Teacher Edition

Glencoe Secondary Mathematics

ALIGNED TO THE

COMMON
CORE
STATE
STANDARDS

Geometry

McGraw Hill Education

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Common Core State Standards

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Common Core State Standards

Welcome to Glencoe Secondary Mathematics to the Common Core

How to Use This Supplement

This supplement is your tour guide to understanding how Glencoe Secondary Mathematics programs teach the new Common Core State Standards. Its purpose is to help make a smooth transition from your state standards to the new Common Core State Standards..

Crosswalk

The crosswalk is your guide to understanding how to use your current *Glencoe Geometry* program with this supplement to create a Common Core State Standards curriculum. Pages vi–xii show you which lessons in your textbook should be kept, which can be considered optional, which lessons have additional available content, and how the new material fits into the flow of the chapters you already use.


Common Core State Standards

Crosswalk

How Do I Use This Crosswalk?

The organization of this crosswalk is to ensure coverage of all Common Core State Standards in the Geometry Pathway using *Glencoe Geometry* and *Glencoe Geometry to the Common Core*. Your *Glencoe Geometry* table of contents has been updated to show where to teach the new supplement lessons and which current lessons can be omitted.

Substitute Copy Lessons highlighted in yellow have small patch substitutions that can be found in this booklet.

Lesson	Lesson Title	Common Core State Standards	Page(s)
Chapter 0 Preparing for Geometry			
0-3	Simple Probability	S.MD.6, S.MD.7	P8–P9
Chapter 1 Tool of Geometry			
1-1	Points, Lines, and Planes	G.CO.1	5–12
Extend 1-1	Geometry Lab: Describing What You See	G.MG.1	13
1-2	Linear Measure	G.CO.1, G.CO.12	14–21
Extend 1-2	Extension Lesson: Precision and Accuracy		22–24
1-3	Distance and Midpoints	G.CO.1, G.CO.12, G.GPE.6	25–35
1-4	Angle Measure	G.CO.1, G.CO.12	36–44
1-5	Angle Relationships	Preparation for G.SRT.7	46–54
Extend 1-5	Geometry Lab: Constructing Perpendiculars	G.CO.12	55
1-6	Two-Dimensional Figures	G.GPE.7	56–64
Extend 1-6	Geometry Software Lab: Two-Dimensional Figures	G.CO.12	65–66
1-7	Three-Dimensional Figures	G.GMD.3	67–74
Extend 1-7	Geometry Lab: Orthographic Drawings and Nets	*Use CCSS Lab 1 in place of this lab*	75
CCSS Lab 1	Geometry Lab: Two-Dimensional Representations of Three-Dimensional Objects	G.MG.1	CCSS 5–7
Chapter 2 Reasoning and Proof			
2-1	Inductive Reasoning and Conjecture		89–96

Leave This Out Lessons highlighted in gray should be considered optional because these concepts are not included in the Geometry Pathway.

Add This In Lessons and labs highlighted in blue can be found in this booklet.

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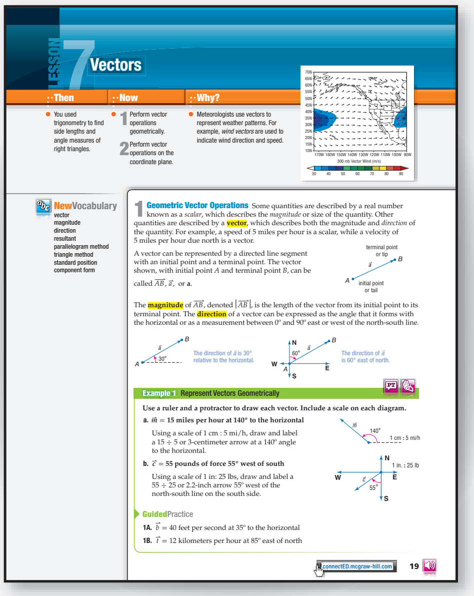
Correlations

Glencoe Geometry and *Glencoe Secondary Mathematics to the Common Core* align your curriculum with the Common Core State Standards and the Traditional Geometry Pathway. You can use pages xiii-xviii to map each standard to the lesson(s) that address each standard.

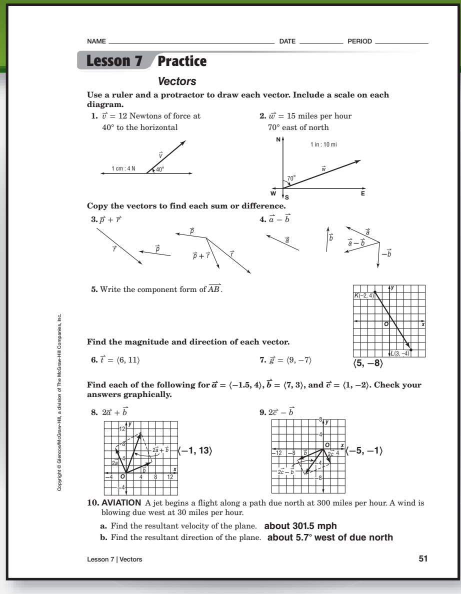

LESSON 7 **Vectors**

In This Booklet

Glencoe Secondary Mathematics to the Common Core contains additional lessons and labs to address the Common Core State Standards and the Traditional Geometry Pathway. (See pages 1–45.) You can also find copy for patch substitutions that can help you better meet the Common Core State Standards using your existing program. (See pages 46–49.) Refer to the Crosswalk on pages vi-xii for appropriate placement of this content in your *Glencoe Geometry* textbook.

Homework Practice

Pages 50–52 of the Student Edition of *Glencoe Secondary Mathematics to the Common Core* contain homework practice pages for the lessons added to meet the Common Core State Standards.


NAME ____________ DATE ________ PERIOD ________

Lesson 7 Practice

Vectors

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

1. $\vec{v}$ = 12 Newtons of force at 40° to the horizontal
2. $\vec{w}$ = 15 miles per hour 70° east of north

Copy the vectors to find each sum or difference.

3. $\vec{p} + \vec{r}$
4. $\vec{a} - \vec{b}$

5. Write the component form of $\overrightarrow{AB}$. **⟨5, −8⟩**

Find the magnitude and direction of each vector.

6. $\vec{t} = \langle 6, 11 \rangle$
7. $\vec{g} = \langle 9, -7 \rangle$

Find each of the following for $\vec{a} = \langle -1.5, 4 \rangle$, $\vec{b} = \langle 7, 3 \rangle$, and $\vec{c} = \langle 1, -2 \rangle$. Check your answers graphically.

8. $2\vec{a} + \vec{b}$ **⟨−1, 13⟩**
9. $2\vec{c} - \vec{b}$ **⟨−5, −1⟩**

10. **AVIATION** A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.
   a. Find the resultant velocity of the plane. **about 301.5 mph**
   b. Find the resultant direction of the plane. **about 5.7° west of due north**

Lesson 7 | Vectors 51

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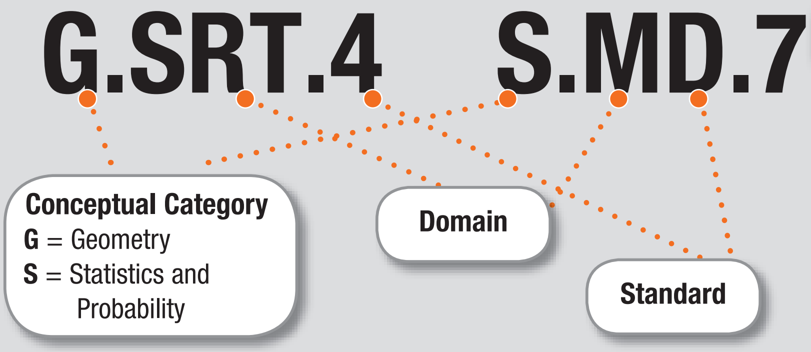
Decoding the Common Core State Standards

This diagram provides clarity for decoding the standard identifiers.



Domain Names	Abbreviations
Congruence	**CO**
Similarity, **R**ight Triangles, and **T**rigonometry	**SRT**
Circles	**C**
Expressing **G**eometric **P**roperties with **E**quations	**GPE**
Geometric **M**easurement and **D**imension	**GMD**
Modeling with **G**eometry	**MG**
Conditional **P**robability and the Rules of Probability	**CP**
Using Probability to **M**ake **D**ecisions	**MD**

You can choose how to use *Glencoe Secondary Mathematics to the Common Core* in your classroom.

- To print, go to connectED.mcgraw-hill.com.
- To display for whole class instruction, go to connectED.mcgraw-hill.com.
- To order print copies of the Student Edition, contact your local sales representative.

Common Core State Standards

Crosswalk

How Do I Use This Crosswalk?

The organization of this crosswalk is to ensure coverage of all Common Core State Standards in the Geometry Pathway using *Glencoe Geometry* and *Glencoe Geometry to the Common Core*. Your *Glencoe Geometry* table of contents has been updated to show where to teach the new supplement lessons and which current lessons can be omitted.

Substitute Copy
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Chapter 0 Preparing for Geometry			
0-3	Simple Probability	S.MD.6, S.MD.7	P8–P9
Chapter 1 Tool of Geometry			
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Extend 1-1	Geometry Lab: Describing What You See	G.MG.1	13
1-2	Linear Measure	G.CO.1, G.CO.12	14–21
Extend 1-2	Extension Lesson: Precision and Accuracy		22–24
1-3	Distance and Midpoints	G.CO.1, G.CO.12, G.GPE.6	25–35
1-4	Angle Measure	G.CO.1, G.CO.12	36–44
1-5	Angle Relationships	Preparation for G.SRT.7	46–54
Extend 1-5	Geometry Lab: Constructing Perpendiculars	G.CO.12	55
1-6	Two-Dimensional Figures	G.GPE.7	56–64
Extend 1-6	Geometry Software Lab: Two-Dimensional Figures	G.CO.12	65–66
1-7	Three-Dimensional Figures	G.GMD.3	67–74
Extend 1-7	Geometry Lab: Orthographic Drawings and Nets	*Use CCSS Lab 1 in place of this lab.*	75
CCSS Lab 1	Geometry Lab: Two-Dimensional Representations of Three-Dimensional Objects	G.MG.1	CCSS 5–7

Lesson	Lesson Title	Common Core State Standards	Page(s)
Chapter 2 Reasoning and Proof			
2-1	Inductive Reasoning and Conjecture		89–96

Leave This Out
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Add This In
Lessons and labs highlighted in blue can be found in this booklet.

Lesson	Lesson Title	CCSS Common Core State Standards	Page(s)
2-2	Logic		97–104
2-3	Conditional Statements		105–113
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Lesson	Lesson Title	Common Core State Standards	Page(s)
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Lesson	Lesson Title	CCSS Common Core State Standards	Page(s)
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11-4	Areas of Regular Polygons and Composite Figures	G.MG.3	791-799
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Common Core State Standards

Traditional Geometry Pathway, Correlated to *Glencoe Geometry*, Common Core Edition

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Geometry		
Congruence G-CO		
Experiment with transformations in the plane. **1.** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	1-1, 1-2, 1-3, 1-4, 3-1, 3-2, 10-1	5–12, 14–21, 25–35, 36–44, 171–176, 178–184, 683–691
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	4-7, 7-6, 9-1, 9-2, Explore 9-3, 9-3, Explore 9-4, 9-4, 9-6	294–300, 505–511, 615–623, 624–630, 631, 632–638, 640, 641–649, 660–667
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	9-5	653–659
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	9-1, 9-2, 9-3, Explore 9-4, 9-4	615–623, 624–630, 632–638, 640, 641–649
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	Explore 4-7, 9-1, 9-2, Explore 9-3, 9-3, Explore 9-4, 9-4	292–293, 615–623, 624–630, 631, 632–638, 640, 641–649
Understand congruence in terms of rigid motions. **6.** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Explore 4-7, 4-7, 9-1, 9-2, 9-3, 9-4 CCSS Lab 11	292–293, 294–300, 615–623, 624–630, 632–638, 641–649 CCSS 33–34
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	4-3, Explore 4-7, 4-7, 9-1, 9-2, 9-3, 9-4 CCSS Lab 11	253–261, 292–293, 294–300, 615–623, 624–630, 632–638, 641–649 CCSS 33–34
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	4-7 CCSS Lab 11	294–300 CCSS 33–34
Prove geometric theorems. **9.** Prove theorems about lines and angles.	2-7, 2-8, 3-2, 3-5, 5-1	142–148, 149–157, 178–184, 205–212, 322–331

Standards	Student Edition Lesson(s)	Student Edition Page(s)
10. Prove theorems about triangles.	4-2, 4-3, 4-4, 4-5, 4-6, 4-8, 5-1, 5-2, 5-3, 5-4, 5-5, 5-6, 7-4, Explore 8-2	244–252, 253–261, 262–270, 273–280, 283–291, 301–307, 322–331, 333–341, 342–349, 351–358, 360–366, 367–376, 484–493, 540
11. Prove theorems about parallelograms.	6-2, 6-3, 6-4, 6-5	399–407, 409–417, 419–425, 426–434
Make geometric constructions. **12.** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	1-2, 1-3, 1-4, Extend 1-5, Extend 1-6, 2-7, Explore 3-2, 3-5, 3-6, 4-1, Explore 4-2, 4-4, Extend 4-4, 4-5, 4-6, Explore 5-5, Explore 6-3, 6-3, 6-4, 6-5, 7-4, 9-1, Explore 9-3, 10-3, 10-5, Extend 10-5 CCSS Lab 2, CCSS Lab 3, CCSS Lab 9	14–21, 25–35, 36–44, 55, 65–66, 142–148, 177, 205–212, 213–222, 235–242, 243, 262–270, 271, 273–280, 283–291, 359, 408, 409–417, 419–425, 426–434, 484–493, 615–623, 631, 701–708, 718–725, 726 CCSS 4, CCSS 5, CCSS 29–30
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Extend 10-5	726
Similarity, Right Triangles, and Trigonometry G-SRT		
Understand similarity in terms of similarity transformations. **1.** Verify experimentally the properties of dilations given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	9-6 CCSS Lab 11	660–667 CCSS 33–34
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	9-6 CCSS Lab 11	660–667 CCSS 33–34
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	7-2, 7-3, 7-6 CCSS Lab 11	465–473, 474–483, 505–511 CCSS 33–34
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	7-3, 7-6 CCSS Lab 11	474–483, 505–511 CCSS 33–34

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Prove theorems involving similarity. **4.** Prove theorems about triangles.	7-3, 7-4, 7-5, 8-1	474–483, 484–493, 495–502, 531–539
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	4-3, 4-4, Extend 4-4, 4-5, Extend 4-5, 7-3, 7-4, 7-5, 7-6, 8-1	253–261, 262–270, 271, 273–280, 281, 474–483, 484–493, 495–502, 505–511, 531–539
Define trigonometric ratios and solve problems involving right triangles. **6.** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	8-3, Explore 8-4, 8-4, Extend 8-4	552–560, 561, 562–571, 572
7. Explain and use the relationship between the sine and cosine of complementary angles.	8-4	562–571
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	8-2, 8-4, 8-5 CCSS Lesson 5	541–549, 562–571, 574–581 CCSS 8–16
Apply trigonometry to general triangles. **9.** (+) Derive the formula $A = ab \sin (C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	CCSS Lesson 5	CCSS 8–16
10.(+) Prove the Laws of Sines and Cosines and use them to solve problems.	CCSS Lesson 5	CCSS 8–16
11.(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	CCSS Lesson 5, CCSS Lab 6	CCSS 8–16, CCSS 17–18
Circles G-C		
Understand and apply theorems about circles. **1.** Prove that all circles are similar.	10-1	683–691
2. Identify and describe relationships among inscribed angles, radii, and chords.	10-1, 10-2, 10-3, 10-4, 10-5	683–691, 692–700, 701–708, 709–716, 718–725
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	10-4, Extend 10-5	709–716, 726
4. (+) Construct a tangent line from a point outside a given circle to the circle.	10-5	718–725
Find arc lengths and areas of sectors of circles. **5.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	10-2, 11-3	692–700, 782–788

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Expressing Geometric Properties with Equations G-GPE		
Translate between the geometric description and the equation for a conic section. **1.** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	CCSS Lesson 12	CCSS 35–40
2. Derive the equation of a parabola given a focus and directrix.	CCSS Lab 13	CCSS 41–42
Use coordinates to prove simple geometric theorems algebraically. **4.** Use coordinates to prove simple geometric theorems algebraically.	4-8, 6-2, 6-3, 6-4, 6-5, 6-6 CCSS Lesson 12	301–307, 399–407, 409–417, 419–425, 426–434, 435–444 CCSS 34–40
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Explore 3-3, 3-3, 3-4, Extend 3-4 CCSS Lab 4	185, 186–194, 196–203, 204 CCSS 6–7
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	1-3, 7-4, 9-6 CCSS Lesson 7, CCSS Lesson 12	25–35, 484–493, 660–667 CCSS 19–26, CCSS 35–40
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	1-6, 11-1	56–64, 763–770

Standards	Student Edition Lesson(s)	Student Edition Page(s)
Geometric Measurement and Dimension G-GMD		
Explain volume formulas and use them to solve problems. **1.** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	10-1, 11-3, 12-4, 12-5, 12-6	683–691, 782–788, 847–854, 857–863, 864–871
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	1-7, 12-4, 12-5, 12-6	67–74, 847–854, 864–871
Visualize relationships between two-dimensional and three-dimensional objects. **4.** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	12-1 CCSS Lab 8	823–828 CCSS 27–28
Modeling with Geometry G-MG		
Apply geometric concepts in modeling situations. **1.** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	Throughout the text; for example, Extend 1-1, 6-1, 11-5, 12-3 CCSS Lab 1	Throughout the text; for example, 13, 389–397, 802–808, 838–846 CCSS 1–3

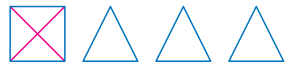
Standards	Student Edition Lesson(s)	Student Edition Page(s)
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	12-4, 12-5 CCSS Lab 14	847–854, 857–863 CCSS 43
3. Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★	2-5, 3-6, 5-1, 5-2, 5-5, 6-6, 7-1, 7-7, 8-2, 10-3, 11-2, 11-4, 12-2, 12-4, 12-6, 13-4	125–132, 213–222, 322–331, 333–341, 360–366, 435–444, 457–463, 512–517, 541–549, 701–708, 773–780, 791–799, 830–837, 847–854, 864–871, 923–930

Statistics and Probability		
Conditional Probability and the Rules of Probability S-CP		
Understand independence and conditional probability and use them to interpret data. 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	13-5, 13-6	931–937, 938–945
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	13-5	931–937
3. Understand the conditional probability of *A* given *B* as , and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	13-5	931–937
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	CCSS Lab 15	CCSS 44–45
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	13-5	931–937
Use the rules of probability to compute probabilities of compound events in a uniform probability model. 6. Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model.	13-5 CCSS Lab 15	931–937 CCSS 44–45
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	13-6	938–945
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.	13-5	931–937

Standards	Student Edition Lesson(s)	Student Edition Page(s)
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	13-2	906–914
Using Probability to Make Decisions S-MD		
Use probability to evaluate outcomes of decisions. **6.** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	0-3, 13-4	P8–P9, 923–930
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	0-3, 13-3, 13-5	P8–P9, 915–921, 931–937

LAB 1 Geometry Lab
Two-Dimensional Representations of Three-Dimensional Objects

If you see a three-dimensional object from only one viewpoint, you may not know its true shape. Here are four views of a square pyramid.

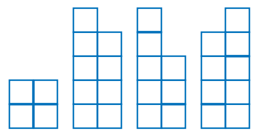


The two-dimensional views of the top, left, front, and right sides of an object are called an **orthographic drawing**.

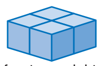
Common Core State Standards
G.MG.1

Activity 1

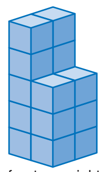
Make a model of a figure for the orthographic drawing shown.



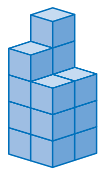
Step 1 Start with a base that matches the top view.



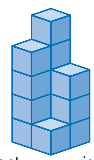
Step 2 The front view indicates that the front left side is 5 blocks high and that the right side is 3 blocks high. However, the dark segments indicate breaks in the surface.



Step 3 The break on the left side of the front view indicates that the back left column is 5 blocks high, but that the front left column is only 4 blocks high, so remove 1 block from the front left column.



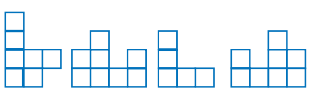
Step 4 The break on the right side of the front view indicates that the back right column is 3 blocks high, but that the front right column is only 1 block high, so remove 2 blocks from the front right column.



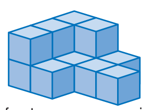
Step 5 Use the left and right views and the breaks in those views to confirm that you have made the correct figure.

Model and Analyze

1. Make a model of a figure for the orthographic drawing shown.



2. Make an orthographic drawing of the figure shown.



(continued on the next page)

1 Focus

Objective Use orthographic views and nets to represent and construct three-dimensional figures.

Materials

- ruler, scissors, tape
- a large sheet of paper

2 Teach

Working in Cooperative Groups

Arrange students in groups of 3 or 4, mixing abilities. Then have groups complete Activities 1–3.

Ask:

- In your own words, what does the prefix *ortho* mean?
- What two shapes make up a triangular prism?
- What three-dimensional figure has the same orthographic drawing?

Practice Have students complete Exercises 1–2.

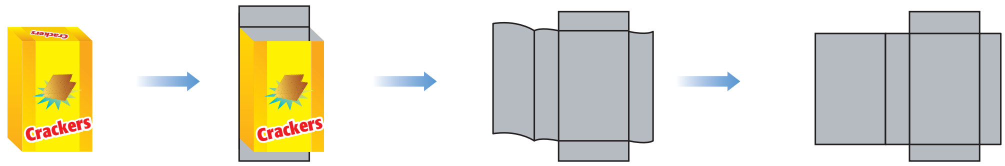
Focus on Mathematical Content

A net for a sphere can be created using several adjoining pointed ellipses or by creating a polyhedron with a large number of sides. However, since paper can only curve in one direction, it is impossible to make a perfect sphere. Thus, the spheres made from nets will be approximations.

Geometry Lab

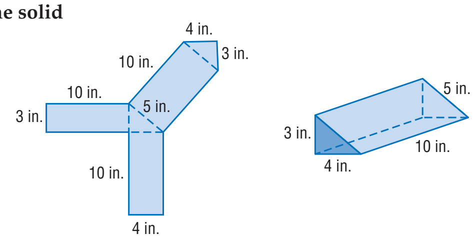
Two-Dimensional Representations of Three-Dimensional Objects *Continued*

If you cut a cardboard box at the edges and lay it flat, you will have a two-dimensional diagram called a **net** that you can fold to form a three-dimensional solid.



Activity 2

Make a model of a figure for the given net. Then identify the solid formed, and find its surface area.



Use a large sheet of paper, a ruler, scissors, and tape. Draw the net on the paper. Cut along the solid lines. Fold the pattern on the dashed lines and secure the edges with tape. This is the net of a triangular prism.

Use the net to find the surface area T.

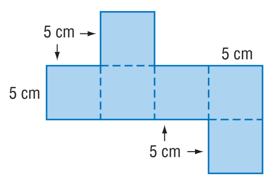
$T = 2\left[\frac{1}{2}(4)(3)\right] + 4(10) + 3(10) + 5(10)$ Area of two congruent triangles plus area of three rectangles

$= 12 + 40 + 30 + 50$ or 132 in^2 Simplify.

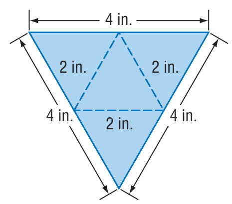
Model and Analyze

Make a model of a figure for each net. Then identify the solid formed and find its surface area. If the solid has more than one name, list both.

3.



4.



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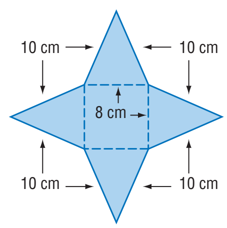
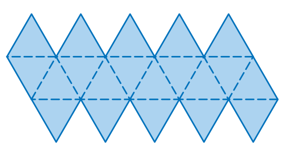


Identify the Platonic Solid that can be formed by the given net.

6.



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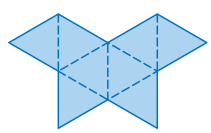

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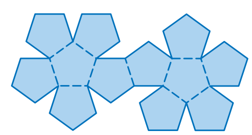
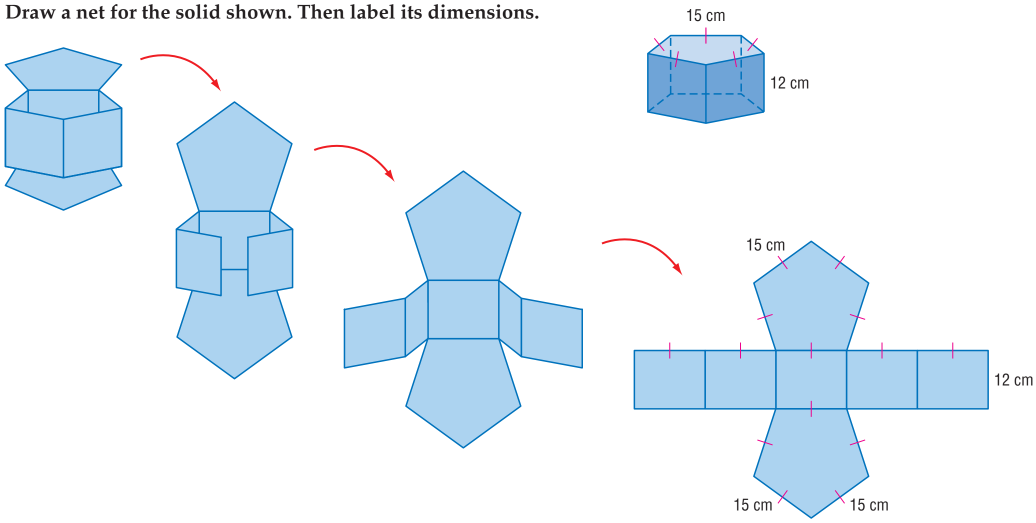


2 | Lab 1 | Geometry Lab: Two-Dimensional Representations of Three-Dimensional Objects

To draw the net of a three-dimensional solid, visualize cutting the solid along one or more of its edges, opening up the solid, and flattening it completely.

Activity 3

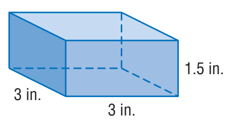
Draw a net for the solid shown. Then label its dimensions.



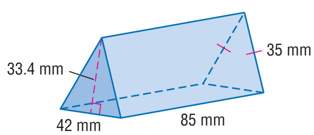
Model and Analyze

Draw a net for each solid. Then label its dimensions.

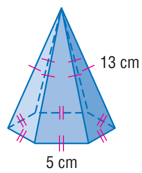
9.



10.



11.



12. PACKAGING A can of pineapple is shown.

a. What shape are the top and bottom of the can?

b. If you remove the top and bottom and then make a vertical cut down the side of the can, what shape will you get when you uncurl the remaining body of the can and flatten it?

c. If the diameter of the can is 3 inches and its height is 2 inches, draw a net of the can and label its dimensions. Explain your reasoning.

3 Assess

Formative Assessment

Use Exercises 1 and 2 to assess whether students comprehend how to make a net for a three-dimensional figure.

From Concrete to Abstract

Have students pick three objects in the classroom, and then make an orthographic drawing and a net for each object. Then have each student present their drawing, and let the class guess the object.

1 Focus

Objective Construct perpendicular bisectors and angle bisectors of triangles.

Teaching Tip

The activity demonstrates two different constructions on an acute scalene triangle. Students could use patty paper to draw and trace two acute scalene triangles with the same side lengths, angle measures, and orientation in three different places on one sheet of paper. When students are finished with the constructions, they can see the differences between the perpendicular and angle bisectors for the same triangle.

Alternative Method

The constructions presented in this lesson can also be completed using classic ruler and compass methods.

2 Teach

Working in Cooperative Groups

Arrange students in groups of 3 with mixed abilities. Each student completes one of the three steps in the construction activities. Rotate the steps for Constructions 1 and 2.

Practice Have the students complete Exercise 1 while performing the activities.

LAB 2 Geometry Lab: Constructing Bisectors

Paper folding can be used to construct special segments in triangles.

Common Core State Standards
G.CO.12

Construction Perpendicular Bisector

Construct a perpendicular bisector of the side of a triangle.

Step 1

Draw, label, and cut out $\triangle MPQ$.

Step 2

Fold the triangle in half along $\overline{MQ}$ so that vertex M touches vertex Q.

Step 3

Use a straightedge to draw $\overline{AB}$ along the fold. $\overline{AB}$ is the perpendicular bisector of $\overline{MQ}$.

An angle bisector in a triangle is a line containing a vertex of the triangle and bisecting that angle.

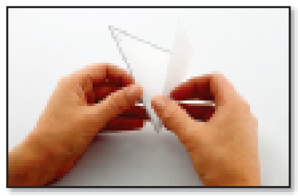
Construction Angle Bisector

Construct an angle bisector of a triangle.

Step 1

Draw, label, and cut out $\triangle ABC$.

Step 2



Fold the triangle in half through vertex A, such that sides $\overline{AC}$ and $\overline{AB}$ are aligned.

Step 3

Label point L at the crease along edge $\overline{BC}$. Use a straightedge to draw $\overline{AL}$ along the fold. $\overline{AL}$ is an angle bisector of $\triangle ABC$.

Model and Analyze

1. Construct the perpendicular bisectors and angle bisectors of the other two sides and angles of $\triangle MPQ$. What do you notice about their intersections?

Repeat the two constructions for each type of triangle.

2. acute
3. obtuse
4. right

3 Assess

Formative Assessment

Use Exercises 2–4 to assess whether students understand the concept and construction of perpendicular and angle bisectors.

From Concrete to Abstract

Give students the three types of triangles mentioned in Exercises 2–4. Tell them that you want to have each triangle balance on a pencil. Have them pick a construction method and explain.

4 | Lab 2 | Geometry Lab: Constructing Bisectors

LAB 3 Geometry Lab
Constructing Medians and Altitudes

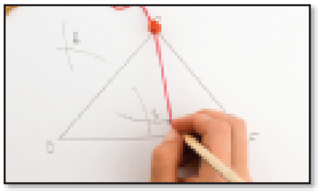
A *median* of a triangle is a segment with endpoints that are a vertex and the midpoint of the side opposite that vertex. You can use the construction for the midpoint of a segment to construct a median.

Wrap the end of string around a pencil. Use a thumbtack to fix the string to a vertex.

Common Core State Standards
G.CO.12

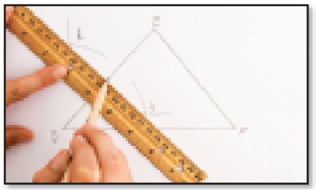
Construction 1 Median of a Triangle

Step 1



Place the thumbtack on vertex E and then on vertex D to draw intersecting arcs above and below $\overline{DE}$. Label the points of intersection R and S.

Step 2



Use a straightedge to find the point where $\overline{RS}$ intersects $\overline{DE}$. Label the point M. This is the midpoint of $\overline{DE}$.

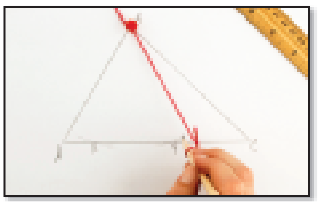
Step 3

Draw a line through F and M. $\overline{FM}$ is a median of $\triangle DEF$.

An *altitude* of a triangle is a segment from a vertex of the triangle to the opposite side and is perpendicular to the opposite side.

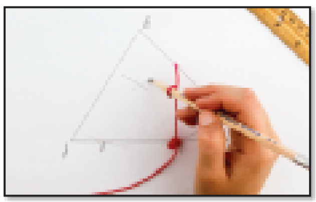
Construction 2 Altitude of a Triangle

Step 1



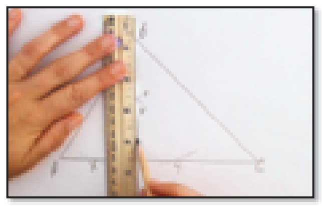
Place the thumbtack on vertex B and draw two arcs intersecting $\overline{AC}$. Label the points where the arcs intersect the sides as X and Y.

Step 2



Adjust the length of the string so that it is greater than $\frac{1}{2}XY$. Place the tack on X and draw an arc above $\overline{AC}$. Use the same length of string to draw an arc from Y. Label the points of intersection of the arcs H.

Step 3



Use a straightedge to draw $\overleftrightarrow{BH}$. Label the point where $\overline{BH}$ intersects $\overline{AC}$ as D. $\overline{BD}$ is an altitude of $\triangle ABC$ and is perpendicular to $\overline{AC}$.

Model and Analyze

1. Construct the medians of the other two sides of $\triangle DEF$. What do you notice about the medians of a triangle?
2. Construct the altitudes to the other two sides of $\triangle ABC$. What do you observe?

1 Focus

Objective Construct medians and altitudes of triangles.

Materials for Each Group

- straightedge
- string
- thumbtack

Teaching Tip

The activity demonstrates two different constructions on an acute scalene triangle. Students could use patty paper to draw and trace two acute scalene triangles with the same side lengths, angle measures, and orientation in three different places on one sheet of paper. When students are finished with the constructions, they can see the differences between the medians and altitudes for the same triangle.

Alternative Method

The constructions presented in this lesson can also be completed using classic ruler and compass methods.

2 Teach

Working in Cooperative Groups

Arrange students in groups of 3 with mixed abilities. Each student is to pick one of the three steps in the construction activities. Rotate the steps for Constructions 1 and 2.

Practice Have students complete Exercises 1 and 2.

From Concrete to Abstract

Have students compare the intersections of the medians and altitudes they constructed with the incenter and circumcenter of the triangle.

3 Assess

Formative Assessment

Use Exercises 1 and 2 to assess whether students understand the construction of medians and altitudes.

1 Focus

Objective Use similar triangles to prove the slope criteria for perpendicular and parallel lines.

Materials for Each Group

- compass
- straightedge

Teaching Tip

Ask students what techniques (AA, SSS, SAS Similarity) they have learned thus far that could be used to prove that two triangles are similar.

2 Teach

Working in Cooperative Groups

Arrange students in groups of 2, mixing abilities. Then have students complete the activity.

Practice Have students complete Exercises 1 and 2.

Focus on Mathematical Content

Finding Slope In Activity 1, the slope of $\overleftrightarrow{AC}$ is negative because it is the *rise* from *A* to *B* in the *negative* direction over the *run* from *B* to *C* in the *positive* direction.

LAB 4 Geometry Lab
Proofs of Perpendicular and Parallel Lines

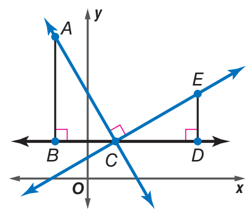
You have learned that two straight lines that are neither horizontal nor vertical are perpendicular if and only if the product of their slopes is –1. In this activity, you will use similar triangles to prove the first half of this theorem: if two straight lines are perpendicular, then the product of their slopes is –1.

Common Core State Standards
G.GPE.5

Activity 1 Perpendicular Lines

Given: Slope of $\overleftrightarrow{AC} = m_1$, slope of $\overleftrightarrow{CE} = m_2$, and $\overleftrightarrow{AC} \perp \overleftrightarrow{CE}$.
Prove: $m_1 m_2 = -1$

Step 1 On a coordinate plane, construct $\overleftrightarrow{AC} \perp \overleftrightarrow{CE}$ and transversal $\overleftrightarrow{BD}$ parallel to the *x*-axis through *C*. Then construct right $\triangle ABC$ such that $\overline{AC}$ is the hypotenuse and right $\triangle EDC$ such that $\overline{CE}$ is the hypotenuse. The legs of both triangles should be parallel to the *x*-and *y*-axes, as shown.



Step 2 Find the slopes of $\overleftrightarrow{AC}$ and $\overleftrightarrow{CE}$.

Slope of $\overleftrightarrow{AC}$

$m_1 = \frac{\text{rise}}{\text{run}}$ Slope Formula

$= \frac{-AB}{BC}$ or $-\frac{AB}{BC}$ rise = $-AB$, run = BC

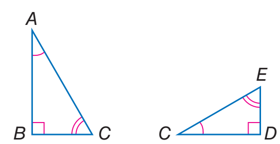
Slope of $\overleftrightarrow{CE}$

$m_2 = \frac{\text{rise}}{\text{run}}$ Slope Formula

$= \frac{DE}{CD}$ rise = DE, run = CD

Step 3 Show that $\triangle ABC \sim \triangle CDE$.

Since $\triangle ACB$ is a right triangle with right angle *B*, $\angle BAC$ is complementary to $\angle ACB$. It is given that $\overleftrightarrow{AC} \perp \overleftrightarrow{CE}$, so we know that $\angle ACE$ is a right angle. By construction, $\angle BCD$ is a straight angle. So, $\angle ECD$ is complementary to $\angle ACB$. Since angles complementary to the same angle are congruent, $\angle BAC \cong \angle ECD$. Since right angles are congruent, $\angle B \cong \angle D$. Therefore, by AA Similarity, $\triangle ABC \sim \triangle CDE$.



Step 4 Use the fact that $\triangle ABC \sim \triangle CDE$ to show that $m_1 m_2 = -1$.

Since $m_1 = -\frac{AB}{BC}$ and $m_2 = \frac{DE}{CD}$, $m_1 m_2 = \left(-\frac{AB}{BC}\right)\left(\frac{DE}{CD}\right)$. Since two similar polygons have proportional sides, $\frac{AB}{BC} = \frac{CD}{DE}$. Therefore, by substitution, $m_1 m_2 = \left(-\frac{CD}{DE}\right)\left(\frac{DE}{CD}\right)$ or -1.

6 | Lab 4 | Geometry Lab: Proofs of Perpendicular and Parallel Lines

Model

1. **PROOF** Use the diagram from Activity 1 to prove the second half of the theorem.

   Given: Slope of $\overleftrightarrow{CE} = m_1$, slope of $\overleftrightarrow{AC} = m_2$, and $m_1 m_2 = -1$. $\triangle ABC$ is a right triangle with right angle B. $\triangle CDE$ is a right triangle with right angle D.

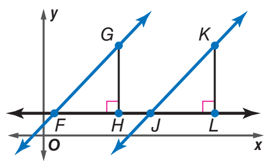
   Prove: $\overleftrightarrow{CE} \perp \overleftrightarrow{AC}$

You can also use similar triangles to prove statements about parallel lines.

Activity 2 Parallel Lines

Given: Slope of $\overleftrightarrow{FG} = m_1$, slope of $\overleftrightarrow{JK} = m_2$, and $m_1 = m_2$. $\triangle FHG$ is a right triangle with right angle H. $\triangle JLK$ is a right triangle with right angle L.

Prove: $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$



Step 1 On a coordinate plane, construct $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$, right $\triangle FHG$, and right $\triangle JLK$. Then draw horizontal transversal $\overleftrightarrow{FL}$, as shown.

Step 2 Find the slopes of $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$.

Slope of $\overleftrightarrow{FG}$

$m_1 = \frac{\text{rise}}{\text{run}}$ Slope Formula

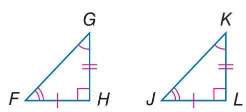
$= \frac{GH}{HF}$ rise $= GH$, run $= HF$

Slope of $\overleftrightarrow{JK}$

$m_2 = \frac{\text{rise}}{\text{run}}$ Slope Formula

$= \frac{KL}{LJ}$ rise $= KL$, run $= LJ$

Step 3 Show that $\triangle FHG \sim \triangle JLK$.



It is given that $m_1 = m_2$. By substitution, $\frac{GH}{HF} = \frac{KL}{LJ}$. This ratio can be rewritten as $\frac{GH}{KL} = \frac{HF}{LJ}$. Since $\angle H$ and $\angle L$ are right angles, $\angle H \cong \angle L$. Therefore, by SAS similarity, $\triangle FHG \sim \triangle JLK$.

Step 4 Use the fact that $\triangle FHG \sim \triangle JLK$ to prove that $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$.

Corresponding angles in similar triangles are congruent, so $\angle GFH \cong \angle KJL$. From the definition of congruent angles, $m\angle GFH = m\angle KJL$ (or $\angle GFH \cong \angle KJL$). By definition, $\angle KJH$ and $\angle KJL$ form a linear pair. Since linear pairs are supplementary, $m\angle KJH + m\angle KJL = 180$. So, by substitution, $m\angle KJH + m\angle GFH = 180$. By definition, $\angle KJH$ and $\angle GFH$ are supplementary. Since $\angle KJH$ and $\angle GFH$ are supplementary and are consecutive interior angles, $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$.

Model

2. **PROOF** Use the diagram from Activity 2 to prove the following statement.

   Given: Slope of $\overleftrightarrow{FG} = m_1$, slope of $\overleftrightarrow{JK} = m_2$, and $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$.

   Prove: $m_1 = m_2$

3 Assess

Formative Assessment

Use Exercises 1 and 2 to assess whether students understand how to prove the slope criteria for perpendicular and parallel lines.

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1 Focus

VerticalAlignment

Before Lesson 5 Use trigonometric ratios to solve right triangles.

Lesson 5 Use the Law of Sines and Law of Cosines to solve a triangle.

After Lesson 5 Use trigonometric functions to solve problems with vectors.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- What method can be used to find the height of a tree?
- What measurements do you need to find the height of a tree?
- If the tree is leaning at an angle, why can you not use right triangle trigonometry?

LESSON 5 The Law of Sines and Law of Cosines

Then

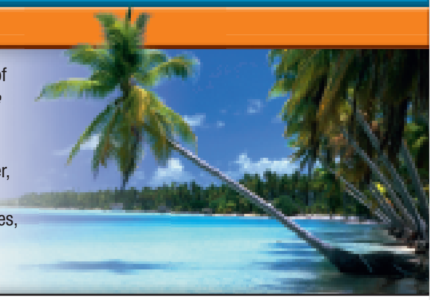
- You used trigonometric ratios to solve right triangles.

Now

1. Use the Law of Sines to solve triangles.
2. Use the Law of Cosines to solve triangles.

Why?

- You have learned that the height or length of a tree can be calculated using *right triangle trigonometry* if you know the angle of elevation to the top of the tree and your distance from the tree. Some trees, however, grow at an angle or lean due to weather damage. To calculate the length of such trees, you must use other forms of trigonometry.



NewVocabulary
Law of Sines
Law of Cosines

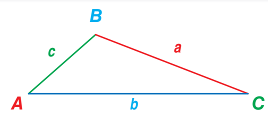
1 Law of Sines In Lesson 8-4, you used trigonometric ratios to find side lengths and acute angle measures in *right* triangles. To find measures for nonright triangles, the definitions of sine and cosine can be extended to obtuse angles.

The **Law of Sines** can be used to find side lengths and angle measures for nonright triangles.

Theorem 8.10 Law of Sines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



You will prove one of the proportions for Theorem 8.10 in Exercise 45.

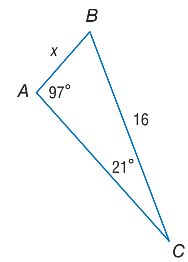
You can use the Law of Sines to solve a triangle if you know the measures of two angles and any side (AAS or ASA).

Example 1 Law of Sines (AAS)

Find x. Round to the nearest tenth.

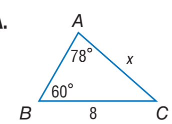
We are given the measures of two angles and a nonincluded side, so use the Law of Sines to write a proportion.

$\frac{\sin A}{a} = \frac{\sin C}{c}$	Law of Sines
$\frac{\sin 97°}{16} = \frac{\sin 21°}{x}$	$m\angle A = 97$, $a = 16$, $m\angle C = 21$, $c = x$
$x \sin 97° = 16 \sin 21°$	Cross Products Property
$x = \frac{16 \sin 21°}{\sin 97°}$	Divide each side by sin 97°.
$x \approx 5.8$	Use a calculator.

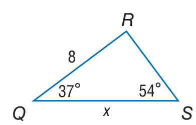


GuidedPractice

1A.



1B.



Ben Mangor/SuperStock

Common Core State Standards
G.SRT.8, G.SRT.9, G.SRT.10, G.SRT.11

StudyTip

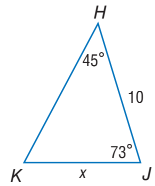
Ambiguous Case You can sometimes use the Law of Sines to solve a triangle if you know the measures of two sides and a nonincluded angle (SSA). However, these three measures do not always determine exactly one triangle. You will learn more about this *ambiguous case* in Extend 8-6.

If given ASA, use the Triangle Angle Sum Theorem to first find the measure of the third angle.

Example 2 Law of Sines (ASA)

Find x. Round to the nearest tenth.

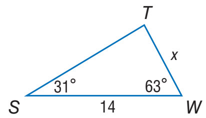
By the Triangle Angle Sum Theorem, $m\angle K = 180 - (45 + 73)$ or 62.



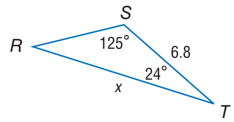
$\frac{\sin H}{h} = \frac{\sin K}{k}$	Law of Sines
$\frac{\sin 45°}{x} = \frac{\sin 62°}{10}$	$m\angle H = 45$, $h = x$, $m\angle K = 62$, $k = 10$
$10 \sin 45° = x \sin 62°$	Cross Products Property
$\frac{10 \sin 45°}{\sin 62°} = x$	Divide each side by sin 62°.
$x \approx 8.0$	Use a calculator.

GuidedPractice

2A.



2B.



2 Law of Cosines

When the Law of Sines cannot be used to solve a triangle, the Law of Cosines may apply.

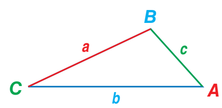
Theorem 8.11 Law of Cosines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$a^2 = b^2 + c^2 - 2bc \cos A$,

$b^2 = a^2 + c^2 - 2ac \cos B$, and

$c^2 = a^2 + b^2 - 2ab \cos C$.



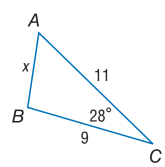
You will prove one of the equations for Theorem 8.11 in Exercise 46.

You can use the **Law of Cosines** to solve a triangle if you know the measures of two sides and the included angle (SAS).

Example 3 Law of Cosines (SAS)

Find x. Round to the nearest tenth.

We are given the measures of two sides and their included angle, so use the Law of Cosines.



$c^2 = a^2 + b^2 - 2ab \cos C$	Law of Cosines
$x^2 = 9^2 + 11^2 - 2(9)(11) \cos 28°$	Substitution
$x^2 = 202 - 198 \cos 28°$	Simplify.
$x = \sqrt{202 - 198 \cos 28°}$	Take the square root of each side.
$x \approx 5.2°$	Use a calculator.

WatchOut!

Order of operations Remember to follow the order of operations when simplifying expressions. Multiplication or division must be performed before addition or subtraction. So, $202 - 198 \cos 28°$ *cannot* be simplified to $4 \cos 28°$.

1 The Law of Sines

Examples 1 and 2 show how to use the Law of Sines to find the missing measures of a triangle.

Formative Assessment

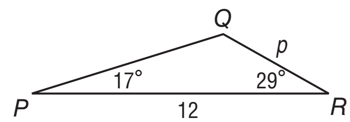
Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Tips for New Teachers

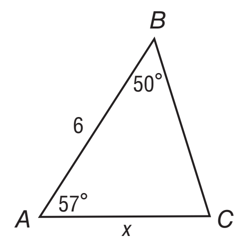
Time-Saver In Example 2, students can also write $k \sin H = h \sin K$ and then substitute values to solve.

Additional Examples

1 Find p. Round to the nearest tenth.



2 Find x. Round to the nearest tenth.



Additional Examples also in Interactive Classroom PowerPoint® Presentations

IWB Interactive White Board READY

2 Law of Cosines

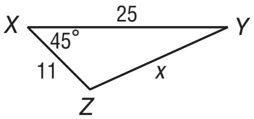
Examples 3 and 4 show how to use the Law of Cosines to find the missing measures of a triangle. **Examples 5 and 6** show how to use the Law of Sines and Law of Cosines to solve a triangle.

DifferentiatedInstruction AL OL

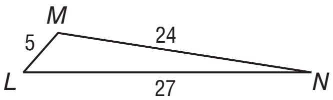
Interpersonal Learners Organize students into groups of three students to work through exercises. One group member can select the exercise. The second member sets up the equation for the Law of Sines and fills in the values. The third member uses a calculator to solve the problem. Group members rotate tasks so each member participates in each responsibility.

Additional Examples

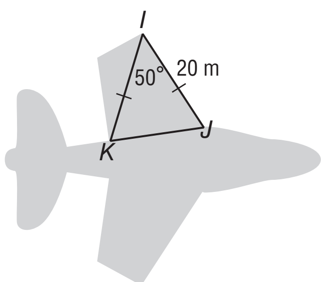
3 Find x. Round to the nearest tenth.



4 Find $m\angle L$. Round to the nearest degree.



5 **AIRCRAFT** From the diagram of the airplane shown, determine the approximate width of each wing. Round to the nearest tenth meter.



Tips for New Teachers

Reasoning Encourage students to recognize a pattern in the equations for the Law of Cosines. Show them that they can memorize one equation and obtain the other two equations by changing the letters. For example, in $a^2 = b^2 + c^2 - 2bc \cos A$, if a and b are reversed, the equation becomes $b^2 = a^2 + c^2 - 2ac \cos B$.

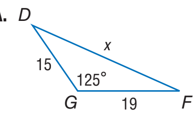
StudyTip

Obtuse Angles There are also values for sin *A*, cos *A*, and tan *A* when $A \geq 90°$. Values of the ratios for these angles can be found using the trigonometric functions on your calculator.

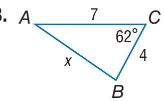
GuidedPractice

Find x. Round to the nearest tenth.

3A.



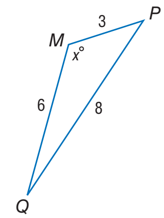
3B.



You can also use the Law of Cosines if you know three side measures (SSS).

Example 4 Law of Cosines (AAS or ASA)

Find x. Round to the nearest degree.



$m^2 = p^2 + q^2 - 2pq \cos M$ — Law of Cosines

$8^2 = 6^2 + 3^2 - 2(6)(3) \cos x°$ — Substitution

$64 = 45 - 36 \cos x°$ — Simplify.

$19 = -36 \cos x°$ — Subtract 45 from each side.

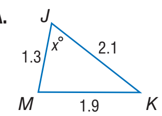
$\frac{19}{-36} = \cos x°$ — Divide each side by −36.

$x = \cos^{-1}\left(-\frac{19}{36}\right)$ — Use the inverse cosine ratio.

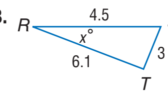
$x \approx 122$ — Use a calculator.

GuidedPractice

4A.



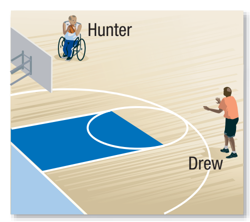
4B.



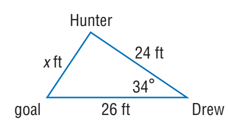
You can use the Law of Sines and Law of Cosines to solve direct and indirect measurement problems.

Real-World Example 5 Indirect Measurement

BASKETBALL **Drew and Hunter are playing basketball. Drew passes the ball to Hunter when he is 26 feet from the goal and 24 feet from Hunter. How far is Hunter from the goal if the angle from the goal to Drew and then to Hunter is 34°?**



Draw a diagram. Since we know two sides of a triangle and the included angle, use the Law of Cosines.

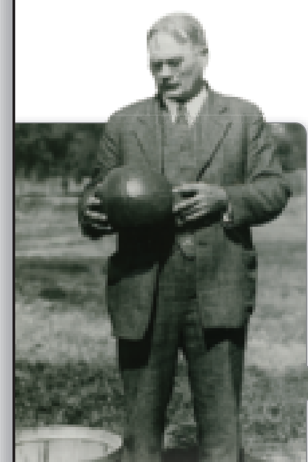


$x^2 = 24^2 + 26^2 - 2(24)(26) \cos 34°$ — Law of Cosines

$x = \sqrt{1252 - 1248 \cos 34°}$ — Simplify and take the positive square root of each side.

$x \approx 15$ — Use a calculator.

Hunter is about 15 feet from the goal when he takes his shot.



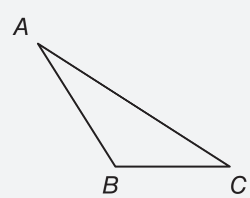
Real-WorldLink

The first game of basketball was played at a YMCA in Springfield, Massachusetts, on December 1, 1891. James Naismith, a physical education instructor, invented the sport using a soccer ball and two half-bushel peach baskets, which is how the name *basketball* came about.

Source: *Encyclopaedia Britannica.*

Focus on Mathematical Content

Check for Reasonableness Consider $\triangle ABC$ with $m\angle A = 25$, $BC = 8$, and $AC = 16$. To find $m\angle B$, we apply the Law of Sines. Thus, $\frac{\sin 25}{8} = \frac{\sin B}{16}$. Solving for B, we get $\sin^{-1} 0.845 = B$. A calculator returns a value of 57.7 for $m\angle B$, but this answer is not reasonable, since $\angle B$ is an obtuse angle. Point out that the sine of an angle x is also equal to the sine of its supplement; that is, $\sin x = \sin (180 - x)$. When finding the measure of this obtuse angle, students will need to find the supplement of their answer. Thus, $m\angle B = 180 - 57.7$ or 122.3.



Guided Practice

5. **LANDSCAPING** At 10 feet away from the base of a tree, the angle the top of a tree makes with the ground is 61°. If the tree grows at an angle of 78° with respect to the ground, how tall is the tree to the nearest foot?

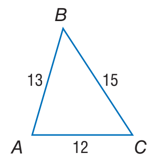
When solving right triangles, you can use sine, cosine, or tangent. When solving other triangles, you can use the Law of Sines or the Law of Cosines, depending on what information is given.

Reading Math

Solve a Triangle Remember that to *solve* a triangle means to find all of the missing side measures and/or angle measures.

Example 6 Solve a Triangle

Solve triangle *ABC*. Round to the nearest degree.



Since $13^2 + 12^2 \neq 15^2$, this is not a right triangle. Since the measures of all three sides are given (SSS), begin by using the Law of Cosines to find $m\angle A$.

$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$15^2 = 12^2 + 13^2 - 2(12)(13) \cos A$	$a = 15$, $b = 12$, and $c = 13$
$225 = 313 - 312 \cos A$	Simplify.
$-88 = -312 \cos A$	Subtract 313 from each side.
$\frac{-88}{-312} = \cos A$	Divide each side by −312.
$m\angle A = \cos^{-1} \frac{88}{312}$	Use the inverse cosine ratio.
$m\angle A \approx 74$	Use a calculator.

Use the Law of Sines to find $m\angle B$.

$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 74°}{15} \approx \frac{\sin B}{12}$	$m\angle A \approx 74$, $a = 15$, and $b = 12$
$12 \sin 74° = 15 \sin B$	Cross Products Property
$\frac{12 \sin 74°}{15} = \sin B$	Divide each side by 15.
$m\angle B = \sin^{-1} \frac{12 \sin 74°}{15}$	Use the inverse sine ratio.
$m\angle B \approx 50$	Use a calculator.

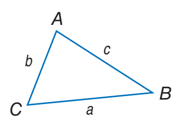
By the Triangle Angle Sum Theorem, $m\angle C \approx 180 - (74 + 50)$ or 56. Therefore $m\angle A \approx 74$, $m\angle B \approx 50$, and $m\angle C \approx 56$.

Watch Out

Rounding When you round a numerical solution and then use it in later calculations, your answers may be inaccurate. Wait until after you have completed all of your calculations to round.

Guided Practice

Solve triangle *ABC* using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

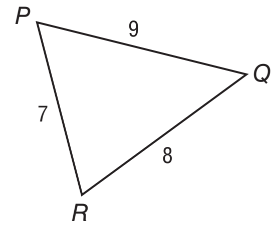


6A. $b = 10.2$, $c = 9.3$, $m\angle A = 26$

6B. $a = 6.4$, $m\angle B = 81$, $m\angle C = 46$

Additional Example

6 Solve triangle *PQR*. Round to the nearest degree.



Teach with Tech

Document Camera Assign several word problems to the class, giving students sufficient time to work through the problems. Then choose several students to share and explain their work to the class. Be sure the students sketch a diagram and explain how they decided whether to use the Law of Sines or the Law of Cosines to solve the problem.

Differentiated Instruction OL BL

Extension Have the students draw right triangles. Then have them use rulers and protractors to find the length of the hypotenuse and the measure of an acute angle. The students should use sine and cosine ratios to find the lengths of the legs of the triangles. Have them check their answers by measuring the legs directly.

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3 Practice

Formative Assessment

Use Exercises 1–11 to check for understanding.

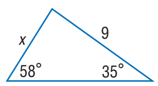
Then use the chart at the bottom of this page to customize assignments for your students.

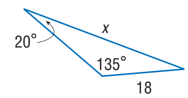
ConceptSummary Solving a Triangle

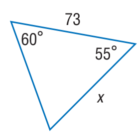
To solve . . .	Given	Begin by using . . .
a right triangle	leg-leg (LL) hypotenuse-leg (HL) acute angle-hypotenuse (AH) acute angle-leg (AL)	tangent ratio sine or cosine ratio sine or cosine ratio sine, cosine, or tangent ratios
any triangle	angle-angle-side (AAS) angle-side-angle (ASA) side-angle-side (SAS) side-side-side (SSS)	Law of Sines Law of Sines Law of Cosines Law of Cosines

Check Your Understanding

Examples 1–2 Find x. Round angle measures to the nearest degree and side measures to the nearest tenth.

1.


2.


3.


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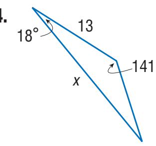
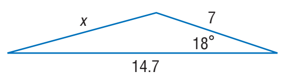


Examples 3–4

5.


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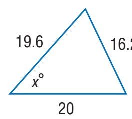
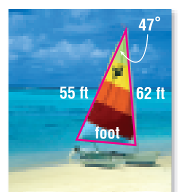
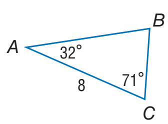


Example 5

7. SAILING Determine the length of the bottom edge, or foot, of the sail.



Example 6 Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

8.


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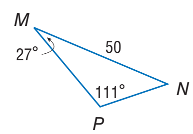

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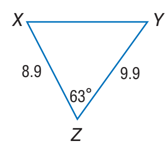


11. Solve $\triangle DEF$ if $DE = 16$, $EF = 21.6$, $FD = 20$.

Differentiated Homework Options

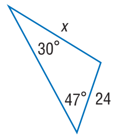
Level	Assignment	Two-Day Option	
AL Basic	12–42, 54, 56–58	13–41 odd	12–42 even, 54, 56–58
OL Core	11–49 odd, 51–54, 56–58	12–42	43–54, 56–58
BL Advanced	43–58		

12 | Lesson 5 | The Law of Sines and Law of Cosines

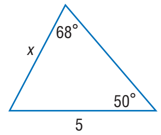
Practice and Problem Solving

Examples 1–2 **Find x. Round side measures to the nearest tenth.**

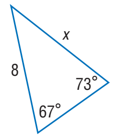
12.



13.



14.



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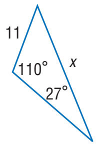

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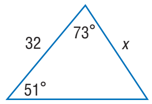
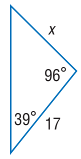


17.



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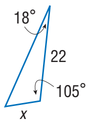

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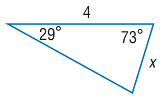
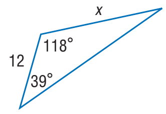
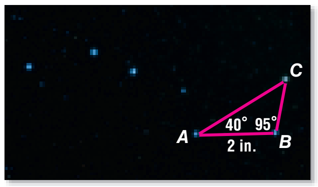


20.

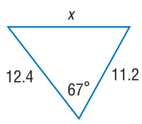


21. ASTRONOMY Angelina is looking at the Big Dipper through a telescope. From her view, the cup of the constellation forms a triangle that has measurements shown on the diagram at the right. Use the Law of Sines to determine distance between A and C.

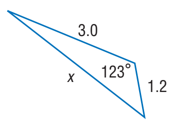


Examples 3–4 **Find x. Round angle measures to the nearest degree and side measures to the nearest tenth.**

22.



23.



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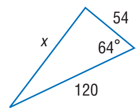

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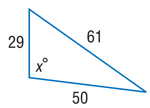

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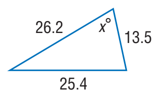

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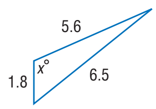
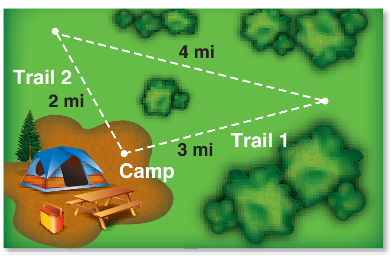


28. HIKING A group of friends who are camping decide to go on a hike. According to the map shown at the right, what is the measure of the angle between Trail 1 and Trail 2?



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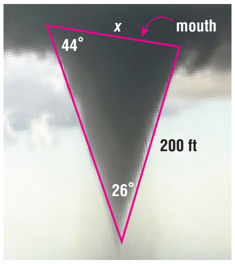
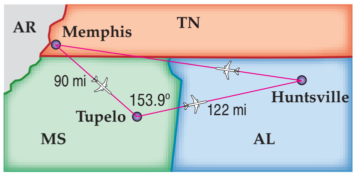
Example 5

29. **TORNADOES** Find the width of the mouth of the tornado shown below.



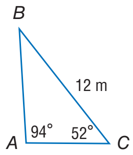
30. **TRAVEL** A pilot flies 90 miles from Memphis, Tennessee, to Tupelo, Mississippi, to Huntsville, Alabama, and finally back to Memphis. How far is Memphis from Huntsville?



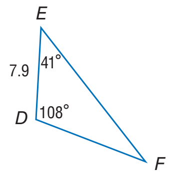
Example 6

Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

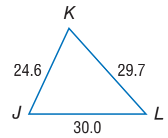
31.



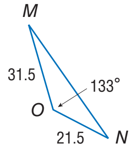
32.



33.



34.



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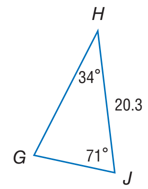

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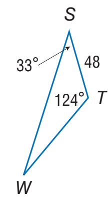

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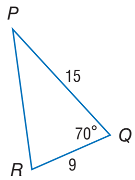

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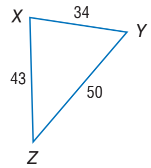

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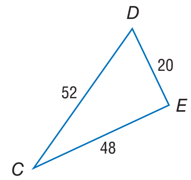
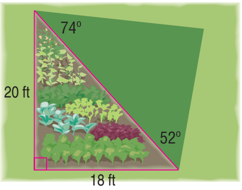


40. Solve $\triangle JKL$ if $JK = 33$, $KL = 56$, $LJ = 65$.

41. Solve $\triangle ABC$ if $m\angle B = 119$, $m\angle C = 26$, $CA = 15$.

42. Solve $\triangle XYZ$ if $XY = 190$, $YZ = 184$, $ZX = 75$.

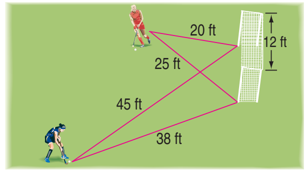
43. **GARDENING** Crystal has an organic vegetable garden. She wants to add another triangular section so that she can start growing tomatoes. If the garden and neighboring space have the dimensions shown, find the perimeter of the new garden to the nearest foot.



Jason Lafontaine/Painet Inc.

14 | Lesson 5 | The Law of Sines and Law of Cosines

44. FIELD HOCKEY Alyssa and Nari are playing field hockey. Alyssa is standing 20 feet from one post of the goal and 25 feet from the opposite post. Nari is standing 45 feet from one post of the goal and 38 feet from the other post. If the goal is 12 feet wide, which player has a greater chance to make a shot? What is the measure of the player's angle?



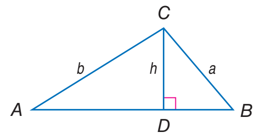
45. PROOF Justify each statement for the derivation of the Law of Sines.

Given: $\overline{CD}$ is an altitude of $\triangle ABC$.

Prove: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Proof:

Statements	Reasons
a. $\sin A = \frac{h}{b}, \sin B = \frac{h}{a}$	**a.** ___?___
b. $b \sin A = h, a \sin B = h$	**b.** ___?___
c. $b \sin A = a \sin B$	**c.** ___?___
d. $\frac{\sin A}{a} = \frac{\sin B}{b}$	**d.** ___?___



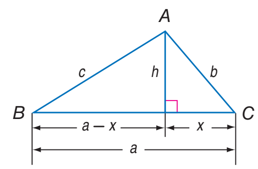
46. PROOF Justify each statement for the derivation of the Law of Cosines.

Given: h is an altitude of $\triangle ABC$.

Prove: $c^2 = a^2 + b^2 - 2ab \cos C$

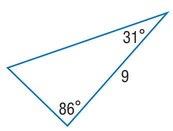
Proof:

Statements	Reasons
a. $c^2 = (a - x)^2 + h^2$	**a.** ___?___
b. $c^2 = a^2 - 2ax + x^2 + h^2$	**b.** ___?___
c. $x^2 + h^2 = b^2$	**c.** ___?___
d. $c^2 = a^2 - 2ax + b^2$	**d.** ___?___
e. $\cos C = \frac{x}{b}$	**e.** ___?___
f. $b \cos C = x$	**f.** ___?___
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	**g.** ___?___
h. $c^2 = a^2 + b^2 - 2ab \cos C$	**h.** ___?___



Find the perimeter of each figure. Round to the nearest tenth.

47.



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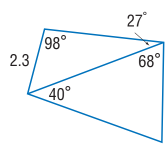

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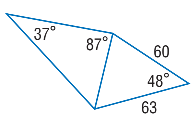
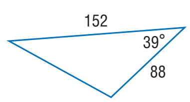


50.



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Multiple Representations

In Exercise 53, students use geometric sketches, algebraic equations, and numeric calculations to investigate the areas of triangles.

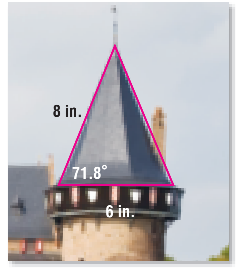
WatchOut!

Error Analysis In Exercise 54, Mike did not apply the Law of Sines correctly. It is important to be careful that the correct proportions are used to compare the angles and side lengths.

4 Assess

Crystal Ball Students know that there are three types of triangles that can be solved using the Law of Sines (AAS, ASA and SSA). Have students list the other types of triangles that may be solved using the Law of Cosines.

51. MODELS Vito is working on a model castle. Find the length of the missing side (in inches) using the diagram at the right.



52. COORDINATE GEOMETRY Find the measure of the largest angle in $\triangle ABC$ with coordinates $A(-3, 6)$, $B(4, 2)$, and $C(-5, 1)$. Explain your reasoning.

53. MULTIPLE REPRESENTATIONS In this problem, you will use trigonometry to find the area of a triangle.

a. Geometric Draw an acute, scalene $\triangle ABC$ including an altitude of length h originating at vertex A.

b. Algebraic Use trigonometry to represent h in terms of $m\angle B$.

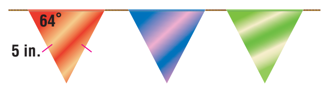
c. Algebraic Write an equation to find the area of $\triangle ABC$ using trigonometry.

d. Numerical If $m\angle B$ is 47, $AB = 11.1$, $BC = 14.1$, and $CA = 10.4$, find the area of $\triangle ABC$. Round to the nearest tenth.

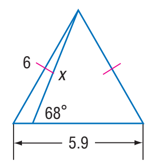
e. Analytical Write an equation to find the area of $\triangle ABC$ using trigonometry in terms of a different angle measure.

H.O.T. Problems Use Higher-Order Thinking Skills

54. ERROR ANALYSIS Colleen and Mike are planning a party. Colleen wants to sew triangular decorations and needs to know the perimeter of one of the triangles to buy enough trim. The triangles are isosceles with angle measurements of 64° at the base and side lengths of 5 inches. Colleen thinks the perimeter is 15.7 inches and Mike thinks it is 15 inches. Is either of them correct?



55. CHALLENGE Find the value of x in the figure at the right.



56. REASONING Explain why the Pythagorean Theorem is a specific case of the Law of Cosines.

57. OPEN ENDED Draw and label a triangle that can be solved:

a. using only the Law of Sines.

b. using only the Law of Cosines.

58. WRITING IN MATH What methods can you use to solve a triangle?

Lenz, G/Arco Images/age fotostock

Follow-up

Students have explored the Law of Sines and Law of Cosines.

Ask:

- What are the strengths and weakness of the Law of Sines and Law of Cosines?

LAB 6

Geometry Lab

The Ambiguous Case

From your work with congruent triangles, you know that three measures determine a unique triangle when the measures are

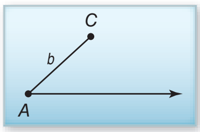
- three sides (SSS),
- two sides and an included angle (SAS),
- two angles and an included side (ASA), or
- two angles and a nonincluded side (AAS).

Common Core State Standards
G.SRT.11

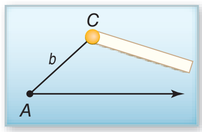
A unique triangle is not necessarily determined by three angles (AAA) or by two sides and a nonincluded angle. In this lab, you will investigate how many triangles are determined by this last case (SSA), called the **ambiguous case**.

Activity 1 The Ambiguous Case (SSA): $\angle A$ is Acute

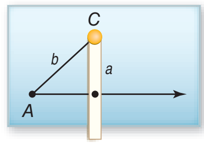
Step 1 On a 5″ × 8″ notecard, draw and label $\overline{AC}$ and a ray extending from A to form an acute angle. Label side $\overline{AC}$ as b.



Step 2 Using a brass fastener, attach one end of a half-inch strip of cardstock to the notecard at C. The strip should be longer than b. This represents side a.



Step 3 Position side a so that it is perpendicular to the ray. Make a black mark on the strip at the point where it touches the ray.



Model and Analyze

1. If a has the given length, how many triangles can be formed? (*Hint*: Rotate the strip to see if the mark can intersect the ray at any other locations to form a different triangle.)
2. Show that if side a is perpendicular to the third side of the triangle, then $a = b \sin A$.

Determine the number of triangles that can be formed given each of the modifications to a in Activity 1.

3. $a < b \sin A$ (*Hint*: Make a green mark above the black mark on the strip, and try to form triangle(s) using this new length for a.)
4. $a = b$ (*Hint*: Rotate the strip so that it lies on top of $\overline{AC}$ and mark off this length in red. Then rotate the strip to try to form triangle(s) using this new length for a.)
5. $a < b$ and $a > b \sin A$ (*Hint*: Make a blue mark between the black and the red marks. Then rotate the strip to try to form triangle(s) using this new length for a.)
6. $a > b$ (*Hint*: Rotate the strip to try to form triangle(s) using the entire length of the strip as the length for a.)

Use your results from Exercises 1–6 to determine whether the given measures define *0, 1, 2,* or *infinitely many* acute triangles. Justify your answers.

7. $a = 14, b = 16, m\angle A = 55$
8. $a = 7, b = 11, m\angle A = 68$
9. $a = 22, b = 25, m\angle A = 39$
10. $a = 13, b = 12, m\angle A = 81$
11. $a = 10, b = 10, m\angle A = 45$
12. $a = 6, b = 9, m\angle A = 24$

(continued on the next page)

1 Focus

Objective

- Determine whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Materials for Each Group

- 5″ × 8″ notecard
- $\frac{1}{2}$-inch strip of cardstock that is about 6 inches long
- brass fasteners
- colored pencils or markers

Teaching Tips

- To refresh students memories about SSS, SAS, ASA, and AAS each determining a unique triangle, you may wish to review triangle congruence constructions using these measures.
- In Exercise 5, many students will see only one triangle that can be formed. Encourage students to rotate side *a* in each Exercise to be sure they have found all possible triangles.

2 Teach

Working in Cooperative Groups

Organize students into groups of 2 or 3, mixing abilities. Then have groups complete Activities 1 and 2 and Exercises 1–6, 13, and 17.

Ask:

- Before calculating any measures, what is the first thing you should note for each problem in Exercises 18–23?

Practice Have students complete Exercises 7–12, 14–16, and 18–23.

3 Assess

Formative Assessment

Use Exercise 24 to assess whether students understand how to determine whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

From Concrete to Abstract

Ask students to summarize what they have learned about the ambiguous case for triangle measures.

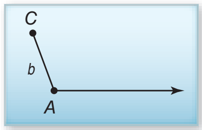
Geometry Lab

The Ambiguous Case *Continued*

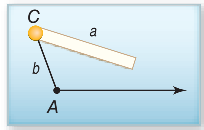
In the next activity, you will investigate how many triangles are determined for the ambiguous case when the angle given is obtuse.

Activity 2 The Ambiguous Case (SSA): $\angle A$ is Obtuse

Step 1 On a 5″ × 8″ notecard, draw and label $\overline{AC}$ and a ray extending from A to form an obtuse angle. Label side $\overline{AC}$ as b.



Step 2 Using a brass fastener, attach one end of a half-inch strip of cardstock to the notecard at C. The strip should be longer than b. This represents side a.



Model and Analyze

13. How many triangles can be formed if $a = b$? if $a < b$? if $a > b$?

Use your results from Exercise 13 to determine whether the given measures define *0, 1, 2,* or *infinitely many* obtuse triangles. Justify your answers.

14. $a = 10, b = 8, m\angle A = 95$ **15.** $a = 13, b = 17, m\angle A = 100$ **16.** $a = 15, b = 15, m\angle A = 125$

17. Explain why three angle measures do not determine a unique triangle. How many triangles are determined by three angles measures?

Determine whether the given measures define *0, 1, 2,* or *infinitely many* triangles. Justify your answers.

18. $a = 25, b = 21, m\angle A = 39$

19. $m\angle A = 41, m\angle B = 68, m\angle C = 71$

20. $a = 17, b = 15, m\angle A = 128$

21. $a = 13, b = 17, m\angle A = 52$

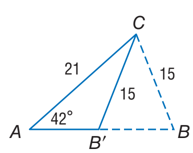
22. $a = 5, b = 9, c = 6$

23. $a = 10, b = 15, m\angle A = 33$

24. **OPEN ENDED** Give measures for a, b, and an acute $\angle A$ that define

a. 0 triangles. **b.** exactly one triangle. **c.** two triangles.

25. **CHALLENGE** Find both solutions for $\triangle ABC$ if $a = 15, b = 21, m\angle A = 42$. Round angle measures to the nearest degree and side measures to the nearest tenth.



- For Solution 1, assume that $\angle B$ is acute, and use the Law of Sines to find $m\angle B$ Then find $m\angle C$. Finally, use the Law of Sines again to find c.
- For Solution 2, assume that $\angle B$ is obtuse. Let this obtuse angle be $\angle B'$. Use $m\angle B$ you found in Solution 1 and the diagram shown to find $m\angle B'$. Then find $m\angle C$. Finally, use the Law of Sines to find c.

18 | Lab 6 | Geometry Lab: The Ambiguous Case

LESSON 7 Vectors

Then

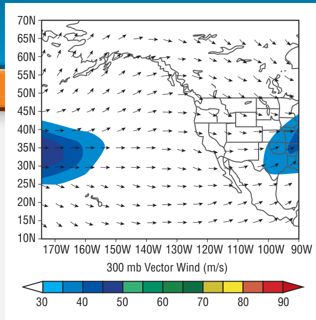
- You used trigonometry to find side lengths and angle measures of right triangles.

Now

1. Perform vector operations geometrically.
2. Perform vector operations on the coordinate plane.

Why?

- Meteorologists use vectors to represent weather patterns. For example, *wind vectors* are used to indicate wind direction and speed.



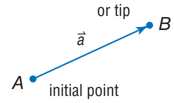
NewVocabulary
vector
magnitude
direction
resultant
parallelogram method
triangle method
standard position
component form

Common Core State Standards
G.GPE.6

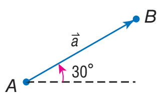
1 Geometric Vector Operations

Some quantities are described by a real number known as a *scalar*, which describes the *magnitude* or size of the quantity. Other quantities are described by a **vector**, which describes both the magnitude and *direction* of the quantity. For example, a speed of 5 miles per hour is a scalar, while a velocity of 5 miles per hour due north is a vector.

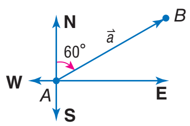
A vector can be represented by a directed line segment with an initial point and a terminal point. The vector shown, with initial point A and terminal point B, can be called $\overrightarrow{AB}$, $\vec{a}$, or **a**.



The **magnitude** of $\overrightarrow{AB}$, denoted $|\overrightarrow{AB}|$, is the length of the vector from its initial point to its terminal point. The **direction** of a vector can be expressed as the angle that it forms with the horizontal or as a measurement between 0° and 90° east or west of the north-south line.



The direction of $\vec{a}$ is 30° relative to the horizontal.



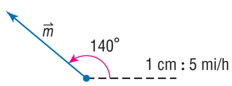
The direction of $\vec{a}$ is 60° east of north.

Example 1 Represent Vectors Geometrically

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

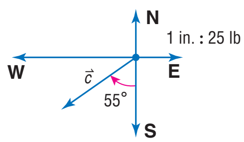
a. $\vec{m}$ **= 15 miles per hour at 140° to the horizontal**

Using a scale of 1 cm : 5 mi/h, draw and label a 15 ÷ 5 or 3-centimeter arrow at a 140° angle to the horizontal.



b. $\vec{c}$ **= 55 pounds of force 55° west of south**

Using a scale of 1 in: 25 lbs, draw and label a 55 ÷ 25 or 2.2-inch arrow 55° west of the north-south line on the south side.



GuidedPractice

1A. $\vec{b}$ = 40 feet per second at 35° to the horizontal

1B. $\vec{t}$ = 12 kilometers per hour at 85° east of north

1 Focus

VerticalAlignment

Before Lesson 7 Use trigonometry to find side lengths and angle measures of right triangles.

Lesson 7 Perform vector operations geometrically and on the coordinate plane.

After Lesson 7 Use and extend similarity properties and transformations.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- Why do you think that vectors are used to represent wind direction and speed?
- What types of quantities do you think that vectors could be used to represent?

Geometric Vector Operations

Example 1 shows how to represent a vector geometrically. **Example 2** shows how to find the resultant of two vectors.

Formative Assessment

Use the Guided Practice exercises after each example to determine students' understanding of concepts.

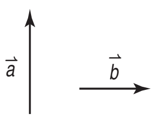
Additional Examples

1 Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

a. $\vec{s} = 80$ meters at 24° west of north

b. $\vec{d} = 16$ yards per second at 165° to the horizontal

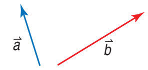
2 Copy the vectors. Then find $\vec{a} - \vec{b}$.



The sum of two or more vectors is a single vector called the **resultant**.

KeyConcept Vector Addition

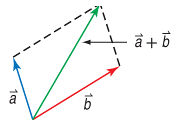
To find the resultant of $\vec{a}$ and $\vec{b}$, use one of the following methods.



Parallelogram Method

Step 1 Translate $\vec{b}$ so that the tail of $\vec{b}$ touches the tail of $\vec{a}$.

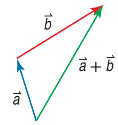
Step 2 Complete the parallelogram. The resultant is the indicated diagonal of the parallelogram.



Triangle Method

Step 1 Translate $\vec{b}$ so that the tail of $\vec{b}$ touches the tip of $\vec{a}$.

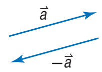
Step 2 Draw the resultant vector from the tail of $\vec{a}$ to the tip of $\vec{b}$.



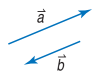
StudyTip

Types of Vectors

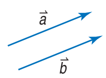
Parallel vectors have the same or opposite direction but not necessarily the same magnitude.



Opposite vectors have the same magnitude but *opposite* direction.



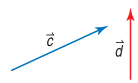
Equivalent vectors have the same magnitude and direction.



Example 2 Find the Resultant of Two Vectors

Copy the vectors. Then find $\vec{c} - \vec{d}$.

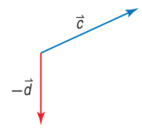
Subtracting a vector is equivalent to adding its opposite vector.



Parallelogram Method

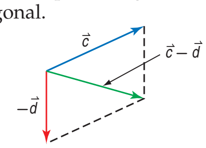
Step 1

Copy $\vec{c}$ and $\vec{d}$. Draw $-\vec{d}$, and translate it so that its tail touches the tail of $\vec{c}$.



Step 2

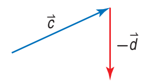
Complete the parallelogram. Then draw the diagonal.



Triangle Method

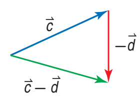
Step 1

Copy $\vec{c}$ and $\vec{d}$. Draw $-\vec{d}$, and translate it so that its tail touches the tip of $\vec{c}$.



Step 2

Draw the resultant vector from the tail of $\vec{c}$ to the tip of $-\vec{d}$.



Both methods produce the same resultant vector $\vec{c} - \vec{d}$. You can use a ruler and a protractor to measure the magnitude and direction of each vector to verify your results.

GuidedPractice

2A. Find $\vec{c} + \vec{d}$.

2B. Find $\vec{d} - \vec{c}$.

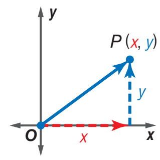
DifferentiatedInstruction OL BL

Extension The vector $k\vec{v}$ is a dilation of the vector $\vec{v}$ with scale factor k. What is the direction of $k\vec{v}$ when $k > 0$? $k < 0$? When $k > 0$, $k\vec{v}$ has the same direction as $\vec{v}$.

2 Vectors on the Coordinate Plane

Vectors can also be represented on the coordinate plane.

A vector is in **standard position** if its initial point is at the origin. In this position, a vector can be uniquely described by its terminal point $P(x, y)$.



To describe a vector with any initial point, you can use the **component form** $\langle x, y\rangle$, which describes the vector in terms of its horizontal component x and vertical component y.

To write the component form of a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), find $\langle x_2 - x_1, y_2 - y_1\rangle$.

Example 3 Write a Vector in Component Form

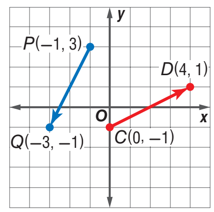
PT

Write the component form of $\overrightarrow{CD}$.

$\overrightarrow{CD} = \langle x_2 - x_1, y_2 - y_1\rangle$ — Component form of a vector

$= \langle 4 - 0, 1 - (-1)\rangle$ — $(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (4, 1)$

$= \langle 4, 2\rangle$ — Simplify.



Guided Practice

3. Write the component form of $\overrightarrow{PQ}$.

The magnitude of a vector on the coordinate plane can be found by using the Distance Formula, and the direction can be found by using trigonometric ratios.

Example 4 Find the Magnitude and Direction of a Vector

PT

Find the magnitude and direction of $\vec{r} = \langle -4, -5\rangle$.

Step 1 Use the Distance Formula to find the magnitude.

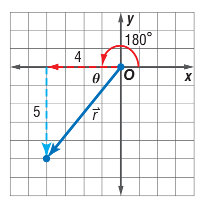
$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{(-4 - 0)^2 + (-5 - 0)^2}$ — $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-4, -5)$

$= \sqrt{41}$ or about 6.4 — Simplify.

Step 2 Use trigonometry to find the direction.

Graph $\vec{r}$, its horizontal component, and its vertical component. Then use the inverse tangent function to find θ.



$\tan \theta = \frac{5}{4}$ — $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$\theta = \tan^{-1} \frac{5}{4}$ — Def. of inverse tangent

$\theta \approx 51.3°$ — Use a calculator.

The direction of $\vec{r}$ is the angle that it makes with the positive x-axis, which is about $180° + 51.3°$ or $231.3°$.

So, the magnitude of $\vec{r}$ is about 6.4 units and the direction is at an angle of about 231.3° to the horizontal.

Guided Practice

4. Find the magnitude and direction of $\vec{p} = \langle -1, 4\rangle$.

Study Tip

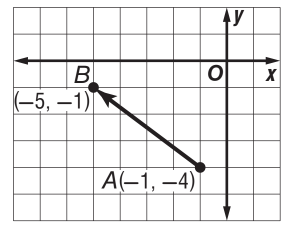
Direction Angles Vectors in standard position that lie in the third or fourth quadrants will have direction angles greater than 180°.

Vectors on the Coordinate Plane

Example 3 shows how to write a vector in component form. **Example 4** shows how to find the magnitude and direction of a vector. **Example 5** shows how to operate with vectors in the coordinate plane.

Additional Examples

3 Write the component form of $\overrightarrow{AB}$.



4 Find the magnitude and direction of $\vec{t} = \langle 7, -5\rangle$.

Watch Out!

Vector Notation Remind students that the notation $\langle a, b\rangle$ is used to represent the component form of a vector, while the notation (a, b) is used to represent a point such as the initial or terminal point of the vector.

Teach with Tech

Interactive Whiteboard Display a coordinate plane on the board. Draw a vector in standard position and show students how to find its magnitude and direction. Then drag the vector to other locations on the plane and find the magnitude and direction of the vector again. Explain to students that the location of the vector on the coordinate plane does not change its magnitude or direction.

Additional Example

5 Find each of the following for $\vec{x} = \langle -1, 4 \rangle$, $\vec{y} = \langle 3, 1 \rangle$, and $\vec{z} = \langle -2, -4 \rangle$.

a. $\vec{x} + \vec{y}$

b. $\vec{z} - \vec{x}$

c. $2\vec{y} + \vec{z}$

You can use the properties of real numbers to add vectors, subtract vectors, and multiply vectors by scalars.

KeyConcept Vector Operations

If $\langle a, b \rangle$ and $\langle c, d \rangle$ are vectors and k is a scalar, then the following are true.

Vector Addition	$\langle a, b \rangle + \langle c, d \rangle = \langle a + c, b + d \rangle$
Vector Subtraction	$\langle a, b \rangle - \langle c, d \rangle = \langle a - c, b - d \rangle$
Scalar Multiplication	$k\langle a, b \rangle = \langle ka, kb \rangle$

PT

Example 5 Operations with Vectors

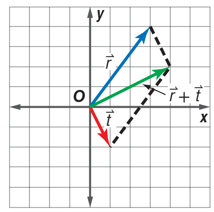
Find each of the following for $\vec{r} = \langle 3, 4 \rangle$, $\vec{s} = \langle 5, -1 \rangle$, and $\vec{t} = \langle 1, -2 \rangle$. Check your answers graphically.

a. $\vec{r} + \vec{t}$

Solve Algebraically

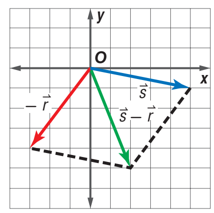
$$\vec{r} + \vec{t} = \langle 3, 4 \rangle + \langle 1, -2 \rangle$$
$$= \langle 3 + 1, 4 + (-2) \rangle$$
$$= \langle 4, 2 \rangle$$

Check Graphically



b. $\vec{s} - \vec{r}$

$$\vec{s} - \vec{r} = \vec{s} + (-\vec{r})$$
$$= \langle 5, -1 \rangle + \langle -3, -4 \rangle$$
$$= \langle 5 + (-3), -1 + (-4) \rangle$$
$$= \langle 2, -5 \rangle$$

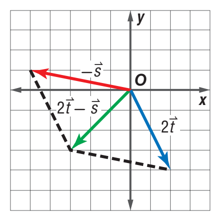


StudyTip

Vector Subtraction To represent vector subtraction graphically, graph the opposite of the vector that is being subtracted. For instance, in Example 5b, the opposite of $\vec{r} = \langle 3, 4 \rangle$ is $-\vec{r} = \langle -3, -4 \rangle$.

c. $2\vec{t} - \vec{s}$

$$2\vec{t} - \vec{s} = 2\vec{t} + (-\vec{s})$$
$$= 2\langle 1, -2 \rangle + \langle -5, 1 \rangle$$
$$= \langle 2, -4 \rangle + \langle -5, 1 \rangle$$
$$= \langle 2 + (-5), -4 + 1 \rangle$$
$$= \langle -3, -3 \rangle$$



StudyTip

Scalar Multiplication The graph of a vector $k\langle a, b \rangle$ is a dilation of the vector $\langle a, b \rangle$ with scale factor k. For instance, in Example 5c, $2\vec{t} = \langle 2, -4 \rangle$ is a dilation of $\vec{t} = \langle 1, -2 \rangle$ with scale factor 2.

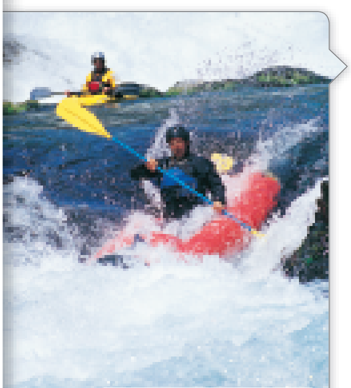
GuidedPractice

5A. $\vec{t} - \vec{r}$ **5B.** $\vec{s} + 2\vec{t}$ **5C.** $\vec{s} - \vec{t}$

DifferentiatedInstruction AL OL

Intrapersonal Learners Have students create and plot their own examples of a pair of equivalent vectors, a pair of parallel vectors, and a vector that is multiplied by a constant. Tell students to label the magnitude and direction of each vector and find the component form of each vector. Students can then create an example of a translation with vectors and vector addition on another sheet of paper.

You can use vectors to solve real-world problems.



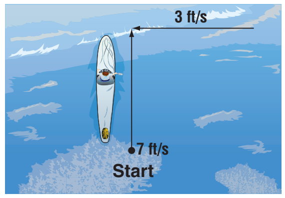
Real-WorldLink

Approximately 47% of kayakers participate in the sport one to three times per year.

Source: Outdoor Industry Association

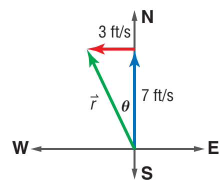
Real-World Example 6 Vector Applications

KAYAKING Trey is paddling due north in a kayak at 7 feet per second. The river is moving with a velocity of 3 feet per second due west. What is the resultant speed and direction of the kayak to an observer on shore?



Step 1 Draw a diagram. Let $\vec{r}$ represent the resultant vector.

The component form of the vector representing the paddling velocity is $\langle 0, 7\rangle$, and the component form of the vector representing the velocity of the river is $\langle -3, 0\rangle$.



The resultant vector is $\langle 0, 7\rangle + \langle -3, 0\rangle$ or $\langle -3, 7\rangle$. This vector represents the resultant velocity of the kayak, and its magnitude represents the resultant speed.

Step 2 Use the Distance Formula to find the resultant speed.

$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-3 - 0)^2 + (7 - 0)^2}$ $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-3, 7)$

$= \sqrt{58}$ or about 7.6 Simplify.

Step 3 Use trigonometry to find the resultant direction.

$\tan \theta = \frac{3}{7}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$\theta = \tan^{-1} \frac{3}{7}$ Def. of inverse tangent

$\theta \approx 23.2°$ Use a calculator.

The direction of $\vec{r}$ is about 23.2° west of north.

Therefore, the resultant speed of the kayak is about 7.6 feet per second at an angle of about 23.2° west of north.

GuidedPractice

6. **KAYAKING** Suppose Trey starts paddling due south at a speed of 8 feet per second. If the river is flowing at a velocity of 2 feet per second due west, what is the resultant speed and direction of the kayak?

Javier Pierini/Digital Vision/Getty Images

Example 6 shows how to use vectors to solve real-world problems.

Additional Example

6 **CANOEING** Suppose a person is canoeing due east across a river at 4 miles per hour. If the river is flowing south at 3 miles per hour, what is the resultant velocity of the canoe?

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3 Practice

Formative Assessment

Use Exercises 1–11 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Check Your Understanding

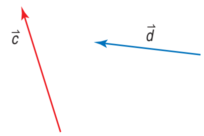
Example 1 **Use a ruler and a protractor to draw each vector. Include a scale on each diagram.**

1. $\vec{w}$ = 75 miles per hour 40° east of south

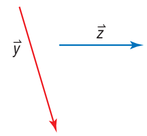
2. $\vec{h}$ = 46 feet per second 170° to the horizontal

Example 2 **Copy the vectors. Then find each sum or difference.**

3. $\vec{c} + \vec{d}$

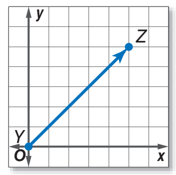


4. $\vec{y} - \vec{z}$

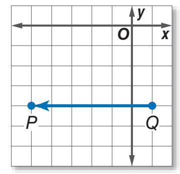


Example 3 **Write the component form of each vector.**

5.



6.



Example 4 **Find the magnitude and direction of each vector.**

7. $\vec{t} = \langle 2, -4 \rangle$

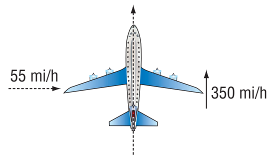
8. $\vec{f} = \langle -6, -5 \rangle$

Example 5 **Find each of the following for $\vec{a} = \langle -4, 1 \rangle$, $\vec{b} = \langle -1, -3 \rangle$, and $\vec{c} = \langle 3, 5 \rangle$. Check your answers graphically.**

9. $\vec{c} + \vec{a}$

10. $2\vec{b} - \vec{a}$

Example 6 **11.** **TRAVEL** A plane is traveling due north at a speed of 350 miles per hour. If the wind is blowing from the west at a speed of 55 miles per hour, what is the resultant speed and direction that the airplane is traveling?



Practice and Problem Solving

Example 1 **Use a ruler and a protractor to draw each vector. Include a scale on each diagram.**

12. $\vec{g}$ = 60 inches per second at 145° to the horizontal

13. $\vec{n}$ = 8 meters at an angle of 24° west of south

14. $\vec{a}$ = 32 yards per minute at 78° to the horizontal

15. $\vec{k}$ = 95 kilometers per hour at angle of 65° east of north

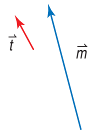
Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	12–42, 49–53	13–41 odd	12–45 even, 49–53
OL Core	13–45 odd, 46–47	12–42	43–53
BL Advanced	43–53		

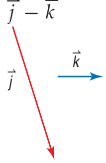
24 | Lesson 7 | Vectors

Example 2 Copy the vectors. Then find each sum or difference.

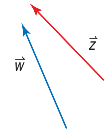
16. $\vec{t} - \vec{m}$



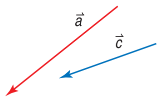
17. $\vec{j} - \vec{k}$



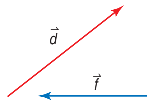
18. $\vec{w} + \vec{z}$



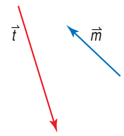
19. $\vec{c} + \vec{a}$



20. $\vec{d} - \vec{f}$

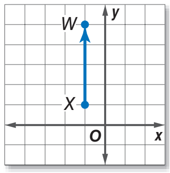


21. $\vec{t} + \vec{m}$

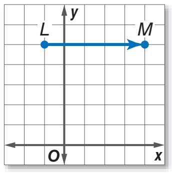


Example 3 Write the component form of each vector.

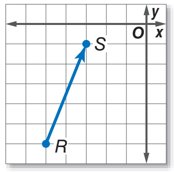
22.



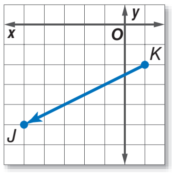
23.



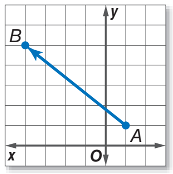
24.



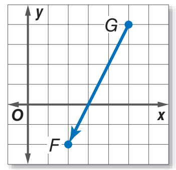
25.



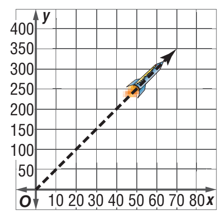
26.



27.



28. **FIREWORKS** The ascent of a firework shell can be modeled using a vector. Write a vector in component form that can be used to describe the path of the firework shown.



Example 4 Find the magnitude and direction of each vector.

29. $\vec{c} = \langle 5, 3 \rangle$

30. $\vec{m} = \langle 2, 9 \rangle$

31. $\vec{z} = \langle -7, 1 \rangle$

32. $\vec{d} = \langle 4, -8 \rangle$

33. $\vec{k} = \langle -3, -6 \rangle$

34. $\vec{q} = \langle -9, -4 \rangle$

Example 5 Find each of the following for $\vec{a} = \langle -3, -5 \rangle$, $\vec{b} = \langle 2, 4 \rangle$, and $\vec{c} = \langle 3, -1 \rangle$. Check your answers graphically.

35. $\vec{b} + \vec{c}$

36. $\vec{c} + \vec{a}$

37. $\vec{b} - \vec{c}$

38. $\vec{a} - \vec{c}$

39. $2\vec{c} - \vec{a}$

40. $2\vec{b} + \vec{c}$

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4 Assess

Name the Math Students can practice naming vector parts and explaining how vectors apply to real-world situations. Select examples or create examples, and select students to use the vocabulary terms and concepts of this lesson to analyze the problems aloud in class.

41. **HIKING** Amy hiked due east for 2 miles and then hiked due south for 3 miles.

a. Draw a diagram to represent the situation, where $\vec{r}$ is the resultant vector.

b. How far and in what direction is Amy from her starting position?

42. **EXERCISE** A runner's velocity is 6 miles per hour due east, with the wind blowing 2 miles per hour due north.

a. Draw a diagram to represent the situation, where $\vec{r}$ is the resultant vector.

b. What is the resultant velocity of the runner?

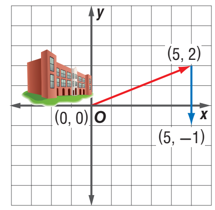
Find each of the following for $\vec{f} = \langle -4, -2 \rangle$, $\vec{g} = \langle 6, 1 \rangle$, and $\vec{h} = \langle 2, -3 \rangle$.

43. $\vec{f} + \vec{g} + \vec{h}$ **44.** $\vec{h} - 2\vec{f} + \vec{g}$ **45.** $2\vec{g} - 3\vec{f} + \vec{h}$

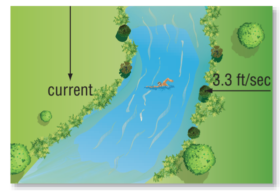
46. **HOMECOMING** Nikki is on a committee to help plan her school's homecoming parade. The parade starts at the high school and continues as shown.

a. Find the magnitude and direction of the vector formed with an initial point at the school and terminal point at the end of the parade.

b. Find the length of the parade if 1 unit = 0.25 mile.



47. **SWIMMING** Jonas is swimming from the east bank to the west bank of a stream at a speed of 3.3 feet per second. The stream is 80 feet wide and flows south. If Jonas crosses the stream in 20 seconds, what is the speed of the current?



H.O.T. Problems Use Higher-Order Thinking Skills

48. **CHALLENGE** Find the coordinates of point *P* on *AB* that partitions the segment into the given ratio *AP* to *PB*.

a. A(0, 0), B(0, 6), 2 to 1 **b.** A(0, 0), B(-15, 0), 2 to 3

49. **REASONING** Are parallel vectors *sometimes*, *always*, or *never* opposite vectors? Explain.

PROOF **Prove each vector property. Let $\vec{a} = \langle x_1, y_1 \rangle$ and $\vec{b} = \langle x_2, y_2 \rangle$.**

50. commutative: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

51. scalar multiplication: $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$, where k is a scalar

52. **OPEN ENDED** Draw a set of parallel vectors.

a. Find the sum of the two vectors. What is true of the direction of the vector representing the sum?

b. Find the difference of the two vectors. What is true of the direction of the vector representing the difference?

53. **WRITING IN MATH** Compare and contrast the parallelogram and triangle methods of adding vectors.

26 | Lesson 7 | Vectors

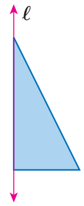
LAB 8 Geometry Lab
Solids of Revolution

A solid of revolution is a three-dimensional figure obtained by rotating a plane figure or curve about a line.

Common Core State Standards
G.GMD.4

Activity 1

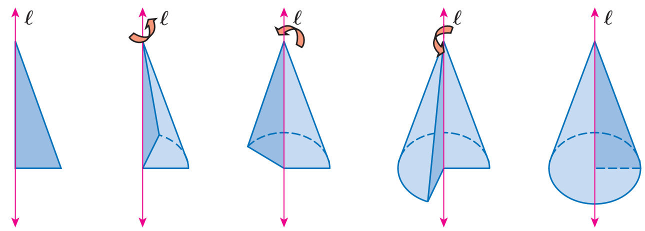
Identify and sketch the solid formed by rotating the right triangle shown about line ℓ.



Step 1 Copy the triangle onto card stock or heavy construction paper and cut it out.

Step 2 Use tape to attach the triangle to a dowel rod or straw.

Step 3 Rotate the end of the straw quickly between your hands and observe the result.

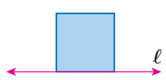


The blurred image you observe is that of a cone.

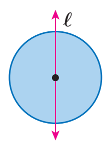
Model and Analyze

Identify and sketch the solid formed by rotating the two-dimensional shape about line ℓ.

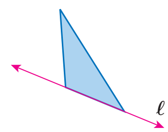
1.



2.



3.

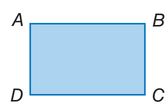


4. Sketch and identify the solid formed by rotating the rectangle shown about the line containing

a. side $\overline{AB}$.

b. side $\overline{AD}$.

c. the midpoints of sides $\overline{AB}$ and $\overline{AD}$.



5. DESIGN Draw a two-dimensional figure that could be rotated to form the vase shown, including the line in which it should be rotated.

6. REASONING *True* or *false:* All solids can be formed by rotating a two-dimensional figure. Explain your reasoning.

1 Focus

Objective Identify and sketch solids formed by revolving two-dimensional figures about lines.

Materials

- straws or dowel rods
- card stock or heavy construction paper
- graphing paper

Teaching Tip

Have students predict what solid will result before doing each activity and exercise.

2 Teach

Working in Cooperative Groups

Have students work in mixed ability pairs, taking turns rotating the figure. Encourage students to draw, cut out, attach, and rotate shapes to verify the sketches they generate for each exercise.

Practice Have students complete Activities 1 and 2 and Exercises 1–5, 7–12, and 14.

3 Assess

Formative Assessment

Use Exercises 6 and 13 to assess each student's understanding of solids of revolution.

From Concrete to Abstract

Ask students to summarize what they learned about solids of revolution and to suggest ways in which they might be used in the real world.

Geometry Lab

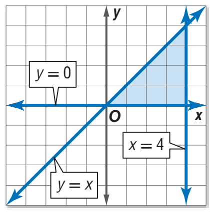
Solids of Revolution *Continued*

In calculus, you will be asked to find the volumes of solids generated by revolving a region on the coordinate plane about the x- or y-axis. An important first step in solving these problems is visualizing the solids formed.

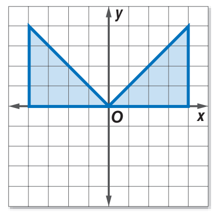
Activity 2

Sketch the solid that results when the region enclosed by $y = x$, $x = 4$, and $y = 0$ is revolved about the y-axis.

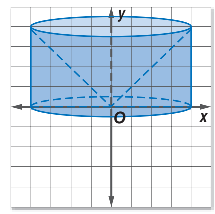
Step 1 Graph each equation to find the region to be rotated.



Step 2 Reflect the region about the y-axis.



Step 3 Connect the vertices of the right triangle using curved lines.



The solid is a cylinder with a cone cut out of its center.

Model and Analyze

Sketch the solid that results when the region enclosed by the given equations is revolved about the y-axis.

7. $y = -x + 4$
$x = 0$
$y = 0$

8. $y = x^2$
$x = 0$
$y = 4$

9. $y = x^2$
$y = 2x$

Sketch the solid that results when the region enclosed by the given equations is revolved about the x-axis.

10. $y = -x + 4$
$x = 0$
$y = 0$

11. $y = x^2$
$x = 0$
$y = 4$

12. $y = x^2$
$y = 2x$

13. **OPEN ENDED** Graph a region in the first quadrant of the coordinate plane.

a. Sketch the graph of the region when revolved about the y-axis.

b. Sketch the graph of the region when revolved about the x-axis.

14. **CHALLENGE** Find equations that enclose a region such that when rotated about the x-axis, a solid is produced with a volume of 18π cubic units.

28 | Lab 8 | Geometry Lab: Solids of Revolution

LAB 9

Geometry Lab

Exploring Constructions with a Reflective Device

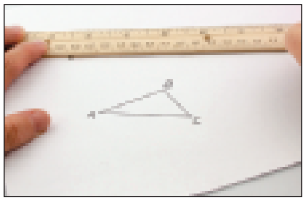
A reflective device is a tool made of semitransparent plastic that reflects objects. It works best if you lay it on a flat service in a well-lit room. You can use a reflective device to transform geometric objects.

Common Core State Standards
G.CO.12

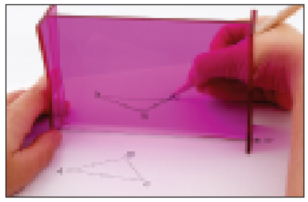
Activity 1 Reflect a Triangle

Use a reflective device to reflect △*ABC* in *w*. Label the reflection △*A′B′C′*.

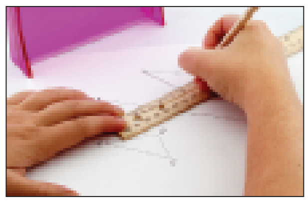
Step 1 Draw △*ABC* and the line of reflection *w*.



Step 2 With the reflective device on line *w*, draw points for the vertices of the reflection.



Step 3 Use a straightedge to connect the points to form △*A′B′C′*.

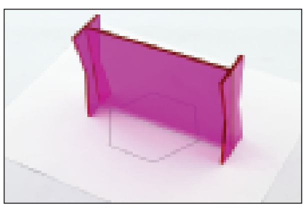


We have used a compass, straightedge, string, and paper folding to make geometric constructions. You can also use a reflective device for constructions.

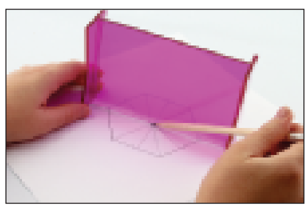
Activity 2 Construct Lines of Symmetry

Use a reflective device to construct the lines of symmetry for a regular hexagon.

Step 1 Draw a regular hexagon. Place the reflective device on the shape and move it until one half of the shape matches the reflection of the other half. Draw the line of symmetry.



Step 2 Repeat Step 1 until you have found all the lines of symmetry.



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1 Focus

Objective Use a reflective device for geometric constructions.

Materials

- reflective device
- straightedge

Teaching Tip

Reflective devices work best when used in a well-lit room on a flat surface. Students can use the edge of the reflective device as a straightedge.

Alternative Method

The constructions presented in this lab can also be completed using classic ruler and compass methods.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activity 1 and Activity 2 as a class. Then ask students to work with their partners to complete Activity 3 and Activity 4.

Practice Have students complete Exercise 1.

3 Assess

Formative Assessment

Use Exercise 2 to assess each student's ability to complete a construction with a reflective device.

From Concrete to Abstract

Ask students to summarize how they can utilize a reflective device for geometric constructions.

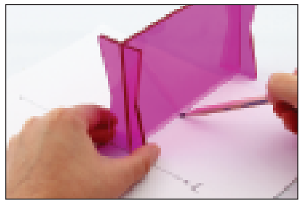
Geometry Lab

Exploring Constructions with a Reflective Device *Continued*

Activity 3 Construct a Parallel line

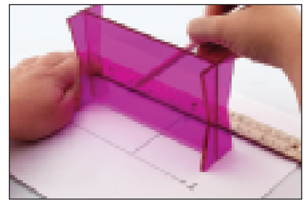
Use a reflective device to reflect line ℓ to line *m* that is parallel and passes through point *P*.

Step 1



Draw line ℓ and point *P*. Place a short side of the reflective device on line ℓ and the long side on point *P*. Draw a line. This line is perpendicular to ℓ through *P*.

Step 2



Place the reflective device so that the perpendicular line coincides with itself and the reflection of line ℓ passes through point *P*. Use a straightedge to draw the parallel line *m* through *P*.

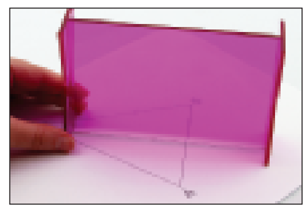
In Explore Lesson 5-1, we constructed perpendicular bisectors with paper folding. You can also use a reflective device to construct perpendicular bisectors of a triangle.

Activity 4 Construct Perpendicular Bisectors

Use a reflective device to find the circumcenter of △*ABC*.

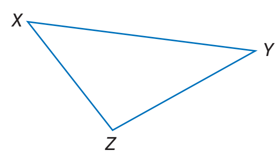
Step 1 Draw △*ABC*. Place the reflective device between *A* and *B* and adjust it until *A* and *B* coincide. Draw the line of symmetry.

Step 2 Repeat Step 1 for sides $\overline{AC}$ and $\overline{BC}$. Then place a point at the intersection of the three perpendicular bisectors. This is the circumcenter of the triangle.



Model and Analyze

1. How do you know that the steps in Activity 4 give the actual perpendicular bisector and the circumcenter of △*ABC*?



2. Construct the angle bisectors and find the incenter of △*XYZ*. Describe how you used the reflective device for the construction.

LAB 10 Graphing Technology Lab
Dilations

You can use TI-Nspire Technology to explore properties of dilations.

Common Core State Standards
G.SRT.1

Activity 1 Dilation of a Triangle

Dilate a triangle by a scale factor of 1.5.

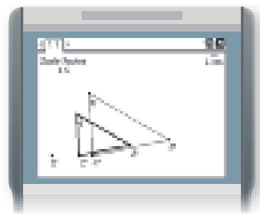
Step 1 Add a new **Geometry** page. Then, from the **Points & Lines** menu, use the **Point** tool to add a point and label it *X*.

Step 2 From the **Shapes** menu, select **Triangle** and specify three points. Label the points *A*, *B*, and *C*.

Step 3 From the **Actions** menu, use the **Text** tool to separately add the text *Scale Factor* and *1.5* to the page.

Step 4 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\triangle ABC$, and the text *1.5*.

Step 5 Label the points on the image *A′*, *B′*, and *C′*.



Analyze the Results

1. Using the **Slope** tool on the **Measurement** menu, describe the effect of the dilation on $\overline{AB}$. That is, how are the lines through $\overline{AB}$ and $\overline{A'B'}$ related?
2. What is the effect of the dilation on the line passing through side $\overline{CA}$?
3. What is the effect of the dilation on the line passing through side $\overline{CB}$?

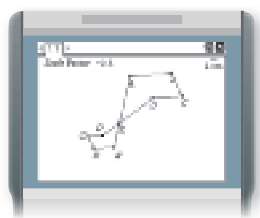
Activity 2 Dilation of a Polygon

Dilate a polygon by a scale factor of −0.5.

Step 1 Add a new **Geometry** page and draw polygon *ABCDX* as shown. Add the text *Scale Factor* and *−0.5* to the page.

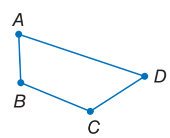
Step 2 From the **Transformation** menu, select **Dilation**. Then select point *X*, polygon *ABCDX*, and the text *−0.5*.

Step 3 Label the points on the image *A′*, *B′*, *C′*, and *D′*.



Model and Analyze

4. Analyze the effect of the dilation in Activity 2 on sides that are on lines passing through the center of the dilation.
5. Analyze the effect of a dilation of trapezoid *ABCD* shown with a scale factor of 0.75 and the center of the dilation at *A*.
6. **MAKE A CONJECTURE** Describe the effect of a dilation on lines that pass through the center of a dilation and lines that do not pass through the center of a dilation.



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1 Focus

Objective Use graphing technology to explore the properties of dilations.

Materials

- TI-Nspire technology

Teaching Tip

- Explain to students that their measurements will not be the same as the measurements on the screen.
- When dilating the figures, it is important that the display confirms the selection prior to pressing the enter key.
- If the transformation causes the figure to move off the screen, move the cursor to a blank place on the screen and hold down on the center of the touchpad until the hand closes. Drag until the figure is on the screen.
- To use the **Slope** tool on the **Measurement** menu, students will need to place a line segment on top of the side of the triangle.

Alternative Method

The activities presented in this lesson can also be completed using Geometer's Sketchpad software or Cabri Jr. on a TI-84.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activity 1 as a class. Then ask students to work with their partners to complete Activities 2 and 3.

Practice Have students complete Exercises 7–9.

3 Assess

Formative Assessment

Use Exercises 10–12 to assess each student's knowledge of the effects of dilating figures.

From Concrete to Abstract

Ask students to summarize the effects of dilations on line segments and on figures.

Graphing Technology Lab

Dilations *Continued*

Activity 3 Dilation of a Segment

Dilate a segment $\overline{AB}$ by the indicated scale factor.

a. scale factor: 0.75

Step 1 On a new **Geometry** page, draw a line segment using the **Points & Lines** menu. Label the endpoints *A* and *B*. Then add and label a point *X*.

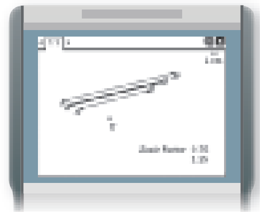
Step 2 Add the text *Scale Factor* and *0.75* to the page.

Step 3 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\overline{AB}$, and the text *0.75*.

Step 4 Label the dilated segment $\overline{A'B'}$.

b. scale factor: 1.25

Step 1 Add the text *1.25* to the page.



Step 2 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\overline{AB}$, and the text 1.25.

Step 3 Label the dilated segment $\overline{A''B''}$.

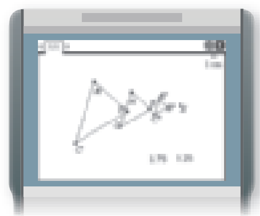
Model and Analyze

7. Using the **Length** tool on the **Measurement** menu, find the measures of $\overline{AB}$, $\overline{A'B'}$, and $\overline{A''B''}$.

8. What is the ratio of *A′B′* to *AB*? What is the ratio of *A″B″* to *AB*?

9. What is the effect of the dilation with scale factor 0.75 on segment $\overline{AB}$? What is the effect of the dilation with scale factor 1.25 on segment $\overline{AB}$?

10. Dilate segment $\overline{AB}$ in Activity 3 by scale factors of −0.75 and −1.25. Describe the effect on the length of each dilated segment.



11. **MAKE A CONJECTURE** Describe the effect of a dilation on the length of a line segment.

12. Describe the dilation from $\overline{AB}$ to $\overline{A'B'}$ and $\overline{A'B'}$ to $\overline{A''B''}$ in the triangles shown.

32 | Lab 10 | Graphing Technology Lab: Dilations

LAB 11

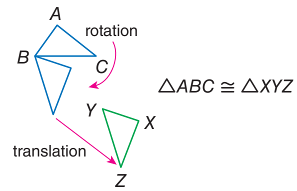
Geometry Lab
Establishing Triangle Congruence and Similarity

In Chapter 4, two triangles were defined to be congruent if all of their corresponding parts were congruent and the criteria for proving triangle congruence (SAS, SSS, and ASA) were presented as postulates. Triangle congruence can also be defined in terms of rigid motions (reflections, translations, rotations).

Common Core State Standards
G.CO.6, G.CO.7, G.CO.8, G.SRT.2, G.SRT.3

The **principle of superposition** states that two figures are congruent if and only if there is a rigid motion or a series of rigid motions that maps one figure exactly onto the other. We can use the following assumed properties of rigid motions to establish the SAS, SSS, and ASA criteria for triangle congruence.

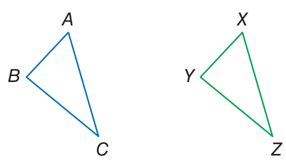
- The distance between points is preserved. Sides are mapped to sides of the same length.
- Angle measures are preserved. Angles are mapped to angles of the same measure.



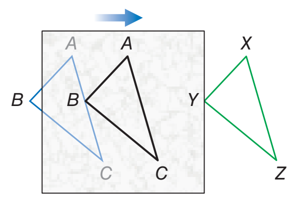
Activity 1 Establish Congruence

Use a rigid motion to map side $\overline{AB}$ of $\triangle ABC$ onto side $\overline{XY}$ of $\triangle XYZ$, $\angle A$ onto $\angle X$, and side $\overline{AC}$ onto side $\overline{XZ}$.

Step 1 Copy the triangles below onto a sheet of paper.



Step 2 Copy $\triangle ABC$ onto a sheet of tracing paper and label. Translate the paper until $\overline{AB}$, $\angle A$, and $\overline{AC}$ lie exactly on top of $\overline{XY}$, $\angle X$, and $\overline{XZ}$.



Analyze the Results

1. Use this activity to explain how the SAS criterion for triangle congruence follows from the definition of congruence in terms of rigid motions. (*Hint*: Extend lines on the tracing paper.)

2. Use the principle of superposition to explain why two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Using the same triangles shown above, describe the steps in an activity to illustrate the indicated criterion for triangle congruence. Then explain how this criterion follows from the principle of superposition.

3. SSS

4. ASA

(continued on the next page)

1 Focus

Objective

- Explore how triangle congruence and similarity follow from an understanding of transformations.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activity 1 as a class. Then ask students to work with their partners to complete Exercises 1–3.

Practice Have students complete Exercise 4.

3 Assess

Formative Assessment

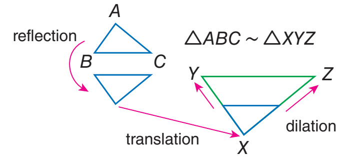
Use Exercises 5–9 to assess whether students comprehend how to use rigid motions and dilations to establish triangle congruence and similarity.

From Concrete to Abstract

Have students describe the transformation(s) and/or dilation necessary to map $\triangle ABC$ onto $\triangle XYZ$ in Exercises 7–9 or explain how they know that $\triangle ABC$ cannot be mapped onto $\triangle XYZ$.

Geometry Lab
Establishing Triangle Congruence and Similarity *Continued*

Two figures are similar if there is a rigid motion, or a series of rigid motions, followed by a dilation, or vice versa, that map one figure exactly onto the other. We can use the following assumed properties of dilations to establish the AA criteria for triangle similarity.



- Angle measures are preserved. Angles are mapped to angles of the same measure.
- Lines are mapped to parallel lines and sides are mapped to parallel sides that are longer or shorter in the ratio given by the scale factor.

Activity 2 Establish Similarity

Use a rigid motion followed by a dilation to map $\angle B$ onto $\angle Y$ and $\angle A$ onto $\angle X$.

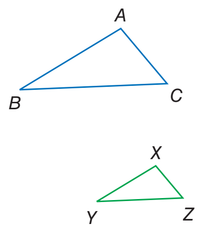
Step 1 Copy the triangles below onto a sheet of paper.

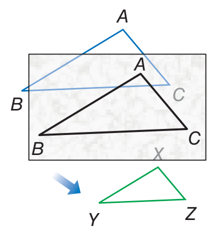
Step 2 Copy $\triangle ABC$ onto tracing paper and label.

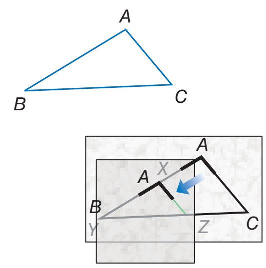
Step 3 Translate the paper until $\angle B$ lies exactly on top of $\angle Y$. Tape this paper down so that it will not move.

Step 4 On another sheet of tracing paper, copy and label $\angle A$.

Step 5 Translate this second sheet of tracing paper along the line from A to Y on the first sheet, until this second $\angle A$ lies exactly on top of $\angle X$.







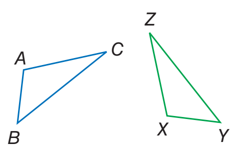
Analyze the Results

5. Use this activity to explain how the AA criterion for triangle similarity follows from the definition of similarity in terms of dilations. (*Hint*: Use parallel lines.)

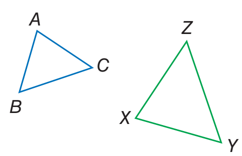
6. Use the definition of similarity in terms of transformations to explain why two triangles are similar if all corresponding pairs of angles are congruent and all corresponding pairs of sides are proportional.

Use a series of rigid motions and/or dilations to determine whether $\triangle ABC$ and $\triangle XYZ$ are *congruent, similar,* or *neither.*

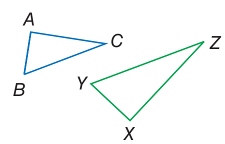
7.



8.

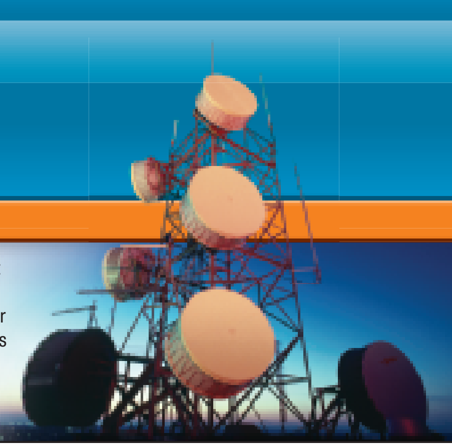


9.



34 | Lab 11 | Geometry Lab: Establishing Triangle Congruence and Similarity

LESSON 12 Equations of Circles



Then

- You wrote equations of lines using information about their graphs.

Now

1. Write the equation of a circle.
2. Graph a circle on the coordinate plane.

Why?

- Telecommunications towers emit radio signals that are used to transmit cellular calls. Each tower covers a circular area, and towers are arranged so that a signal is available at any location in the coverage area.

New Vocabulary

compound locus

Common Core State Standards

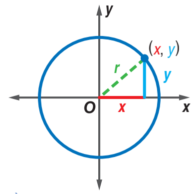
G.GPE.1, G.GPE.4, G.GPE.6

1 Equation of a Circle

Since all points on a circle are equidistant from the center, you can find an equation of a circle by using the Distance Formula.

Let (x, y) represent a point on a circle centered at the origin. Using the Pythagorean Theorem, $x^2 + y^2 = r^2$.

Now suppose that the center is not at the origin, but at the point (h, k). You can use the Distance Formula to develop an equation for the circle.

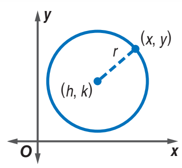


$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$r = \sqrt{(x - h)^2 + (y - k)^2}$	$d = r$, $(x_1, y_1) = (h, k)$, $(x_2, y_2) = (x, y)$
$r^2 = (x - h)^2 + (y - k)^2$	Square each side.

Key Concept Equation of a Circle in Standard Form

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

The standard form of the equation of a circle is also called the *center-radius* form.



Example 1 Write an Equation Using the Center and Radius

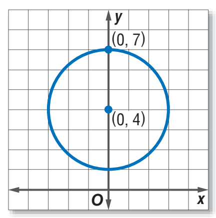
Write the equation of each circle.

a. **center at (1, −8), radius 7**

$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$(x - 1)^2 + [y - (-8)]^2 = 7^2$	$(h, k) = (1, -8)$, $r = 7$
$(x - 1)^2 + (y + 8)^2 = 49$	Simplify.

b. **the circle graphed at the right**

The center is at (0, 4) and the radius is 3.



$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$(x - 0)^2 + (y - 4)^2 = 3^2$	$(h, k) = (0, 4)$, $r = 3$
$x^2 + (y - 4)^2 = 9$	Simplify.

Guided Practice

1A. center at origin, radius $\sqrt{10}$

1B. center at (4, −1), diameter 8

1 Focus

Vertical Alignment

Before Lesson 12 Write equations of lines using information about their graphs.

Lesson 12 Write the equation of a circle.
Graph a circle on the coordinate plane.

After Lesson 12 Extend similarity properties and transformations to explore and justify conjectures about geometric figures.

2 Teach

Scaffolding Questions

Have students read the **Why?** section of the lesson.

Ask:

- Where is the tower located in relationship to the area it covers?
- What does the distance from the tower to the farthest point of the service area represent?
- A certain cellular tower sends out a signal with a 15-mile radius. To increase the service area by 50%, how many more miles does the signal from the tower have to reach?

1 Equation of a Circle

Examples 1 and 2 show how to use information given about a circle to find the equation of a circle.

Formative Assessment

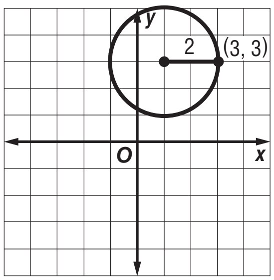
Use the Guided Practice exercises after each example to determine students' understanding of concepts.

Additional Examples

1 Write the equation of each circle.

a. center at (3, –3), radius 6

b. the circle graphed below



2 Write the equation of the circle with center at (–3, –2), that passes through (1, –2).

Additional Examples also in Interactive Classroom PowerPoint® Presentations

IWB Interactive White Board READY

Example 2 Write an Equation Using the Center and a Point

Write the equation of the circle with center at (–2, 4), that passes through (–6, 7).

Step 1 Find the distance between the points to determine the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[-6 - (-2)]^2 + (7 - 4)^2}$ $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (-6, 7)$

$= \sqrt{25}$ or 5 Simplify.

Step 2 Write the equation using $h = -2$, $k = 4$, and $r = 5$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$[x - (-2)]^2 + (y - 4)^2 = 5^2$ $h = -2$, $k = 4$, and $r = 5$

$(x + 2)^2 + (y - 4)^2 = 25$ Simplify.

GuidedPractice

2. Write the equation of the circle with center at (–3, –5) that passes through (0, 0).

2 Graph Circles

You can use the equation of a circle to graph it on a coordinate plane. To do so, you may need to write the equation in standard form first.

Example 3 Graph a Circle

The equation of a circle is $x^2 + y^2 - 8x + 2y = -8$. State the coordinates of the center and the measure of the radius. Then graph the equation.

Write the equation in standard form by completing the square.

$x^2 + y^2 - 8x + 2y = -8$ Original equation

$x^2 - 8x + y^2 + 2y = -8$ Isolate and group like terms.

$x^2 - 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1$ Complete the squares.

$(x - 4)^2 + (y + 1)^2 = 9$ Factor and simplify.

$(x - 4)^2 + [y - (-1)]^2 = 3^2$ Write +1 as – (–1) and 9 as 3^2.

> **StudyTip**
>
> **Completing the Square** To complete the square for any quadratic expression of the form $x^2 + bx$, follow these steps.
>
> **Step 1** Find one half of b.
>
> **Step 2** Square the result in Step 1.
>
> **Step 3** Add the result of Step 2 to $x^2 + bx$.

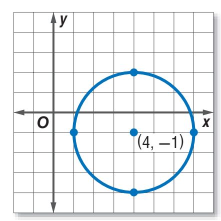
With the equation now in standard form, you can identify h, k, and r.

$(x - 4)^2 + [y - (-1)]^2 = 3^2$

↑ ↑ ↑

$(x - h)^2 + (y - k)^2 = r^2$

So, $h = 4$, $k = -1$, and $r = 3$. The center is at (4, –1), and the radius is 3. Plot the center and four points that are 3 units from this point. Sketch the circle through these four points.



GuidedPractice

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

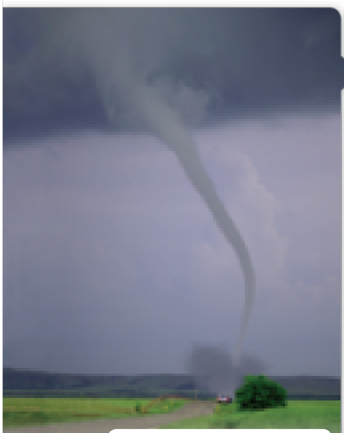
3A. $x^2 + y^2 - 4 = 0$

3B. $x^2 + y^2 + 8x - 14y + 40 = 0$

Teach with Tech

Interactive Whiteboard Display a coordinate plane on the board. Draw a circle centered at the origin. Display the coordinates of the center, the length of the radius, and the equation of the circle. Have students take turns moving the circle or changing the length of the radius. Save the results and distribute them to the class.

> **WatchOut!**
>
> **Distance Formula** When using the distance formula, remind students to be careful to keep the x- and y-coordinates in the correct order and to keep track of their signs.



Real-WorldLink

About 1000 tornadoes are reported across the United States each year. The most violent tornadoes have wind speeds of 250 mph or more. Damage paths can be a mile wide and 50 miles long.

Source: National Oceanic & Atmospheric Administration

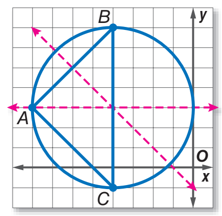
Real-World Example 4 Use Three Points to Write an Equation

TORNADOES **Three tornado sirens are placed strategically on a circle around a town so they can be heard by all. Write the equation of the circle on which they are placed if the coordinates of the sirens are $A(-8, 3)$, $B(-4, 7)$, and $C(-4, -1)$.**

Understand You are given three points that lie on a circle.

Plan Graph $\triangle ABC$. Construct the perpendicular bisectors of two sides to locate the center of the circle. Then find the radius.

Use the center and radius to write an equation.



Solve The center appears to be at $(-4, 3)$. The radius is 4. Write an equation.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$[x - (-4)]^2 + (y - 3)^2 = 4^2$$

$$(x + 4)^2 + (y - 3)^2 = 16$$

Check Verify the center by finding the equations of the two bisectors and solving the system of equations. Verify the radius by finding the distance between the center and another point on the circle. ✓

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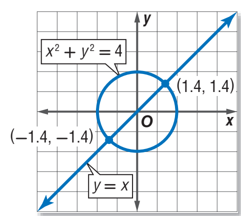
4. Write an equation of a circle that contains $R(1, 2)$, $S(-3, 4)$, and $T(-5, 0)$.

A line can intersect a circle in at most two points. You can find the point(s) of intersection between a circle and a line by applying techniques used to find the intersection between two lines and techniques used to solve quadratic equations.

Example 5 Intersections with Circles

Find the point(s) of intersection between $x^2 + y^2 = 4$ and $y = x$.

Graph these equations on the same coordinate plane. The points of intersection are solutions of both equations. You can estimate these points on the graph to be at about $(-1.4, -1.4)$ and $(1.4, 1.4)$. Use substitution to find the coordinates of these points algebraically.



$x^2 + y^2 = 4$	Equation of circle
$x^2 + x^2 = 4$	Since $y = x$, substitute x for y.
$2x^2 = 4$	Simplify.
$x^2 = 2$	Divide each side by 2.
$x = \pm\sqrt{2}$	Take the square root of each side.

So $x = \sqrt{2}$ or $x = -\sqrt{2}$. Use the equation $y = x$ to find the corresponding y-values.

$y = x$	Equation of line	$y = x$
$y = \sqrt{2}$	$x = \sqrt{2}$ or $x = -\sqrt{2}$	$y = -\sqrt{2}$

The points of intersection are located at $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$ or at about $(-1.4, -1.4)$ and $(1.4, 1.4)$. Check these solutions in both of the original equations.

StudyTip

Quadratic Techniques In addition to taking square roots, other quadratic techniques that you may need to apply in order to solve equations of the form $ax^2 + bx + c = 0$ include completing the square, factoring, and the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Warren Faidley/CORBIS

GuidedPractice

5. Find the point(s) of intersection between $x^2 + y^2 = 8$ and $y = -x$.

2 Graph Circles

Examples 3 and 4 show how to analyze the equation of a circle that will help graph the circle on a coordinate plane. **Example 5** shows how to find the point(s) of intersection between a circle and a line.

Additional Examples

3 The equation of a circle is $x^2 - 4x + y^2 + 6y = -9$. State the coordinates of the center and the measure of the radius. Then graph the equation.

4 **ELECTRICITY** Strategically located substations are extremely important in the transmission and distribution of a power company's electric supply. Suppose three substations are modeled by the points $D(3, 6)$, $E(-1, 1)$, and $F(3, -4)$. Determine the location of a town equidistant from all three substations, and write an equation for the circle.

5 Find the point(s) of intersection between $x^2 + y^2 = 32$ and $y = x + 8$.

DifferentiatedInstruction OL BL

Logical Learners Explain that students will rely heavily on their geometric knowledge and reasoning skills to solve the problems in this lesson. Allow students to explain how to explore and collaborate as they work through examples and exercises. Students need to recall definitions, concepts, and theorems to help explain why they use certain methods to solve problems.

3 Practice

Formative Assessment

Use Exercises 1–12 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Exercise Alert

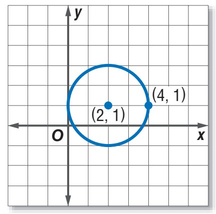
Graph Paper Exercises 23–28, 39, 40, 42, and 46 require the use of graph paper.

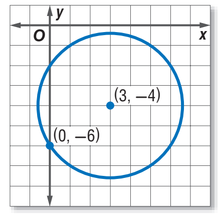
Multiple Representations

In Exercise 42, students use a table, graphs, and verbal descriptions to investigate the compound locus of a pair of points.

Check Your Understanding

Examples 1–2 **Write the equation of each circle.**

1. center at (9, 0), radius 5
2. center at (3, 1), diameter 14
3. center at origin, passes through (2, 2)
4. center at (–5, 3), passes through (1, –4)
5. 

6. 

Example 3 **For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.**

7. $x^2 - 6x + y^2 + 4y = 3$
8. $x^2 + (y + 1)^2 = 4$

Example 4

9. **RADIOS** Three radio towers are modeled by the points $R(4, 5)$, $S(8, 1)$, and $T(-4, 1)$. Determine the location of another tower equidistant from all three towers, and write an equation for the circle.

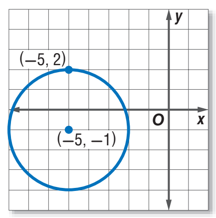
10. **COMMUNICATION** Three cell phone towers can be modeled by the points $X(6, 0)$, $Y(8, 4)$, and $Z(3, 9)$. Determine the location of another cell phone tower equidistant from the other three, and write an equation for the circle.

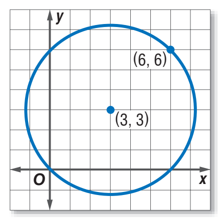
Example 5 **Find the point(s) of intersection, if any, between each circle and line with the equations given.**

11. $(x - 1)^2 + y^2 = 4$
    $y = x + 1$
12. $(x - 2)^2 + (y + 3)^2 = 18$
    $y = -2x - 2$

Practice and Problem Solving

Examples 1–2 **Write the equation of each circle.**

13. center at origin, radius 4
14. center at (6, 1), radius 7
15. center at (–2, 0), diameter 16
16. center at (8, –9), radius $\sqrt{11}$
17. center at (–3, 6), passes through (0, 6)
18. center at (1, –2), passes through (3, –4)
19. 

20. 

21. **WEATHER** A Doppler radar screen shows concentric rings around a storm. If the center of the radar screen is the origin and each ring is 15 miles farther from the center, what is the equation of the third ring?

22. **GARDENING** A sprinkler waters a circular area that has a diameter of 10 feet. The sprinkler is located 20 feet north of the house. If the house is located at the origin, what is the equation for the circle of area that is watered?

Differentiated Homework Options

Level	Assignment	Two-Day Option	
AL Basic	13–34, 45–47, 50	13–33 odd	14–42 even, 45–47, 50
OL Core	13–35 odd, 37–42, 44–50	13–34	35–43, 45–47, 50
BL Advanced	35–50		

Example 3 For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

23. $x^2 + y^2 = 36$

24. $x^2 + y^2 - 4x - 2y = -1$

25. $x^2 + y^2 + 8x - 4y = -4$

26. $x^2 + y^2 - 16x = 0$

Example 4 Write an equation of a circle that contains each set of points. Then graph the circle.

27. $A(1, 6), B(5, 6), C(5, 0)$

28. $F(3, -3), G(3, 1), H(7, 1)$

Example 5 Find the point(s) of intersection, if any, between each circle and line with the equations given.

29. $x^2 + y^2 = 5$
$y = \frac{1}{2}x$

30. $x^2 + y^2 = 2$
$y = -x + 2$

31. $x^2 + (y + 2)^2 = 8$
$y = x - 2$

32. $(x + 3)^2 + y^2 = 25$
$y = -3x$

33. $x^2 + y^2 = 5$
$y = 3x$

34. $(x - 1)^2 + (y - 3)^2 = 4$
$y = -x$

Write the equation of each circle.

35. a circle with a diameter having endpoints at (0, 4) and (6, −4)

36. a circle with $d = 22$ and a center translated 13 units left and 6 units up from the origin

37. **MODEL ROCKETS** Different-sized engines will launch model rockets to different altitudes. The higher a rocket goes, the larger the circle of possible landing sites becomes. Under normal wind conditions, the landing radius is three times the altitude of the rocket.

a. Write the equation of the landing circle for a rocket that travels 300 feet in the air.

b. What would be the radius of the landing circle for a rocket that travels 1000 feet in the air? Assume the center of the circle is at the origin.

38. **SKYDIVING** Three of the skydivers in the circular formation shown have approximate coordinates of $G(13, -2)$, $H(-1, -2)$, and $J(6, -9)$.

a. What are the approximate coordinates of the center skydiver?

b. If each unit represents 1 foot, what is the diameter of the skydiving formation?

39. **DELIVERY** Pizza and Subs offers free delivery within 6 miles of the restaurant. The restaurant is located 4 miles west and 5 miles north of Consuela's house.

a. Write and graph an equation to represent this situation if Consuela's house is at the origin of the coordinate system.

b. Can Consuela get free delivery if she orders pizza from Pizza and Subs? Explain.

40. **INTERSECTIONS OF CIRCLES** Graph $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$ on the same coordinate plane.

a. Estimate the point(s) of intersection between the two circles.

b. Solve $x^2 + y^2 = 4$ for y.

c. Substitute the value you found in part **b** into $(x - 2)^2 + y^2 = 4$ and solve for x.

d. Substitute the value you found in part **c** into $x^2 + y^2 = 4$ and solve for y.

e. Use your answers to parts **c** and **d** to write the coordinates of the points of intersection. Compare these coordinates to your estimate from part **a**.

f. Verify that the point(s) you found in part **d** lie on both circles.

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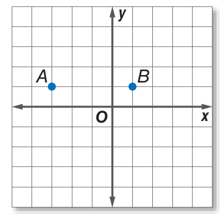
4 Assess

Name the Math Let students take turns saying an equation of a circle. Then they should name the centers of the circles, and state the lengths of the radii.

41. Prove or disprove that the point $(1, 2\sqrt{2})$ lies on a circle centered at the origin and containing the point $(0, -3)$.

42. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a compound locus for a pair of points. A **compound locus** satisfies more than one distinct set of conditions.



a. Tabular Choose two points A and B in the coordinate plane. Locate 5 coordinates from the locus of points equidistant from A and B.

b. Graphical Represent this same locus of points by using a graph.

c. Verbal Describe the locus of all points equidistant from a pair of points.

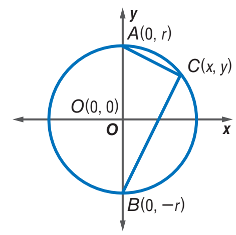
d. Graphical Using your graph from part **b**, determine and graph the locus of all points in a plane that are a distance of AB from B.

e. Verbal Describe the locus of all points in a plane equidistant from a single point. Then describe the locus of all points that are both equidistant from A and B and are a distance of AB from B. Describe the graph of the compound locus.

43. A circle with a diameter of 12 has its center in the second quadrant. The lines $y = -4$ and $x = 1$ are tangent to the circle. Write an equation of the circle.

H.O.T. Problems Use Higher-Order Thinking Skills

44. **CHALLENGE** Write a coordinate proof to show that if an inscribed angle intercepts the diameter of a circle, as shown, the angle is a right angle.



45. **REASONING** A circle has the equation $(x - 5)^2 + (y + 7)^2 = 16$. If the center of the circle is shifted 3 units right and 9 units up, what would be the equation of the new circle? Explain your reasoning.

46. **OPEN ENDED** Graph three noncollinear points and connect them to form a triangle. Then construct the circle that circumscribes it.

47. **WRITING IN MATH** Seven new radio stations must be assigned broadcast frequencies. The stations are located at $A(9, 2)$, $B(8, 4)$, $C(8, 1)$, $D(6, 3)$, $E(4, 0)$, $F(3, 6)$, and $G(4, 5)$, where 1 unit = 50 miles.

a. If stations that are more than 200 miles apart can share the same frequency, what is the least number of frequencies that can be assigned to these stations?

b. Describe two different beginning approaches to solving this problem.

c. Choose an approach, solve the problem, and explain your reasoning.

CHALLENGE Find the coordinates of point P on $\overrightarrow{AB}$ that partitions the segment into the given ratio AP to PB.

48. $A(0, 0)$, $B(3, 4)$, 2 to 3

49. $A(0, 0)$, $B(-8, 6)$, 4 to 1

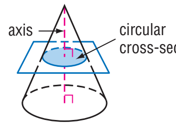
50. **WRITING IN MATH** Describe how the equation for a circle changes if the circle is translated a units to the right and b units down.

Differentiated Instruction OL BL

Extension What is the relationship between concentric circles with the same radius? Explain.

LAB 13 Geometry Lab
Parabolas

A circle is one type of cross-section of a right circular cone. Such cross-sections are called **conic sections** or **conics**. A circular cross-section is formed by the intersection of a cone with a plane that is perpendicular to the axis of the cone. You can find other conic sections using concrete models of cones.

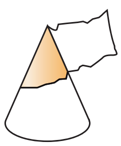


Common Core State Standards
G.GPE.2

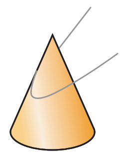
Activity 1 Intersection of Cone and Plane

Sketch the intersection of a cone and a plane that lies at an angle to the axis of the cone but does not pass through its base.

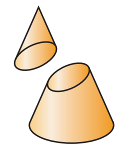
Step 1 Fill a conical paper cup with modeling compound. Then peel away the cup.



Step 2 Draw dental floss through the cone model at an angle to the axis that does not pass through the base.

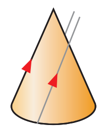


Step 3 Pull the pieces of the cone apart and trace the cross-section onto your paper.



Model and Analyze

1. The conic section in Activity 1 is called an ellipse. What shape is an ellipse?
2. Repeat Activity 1, drawing the dental floss through the model at an angle parallel to an imaginary line on the side of the cone through the cone's base. Describe the resulting shape.

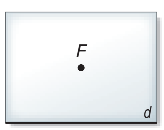


The conic section you found in Exercise 2 is called a **parabola**. In Algebra 1, a parabola was defined as the shape of the graph of a quadratic function, such as $y = x^2$. Like a circle and all conics, a parabola can also be defined as a locus of points. You can explore the loci definition of a parabola using paper folding.

Activity 2 Shape of Parabola

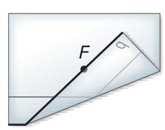
Use paper folding to approximate the shape of a parabola.

Step 1 Mark and label the bottom edge of a rectangular piece of wax paper *d*. Label a point *F* at the center.



Step 2 Fold *d* up so that it touches *F*. Make a sharp crease. Then open the paper and smooth it flat.

Step 3 Repeat Step 2 at least 20 times, folding the paper to a different point on *d* each time. Trace the curve formed.



(continued on the next page)

1 Focus

Objectives
- Identify conic sections.
- Translate between the geometric description and the equation for a parabola.
- Determine intersections between lines and parabolas.

Materials for Each Group
- modeling clay
- wax paper
- graph paper

Teaching Tip
If time or materials is an issue, you may wish to complete Activity 1 as a class by finding and using an online conic section applet. Activity 2 can also be completed as a whole-class activity using a graphing calculator.

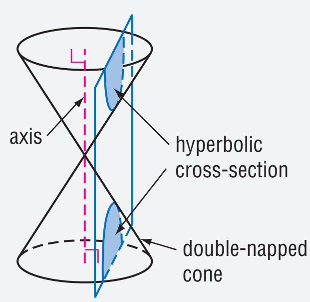
2 Teach

Working in Cooperative Groups
Organize students into groups of 2 or 3, mixing abilities. Then have groups complete Activities 1–3 and Exercises 1–8.

Ask:
- Will the graph of a parabola ever intersect its directrix?
- In Algebra 1, you learned that a parabola is symmetric with respect to its axis of symmetry, which passes through its vertex. What is the relationship between the directrix and axis of symmetry?
- What is the relationship between the vertex and axis of symmetry?
- Where is the vertex located in relationship to the focus and the directrix?

Focus on Mathematical Content

Double-Napped Cone In later courses, students will learn that conic sections are figures formed when a plane intersects a double-napped cone. Using this definition, students will explore another conic section called the *hyperbola*, which is formed by the intersection of a double-napped cone and a plane that is parallel to the axis of the cone.



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Teaching Tips

- For Exercise 3, encourage students to use paper folding to compare $\overline{PF}$ to $\overline{PD}$.
- In Lesson 10-8, students used techniques for solving systems of equations and quadratic equations to find the intersection(s) between circles and lines. Tell students that these same techniques can be used to find the intersection(s) between parabolas and lines in Exercises 13–16.

Practice Have students complete Exercises 9–16.

3 Assess

Formative Assessment

Use Exercises 9 and 14 to assess whether students understand how to find the equation of a parabola given a focus and directrix and how to find the intersection(s) of a parabola and a line algebraically.

From Concrete to Abstract

Ask students to summarize what they have learned about the geometric definition of a parabola and how it relates to the algebraic definition they learned in Algebra 1.

Geometry Lab

Parabolas *Continued*

Model and Analyze

3. Label a point P on the parabola and draw $\overline{PF}$. Then use a protractor to find a point D on line d such that $\overline{PD} \perp d$. Describe the relationship between $\overline{PF}$ and $\overline{PD}$.

Repeat Activity 2, making the indicated change on a new piece of wax paper. Describe the effect on the parabola formed.

4. Place line d along the edge above point F.

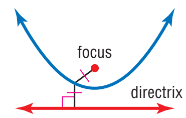
5. Place line d along the edge to the right of point F.

6. Place line d along the edge to left of point F.

7. Place point F closer to line d.

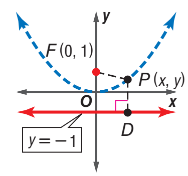
8. Place point F farther away from line d.

Geometrically, a parabola is the locus of all points in a plane equidistant from a fixed point, called the **focus**, and a fixed line, called the **directrix**. Recall that the distance between a fixed point and a line is the length of the segment perpendicular to the line through that point. You can find an equation of a parabola on the coordinate plane using its locus definition and the Distance Formula.



Activity 3 Equation of Parabola

Find an equation of the parabola with focus at (0, 1) and directrix $y = -1$.



Step 1 Graph $F(0, 1)$ and $y = -1$. Sketch a U-shaped curve for the parabola between the point and line as shown. Label a point $P(x, y)$ on the curve.

Step 2 Label a point D on $y = -1$ such that $\overline{PD}$ is perpendicular to the line $y = -1$. The coordinates of this point must therefore be $D(x, -1)$.

Step 3 Use the Distance Formula to find PD and PF.

$$PD = \sqrt{(x - x)^2 + [y - (-1)]^2} \quad D(x, -1), P(x, y), F(0, 1)$$
$$= \sqrt{(y + 1)^2} \quad \text{Simplify.}$$

$$PF = \sqrt{(x - 0)^2 + (y - 1)^2}$$
$$= \sqrt{x^2 + (y - 1)^2}$$

Step 4 Since $PD = PF$, set these expressions equal to each other.

$$\sqrt{(y + 1)^2} = \sqrt{x^2 + (y - 1)^2} \quad PD = PF$$
$$(y + 1)^2 = x^2 + (y - 1)^2 \quad \text{Square each side.}$$
$$y^2 + 2y + 1 = x^2 + y^2 - 2y + 1 \quad \text{Square each binomial.}$$
$$4y = x^2 \text{ or } y = \frac{1}{4}x^2 \quad \text{Subtract } y^2 - 2y + 1 \text{ from each side.}$$

An equation of the parabola with focus at (0, 1) and directrix $y = -1$ is $y = \frac{1}{4}x^2$.

Model and Analyze

Find an equation of the parabola with the focus and directrix given.

9. $(0, -2), y = 2$

10. $\left(0, \frac{1}{2}\right), y = -\frac{1}{2}$

11. $(1, 0), x = -1$

12. $(-3, 0), x = 3$

A line can intersect a parabola in 0, 1, or 2 points. Find the point(s) of intersection, if any, between each parabola and line with the given equations.

13. $y = x^2, y = x + 2$

14. $y = 2x^2, y = 4x - 2$

15. $y = -3x^2, y = 6x$

16. $y = -(x + 1)^2, y = -x$

Differentiated Instruction BL

Extension Find a general equation for a parabola with focus $(0, a)$ and directrix $y = -a$. Then find the general equation of a parabola with focus $(a, 0)$ and directrix $x = -a$. What does a represent in each equation?

LAB 14 Geometry Lab
Population Density

After data are collected for the U.S. census, the population density is calculated for states, major cities, and other areas. **Population density** is the measurement of population per unit of area.

Common Core State Standards
G.MG.2

Activity 1 Calculate Population Density

Find the population density for the borough of Queens using the data in the table.

Borough	Population	Land Area (mi^2)
Brooklyn	2,465,326	70.61
Manhattan	1,537,195	22.96
Queens	2,229,379	109.24
Staten Island	443,728	58.48
The Bronx	1,332,650	42.03

Calculate population density with the formula population density $= \frac{\text{population}}{\text{land area}}$.

The population density of Queens would be $\frac{2,229,379}{109.24}$ or about 20,408 people per square mile.

Model and Analyze

1. Find the population densities for Brooklyn, Manhattan, Staten Island and the Bronx. Round to the nearest person. Of the five boroughs, which have the highest and the lowest population densities?

Activity 2 Use Population Density

PT

In a proposal to establish a new rustic campgound at Yellowstone National Park, there is a concern about the number of wolves in the area. At last report, there were 98 wolves in the park. The new campgound will be accepted if there are fewer than 2 wolves in the campground. Use the data in the table to determine if the new campgound can be established.

Location	Size
Area of park	3472 mi^2
Area of new campground	10 acres

Step 1 Find the density of wolves in the park.
$98 \div 3472 = 0.028$ wolves per square mile

Step 2 Find the density of wolves in the proposed campgound. First convert the size of the campgound to square miles. If 1 acre is equivalent to 0.0015625 square mile, then 10 acres is 0.015625 square mile. The potential number of wolves in the proposed site is $0.015625 \cdot 0.028$ or 0.0004375 wolves.

Step 3 Since 0.0004375 is fewer than 2, the proposed campgound can be accepted.

Exercises

2. Find the population density of gaming system owners if there are 436,000 systems in the United States and the area of the United States is 3,794,083 square miles.

3. The population density of the burrowing owl in Cape Coral, Florida, is 8.3 pairs per square mile. A new golf club is planned for a 2.4-square-mile site where the owl population is estimated to be 17 pairs. Would Lee County approve the proposed club if their policy is to decline when the estimated population density of owls is below the average density? Explain.

1 Focus

Objective Explore population density.

2 Teach

Working in Cooperative Groups

Divide the class into pairs. Work through Activities 1 and 2 as a class. Then ask students to work with their partners to complete Exercise 1.

Practice Have students complete Exercise 2.

3 Assess

Formative Assessment

Use Exercise 3 to assess each student's understanding of modeling with population density.

From Concrete to Abstract

Ask students to summarize how they can use population density to support a claim.

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1 Focus

Objective Use two-way frequency tables to find marginal, joint, and relative frequencies.

Teaching Tip

Either of the tables in Activities 1 or 2 can be used to compute conditional probabilities like the one asked for in Activity 3. To find the conditional probability that a student attends the prom given that he or she is a senior using the table in Activity 1, divide the number of seniors attending the prom, 44, by the total number of seniors, 76. However, converting the two-way frequency table to a two-way relative frequency table will give you more information and allow you to answer questions more quickly.

2 Teach

Working in Cooperative Groups

Have student work in mixed ability pairs. Have students alternate the steps of the activities.

LAB 15 Geometry Lab: Two-Way Frequency Tables

A **two-way frequency table** or *contingency table* is used to show the frequencies of data from a survey or experiment classified according to two variables, with the rows indicating one variable and the columns indicating the other.

Common Core State Standards
S.CP.4, S.CP.6

PT

Activity 1 Two-Way Frequency Table

PROM Michael asks a random sample of 160 upperclassmen at his high school whether or not they plan to attend the prom. He finds that 44 seniors and 32 juniors plan to attend the prom, while 25 seniors and 59 juniors do not plan to attend. Organize the responses into a two-way frequency table.

Step 1 Identify the variables. The students surveyed can be classified according *class* and *attendance*. Since the survey included only upperclassmen, the variable *class* has two categories: senior or junior. The variable *attendance* also has two categories: attending or not attending the prom.

Step 2 Create a two-way frequency table. Let the rows of the table represent *class* and the columns represent *attendance*. Then fill in the cells of the table with the information given.

Step 3 Add a *Totals* row and a *Totals* column to your table and fill in these cells with the correct sums.

Class	Attending the Prom	Not Attending the Prom	Totals
Senior	44	32	76
Junior	25	59	84
Totals	69	91	160

The frequencies reported in the *Totals* row and *Totals* column are called **marginal frequencies**, with the bottom rightmost cell reporting the total number of observations. The frequencies reported in the interior of the table are called **joint frequencies**. These show the frequencies of all possible combinations of the categories for the first variable with the categories for the second variable.

Analyze the Results

1. How many seniors were surveyed?
2. How many of the students that were surveyed plan to attend the prom?

A **relative frequency** is the ratio of the number of observations in a category to the total number of observations.

Activity 2 Two-Way Relative Frequency Table

PROM Convert the table from Activity 1 to a table of relative frequencies.

Step 1 Divide the frequency reported in each cell by the total number of respondents, 160.

Class	Attending the Prom	Not Attending the Prom	Totals
Senior	$\frac{44}{160}$	$\frac{32}{160}$	$\frac{76}{160}$
Junior	$\frac{25}{160}$	$\frac{59}{160}$	$\frac{84}{160}$
Totals	$\frac{69}{160}$	$\frac{91}{160}$	$\frac{160}{160}$

Step 2 Write each fraction as a percent rounded to the nearest tenth.

Class	Attending the Prom	Not Attending the Prom	Totals
Senior	27.5%	20%	47.5%
Junior	15.6%	36.9%	52.5%
Totals	43.1%	56.9%	100%

44 | Lab 15 | Geometry Lab: Two-Way Frequency Tables

Practice Have students complete Activities 1–4 and Exercises 1–6.

You can use joint and marginal relative frequencies to approximate conditional probabilities.

Activity 3 Conditional Probabilities

PROM **Using the table from Activity 2, find the probability that a surveyed upperclassman plans to attend the prom given that he or she is a junior.**

The probability that a surveyed upperclassman plans to attend the prom given that he or she is a junior is the conditional probability $P(\text{attending the prom} \mid \text{junior})$.

$$P(\text{attending the prom} \mid \text{junior}) = \frac{P(\text{attending the prom and junior})}{P(\text{junior})}$$ Conditional Probability

$$\approx \frac{0.156}{0.525} \text{ or } 29.7\%$$ P(attending the prom and junior) = 15.6% or 0.156, P(junior) = 52.5% or 0.525

Analyze and Apply

Refer to Activities 2 and 3.

3. If there are 285 upperclassmen, about how many would you predict plan to attend the prom?

4. Find the probability that a surveyed student is a junior and does not plan to attend the prom.

5. Find the probability that a surveyed student is a senior given that he or she plans to attend the prom.

6. What is a possible trend you notice in the data?

When survey results are classified according to variables, you may want to decide whether these variables are independent of each other. Variable A is considered independent of variable B if $P(A \text{ and } B) = P(A) \cdot P(B)$. In a two-way frequency table, you can test for the independence of two variables by comparing the joint relative frequencies with the products of the corresponding marginal relative frequencies.

Activity 4 Independence of Events

PROM **Use the relative frequency table from Activity 3 to determine whether prom attendance is independent of class.**

Calculate the expected joint relative frequencies if the two variables were independent. Then compare them to the actual relative frequencies.

For example, if 47.5% of respondents were seniors and 43.1% of respondents plan to attend the prom, then one would expect 47.5% • 43.1% or about 20.5% of respondents are seniors who plan to attend the prom.

Since the expected and actual joint relative frequencies are not the same, prom attendance for these respondents is not independent of class.

Class	Attending the Prom	Not Attending the Prom	Totals
Senior	27.5% (20.5%)	20% (27%)	47.5%
Junior	15.6% (22.6%)	36.9% (29.9%)	52.5%
Totals	43.1%	56.9%	100%

Note: The numbers in parentheses are the expected relative frequencies.

COLLECT DATA **Design and conduct a survey of students at your school. Create a two-way relative frequency table for the data. Use your table to decide whether the data you collected indicate an independent relationship between the two variables. Explain your reasoning.**

7. student gender and whether a student's car insurance is paid by the student or the student's parent(s)

8. student gender and whether a student buys or brings his or her lunch

3 Assess

Formative Assessment

Use Exercises 7 and 8 to assess each student's ability to use two-way frequency tables to determine if events are independent.

From Concrete to Abstract

Ask students to summarize what they learned about two-way frequency tables.

Additional Exercises

Use with Lesson 0-3.

1. **DECISION MAKING** You and two of your friends have pooled your money to buy a new video game. Describe a method that could be used to make a fair decision as to who gets to play the game first.

2. **DECISION MAKING** A new study finds that the incidence of heart attack while taking a certain diabetes drug is less than 5%. Should a person with diabetes take this drug? Should they take the drug if the risk is less than 1%? Explain your reasoning.

Use with Lesson 3-4.

3. **MULTIPLE REPRESENTATIONS** In Algebra 1, you learned that the solution of a system of two linear equations is an ordered pair that is a solution of both equations. Consider lines q, r, s, and t with the equations given.

   line q: $y = 3x + 2$
   line r: $y = 0.5x - 3$
   line s: $2y = x - 6$
   line t: $y = 3x - 3$

   a. TABULAR Make a table of values for each equation for $x = -3, -2, -1, 0, 1, 2$, and 3. Which pairs of lines appear to represent a system of equations with one solution? no solution? infinitely many solutions? Use your tables to explain your reasoning.

   b. GRAPHICAL Graph the equations on the same coordinate plane. Describe the geometric relationship between each pair of lines, including points of intersection.

   c. ANALYTICAL How could you have determined your answers to part a using only the equations of the lines?

   d. VERBAL Explain how to determine whether a given system of two linear equations has one solution , no solution, or infinitely many solutions using a table, a graph, or the equations of the lines.

Use with Lesson 7-1.

4. **DESIGN** In a golden rectangle, the ratio of the length to the width is about 1.618. This is known as the *golden ratio*.

   a. A standard television screen has an aspect ratio of 4:3, while a high- definition television screen has an aspect ratio of 16:9. Is either type of screen a golden rectangle? Explain.

   b. The golden ratio can also be used to determine column layouts for Web pages. Consider a site with two columns, the left for content and the right as a sidebar. The ratio of the left to right column widths is the golden rectangle. Determine the width of each column if the page is 960 pixels wide.

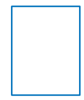
Use with Lesson 8-4.

5. **REASONING** What is the relationship between the sine and cosine of complementary angles? Explain your reasoning and use the relationship to find cos 50 if sin 40 ≈ 0.64.

Use with Lesson 9-5.

State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

7.



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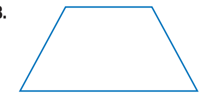
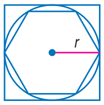


Use with Lesson 10-1.

9. **REASONING** In the figure, a circle with radius r is inscribed in a regular polygon and circumscribed about another.



   a. What are the perimeters of the circumscribed and inscribed polygons in terms of r? Explain.

   b. Is the circumference of the circle C greater or less than the perimeter of the circumscribed polygon? the inscribed polygon? Write a compound inequality comparing C to these perimeters.

   c. Rewrite the inequality from part b in terms of the diameter d of the circle and interpret its meaning.

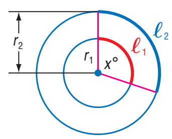
   d. As the number of sides of both the circumscribed and inscribed polygons increase, what will happen to the upper and lower limits of the inequality from part c, and what does this imply?

10. Use the locus definition of a circle and dilations to prove that all circles are similar.

Use with Lesson 10-2.

11. **ARC LENGTH AND RADIAN MEASURE** In this problem, you will use concentric circles to show the length of the arc intercepted by a central angle of a circle is dependent on the circle's radius.

   a. Compare the measures of arc ℓ_1 and arc ℓ_2. Then compare the lengths of arc ℓ_1 and arc ℓ_2. What do these two comparisons suggest?



   b. Use similarity transformations (dilations) to explain why the length of an arc ℓ intercepted by a central angle of a circle is proportional to the circle's radius r. That is, explain why we can say that for this diagram, $\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2}$.

   c. Write expressions for the lengths of arcs ℓ_1 and ℓ_2. Use these expressions to identify the constant of proportionality k in $\ell = kr$.

   d. The expression that you wrote for k in part c gives the *radian measure* of an angle. Use it to find the radian measure of an angle measuring 90°.

46 | Additional Exercises

Additional Exercises

Use with Extend 10-5.

12. Draw a right triangle and inscribe a circle in it.

13. Inscribe a regular hexagon in a circle. Then inscribe an equilateral triangle in a circle. (*Hint:* The first step of each construction is identical to Step 1 in Activity 2.)

14. Inscribe a square in a circle. Then circumscribe a square about a circle.

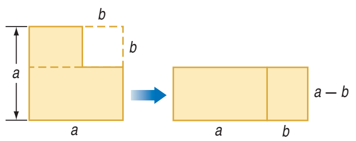
15. **CHALLENGE** Circumscribe a regular hexagon about a circle.

Use with Lesson 11-3.

18. **CHALLENGE** Derive the formula for the area of a sector of a circle using the formula for arc length.

Use with Lesson 11-4.

19. **WRITING IN MATH** Consider the sequence of area diagrams shown.



a. What algebraic theorem do the diagrams prove? Explain your reasoning.

b. Create your own sequence of diagrams to prove a different algebraic theorem.

20. **CARPETING** Ignacio's family is getting new carpet in their family room, and they want to determine how much the project will cost.

a. Use the floor plan shown to find the area to be carpeted.

b. If the carpet costs \$4.86 per square yard, how much will the project cost?

21. **FLOORING** JoAnn wants to lay 12" × 12" tile on her bathroom floor.

a. Find the area of the bathroom floor in her apartment floor plan.

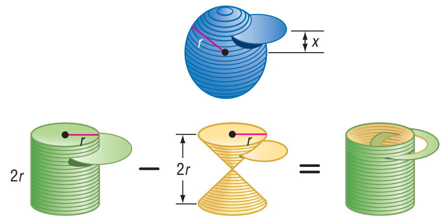
b. If the tile comes in boxes of 15, and JoAnn buys no extra tile, how many boxes will she need?

22. **DESIGN** A standard juice box holds 8 fluid ounces.

a. Sketch designs for three different juice containers that will each hold 8 fluid ounces. Label dimensions in centimeters. At least one container should be cylindrical. (*Hint:* 1 fl oz ≈ 29.57353 cm^3)

b. For each container in part a, calculate the surface area to volume (cm^2 per fl oz) ratio. Use these ratios to decide which of your containers can be made for the lowest materials cost. What shape container would minimize this ratio, and would this container be the cheapest to produce? Explain your reasoning.

Use with Lesson 12-6.



a. Find the radius of the disc from the sphere in terms of its distance x above the sphere's center. (*Hint:* Use the Pythagorean Theorem.)

b. If the disc from the sphere has a thickness of y units, find its volume in terms of x and y.

c. Show that this volume is the same as that of the hollowed-out disc with thickness of y units that is x units above the center of the cylinder and cone.

d. Since the expressions for the discs at the same height are the same, what guarantees that the hollowed-out cylinder and sphere have the same volume?

e. Use the formulas for the volumes of a cylinder and a cone to derive the formula for the volume of the hollowed-out cylinder and thus, the sphere.

23. **INFORMAL PROOF** A sphere with radius r can be thought of as being made up of a large number of discs or thin cylinders. Consider the disc shown that is x units above or below the center of the sphere. Also consider a cylinder with radius r and height $2r$ that is hollowed out by two cones of height and radius r.

Use with Lesson 13-3.

24. **DECISION MAKING** Meleah's flight was delayed and she is running late to make it to a national science competition. She is planning on renting a car at the airport and prefers car rental company A over car rental company B. The courtesy van for car rental company A arrives every 7 minutes, while the courtesy van for car rental company B arrives every 12 minutes.

a. What is the probability that Meleah will have to wait 5 minutes or less to see each van? Explain your reasoning. (*Hint:* Use an area model.)

b. What is the probability that Meleah will have to wait 5 minutes or less to see one of the vans? Explain your reasoning.

c. Meleah can wait no more than 5 minutes without risking being late for the competition. If the van from company B should arrive first, should she wait for the van from company A or take the van from company B? Explain your reasoning.

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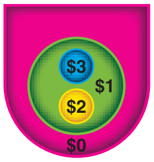
Additional Exercises

Use with Lesson 13-4.

25. DECISION MAKING The object of the game shown is to win money by rolling a ball up an incline into regions with different payoff values. The probability that Susana will get $0 in a roll is 55%, $1 is 20%, $2 is 20%, and $3 is 5%.



a. Suppose Susana pays $1 to play. Calculate the expected payoff, which is the expected value minus the cost to play, for each roll.

b. Design a simulation to estimate Susana's average payoff for this game after she plays 10 times.

c. Should Susana play this game? Explain your reasoning.

26. DECISION MAKING A lottery consists of choosing 5 winning numbers from 31 possible numbers (0–30). The person who matches all 5 numbers, in any sequence, wins $1 million.

a. If a lottery ticket costs $1, should you play? Explain your reasoning by computing the expected payoff value, which is the expected value minus the ticket cost.

b. Would your decision to play change if the winnings increased to $5 million? if the winnings were only $0.5 million, but you chose from 21 instead of 31 numbers? Explain.

Use with Lesson 13-5.

27. DECISION MAKING As the manager of a successful widget factory, you are trying to decide whether you should expand the business. If you do not expand and the economy remains good, you expect $2 million in revenue next year. If the economy is bad, you expect $0.5 million. The cost to expand is $1 million, but the expected revenue after the expansion is $4 million in a good economy and $1 million in a bad economy. You assume that the chances of a good and a bad economy are 30% and 70%, respectively. What should you do? Use a probability tree to explain your reasoning.

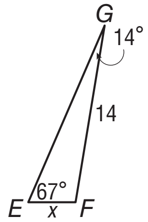
48 | Additional Exercises

Lesson 5 Practice

The Law of Sines and Law of Cosines

Find *x*. Round angle measures to the nearest degree and side lengths to the nearest tenth.

1.



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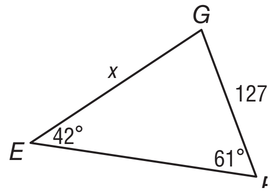
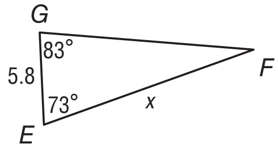


166, 0

3.



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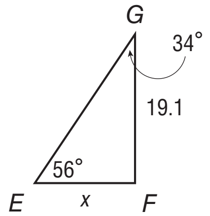
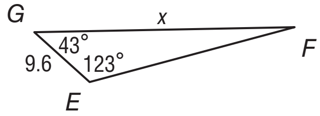


12.9

5.



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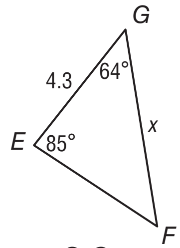

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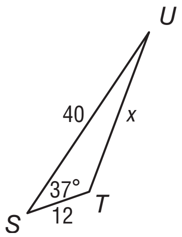

31.3

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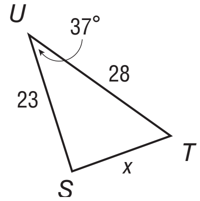
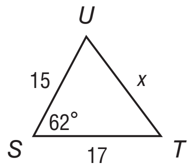


16.9

9.



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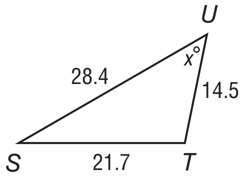

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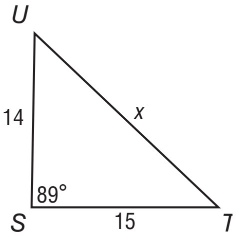
11.



20.3

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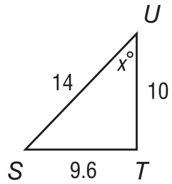

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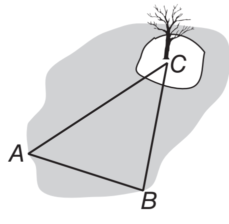
13. INDIRECT MEASUREMENT To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location *A* to location *B* is 85 meters. The measures of the angles at *A* and *B* are 51° and 83°, respectively. What is the distance from the edge of the lake at *B* to the tree on the island at *C*?



about 91.8 m

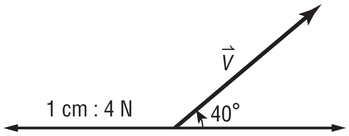
NAME _______________________________ DATE ____________ PERIOD ____________

Lesson 7 Practice

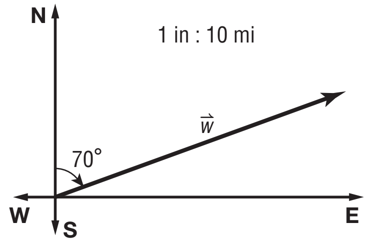
Vectors

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

1. $\vec{v}$ = 12 Newtons of force at 40° to the horizontal

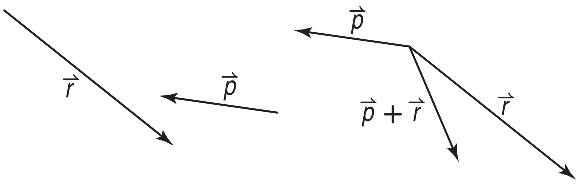


2. $\vec{w}$ = 15 miles per hour 70° east of north

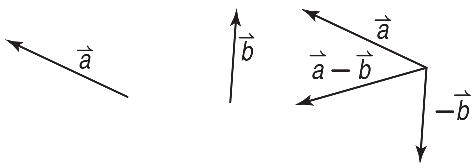


Copy the vectors to find each sum or difference.

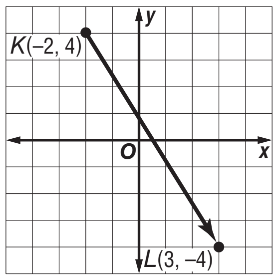
3. $\vec{p} + \vec{r}$



4. $\vec{a} - \vec{b}$



5. Write the component form of $\overrightarrow{AB}$.



⟨5, –8⟩

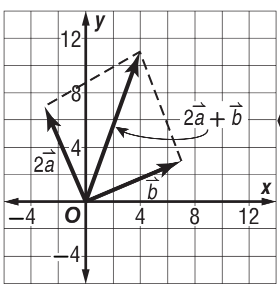
Find the magnitude and direction of each vector.

6. $\vec{t} = \langle 6, 11 \rangle$

7. $\vec{g} = \langle 9, -7 \rangle$

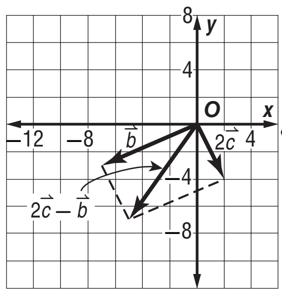
Find each of the following for $\vec{a} = \langle -1.5, 4 \rangle$, $\vec{b} = \langle 7, 3 \rangle$, and $\vec{c} = \langle 1, -2 \rangle$. Check your answers graphically.

8. $2\vec{a} + \vec{b}$



⟨–1, 13⟩

9. $2\vec{c} - \vec{b}$



⟨–5, –1⟩

10. AVIATION A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.

a. Find the resultant velocity of the plane. **about 301.5 mph**

b. Find the resultant direction of the plane. **about 5.7° west of due north**

Lesson 12 Practice

Equations of Circles

Write the equation of each circle.

1. center at (0, 0), diameter 18
   $x^2 + y^2 = 81$

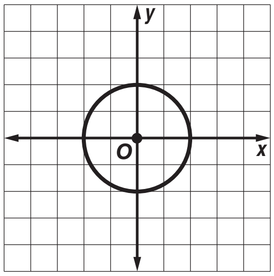
2. center at (−7, 11), radius 8
   $(x + 7)^2 + (y - 11)^2 = 64$

3. center at (−1, 8), passes through (9, 3)
   $(x + 1)^2 + (y - 8)^2 = 125$

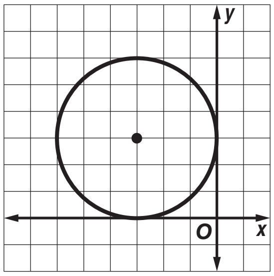
4. center at (−3, −3), passes through (−2, 3)
   $(x + 3)^2 + (y + 3)^2 = 37$

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

5. $x^2 + y^2 - 4 = 0$ **(0, 0); 2**

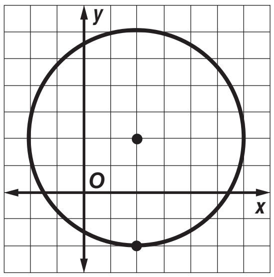


6. $x^2 + y^2 + 6x - 6y + 9 = 0$ **(−3, 3); 3**



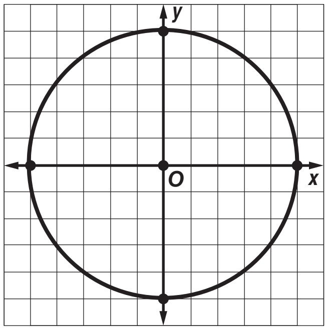
Write an equation of a circle that contains each set of points. Then graph the circle.

7. $A(-2, 2)$, $B(2, -2)$, $C(6, 2)$



$(x - 2)^2 + (y - 2)^2 = 16$

8. $R(5, 0)$, $S(-5, 0)$, $T(0, -5)$



$x^2 + y^2 = 25$

Find the point(s) of intersection, if any, between each circle and line with the equations given.

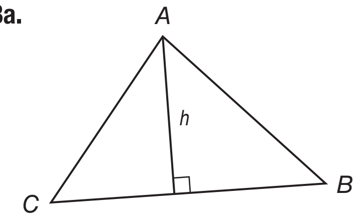
9. $x^2 + y^2 = 25;\ y = x$
   $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right), \left(-\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

10. $(x + 4)^2 + (y - 3)^2 = 25;\ y = x + 2$
    (−4, −2), (1, 3)

11. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents one of the concentric circles of seismic waves of the earthquake.
    $(x - 50)^2 + y^2 = 2500$

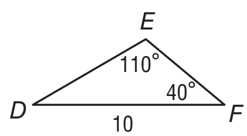
Lesson 5

53a.

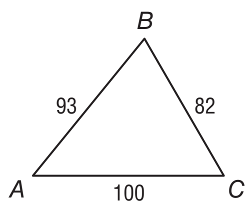


56. The Law of Cosines relates the measures of two sides *a* and *b* of any triangle to the measure of the third side *c* given the measure of the angle *C* between those two sides using the equation $a^2 + b^2 - 2ab \cos C = c^2$. When $m\angle C = 90$, we have $a^2 + b^2 - 2ab \cos 90° = c^2$. Since cos 90° is 0, this equation simplifies to $a^2 + b^2 - 2ab(0) = c^2$ or $a^2 + b^2 = c^2$, which is the same as the Pythagorean Theorem. Therefore, the Pythagorean Theorem is a specific case of the Law of Cosines.

57a. Sample answer:

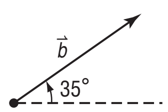


57b. Sample answer:



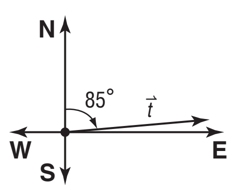
Lesson 7 (Guided Practice)

1A.



1 cm : 20 ft/s

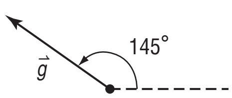
1B.



1 in. : 10 km/h

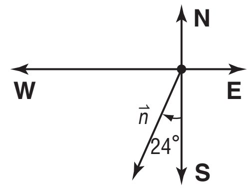
Lesson 7

12.



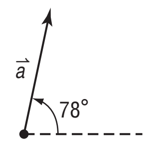
1 in. : 30 in./s

13.



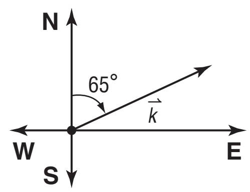
1 in. : 5 m

14.



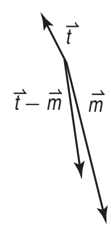
1 cm : 10 yd/min

15.

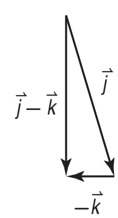


1 in. : 50 km/h

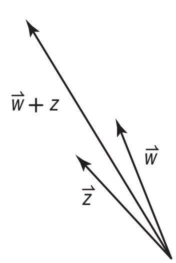
16.



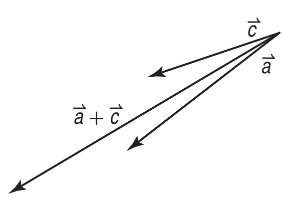
17.



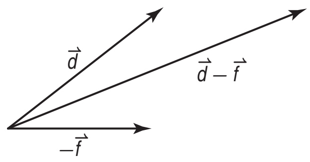
18.



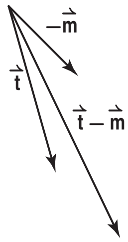
19.



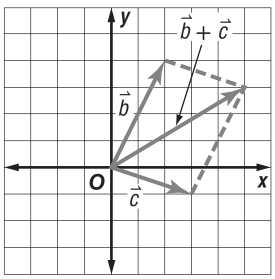
20.



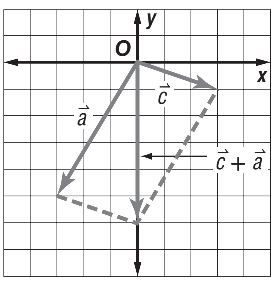
21.



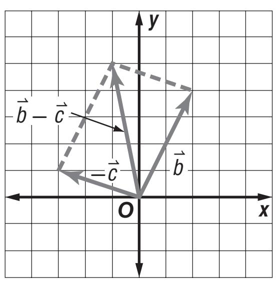
35. ⟨5, 3⟩



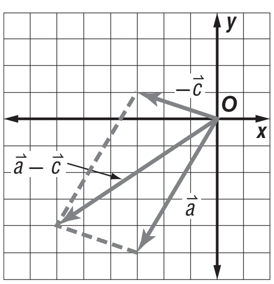
36. ⟨0, −6⟩



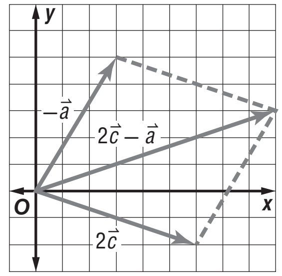
37. ⟨−1, 5⟩



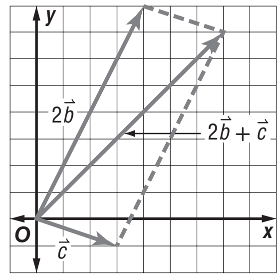
38. ⟨−6, −4⟩



39. ⟨9, 3⟩



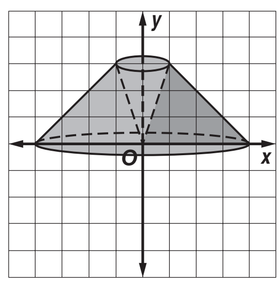
40. ⟨7, 7⟩



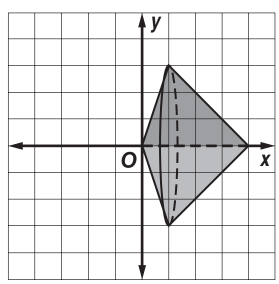
53. The initial point of the resultant starts at the initial point of the first vector in both methods. However, in the parallelogram method, both vectors start at the same initial point, whereas, in the triangle method, the resultant connects the initial point of the first vector and the terminal point of the second. The resultant is the diagonal of the parallelogram formed using the parallelogram method.

Lab 8

13a. Sample answer:

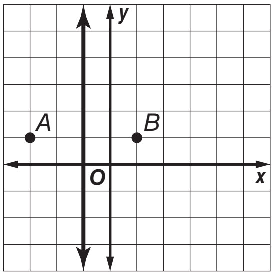


13b. Sample answer:

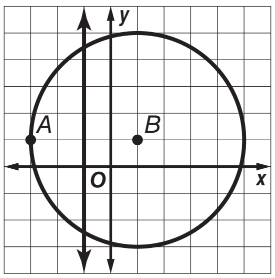


Lesson 12

42b. Sample answer:



42d. Sample answer:



44. Given: $\overline{AB}$ is a diameter of O, and C is a point on O.

Prove: $\angle ABC$ is a right angle.

Proof:

$\overline{AC}$ has slope $\frac{y-r}{x}$, and $\overline{CB}$ has slope $\frac{y-(-r)}{x}$ or $\frac{y+r}{x}$.

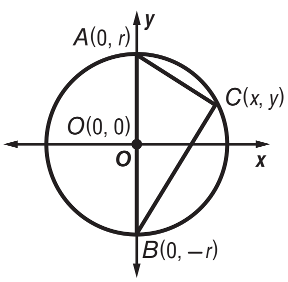
$$\frac{y-r}{x} \cdot \frac{y+r}{x} = \frac{y^2-r^2}{x} \quad \text{Multiply.}$$

$$= \frac{y^2-(x^2+y^2)}{x^2} \quad r^2 = x^2 + y^2$$

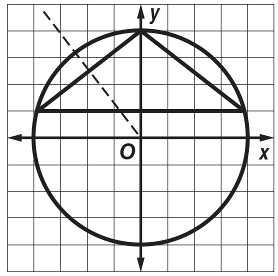
$$= \frac{y^2-x^2-y^2}{x^2} \quad -(x^2+y^2) = -x^2 - y^2$$

$$= \frac{-x^2}{x^2} \text{ or } -1 \quad \text{Simplify.}$$

Since the product of the slope of $\overline{AC}$ and $\overline{CB}$ is -1, $\overline{AC} \perp \overline{CB}$ and $\angle ACB$ is a right angle.



46. Sample answer:



47a. 4

47b-c. Method 1: Draw circles centered on each station that have a radius of 4 units. Stations outside of a circle can have the same frequency as the station at the center of the circle. Method 2: Use the Pythagorean theorem to identify stations that are more than 200 miles apart. Using Method 2, plot the points representing the stations on a graph. Stations that are more than 4 units apart on the graph will be more than 200 miles apart and will thus be able to use the same frequency. Assign station A to the first frequency. Station B is within 4 units of station A, so it must be assigned the second frequency. Station C is within 4 units of both stations A and B, so it must be assigned a third frequency. Station D is also within 4 units of stations A, B, and C, so it must be assigned a fourth frequency, Station E $\sqrt{29}$ or about 5.4 units away from station A, so it can share the first frequency. Station F is $\sqrt{29}$ or about 5.4 units away from station B, so it can share the second frequency. Station G is $\sqrt{32}$ or about 5.7 units away from station C, so it can share the third frequency. Therefore, the least number of frequencies that can be assigned is 4.

Additional Exercises, Lesson 0-3

1. Sample answer: Assign each friend a different colored marble: red, blue, or green. Place all the marbles in a bag and without looking, select a marble from the bag. Whoever's marble is chosen gets to go first.
2. Sample answer: With either a less than 5% or 1% chance of having a heart attack, a person would still need to weigh the benefits of the drug versus the small chance of having a heart attack. A chance of less than 1% versus a chance of less than 5% should make a person who is considering taking the drug more likely to risk taking the drug to control their diabetes.

Additional Exercises, Lesson 3-4

3a.

line q $y = 3x + 2$	
x	y
−3	−7
−2	−4
−1	−1
0	2
1	5
2	8
3	11

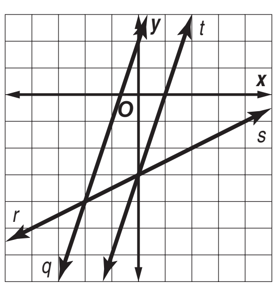
line r $y = 0.5x - 3$	
x	y
−3	−4.5
−2	−4
−1	−3.5
0	−3
1	−2.5
2	−2
3	−1.5

line s $2y = x - 6$	
x	y
−3	−4.5
−2	−4
−1	−3.5
0	−3
1	−2.5
2	−2
3	−1.5

line t $y = 3x - 3$	
x	y
−3	−12
−2	−9
−1	−6
0	−3
1	0
2	3
3	6

Sample answer: The system of equations represented by lines *q* and *r* and by lines *q* and *s* each appear to have one solution, since each pairing of tables has the ordered pair (−2, −4) in common. The systems of equations represented by lines *r* and *t* and by lines *s* and *t* each appear to have one solution, since each pairing of tables has the ordered pair (0, −3) in common. The system of equations represented by lines *q* and *t* appears to have no solution, since the *y*-values of the ordered pairs with the same *x*-values will always differ by 5. The system of equations represented by lines r and s appears to have infinitely many solutions since the pair of tables has all ordered pairs in common.

3b. Sample answer: Lines *q* and *t* are parallel. Lines *r* and *s* coincide. Lines *q* and *r* intersect at point (−2, −4). Lines *r* and *t* intersect at point (0, −3).



3c. Sample answer: Compare the slopes of the lines and their *y*-intercepts. Line *q* has slope 3 and *y*-intercept 2, line *r* has slope 0.5 and *y*-intercept −3, line s has slope 0.5 and *y*-intercept −3, and line *t* has slope 3 and *y*-intercept −3. Since lines *q* and *r* have different slopes and lines *r* and *t* have different slopes, each pair of lines intersect, and therefore each related system of equations has one solution. Since lines *q* and *t* have the same slope but different *y*-intercepts, the lines are parallel, and therefore the related system of equations has no solution. Since lines *r* and *s* have the same slope and the same *y*-intercept, the lines coincide, and therefore the related system of equations has infinitely many solutions.

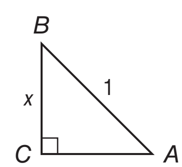
3d. Sample answer: A system of equations that has one solution will have only one ordered pair that is common to each table of values, a graph of intersecting lines, and equations that have different slopes. A system of equations that has no solution will have not have any ordered pairs common to each table of values, a graph of parallel lines, and equations that have the same slope, but different y-intercepts. A system of equations that has infinitely many solutions will have identical tables of values, a graph of coinciding lines, and equations that have the same slope and the same y-intercept.

Additional Exercises, Lesson 7-1

4a. No; the HDTV aspect ratio is 1.77778 and the standard aspect ratio is 1.33333. Neither television set is a golden rectangle since the ratios of the lengths to the widths are not the golden ratio.

Additional Exercises, Lesson 8-4

5. In the diagram, $\sin A = x$ and $\cos B = x$; therefore $\sin A = \cos B$.



Since the acute angles of a right triangle are complementary, $m\angle B = 90 - m\angle A$. By substitution, $\sin A = \cos (90 - A)$. Since $\sin A = x$, $\cos (90 - A) = x$ by substitution. Applying this relationship, if $\sin 40 \approx 0.64$, then $\cos (90 - 40) \approx 0.64$. Since $90 - 40 = 50$, $\cos 50 \approx 0.64$.

Additional Exercises, Extend 10-1

9a. 8r and 6r; Twice the radius of the circle, $2r$ is the side length of the square, so the perimeter of the square is $4(2r)$ or 8r. The regular hexagon is made up of six equilateral triangles with side length r, so the perimeter of the hexagon is $6(r)$ or $6r$.

9b. less; greater; $6r < C < 8r$

9c. $3d < C < 4d$; The circumference of the circle is between 3 and 4 times its diameter.

9d. These limits will approach a value of π, implying that $C = \pi d$.

10. A circle is a locus of points in a plane equidistant from a given point. For any two circles $\odot A$ and $\odot B$ there exists a translation that maps center A onto center B, moving $\odot A$ so that it is concentric with $\odot B$. There also exists a dilation with scale factor k such that each point that makes up $\odot A$ is moved to be the same distance from center A as the points that make up $\odot B$ are from center B. Therefore, $\odot A$ is mapped onto $\odot B$. Since there exists a rigid motion followed by a scaling that maps $\odot A$ onto $\odot B$, the circles are similar. Thus, all circles are similar.

Additional Exercises, Lesson 10-2

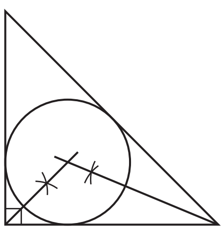
11a. $m\ell_1 = m\ell_2$; $\ell_1 < \ell_2$; these comparisons suggest that arc measure is not affected by the size of the circle, but arc length is affected.

11b. Since all circles are similar, the larger circle is a dilation of the smaller by some factor k, so $r_2 = kr_1$ or $k = \frac{r_2}{r_1}$. Likewise, the arc intercepted on the larger circle is a dilation of the arc intercepted on the smaller circle, so $\ell_2 = k\ell_1$ or $k = \frac{\ell_2}{\ell_1}$. Thus $\frac{r_2}{r_1} = \frac{\ell_2}{\ell_1}$ or $\frac{\ell}{r_1} = \frac{\ell_2}{r_1}$.

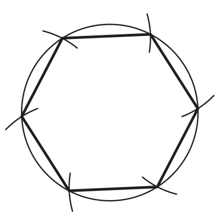
11c. $\ell_1 = \frac{\pi r_1 x}{180}$ and $\ell_2 = \frac{\pi r_2 x}{180}$; $k = \frac{\pi x}{180}$

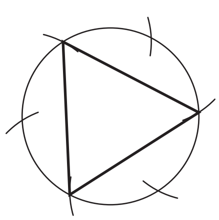
Additional Exercises, Extend 10-5

12. Sample answer:

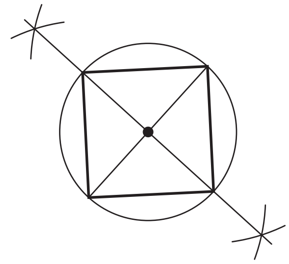


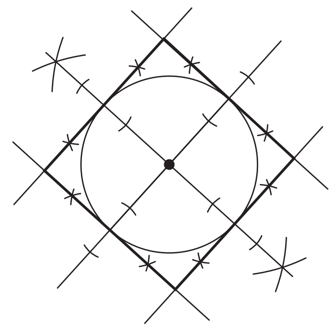
13. Sample answers:



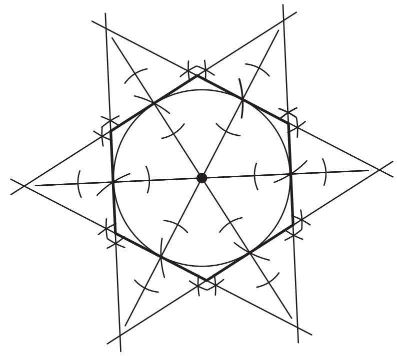


14. Sample answers:



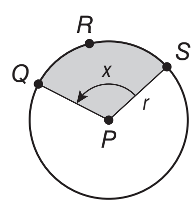


15. Sample answer:



Additional Exercises, Lesson 11-3

18. The ratio of the area of a sector to the area of a whole circle is equal to the ratio of the corresponding arc length to the circumference of the circle. Let A represent the area of the sector.



$\frac{A}{\pi r^2} = \frac{\text{length of } QRS}{2\pi r}$ $\quad \frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc length}}{\text{circumference of circle}}$

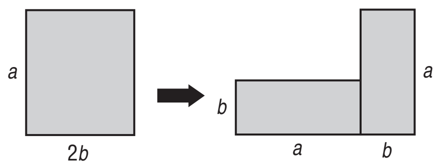
$\frac{A}{\pi r^2} = \frac{\frac{\pi r x}{180}}{2\pi r}$ $\quad$ The length of QRS is $\frac{\pi r x}{180}$.

$A = \frac{\pi r^2 x}{360}$ $\quad$ Solve for A.

Additional Exercises, Lesson 11-4

19a. $a^2 - b^2 = (a + b)(a - b)$; Sample answer: The area of the first figure is equal to the area of the larger square a^2 minus the area of the smaller square b^2 or $a^2 - b^2$. The area of the second figure is equal to the area of a rectangle with side lengths $a + b$ and $a - b$ or $(a + b)(a - b)$. Since the figures are composed of congruent shapes, the areas are equal, so $a^2 - b^2 = (a + b)(a - b)$.

19b. Sample answer: $2ab = ab + ab$



Additional Exercises, Lesson 12-6

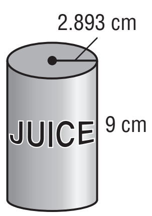
22b. $\pi\left(\sqrt{r^2 - x^2}\right)^2 - y$ or $\pi yr^2 - \pi yx^2$

22c. The volume of the disc from the cylinder is $\pi r^2 y$ or πyr^2. The volume of the disc from the two cones is $\pi x^2 y$ or πyx^2. Subtract the volumes of the discs from the cylinder and cone to get $\pi yr^2 - \pi yx^2$, which is the expression for the volume of the disc from the sphere at height x.

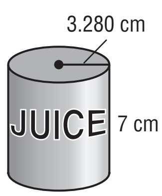
22d. Cavalieri's Principle

22e. The volume of the cylinder is $\pi r^2(2r)$ or $2\pi r^3$. The volume of one cone is $\frac{1}{3}\pi r^2(r)$ or $\frac{1}{3}\pi r^3$, so the volume of the double napped cone is $2 \cdot \frac{1}{3}\pi r^3$ or $\frac{2}{3}\pi r^3$. Therefore, the volume of the hollowed out cylinder, and thus the sphere, is $2\pi r^3 - \frac{2}{3}\pi r^3$ or $\frac{4}{3}\pi r^3$.

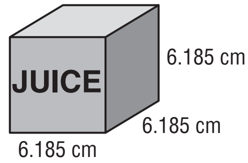
23a. Sample answers:



Container A



Container B



Container C

23b. Sample answers: Container A, ≈ 27.02 cm^2 per fl oz; Container B, ≈ 26.48 cm^2 per fl oz; Container C, ≈ 28.69 cm^2 per fl oz; Of these three, Container B can be made for the lowest materials cost. The lower the surface area to volume ratio, the less packaging used for each fluid ounce of juice it holds. A spherical container with $r = 3.837$ cm would minimize this cost since it would have the least surface area to volume ratio of any shape, ≈ 23.13 cm per fl oz. However, a spherical container would likely be more costly to manufacture than a rectangular container since specially made machinery would be necessary.

Additional Exercises, Lesson 13-3

24a. On a coordinate plane, graph $x = 7$ and shade between this line and the y-axis to represent the possible waiting times for the company A van. Graph $y = 12$ and shade between this line and the x-axis to represent the possible waiting times for the company B van. The area of the rectangle formed by the intersection is 84 units^2. Then graph $x = 5$ and $y = 5$ and shade the region bounded by these lines and the axes to represent the possible waiting times of 5 minutes or less for both vans. The area of the square is 25 square units. So the geometric probability is $\frac{25}{84}$ or about 30%.

24b. On a coordinate plane, graph the lines $x = 7$ and $y = 12$ and shade as before. The area of this rectangle is 84 units^2. Then graph $x = 7$ and $y = 5$. Shade the region bounded by the lines $y = 5$, $x = 7$, and the axes to represent the possible waiting times of 5 minutes or less for the company A van. The area of this rectangle is 35 unit^2. Shade the region bounded by the lines $x = 5$, $y = 12$, and the axes to represent the possible waiting times of 5 minutes or less for the company B van. The area of this rectangle is 60 unit^2. In each of these rectangles, the waiting time of 5 minutes or less for both vans has been counted twice. So the geometric probability is $\frac{60}{84} + \frac{35}{84} - \frac{25}{84} = \frac{70}{84}$, or about 83%.

24c. Sample answer: Since the chance of Meleah waiting 5 minutes or less to see the vans from both company A and B is only 30%, Meleah should take the van from company B.

Additional Exercises, Lesson 13-4

25a. −$0.25

25b. Sample answer: Use a random number generator to generate integers 1 though 20, where 1–11 represents $0, 12–15 represents $2, 16–19 represents $3, and 20 represents $4. The average payoff after 10 trials is $0.90 − $1 or −$0.10.

25c. Sample answer: No, both the theoretical and simulated expected payoffs are less than what it costs to play the game.

26a. Sample answer: The expected value is $\$1{,}000{,}000\left(\frac{1}{{}_{31}C_5}\right) \approx \$5.89$. The expected payoff is $5.89 − $1 or $4.89, which means that on average, a person can expect to gain $4.89 for every $1 he or she spends to buy a ticket. This is a low payoff value, so I may not play.

26b. Sample answer: The expected value is $\$5{,}000{,}000\left(\frac{1}{{}_{31}C_5}\right) \approx \$29.43$. The expected payoff is S29.43 − $1 or $28.43. This is a higher payoff, so I would play. The expected value from playing for $0.5 million but only having to choose from 21 numbers is $500{,}000\left(\frac{1}{{}_{21}C_5}\right) \approx \$24.57$. The expected payoff is $24.57 − $1 or $23.57. This is still a higher payoff value, so I would play.

Additional Exercises, Lesson 13-5

27. Sample answer: The expected value of choosing to expand is $1.2M + $0.7M or $1.9M, and the expected value of choosing not 10 expand is $0.95M. When we subtract the costs of expanding and of not expanding, we find the net expected value of expanding is $1 .9M − $1 M or $0.9M and the net expected value of not expanding is $0.95M − $0 or $0.9. Since $0.9M < $0.95M, you should not expand the business.