums had a mean run time of 61.3 minutes with a standard deviation of 5.2 minutes. Use a 95% confidence level to calculate the maximum error of estimate. **1.17**

Example 2

12. H_0: $\mu = 27$ (claim); H_a: $\mu \ne 27$

10. **COLLEGE** A poll of 218 students at a university showed that they spend 11.8 hours per week studying. The standard deviation is 3.7 hours. Determine a 90% confidence interval for the population mean. **$11.39 \le \mu \le 12.21$**

Example 3

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim. **11. H_0: $\mu \ge 6$ (claim) ; H_a: $\mu < 6$**

15. H_0: $\mu \ge 30$; H_a: $\mu < 30$ (claim); Do not reject H_0; There is not enough evidence to support the pizza chain's claim of a delivery time of less than 30 minutes cannot be rejected.

11. Julian sends at least six text messages to his best friend every day.
12. A car company states that one of their vehicles gets 27 miles per gallon.
13. A company advertisement states that it takes no more than 2 hours to paint a 200-square-foot room. **H_0: $\mu \le 2$ (claim); H_a: $\mu > 2$**
14. A singer plays at least 18 songs at every concert. **H_0: $\mu \ge 18$ (claim) ; H_a: $\mu < 18$**

Identify the hypotheses and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

Example 4

15. **PIZZA** A pizza chain promises a delivery time of less than 30 minutes. Using a sample of 38 deliveries, Chelsea calculated a mean delivery time of 29.6 minutes with a standard deviation of 3.9 minutes. Test the hypothesis at 1% significance.

3 Practice

Formative Assessment

Use Exercises 1–8 to check for understanding.

Use the chart on the bottom of this page to customize assignments for your students.

Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	9–16, 20, 22–24	9–15 odd	2–16 even, 20, 22–24
OL Core	9–15 odd, 16–20, 22–24	9–16	17–20, 22–24
BL Advanced	17–24		

4 Assess

Yesterday's News Ask students to describe how their study of normal distributions relates to their study of confidence intervals and hypothesis testing.

Additional Answers

17. H_0: $\mu = 12$; H_a: $\mu \neq 12$; The mean of the sample data is 12.89 with a standard deviation of about 1.06. The *z*-statistic is about 5.31, which falls in the critical region at 1% significance. Therefore, the null hypothesis is rejected and the company should not make the claim on the label.

18. H_0: $\mu = 20$; H_a: $\mu \neq 20$; The mean of the sample data is 19.175 with a standard deviation of about 1.38. The *z*-statistic is about −3.78, which falls in the critical region at 1% significance. Therefore, the null hypothesis is rejected and the company should not make the claim on the label.

20. Neither; Sample answer: The null hypothesis is a statement of equality, so Tim is incorrect. The claim is that the delivery time is less than or equal to 45, so Judie is also incorrect.

24. Sample answer: A pizza company claims to put 100 pepperonis on its large pizza. H_0: $\mu = 100$ and H_1: $\mu > 100$; data collected: 100, 102, 101, 101, 100, 99, 99, 103, 102, 103, 103, 101, 102, 99, 105, 103, 102, 100, 101, 104; $\bar{x} = 101.5$, $s = 1.70$; a 95% confidence interval is $100.74 < \bar{x} < 102.26$. H_0 does not fall within the confidence interval, so we reject the null hypothesis and accept the alternative hypothesis.

Example 5

16. H_0: $\mu = 24$ (claim), H_a: $\mu \neq 24$; Reject H_0; The company's claim that each package contains 24 slices is rejected.

16. CHEESE A company claims that each package of cheese contains exactly 24 slices. Using a sample of 93 packages, Mr. Matthews calculated a mean of 24.1 slices with a standard deviation of 0.5. Test the hypothesis at 5% significance.

B **17. DECISION MAKING** The number of peaches in 40 random cans is shown below. Should the manufacturer place a label on the can promising 12 peaches in every can? Explain your reasoning. **See margin.**

13, 14, 13, 14, 12, 12, 12, 11, 15, 12, 13, 13, 14, 13, 14, 12, 15, 11, 11, 14, 13, 14, 14, 13, 12, 12, 12, 12, 13, 13, 11, 14, 14, 13, 14, 13, 13, 14, 12, 12

18. COOKIES The number of chocolate chips in 40 random cookies is shown below. Should the manufacturer place a label on the package promising 20 chips on every cookie? Explain your reasoning. **See margin.**

21, 19, 20, 20, 19, 19, 18, 21, 19, 17, 19, 18, 18, 20, 20, 19, 18, 20, 19, 20, 21, 21, 19, 17, 17, 18, 19, 19, 20, 17, 22, 21, 21, 20, 19, 18, 19, 17, 17, 21

19. MULTIPLE REPRESENTATIONS In this problem, you will explore how the confidence interval is affected by the sample size and the confidence level. Consider a sample of data where $\bar{x} = 25$ and $s = 3$.

a. **GRAPHICAL** Graph the 90% confidence interval for $n = 50$, 100, and 250 on a number line.

b. **ANALYTICAL** How does the sample size affect the confidence interval?

c. **GRAPHICAL** Graph the 90%, 95%, and 99% confidence intervals for $n = 150$.

d. **ANALYTICAL** How does the confidence level affect the confidence interval?

e. **ANALYTICAL** How does decreasing the size of the confidence interval affect the accuracy of the confidence interval?

a–e. See Answer Appendix.

H.O.T. Problems Use Higher-Order Thinking Skills

C **20. ERROR ANALYSIS** Tim and Judie want to test whether a delivery service meets their promised time of 45 minutes or less. Their hypotheses are shown below. Is either of them correct? **See margin.**

Tim	Judie
H_0: $\mu < 45$ (claim)	H_0: $\mu \leq 45$
H_a: $\mu \geq 45$	H_a: $\mu > 45$ (claim)

21. CHALLENGE A 95% confidence interval for the mean weight of a 20-ounce box of cereal was $19.932 \leq \mu \leq 20.008$ with a sample standard deviation of 0.128 ounces. Determine the sample size that led to this interval. **44**

22. Sample answer: Always; if the null hypothesis value of μ falls within the confidence interval, then it is not rejected.

22. REASONING Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

If a confidence interval contains the H_0 value of μ, then it is not rejected.

23. WRITING IN MATH How can a statistical test be used in a decision-making process? **Sample answer: You can use a statistical test to help you to determine the strength of your decision.**

24. OPEN ENDED Design and conduct your own research study, and draw conclusions based on the results of a hypothesis test. Write a brief summary of your findings. **See margin.**

Differentiated Instruction BL

Extension When the null hypothesis is rejected when it is actually true, the result is called a Type I error. When the null hypothesis is not rejected when it is actually false, the result is called a Type II error. Have students explore the risks of making Type I and Type II errors with different significant levels. What conclusions can they make? The risk of making a Type I error is identical to the significance level. Reducing the risk of a Type I error increases the risk of a Type II error by widening the acceptance level.

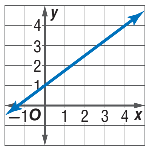
Additional Exercises

Use with Lesson 2-2.

1. **WHICH ONE DOESN'T BELONG?** Of the four items shown, identify the one that does not belong. Explain your reasoning. **$y = 2xy$; Sample answer; $y = 2xy$ is not a linear function.**

$y = 2x + 3$

x	y
0	4
1	2
2	0
3	−2



$y = 2xy$

Use with Lesson 2-5.

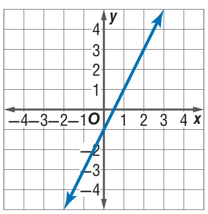
2. **WRITING IN MATH** What are the strengths and weaknesses of using a regression equation to approximate data? **See Answer Appendix.**

Use with Explore 2-7.

3. What are the domain and range of functions of the form $f(x) = mx + b$, where $m \neq 0$? **all real numbers**

Use with Lesson 2-7.

4. **ERROR ANALYSIS** Kimi thinks that the graph and table below are representations of the same linear relation. Carla disagrees. Who is correct? Explain your reasoning.

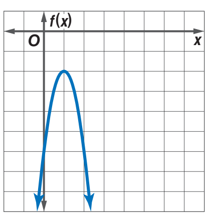


x	y
0	−1
1	1
2	3
3	5

See Answer Appendix.

Use with Lesson 5-1.

5. **ERROR ANALYSIS** Trent thinks that the function $f(x)$ graphed below and the function $g(x)$ described next to it have the same maximum. Madison thinks that $g(x)$ has the greater maximum. Is either of them correct? Explain your reasoning. **See Answer Appendix.**



$g(x)$ is a quadratic function with roots of 4 and 2 and a y-intercept of −8.

Use with Lesson 6-3.

6. **CHALLENGE** Of $f(x)$ and $g(x)$, which function has more potential real roots? What is the degree of that function? **See Answer Appendix.**

x	−24	−18	−12	−6	0	6	12	18	24
$f(x)$	−8	−1	3	−2	4	7	−1	−8	5

$$g(x) = x^4 + x^3 - 13x^2 + x + 4$$

Use with Lesson 6-7.

Sketch the graph of each function using its zeros.

7. $f(x) = x^3 - 5x^2 - 2x + 24$
8. $f(x) = 4x^3 + 2x^2 - 4x - 2$
9. $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$
10. $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$

7-10 See Answer Appendix.

Use with Lesson 7-1.

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in domain or range.

11-20 See Answer Appendix.

11. $f(x) = x + 2$, $g(x) = 3x - 1$
12. $f(x) = x^2 - 5$, $g(x) = -x + 8$
13. $f(x) = 2x$, $g(x) = -4x + 5$
14. $f(x) = x - 1$, $g(x) = 5x - 2$
15. $f(x) = x^2$, $g(x) = -x + 1$
16. $f(x) = 3x$, $g(x) = -2x + 6$
17. $f(x) = x - 2$, $g(x) = 2x - 7$
18. $f(x) = x^2$, $g(x) = x - 5$
19. $f(x) = -x^2 + 6$, $g(x) = 2x^2 + 3x - 5$
20. $f(x) = 3x^2 - 4$, $g(x) = x^2 - 8x + 4$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

21. $f = \{(2, 5), (6, 10), (12, 9), (7, 6)\}$
$g = \{(9, 11), (6, 15), (10, 13), (5, 8)\}$
22. $f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}$
$g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}$
23. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
$g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$
24. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
$g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$
25. $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$
$g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$
26. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
$g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$
27. $f = \{(-15, -5), (-4, 12), (1, 7), (3, 9)\}$
$g = \{(3, -9), (7, 2), (8, -6), (12, 0)\}$
28. $f = \{(-1, 11), (2, -2), (5, -7), (4, -4)\}$
$g = \{(5, -4), (4, -3), (-1, 2), (2, 3)\}$

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Additional Exercises

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

29. $f = \{(7, -3), (-10, -3), (-7, -8), (-3, 6)\}$
$g = \{(4, -3), (3, -7), (9, 8), (-4, -4)\}$

30. $f = \{(1, -1), (2, -2), (3, -3), (4, -4)\}$
$g = \{(1, -4), (2, -3), (3, -2), (4, -1)\}$

31. $f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\}$
$g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\}$

32. $f = \{(12, -3), (9, -2), (8, -1), (6, 3)\}$
$g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\}$

33. $f(x) = -3x$
$g(x) = 5x - 6$

34. $f(x) = x + 4$
$g(x) = x^2 + 3x - 10$

35. $f(x) = 2x$
$g(x) = x + 5$

36. $f(x) = -3x$
$g(x) = -x + 8$

37. $f(x) = x + 5$
$g(x) = 3x - 7$

38. $f(x) = x - 4$
$g(x) = x^2 - 10$

39. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

40. $f(x) = 2x^2 - x + 1$
$g(x) = 4x + 3$

41. $f(x) = 4x - 1$
$g(x) = x^3 + 2$

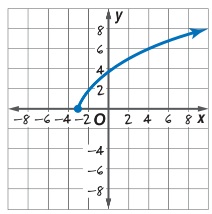
42. $f(x) = x^2 + 3x + 1$
$g(x) = x^2$

43. $f(x) = 2x^2$
$g(x) = 8x^2 + 3x$

21-42 See Answer Appendix.

Use with Lesson 7-3.

44. **ERROR ANALYSIS** Cleveland thinks the graph and the equation represent the same function. Molly disagrees. Who is correct? Explain your reasoning.



$y = \sqrt{5x + 10}$

See Answer Appendix.

Use with Lesson 8-1.

45. **PHONES** The function $P(x) = 2.28(0.9^x)$ can be used to model the number of pay phones in millions x years since 1999.

a. Classify the function as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function. **See Answer Appendix.**

b. Explain what the $P(x)$-intercept and the asymptote represent in this situation. **See Answer Appendix.**

46. **HEALTH** Each day, 10% of a certain drug dissipates from system. **a-c See Answer Appendix.**

a. Classify the function representing this situation as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.

b. How much of the original amount remains in the system after 9 days?

c. If a second dose should not be taken if more than 50% of the original amount is in the system, when should the label say it is safe to redose? Design the label and explain your reasoning.

47. **NUMBER THEORY** A sequence of numbers follows a pattern in which the next number is 125% of the previous number. The first number in the pattern is 18.

a. Write the function that represents the situation. **$f(x) = 18(1.25)^{x-1}$**

b. Classify the function as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function for the first 10 numbers.

c. What is the value of the tenth number? Round to the nearest whole number. **134**

b. See Answer Appendix.

48. **ERROR ANALYSIS** Vince and Grady were asked to graph the following functions. Vince thinks they are the same, but Grady disagrees. Who is correct? Explain your reasoning.

x	y
0	2
1	1
2	0.5
3	0.25
4	0.125
5	0.0625
6	0.03125

an exponential function with rate of decay of $\frac{1}{2}$ and an initial amount of 2

Vince; the graphs of the function would be the same

Use with Extend 8-3.

49. Write the equation of best fit. What are the domain and range? **Sample answer: $y = 46.17(1.10^x)$; $D = \{x \mid x \geq 0\}$; $R = \{y \mid y \geq 46.47\}$**

Use with Extend 8-8.

50. Do you think the results of the experiment would change if you used an insulated container for the water? What part of the function will change, the constant or the rate of decay? Repeat the experiment to verify your conjecture.

51. How might the results of the experiment change if you added ice to the water? What part of the function will change, the constant or the rate of decay? Repeat the experiment to verify your conjecture.

50-51. See Answer Appendix.

Use with Lesson 9-2.

52. **REASONING** The sum of any two rational numbers is always a rational number. So, the set of rational numbers is said to be closed under addition. Determine whether the set of rational expressions is closed under addition, subtraction, multiplication, and division by a nonzero rational expression. Justify your reasoning.
See Answer Appendix.

66 | Additional Exercises

Additional Exercises

Use with Lesson 9-3.

53. **MULTIPLE REPRESENTATIONS** In this problem you will investigate the similarities and differences between power functions with positive and negative exponents.

a. **TABULAR** Make a table of values for $a(x) = x^2$, $b(x) = x^{-2}$, $c(x) = x^3$, and $d(x) = x^{-3}$.

b. **GRAPHICAL** Graph $a(x)$ and $b(x)$ on the same coordinate plane.

c. **VERBAL** Compare the domain, range, end behavior, and behavior at $x = 0$ for $a(x)$ and $b(x)$.

d. **GRAPHICAL** Graph $c(x)$ and $d(x)$ on the same coordinate plane.

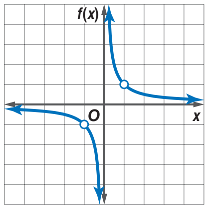
e. **VERBAL** Compare the domain, range, end behavior, and behavior at $x = 0$ for $c(x)$ and $d(x)$.

f. **ANALYTICAL** What conclusions can you make about the similarities and differences between power functions with positive and negative exponents?

a-f. See Answer Appendix.

Use with Lesson 9-4.

54. **CHALLENGE** Compare and contrast $g(x) = \frac{x^2 - 1}{x(x^2 - 2)}$ and the function $f(x)$ shown in the graph.
See Answer Appendix.



Use with Lesson 10-6.

55. **CHALLENGE** When a plane passes through the vertex of a cone, a *degenerate* conic is formed.

a. Determine the type of conic represented by $4x^2 + 8y^2 = 0$. **ellipse**

b. Graph the conic. **See Answer Appendix.**

c. Describe the difference between this degenerate conic and a standard conic of the same type with $A = 4$ and $B = 8$. **See Answer Appendix.**

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Lesson 3 Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. 7, 2
$x^2 - 9x + 14 = 0$

2. 0, 3
$x^2 - 3x = 0$

3. −5, 8
$x^2 - 3x - 40 = 0$

4. −7, −8
$x^2 + 15x + 56 = 0$

5. −6, −3
$x^2 + 9x + 18 = 0$

6. 3, −4
$x^2 + x - 12 = 0$

7. $1, \frac{1}{2}$
$2x^2 - 3x + 1 = 0$

8. $\frac{1}{3}, 2$
$3x^2 - 7x + 2 = 0$

9. $0, -\frac{7}{2}$
$2x^2 + 7x = 0$

Factor each polynomial.

10. $r^3 + 3r^2 - 54r$
$r(r + 9)(r - 6)$

11. $8a^2 + 2a - 6$
$2(4a - 3)(a + 1)$

12. $c^2 - 49$
$(c - 7)(c + 7)$

13. $x^3 + 8$
$(x + 2)(x^2 - 2x + 4)$

14. $16r^2 - 169$
$(4r + 13)(4r - 13)$

15. $b^4 - 81$
$(b^2 + 9)(b + 3)(b - 3)$

Solve each equation by factoring.

16. $x^2 - 4x - 12 = 0$ $\{6, -2\}$

17. $x^2 - 16x + 64 = 0$ $\{8\}$

18. $x^2 - 6x + 8 = 0$ $\{2, 4\}$

19. $x^2 + 3x + 2 = 0$ $\{-2, -1\}$

20. $x^2 - 4x = 0$ $\{0, 4\}$

21. $7x^2 = 4x$ $\left\{0, \frac{4}{7}\right\}$

22. $10x^2 = 9x$ $\left\{0, \frac{9}{10}\right\}$

23. $x^2 = 2x + 99$ $\{-9, 11\}$

24. $x^2 + 12x = -36$ $\{-6\}$

25. $5x^2 - 35x + 60 = 0$ $\{3, 4\}$

26. $36x^2 = 25$ $\left\{\frac{5}{6}, -\frac{5}{6}\right\}$

27. $2x^2 - 8x - 90 = 0$ $\{9, -5\}$

28. NUMBER THEORY Find two consecutive even positive integers whose product is 624.
24, 26

29. NUMBER THEORY Find two consecutive odd positive integers whose product is 323.
17, 19

30. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet. **7 ft by 9 ft**

31. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced? **2 in.**

Lesson 9 Practice

Designing a Study

Determine whether each situation calls for a *survey,* an *experiment*, or an *observational study.* Explain your reasoning.

1. You want to compare the health of students who walk to school to the health of students who ride the bus.

Observational study; the health of students who walk to school and who ride the bus will be observed and compared without them being affected by the study.

2. You want to find out if people who eat a candy bar immediately before a math test get higher scores than people who do not.

Experiment; a group of students will need to eat a candy bar before a math test, which means that members of the sample will be affected by the study.

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

3. What is your current age?

unbiased

4. Do you think teachers should be required to attend all home and away football games?

Biased; the question addresses more than one issue but allows for only one answer.

5. Do you agree or disagree with the following statement?
Teachers should not be required to not supervise students during lunch.

Biased; the question is confusing because it introduces a double-negative.

6. Most teenagers text message during class. Are you one of them?

Biased; the question encourages student to respond yes.

7. A research group wants to conduct an experiment to test the claim that student who use laptops in class have higher standardized test scores. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Objective: to determine whether a student who uses a laptop in class has higher standardized test scores than a student who does not use a laptop in class; population: all high school students; experimental group: class of 25 students given laptops to use in class; control group: class of 25 students who do not have access to laptops in class ; sample procedure: Randomly select the 25 high school students of the same grade level for each group. At the end of the school year, give each class the same standardized test and compare the results.

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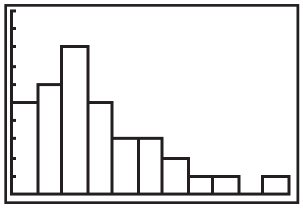
Lesson 10 Practice

Distributions of Data

1. KENNAL The manager of a kennel records the weights for a sample of dogs currently being housed.

Weight (pounds)
31, 67, 8, 37, 12, 87, 14, 34, 105, 57, 42, 8, 16, 54, 17, 20, 72, 23, 27, 63, 24, 52, 14, 44, 27, 5, 28, 22, 33, 15, 6, 36, 41, 21, 46

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution. **positively skewed**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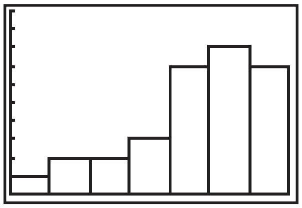
b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice. **Sample answer: The distribution is skewed, so use the five-number summary. The range is 4 to 105 pounds. The median weight is 27 pounds, and half of the dogs' weights are between 15 and 46 pounds.**

2. CAMP The enrollment for a biannual computer camp over the past 15 years is shown.

Number of Participants
45, 68, 55, 25, 48, 36, 61, 52, 31, 8, 41, 58, 40, 55, 68, 47, 60, 28, 44, 63, 18, 68, 50, 57, 37, 16, 56, 40, 50, 68

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
negatively skewed

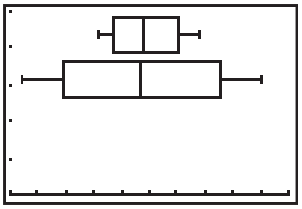

[0, 70] scl: 10 by [0, 10] scl: 1

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice. **Sample answer: The distribution is skewed, so use the five-number summary. The range is 8 to 68 participants. The median is 49, and half of the camps had between 37 and 48 participants.**

3. TEMPERATURES The monthly average low temperatures for two cities are shown.

Astoria, OR	Boston, MA
36, 51, 37, 42, 54, 39, 53, 42, 46, 38, 50, 47	22, 57, 46, 24, 31, 41, 64, 50, 28, 59, 65, 38

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution. **both symmetric**


[20, 70] scl: 5 by [0, 5] scl: 1

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean for Astoria is about 44.6 with standard deviation of about 6.2. The mean for Boston is about 43.8 with standard deviation of about 14.8. While the average low temperatures for the cities are approximately equal, the greater standard deviation for Boston means that Boston's low temperatures have a greater variability than Astoria's temperatures.

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Lesson 11 Practice

Probability Distributions

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

1. the number of bytes in the memory of a computer **discrete**

2. the world population **discrete**

3. the mass of a banana **continuous**

4. the speed of a car **continuous**

5. **COINS** A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

   a. P(2 pennies) $\frac{1}{100}$ **b.** P(2 dimes) $\frac{1}{50}$ **c.** P(1 nickel and 1 dime) $\frac{8}{75}$

   d. P(1 quarter and 1 penny) $\frac{1}{10}$ **e.** P(1 quarter and 1 nicke l) $\frac{4}{15}$ **f.** P(2 dimes and 1 quarter) **0**

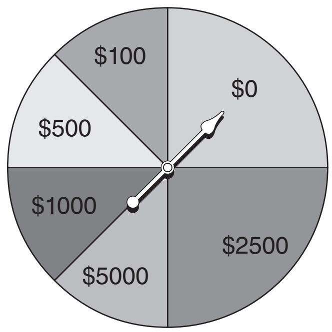
6. **CARDS** Chuck is drawing a card from a special deck that includes the following cards.

Card Value	1	2	3	4	5	6	7
Frequency	6	10	9	4	8	7	6

   What is the expected value of the drawn card? **3.86**

7. **GAMES** A contestant won two spins of the wheel.

   a. Construct a relative-frequency table.



Sum ($)	0	100	200	500	600	1000	1100	1500	2000	2500
Relative Frequency	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{5}{64}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{8}$
Sum ($)	2600	3000	3500	5000	5100	5500	6000	7500	10,000	
Relative Frequency	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{64}$	

   b. What is the expected value of two spins? **$1825**

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Lesson 12 Practice

The Binomial Distribution

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state *n*, *p*, and *q*.

1. You randomly remove one card from a deck to see if it is a heart. You place the card back in the deck and repeat the process five times. **Sample answer: This experiment can be reduced to a binomial experiment. Success is a heart, failure is not a heart, a trial is removing a card, and the random variable is the number of hearts; $n = 5$, $p = 0.25$, $q = 0.75$.**

2. A bag has 8 blue chips, 5 red chips, and 8 white chips. Four chips are removed without replacement to see how many red chips are removed. **Sample answer: This experiment cannot be reduced to a binomial experiment because the events are not independent. The probability of choosing a red chip changes after each selection.**

3. BOARD GAME When Tarin and Sam play a certain board game, the probability that Tarin will win a game is $\frac{3}{4}$. If they play 5 games, find each probability.

a. *P*(Sam wins only once)

0.396 or 39.6%

b. *P*(Tarin wins exactly twice)

0.088 or 8.8%

c. *P*(Sam wins exactly 3 games)

0.088 or 8.8%

d. *P*(Sam wins at least 1 game)

0.763 or 76.3%

e. *P*(Tarin wins at least 3 games)

0.896 or 89.6%

f. *P*(Tarin wins at most 2 games)

0.104 or 10.4%

4. SAFETY In August 2001, the American Automobile Association reported that 73% of Americans use seat belts. In a random selection of 10 Americans in 2001, what is the probability that exactly half of them use seat belts? **0.075 or 7.5%**

5. HEALTH In 2001, the American Heart Association reported that 50 percent of the Americans who receive heart transplants are ages 50–64 and 20 percent are ages 35–49.

a. In a randomly selected group of 10 heart transplant recipients, what is the probability that at least 8 of them are ages 50–64? **0.055 or 5.5%**

b. In a randomly selected group of 5 heart transplant recipients, what is the probability that 2 of them are ages 35–49? **0.205 or 20.5%**

Lesson 13 Practice

The Normal Distribution

A normal distribution has a mean of 186.4 and a standard deviation of 48.9.

1. What range of values represents the middle 99.7% of the data? **$39.7 < X < 333.1$**

2. What percent of data will be more than 235.3? **16%**

3. What range of values represents the upper 2.5% of the data? **$X > 284.2$**

Find the missing variable. Indicate the position of *X* in the distribution.

4. σ if $\mu = 19, X = 21$, and $z = 1.3$ **1.54**

5. μ if $\sigma = 9.8, X = 55.4$, and $z = -1.32$ **68.34**

6. X if $z = -2.19, \mu = 68.2$, and $\sigma = 11.6$ **42.80**

7. z if $\mu = 112.4, X = 119.2$, and $\sigma = 11.9$ **3.58**

8. **TESTING** The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.
   a. About what percent of the scores are between 70 and 80? **4.5%**
   b. About what percent of the scores are between 85 and 115? **77.7%**
   c. About what percent of the scores are over 115? **11.1%**
   d. About what percent of the scores are lower than 90 or higher than 100? **41.6%**
   e. If 80 people take the test, how many would you expect to score higher than 130? **about 1**
   f. If 75 people take the test, how many would you expect to score lower than 75? **about 2**

9. **TEMPERATURE** The daily July surface temperature of a lake at a resort has a mean of 82˚ and a standard deviation of 4.2˚. If you prefer to swim when the temperature is at least 80˚, about what percent of the days does the temperature meet your preference? **68.3%**

Lesson 15 Practice

Confidence Intervals and Hypothesis Testing

Find a 99% confidence interval for each of the following. Round to the nearest tenth if necessary.

1. $\bar{x} = 56$, $s = 2$, and $n = 50$
55.3 ≤ $\bar{x}$ ≤ 56.7

2. $\bar{x} = 99$, $s = 22$, and $n = 121$
93.8 ≤ $\bar{x}$ ≤ 104.2

3. $\bar{x} = 34$, $s = 4$, and $n = 200$
33.3 ≤ $\bar{x}$ ≤ 34.7

4. $\bar{x} = 12$, $s = 4.5$, and $n = 100$
10.8 ≤ $\bar{x}$ ≤ 13.2

5. $\bar{x} = 37$, $s = 2.5$, and $n = 50$
36.1 ≤ $\bar{x}$ ≤ 37.9

6. $\bar{x} = 78$, $s = 2$, and $n = 225$
77.7 ≤ $\bar{x}$ ≤ 78.3

7. $\bar{x} = 36$, $s = 6$, and $n = 36$
33.4 ≤ $\bar{x}$ ≤ 38.6

8. $\bar{x} = 121$, $s = 2.5$, and $n = 100$
120.4 ≤ $\bar{x}$ ≤ 121.6

Test each null hypothesis at 1% significance. Write *reject* or *fail to reject*.

9. $H_0 \geq 200.1$, $H_a < 200.1$, $n = 50$, $\bar{x} = 200$, and $s = 2$ **fail to reject**

10. $H_0 \geq 75.6$, $H_a < 75.6$, $n = 100$, $\bar{x} = 77$, and $s = 7$ **fail to reject**

11. $H_0 \geq 89.3$, $H_a < 89.3$, $n = 100$, $\bar{x} = 89$ and $s = 1.5$ **fail to reject**

12 $H_0 \geq 75$, $H_a < 75$, $n = 150$, $\bar{x} = 74.2$, and $s = 2.5$ **reject**

13. $H_0 \geq 121$, $H_a < 121$, $n = 64$, $\bar{x} = 120$, and $s = 2$ **reject**

14. $H_0 \leq 198.5$, $H_a > 198.5$, $n = 100$, $\bar{x} = 200$, and $s = 7.5$ **fail to reject**

15. $H_0 \leq 38.5$, $H_a > 38.5$, $n = 50$, $\bar{x} = 40$, and $s = 4.5$ **reject**

16. $H_0 \geq 112.5$, $H_a < 112.5$, $n = 100$, $\bar{x} = 110.5$, and $s = 10$ **fail to reject**

17. RUNNING Josh and his sister Megan run together each morning and do not use a stopwatch to keep track of their time. Josh thinks they usually run the mile under 7 minutes, while Megan thinks it takes them longer. They borrow a stopwatch and time themselves each day for 20 days. Their mean time to run one mile is 7.4 minutes with a standard deviation of 0.2 minutes. Test Megan's hypothesis at 10% significance.
fail to reject

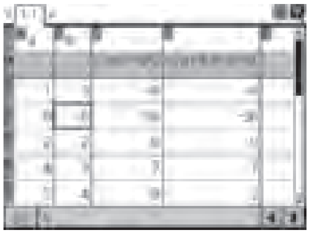
18. QUALITY CONTROL Kim is a quality tester for a tropical fruit company. The company claims that their canned pineapple stays fresh for at least 16 hours after opening. Kim tests 15 different cans to see if they actually stay fresh for at least 16 hours. Use the data below to conduct a hypothesis test at 5% significance.

Number of Hours Each Can Stays Fresh				
12	14	7	12	10
12	12	13	16	9
5	11	19	18	6

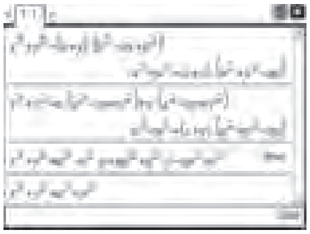
reject

Lab 6

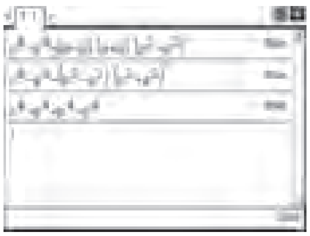
1. Sample answer:



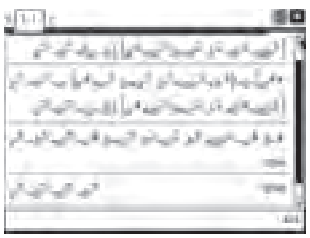
2. Sample answer:



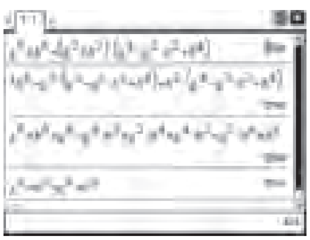
3. Sample answer:



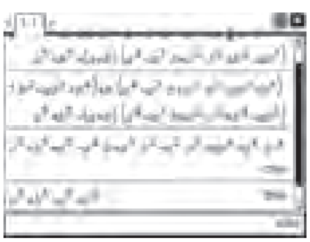
4. Sample answer:



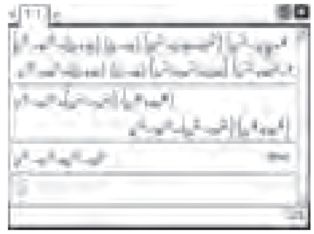
5. Sample answer:



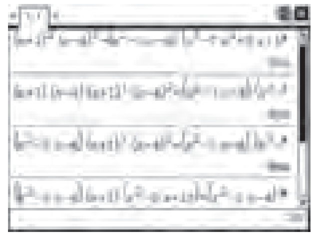
6. Sample answer:



7. Sample answer:

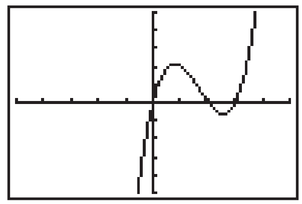


8. Sample answer:



Lab 7

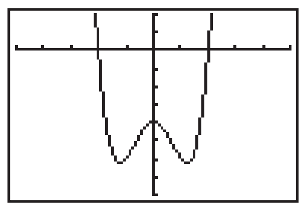
1.



[−5, 5] scl: 1 by [−5, 5] scl: 1

$x = 0, 2, 3$; rel. max. at (0.78, 2.11), min. at (2.55, −0.63); 3 roots found; y-int. 0; no symmetry

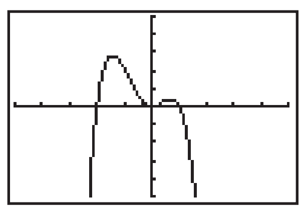
2.



[−5, 5] scl: 1 by [−8, 2] scl: 1

$x = -2, 3$; rel. max. at (0, −4), min. at (−1.22, −6.25), (−1.22, −6.25); 2 roots found, either multiple roots or 2 imaginary roots; y-int. 0; line of symmetry $x = 0$

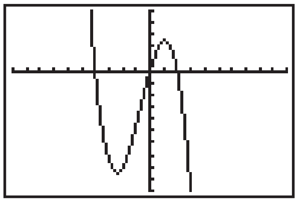
3.



[−5, 5] scl: 1 by [−5, 5] scl: 1

$x = -2, 0, 1$; max. at (−1.44, 2.8), rel. max. at (0.69, 0.40), min. at (0, 0); 3 roots found, either multiple roots or 1 imaginary root; y-int. 0; no symmetry

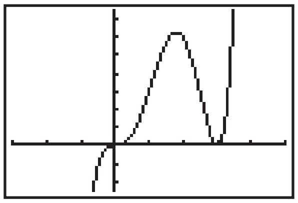
4.



[−10, 10] scl: 1 by [−40, 20] scl: 4

$x = -4, 0, 3$; rel. max. at (1.10, 10.10), rel. min. at (−2.43, −33.8); 3 roots found; y-int. 0; no symmetry

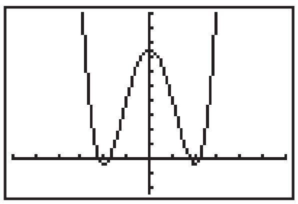
5.



[−3, 5] scl: 1 by [−10, 30] scl: 4

$x = 0, 3$; rel. max. at (1.80, 25.19), rel. min. at (3, 0); 2 roots found, either multiple roots or 3 imaginary roots; y-int. 0; no symmetry

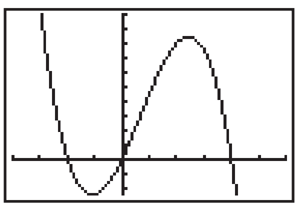
6.



[−6, 6] scl: 1 by [−5, 20] scl: 2

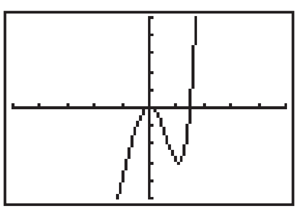
$x = -2.23, -1.73, 4, 1.73, 2.23$; rel. max. at (0, 15), min. at (−2, −1), (−2, 1); 4 roots found; y-int. 15; line of symmetry of $x = 0$

7.



$x = -2, 0, 4$; rel. max. at (2.43, 16.90), rel. min. at (−1.10, −5.05); 3 roots found, either multiple root or 1 imaginary root; y-int. 0; no symmetry

8.



[−5, 5] scl: 1 by [−10, 10] scl: 2

$x = 0, 1.5$; rel. max. at (0, 0), min. at (1.07, −6.11); 2 roots found, either multiple roots or 3 imaginary roots; y-int. 0; no symmetry

Lesson 8

3. Observation study; sample answer: The scores of the participants are observed and compared without them being affected by the study.

4. Experiment; sample answer: A sample of dyed shirts will need to be tested, which means that the members of the sample will be affected by the study.

7. objective: to determine how many people in the U.S. are interested in purchasing a hybrid vehicle; population: the people surveyed; sample survey questions: Do you currently own a hybrid vehicle? Are you planning on purchasing a hybrid vehicle?

8. Sample answer: The flaw is that the experimental group consists of students, and the control group consists of instructors. On average, college students are younger than their instructors, and thus, are more likely to score higher on a physical test whether given a vitamin or not.

9. objective: to determine whether the protein shake helps athletes recover from exercise; population: all athletes; experiment group: athletes given the protein shake; control group: athletes given a placebo; sample procedure: The researchers could randomly divide the athletes into two groups: an experimental group given the protein shake and a control group given the placebo. Next, they could have the athletes exercise and then drink the protein shake or placebo. Later, the researchers could interview the athletes to see how they feel.

10. survey; sample: customers that take the online survey; population: all customers

11. observational study; sample: physics students selected; population: all college students that take a physics course

12. experiment; sample: adults participating in the study; population: all adults

13. survey; sample: people that receive the questionnaire; population: all viewers

24a. objective: to determine whether a mixture of salt and lemon juice will remove copper oxide from copper objects; population: all copper objects; experimental group: pennies that are submerged in the mixture; control group: pennies that are submerged in a placebo mixture; sample procedure: Pedro could randomly assign the pennies into two groups, and create the lemon and salt mixture. Next, he could submerge the experimental group of pennies in the mixture for a certain period of time, remove them, and then visually compare the two groups of pennies.

24b. Sample answer: The pennies should be chosen so that they have roughly the same amount of copper oxide and are from the same time period. Using these guidelines could eliminate bias regarding the initial condition and composition, so that the conditions of the experimental and control groups are exactly the same at the start of the experiment.

27a. See students' work.

27b. Sample answer for Product A: ≈63.3%

Product A	
Number	**Frequency**
0–6	𝍸 𝍸 𝍸 ||||
7–9	𝍸 𝍸 |

Sample answer for Product B: ≈76.7%

Product B	
Number	**Frequency**
0–7	𝍸 𝍸 𝍸 𝍸 |||
8–9	𝍸 ||

27c. Sample answer: Yes; the probability that Product B is effective is 14.4% higher than that of Product A.

27d. Sample answer: It depends on what the product is and how it is being used. For example, if the product is a pencil sharpener, then the lower price may be more important than the effectiveness, and therefore, might not justify the price difference. However, if the product is a life-saving medicine, the effectiveness may be more important than the price, and therefore, might justify the price difference.

28. False; sample answer: A sample statistic is used to estimate a population parameter.

30. Sample answer:
objective: Determine the average amount of time that students spend studying at the library.
population: All students that study at the library.
sample: 30 randomly selected students studying at the library during a given week.

Study Time (minutes)				
38	16	45	41	63
18	20	17	8	15
41	28	55	19	15
30	11	20	79	24
78	24	26	32	19

31. Sample answer: the sampling method used, the type of sample that was selected, the type of study performed, the survey question(s) that were asked or procedures that were used

32a. Sample answer: In a convenience sample, members are selected based on the convenience of the researcher. One example is handing a survey to shoppers as they walk out of the mall. This method could result in bias if the members of the population who are readily available to be sampled are not representative of the entire population.

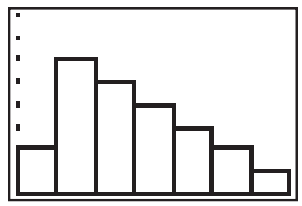
32b. Sample answer: In a self-selected sample, members volunteer to be in the sample. This method could result in bias if certain groups of people in the population choose not to volunteer.

32c. Sample answer: In a stratified sample, the population is first divided into similar, nonoverlapping groups, and members are then randomly selected from each group. This method could result in bias if the entire population is not represented when divided into groups or if the members are not randomly selected from each group.

32d. Sample answer: In a systematic sample, a rule is used to select the members. This method could result in bias if the rule does not include everyone in the population.

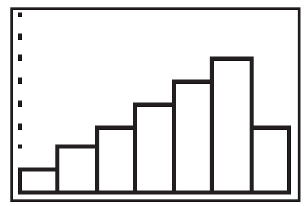
Lesson 10

3a. Mrs. Johnson's Class



[5, 40] scl: 5 by [0, 8] scl: 1

Mr. Edmunds' Class

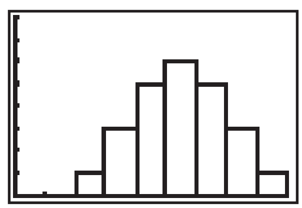


[5, 40] scl: 5 by [0, 8] scl: 1

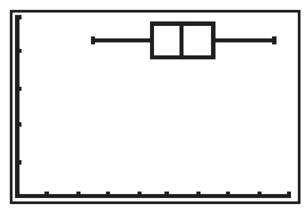
Mrs. Johnson's class, positively skewed; Mr. Edmunds' class, negatively skewed

3b. Sample answer: The distributions are skewed, so use the five-number summaries. The range for both classes is the same. However, the median for Mrs. Johnson's class is 17 and the median for Mr. Edmunds' class is 28. The lower quartile for Mr. Edmunds' class is 20. Since this is greater than the median for Mrs. Johnson's class, this means that 75% of the data from Mr. Edmunds' class is greater than 50% of the data from Mrs. Johnson's class. Therefore, we can conclude that the students in Mr. Edmunds' class had slightly higher sales overall than the students in Mrs. Johnson's class.

6a.



[0, 18] scl: 2 by [0, 8] scl: 1



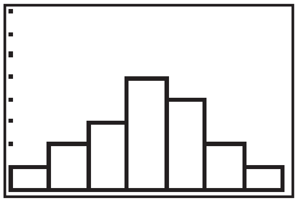
[0, 18] scl: 2 by [0, 5] scl: 1

symmetric

6b. Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean number of movies watched was about 10.7 with standard deviation of about 3 movies.

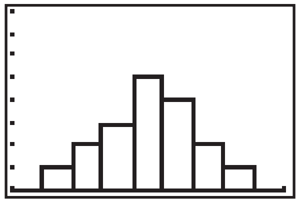
7a.

Sophomore Year



[1200, 1900] scl: 100 by [0, 8] scl: 1

Junior Year



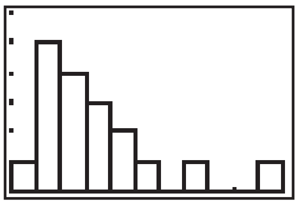
[1300, 2200] scl: 100 by [0, 8] scl: 1

both symmetric

7b. Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean score for sophomore year is about 1552.9 with standard deviation of about 147.2. The mean score for junior year is about 1753.8 with standard deviation of about 159.1. We can conclude that the scores and the variation of the scores from the mean both increased from sophomore year to junior year.

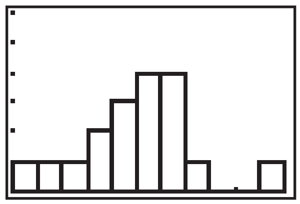
8a.

Yorkshire



[50, 105] scl: 5 by [0, 6] scl: 1

Applewood

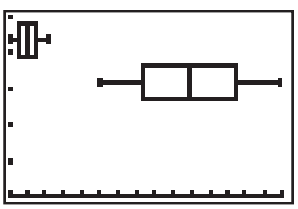


[35, 90] scl: 5 by [0, 6] scl: 1

Yorkshire, positively skewed; Applewood, negatively skewed

8b. Sample answer: The distributions are skewed, so use the five-number summaries. The median for Yorkshire is 63.5, and the median for Applewood is 60. The lower quartile for Applewood is 52, while the minimum for Yorkshire is 53. This means that 25% of the incomes for Applewood are lower than any of the incomes for Yorkshire. Also, the upper 25% of incomes for Yorkshire is between 72 and 103, while the upper 25% of incomes for Applewood is between 65 and 87. We can conclude that the incomes for the households in Yorkshire are greater than the incomes for the households in Applewood.

9a.

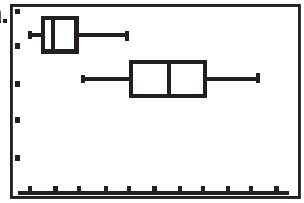


[3000, 18,000] scl: 1000 by [0, 5] scl: 1

both symmetric

9b. Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean for the public colleges is $4037.50 with standard deviation of about $621.93. The mean for private colleges is about $12,803.11 with standard deviation of about $2915.20. We can conclude that not only is the average cost of private schools far greater than the average cost of public schools, but the variation of the costs from the mean is also much greater.

10a.

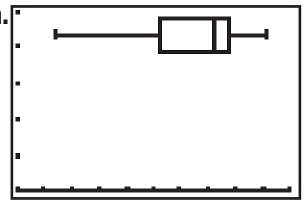


[225, 775] scl: 50 by [0, 5] scl: 1

boys, positively skewed; girls, symmetric

10b. Sample answer: One distribution is skewed and the other is symmetric, so use the five-number summaries. The maximum value for the boys is 450. The lower quartile for the girls is 453. This means that 75% of the data for the girls is higher than any data for the boys. We can conclude that 75% of the girls spent more money on prom than any of the boys.

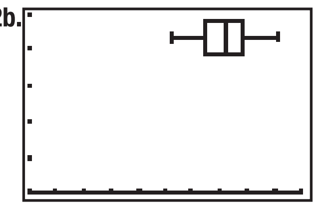
12a.



[50, 100] scl: 5 by [0, 5] scl: 1

Sample answer: The distribution is negatively skewed, so use the five-number summary. The scores range from 57 to 96. The median is 86, and half of the data are between 76 and 89.

12b.



[50, 100] scl: 5 by [0, 5] scl: 1

Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean is about 85.6 with standard deviation of about 5.9.

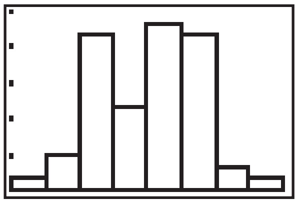
14. Sample answer: Since the distribution has two clusters, an overall summary of center and spread would give an inaccurate depiction of the data. Instead, summarize the center and spread of each cluster individually using its mean and standard deviation.

15. Sample answer: The heights of the players on the Pittsburgh Steelers roster appear to represent a normal distribution.

Heights of the Players on the 2009 Pittsburgh Steelers Roster (inches)

75	74	71	70	74	75	77	72
71	72	70	70	75	78	71	75
77	71	69	70	77	75	74	73
77	71	73	76	76	74	72	75
75	70	70	74	73	76	79	73
71	69	70	77	77	80	75	77
67	74	69	76	77	76		

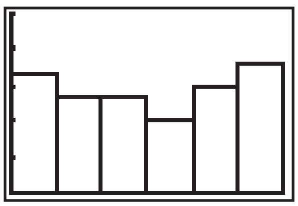
The mean of the data is about 73.61 in. or 6 ft 1.61 in. The standard deviation is about 2.97 in.



[66, 82] scl: 2 by [0, 15] scl: 3

The birth months of the players do not display central tendency.

Birth Months of the Players on the 2009 Pittsburgh Steelers Roster							
1	12	10	3	11	1	10	5
4	8	9	11	1	1	11	5
8	6	11	4	3	4	8	5
3	7	2	1	11	4	3	2
1	1	6	1	6	8	11	9
3	3	1	6	9	1	9	9
6	5	10	11	11	12		



[0, 12] scl: 2 by [0, 15] scl: 3

16. Sample answer: The distribution for a data set is positively skewed when the majority of the data is on the left of the mean and a tail appears to the right of the mean. An example is when the data set includes the height of everyone in an elementary school, most of the data will be on the left side (the students), while a comparatively small amount will be on the right (the teachers and staff). The distribution for a data set is negatively skewed when the majority of the data is on the right of the mean and a tail appears to the left of the mean. An example is when the batting averages of a baseball lineup are listed, most of the data will be at a certain level while the pitcher will typically be much lower. The distribution for a data set is symmetric when the data are evenly distributed on both sides of the mean. An example is when test scores are calculated for an entire state, most of the students will place in the middle, while some will place above or below.

Lesson 11 (Guided Practice)

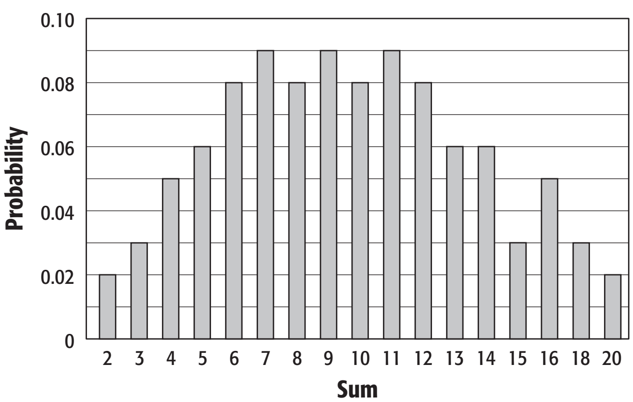
1A. The random variable X is the distance traveled for each throw. Distance can be anywhere within a certain range. Therefore, X is continuous.

1B. The random variable X is the ages of the counselors. The ages are countable, so X is discrete.

2A.

Sum	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20
Frequency	1	2	3	4	5	6	5	6	5	6	5	4	4	2	3	2	1
Relative	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{3}{64}$	$\frac{1}{16}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{5}{64}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{3}{64}$	$\frac{1}{32}$	$\frac{1}{64}$

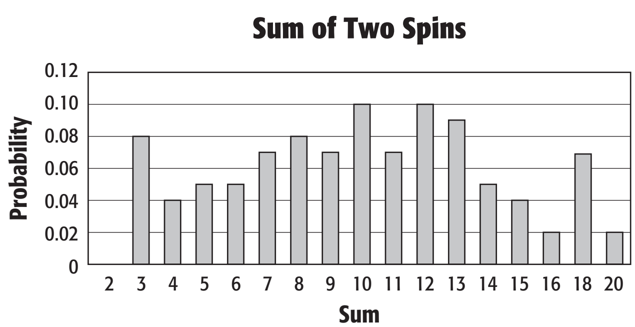
2B.



3A.

Sum	Frequency	Relative Frequency
2	0	0
3	8	0.08
4	4	0.04
5	5	0.05
6	5	0.05
7	7	0.07
8	8	0.08
9	7	0.07
10	10	0.10
11	7	0.07
12	10	0.10
13	9	0.09
14	5	0.05
15	4	0.04
16	2	0.02
18	7	0.07
20	2	0.02

3B.

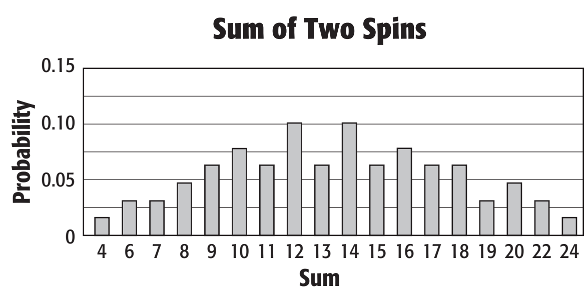


Lesson 11

5a.

Sum	Frequency	Relative Frequency
4	1	$\frac{1}{64}$
6	2	$\frac{1}{32}$
7	2	$\frac{1}{32}$
8	3	$\frac{3}{64}$
9	4	$\frac{1}{16}$
10	5	$\frac{5}{64}$
11	4	$\frac{1}{16}$
12	7	$\frac{7}{64}$
13	4	$\frac{1}{16}$
14	7	$\frac{7}{64}$
15	4	$\frac{1}{16}$
16	5	$\frac{5}{64}$
17	4	$\frac{1}{16}$
18	4	$\frac{1}{16}$
19	2	$\frac{1}{32}$
20	3	$\frac{3}{64}$
22	2	$\frac{1}{32}$
24	1	$\frac{1}{64}$

5b.



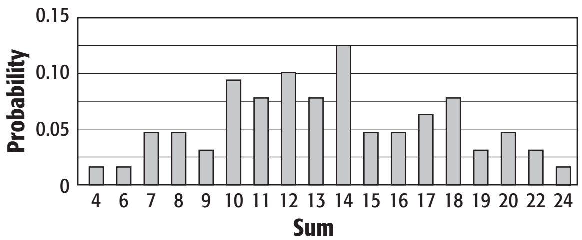
Answer Appendix

5c.

Sum	Frequency	Relative Frequency
4	1	$\frac{1}{64}$
6	1	$\frac{1}{64}$
7	3	$\frac{3}{64}$
8	3	$\frac{3}{64}$
9	2	$\frac{1}{32}$
10	6	$\frac{3}{32}$
11	5	$\frac{5}{64}$
12	7	$\frac{7}{64}$
13	5	$\frac{5}{64}$
14	8	$\frac{1}{8}$
15	3	$\frac{3}{64}$
16	3	$\frac{3}{64}$
17	4	$\frac{1}{16}$
18	5	$\frac{5}{64}$
19	2	$\frac{1}{32}$
20	3	$\frac{3}{64}$
22	2	$\frac{1}{32}$
24	1	$\frac{1}{64}$

5d.

Sum of Two Spins

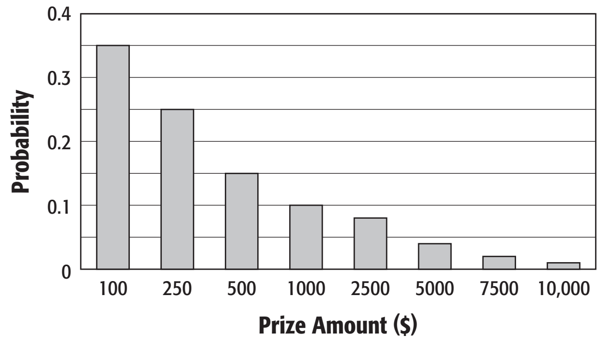


10a.

Prize, *X*	P(*X*)
$100	0.35
$250	0.25
$500	0.15
$1000	0.10
$2500	0.08
$5000	0.04
$7500	0.02
$10,000	0.01

10b.

Prizes Won

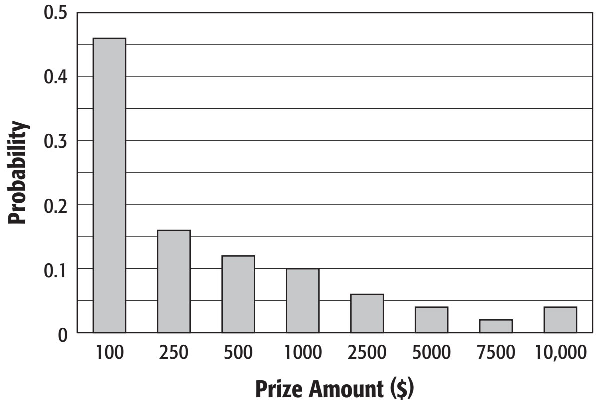


10c.

Prize, *X*	Frequency	Relative Frequency
$100	23	0.46
$250	8	0.16
$500	6	0.12
$1000	5	0.10
$2500	3	0.06
$5000	2	0.04
$7500	1	0.02
$10,000	2	0.04

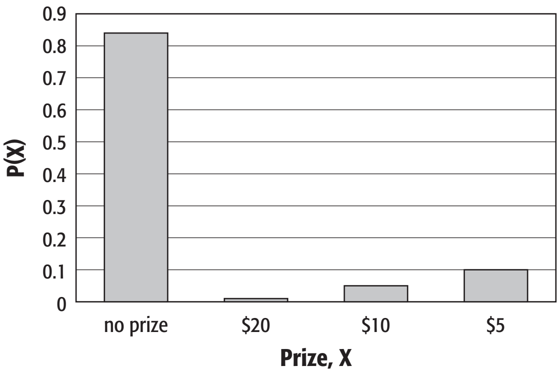
10d.

Prizes Won



13a.

Probability to Win Each Prize



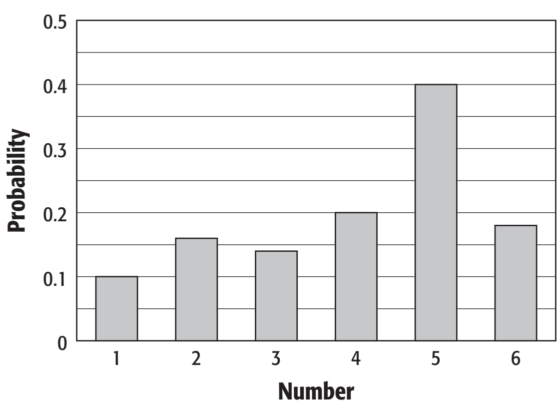
14a. 4.2; Sample answer: The expected number is 4.2, so we can expect there to be 4 students running. We cannot have 0.2 people, so we round to the nearest whole number.

14b. Sample answer:

Number of Students, *X*	Frequency	Relative Frequency
1	5	0.10
2	8	0.16
3	7	0.14
4	1	0.02
5	20	0.40
6	9	0.18

14c. Sample answer:

Number of Students Running

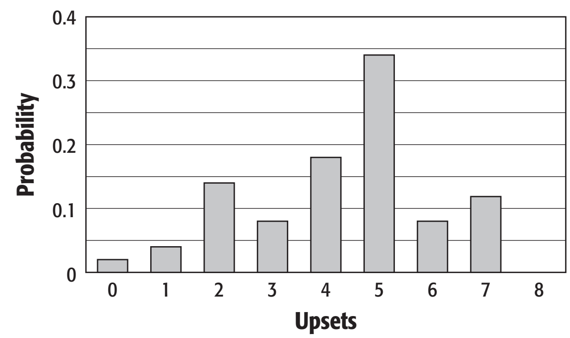


15c.

Number of Students, *X*	Frequency	Relative Frequency
0	1	0.02
1	2	0.04
2	7	0.14
3	4	0.08
4	9	0.18
5	17	0.34
6	4	0.08
7	6	0.12
8	0	0

15d.

Number of Upsets



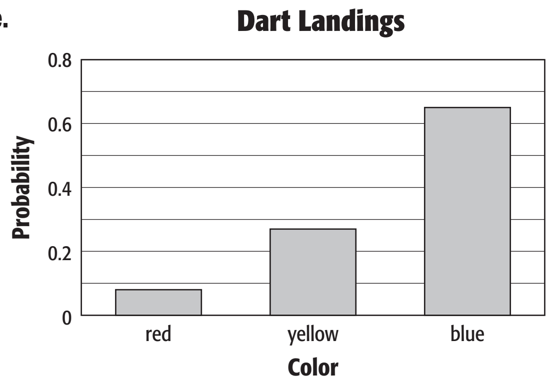
16c. Sample answer: The standard deviation of the probability distribution for the Glee Club raffle is about half the standard deviation for the French Club raffle, so the Glee Club raffle is less risky. Since they have similar expected values, the riskier raffle will also have the potential to win more, so both raffles have good and bad qualities. It is up to the individual participant to decide which one to choose.

17. Sample answer: The expected value of Funds A and B is $595 and $540, respectively. The standard deviation for Fund A is about 951.6, while the standard deviation for Fund B is about 941.5. Since the standard deviations are about the same, the funds have about the same amount of risk. Therefore, with a higher expected value, Fund A is the better investment.

18d.

Color	Frequency	Relative Frequency
red	8	0.08
yellow	27	0.27
blue	65	0.65

18e.



21. Sample answer: A spinner with 5 equal-sided areas shaded red, blue, yellow, green, and brown.

Color	red	blue	yellow	green	brown
Probability	0.2	0.2	0.2	0.2	0.2

22. Sample answer: False; a random variable X representing the number of Web sites on the Internet at any given time is infinite and countable. However, there cannot be 1.3 Web sites, so it is also discrete.

23. Sample answer: A discrete probability distribution can be the uniform distribution of the roll of a die. In this type of distribution, there are only a finite number of possibilities. A continuous probability distribution can be the distribution of the lives of 400 batteries. In this distribution, there are an infinite number of possibilities.

24. Sample answer: Since the investments have identical expected values, an investor would expect to earn the same amount of money on each investment. However, since they have significantly different standard deviations, the investment with the higher standard deviation is much riskier than the other investment. The greater standard deviation indicates a greater variability, so the riskier investment will have an opportunity to earn more money than the other investment, but it will also have an opportunity to lose more as well.

Lesson 12 (Guided Practice)

1A. This experiment can be reduced to a binomial experiment. Success is yes, failure is no, a trial is asking a student, and the random variable is the number of yeses; $n = 75$, $p = 0.34$, $q = 0.66$.

1B. This experiment cannot be reduced to a binomial experiment because the events are not independent. The probability of choosing an ace changes after each selection.

2. Sample answer:
Step 1 A trial is drawing a card from a deck. The simulation will consist of 26 trials.
Step 2 A success is drawing an even-numbered card. The probability of success is $\frac{5}{13}$ and the probability of failure is $\frac{8}{13}$.
Step 3 The random variable X represents the number of even-numbered cards drawn in 20 trials.
Step 4 Use a random number generator. Let 0–4 represent drawing an even-numbered card. Let 5–12 represent drawing all other cards. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Even-Numbered Card	𝍸 𝍸	10
Other Cards	𝍸 𝍸	10

The experimental probability is $\frac{10}{20}$ or 50%. This is greater than the theoretical probability of $\frac{5}{13}$ or about 38.5%.

Lesson 12

6a. 0 in favor, 0.00001 or 0.001%;
1 in favor, 0.00039 or 0.039%;
2 in favor, 0.00549 or 0.549%;
3 in favor, 0.04145 or 4.145%;
4 in favor, 0.17618 or 17.618%;
5 in favor, 0.39933 or 39.933%;
6 in favor, 0.37715 or 37.715%

7. This experiment can be reduced to a binomial experiment. Success is a day that it rains, failure is a day it does not rain, a trial is a day, and the random variable X is the number of days it rains; $n =$ the number of days in the month, $p = 0.35$, $q = 0.65$.

8. This experiment cannot be reduced to a binomial experiment because there are more than two possible outcomes.

9. This experiment cannot be reduced to a binomial experiment because the events are not independent. The probability of choosing the hat that covers the ball changes after each selection.

10. Sample answer:
Step 1 A trial is rolling two dice. The simulation will consist of 25 trials.
Step 2 A success is rolling a 7. The probability of success is $\frac{1}{6}$ and the probability of failure is $\frac{5}{6}$.
Step 3 The random variable X represents the number of times a 7 is rolled in 25 trials.
Step 4 Use a random number generator. Let 0 represent rolling a 7. Let 1–5 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Rolling a 7	||||	4
Other Outcomes	𝍸 𝍸 𝍸 𝍸		21

The experimental probability is $\frac{4}{25}$ or 16%. This is approximately equal to the theoretical probability of $\frac{1}{6}$ or about 16.7%.

11. Sample answer:

Step 1 A trial is pulling out a marble. The simulation will consist of 20 trials.

Step 2 A success is pulling out a red marble. The probability of success is $\frac{5}{12}$ and the probability of failure is $\frac{7}{12}$.

Step 3 The random variable X represents the number of red marbles pulled out in 20 trials.

Step 4 Use a random number generator. Let 0–4 represent pulling out a red marble. Let 5–11 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Red Marble	𝍸 𝍸	10
Other Outcomes	𝍸 𝍸	10

The experimental probability is $\frac{10}{20}$ or 50%. This is greater than the theoretical probability of $\frac{5}{12}$ or about 41.7%.

12. Sample answer:

Step 1 A trial is spinning the spinner. The simulation will consist of 25 trials.

Step 2 A success is the spinner landing on an even number. The probability of success is $\frac{2}{5}$ and the probability of failure is $\frac{3}{5}$.

Step 3 The random variable X represents the number of times the spinner stops on an even number in 25 trials.

Step 4 Use a random number generator. Let 0–1 represent the spinner stopping on an even number. Let 2–4 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Even Number	𝍸 𝍸 |	11
Other Outcomes	𝍸 𝍸 ||||	14

The experimental probability is $\frac{11}{25}$ or 44%. This is slightly greater than the theoretical probability of $\frac{2}{5}$ or 40%.

13. Sample answer:

Step 1 A trial is drawing a card from a deck. The simulation will consist of 20 trials.

Step 2 A success is drawing a face card. The probability of success is $\frac{3}{13}$ and the probability of failure is $\frac{10}{13}$.

Step 3 The random variable X represents the number of face cards drawn in 20 trials.

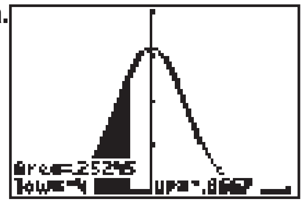
Step 4 Use a random number generator. Let 0–2 represent drawing a face card. Let 3–12 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Even Number	||	2
Other Outcomes	𝍸 𝍸 𝍸 |||	18

The experimental probability is $\frac{2}{20}$ or 10%. This is less than the theoretical probability of $\frac{3}{13}$ or about 23.1%.

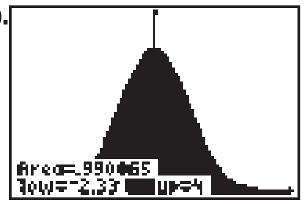
Lesson 13

16a.



[−4, 4] scl: 1 by [0, .5] scl: 0.125

16b.



[−4, 4] scl: 1 by [0, .5] scl: 0.125

19c. Sample answer: I would expect people with several traffic citations to lie to the far right of the distribution where insurance costs are highest, because I think insurance companies would charge them more.

19d. Sample answer: As the probability of an accident occurring increases, the more an auto insurance company is going to charge. I think auto insurance companies would charge younger people more than older people because they have not been driving as long. I think they would charge more for expensive cars and sports cars and less for cars that have good safety ratings. I think they would charge a person less if they have a good driving record and more if they have had tickets and accidents.

20a. Math: 1.03; Science: 0.81; Social Studies: 1.21

20b. Math: 84.8%; Science: 79.1%; Social Studies: 88.7%

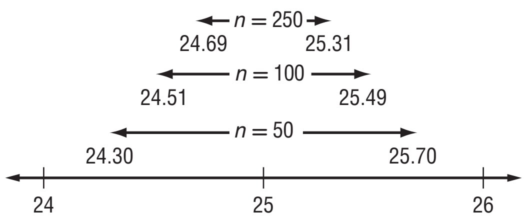
20c. Social Studies; Sample answer: When the distributions are standardized, Nikki's relative scores on each test are 84.8%, 79.1%, and 88.7%. These standardized scores are Nikki's scores in relation to the population of scores for each individual test. Therefore, her 86% on the Social Studies test was better than 88.7% of the other test takers' scores on that test. This is the highest percentage of the three tests, so Nikki performed the best in Social Studies.

25. Sample answer: The scores per team in each game of the first round of the 2010 NBA playoffs. The mean is 96.56 and the standard deviation is 11.06. The middle 68% of the distribution is $85.50 < X < 107.62$. The middle 95% is $74.44 < X < 118.68$. The middle 99.7% is $63.38 < X < 129.74$.

26. Sample answer: The *z*-value represents the position of a value *X* in a normal distribution. A *z*-value of 1.43 means that the corresponding data value *X* is 1.43 standard deviations to the right of the mean in the distribution. An interval of all of the values greater than *X* in the distribution will be represented by the area under the curve from $z = 1.43$ to $z = 4$. This area is equivalent to the probability of the interval occurring (a random data value falling within the interval).

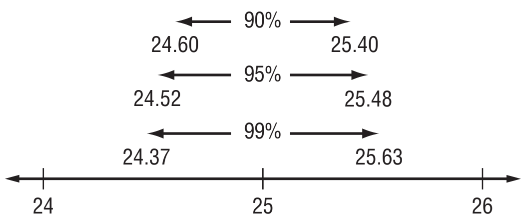
Lesson 15

19a.



19b. Sample answer: With everything else held constant, increasing the same size will decrease the size of the confidence interval.

19c.



19d. Sample answer: With everything else held constant, increasing the confidence level will increase the size of the confidence interval.

19e. Sample answer: Expanding the confidence interval reduces the accuracy of the estimate. So decreasing the size of the confidence interval increases the accuracy of the estimate.

Additional Exercises, Lesson 2-5

2. Sample answer: A strength of using regression equations is that the regression equation can be used to make predictions when the values fall close to the domain of the original data set. A weakness of using regression equations is that they assume that a trend in the original data set will continue, and they are very sensitive to outliers. Both weaknesses can make predictions inaccurate.

Additional Exercises, Lesson 2-7

4. Kimi; sample answer: Each point in the table corresponds to a point on the graph. Therefore they represent the same relation.

Additional Exercises, Lesson 5-1

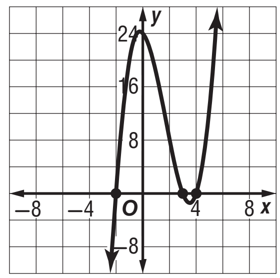
1. Madison; sample answer: The function graphed has a maximum of −2. The function described in words has a maximum of 1.

Additional Exercises, Lesson 6-3

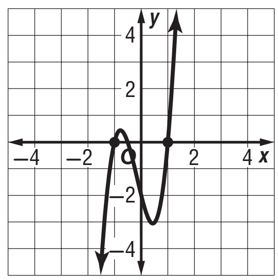
6. $f(x)$; $f(x)$ has a has potential for 5 or more real roots and a degree of 5 or more. $g(x)$ has a potential for 4 real roots and a degree of 4.

Additional Exercises, Lesson 6-7

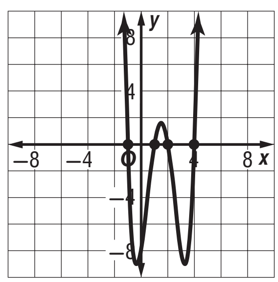
7.



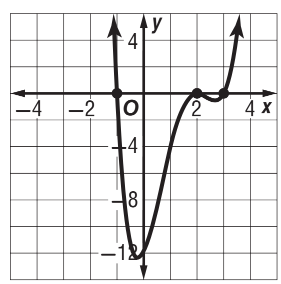
8.



9.



10.



Additional Exercises, Lesson 7-1

11–43. Domains and ranges are all real numbers unless otherwise specified.

11. $(f + g)(x) = 4x + 1$; $(f - g)(x) = -2x + 3$; $(f \cdot g)(x) = 3x^2 + 5x - 2$; $\left(\frac{f}{g}\right)(x) = \frac{x+2}{3x-1}, x \neq \frac{1}{3}$

12. $(f+g)(x) = x^2 - x + 3$; $(f-g)(x) = x^2 + x - 13$;
$(f \cdot g)(x) = -x^3 + 8x^2 + 5x - 40$; $\left(\frac{f}{g}\right)(x) = \frac{x^2-5}{-x+8}, x \neq 8$

13. $(f+g)(x) = -2x + 5$; $(f-g)(x) = 6x - 5$;
$(f \cdot g)(x) = -8x^2 + 10x$; $\left(\frac{f}{g}\right)(x) = \frac{2x}{-4x+5}, x \neq \frac{5}{4}$

14. $(f+g)(x) = 6x - 3$; $(f-g)(x) = -4x + 1$;
$(f \cdot g)(x) = 5x^2 - 7x + 2$; $\left(\frac{f}{g}\right)(x) = \frac{x-1}{5x-2}, x \neq \frac{2}{5}$

15. $(f+g)(x) = x^2 - x + 1$; $(f-g)(x) = x^2 + x - 1$;
$(f \cdot g)(x) = -x^3 + x^2$; $\left(\frac{f}{g}\right)(x) = \frac{x^2}{-x+1}, x \neq 1$

16. $(f+g)(x) = x + 6$; $(f-g)(x) = 5x - 6$;
$(f \cdot g)(x) = -6x^2 + 18x$; $\left(\frac{f}{g}\right)(x) = \frac{3x}{-2x+6}, x \neq 3$

17. $(f+g)(x) = 3x - 9$; $(f-g)(x) = -x + 5$;
$(f \cdot g)(x) = 2x^2 - 11x + 14$; $\left(\frac{f}{g}\right)(x) = \frac{x-2}{2x-7}, x \neq \frac{7}{2}$

18. $(f+g)(x) = x^2 + x - 5$; $(f-g)(x) = x^2 - x + 5$;
$(f \cdot g)(x) = x^3 - 5x^2$; $\left(\frac{f}{g}\right)(x) = \frac{x^2}{x-5}, x \neq 5$

19. $(f+g)(x) = x^2 + 3x + 1$; $(f-g)(x) = -3x^2 - 3x + 11$;
$(f \cdot g)(x) = -2x^4 - 3x^3 + 17x^2 + 18x - 30$;
$\left(\frac{f}{g}\right)(x) = \frac{-x^2+6}{2x^2+3x-5}, x \neq 1 \text{ or } -\frac{5}{2}$

20. $(f+g)(x) = 4x^2 - 8x$; $(f-g)(x) = 2x^2 + 8x - 8$;
$(f \cdot g)(x) = 3x^4 - 24x^3 + 8x^2 + 32x - 16$;
$\left(\frac{f}{g}\right)(x) = \frac{3x^2-4}{x^2-8x+4}, x \neq 4 \pm 2$

21. $f \circ g$ is undefined, D = ∅, R = ∅; $g \circ f$ = {(2, 8), (6, 13), (12, 11), (7, 15)}, D = {2, 6, 7, 12}, R = {8, 11, 13, 15}.

22. $f \circ$ g is undefined, D = ∅, R = ∅;
$g \circ f$ = {(0, 2)}, D = {0}, R = {2}.

23. $f \circ g$ = {(−4 , 4)}, D = {−4}, R = {4}; $g \circ f$ = {(−8, 0), (0, −4), (2, −5), (−6, −1)}, D = {−6, 0, 2}, R = {−5, −4, −1, 0}

24. $f \circ g$ = {(6, 12)}, D = {6}, R = {12}; $g \circ f$ = {(−7, 5), (4, 1), (−3, 8)}, D = {−7, −3, 4}, R = {1, 5, 8}

25. $f \circ g$ is undefined, D = ∅, R = ∅;
$g \circ f$ is undefined, D = ∅, R = ∅.

26. $f \circ g$ is undefined, D = ∅, R = ∅; $g \circ f$ = {(−4, 9), (0, 1), (−6, 13), (2, −3)}, D = {−6, −4, 0, 2}, R = {−3, 1, 9, 13}.

27. $f \circ g$ is undefined, D = ∅, R = ∅;
$g \circ f$ = {(−4, 0), (1, 2)}, D = {−4, 1}, R = {0, 2}.

28. $f \circ g$ = {(−1, −2)}, D = {−1}, R = { −2};
$g \circ f$ is undefined, D = ∅, R = ∅.

29. $f \circ g$ = {(4, 6), (3, −8)}, D = {3, 4}, R = {−8, 6};
$g \circ f$ is undefined, D = ∅, R = ∅.

30. $f \circ g$ is undefined, D = ∅, R = ∅;
$g \circ f$ is undefined, D = ∅, R = ∅.

31. f $\circ$ g = {(3, −1), (6, 11)}, D = { 3, 6}, R = {−1, 11};
$g \circ f$ = {(−4, 5), (−2, 4), (−1, 8)}, D = { −4, −2, −1}, R = {4, 5, 8}

32. $f \circ g$ = {(−2, 3), (−4, −1)}, D = {−4, −2}, R = {−1, 3};
$g \circ f$ = {(12, −1), (9, 6), (8, 5)}, D = {8, 9, 12}, R = {−1, 5, 6}

33. $[f \circ g](x) = -15x + 18$, R = {all multiples of 3};
$[g \circ f](x) = -15x - 6$

34. $[f \circ g](x) = x^2 + 3x - 6$; $[g \circ f](x) = x^2 + 11x + 18$

35. $[f \circ g](x) = 2x + 10$, R = {all even numbers};
$[g \circ f](x) = 2x + 5$, R = {all odd numbers}

36. $[f \circ g](x) = 3x - 24$, R = {all multiples of 3};
$[g \circ f](x) = 3x + 8$

37. $[f \circ g](x) = 3x - 2$; $[g \circ f](x) = 3x + 8$

38. $[f \circ g](x) = x^2 - 14$, R = $\{y \mid y \geq -14\}$;
$[g \circ f](x) = x^2 - 8x + 6$, R = $\{y \mid y \geq -10\}$

39. $[f \circ g](x) = x^2 - 6x - 2$, R = $\{y \mid y \geq -11\}$;
$[g \circ f](x) = x^2 + 6x - 8$, R = $\{y \mid y \geq -17\}$

40. $[f \circ g](x) = 32x^2 + 44x + 16$, R = $\{y \mid y \geq 0.875\}$;
$[g \circ f](x) = 8x^2 - 4x + 7$, R = $\{y \mid y \geq 6.5\}$

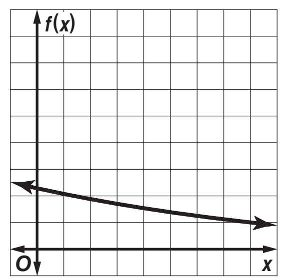
41. $[f \circ g](x) = 4x^3 + 7$; $[g \circ f](x) = 64x^3 - 48x^2 + 12x + 1$

42. $[f \circ g](x) = x^4 + 3x^2 + 1$, R = $\{y \mid y \geq 1\}$;
$[g \circ f](x) = x^4 + 6x^3 + 11x^2 + 6x + 1$, R = $\{y \mid y \geq 0\}$

43. $[f \circ g](x) = 128x^4 + 96x^3 + 18x^2$, R = $\{y \mid y \geq 0\}$;
$[g \circ f](x) = 32x^4 + 6x^2$, R = $\{y \mid y \geq 0\}$

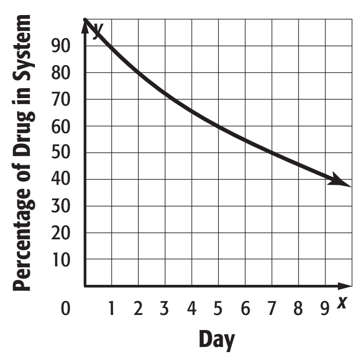
Additional Exercises, Lesson 8-3

45a. decay; 0.9



45b. The $P(x)$-intercept represents the number of pay phones in 1999. The asymptote is the x-axis. The number of pay phones can approach 0, but will never equal 0. This makes sense as there will probably always be a need for some pay phones.

46a. decay; 0.9

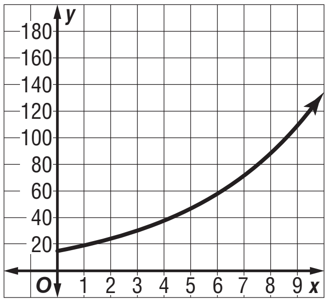


Answer Appendix

46b. a little less than 40%

46c. Sample answer: The 7th day; see students' work; there is 30% of the amount in the system after about 6.6 days.

47b. growth; 1.25



Additional Exercises, Extend 8-8

50. Sample answer: Yes; the thermal container will slow the rate of cooling. This affects the rate of decay in the function.

51. Sample answer: The ice will speed the rate of cooling. This affects the rate of decay in the function.

Additional Exercises, Lesson 9-2

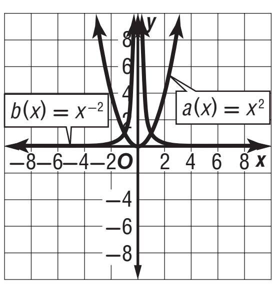
52. Sample answer: The set of rational expressions is closed under all of these operations because the sum, difference, product, and quotient of two rational expressions is a rational expression.

Additional Exercises, Lesson 9-3

53a.

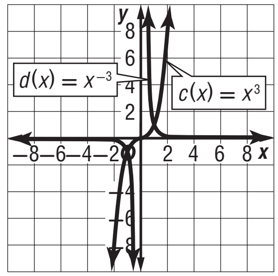
x	$a(x) = x^2$	$b(x) = x^2$	$c(x) = x^2$	$d(x) = x^2$
−4	16	$\frac{1}{16}$	−64	$-\frac{1}{64}$
−3	9	$\frac{1}{9}$	−27	$-\frac{1}{27}$
−2	4	$\frac{1}{4}$	−8	$-\frac{1}{8}$
−1	1	1	−1	−1
0	0	undefined	0	undefined
1	1	1	1	1
2	4	$\frac{1}{4}$	8	$\frac{1}{8}$
3	9	$\frac{1}{9}$	27	$\frac{1}{27}$
4	16	$\frac{1}{16}$	64	$\frac{1}{64}$

53b.



53c. $a(x)$: D = {all real numbers}, R = $\{a(x) \mid a(x) \geq 0\}$; $x \to -\infty$, $a(x) \to \infty$, $x \to -\infty$, $a(x) \to \infty$; At $x = 0$, $a(x) = 0$, so there is a zero at $x = 0$. $b(x)$: D = $\{x \mid x \neq 0\}$, R $\{b(x) \mid b(x) > 0\}$; $x \to -\infty$, $a(x) \to 0$, $x \to \infty$, $a(x) \to 0$; At $x = 0$, $b(x)$ is undefined, so there is an asymptote at $x = 0$.

53d.



53e. $c(x)$: D = {all real numbers}, R = {all real numbers}; $x \to -\infty$, $a(x) \to -\infty$, $x \to \infty$, $a(x) \to \infty$; At $x = 0$, $a(x) = 0$, so there is a zero at $x = 0$. $d(x)$: D = $\{x \mid x \neq 0\}$, R $\{b(x) \mid b(x) \neq 0\}$; $x \to -\infty$, $a(x) \to 0$, $x \to \infty$, $a(x) \to 0$; At $x = 0$, $b(x)$ is undefined, so there is an asymptote at $x = 0$.

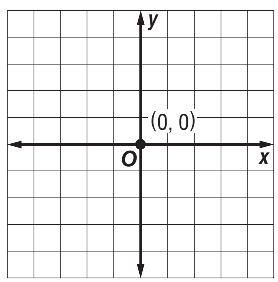
53f. For two power functions $f(x) = ax^n$ and $g(x) = ax^{-n}$, for every x, $f(x)$ and $g(x)$ are reciprocals. The domains are similar except that for $g(x)$, $x \neq 0$. Additionally, whenever $f(x)$ has a zero, $g(x)$ is undefined.

Additional Exercises, Lesson 9-4

54. Similarities: Both have vertical asymptotes at $x = 0$. Both approach 0 as x approaches $-\infty$ and approaches 0 as x approaches ∞. Differences: $f(x)$ has holes at $x = 1$ and $x = -1$ where as $g(x)$ has vertical asymptotes at $x = \sqrt{2}$ and $x = -\sqrt{2}$. $f(x)$ has no zeros and $g(x)$ has at zeros at $x = 1$ and $x = -1$

Additional Exercises, Lesson 10-6

55b.



55c. The standard conic is the graph of an ellipse and the degenerate conic is the graph of a single point.