Glencoe Secondary Mathematics

ALIGNED TO THE COMMON CORE STATE STANDARDS

Algebra 2

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Send all inquiries to:
McGraw-Hill Education
STEM Learning Solutions Center
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Columbus, OH 43240

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Common Core State Standards

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LAB 1

Algebra Lab
Correlation and Causation

You have learned that the correlation coefficient measures how well an equation fits a set of data. A correlation coefficient close to 1 or -1 indicates a high correlation. However, this does not imply causation. When there is **causation**, one data set is the direct cause of the other data set. When there is a correlation between two sets of data, the data sets are related.

> **NEWS**
>
> **Cell Phones Cause Brain Tumors!**
>
> Studies have shown that brain tumors are linked to cell phone use. One particular study showed a very strong correlation between the two events.

The news clip to the right uses correlation to imply causation. While a study may have found a high correlation between these two variables, this does not mean that cell phone use causes brain tumors. Other variables, such as an individual's family history, diet, and environment could also have an effect on the formation of brain tumors.

A **lurking variable** affects the relationship between two other variables, but is not included in the study that compares them.

EXAMPLE 1

Determine whether the following correlations possibly show causation. Write *yes* or *no*. If not, identify other lurking variables.

a. Studies have shown that students are less energetic after they eat lunch.

No; students could have had gym class before lunch, or the class after lunch could be one that they do not find interesting.

b. If Kevin lifts weights, he will make the football team.

No; Kevin may need to maintain a certain grade-point average to make the team. He also needs to be talented enough to make the team.

c. When the Sun is visible, we have daylight.

Yes; no other variables cause daylight.

Exercises

Determine whether the following correlations possibly show causation. Write *yes* or *no*. If not, identify other lurking variables.

1. If Allison studies, she will get an A.
2. When Lisa exercises, she is in a better mood.
3. The gun control laws have reduced violent crime.
4. Smoking causes lung cancer.
5. If we have a Level 2 snow emergency, we do not have school.
6. Reading more increases one's intelligence.

Think About It

7. What do you think must be done to show that a correlation between two variables actually shows causation?

(continued on the next page)

Algebra Lab
Correlation and Causation *Continued*

Statistics are often provided that show a correlation between two events and causation is implied, but not confirmed. The best method to find evidence that x causes y is to actually perform multiple experiments with large sample sizes. When an experiment is not available, the only option is to determine if causation is *possible.*

EXAMPLE 2

The table shows IQ scores for ten high school students and their corresponding grade-point averages.

IQ	GPA	IQ	GPA
106	3.4	118	3.9
110	3.6	107	3.3
109	3.5	111	3.6
98	2.9	109	3.7
115	3.8	102	3.1

a. Calculate the correlation coefficient.
The value of r is about 0.96.

b. Describe the correlation.
There is a strong positive correlation.

c. Is there causation? Explain your reasoning.
There is no causation. Many other factors determine grade-point averages, including effort, study habits, knowledge of the material in the particular classes, and extracurricular activities.

EXAMPLE 3

The table shows the approximate boiling temperatures of water at different altitudes.

Altitude (ft)	Temp. (°F)
0	212
984	210
2000	208
3000	206
5000	203
7500	198
10,000	194
20,000	178
26,000	168

a. Calculate the correlation coefficient.
The value of r is about -0.99.

b. Describe the correlation.
There is a strong negative correlation.

c. Is there causation? Explain your reasoning.
There appears to be causation. Altitude is a direct cause for the boiling temperature of water to decrease.

Exercises

Calculate the correlation coefficient and describe the correlation. Determine if causation is possible. Explain your reasoning.

8.

Car Value ($)	Miles per Gallon	Car Value ($)	Miles per Gallon
14,000	55	25,000	33
16,000	48	29,000	29
18,500	37	32,000	19
20,000	35	35,000	26
22,500	29	40,000	33

9.

Temp. (°F)	Ice Cream Sales ($)	Temp. (°F)	Ice Cream Sales ($)
76	850	85	905
78	885	86	1005
80	875	87	1060
82	920	88	1105
84	955	89	1165

10.

Year	Sales via Internet ($)	Year	Sales via Internet ($)
2001	1923	2006	7067
2002	2806	2007	8568
2003	3467	2008	9991
2004	4848	2009	10,587
2005	5943	2010	12,695

11.

Number of Rings from Pith to Bark	Age of Tree (yr.)	Number of Rings from Pith to Bark	Age of Tree (yr.)
5	5	30	30
10	10	40	40
15	15	50	50
20	20	60	60

LAB 2

Graphing Technology Lab
Solving Quadratic Equations by Graphing

You can use a TI-83/84 Plus graphing calculator to solve quadratic equations.

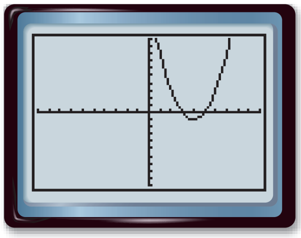
Activity Solving Quadratic Equations

Solve $x^2 - 8x + 15 = 0$.

Step 1 Let **Y1** = $x^2 - 8x + 15$ and **Y2** = 0.

Step 2 Graph **Y1** and **Y2** in the standard viewing window.

KEYSTROKES: [Y=] [X,T,θ,n] [x^2] [(–)] 8 [X,T,θ,n] [+] 15 [ENTER] 0 [ZOOM] 6



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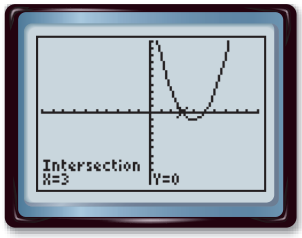
Step 3 To find the x-intercepts, determine the points where **Y1** = **Y2**.

KEYSTROKES: [2nd] [CALC] 5

Press [ENTER] for the first equation. Press [ENTER] for the second equation.

Move the cursor as close to the left x-intercept and press [ENTER].



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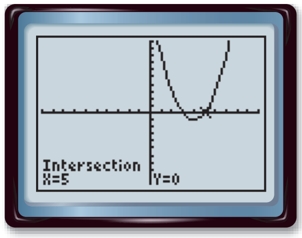
Find the second x-intercept.

KEYSTROKES: [2nd] [CALC] 5

Press [ENTER] for the first equation. Press [ENTER] for the second equation.

Move the cursor as close to the right x-intercept and press [ENTER].



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The x-intercepts are 3 and 5, so $x = 3$ and $x = 5$.

Exercises

Solve each equation. Round to the nearest tenth if necessary.

1. $x^2 - 7x + 12 = 0$

2. $x^2 + 5x + 6 = 0$

3. $x^2 - 3 = 2x$

4. $x^2 + 5x + 6 = 12$

5. $x^2 + 5x = 0$

6. $x^2 - 4 = 0$

7. $x^2 + 8x + 16 = 0$

8. $x^2 - 10x = -25$

9. $9x^2 + 48x + 64 = 0$

10. $2x^2 + 3x - 1 = 0$

11. $5x^2 - 7x = -2$

12. $6x^2 + 2x + 1 = 0$

LESSON 3 Solving Quadratic Equations by Factoring

Then

- You found the greatest common factors of sets of numbers.

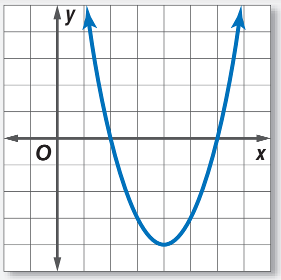
Now

1. Write quadratic equations in intercept form.
2. Solve quadratic equations by factoring.

Why?

- The **factored form** of a quadratic equation is $0 = a(x - p)(x - q)$. In the equation, p and q represent the x-intercepts of the graph of the equation.

The x-intercepts of the graph at the right are 2 and 6. In this lesson, you will learn how to change a quadratic equation in factored form into standard form and vice versa.



Standard Form	Factored Form	Related Graph
$0 = x^2 - 8x + 12$	$0 = (x - 6)(x - 2)$ (Factors)	2 and 6 are x-intercepts.

NewVocabulary
factored form
FOIL method

1 Factored Form

You can use the FOIL method to write a quadratic equation that is in factored form in standard form. The **FOIL method** uses the Distributive Property to multiply binomials.

KeyConcept FOIL Method for Multiplying Binomials

Words To multiply two binomials, find the sum of the products of *F* the *First terms*, *O* the *Outer terms*, *I* the *Inner terms*, and *L* the *Last terms*.

Examples

	Product of First Terms		Product of Outer Terms		Product of Inner Terms		Product of Last Terms
$(x - 6)(x - 2) =$ (F, O, I, L)	$(x)(x)$	$+$	$(x)(-2)$	$+$	$(-6)(x)$	$+$	$(-6)(-2)$

$$= x^2 - 2x - 6x + 12 \text{ or } x^2 - 8x + 12$$

Example 1 Translate Sentences into Equations

Write a quadratic equation in standard form with $-\frac{1}{3}$ and 6 as its roots.

$(x - p)(x - q) = 0$	Write the pattern.
$\left[x - \left(-\frac{1}{3}\right)\right](x - 6) = 0$	Replace p with $-\frac{1}{3}$ and q with 6.
$\left(x + \frac{1}{3}\right)(x - 6) = 0$	Simplify.
$x^2 - \frac{17}{3}x - 2 = 0$	Multiply.
$3x^2 - 17x - 6 = 0$	Multiply each side by 3 so that b and c are integers.

GuidedPractice

1. Write a quadratic equation in standard form with $\frac{3}{4}$ and -5 as its roots.

2 Solve Equations by Factoring

Solving quadratic equations by factoring is an application of the Zero Product Property.

KeyConcept Zero Product Property

Words	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.
Example	If $(x + 3)(x - 5) = 0$, then $x + 3 = 0$ or $x - 5 = 0$.

PT

Example 2 Factor the GCF

Solve $16x^2 + 8x = 0$.

$16x^2 + 8x = 0$	Original equation.
$8x(2x) + 8x^2(1) = 0$	Factor the GCF.
$8x(2x + 1) = 0$	Distributive Property
$8x = 0$ or $2x + 1 = 0$	Zero Product Property
$x = 0$ $\quad 2x = -1$	Solve both equations.
$x = -\frac{1}{2}$	

GuidedPractice **Solve each equation.**

2A. $20x^2 + 15x = 0$ **2B.** $4y^2 + 16y = 0$ **2C.** $6a^5 + 18a^4 = 0$

ReviewVocabulary

perfect square a number with a positive square root that is a whole number

Trinomials and binomials that are perfect squares have special factoring rules. In order to use these rules, the first and last terms need to be perfect squares and the middle term needs to be twice the product of the square roots of the first and last terms.

PT

Example 3 Perfect Squares and Differences of Squares

Solve each equation.

a. $x^2 + 16x + 64 = 0$

$x^2 = (x)^2$; $64 = (8)^2$	First and last terms are perfect squares.
$16x = 2(x)(8)$	Middle term equals $2ab$.

$x^2 + 16x + 64$ is a perfect square trinomial.

$x^2 + 16x + 64 = 0$	Original equation
$(x + 8)^2 = 0$	Factor using the pattern.
$x + 8 = 0$	Take the square root of each side.
$x = -8$	Solve.

b. $x^2 = 64$

$x^2 = 64$	Original equation
$x^2 - 64 = 0$	Subtract 64 from each side.
$x^2 - (8)^2 = 0$	Write in the form $a^2 - b^2$.
$(x + 8)(x - 8) = 0$	Factor the difference of squares.
$x + 8 = 0$ or $x - 8 = 0$	Zero Product Property
$x = -8$ $\quad x = 8$	Solve.

StudyTip

Square Roots By inspection, notice that the square root of 64 is −8 and 8. Also, for $x^2 = 4$, the solutions would be −2 and 2.

GuidedPractice

3A. $4x^2 - 12x + 9 = 0$ **3B.** $81x^2 - 9x = 0$ **3C.** $6a^2 - 3a = 0$

StudyTip

Trinomials If values for m and p exist, then the trinomial can always be factored.

A special pattern is used when factoring trinomials of the form $ax^2 + bx + c$. First, multiply the values of a and c. Then, find two values, m and p, such that their product equals ac and their sum equals b.

Consider $6x^2 + 13x - 5$: $ac = 6(-5) = -30$.

Factors of −30	Sum	Factors of −30	Sum
1, −30	−29	−1, 30	29
2, −15	−13	**−2, 15**	**13**
3, −10	−7	−3, 10	7
5, −6	−1	−5, 6	1

Now the middle term, $13x$, can be rewritten as $-2x + 15x$.

This polynomial can now be factored by grouping.

$6x^2 + 13x - 5 = 6x^2 + mx + px - 5$	Write the pattern.
$= 6x^2 - 2x + 15x - 5$	$m = -2$ and $p = 15$
$= (6x^2 - 2x) + (15x - 5)$	Group terms.
$= 2x(3x - 1) + 5(3x - 1)$	Factor the GCF.
$= (2x + 5)(3x - 1)$	Distributive Property

Example 4 Factor Trinomials

Solve each equation.

a. $x^2 + 9x + 20 = 0$

$ac = 20$ $a = 1, c = 20$

Factors of 20	Sum	Factors of 20	Sum
1, 20	21	−1, −20	−21
2, 10	12	−2, −10	−12
4, 5	**9**	−4, −5	−9

StudyTip

Trinomials It does not matter if the values of m and p are switched when grouping.

$x^2 + 9x + 20 = 0$	Original expression
$x^2 + mx + px + 20 = 0$	Write the pattern.
$x^2 + 4x + 5x + 20 = 0$	$m = 4$ and $p = 5$
$(x^2 + 4x) + (5x + 20) = 0$	Group terms with common factors.
$x(x + 4) + 5(x + 4) = 0$	Factor the GCF from each grouping.
$(x + 5)(x + 4) = 0$	Distributive Property
$x + 5 = 0$ or $x + 4 = 0$	Zero Product Property
$x = -5$ $x = -4$	Solve each equation.

b. $6y^2 - 23y + 20 = 0$

$ac = 120$	$a = 6, c = 20$
$m = -8, p = -15$	$-8(-15) = 120; -8 + (-15) = -23$
$6y^2 - 23y + 20 = 0$	Original equation
$6y^2 + my + py + 20 = 0$	Write the pattern.
$6y^2 - 8y - 15y + 20 = 0$	$m = -8$ and $p = -15$
$(6y^2 - 8y) + (-15y + 20) = 0$	Group terms with common factors.
$2y(3y - 4) - 5(3y - 4) = 0$	Factor the GCF from each grouping.
$(2y - 5)(3y - 4) = 0$	Distributive Property
$2y - 5 = 0$ or $3y - 4 = 0$	Zero Product Property
$2y = 5$ $3y = 4$	Solve both equations.
$y = \frac{5}{2}$ $y = \frac{4}{3}$	

GuidedPractice

4A. $x^2 - 11x + 30 = 0$

4B. $x^2 - 4x - 21 = 0$

4C. $15x^2 - 8x + 1 = 0$

4D. $-12x^2 + 8x + 15 = 0$

Stu Forster/Getty Images Sport/Getty Images

Real-World Link

Cuba's Osleidys Menendez broke the javelin world record in 2002 with a distance of 234 feet 8 inches.

Source: *New York Times*

Real-World Example 5 Solve Equations by Factoring

TRACK AND FIELD **The height of a javelin in feet is modeled by $h(t) = -16t^2 + 79t + 5$, where t is the time in seconds after the javelin is thrown. How long is it in the air?**

To determine how long the javelin is in the air, we need to find when the height equals 0. We can do this by solving $-16t^2 + 79t + 5 = 0$.

$-16t^2 + 79t + 5 = 0$	Original equation
$m = 80;\ p = -1$	$-16(5) = -80,\ 80 \cdot (-1) = -80,\ 80 + (-1) = 79$
$-16t^2 + 80t - t + 5 = 0$	Write the pattern.
$(-16t^2 + 80t) + (-t + 5) = 0$	Group terms with common factors.
$16t(-t + 5) + 1(-t + 5) = 0$	Factor GCF from each group.
$(16t + 1)(-t + 5) = 0$	Distributive Property
$16t + 1 = 0$ or $-t + 5 = 0$	Zero Product Property
$16t = -1$ $\quad -t = -5$	Solve both equations.
$t = -\frac{1}{16}$ $\quad t = 5$	Solve.

CHECK We have two solutions.

- The first solution is negative and since time cannot be negative, this solution can be eliminated.
- The second solution of 5 seconds seems reasonable for the time a javelin spends in the air.
- The answer can be confirmed by substituting back into the original equation.

$$-16t^2 + 79t + 5 = 0$$
$$-16(5)^2 + 79(5) + 5 \stackrel{?}{=} 0$$
$$-400 + 395 + 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

The javelin is in the air for 5 seconds.

GuidedPractice

5. **BUNGEE JUMPING** Juan recorded his brother bungee jumping from a height of 1100 feet. At the time the cord lifted his brother back up, he was 76 feet above the ground. If Juan started recording as soon as his brother fell, how much time elapsed when the cord snapped back? Use $f(t) = -16t^2 + c$, where c is the height in feet.

Check Your Understanding

Example 1 **Write a quadratic equation in standard form with the given root(s).**

1. $-8, 5$ **2.** $\frac{3}{2}, \frac{1}{4}$ **3.** $-\frac{2}{3}, \frac{5}{2}$

Examples 2–4 **Factor each polynomial.**

4. $35x^2 - 15x$ **5.** $18x^2 - 3x + 24x - 4$ **6.** $x^2 - 12x + 32$

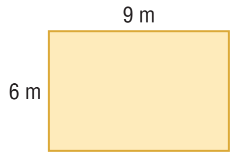
7. $x^2 - 4x - 21$ **8.** $2x^2 + 7x - 30$ **9.** $16x^2 - 16x + 3$

Example 5 **Solve each equation.**

10. $x^2 - 36 = 0$ **11.** $12x^2 - 18x = 0$ **12.** $12x^2 - 2x - 2 = 0$

13. $x^2 - 9x = 0$ **14.** $x^2 - 3x - 28 = 0$ **15.** $2x^2 - 24x = -72$

16. **GARDENING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then?



Practice and Problem Solving

Example 1 **Write a quadratic equation in standard form with the given root(s).**

17. 7 **18.** $-5, \frac{1}{2}$ **19.** $\frac{1}{5}, 6$

Examples 2–4 **Factor each polynomial.**

20. $40a^2 - 32a$ **21.** $51c^3 - 34c$ **22.** $32xy + 40bx - 12ay - 15ab$

23. $3x^2 - 12$ **24.** $15y^2 - 240$ **25.** $48cg + 36cf - 4dg - 3df$

26. $x^2 + 13x + 40$ **27.** $x^2 - 9x - 22$ **28.** $3x^2 + 12x - 36$

29. $15x^2 + 7x - 2$ **30.** $4x^2 + 29x + 30$ **31.** $18x^2 + 15x - 12$

32. $8x^2z^2 - 4xz^2 - 12z^2$ **33.** $9x^2 - 25$ **34.** $18x^2y^2 - 24xy^2 + 36y^2$

Example 3 **Solve each equation.**

35. $15x^2 - 84x - 36 = 0$ **36.** $12x^2 + 13x - 14 = 0$ **37.** $12x^2 - 108x = 0$

38. $x^2 + 4x - 45 = 0$ **39.** $x^2 - 5x - 24 = 0$ **40.** $x^2 = 121$

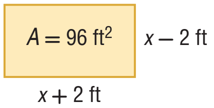
41. $x^2 + 13 = 17$ **42.** $-3x^2 - 10x + 8 = 0$ **43.** $-8x^2 + 46x - 30 = 0$

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.

GEOMETRY **Find x and the dimensions of each rectangle.**

46.



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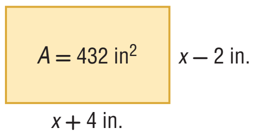

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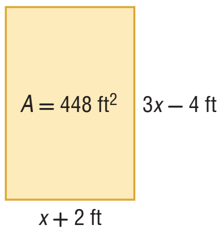


Solve each equation by factoring.

49. $12x^2 - 4x = 5$ **50.** $5x^2 = 15x$ **51.** $16x^2 + 36 = -48x$

52. $75x^2 - 60x = -12$ **53.** $4x^2 - 144 = 0$ **54.** $-7x + 6 = 20x^2$

55. MOVIE THEATER A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where x is the number of movie screens, and $P(x)$ is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.

Write a quadratic equation in standard form with the given root(s).

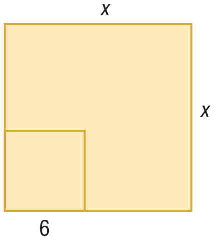
56. $-\frac{4}{7}, \frac{3}{8}$ **57.** 3.4, 0.6 **58.** $\frac{2}{11}, \frac{5}{9}$

Solve each equation by factoring.

59. $10x^2 + 25x = 15$ **60.** $27x^2 + 5 = 48x$ **61.** $x^2 + 0.25x = 1.25$

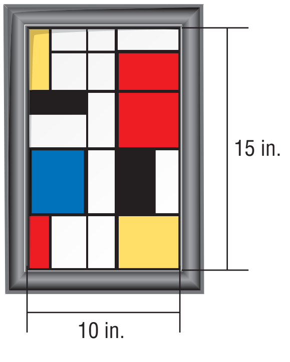
62. $48x^2 - 15 = -22x$ **63.** $3x^2 + 2x = 3.75$ **64.** $-32x^2 + 56x = 12$

65. DESIGN A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression.



66. FINANCIAL LITERACY After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by $P(x) = -16x^2 + 368x - 2035$, where x is the price of each Web site and $P(x)$ is the company's profit. Determine the price range of the Web sites that will be profitable for the company.

67. PAINTINGS Enola wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included?



68. MULTIPLE REPRESENTATIONS In this problem, you will consider $a(x - p)(x - q) = 0$.

a. Graphical Graph the related function for $a = 1$, $p = 2$, and $q = -3$.

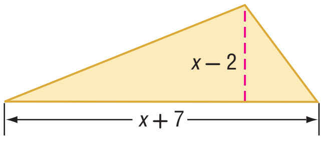
b. Analytical What are the solutions of the equation?

c. Graphical Graph the related functions for $a = 4, -3$, and $\frac{1}{2}$ on the same graph.

d. Verbal What are the similarities and differences between the graphs?

e. Verbal What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

69. GEOMETRY The area of the triangle is 26 square centimeters. Find the length of the base.



70. **SOCCER** When a ball is kicked in the air, its height in meters above the ground can be modeled by $h(t) = -4.9t^2 + 14.7t$ and the distance it travels can be modeled by $d(t) = 16t$, where t is the time in seconds.

a. How long is the ball in the air?

b. How far does it travel before it hits the ground? (*Hint*: Ignore air resistance.)

c. What is the maximum height of the ball?

Factor each polynomial.

71. $18a - 24ay + 48b - 64by$

72. $3x^2 + 2xy + 10y + 15x$

73. $6a^2b^2 - 12ab^2 - 18b^3$

74. $12a^2 - 18ab + 30ab^3$

75. $32ax + 12bx - 48ay - 18by$

76. $30ac + 80bd + 40ad + 60bc$

77. $5ax^2 - 2by^2 - 5ay^2 + 2bx^2$

78. $12c^2x + 4d^2y - 3d^2x - 16c^2y$

H.O.T. Problems Use Higher-Order Thinking Skills

79. **ERROR ANALYSIS** Gwen and Morgan are solving $-12x^2 + 5x + 2 = 0$. Is either of them correct? Explain your reasoning.

Gwen

$$-12x^2 + 5x + 2 = 0$$
$$-12x^2 + 8x - 3x + 2 = 0$$
$$4x(-3x + 2) - (3x + 2) = 0$$
$$(4x - 1)(3x + 2) = 0$$
$$x = \frac{1}{4} \text{ or } -\frac{2}{3}$$

Morgan

$$-12x^2 + 5x + 2 = 0$$
$$-12x^2 + 8x - 3x + 2 = 0$$
$$4x(-3x + 2) + (-3x + 2) = 0$$
$$(4x + 1)(-3x + 2) = 0$$
$$x = -\frac{1}{4} \text{ or } \frac{2}{3}$$

80. **CHALLENGE** Solve $3x^6 - 39x^4 + 108x^2 = 0$ by factoring.

81. **CHALLENGE** The rule for factoring a difference of cubes is shown below. Use this rule to factor $40x^5 - 135x^2y^3$.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

82. **OPEN ENDED** Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

83. **CHALLENGE** For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the x-intercepts p and q.

84. **WRITE A QUESTION** A classmate is using the guess-and-check strategy to factor trinomials of the form $x^2 + bx + c$. Write a question to help him think of a way to use that strategy for $ax^2 + bx + c$.

85. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

In a quadratic equation in standard form where a, b, and c are integers, if b is odd, then the quadratic cannot be a perfect square trinomial.

86. **WRITING IN MATH** Explain how to factor a trinomial in standard form with $a > 1$.

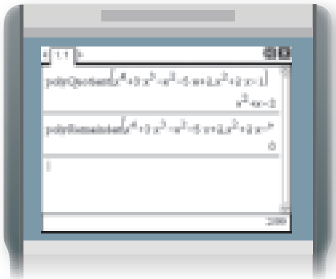
LAB 4

Graphing Technology Lab
Dividing Polynomials

Long division and synthetic division are two alternatives for dividing polynomials with linear divisors. You can use a graphing calculator with a computer algebra system (CAS) to divide polynomials with any divisor.

Activity 1 Divide Polynomials Without Remainders

Use CAS to find $(x^4 + 3x^3 - x^2 - 5x + 2) \div (x^2 + 2x - 1)$.



Step 1 Add a new **Calculator** page on the TI-Nspire.

Step 2 From the menu, select **Algebra**, then **Polynomial Tools** and **Quotient of Polynomial**.

Step 3 Type the dividend, a comma, and the divisor.

The CAS indicates that $(x^4 + 3x^3 - x^2 - 5x + 2) \div (x^2 + 2x - 1)$ is $x^2 + x - 2$.

Step 4 To verify that there is no remainder, select **Remainder of a Polynomial** from the **Algebra, Polynomial Tools** menu then type the dividend, a comma, and the divisor.

In Activity 1, there was no remainder. But in many cases, there will be a remainder.

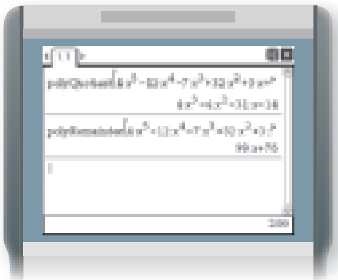
Activity 2 Divide Polynomials With Remainders

Use CAS to find $(4x^5 - 12x^4 - 7x^3 + 32x^2 + 3x + 20) \div (x^2 - 2x + 4)$.

Step 1 Add a new **Calculator** page on the TI-Nspire.

Step 2 From the menu, select **Algebra, Polynomial Tools,** and **Quotient of Polynomial**.

Step 3 Type the dividend, a comma, and the divisor.



The CAS indicates that $(4x^5 - 12x^4 - 7x^3 + 32x^2 + 3x + 20) \div (x^2 - 2x + 4)$ is $4x^3 - 4x^2 - 31x + 14$.

We need to determine whether there is a remainder.

Step 4 Use the **Remainder of a Polynomial** option from the **Algebra, Polynomial Tools** menu to determine the remainder. Then type the dividend, a comma, and the divisor.

The remainder is $99x + 76$.
Therefore, $(4x^5 - 12x^4 - 7x^3 + 32x^2 + 3x + 20) \div (x^2 - 2x + 4)$ is $4x^3 - 4x^2 - 31x - 14 + \dfrac{99x + 76}{x^2 - 2x + 4}$.

(continued on the next page)

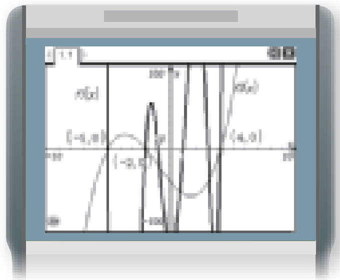
Dividing Polynomials *Continued*

You can also use a graphing calculator to determine roots of a polynomial so you can divide with synthetic division.

Activity 3 Divide with Synthetic Division

Use synthetic division to find $(x^6 - 28x^4 + 14x^3 + 147x^2 - 14x - 120) \div (x^3 + 3x^2 - 18x - 40)$.

Step 1 Graph the dividend as $f1(x)$ and the divisor as $f2(x)$ on the same calculator page. Use the **intersection points** tool from **Points and Lines** menu to find where the graphs have the same x-intercepts.

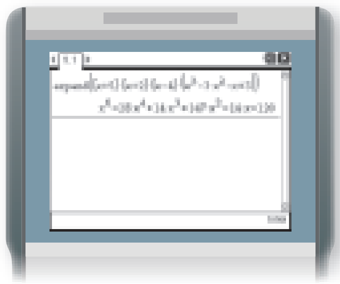


[−10, 10] scl: 1 by [−100, 100] scl: 10

Step 2 Use the roots from Step 1 as the divisors for synthetic division.

−5	1	0	−28	14	147	−14	−120
		−5	25	15	−145	−10	120
	1	−5	−3	29	2	−24	0
−2	1	−5	−3	29	2	−24	
		−2	14	−22	−14	24	
	1	−7	11	7	−12	0	
4	1	−7	11	7	−12		
		4	−12	−4	12		
	1	−3	−1	3	0		

Step 3 Use the **Expand** function to verify that $x^3 - 3x^2 - x + 3$ is the quotient when −5, −2, and 4 are roots.



Thus, $(x^6 - 28x^4 + 14x^3 + 147x^2 - 14x - 120) \div (x^3 + 3x^2 - 18x - 40)$ is $x^3 - 3x^2 - x + 3$.

Exercises

Find each quotient.

1. $(2x^4 + x^3 - 8x^2 + 17x - 12) \div (x^2 + 2x - 3)$

2. $(x^4 + 7x^3 + 8x^2 + x - 12) \div (x^2 + 3x - 4)$

3. $(9x^5 - 9x^3 - 5x^2 + 5) \div (9x^3 - 5)$

4. $(x^5 - 8x^4 + 10x^3 + 14x^2 + 61x - 30) \div (x^2 - 5x + 3)$

5. $(2x^6 + 2x^5 - 4x^4 - 18x^3 - 16x^2 + 8x + 16) \div (2x^3 + 2x^2 - 4x - 2)$

6. $(6x^6 - 2x^5 - 14x^4 + 10x^3 - 4x^2 - 28x - 5) \div (3x^3 - x^2 - 7x - 1)$

7. Use synthetic division to find $(x^6 - 7x^5 - 21x^4 + 175x^3 + 56x^2 - 924x + 720) \div (x^3 - 5x^2 - 12x + 36)$.

LAB 5

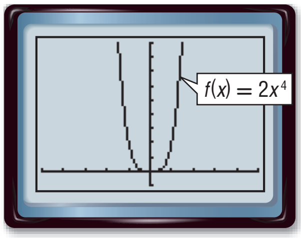
Graphing Technology Lab
Power Functions

A **power function** is any function of the form $f(x) = ax^n$, where a and n are nonzero constant real numbers. A power function in which n is a positive integer is called a **monomial function**.

Activity 1 $y = ax^4$

Graph $f(x) = 2x^4$. Analyze the graph.

Step 1 Graph the function.



[−5, 5] scl: 1 by [−1, 9] scl: 1

Step 2 Analyze the graph.

The graph resembles the graph of $g(x) = x^2$, but it flattens out more at its vertex.

Analyze the Results

1. What assumptions can you make about the graph of $f(x) = 2x^4$ as x becomes more positive or more negative? Use the TABLE feature to evaluate the function for values of x to confirm this assumption, if necessary.

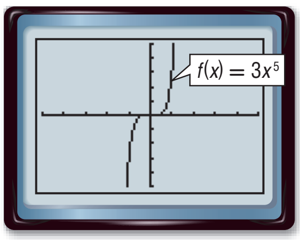
2. State the domain and range of $f(x) = 2x^4$. Compare these values with the domain and range of $g(x) = x^2$.

3. What other characteristics does the graph of $f(x) = 2x^4$ share with the graph of $g(x) = x^2$?

Activity 2 $y = ax^5$

Graph $f(x) = 3x^5$. Analyze the graph.

Step 1 Graph the function.



[−5, 5] scl: 1 by [−5, 5] scl: 1

Step 2 Analyze the graph.

The graph resembles the graph of $f(x) = x^3$, but it flattens out more as the graph approaches the origin.

(continued on the next page)

Graphing Technology Lab
Power Functions *Continued*

Analyze the Results

4. What assumptions can you make about the graph of $f(x) = 3x^5$ as x becomes more positive or more negative? Use the **TABLE** feature to evaluate the function for values of x to confirm this assumption, if necessary.

5. State the domain and range of $f(x) = 3x^5$. Compare these values with the domain and range of $g(x) = x^3$.

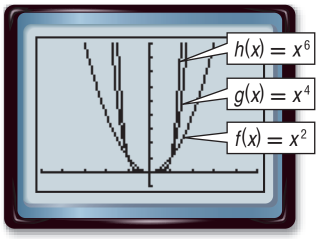
6. What other characteristics does the graph of $f(x) = 3x^5$ share with the graph of $g(x) = x^3$?

7. Compare the characteristics of the graph $h(x) = -3x^5$ with the graph of $f(x) = 3x^5$. What conclusions can you make about $f(x) = ax^5$ when a is positive and when a is negative?

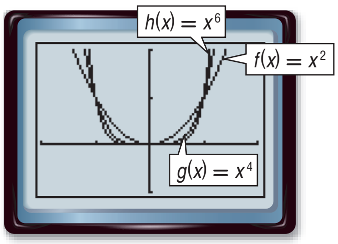
Activity 3 $y = x^n$, where n is even

Graph $f(x) = x^2$, $g(x) = x^4$, and $h(x) = x^6$ on the same screen. Analyze the graphs.

Step 1 Graph the function.



[−5, 5] scl: 1 by [−1, 9] scl: 1



[−2, 2] scl: 1 by [−1, 2] scl: 1

Step 2 The graphs are all U-shaped, but the widths are different. The graphs also differ around the origin. If you zoom in around the origin, you can see this more clearly. You can do this by using the **ZOOM** feature or by adjusting the window manually.

Analyze the Results

8. What assumptions can you make about the graphs of $a(x) = x^8$, $b(x) = x^{10}$, and so on?

9. Identify the common characteristics of the graphs of power functions in which the power is an even number.

10. Graph $f(x) = x^3$, $g(x) = x^5$, and $h(x) = x^7$ on the same graph.

11. What assumptions can you make about the graphs of $a(x) = x^9$, $b(x) = x^{11}$, and so on?

12. Identify the common characteristics of the graphs of power functions in which the power is an odd number.

13. Graph $f(x) = x^4$ and $g(x) = -x^4$ on the same graph.

14. Graph $f(x) = x^5$ and $g(x) = -x^5$ on the same graph.

15. What assumptions can you make about the effects of a negative value of a in $f(x) = ax^n$ when n is even? when n is odd?

LAB 6

Graphing Technology Lab
Polynomial Identities

An **identity** is an equation that is satisfied by any numbers that replace the variables. Thus, a **polynomial identity** is a polynomial equation that is true for any values that are substituted for the variables.

You can use a table or a spreadsheet on your graphing calculator to determine whether a polynomial equation may be an identity.

Activity 1 Use a Table

Determine whether $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ may be an identity.

Step 1 Add a new **List/Spreadsheet** page on the TI-Nspire. Label column A x and column B y. Type any values in columns A and B.

Step 2 Move the cursor to the formula row in column C and type $= x^3 - y^3$. In column D, type $= (x - y)(x^2 + xy + y^2)$.

No matter what values are entered for x and y in column A and B, the values columns C and D are the same. Thus, the equation may be an identity.

If you want to prove that an equation is an identity, you need to show that it is true for all values of the variables.

KeyConcept Verifying Identities by Transforming One Side

Step 1 Simplify one side of an equation until the two sides of the equation are the same. It is often easier to work with the more complicated side of the equation.

Step 2 Transform that expression into the form of the simpler side.

Activity 2 Transform One Side

Prove that $(x + y)^2 = x^2 + 2xy + y^2$ is an identity.

$(x + y)^2 \stackrel{?}{=} x^2 + 2xy + y^2$	Original equation
$(x + y)(x + y) \stackrel{?}{=} x^2 + 2xy + y^2$	Write $(x + y)^2$ as two factors
$x^2 + xy + xy + y^2 \stackrel{?}{=} x^2 + 2xy + y^2$	FOIL Method
$x^2 + 2xy + y^2 = x^2 + 2xy + y^2$ ✓	Simplify.

Thus, the identity $(x + y)^2 = x^2 + 2xy + y^2$ is verified.

You can also use a TI-Nspire with a computer algebra system (CAS) to prove an identity.

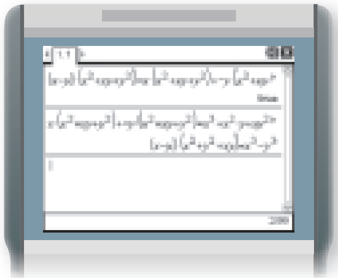
Activity 3 Use CAS

Prove that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ is an identity.

Step 1 Add a new **Calculator** page on the TI-Nspire CAS. Simplify the right side of the equation one step at a time.

Step 2 Enter the right side of the equation and then distribute.

Step 3 Multiply next. The CAS system will do the final simplification step.



The final step shown on the CAS screen is the results in $x^3 - y^3$. Thus, the identity has been proved.

You can also prove identities by transforming each side of the equation.

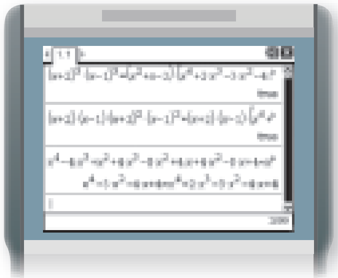
Activity 4 Use CAS to Transform Each Side

Prove that $(x + 2)^3(x - 1)^3 = (x^2 + x - 2)(x^4 + 2x^3 - 3x^2 - 4x + 4)$ is an identity.

Add a new **Calculator** page on the TI-Nspire. Simplify the left and the right sides of the equation simultaneously.

The CAS will indicate if the changes are true, otherwise it will simplify for you.

The CAS system will do the final simplification step. The identity $(x + 2)^3 (x - 1)^3 = (x^2 + x - 2)(x^4 + 2x^3 - 3x^2 - 4x + 4)$ has been proved.



Analyze

1. Use CAS to prove $a^2 - b^2 = (a + b)(a - b)$.

Exercises

Use CAS to prove each identity.

2. $x^3 + y^3 = (x + y)(x^2 - xy + y^3)$
3. $p^4 - q^4 = (p - q)(p + q)(p^2 + q^2)$
4. $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$
5. $g^6 + h^6 = (g^2 + h^2)(g^4 - g^2h^2 + h^4)$
6. $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$
7. $u^6 - w^6 = (u + w)(u - w)(u^2 + vw + w^2)(u^2 - vw + w^2)$
8. $(x + 1)^2(x - 4)^3 = (x^2 - 3x - 4)(x^3 - 7x^2 + 8x + 16)$

LAB 7

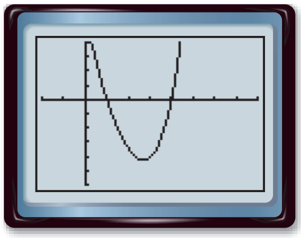
Graphing Technology Lab
Analyzing Polynomial Functions

You can use graphing technology to help you identify zeros, maximum and minimum points, multiplicity of zeros, *y*-intercepts, and symmetry of polynomial functions.

Activity Identify Polynomial Characteristics

Graph each function. Identify the zeros, maximum and minimum points, multiplicity of zeros, *y*-intercepts, and symmetry.

a. $g(x) = 3x^4 - 15x^3 + 87x^2 - 375x + 300$



[−2, 8] scl: 1 by [−300, 200] scl: 50

Step 1 Graph the equation.

Step 2 Use 2nd [CALC] **zero** to find the zeros at $x = 1$ and $x = 4$.

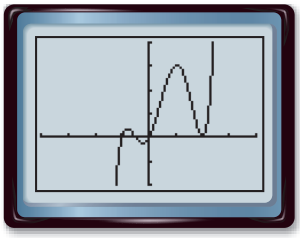
Step 3 Use 2nd [CALC] **minimum** to find the relative minimum at (2.68, −214.11). There is no relative maximum point.

Step 4 $g(x)$ has degree 4 and can have at most 4 zeros. Two were found through graphing. The other two roots are either multiple roots or imaginary roots.

Step 5 Use 2nd [CALC] **value** 0 to find the *y*-intercept, 300.

Step 6 The line of symmetry passes through the vertex. Its equation is $x = 2.68$.

b. $f(x) = 2x^5 - 5x^4 - 3x^3 + 8x^2 + 4x$



[−4, 4] scl: 1 by [−4, 8] scl: 2

Step 1 Graph the equation.

Step 2 Locate the zeros at $x = -1$, $x = 0$, $x = \frac{1}{2}$ and $x = 2$.

Step 3 Find the relative maxima at (−0.81, 0.75) and (1.04, 6.02) and the relative minima at (−0.24, −0.48) and (2, 0).

Step 4 $f(x)$ has degree 5 and can have at most 5 zeros. Four roots were found through graphing. The other root is either a multiple root or an imaginary root. In this case, there is a double root at $x = 2$.

Step 5 The *y*-intercept is 0 because the graph goes through the origin.

Step 6 There is no symmetry.

Exercises

Graph each function. Identify the zeros, maximum and minimum points, multiplicity of zeros, *y*-intercepts, and symmetry.

1. $f(x) = x^3 - 5x^2 + 6x$

2. $g(x) = x^4 - 3x^2 - 4$

3. $k(x) = -x^4 - x^3 + 2x^2$

4. $f(x) = -2x^3 - 4x^2 + 16x$

5. $g(x) = 3x^5 - 18x^4 + 27x^3$

6. $k(x) = x^4 - 8x^2 + 15$

7. $f(x) = -x^3 + 2x^2 + 8x$

8. $g(x) = x^5 + 3x^4 - 10x^2$

LESSON 8 Designing a Study

Then

- You identified various sampling techniques.

Now

1. Classify study types.
2. Design statistical studies.

Why?

- According to a recent study, 88% of teen cell phone users in the U.S. send text messages, and one in three teens sends more than 100 texts per day.

NewVocabulary
parameter
statistic
bias
random sample
survey
experiment
observational study

1 Classifying Studies

In a statistical study, data are collected and used to answer questions about a population characteristic or **parameter**. Due to time and money constraints, it may be impractical or impossible to collect data from each member of a population. Therefore, in many studies, a sample of the population is taken, and a measure called a **statistic** is calculated using the data. The sample statistic, such as the sample mean or sample standard deviation, is then used to make inferences about the population parameter.

The steps in a typical statistical study are shown below.

Identify the objective of the study. → Choose a sample, and collect data. → Organize the data, and calculate sample statistics. → Make inferences, and draw conclusions about the population.

To obtain good information and draw accurate conclusions about a population, it is important to select an *unbiased* sample. A **bias** is an error that results in a misrepresentation of members of a population. A poorly chosen sample can cause biased results. To reduce the possibility of selecting a biased sample, a **random sample** can be taken, in which members of the population are selected entirely by chance.

You will review other sampling methods in Exercise 32.

The following study types can be used to collect sample information.

KeyConcept Study Types

Definition	Example
In a **survey**, data are collected from responses given by members of a population regarding their characteristics, behaviors, or opinions.	To determine whether the student body likes the new cafeteria menu, the student council asks a random sample of students for their opinion.
In an **experiment**, the sample is divided into two groups: • an *experimental group* that undergoes a change, and • a *control group* that does not undergo the change. The effect on the experimental group is then compared to the control group.	A restaurant is considering creating meals with chicken instead of beef. They randomly give half of a group of participants meals with chicken and the other half meals with beef. Then they ask how they like the meals.
In an **observational study**, members of a sample are measured or observed without being affected by the study.	Researchers at an electronics company observe a group of teenagers using different laptops and note their reactions.

Radius Images/Jupiterimages

Example 1 Classify Study Types

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

a. MUSIC A record label wants to test three designs for an album cover. They randomly select 50 teenagers from local high schools to view the covers while they watch and record their reactions.

This is an observational study, because the company is going to observe the teens without them being affected by the study. The sample is the 50 teenagers selected, and the population is all potential purchasers of this album.

StudyTip

Census
A *census* is a survey in which each member of a population is questioned. Therefore, when a census is conducted, there is no sample.

b. RECYCLING The city council wants to start a recycling program. They send out a questionnaire to 200 random citizens asking what items they would recycle.

This is a survey, because the data are collected from participants' responses in the questionnaire. The sample is the 200 people who received the questionnaire, and the population is all of the citizens of the city.

GuidedPractice

1A. RESEARCH Scientists study the behavior of one group of dogs given a new heartworm treatment and another group of dogs given a false treatment or *placebo.*

1B. YEARBOOKS The yearbook committee conducts a study to determine whether students would prefer to have a print yearbook or both print and digital yearbooks.

To determine when to use a survey, experiment, or observational study, think about how the data will be obtained and whether or not the participants will be affected by the study.

Example 2 Choose a Study Type

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

a. MEDICINE A pharmaceutical company wants to test whether a new medicine is effective.

The treatment will need to be tested on a sample group, which means that the members of the sample will be affected by the study. Therefore, this situation calls for an experiment.

b. ELECTIONS A news organization wants to randomly call citizens to gauge opinions on a presidential election.

This situation calls for a survey because members of the sample population are asked for their opinion.

GuidedPractice

2A. RESEARCH A research company wants to study smokers and nonsmokers to determine whether 10 years of smoking affects lung capacity.

2B. PETS A national pet chain wants to know whether customers would pay a small annual fee to participate in a rewards program. They randomly select 200 customers and send them questionnaires.

2 Designing Studies

The questions chosen for a survey or procedures used in an experiment can also introduce bias, and thus, affect the results of the study.

A survey question that is poorly written may result in a response that does not accurately reflect the opinion of the participant. Therefore, it is important to write questions that are clear and precise. Avoid survey questions that:

- are confusing or wordy
- cause a strong reaction
- encourage a certain response
- address more than one issue

Questions can also introduce bias if there is not enough information given for the participant to give an accurate response.

Example 3 Identify Bias in Survey Questions

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

a. Don't you agree that the cafeteria should serve healthier food?

This question is biased because it encourages a certain response. The phrase "don't you agree" encourages you to agree that the cafeteria should serve healthier food.

b. How often do you exercise?

This question is unbiased because it is clearly stated and does not encourage a certain response.

GuidedPractice

3A. How many glasses of water do you drink a day?

3B. Do you prefer watching exciting action movies or boring documentaries?

When designing a survey, clearly state the objective, identify the population, and carefully choose unbiased survey questions.

Real-World Example 4 Design a Survey

TECHNOLOGY Jim is writing an article for his school newspaper about online courses. He wants to conduct a survey to determine how many students at his school would be interested in taking an online course from home. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Step 1 State the objective of the survey.

The objective of the survey is to determine students' interest in taking an online course from home.

Step 2 Identify the population.

The population is the student body.

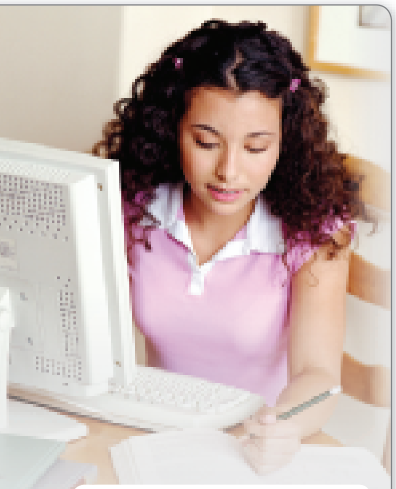
Step 3 Write unbiased survey questions.

Possible survey questions:

- "Do you have Internet access at home?"
- "If offered, would you take an online course?"

GuidedPractice

4. TECHNOLOGY In a follow-up article, Jim decides to conduct a survey to determine how many teachers from his school with at least five years of experience would be interested in teaching an online course. State the objective of the survey, suggest a population, and write two unbiased survey questions.



Real-WorldLink

Online Courses In 2009, about 1.2 million students took at least one online course.

Source: International Association for K-12 Online Learning

Ephraim Ben-Shimon/Corbis

To avoid introducing bias in experiments, the experimental and control groups should be randomly selected and the experiment should be designed so that everything about the two groups is alike (except for the treatment or procedure).

StudyTip

Bias in Experiments
An experiment is biased when the participants know which group they are in.

Example 5 Identify Flaws in Experiments

Identify any flaws in the design of the experiment, and describe how they could be corrected.

Experiment: **An electronics company wants to test whether using a new graphing calculator increases students' test scores. A random sample is taken. Calculus students in the experimental group are given the new calculator to use, and Algebra 2 students in the control group are asked to use their own calculator.**

Results: **When given the same test, the experimental group scored higher than the control group. The company concludes that the use of this calculator increases test scores.**

Calculus students are more likely to score higher when given the same test as Algebra 2 students. Therefore, the flaw is that the experimental group consists of Calculus students and the control group consists of Algebra 2 students. This flaw could be corrected by selecting a random sample of all Calculus or all Algebra 2 students.

GuidedPractice

5. Experiment: A research firm tests the effectiveness of a de-icer on car locks. They use a random sample of drivers in California and Minnesota for the control and experimental groups.

Results: They concluded that the de-icer is effective.

When designing an experiment, clearly state the objective, identify the population, determine the experimental and control groups, and define the procedure.

Real-World Example 6 Design an Experiment

PLANTS **A research company wants to test the claim of the advertisement shown at the right. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.**

Step 1 **State the objective, and identify the population.**

The objective of the experiment is to determine whether tomato plants given the plant food grow taller in three weeks than tomato plants not given the food. The population is all tomato plants.

Step 2 **Determine the experimental and control groups.**

The experimental group is the tomato plants given the food, and the control group is the tomato plants not given the food.

Step 3 **Describe a sample procedure.**

Measure the heights of the plants in each group, and give the experimental group the plant food. Then, wait three weeks, measure the heights of the plants again, and compare the heights for each group to see if the claim was valid.

GuidedPractice

6. **SPORTS** A company wants to determine whether wearing a new tennis shoe improves jogging time. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Check Your Understanding

Example 1 **Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.**

1. **SCHOOL** A group of high school students is randomly selected and asked to complete the form shown.

Do you agree with the new lunch rules?

- ☐ agree
- ☐ disagree
- ☐ don't care

2. **DESIGN** An advertising company wants to test a new logo design. They randomly select 20 participants and watch them discuss the logo.

Example 2 **Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.**

3. **LITERACY** A literacy group wants to determine whether high school students that participated in a recent national reading program had higher standardized test scores than high school students that did not participate in the program.

4. **RETAIL** The research department of a retail company plans to conduct a study to determine whether a dye used on a new T-shirt will begin fading before 50 washes.

Example 3 **Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.**

5. Which student council candidate's platform do you support?

6. How long have you lived at your current address?

Example 4

7. **HYBRIDS** A car manufacturer wants to determine what the demand in the U.S. is for hybrid vehicles. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Example 5

8. Identify any flaws in the experiment design, and describe how they could be corrected.

Experiment: A research company wants to determine whether a new vitamin boosts energy levels and decides to test the vitamin at a college campus. A random sample is taken. The experimental group consists of students who are given the vitamin, and the control group consists of instructors who are given a placebo.

Results: When given a physical test, the experimental group outperformed the control group. The company concludes that the vitamin is effective.

Example 6

9. **SPORTS** A research company wants to conduct an experiment to test the claim of the protein shake shown. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Practice and Problem Solving

Example 1 **Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.**

10. **FOOD** A grocery store conducts an online study in which customers are randomly selected and asked to provide feedback on their shopping experience.

11. **GRADES** A research group randomly selects 80 college students, half of whom took a physics course in high school, and compares their grades in a college physics course.

12. **HEALTH** A research group randomly chooses 100 people to participate in a study to determine whether eating blueberries reduces the risk of heart disease for adults.

13. **TELEVISION** A television network mails a questionnaire to randomly selected people across the country to determine whether they prefer watching sitcoms or dramas.

Example 2 **Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.**

14. **FASHION** A fashion magazine plans to poll 100 people in the U.S. to determine whether they would be more likely to buy a subscription if given a free issue.

15. **TRAVEL** A travel agency randomly calls 250 U.S. citizens and asks them what their favorite vacation destination is.

16. **FOOD** Chee wants to examine the eating habits of 100 random students at lunch to determine how many students eat in the cafeteria.

17. **ENGINEERING** An engineer is planning to test 50 metal samples to determine whether a new titanium alloy has a higher strength than a different alloy.

Example 3 **Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.**

18. Do you think that the school needs a new gym and football field?

19. Which is your favorite football team, the Dallas Cowboys or the Pittsburgh Steelers?

20. Do you play any extracurricular sports?

21. Don't you agree that students should carpool to school?

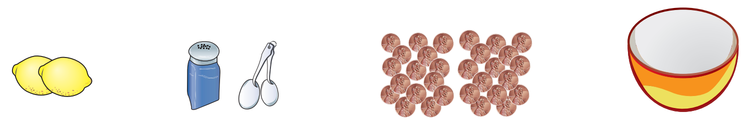
Example 4 **22.** **COLLEGE** A school district wants to conduct a survey to determine the number of juniors in the district who are planning to attend college after high school. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Example 5 **23.** Identify any flaws in the experiment design, and describe how they could be corrected.

Experiment: A supermarket chain wants to determine whether shoppers are more likely to buy sunscreen if it is located near the checkout line. The experimental group consists of a group of stores in the midwest in which the sunscreen was moved next to the checkout line, and the control group consists of stores in Arizona in which the sunscreen was not moved.

Results: The Arizona stores sold more sunscreen than the midwest stores. The company concluded that moving the sunscreen closer to the checkout line did not increase sales.

Example 6 **24.** **CHEMISTRY** In chemistry class, Pedro learned that copper objects become dull over time because the copper reacts with air to form a layer of copper oxide. He plans to use the supplies shown below to determine whether a mixture of lemon juice and salt will remove copper oxide from pennies.



2 lemons 1 teaspoon 30 dull pennies plastic bowl

a. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

b. What factors do you think should be considered when selecting pennies for the experiment? Explain your reasoning.

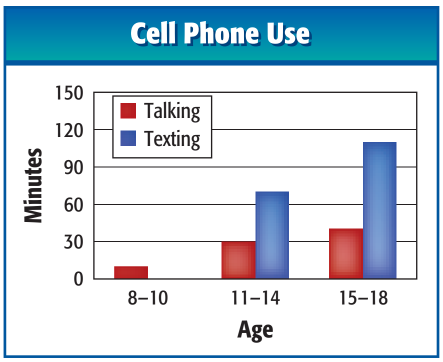
25. **REPORTS** The graph shown is from a report on the average number of minutes 8- to 18-year-olds in the U.S. spend on cell phones each day.

a. Describe the sample and suggest a population.

b. What type of sample statistic do you think was calculated for this report?

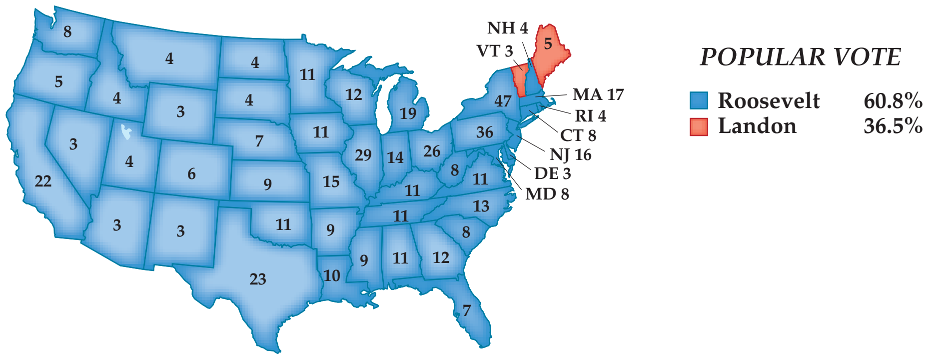
c. Describe the results of the study for each age group.

d. Who do you think would be interested in this type of report? Explain your reasoning.



26. **HISTORY** In 1936, the *Literary Digest* reported the results of a statistical study used to predict whether Alf Landon or Franklin D. Roosevelt would win the presidential election that year. The sample consisted of 2.4 million Americans, including subscribers to the magazine, registered automobile owners, and telephone users. The results concluded that Landon would win 57% of the popular vote. The actual election results are shown.

ELECTORAL VOTE



a. Describe the type of study performed, the sample taken, and the population.

b. How do the predicted and actual results compare?

c. Do you think that the survey was biased? Explain your reasoning.

27. **MULTIPLE REPRESENTATIONS** The results of two experiments concluded that Product A is 70% effective and Product B is 80% effective.

a. **NUMERICAL** To simulate the experiment for Product A, use the random number generator on a graphing calculator to generate 30 integers between 0 and 9. Let 0–6 represent an effective outcome and 7–9 represent an ineffective outcome.

b. **TABULAR** Copy and complete the frequency table shown using the results. from part **a.** Then use the data to calculate the probability that Product A was effective. Repeat to find the probability for Product B.

Product A	
Number	**Frequency**
0–6	
7–9	

c. **ANALYTICAL** Compare the probabilities that you found in part **b.** Do you think that the difference in the effectiveness of each product is significant enough to justify selecting one product over the other? Explain.

d. **LOGICAL** Suppose Product B costs twice as much as Product A. Do you think the probability of the product's effectiveness justifies the price difference to a consumer? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

REASONING **Determine whether each statement is *true* or *false*. If false, explain.**

28. To save time and money, population parameters are used to estimate sample statistics.

29. Observational studies and experiments can both be used to study cause-and-effect relationships.

30. **OPEN ENDED** Design an observational study. Identify the objective of the study, define the population and sample, collect and organize the data, and calculate a sample statistic.

31. **CHALLENGE** What factors should be considered when determining whether a given statistical study is reliable?

32. **WRITING IN MATH** Research each of the following sampling methods. Then describe each method and discuss whether using the method could result in bias.

a. convenience sample

b. self-selected sample

c. stratified sample

d. systematic sample

LAB 9

Graphing Calculator Lab
Simulations and Margin of Error

The Pew Research Center conducted a survey of a random sample of teens and concluded that 43% of all teens who take their cell phones to school text in class on a daily basis. How accurately did their random sample represent all teens?

As you learned in the previous lesson, a survey of a random sample is a valuable tool for generalizing information about a larger population. The program in the following activity makes use of a random number generator (randInt(a, b)) to simulate the results of a random sampling survey.

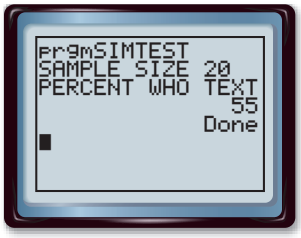
Activity 1 Random Sampling Simulation

Use the following program that simulates the texting survey to measure the percent of teens who text in class for random sample sizes of 20, 50, and 100 students.

Step 1 Input the following program into a graphing calculator.

```
Program:SIMTEST
:Input "SAMPLE SIZE ",S
:0→A
:0→B
:Lbl Z
:A + 1→A
:randInt(1, 100) →C
:If C ≤ 43
:B + 1→B
:If A<S
:Goto Z
:100B/S→P
:Disp "PERCENT WHO TEXT",P
:Stop
```

Step 2 Run 10 trials of the program for each sample size of 20, 50, and 100. Press ENTER to run the program again each time.



Step 3 Record the percent who text for each trial in the table below.

Sample Size	1	2	3	4	5	6	7	8	9	10
20										
50										
100										

Analyze the Results

1. Discard the percent that is farthest from the Pew survey result of 43% for each sample size. What is the range of the remaining nine percents for each sample size?
2. What is the farthest any of these remaining trials is from the 43% for each sample size?
3. The positive or negative of the result found in Exercise 2 is known as the **margin of error**. For your results, which sample size had the smallest margin of error?
4. What would you expect to happen to the margin of error if we used a sample size of 500?

(continued on the next page)

Simulations and Margin of Error *Continued*

Statisticians have found that for large populations, the margin of error for a random sample of size n can be approximated by the following formula.

KeyConcept Margin of Error Formula

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}}(100)$$

Since n is in the denominator, the margin of error will decrease as the size of the random sample increases. This expression can also be used to determine the size of a random sample necessary to achieve a desired level of reliability.

Activity 2 Margin of Error and Sample Size

You are a member of a research team.

a. You need to decide whether to conduct a survey with a margin of error of ±3% or ±2%. What sample size would be needed to achieve each goal?

Set each percent equal to the margin of error formula and solve for n.

$\pm 3\% = \pm\frac{1}{\sqrt{n}}(100)$	Margin of error formula	$\pm 2\% = \pm\frac{1}{\sqrt{n}}(100)$
$0.03\sqrt{n} = 1$	Multiply by $\frac{\sqrt{n}}{100}$.	$0.02\sqrt{n} = 1$
$\sqrt{n} = 33.333$	Divide.	$\sqrt{n} = 50$
$n = 1111.11$	Square each side.	$n = 2500$

A random sample of about 1100 would have a margin of error of about ±3%, while a random sample of 2500 would have a margin of error of ±2%.

b. Suppose the finance director would like to reduce the cost of the survey by using a random sample of 100. What would be the margin of error for this sample size?

Substitute 100 for n in the margin of error formula.

$$\text{margin of error} = \pm\frac{1}{\sqrt{n}}(100) \qquad \text{Margin of error formula}$$

$$= \pm\frac{1}{\sqrt{100}}(100) \text{ or } \pm 10\% \qquad n = 100$$

A random sample of 100 would have a margin of error of ±10%.

Exercises

5. What random sample size would produce a margin of error of ±1%?
6. What margin of error can be expected when using a sample size of 500?
7. What are some reasons that a research center might decide that a survey with a margin of error of ±3% would be more desirable than one with a margin of error of ±2%?
8. What is the range for the percent of students that text in class that the research center can expect from any random survey they conduct with a sample size of 2500?
9. If a survey with a random sample of 2500 students is conducted, is it possible that only 20% of the students could respond that they text during class? If so, how could this be possible?
10. If Step 2 from Activity 1 is repeated using a sample size of 2500 and the range for the percents is found to be 19%–23%, would this result cause you to question the model?

LESSON 10

Distributions of Data

Then

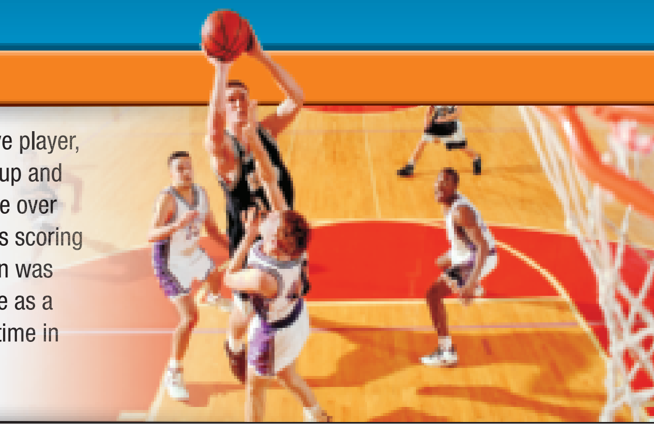
- You calculated measures of central tendency and variation.

Now

1. Use the shapes of distributions to select appropriate statistics.
2. Use the shapes of distributions to compare data.

Why?

- After four games as a reserve player, Craig joined the starting lineup and averaged 18 points per game over the *remaining* games. Craig's scoring average for the *entire* season was less than 18 points per game as a result of the lack of playing time in the first four games.



NewVocabulary

distribution
negatively skewed distribution
symmetric distribution
positively skewed distribution

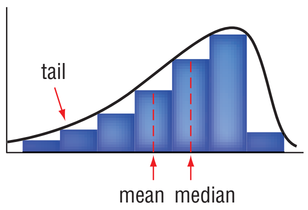
1 Analyzing Distributions

A **distribution** of data shows the observed or theoretical frequency of each possible data value. In Lesson 0-9, you described distributions of sample data using statistics. You used the mean or median to describe a distribution's center and standard deviation or quartiles to describe its spread. Analyzing the shape of a distribution can help you decide which measure of center or spread best describes a set of data.

The shape of the distribution for a set of data can be seen by drawing a curve over its histogram.

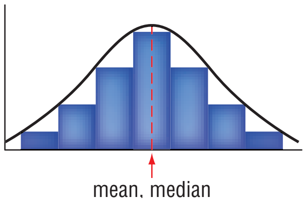
KeyConcept Symmetric and Skewed Distributions

Negatively Skewed Distribution



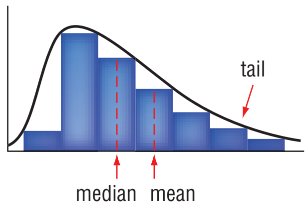
- The mean is less than the median.
- The majority of the data are on the right of the mean.

Symmetric Distribution



- The mean and median are approximately equal.
- The data are evenly distributed on both sides of the mean.

Positively Skewed Distribution



- The mean is greater than the median.
- The majority of the data are on the left of the mean.

When a distribution is symmetric, the mean and standard deviation accurately reflect the center and spread of the data. However, when a distribution is skewed, these statistics are not as reliable. Recall that outliers have a strong effect on the mean of a data set, while the median is less affected. Similarly, when a distribution is skewed, the mean lies away from the majority of the data toward the tail. The median is less affected, so it stays near the majority of the data.

When choosing appropriate statistics to represent a set of data, first determine the skewness of the distribution.

- If the distribution is relatively symmetric, the mean and standard deviation can be used.
- If the distribution is skewed or has outliers, use the five-number summary to describe the center and spread of the data.

Real-WorldLink

The first portable computer, the Osborne I, was available for sale in 1981 for $1795. The computer weighed 24 pounds and included a 5-inch display. Laptops can now be purchased for as little as $250 and can weigh as little as 3 pounds.

Source: Computer History Museum

Real-World Example 1 Describe a Distribution Using a Histogram

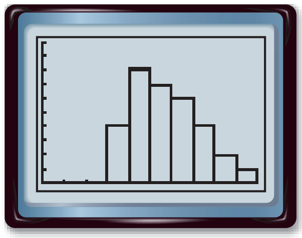
COMPUTERS **The prices for a random sample of personal computers are shown.**

Price (dollars)							
723	605	847	410	440	386	572	523
374	915	734	472	420	508	613	659
706	463	470	752	671	618	538	425
811	502	490	552	390	512	389	621

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

First, press STAT ENTER and enter each data value. Then, press 2nd [STAT PLOT] ENTER ENTER and choose the histogram icon. Finally, adjust the window to the dimensions shown.

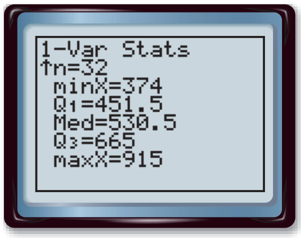
The majority of the computers cost between $400 and $700. Some of the computers are priced significantly higher, forming a tail for the distribution on the right. Therefore, the distribution is positively skewed.



[0, 1000] scl: 100 by [0, 10] scl: 1

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is skewed, so use the five-number summary to describe the center and spread. Press STAT ▶ ENTER ENTER and scroll down to view the five-number summary.



The prices for this sample range from $374 to $915. The median price is $530.50, and half of the computers are priced between $451.50 and $665.

GuidedPractice

1. **RAINFALL** The annual rainfall for a region over a 24-year period is shown below.

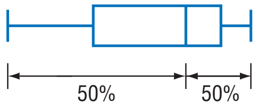
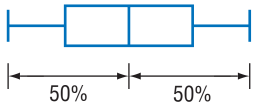
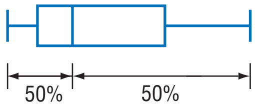
A. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

B. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Annual Rainfall (in.)					
27.2	30.2	35.8	26.1	39.3	20.6
28.9	23.0	32.7	26.8	22.7	25.4
29.6	36.8	33.4	28.4	21.9	20.8
24.7	30.6	27.7	31.4	34.9	37.1

A box-and-whisker plot can also be used to identify the shape of a distribution. The position of the line representing the median indicates the center of the data. The "whiskers" show the spread of the data. If one whisker is considerably longer than the other and the median is closer to the shorter whisker, then the distribution is skewed.

KeyConcept Box-and-Whisker Plots as Distributions

Negatively Skewed	Symmetric	Positively Skewed
		
The data to the left of the median are distributed over a wider range than the data to the right. The data have a tail to the left.	The data are equally distributed to the left and right of the median.	The data to the right of the median are distributed over a wider range than the data to the left. The data have a tail to the right.

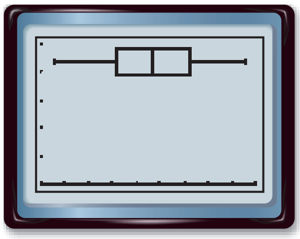
Example 2 Describe a Distribution Using a Box-and-Whisker Plot

HOMEWORK The students in Mr. Fejis' language arts class found the average number of minutes that they each spent on homework each night.

Minutes per Night					
62	53	46	66	38	45
52	46	73	39	42	56
64	54	48	59	70	60
49	54	48	57	70	33

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

Enter the data as **L1**. Press 2nd [STAT PLOT] ENTER ENTER and choose the box plot icon. Adjust the window to the dimensions shown.



[30, 75] scl: 5 by [0, 5] scl: 1

The lengths of the whiskers are approximately equal, and the median is in the middle of the data. This indicates that the data are equally distributed to the left and right of the median. Thus, the distribution is symmetric.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is symmetric, so use the mean and standard deviation to describe the center and spread. The average number of minutes that a student spent on homework each night was 53.5 with standard deviation of about 10.5.

WatchOut!

Standard Deviation Recall from Lesson 0-9 that the formulas for standard deviation for a population σ and for a sample s are slightly different. In Example 2, times for all of the students in Mr. Fejis' class are being analyzed, so use the population standard deviation.

GuidedPractice

2. CELL PHONE Janet's parents have given her a prepaid cell phone. The number of minutes she used each month for the last two years are shown in the table.

Minutes Used per Month			
582	608	670	620
667	598	671	613
537	511	674	627
638	661	642	641
668	673	680	695
658	653	670	688

A. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

B. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

2 Comparing Distributions

To compare two sets of data, first analyze the shape of each distribution. Use the mean and standard deviation to compare two symmetric distributions. Use the five-number summaries to compare two skewed distributions or a symmetric distribution and a skewed distribution.

Example 3 Compare Data Using Histograms

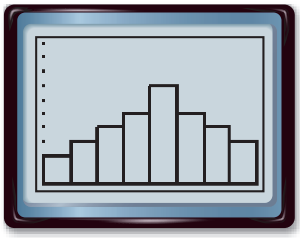
TEST SCORES **Test scores from Mrs. Morash's class are shown below.**

Chapter 3 Test Scores
81, 81, 92, 99, 61, 67, 86, 82, 76, 73, 62, 97, 97, 72, 72, 84, 77, 88, 92, 93, 76, 74, 66, 78, 76, 69, 84, 87, 83, 87, 92, 87, 82

Chapter 4 Test Scores
87, 73, 69, 83, 74, 86, 74, 69, 79, 84, 79, 74, 83, 74, 86, 69, 91, 73, 79, 83, 69, 79, 83, 74, 86, 79, 79, 78, 83, 79, 86, 79, 84

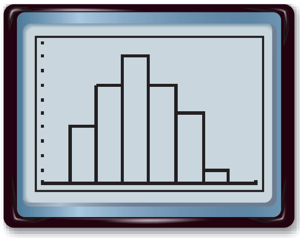
a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

Chapter 3 Test Scores



[60, 100] scl: 5 by [0, 10] scl: 1

Chapter 4 Test Scores



[60, 100] scl: 5 by [0, 10] scl: 1

Both distributions are symmetric.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

The distributions are symmetric, so use the means and standard deviations.

Chapter 3 Test Scores

Chapter 4 Test Scores

The Chapter 4 test scores, while lower in average, have a much smaller standard deviation, indicating that the scores are more closely grouped about the mean. Therefore, the mean for the Chapter 4 test scores is a better representation of the data than the mean for the Chapter 3 test scores.

> **StudyTip**
>
> **Multiple Data Sets** To compare two sets of data, enter one set as **L1** and the other as **L2**. In order to calculate statistics for a set of data in **L2**, press STAT ▶ ENTER 2nd [L2] ENTER.

GuidedPractice

3. TYPING The typing speeds of the students in two classes are shown below.

A. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

B. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

3rd Period (wpm)
23, 38, 27, 28, 40, 45, 32, 33, 34, 27, 40, 22, 26, 34, 29, 31, 35, 33, 37, 38, 28, 29, 39, 42

6th Period (wpm)
38, 26, 43, 46, 23, 24, 27, 36, 22, 21, 26, 27, 31, 32, 27, 25, 23, 22, 28, 29, 28, 33, 23, 24

Box-and-whisker plots can be displayed alongside one another, making them useful for side-by-side comparisons of data.

Example 4 Compare Data Using Box-and-Whisker Plots

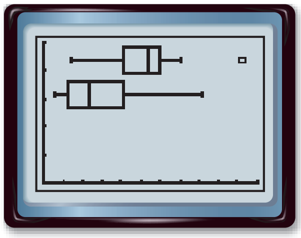
POINTS The points scored per game by a professional football team for the 2008 and 2009 football seasons are shown.

2008							
7	51	24	27	17	35	27	33
28	30	27	21	24	30	14	20

2009							
20	9	3	10	6	14	3	10
3	37	7	21	13	41	20	23

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

Enter the 2008 scores as **L1**. Graph these data as **Plot1** by pressing [2nd] **[STAT PLOT]** [ENTER] [ENTER] and choosing ⊏⊐··. Enter the 2009 scores as **L2**. Graph these data as **Plot2** by pressing [2nd] **[STAT PLOT]** [▼] [ENTER] [ENTER] and choosing ⊏⊐··. For **Xlist**, enter **L2**. Adjust the window to the dimensions shown.



[0, 55] scl: 5 by [0, 5] scl: 1

For the 2008 scores, the left whisker is longer than the right and the median is closer to the right whisker. The distribution is negatively skewed.

For the 2009 scores, the right whisker is longer than the left and the median is closer to the left whisker. The distribution is positively skewed.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

The distributions are skewed, so use the five-number summaries to compare the data.

The lower quartile for the 2008 season and the upper quartile for the 2009 season are both 20.5. This means that 75% of the scores from the 2008 season were greater than 20.5 and 75% of the scores from the 2009 season were less than 20.5.

The minimum of the 2008 season is approximately equal to the lower quartile for the 2009 season. This means that 25% of the scores from the 2009 season are lower than any score achieved in the 2008 season. Therefore, we can conclude that the team scored a significantly higher amount of points during the 2008 season than the 2009 season.

StudyTip

Outliers Recall from Lesson 0-9 that outliers are data that are more than 1.5 times the interquartile range beyond the upper or lower quartile. All outliers should be plotted, but the whiskers should be drawn to the least and greatest values that are not outliers.

GuidedPractice

4. GOLF Robert recorded his golf scores for his sophomore and junior seasons.

A. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

B. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Sophomore Season
42, 47, 43, 46, 50, 47, 52, 45, 53, 55, 48, 39, 40, 49, 47, 50

Junior Season
44, 38, 46, 48, 42, 41, 42, 46, 43, 40, 43, 43, 44, 45, 39, 44

Check Your Understanding

Example 1 **1. EXERCISE** The amount of time that James ran on a treadmill for the first 24 days of his workout is shown.

Time (minutes)											
23	10	18	24	13	27	19	7	25	30	15	22
10	28	23	16	29	26	26	22	12	23	16	27

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Example 2 **2. RESTAURANTS** The total number of times that 20 random people either ate at a restaurant or bought fast food in a month are shown.

Restaurants or Fast Food									
4	7	5	13	3	22	13	6	5	10
7	18	4	16	8	5	15	3	12	6

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Example 3 **3. SALES** The total fundraiser sales for the students in two classes at Cantonville High School are shown.

Mrs. Johnson's Class (dollars)					
6	14	17	12	38	15
11	12	23	6	14	28
16	13	27	34	25	32
21	24	21	17	16	

Mr. Edmunds' Class (dollars)					
29	38	21	28	24	33
14	19	28	15	30	6
31	23	33	12	38	28
18	34	26	34	24	37

a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Example 4 **4. RECYCLING** The weekly totals of recycled paper for the junior and senior classes are shown.

Junior Class (pounds)					
14	24	8	26	19	38
12	15	12	18	9	24
12	21	9	15	13	28

Senior Class (pounds)					
25	31	35	20	37	27
22	32	24	28	18	32
25	32	22	29	26	35

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Practice and Problem Solving

Examples 1–2 **For Exercises 5 and 6, complete each step.**

a. **Use a graphing calculator to create a histogram and a box-and-whisker plot. Then describe the shape of the distribution.**

b. **Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.**

5. **FANTASY** The weekly total points of Kevin's fantasy football team are shown.

Total Points							
165	140	88	158	101	137	112	127
53	151	120	156	142	179	162	79

6. **MOVIES** The students in one of Mr. Peterson's classes recorded the number of movies they saw over the past month.

Movies Seen											
14	11	17	9	6	11	7	8	12	13	10	9
5	11	7	13	9	12	10	9	15	11	13	15

Example 3 **For Exercises 7 and 8, complete each step.**

a. **Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.**

b. **Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.**

7. **SAT** A group of students took the SAT their sophomore year and again their junior year. Their scores are shown.

Sophomore Year Scores					
1327	1663	1708	1583	1406	1563
1637	1521	1282	1752	1628	1453
1368	1681	1506	1843	1472	1560

Junior Year Scores					
1728	1523	1857	1789	1668	1913
1834	1769	1655	1432	1885	1955
1569	1704	1833	2093	1608	1753

8. **INCOME** The total incomes for 18 households in two neighboring cities are shown.

Yorkshire (thousands of dollars)					
68	59	61	78	58	66
56	72	86	58	63	53
68	58	74	60	103	64

Applewood (thousands of dollars)					
52	55	60	61	55	65
65	60	45	37	41	71
50	61	65	66	87	55

Example 4

9. **TUITION** The annual tuitions for a sample of public colleges and a sample of private colleges are shown. Complete each step.

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Public Colleges (dollars)					
3773	3992	3004	4223	4821	3880
3163	4416	5063	4937	3321	4308
4006	3508	4498	3471	4679	3612

Private Colleges (dollars)					
10,766	13,322	12,995	15,377	16,792	9147
15,976	11,084	17,868	7909	12,824	10,377
14,304	10,055	12,930	16,920	10,004	11,806

10. **DANCE** The total amount of money that a random sample of seniors spent on prom is shown. Complete each step.

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Boys (dollars)					
253	288	304	283	348	276
322	368	247	404	450	341
291	260	394	302	297	272

Girls (dollars)					
682	533	602	504	635	541
489	703	453	521	472	368
562	426	382	668	352	587

11. **BASKETBALL** Refer to the beginning of the lesson. The points that Craig scored in the remaining games are shown.

a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.

b. Craig scored 0, 2, 1, and 0 points in the first four games. Use a graphing calculator to create a box-and-whisker plot that includes the new data. Then find the mean and median of the new data set.

c. What effect does adding the scores from the first four games have on the shape of the distribution and on how you should describe the center and spread?

Points Scored			
18	10	18	21
9	25	13	17
17	12	24	19
20	17	27	21

12. **SCORES** Allison's quiz scores are shown.

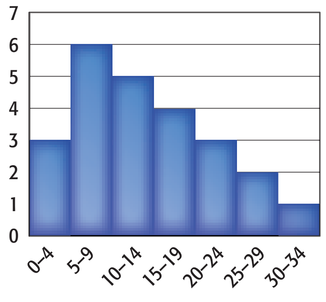
a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread.

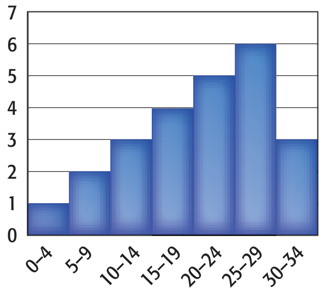
b. Allison's teacher allows students to drop their two lowest quiz scores. Use a graphing calculator to create a box-and-whisker plot that reflects this change. Then describe the center and spread of the new data set.

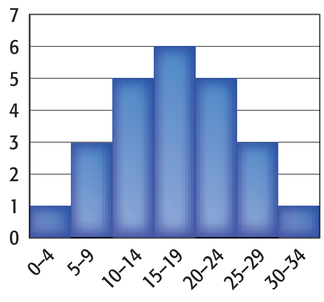
Math Quiz Scores					
83	76	86	82	84	57
86	62	90	96	76	89
76	88	86	86	92	94

H.O.T. Problems Use Higher-Order Thinking Skills

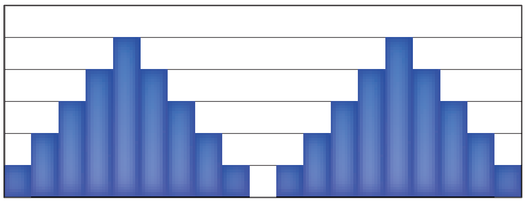
13. **CHALLENGE** Approximate the mean and median for each distribution of data.

a.


b.


c.


14. **REASONING** Distributions of data are not always symmetric or skewed. If a distribution has a gap in the middle, like the one shown, two separate clusters of data may result, forming a *bimodal distribution.* How can the center and spread of a bimodal distribution be described?



15. **OPEN ENDED** Find a real-world data set that appears to represent a symmetric distribution and one that does not. Describe each distribution. Create a visual representation of each set of data.

16. **WRITING IN MATH** Explain the difference between positively skewed, negatively skewed, and symmetric sets of data, and give an example of each.

LESSON 11 Probability Distributions

Then

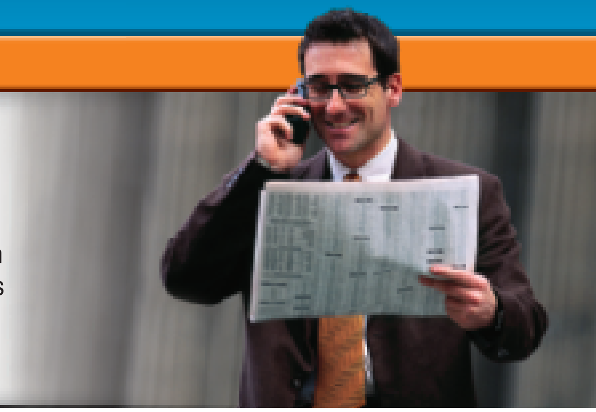
- You used statistics to describe symmetrical and skewed distributions of data.

Now

1. Construct a probability distribution.
2. Analyze a probability distribution and its summary statistics.

Why?

- Mutual funds are professionally managed investments that offer diversity to investors. An accurate analysis of the fund's current and expected performance can help an investor determine if the fund suits their needs.



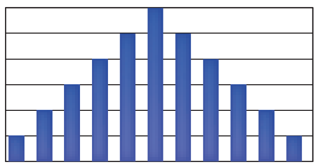
NewVocabulary

random variable
discrete random variable
continuous random variable
probability distribution
theoretical probability distribution
experimental probability distribution
Law of Large Numbers
expected value

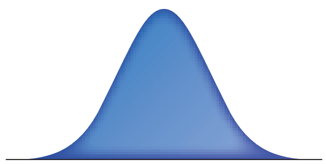
1 Construct a Probability Distribution

A sample space is the set of all possible outcomes in a distribution. Consider a distribution of values represented by the sum of the values on two dice and a distribution of the miles per gallon for a sample of cars.

Sum of Two Dice



Miles Per Gallon



The sum of the values on the dice can be any integer from 2 to 12. So, the sample space is [2, 3, …, 11, 12]. This distribution is *discrete* because the number of possible values in the sample space can be counted.

The distribution of miles per gallon is *continuous.* While the sample space includes any positive value less than a certain maximum (around 100), the data can take on an infinite number of values within this range.

The value of a **random variable** is the numerical outcome of a random event. A random variable can be discrete or continuous. **Discrete random variables** represent countable values. **Continuous random variables** can take on any value.

Example 1 Identify and Classify Random Variables

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

a. the number of songs found on a random selection of mp3 players

The random variable X is the number of songs on any mp3 player in the random selection of players. The number of songs is countable, so X is discrete.

b. the weights of football helmets sent by a manufacturer

The random variable X is the weight of any particular helmet. The weight of any particular helmet can be anywhere within a certain range, typically 6 to 8 pounds. Therefore, X is continuous.

GuidedPractice

1A. the exact distances of a sample of discus throws

1B. the ages of counselors at a summer camp

StudyTip

Discrete vs. Continuous Variables representing height, weight, and capacity will always be continuous variables because they can take on any positive value.

A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. Probability distributions can be represented by tables, equations, or graphs. In this lesson, we will focus on discrete probability distributions.

A probability distribution has the following properties.

KeyConcept Probability Distribution

- A probability distribution can be determined theoretically or experimentally.
- A probability distribution can be discrete or continuous.
- The probability of each value of X must be at least 0 and not greater than 1.
- The sum of all the probabilities for all of the possible values of X must equal 1. That is, $\Sigma P(X) = 1$.

ReviewVocabulary

Theoretical and Experimental Probability Theoretical probability is based on assumptions, and experimental probability is based on experiments. (Lesson 0-5)

A **theoretical probability distribution** is based on what is expected to happen. For example, the distribution for flipping a fair coin is $P(\text{heads}) = 0.5$, $P(\text{tails}) = 0.5$.

Example 2 Construct a Theoretical Probability Distribution

X represents the sum of the values on two dice.

a. Construct a relative-frequency table.

The theoretical probabilities associated with rolling two dice can be described using a relative-frequency table. When two dice are rolled, 36 total outcomes are possible. To determine the relative frequency, or theoretical probability, of each outcome, divide the frequency by 36.

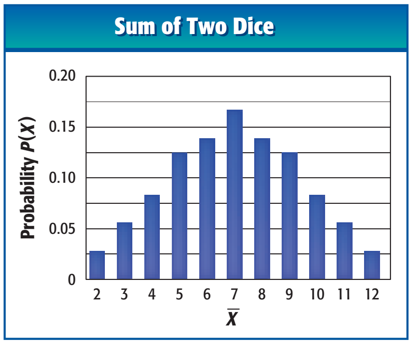
Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	2	3	4	5	6	5	4	3	2	1
Relative Frequency	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Sum: 36

b. Graph the theoretical probability distribution.

The graph shows the probability distribution for the sum of the values on two dice X. The bars are separated on the graph because the distribution is discrete (no other values of X are possible).

Each unique outcome of X is indicated on the horizontal axis, and the probability of each outcome occurring $P(X)$ is indicated on the vertical axis.

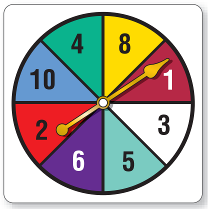


GuidedPractice

2. X represents the sum of the values of two spins of the wheel.

A. Construct a relative-frequency table.

B. Graph the theoretical probability distribution.



An **experimental probability distribution** is a distribution of probabilities estimated from experiments. Simulations can be used to construct an experimental probability distribution. When constructing this type of distribution, use the frequency of occurrences of each observed value to compute its probability.

Example 3 Construct an Experimental Probability Distribution

X represents the sum of the values found by rolling two dice.

a. Construct a relative-frequency table.

Roll two dice 100 times or use a random number generator to complete the simulation and create a simulation tally sheet.

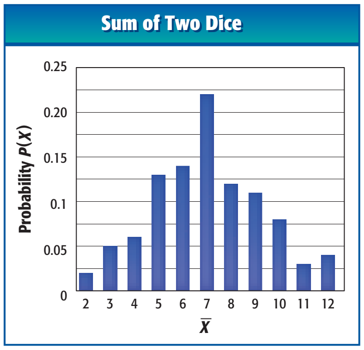
Sum	Tally	Frequency	Sum	Tally	Frequency
2	||	2	8	𝍸 𝍸 ||	12
3	𝍸	5	9	𝍸 𝍸 |	11
4	𝍸 |	6	10	𝍸 |||	8
5	𝍸 𝍸 |||	13	11	|||	3
6	𝍸 𝍸 ||||	14	12	||||	4
7	𝍸 𝍸 𝍸 𝍸 ||	22			

Calculate the experimental probability of each value by dividing its frequency by the total number of trials, 100.

Sum	2	3	4	5	6	7	8	9	10	11	12
Relative Frequency	0.02	0.05	0.06	0.13	0.14	0.22	0.12	0.11	0.08	0.03	0.04

b. Graph the experimental probability distribution.

The graph shows the discrete probability distribution for the sum of the values shown on two dice X.



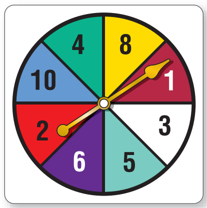
Review Vocabulary

Simulations and Random Number Generators For more practice on simulations and random number generators, see Extend 12-1.

Study Tip

Random Number Generators and Proportions When using a random number generator to simulate events with different probabilities, set up a proportion. For example, suppose there are 3 possible outcomes with probabilities of A: 0.25, B:0.35, and C:0.40. Random numbers 1–25 can represent A, 26–60 represent B, and 61–100 represent C.

Guided Practice

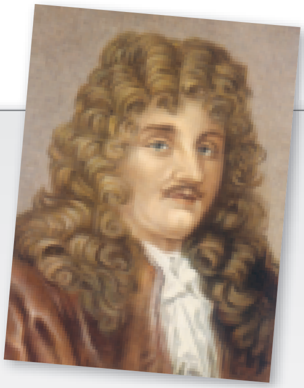


3. X represents the sum of the values of two spins of the wheel.

A. Construct a relative-frequency table for 100 trials.

B. Graph the experimental probability distribution.

Notice that this graph is different from the theoretical graph in Example 2. With small sample sizes, experimental distributions may vary greatly from their associated theoretical distributions. However, as the sample size increases, experimental probabilities will more closely resemble their associated theoretical probabilities. This is due to the **Law of Large Numbers**, which states that the variation in a data set decreases as the sample size increases.



Math HistoryLink

Christian Huygens
(1629–1695) This Dutchman was the first to discuss games of chance. "Although in a pure game of chance the results are uncertain, the chance that one player has to win or to lose depends on a determined value." This became known as the *expected value*.

WatchOut!

Expected Value The expected value is what you *expect* to happen in the long run, not necessarily what *will* happen.

2 Analyze a Probability Distribution

Probability distributions are often used to analyze financial data. The two most common statistics used to analyze a discrete probability distribution are the mean, or expected value, and the standard deviation. The **expected value** $E(X)$ of a discrete random variable of a probability distribution is the weighted average of the variable.

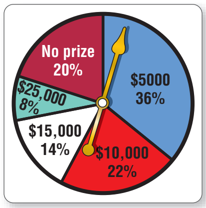
KeyConcept Expected Value of a Discrete Random Variable

Words The expected value of a discrete random variable is the weighted average of the values of the variable. It is calculated by finding the sum of the products of every possible value of X and its associated probability $P(X)$.

Symbols $E(X) = \Sigma\,[X \cdot P(X)]$

Real-World Example 4 Expected Value

GAME SHOWS A game-show contestant has won one spin of the wheel at the right. Find the expected value of his winnings.



Each prize value represents a value of X and each percent represents the corresponding probability $P(X)$. Find $E(X)$.

$$\begin{aligned} E(X) &= \Sigma[X \cdot P(X)] \\ &= 0(0.20) + 25{,}000(0.08) + 15{,}000(0.14) + 10{,}000(0.22) + 5000(0.36) \\ &= 0 + 2000 + 2100 + 2200 + 1800 \\ &= 8100 \end{aligned}$$

The expected value of the contestant's winnings is $8100.

GuidedPractice

4. **PRIZES** Curt won a ticket for a prize. The distribution of the values of the tickets and their relative frequencies are shown. Find the expected value of his winnings.

Value ($)	1	10	100	1000	5000	25,000
Frequency	5000	100	25	5	1	1

Sometimes the expected value does not provide enough information to fully analyze a probability distribution. For example, suppose two wheels had roughly the same expected value. Which one would you choose? Which one is *riskier*? The standard deviation can provide more insight into the expected value of a probability distribution.

The formula for calculating the standard deviation of a probability distribution is similar to the one used for a set of data.

KeyConcept Standard Deviation of a Probability Distribution

Words For each value of X, subtract the mean from X and square the difference. Then multiply by the probability of X. The sum of each of these products is the variance. The standard deviation is the square root of the variance.

Symbols
Variance: $\sigma^2 = \Sigma[[X - E(X)]^2 \cdot P(X)]$
Standard Deviation: $\sigma = \sqrt{\sigma^2}$

Real-World Career

Mutual Fund Manager Mutual fund managers buy and sell fund investments according to the investment objective of the fund. Investment management includes financial statement analysis, asset and stock selection, and monitoring of investments. Certification beyond a bachelor's degree is required.

Source: International Financial Services, London

Real-World Example 5 Standard Deviation of a Distribution

PT

DECISION MAKING **Jimmy is thinking about investing $10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below.**

Fund A	Fund B
50% chance of an $800 profit	30% chance of a $2400 profit
20% chance of a $1200 profit	10% chance of a $1900 profit
20% chance of a $600 profit	40% chance of a $200 loss
10% chance of a $100 loss	20% chance of a $400 loss

a. Find the expected value of each investment.

Fund A: $E(X) = 0.50(800) + 0.20(1200) + 0.20(600) + 0.10(-100)$ or 750

Fund B: $E(X) = 0.30(2400) + 0.10(1900) + 0.40(-200) + 0.20(-400)$ or 750

An investment of $10,000 in Fund A or Fund B will expect to yield $750.

b. Find each standard deviation.

Fund A:

Profit, X	$P(X)$	$[X - E(X)]^2$	$[X - E(X)]^2 \cdot P(X)$
800	0.50	$(800 - 750)^2 = 2500$	$2500 \cdot 0.50 = 1250$
1200	0.20	$(1200 - 750)^2 = 202{,}500$	$202{,}500 \cdot 0.20 = 40{,}500$
600	0.20	$(600 - 750)^2 = 22{,}500$	$22{,}500 \cdot 0.20 = 4500$
−100	0.10	$(-100 - 750)^2 = 722{,}500$	$722{,}500 \cdot 0.10 = 72{,}250$
			$\Sigma[[(X - E(X)]^2 \cdot P(X)] = 118{,}500$
			$\sqrt{118{,}500} \approx 344.2$

Fund B:

Profit, X	$P(X)$	$[X - E(X)]^2$	$[X - E(X)]^2 \cdot P(X)$
2400	0.30	$(2400 - 750)^2 = 2{,}722{,}500$	$2{,}722{,}500 \cdot 0.30 = 816{,}750$
1900	0.10	$(1900 - 750)^2 = 1{,}322{,}500$	$1{,}322{,}500 \cdot 0.10 = 132{,}250$
−200	0.40	$(-200 - 750)^2 = 902{,}500$	$902{,}500 \cdot 0.40 = 361{,}000$
−400	0.20	$(-400 - 750)^2 = 1{,}322{,}500$	$1{,}322{,}500 \cdot 0.20 = 264{,}500$
			$\Sigma[[(X - E(X)]^2 \cdot P(X)] = 1{,}574{,}500$
			$\sqrt{1{,}574{,}500} \approx 1254.8$

c. Which investment would you advise Jimmy to choose, and why?

Jimmy should choose Fund A. While the funds have identical expected values, the standard deviation of Fund B is almost four times the standard deviation for Fund A. This means that the expected value for Fund B will have about four times the variability than Fund A and will be riskier with a greater chance for gains and losses.

StudyTip

Return on Investment When investing $1000 in a product that has a 6% expected return, the investor can expect a 0.06(1000) or $60 profit.

GuidedPractice

5. DECISION MAKING Compare a $10,000 investment in the two funds. Which investment would you recommend, and why?

Fund C	Fund D
30% chance of a $1000 profit	40% chance of a $1000 profit
40% chance of a $500 profit	30% chance of a $600 profit
20% chance of a $100 loss	15% chance of a $100 profit
10% chance of a $300 loss	15% chance of a $200 loss

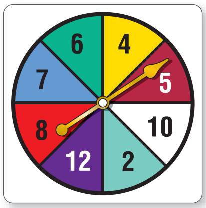
Check Your Understanding

Example 1 **Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.**

1. the number of pages linked to a Web page
2. the number of stations in a cable package
3. the amount of precipitation in a city per month
4. the number of cars passing through an intersection in a given time interval

Examples 2–5

5. X represents the sum of the values of two spins of the wheel.



   a. Construct a relative-frequency table showing the theoretical probabilities.
   b. Graph the theoretical probability distribution.
   c. Construct a relative-frequency table for 100 trials.
   d. Graph the experimental probability distribution.
   e. Find the expected value for the sum of two spins of the wheel.
   f. Find the standard deviation for the sum of two spins of the wheel.

Practice and Problem Solving

Example 1 **Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.**

6. the number of texts received per week
7. the number of diggs (or "likes") for a Web page
8. the height of a plant after a specific amount of time
9. the number of files infected by a computer virus

Examples 2–5

10. **GAME SHOWS** A contestant has won a prize on a game show. The frequency table at the right shows the number of winners for 3200 hypothetical players.

Prize, X	Winners
\$100	1120
\$250	800
\$500	480
\$1000	320
\$2500	256
\$5000	128
\$7500	64
\$10,000	32

   a. Construct a relative-frequency table showing the theoretical probability.
   b. Graph the theoretical probability distribution.
   c. Construct a relative-frequency table for 50 trials.
   d. Graph the experimental probability distribution.
   e. Find the expected value.
   f. Find the standard deviation.

11. **SNOW DAYS** The following probability distribution lists the probable number of snow days per school year at North High School. Use this information to determine the expected number of snow days per year.

Number of Snow Days Per Year									
Days	0	1	2	3	4	5	6	7	8
Probability	0.1	0.1	0.15	0.15	0.25	0.1	0.08	0.05	0.02

12. **CARDS** In a standard deck of 52 cards, there are 4 different suits.
   a. If jacks = 11, queens = 12, kings = 13, and aces = 1, what is the expected value of a card that is drawn from a standard deck?
   b. If you are dealt 7 cards, what is the expected number of spades?

13. **RAFFLES** The table shows the probability distribution for a raffle if 100 tickets are sold for $1 each. There is 1 prize for $20, 5 prizes for $10, and 10 prizes for $5.

Distribution of Prizes				
Prize	no prize	$20	$10	$5
Probability	0.84	0.01	0.05	0.10

a. Construct a relative frequency table.

b. Find the expected value.

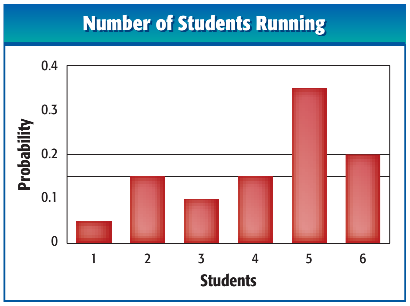
c. Interpret the results you found in part **b**. What can you conclude about the raffle?

14. **STUDENT GOVERNMENT** Based on previous data, the probability distribution of the number of students running for class president is shown.

a. Determine the expected number of students who will run. Interpret your results.

b. Construct a relative-frequency table for 50 trials.

c. Graph the experimental probability distribution.



15. **BASKETBALL** The distribution below lists the probability of the number of major upsets in the first round of a basketball tournament each year.

Number of Upsets Per Year									
Upsets	0	1	2	3	4	5	6	7	8
Probability	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

a. Determine the expected number of upsets. Interpret your results.

b. Find the standard deviation.

c. Construct a relative-frequency table for 50 trials.

d. Graph the experimental probability distribution.

16. **RAFFLES** The French Club sold 500 raffle tickets for $1 each. The first prize ticket will win $100, 2 second prize tickets will each win $10, and 5 third prize tickets each win $5.

a. What is the expected value of a single ticket?

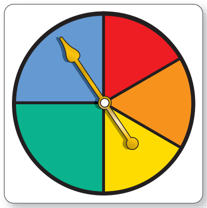
b. Calculate the standard deviation of the probability distribution.

c. **DECISION MAKING** The Glee Club is offering a raffle with a similar expected value and a standard deviation of 2.2. In which raffle should you participate? Explain your reasoning.

17. **DECISION MAKING** Carmen is thinking about investing $10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below. Compare the two investments using the expected value and standard deviation. Which investment would you advise Carmen to choose, and why?

Fund A	Fund B
30% chance of a $1900 profit	40% chance of a $1600 profit
30% chance of a $600 profit	10% chance of a $900 profit
15% chance of a $200 loss	10% chance of a $300 loss
25% chance of a $500 loss	40% chance of a $400 loss

18. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric probability.

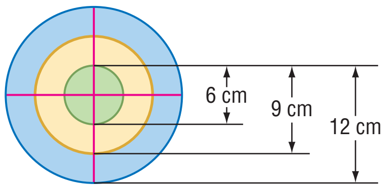


a. **Tabular** The spinner shown has a radius of 2.5 inches. Copy and complete the table below.

Color	Probability	Sector Area	Total Area	$\frac{\text{Sector Area}}{\text{Total Area}}$
red				
orange				
yellow				
green				
blue				

b. **Verbal** Make a conjecture about the relationship between the ratio of the area of the sector to the total area and the probability of the spinner landing on each color.

c. **Analytical** Consider the dartboard shown. Predict the probability of a dart landing in each area of the board. Assume that any dart thrown will land on the board and is equally likely to land at any point on the board.

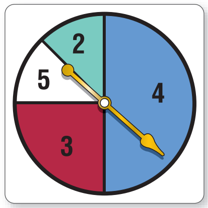


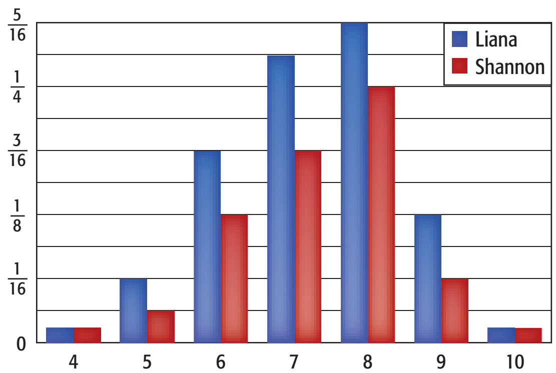
d. **Tabular** Construct a relative-frequency table for throwing 100 darts.

e. **Graphical** Graph the experimental probability distribution.

H.O.T. Problems Use Higher-Order Thinking Skills

19. **ERROR ANALYSIS** Liana and Shannon each created a probability distribution for the sum of two spins on the spinner at the right. Is either of them correct? Explain your reasoning.





20. **REASONING** Determine whether the following statement is *true* or *false*. Explain.

If you roll a die 10 times, you will roll the expected value at least twice.

21. **OPEN ENDED** Create a discrete probability distribution that shows five different outcomes and their associated probabilities.

22. **REASONING** Determine whether the following statement is *true* or *false*. Explain.

Random variables that can take on an infinite number of values are continuous.

23. **OPEN ENDED** Provide examples of a discrete probability distribution and a continuous probability distribution. Describe the differences between them.

24. **WRITING IN MATH** Compare and contrast two investments that have identical expected values and significantly different standard deviations.

LESSON 12

The Binomial Distribution

Then

- You used the Binomial Theorem.

Now

1. Identify and conduct a binomial experiment.
2. Find probabilities using binomial distributions.

Why?

- Jessica forgot to study for her civics quiz. The quiz consists of five multiple-choice questions with each question having four answer choices. Jessica randomly circles an answer for each question. In order to pass, she needs to answer at least four questions correctly.

NewVocabulary
binomial experiment
binomial distribution

1 Binomial Experiments

Each question on a multiple-choice quiz, like the one described above, can be thought of as a trial with two possible outcomes, correct or incorrect. If Jessica guesses on each question, the probability that she answers a question correctly is the same for all five questions.

Jessica's guessing on each question is an example of a binomial experiment. A **binomial experiment** is a probability experiment that satisfies the following conditions.

KeyConcept Binomial Experiments

- There is a fixed number of independent trials n.
- Each trial has only two possible outcomes, success or failure.
- The probability of success p is the same in every trial. The probability of failure q is $1 - p$.
- The random variable X is the number of successes in n trials.

Many probability experiments are or can be reduced to binomial experiments.

Example 1 Identify a Binomial Experiment

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

a. The spinner at the right is spun 20 times to see how many times it lands on red.

This experiment can be reduced to a binomial experiment with success being that the spinner lands on red and failure being any other outcome. Thus, a trial is a spin, and the random variable X represents the number of reds spun. The number of trials n is 20, the probability of success p is $\frac{1}{4}$ or 0.25, and the probability of failure q is $1 - 0.25$ or 0.75.

b. One hundred students are randomly asked their favorite food.

This is not a binomial experiment because there are many possible outcomes.

GuidedPractice

1A. Seventy-five students are randomly asked if they own a car.

1B. Four cards are removed from a deck to see how many aces are selected.

Use the following guidelines when conducting a binomial experiment.

KeyConcept Conducting Binomial Experiments

Step 1 Describe a trial for the situation, and determine the number of trials to be conducted.

Step 2 Define a success, and calculate the theoretical probabilities of success and failure.

Step 3 Describe the random variable X.

Step 4 Design and conduct a simulation to determine the experimental probability.

A binomial experiment can be conducted to compare experimental and theoretical probabilities.

PT

Example 2 Design a Binomial Experiment

Conduct a binomial experiment to determine the probability of drawing an odd-numbered card from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Step 1 A trial is drawing a card from a deck. The number of trials conducted can be any number greater than 0. We will use 52.

Step 2 A success is drawing an odd-numbered card. The odd-numbered cards in a deck are 3, 5, 7, and 9, and they occur once in each of the four suits. Therefore, there are $4 \cdot 4$ or 16 odd-numbered cards in the deck. The probability of drawing an odd-numbered card, or the probability of success, is $\frac{16}{52}$ or $\frac{4}{13}$. The probability of failure is $1 - \frac{4}{13}$ or $\frac{9}{13}$.

Step 3 The random variable X represents the number of odd-numbered cards drawn in 52 trials.

Step 4 Use the random number generator on a calculator to create a simulation. Assign the integers 0–12 to accurately represent the probability data.

Odd-numbered cards	0, 1, 2, 3
Other cards	4, 5, 6, 7, 8, 9, 10, 11, 12

Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Odd-Numbered Card	𝍸 𝍸 II	12
Other Cards	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40

An odd-numbered card was drawn 12 times, so the experimental probability is $\frac{12}{52}$ or about 23.1%. This is less than the theoretical probability of $\frac{16}{52}$ or about 30.8%.

StudyTip

Random Number Generator To generate all 52 random numbers using a graphing calculator, press MATH ◀ 5 and then enter the desired range followed by the number of trials. For example, type (0, 12, 52) for Example 2.

GuidedPractice

2. Conduct a binomial experiment to determine the probability of drawing an even-numbered card from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

2 Binomial Distribution

In the binomial experiment in Example 2, there were 12 successes in 52 trials. If you conducted that same experiment again, there may be any number of successes from 0 to 52. This situation can be represented by a binomial distribution. A **binomial distribution** is a frequency distribution of the probability of each value of X, where the random variable X represents the number of successes in n trials. Because X is a discrete random variable, a binomial distribution is a *discrete probability distribution.*

The probabilities in a binomial distribution can be calculated using the following formula.

StudyTip

Binomial Probability Formula In the Binomial Probability Formula, X represents the number of successes in n trials. Thus, the exponent for q, $n - X$, represents the number of failures in n trials.

KeyConcept Binomial Probability Formula

The probability of X successes in n independent trials is

$$P(X) = {}_nC_X\, p^X q^{n-X},$$

where p is the probability of success of an individual trial and q is the probability of failure on that same individual trial ($q = 1 - p$).

Notice that the Binomial Probability Formula is an adaptation of the Binomial Theorem you have already studied. The expression ${}_nC_X\, p^X q^{n-X}$ represents the $p^X q^{n-X}$ term in the binomial expansion of $(p + q)^n$.

Standardized Test Example 3 Find a Probability

PT

Garrett is selling items from a catalog to raise money for school. He has a 40% chance of making a sale each time he solicits a potential customer. Garrett asks 10 people to purchase an item. Find the probability that 6 people make a purchase.

A 8.6% **B** 11.2% **C** 24% **D** 40%

Read the Test Item

We need to find the probability that 6 people purchase an item. A success is making a sale, so $p = 0.4$, $q = 1 - 0.4$ or 0.6, and $n = 10$.

Solve the Test Item

$P(X) = {}_nC_X\, p^X q^{n-X}$ — Binomial Probability Formula

$P(6) = {}_{10}C_6\,(0.4)^6\,(0.6)^{10-6}$ — $n = 10$, $X = 6$, $p = 0.4$, and $q = 0.6$

≈ 0.111 — Simplify.

The probability of Garrett making six sales is about 0.111 or 11.1%. So, the correct answer is B.

Test-TakingTip

Failures A common error when using the Binomial Probability Formula is to focus on only the successes and to forget the failures. Notice that $P(6) \neq (0.4)^6$ and $P(6) \neq {}_{10}C_6(0.4)^6$.

GuidedPractice

3. TELEMARKETING At Jenny's telemarketing job, 15% of the calls that she makes to potential customers result in a sale. She makes 20 calls in a given hour. What is the probability that 5 calls result in a sale?

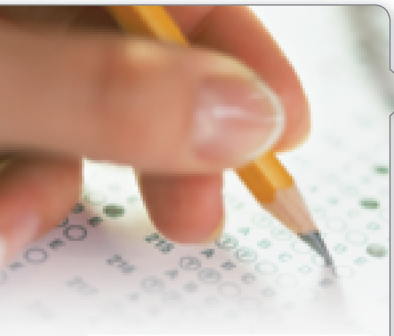
F 6.7% **G** 8.3% **H** 10.3% **J** 11.9%

If, on average, 40% of the people Garrett solicits make a purchase and he solicits 10 people, he can probably expect to make 10(0.40) or 4 sales. This value represents the mean of the binomial distribution. In general, the mean of a binomial distribution can be calculated by the following formula.

KeyConcept Mean of a Binomial Distribution

The mean μ of a binomial distribution is given by $\mu = np$, where n is the number of trials and p is the probability of success.

You can find the probability distribution for a binomial experiment by fully expanding the binomial $(p + q)^n$. A probability distribution can be helpful when solving for problems that allow multiple numbers of successes.



Real-WorldLink

ACT The math portion of the ACT college entrance exam includes 60 multiple-choice questions that each have five answer choices.

Source: ACT

Real-World Example 4 Full Probability Distribution

TEST TAKING **Refer to the beginning of the lesson.**

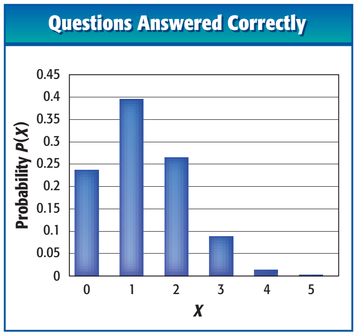
a. Determine the probabilities associated with the number of questions Jessica answered correctly by calculating the probability distribution.

If there are four answer choices for each question, then the probability that Jessica guesses and answers a question correctly is $\frac{1}{4}$ or 0.25. In this binomial experiment, $n = 5$, $p = 0.25$, and $q = 1 - 0.25$ or 0.75. Expand the binomial $(p + q)^n$.

$$(p + q)^n$$
$$= 1p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + 1q^5$$
$$= (0.25)^5 + 5(0.25)^4(0.75) + 10(0.25)^3(0.75)^2 + 10(0.25)^2(0.75)^3 + 5(0.25)(0.75)^4 + (0.75)^5$$
$$\approx 0.001 + 0.015 + 0.089 + 0.264 + 0.396 + 0.237$$

0.1%	1.5%	8.9%	26.4%	39.6%	23.7%
5 correct	4 correct	3 correct	2 correct	1 correct	0 correct

The graph shows the binomial probability distribution for the number of questions that Jessica answered correctly.



b. What is the probability that Jessica passes the quiz?

Jessica must answer at least four questions correctly to pass the quiz. The probability that Jessica answers *at least* four correct is the sum of the probabilities that she answers four or five correct and is about 1.5% + 0.1% or 1.6%. So, Jessica has about a 1.6% chance of passing, which is not likely.

StudyTip

Mean and Expected Value The mean of a binomial distribution can be any positive rational number. The expected value of a binomial distribution, however, should be rounded to the nearest whole number since a fraction of a success is not possible.

c. How many questions should Jessica expect to answer correctly?

Find the mean.

$\mu = np$ — Mean of a Binomial Distribution

$= 5(0.25)$ or 1.25 — $n = 5$ and $p = 0.25$

The mean of the distribution is 1.25. On average, Jessica should expect to answer one question correctly when she guesses on five.

GuidedPractice

4. TEST TAKING Suppose Jessica's civics quiz consisted of five true-or-false questions instead of multiple-choice questions.

A. Determine the probabilities associated with the number of answers Jessica answered correctly by calculating the probability distribution.

B. What is the probability that Jessica passes the quiz?

C. How many questions should Jessica expect to answer correctly?

Check Your Understanding

Example 1 **Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.**

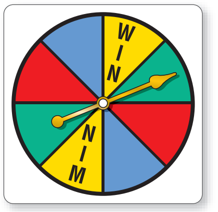
1. A study finds that 58% of people have pets. You ask 100 people how many pets they have.
2. You roll a die 15 times and find the sum of all of the rolls.
3. A poll found that 72% of students plan on going to the homecoming dance. You ask 30 students if they are going to the homecoming dance.

Example 2

4. Conduct a binomial experiment to determine the probability of drawing an ace or a king from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Example 3

5. **GAMES** Aiden has earned five spins of the wheel on the right. He will receive a prize each time the spinner lands on WIN. What is the probability that he receives three prizes?



A 4.2% **C** 7.1%
B 5.8% **D** 8.8%

Example 4

6. **PARKING** A poll at Steve's high school was taken to see if students are in favor of spending class money to expand the junior-senior parking lot. Steve surveyed 6 random students from the population.

Expand the Parking Lot	
favor	85%
oppose	15%

a. Determine the probabilities associated with the number of students that Steve asked who are in favor of expanding the parking lot by calculating the probability distribution.

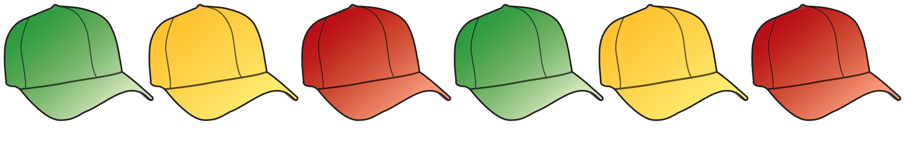
b. What is the probability that no more than 2 people are in favor of expanding the parking lot?

c. How many students should Steve expect to find who are in favor of expanding the parking lot?

Practice and Problem Solving

Example 1 **Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.**

7. There is a 35% chance that it rains each day in a given month. You record the number of days that it rains for that month.
8. A survey found that on a scale of 1 to 10, a movie received a 7.8 rating. A movie theater employee asks 200 patrons to rate the movie on a scale of 1 to 10.
9. A ball is hidden under one of the hats shown below. A hat is chosen, one at a time, until the ball is found.



Example 2

10. **DICE** Conduct a binomial experiment to determine the probability of rolling a 7 with two dice. Then compare the experimental and theoretical probabilities of the experiment.
11. **MARBLES** Conduct a binomial experiment to determine the probability of pulling a red marble from the bag. Then compare the experimental and theoretical probabilities of the experiment.

12. SPINNER Conduct a binomial experiment to determine the probability of the spinner stopping on an even number. Then compare the experimental and theoretical probabilities of the experiment.

13. CARDS Conduct a binomial experiment to determine the probability of drawing a face card out of a standard deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Example 3

14. MP3 PLAYERS According to a recent survey, 85% of high school students own an MP3 player. What is the probability that 6 out of 10 random high school students own an MP3 player?

15. CARS According to a recent survey, 92% of high school seniors own their own car. What is the probability that 10 out of 12 random high school students own their own car?

16. SENIOR PROM According to a recent survey, 25% of high school upperclassmen think that the junior-senior prom is the most important event of the school year. What is the probability that 3 out of 15 random high school upperclassmen think this way?

17. FOOTBALL A certain football team has won 75.7% of their games. Find the probability that they win 7 of their next 12 games.

18. GARDENING Peter is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Peter knows that the probability of having a blue flower is 75%. What is the probability that 20 of the flowers will be blue?

19. FOOTBALL A field goal kicker is accurate 75% of the time from within 35 yards. What is the probability that he makes exactly 7 of his next 10 kicks from within 35 yards?

Range (yd)	Accuracy (%)
0–35	75
35–45	62
45+	20

20. BABIES Mr. and Mrs. Davis are planning to have 3 children. The probability of each child being a boy is 50%. What is the probability that they will have 2 boys?

Example 4

21. LAPTOPS According to a recent survey, 52% of high school students own a laptop. Ten random students are chosen.

a. Determine the probabilities associated with the number of students that own a laptop by calculating the probability distribution.

b. What is the probability that at least 8 of the 10 students own a laptop?

c. How many students should you expect to own a laptop?

22. ATHLETICS A survey was taken to see the percent of students that participate in sports for their school. Six random students are chosen.

Student Athletics	
0 sports	20%
1 sport	55%
2 sports	20%
3+ sports	5%

a. Determine the probabilities associated with the number of students playing in at least one sport by calculating the probability distribution.

b. What is the probability that no more than 2 of the students participated in a sport?

c. How many students should you expect to have participated in at least one sport?

23. **MUSIC** An online poll showed that 57% of adults still own vinyl records. Moe surveyed 8 random adults from the population.

a. Determine the probabilities associated with the number of adults that still own vinyl records by calculating the probability distribution.

b. What is the probability that no less than 6 of the people surveyed still own vinyl records?

c. How many people should Moe expect to still own vinyl records?

A binomial distribution has a 60% rate of success. There are 18 trials.

24. What is the probability that there will be at least 12 successes?

25. What is the probability that there will be 12 failures?

26. What is the expected number of successes?

27. **DECISION MAKING** Six roommates randomly select someone to wash the dishes each day.

a. What is the probability that the same person has to wash the dishes 3 times in a given week?

b. What method can the roommates use to select who washes the dishes each day?

28. **DECISION MAKING** A committee of five people randomly selects someone to take the notes of each meeting.

a. What is the probability that a person takes notes less than twice in 10 meetings?

b. What method can the committee use to select the notetaker each meeting?

c. If the method described in part **b** results in the same person being notetaker for nine straight meetings, would this result cause you to question the method?

Each binomial distribution has n trials and p probability of success. Determine the most likely number of successes.

29. $n = 8, p = 0.6$

30. $n = 10, p = 0.4$

31. $n = 6, p = 0.8$

32. $n = 12, p = 0.55$

33. $n = 9, p = 0.75$

34. $n = 11, p = 0.35$

35. **SWEEPSTAKES** A beverage company is having a sweepstakes. The probabilities of winning selected prizes are shown at the right. If Ernesto purchases 8 beverages, what is the probability that he wins at least one prize?

Odds of Winning	
beverage	1 in 10
CD	1 in 200
hat	1 in 250
MP3 player	1 in 20,000
car	1 in 25,000,000

Each binomial distribution has n trials and p probability of success. Determine the probability of s successes.

36. $n = 8, p = 0.3, s \geq 2$

37. $n = 10, p = 0.2, s > 2$

38. $n = 6, p = 0.6, s \leq 4$

39. $n = 9, p = 0.25, s \leq 5$

40. $n = 10, p = 0.75, s \geq 8$

41. $n = 12, p = 0.1, s < 3$

H.O.T. Problems Use Higher-Order Thinking Skills

42. **CHALLENGE** A poll of students determined that 88% wanted to go to college. Eight random students are chosen. The probability that at least x students want to go to college is about 0.752 or 75.2%. Solve for x.

43. **WRITING IN MATH** What should you consider when using a binomial distribution to make a decision?

44. **OPEN ENDED** Describe a real-world setting within your school or community activities that seems to fit a binomial distribution. Identify the key components of your setting that connect to binomial distributions.

45. **WRITING IN MATH** Describe how binomial distributions are connected to Pascal's triangle.

46. **WRITING IN MATH** Explain the relationship between a binomial experiment and a binomial distribution.

LESSON 13 The Normal Distribution

Then

- You constructed and analyzed discrete probability distributions.

Now

1. Use the Empirical Rule to analyze normally distributed variables.
2. Apply the standard normal distribution and z-values.

Why?

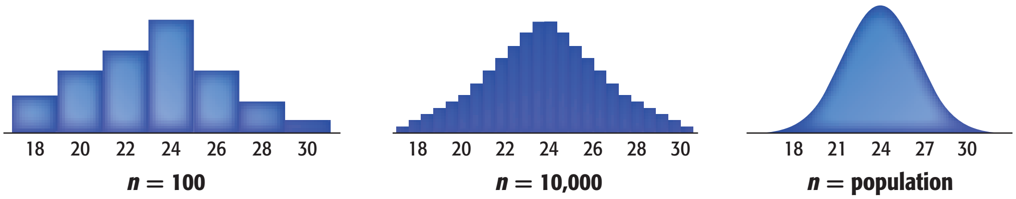
- Extensive observations of Swiss cherry trees found that the mean flowering date is April 21 with a standard deviation of about 10 days. Therefore, 95% of the time, a Swiss cherry tree will have a flowering date between April 1 and May 3.

Frank Krahmer/Photographer's Choice RF/Getty Images

NewVocabulary
normal distribution
Empirical Rule
z-value
standard normal distribution

1 The Normal Distribution

Distributions of mileages of different sample sizes of cars are shown below. As the sample size increases, the distributions become more and more symmetrical and resemble the curve at the right, due to the Law of Large Numbers.



The curve at the right is a **normal distribution**, a continuous, symmetric, bell-shaped distribution of a random variable. It is the most common *continuous probability distribution*. The characteristics of the normal distribution are as follows.

KeyConcept The Normal Distribution

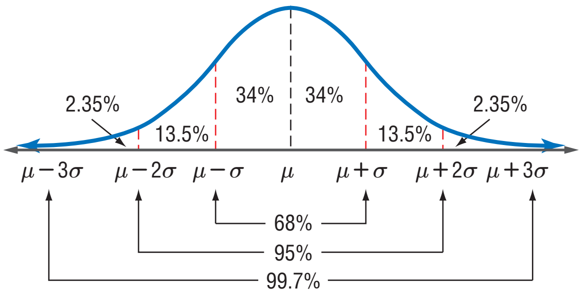
- The graph of the curve is continuous, bell-shaped, and symmetric with respect to the mean.
- The mean, median, and mode are equal and located at the center.
- The curve approaches, but never touches, the *x*-axis.
- The total area under the curve is equal to 1 or 100%.

The area under the normal curve represents the amount of data within a certain interval or the probability that a random data value falls within that interval. The **Empirical Rule** can be used to determine the area under the normal curve at specific intervals.

KeyConcept The Empirical Rule

In a normal distribution with mean μ and standard deviation σ,

- approximately 68% of the data fall within 1σ of the mean,
- approximately 95% of the data fall within 2σ of the mean, and
- approximately 99.7% of the data fall within 3σ of the mean.



StudyTip

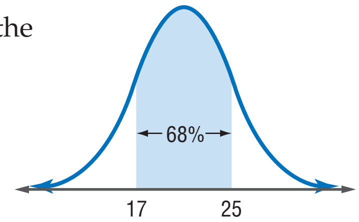
Normal Distributions In all of these cases, the number of data values must be large for the distribution to be approximately normal.

Example 1 Use the Empirical Rule to Analyze Data

A normal distribution has a mean of 21 and a standard deviation of 4.

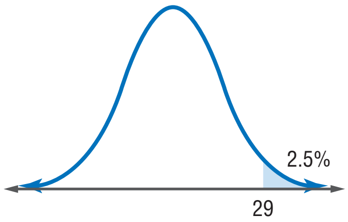
a. Find the range of values that represent the middle 68% of the distribution.

The middle 68% of data in a normal distribution is the range from $\mu - \sigma$ to $\mu + \sigma$. Therefore, the range of values in the middle 68% is $17 < X < 25$.



b. What percent of the data will be greater than 29?

29 is 2σ more than μ. 95% of the data fall between $\mu - 2\sigma$ and $\mu + 2\sigma$, so the remaining data values represented by the two tails covers 5% of the distribution. We are only concerned with the upper tail, so 2.5% of the data will be greater than 29.



GuidedPractice

1. A normal distribution has a mean of 8.2 and a standard deviation of 1.3.

A. Find the range of values that represent the middle 95% of the distribution.

B. What percent of the data will be less than 4.3?

Image Source/Getty Images

Real-World Link

While the average adult American male is 5 feet 10 inches, the average height of adult males in the Netherlands is the highest worldwide, at almost 6 feet 1 inch.

Source: Eurostats Statistical Yearbook

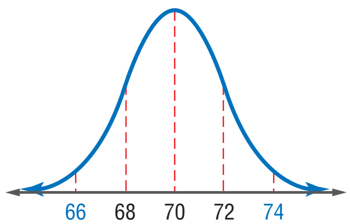
Real-World Example 2 Use the Empirical Rule to Analyze a Distribution

HEIGHTS The heights of 1800 adults are normally distributed with a mean of 70 inches and a standard deviation of 2 inches.

a. About how many adults are between 66 and 74 inches?

66 and 74 are 2σ away from the mean. Therefore, about 95% of the data are between 66 and 74.

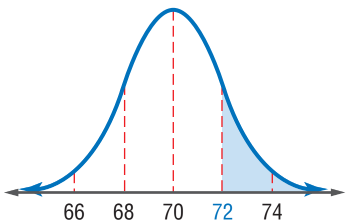
Since $1800 \times 95\% = 1710$, we know that about 1710 of the adults are between 66 and 74 inches tall.



b. What is the probability that a random adult is more than 72 inches tall?

From the curve, values greater than 72 are more than 1σ from the mean. 13.5% are between 1σ and 2σ, 2.35% are between 2σ and 3σ, and 0.15% are greater than 3σ.

So, the probability that an adult selected at random has a height greater than 72 inches is $13.5 + 2.35 + 0.15$ or 16%.



GuidedPractice

2. NETWORKING SITES The number of friends per member in a sample of 820 members is normally distributed with a mean of 38 and a standard deviation of 12.

A. About how many members have between 26 and 50 friends?

B. What is the probability that a random member will have more than 14 friends?

2 Standard Normal Distribution

The Empirical Rule is only useful for evaluating specific values, such as $\mu + \sigma$. Once the data set is *standardized*, however, any data value can be evaluated. Data are standardized by converting them to z-values, also known as z-scores. The **z-value** represents the number of standard deviations that a given data value is from the mean. Therefore, z-values can be used to determine the position of any data value within a set of data.

KeyConcept Formula for z-Values

The z-value for a data value X in a set of normally distributed data is given by $z = \frac{X - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation.

StudyTip

Symmetry The normal distribution is symmetrical, so when you are asked for the middle or outside set of data, the z-values will be opposites.

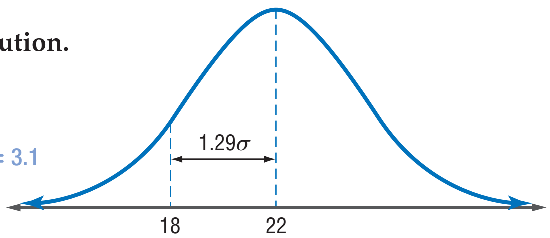
Example 3 Use z-Values to Locate Position

Find z if $X = 18$, $\mu = 22$, and $\sigma = 3.1$. Indicate the position of X in the distribution.

$z = \frac{X - \mu}{\sigma}$ — Formula for z-values

$= \frac{18 - 22}{3.1}$ — $X = 18$, $\mu = 22$, $\sigma = 3.1$

≈ -1.29 — Simplify.



The z-value that corresponds to $X = 18$ is approximately -1.29. Therefore, 18 is about 1.29 standard deviations less than the mean of the distribution.

GuidedPractice

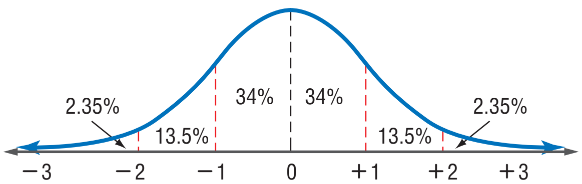
3. Find X if $\mu = 39$, $\sigma = 8.2$, and $z = 0.73$. Indicate the position of X in the distribution.

Any combination of mean and standard deviation is possible for a normally distributed set of data. As a result, there are infinitely many normal probability distributions. This makes comparing two individual distributions difficult. Different distributions *can* be compared, however, once they are standardized using z-values. The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

StudyTip

Standard Normal Distribution The standard normal distribution is the set of all z-values.

KeyConcept Characteristics of the Standard Normal Distribution



- The total area under the curve is equal to 1 or 100%.
- Almost all of the area is between $z = -3$ and $z = 3$.
- The distribution is symmetric.
- The mean is 0, and the standard deviation is 1.

The standard normal distribution allows us to assign actual areas to the intervals created by z-values. The area under the normal curve corresponds to the proportion of data values in an interval as well as the probability of a random data value falling within the interval. For example, the area between $z = 0$ and $z = 1$ is 0.34. Therefore, the probability of a z-value being in this interval is 34%.

Real-WorldLink

Video Uploading According to a recent study, 52% of people who said they upload videos to the Web do it through sites such as Facebook and MySpace. The rest use video-sharing sites like YouTube and Google Video.

Source: *Pew Internet and American Life Project*

Real-World Example 4 Find Probabilities

VIDEOS The number of videos uploaded daily to a video sharing site is normally distributed with μ = 181,099 videos and σ = 35,644 videos. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

a. $P(180{,}000 < X < 200{,}000)$

The question is asking for the percent of days when between 180,000 and 200,000 videos are uploaded. First, find the corresponding z-values for $X = 180{,}000$ and $X = 200{,}000$.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$

$$= \frac{180{,}000 - 181{,}099}{35{,}644} \text{ or about } -0.03 \quad X = 180{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

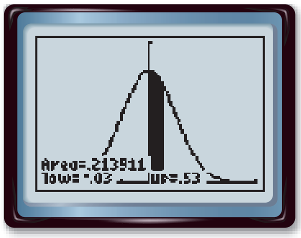
Use 200,000 to find the other z-value.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$

$$= \frac{200{,}000 - 181{,}099}{35{,}644} \text{ or about } 0.53 \quad X = 200{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

The range of z-values that c s to $180{,}000 < X < 200{,}000$ is $-0.03 < z < 0.53$. Find the area under the normal curve within this interval.

You can use a graphing calculator to display the area that corresponds to any z-value by selecting 2nd [DISTR]. Then, under the **DRAW** menu, select **ShadeNorm(lower *z* value, upper *z* value)**. The area between $z = -0.03$ and $z = 0.53$ is about 0.21 as shown in the graph.



[−4, 4] scl: 1 by [0, 0.5] scl: 0.125

Therefore, about 21% of the time, there will be between 180,000 and 200,000 video uploads on a given day.

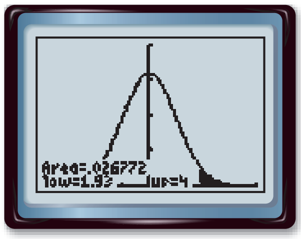
b. $P(X > 250{,}000)$

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$

$$= \frac{250{,}000 - 181{,}099}{35{,}644} \text{ or about } 1.93 \quad X = 250{,}000,\ \mu = 181{,}099, \text{ and } \sigma = 35{,}644$$

Using a graphing calculator, you can find the area between $z = 1.93$ and $z = 4$ to be about 0.027.

Therefore, the probability that more than 250,000 videos will be uploaded is about 2.7%.



[−4, 4] scl: 1 by [0, 0.5] scl: 0.125

StudyTip

Range of *z*-values The majority of data values are within $\pm 4\sigma$ of the mean, so setting the maximum value of z equal to 4 is sufficient in part **b**. Use the window [−4, 4] by [0, 0.5] when using **ShadeNorm.**

GuidedPractice

4. TIRES The life spans of a certain tread of tire are normally distributed with μ = 31,066 miles and σ = 1644 miles. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

A. $P(30{,}000 < X < 32{,}000)$ **B.** $P(X > 35{,}000)$

Another method for calculating the area between two z-values is 2nd [DISTR] **normalcdf**(*lower z value, upper z value*).

Check Your Understanding

Example 1 **A normal distribution has a mean of 416 and a standard deviation of 55.**

1. Find the range of values that represent the middle 99.7% of the distribution.

2. What percent of the data will be less than 361?

Example 2 **3.** **TEXTING** The number of texts sent per day by a sample of 811 teens is normally distributed with a mean of 38 and a standard deviation of 7.

a. About how many teens sent between 24 and 38 texts?

b. What is the probability that a teen selected at random sent less than 818 texts?

Example 3 **Find the missing variable. Indicate the position of X in the distribution.**

4. z if $\mu = 89$, $X = 81$, and $\sigma = 11.5$

5. z if $\mu = 13.3$, $X = 17.2$, and $\sigma = 1.9$

6. X if $z = -1.38$, $\mu = 68.9$, and $\sigma = 6.6$

7. σ if $\mu = 21.1$, $X = 13.7$, and $z = -2.40$

Example 4 **8.** **CONCERTS** The number of concerts attended per year by a sample of 925 teens is normally distributed with a mean of 1.8 and a standard deviation of 0.5. Find each probability. Then use a graphing calculator to sketch the area under each curve.

a. $P(X < 2)$

b. $P(1 < X < 3)$

Practice and Problem Solving

Example 1 **A normal distribution has a mean of 29.3 and a standard deviation of 6.7.**

9. Find the range of values that represent the outside 5% of the distribution.

10. What percent of the data will be between 22.6 and 42.7?

Example 2 **11.** **GYMS** The number of visits to a gym per year by a sample of 522 members is normally distributed with a mean of 88 and a standard deviation of 19.

a. About how many members went to the gym at least 50 times?

b. What is the probability that a member selected at random went to the gym more than 145 times?

Example 3 **Find the missing variable. Indicate the position of X in the distribution.**

12. z if $\mu = 3.3$, $X = 3.8$, and $\sigma = 0.2$

13. z if $\mu = 19.9$, $X = 18.7$, and $\sigma = 0.9$

14. μ if $z = -0.92$, $X = 44.2$, and $\sigma = 8.3$

15. X if $\mu = 138.8$, $\sigma = 22.5$, and $z = 1.73$

Example 4 **16.** **VENDING** A vending machine dispenses about 8.2 ounces of coffee. The amount varies and is normally distributed with a standard deviation of 0.3 ounce. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

a. $P(X < 8)$

b. $P(X > 7.5)$

17. **CAR BATTERIES** The useful life of a certain car battery is normally distributed with a mean of 113,627 miles and a standard deviation of 14,266 miles. The company makes 20,000 batteries a month.

a. About how many batteries will last between 90,000 and 110,000 miles?

b. About how many batteries will last more than 125,000 miles?

c. What is the probability that if you buy a car battery at random, it will last less than 100,000 miles?

18. **FOOD** The shelf life of a particular snack chip is normally distributed with a mean of 173.3 days and a standard deviation of 23.6 days.

a. About what percent of the product lasts between 150 and 200 days?

b. About what percent of the product lasts more than 225 days?

c. What range of values represents the outside 5% of the distribution?

19. FINANCIAL LITERACY The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual's insurance rate. Suppose insurance rates for a sample population are normally distributed.

a. If the mean annual cost per person is $829 and the standard deviation is $115, what is the range of rates you would expect the middle 68% of the population to pay annually?

b. If 900 people were sampled, how many would you expect to pay more than $1000 annually?

c. Where on the distribution would you expect a person with several traffic citations to lie? Explain your reasoning.

d. How do you think auto insurance companies use each factor to calculate an individual's insurance rate?

20. STANDARDIZED TESTS Nikki took three national standardized tests and scored an 86 on all three. The table shows the mean and standard deviation of each test.

	Math	Science	Social Studies
μ	76	81	72
σ	9.7	6.2	11.6

a. Calculate the z-values that correspond to her score on each test.

b. What is the probability of a student scoring an 86 or *lower* on each test?

c. On which test was Nikki's standardized score the highest? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

21. ERROR ANALYSIS A set of normally distributed tree diameters have mean 11.5 centimeters, standard deviation 2.5, and range from 3.6 to 19.8. Monica and Hiroko are to find the range that represents the middle 68% of the data. Is either of them correct? Explain.

Monica	Hiroko
The data span 16.2 cm. 68% of 16.2 is about 11 cm. Center this 11-cm range around the mean of 11.5 cm. This 68% group will range from about 6 cm to about 17 cm.	The middle 68% span from $\mu + \sigma$ to $\mu - \sigma$. So we move 2.5 cm below 11.5 and then 2.5 cm above 11.5. The 68% group will range from 9 cm to 14 cm.

22. CHALLENGE A case of MP3 players has an average battery life of 8.2 hours with a standard deviation of 0.7 hour. Eight of the players have a battery life greater than 9.3 hours. If the sample is normally distributed, how many MP3 players are in the case?

23. REASONING The term *six sigma process* comes from the notion that if one has six standard deviations between the mean of a process and the nearest specification limit, there will be practically no items that fail to meet the specifications. Is this a true assumption? Explain.

24. REASONING *True* or *false*: According to the Empirical Rule, in a normal distribution, most of the data will fall within one standard deviation of the mean. Explain.

25. OPEN ENDED Find a set of real-world data that appears to be normally distributed. Calculate the range of values that represent the middle 68%, the middle 95%, and the middle 99.7% of the distribution.

26. WRITING IN MATH Describe the relationship between the z-value, the position of an interval of X in the normal distribution, the area under the normal curve, and the probability of the interval occurring. Use an example to explain your reasoning.

LAB 14

Spreadsheet Lab
Normal Approximation of Binomial Distributions

In Lesson 11-4, you used a binomial expansion to find a full probability distribution. A spreadsheet can be used to quickly find and graph a full distribution for any number of trials.

Activity 1 Full Probability Distribution

PLAYING CARDS **Tom randomly selects a card from a deck of 52 playing cards, records its suit, and replaces it. Use a spreadsheet to construct and graph a full probability distribution for *X*, the number of hearts that Tom selects if he chooses 4, 20, or 100 cards.**

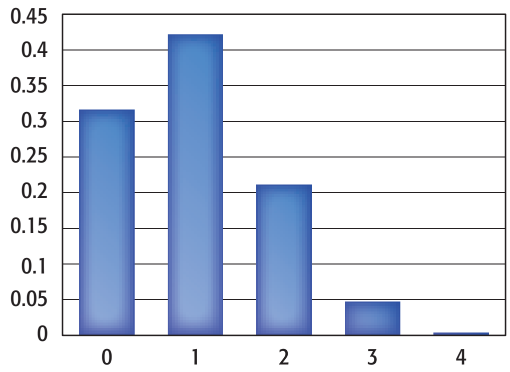
Since one fourth of the cards in a standard deck are hearts, the probability of success is 25% or 0.25, and the probability of failure is 75% or 0.75.

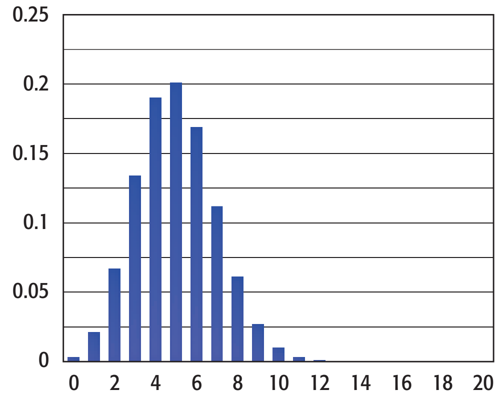
Step 1 Enter the numbers 0 to 4 in column A. In B1, enter the binomial probability formula as =COMBIN(4,A1)*(0.25)^A1*(0.75)^(4−A1). Copy and paste this formula in cells B2:B5.

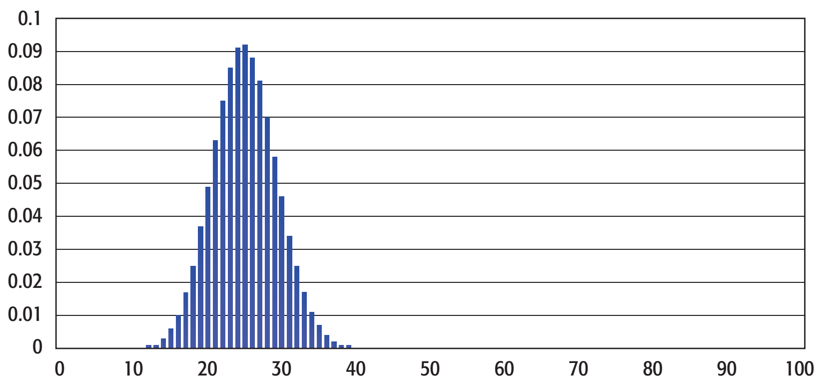
Step 2 Select cells B1:B5, and insert a clustered column bar graph. Use the values in column A for Category *(X)* axis labels.

Step 3 Select cells A1:A5, and autofill the through A21. In C1, enter the formula =COMBIN(20,A1)*(0.25)^A1* (0.75)^(20−A1). Copy this formula in cells C2:C21. Repeat Step 2 to graph this distribution.

Step 4 Autofill column A through A101. In D1, enter the formula =COMBIN(100,A1)*(0.25)^A1* (0.75)^(100−A1). Copy this formula in cells D2:C1011. Repeat Step 2 to graph this distribution.







Notice that the number of trials increases, the shape of the graph of the distribution becomes more symmetric with its center at the mean, $\mu = np$. If the number of trials becomes large enough, the shape of the distribution approaches a normal curve. Therefore, a normal distribution can be used to approximate the binomial distribution.

KeyConcept Normal Approximation of a Binomial Distribution

In a binomial distribution with n trials, a probability of success p, and a probability of failure q, such that $np \geq 5$ and $nq \geq 5$, a binomial distribution can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$.

Once the mean and standard deviation are calculated, a z-value can be determined, and the corresponding probability can be found as demonstrated in the next activity.

Activity 2 Normal Approximation of a Binomial Distribution

JURY DUTY According to a poll, 60% of the registered voters in a city have never been called for jury duty. Mariah conducts a random survey of 300 registered voters. What is the probability that at least 170 of those voters have never been called for jury duty?

This is a binomial experiment with $n = 300$, $p = 0.6$, and $q = 0.4$. Since $np = 300(0.6)$ or 180 and $nq = 300(0.4)$ or 120 are both greater than 5, the normal distribution can be used to approximate the binomial distribution.

Step 1 The mean μ is np or 180. Find the standard deviation σ.

$\sigma = \sqrt{npq}$ — Standard deviation of a binomial distribution

$= \sqrt{300(0.6)(0.4)}$ — $n = 300$, $p = 0.6$, and $q = 0.4$

≈ 8.49 — Simplify.

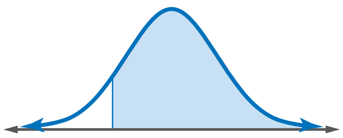
Step 2 Write the problem in probability notation using X. The probability that at least 170 people have never been called for jury duty is $P(X \geq 170)$.

Step 3 Find the corresponding z-value for X.

$z = \frac{X - \mu}{\sigma}$ — Formula for z-values

$= \frac{170 - 180}{8.49}$ — $X = 170$, $\mu = 180$, and $\sigma = 8.49$

≈ -1.18 — Simplify.



Step 4 Use a calculator to find the area under the normal curve to the right of z.

KEYSTROKES: [2nd] [DISTR] 2 [(−)] 1 [.] 18 [,] 4 [)] [ENTER]

The approximate area to the right of z is 0.881. Therefore, the probability that at least 170 of the registered voters have never been called for jury duty is about 88.1%.

Exercises

1. Use a spreadsheet to construct the graph of the full probability distribution for X, the number of times a 3 is rolled from rolling a die 25 times.

2. **SUMMER JOBS** According to an online poll, 80% of high school upperclassmen have summer jobs. Tadeo thinks the number should be lower so he conducts a survey of 480 random upperclassmen. What is the probability that no more than 380 of the surveyed upperclassmen have summer jobs?

3. **WORK** According to an online poll, 28% of adults feel that the standard 40-hour work week should be increased. Sheila interviews 300 adults who work at the mall. What is the probability that more than 80 but fewer than 100 of those surveyed will say that the work week should be increased?

LESSON 15

Confidence Intervals and Hypothesis Testing

Then

You applied the standard normal distribution and *z*-values.

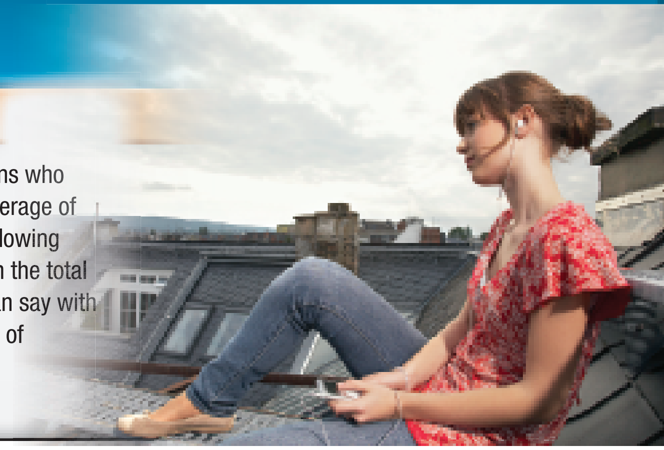
Now

1 Find confidence intervals for normally distributed data.

2 Perform hypothesis tests on normally distributed data.

Why?

In a recent Gallup Poll, 1514 teens who owned an mp3 player had an average of 1033 songs. The poll had the following disclaimer: "For results based on the total sample of national teens, one can say with 95% confidence that the margin of sampling error is ±31 songs."



Fancy Photography/Veer

NewVocabulary

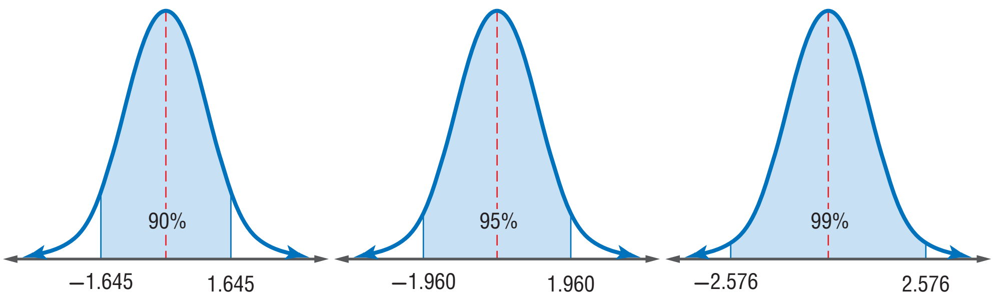
inferential statistics
statistical inference
confidence interval
maximum error of estimate
hypothesis test
null hypothesis
alternative hypothesis
critical region
left-tailed test
two-tailed test
right-tailed test

1 Confidence Intervals

Inferential statistics are used to draw conclusions or **statistical inferences** about a population using a sample. For example, the sample mean of 1033 songs per mp3 player can be used to estimate the population mean.

A **confidence interval** is an estimate of a population parameter stated as a range with a specific degree of certainty. Typically, statisticians use 90%, 95%, and 99% confidence intervals, but any percentage can be considered. In the opening example, we are 95% confident that the population mean is within 31 songs of 1033.

The confidence interval for a normal distribution is equivalent to the area under the standard normal curve between $-z$ and z, as shown. A 95% confidence interval for a population mean implies that we are 95% sure that the mean will fall within the range of z-values.



Suppose you want 95% confidence when conducting an experiment. The corresponding z-value is 1.960, where 2.5% of the area lies to the left of $-z$ and 2.5% lies to the right of z.

To find the confidence interval, use the maximum error of estimate and the sample mean. The **maximum error of estimate** E is the maximum difference between the estimate of the population mean μ and its actual value.

KeyConcept Maximum Error of Estimate

The maximum error of estimate E for a population mean is given by

$$E = z \cdot \frac{s}{\sqrt{n}},$$

where z is the z-value that corresponds to a particular confidence level, s is the standard deviation of the sample, and n is the sample size; $n \geq 30$.

Example 1 Maximum Error of Estimate

SOCIAL NETWORKING A poll of 218 randomly selected members of a social networking Web site showed that they spent an average of 14 minutes per day on the site with a standard deviation of 3.1 minutes. Use a 95% confidence interval to find the maximum error of estimate for the time spent on the site.

In a 95% confidence interval, 2.5% of the area lies in each tail. The corresponding z-value is 1.960.

$$E = z \cdot \frac{s}{\sqrt{n}} \quad \text{Maximum Error of Estimate}$$

$$= 1.960 \cdot \frac{3.1}{\sqrt{218}} \quad z = 1.960,\ s = 3.1, \text{ and } n = 218$$

$$\approx 0.41 \quad \text{Simplify.}$$

This means that you can be 95% confident that the population mean time spent on the site will be within 0.41 minute of the sample mean of 14 minutes.

GuidedPractice

1. **TEACHING** A poll of 184 randomly selected high school teachers showed that they spend an average of 16.8 hours per week grading, planning, and preparing for class. The standard deviation is 2.9 hours. Use a 90% confidence interval to find the maximum error of estimate for the amount of time spent per week.

StudyTip

Critical Values The z-value that corresponds to a particular confidence level is known as the *critical value*. The most commonly used levels and their corresponding z-value are shown below.

Confidence Level	z-Value
90%	1.645
95%	1.960
99%	2.576

Once the maximum error of estimate E is found, a confidence interval CI for the population mean can be determined by adding $\pm E$ to the sample mean.

KeyConcept Confidence Interval for the Population Mean

A confidence interval for a population mean is given by

$$CI = \bar{x} \pm E \text{ or } \bar{x} \pm z \cdot \frac{s}{\sqrt{n}},$$

where $\bar{x}$ is the sample mean and E is the maximum error of estimate.

Real-World Example 2 Confidence Interval

SCHOOL WORK A sample of 200 students was asked the average time they spent on homework each weeknight. The mean time was 52.5 minutes with a standard deviation of 5.1 minutes. Determine a 99% confidence interval for the population mean.

$$CI = \bar{x} \pm z \cdot \frac{s}{\sqrt{n}} \quad \text{Confidence Interval for Population Mean}$$

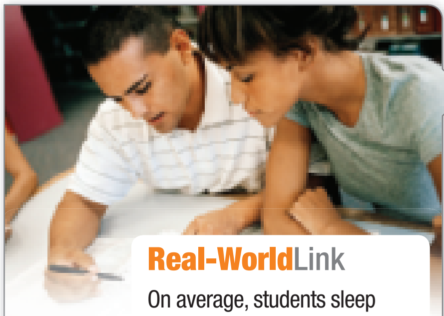
$$= 52.5 \pm 2.576 \cdot \frac{5.1}{\sqrt{200}} \quad \bar{x} = 52.5,\ z = 2.576,\ s = 5.1, \text{ and } n = 200$$

$$\approx 52.5 \pm 0.93 \quad \text{Use a calculator.}$$

The 99% confidence interval is $51.57 \leq \mu \leq 53.43$. Therefore, we are 99% confident that the population mean time is between 51.57 and 53.43 minutes.

GuidedPractice

2. **SCHOOL ATHLETICS** A sample of 224 students showed that they attend an average of 2.6 school athletic events per year with a standard deviation of 0.8. Determine a 90% confidence interval for the population mean.



Real-WorldLink

On average, students sleep 8.1 hours per weekday and spend 7.5 hours on educational activities, such as attending class or doing homework.

Source: Bureau of Labor Statistics

BananaStock/Jupiterimages

2 Hypothesis Testing

While a confidence interval provides an estimate of the population mean, a **hypothesis test** is used to assess a specific claim about the mean. Typical claims are that the mean is equal to, is greater than, or is less than a specific value.

There are two parts to a hypothesis test: the null hypothesis and the alternative hypothesis. The **null hypothesis** H_0 is a statement of equality to be tested. The **alternative hypothesis** H_a is a statement of inequality that is the complement of the null hypothesis. A *claim* can be part of the null or alternative hypothesis.

PT

Example 3 Claims and Hypotheses

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

a. A school administrator thinks it takes less than 3 minutes to evacuate the entire building for a fire drill.

less than 3 minutes	not less than three minutes
$\mu < 3$	$\mu \geq 3$

The claim is $\mu < 3$, and it is the alternative hypothesis because it does not include equality. The null hypothesis is $\mu \geq 3$, which is the complement of $\mu < 3$.

H_0: $\mu \geq 3$ $\qquad$ H_a: $\mu < 3$ (claim)

b. The owner of a deli says that there are 2 ounces of ham in a sandwich.

H_0: $\mu = 2$ (claim); H_a: $\mu \neq 2$

The claim is $\mu = 2$, and it is the null hypothesis because it is an equality. The alternative hypothesis is $\mu \neq 2$, which is the complement of $\mu = 2$.

GuidedPractice

3A. Jill thinks it takes longer than 10 minutes for the school bus to reach school from her stop.

3B. Mindy thinks there aren't 35 peanuts in every package.

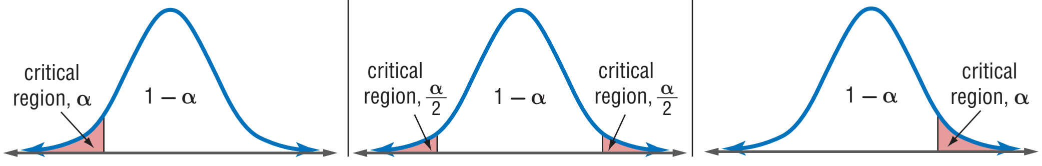
ReadingMath

Significance Level A significance level of 5% means that we need the data to provide evidence against H_0 so strong that the data would occur no more than 5% of the time when H_0 is true.

Once a claim is made, a sample is collected and analyzed, and the null hypothesis is either *rejected* or *not rejected*. From this information, the claim is then rejected or not rejected. This is determined by whether the sample mean falls within the critical region. The **critical region** is the range of values that suggests a significant enough difference to reject the null hypothesis. The critical region is determined by the significance level α, most commonly 1%, 5%, or 10%, and the inequality sign of the alternate hypothesis.

ConceptSummary Tests of Significance

If H_a: $\mu < k$, the hypothesis test is a **left-tailed test**.	If H_a: $\mu \neq k$, the hypothesis test is a **two-tailed test**.	If H_a: $\mu > k$, the hypothesis test is a **right-tailed test**.



The value k represents the claim about the population mean μ.

StudyTip

z-Values and z-Statistics In hypothesis testing, a z-value is used to determine the critical region. A z-statistic is calculated to see if the sample mean falls within that critical region, and thus determines whether or not to reject the null hypothesis.

The type of hypothesis test that we are using in this lesson is known as a z-test. Once the area corresponding to the significance level is determined, a z-statistic for the sample mean is calculated. The z-statistic is the z-value for the sample. When testing at 5% significance, it is only 5% likely for the z-statistic to fall within the critical region *and* for H_0 to be true. Therefore, when the z-statistic falls within the critical region, H_0 is rejected.

Use these steps to test a hypothesis.

KeyConcept Steps for Hypothesis Testing

Step 1 State the hypotheses and identify the claim.

Step 2 Determine the critical value(s) and critical region.

Step 3 Calculate the z-statistic using $z = \dfrac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Step 4 Reject or fail to reject the null hypothesis.

Step 5 Make a conclusion about the claim.

Real-WorldCareer

Politician Politicians are elected officials in the federal, state, and local governments. They influence public decision making and make laws that affect everyone. Many politicians have business, teaching, or legal experience prior to getting involved in politics.

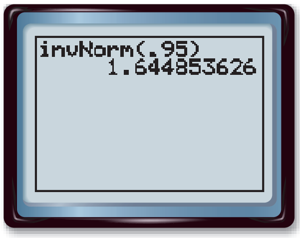
Real-World Example 4 One-Sided Hypothesis Test

STUDENT COUNCIL Lindsey, the president of the student council, has heard complaints that the cafeteria lunch line moves too slowly. The dining services coordinator assures her that the average wait time is 6 minutes or less. Using a sample of 65 customers, Lindsey calculated a mean wait time of 6.3 minutes and a standard deviation of 1.1 minutes. Test the hypothesis at 5% significance.

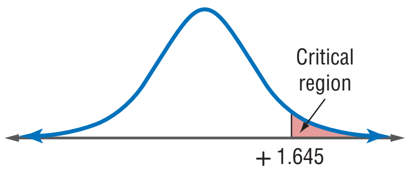
Step 1 State the claim and hypotheses. H_0: $\mu \leq 6$ (claim) and H_a: $\mu > 6$.

Step 2 Determine the critical region.

The alternative hypothesis is $\mu > 6$, so this is a right-tailed test. We are testing at 5% significance, so we need to identify the z-value that corresponds with the upper 5% of the distribution.



KEYSTROKES: 2nd [DISTR] 3 .95 ENTER



Step 3 Calculate the z-statistic for the sample data.

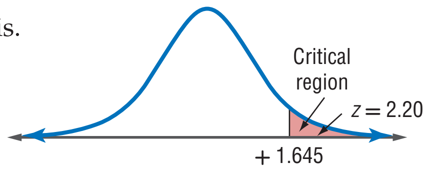
$$z = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$ Formula for z-statistic

$$= \frac{6.3 - 6}{\frac{1.1}{\sqrt{65}}}$$ $\overline{x} = 6.3$, $\mu = 6$, $s = 1.1$, and $n = 65$

$$\approx 2.20$$ Simplify.

Step 4 Decide whether to reject the null hypothesis.

H_0 is rejected because the z-statistic for the sample falls within the critical region.



Step 5 Make a conclusion about the claim.

Therefore, there is enough evidence to reject the claim that the average wait time is 6 minutes or less.

StudyTip

Rejecting the Null Hypothesis When the null hypothesis is rejected, it is implied that the alternative hypothesis is accepted.

GuidedPractice

4. **EXERCISE** Julian thinks it takes less than 60 minutes for people to get in a full workout at the gym. Using a sample of 128 members, he calculated a mean workout time of 58.4 minutes with a standard deviation of 8.9 minutes. Test the hypothesis at 10% significance.

For a two-sided test, the significance level α must be divided by 2 in order to determine the critical value at each tail.

Example 5 Two-Sided Hypothesis Test

ADVERTISEMENTS **Jerrod wants to determine if the advertisement shown is accurate. Using a sample of 42 pizzas, Jerrod calculates a mean of 99.8 pieces and a standard deviation of 0.8. Test the hypothesis at 5% significance.**

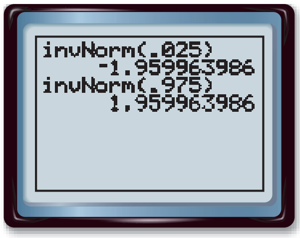
Step 1 State the claim and hypotheses.

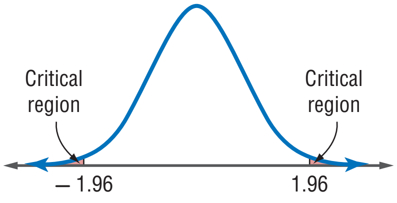
H_0: $\mu = 100$ (claim) and H_a: $\mu \neq 100$.

Step 2 Determine the critical region.

The alternative hypothesis is $\mu \neq 100$, so this is a two-tailed test. We are testing at 5% significance, so we need to identify the z-value that corresponds with upper and lower 2.5% of the distribution.

KEYSTROKES: 2nd [DISTR] 3 0.25 ENTER and 2nd [DISTR] 3 .975 ENTER.





Step 3 Calculate the z-statistic for the sample data.

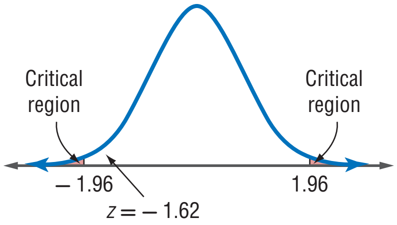
$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$ Formula for z-statistic

$$= \frac{99.8 - 100}{\frac{0.8}{\sqrt{42}}}$$ $\bar{x} = 99.8$, $\mu = 100$, $s = 0.8$, and $n = 42$

$$\approx -1.62$$ Simplify.

Step 4 Reject or fail to reject the null hypothesis.

H_0 is not rejected because the z-value for the sample does not fall within the critical region.



Step 5 Make a conclusion about the claim.

Therefore, there is not enough evidence to reject the claim that there are 100 pieces of pepperoni on every large pizza.

Guided Practice

5. JUICE A manufacturer claims there are 12 ounces of juice in every can. Using a sample of 68 cans, Susan calculated a mean of 11.95 ounces with a standard deviation of 0.15 ounce. Test the hypothesis at 1% significance.

Study Tip

z-Test The z-test can be used when σ is known or when $n \geq 30$.

Watch Out!

Inferential Statistics Inferential statistics are not designed to definitely prove a hypothesis. They depend on probability statements and therefore, may not be correct.

Check Your Understanding

Example 1 **1.** **LUNCH** A sample of 145 high school seniors was asked how many times they go out for lunch per week. The mean number of times was 2.4 with a standard deviation of 0.7. Use a 90% confidence level to calculate the maximum error of estimate.

Example 2 **2.** **PRACTICE** A poll of 233 randomly chosen high school athletes showed that they spend an average of 1.6 hours practicing their sport during the off-season. The standard deviation is 0.5 hour. Determine a 99% confidence interval for the population mean.

Example 3 **Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.**

3. Lori thinks it takes a fast food restaurant less than 2 minutes to serve her meal after she orders it.

4. A snack label states that one serving contains one gram of fat.

5. Mrs. Hart's review game takes at least 20 minutes to complete.

6. The tellers at a bank can complete no more than 18 transactions per hour.

Identify the hypotheses and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

Example 4 **7.** **COMPACT DISCS** A manufacturer of blank compact discs claims that each disc can hold at least 84 minutes of music. Using a sample of 219 compact discs, Cayla calculated a mean time of 84.1 minutes per disc with a standard deviation of 1.9 minutes. Test the hypothesis at 5% significance.

Example 5 **8.** **GOLF TEES** A company claims that each golf tee they produce is 5 centimeters in length. Using a sample of 168 tees, Angelene calculated a mean of 5.1 centimeters with a standard deviation of 0.3. Test the hypothesis at 10% significance.

Practice and Problem Solving

Example 1 **9.** **MUSIC** A sample of 76 albums had a mean run time of 61.3 minutes with a standard deviation of 5.2 minutes. Use a 95% confidence level to calculate the maximum error of estimate.

Example 2 **10.** **COLLEGE** A poll of 218 students at a university showed that they spend 11.8 hours per week studying. The standard deviation is 3.7 hours. Determine a 90% confidence interval for the population mean.

Example 3 **Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.**

11. Julian sends at least six text messages to his best friend every day.

12. A car company states that one of their vehicles gets 27 miles per gallon.

13. A company advertisement states that it takes no more than 2 hours to paint a 200-square-foot room.

14. A singer plays at least 18 songs at every concert.

Identify the hypotheses and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

Example 4 **15.** **PIZZA** A pizza chain promises a delivery time of less than 30 minutes. Using a sample of 38 deliveries, Chelsea calculated a mean delivery time of 29.6 minutes with a standard deviation of 3.9 minutes. Test the hypothesis at 1% significance.

Example 5

16. **CHEESE** A company claims that each package of cheese contains exactly 24 slices. Using a sample of 93 packages, Mr. Matthews calculated a mean of 24.1 slices with a standard deviation of 0.5. Test the hypothesis at 5% significance.

17. **DECISION MAKING** The number of peaches in 40 random cans is shown below. Should the manufacturer place a label on the can promising 12 peaches in every can? Explain your reasoning.

13, 14, 13, 14, 12, 12, 12, 11, 15, 12, 13, 13, 14, 13, 14, 12, 15, 11, 11, 14,
13, 14, 14, 13, 12, 12, 12, 12, 13, 13, 11, 14, 14, 13, 14, 13, 13, 14, 12, 12

18. **COOKIES** The number of chocolate chips in 40 random cookies is shown below. Should the manufacturer place a label on the package promising 20 chips on every cookie? Explain your reasoning.

21, 19, 20, 20, 19, 19, 18, 21, 19, 17, 19, 18, 18, 20, 20, 19, 18, 20, 19, 20,
21, 21, 19, 17, 17, 18, 19, 19, 20, 17, 22, 21, 21, 20, 19, 18, 19, 17, 17, 21

19. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how the confidence interval is affected by the sample size and the confidence level. Consider a sample of data where $\overline{x} = 25$ and $s = 3$.

a. **GRAPHICAL** Graph the 90% confidence interval for $n = 50$, 100, and 250 on a number line.

b. **ANALYTICAL** How does the sample size affect the confidence interval?

c. **GRAPHICAL** Graph the 90%, 95%, and 99% confidence intervals for $n = 150$.

d. **ANALYTICAL** How does the confidence level affect the confidence interval?

e. **ANALYTICAL** How does decreasing the size of the confidence interval affect the accuracy of the confidence interval?

H.O.T. Problems Use Higher-Order Thinking Skills

20. **ERROR ANALYSIS** Tim and Judie want to test whether a delivery service meets their promised time of 45 minutes or less. Their hypotheses are shown below. Is either of them correct?

Tim	Judie
$H_0: \mu < 45$ (claim)	$H_0: \mu \leq 45$
$H_a: \mu \geq 45$	$H_a: \mu > 45$ (claim)

21. **CHALLENGE** A 95% confidence interval for the mean weight of a 20-ounce box of cereal was $19.932 \leq \mu \leq 20.008$ with a sample standard deviation of 0.128 ounces. Determine the sample size that led to this interval.

22. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

If a confidence interval contains the H_0 value of μ, then it is not rejected.

23. **WRITING IN MATH** How can a statistical test be used in a decision-making process?

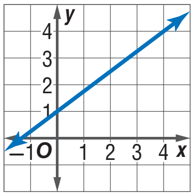
24. **OPEN ENDED** Design and conduct your own research study, and draw conclusions based on the results of a hypothesis test. Write a brief summary of your findings.

Use with Lesson 2-2.

1. **WHICH ONE DOESN'T BELONG?** Of the four items shown, identify the one that does not belong. Explain your reasoning.

$y = 2x + 3$

x	y
0	4
1	2
2	0
3	−2



$y = 2xy$

Use with Lesson 2-5.

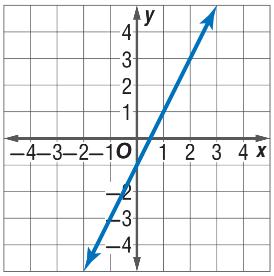
2. **WRITING IN MATH** What are the strengths and weaknesses of using a regression equation to approximate data?

Use with Explore 2-7.

3. What are the domain and range of functions of the form $f(x) = mx + b$, where $m \neq 0$?

Use with Lesson 2-7.

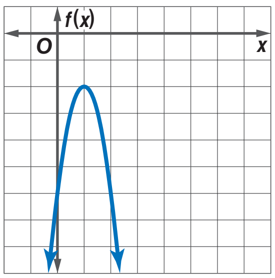
4. **ERROR ANALYSIS** Kimi thinks that the graph and table below are representations of the same linear relation. Carla disagrees. Who is correct? Explain your reasoning.



x	y
0	−1
1	1
2	3
3	5

Use with Lesson 5-1.

5. **ERROR ANALYSIS** Trent thinks that the function $f(x)$ graphed below and the function $g(x)$ described next to it have the same maximum. Madison thinks that $g(x)$ has the greater maximum. Is either of them correct? Explain your reasoning.



$g(x)$ is a quadratic function with roots of 4 and 2 and a y-intercept of −8.

Use with Lesson 6-3.

6. **CHALLENGE** Of $f(x)$ and $g(x)$, which function has more potential real roots? What is the degree of that function?

x	−24	−18	−12	−6	0	6	12	18	24
$f(x)$	−8	−1	3	−2	4	7	−1	−8	5

$$g(x) = x^4 + x^3 - 13x^2 + x + 4$$

Use with Lesson 6-7.

Sketch the graph of each function using its zeros.

7. $f(x) = x^3 - 5x^2 - 2x + 24$
8. $f(x) = 4x^3 + 2x^2 - 4x - 2$
9. $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$
10. $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$

Use with Lesson 7-1.

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in domain or range.

11. $f(x) = x + 2$
    $g(x) = 3x - 1$
12. $f(x) = x^2 - 5$
    $g(x) = -x + 8$
13. $f(x) = 2x$
    $g(x) = -4x + 5$
14. $f(x) = x - 1$
    $g(x) = 5x - 2$
15. $f(x) = x^2$
    $g(x) = -x + 1$
16. $f(x) = 3x$
    $g(x) = -2x + 6$
17. $f(x) = x - 2$
    $g(x) = 2x - 7$
18. $f(x) = x^2$
    $g(x) = x - 5$
19. $f(x) = -x^2 + 6$
    $g(x) = 2x^2 + 3x - 5$
20. $f(x) = 3x^2 - 4$
    $g(x) = x^2 - 8x + 4$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

21. $f = \{(2, 5), (6, 10), (12, 9), (7, 6)\}$
    $g = \{(9, 11), (6, 15), (10, 13), (5, 8)\}$
22. $f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}$
    $g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}$
23. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
    $g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$
24. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
    $g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$
25. $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$
    $g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$
26. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
    $g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$
27. $f = \{(-15, -5), (-4, 12), (1, 7), (3, 9)\}$
    $g = \{(3, -9), (7, 2), (8, -6), (12, 0)\}$
28. $f = \{(-1, 11), (2, -2), (5, -7), (4, -4)\}$
    $g = \{(5, -4), (4, -3), (-1, 2), (2, 3)\}$

Additional Exercises

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

29. $f = \{(7, -3), (-10, -3), (-7, -8), (-3, 6)\}$
$g = \{(4, -3), (3, -7), (9, 8), (-4, -4)\}$

30. $f = \{(1, -1), (2, -2), (3, -3), (4, -4)\}$
$g = \{(1, -4), (2, -3), (3, -2), (4, -1)\}$

31. $f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\}$
$g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\}$

32. $f = \{(12, -3), (9, -2), (8, -1), (6, 3)\}$
$g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\}$

33. $f(x) = -3x$
$g(x) = 5x - 6$

34. $f(x) = x + 4$
$g(x) = x^2 + 3x - 10$

35. $f(x) = 2x$
$g(x) = x + 5$

36. $f(x) = -3x$
$g(x) = -x + 8$

37. $f(x) = x + 5$
$g(x) = 3x - 7$

38. $f(x) = x - 4$
$g(x) = x^2 - 10$

39. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

40. $f(x) = 2x^2 - x + 1$
$g(x) = 4x + 3$

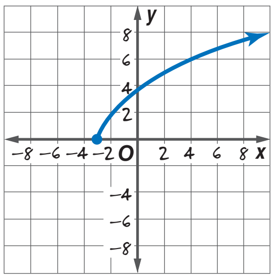
41. $f(x) = 4x - 1$
$g(x) = x^3 + 2$

42. $f(x) = x^2 + 3x + 1$
$g(x) = x^2$

43. $f(x) = 2x^2$
$g(x) = 8x^2 + 3x$

Use with Lesson 7-3.

44. **ERROR ANALYSIS** Cleveland thinks the graph and the equation represent the same function. Molly disagrees. Who is correct? Explain your reasoning.



$y = \sqrt{5x + 10}$

Use with Lesson 8-1.

45. **PHONES** The function $P(x) = 2.28(0.9^x)$ can be used to model the number of pay phones in millions x years since 1999.

a. Classify the function as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.

b. Explain what the $P(x)$-intercept and the asymptote represent in this situation.

46. **HEALTH** Each day, 10% of a certain drug dissipates from system.

a. Classify the function representing this situation as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.

b. How much of the original amount remains in the system after 9 days?

c. If a second dose should not be taken if more than 50% of the original amount is in the system, when should the label say it is safe to redose? Design the label and explain your reasoning.

47. **NUMBER THEORY** A sequence of numbers follows a pattern in which the next number is 125% of the previous number. The first number in the pattern is 18.

a. Write the function that represents the situation.

b. Classify the function as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function for the first 10 numbers.

c. What is the value of the tenth number? Round to the nearest whole number.

48. **ERROR ANALYSIS** Vince and Grady were asked to graph the following functions. Vince thinks they are the same, but Grady disagrees. Who is correct? Explain your reasoning.

x	y
0	2
1	1
2	0.5
3	0.25
4	0.125
5	0.0625
6	0.03125

an exponential function with rate of decay of $\frac{1}{2}$ and an initial amount of 2

Use with Extend 8-3.

49. Write the equation of best fit. What are the domain and range?

Use with Extend 8-8.

50. Do you think the results of the experiment would change if you used an insulated container for the water? What part of the function will change, the constant or the rate of decay? Repeat the experiment to verify your conjecture.

51. How might the results of the experiment change if you added ice to the water? What part of the function will change, the constant or the rate of decay? Repeat the experiment to verify your conjecture.

Use with Lesson 9-2.

52. **REASONING** The sum of any two rational numbers is always a rational number. So, the set of rational numbers is said to be closed under addition. Determine whether the set of rational expressions is closed under addition, subtraction, multiplication, and division by a nonzero rational expression. Justify your reasoning.

Additional Exercises

Use with Lesson 9-3.

53. **MULTIPLE REPRESENTATIONS** In this problem you will investigate the similarities and differences between power functions with positive and negative exponents.

a. **TABULAR** Make a table of values for $a(x) = x^2$, $b(x) = x^{-2}$, $c(x) = x^3$, and $d(x) = x^{-3}$.

b. **GRAPHICAL** Graph $a(x)$ and $b(x)$ on the same coordinate plane.

c. **VERBAL** Compare the domain, range, end behavior, and behavior at $x = 0$ for $a(x)$ and $b(x)$.

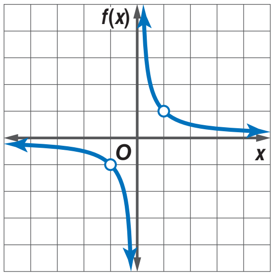
d. **GRAPHICAL** Graph $c(x)$ and $d(x)$ on the same coordinate plane.

e. **VERBAL** Compare the domain, range, end behavior, and behavior at $x = 0$ for $c(x)$ and $d(x)$.

f. **ANALYTICAL** What conclusions can you make about the similarities and differences between power functions with positive and negative exponents?

Use with Lesson 9-4.

54. **CHALLENGE** Compare and contrast $g(x) = \dfrac{x^2 - 1}{x(x^2 - 2)}$ and the function $f(x)$ shown in the graph.



Use with Lesson 10-6.

55. **CHALLENGE** When a plane passes through the vertex of a cone, a *degenerate* conic is formed.

a. Determine the type of conic represented by $4x^2 + 8y^2 = 0$.

b. Graph the conic.

c. Describe the difference between this degenerate conic and a standard conic of the same type with $A = 4$ and $B = 8$.

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Lesson 3 Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. 7, 2

2. 0, 3

3. −5, 8

4. −7, −8

5. −6, −3

6. 3, −4

7. $1, \frac{1}{2}$

8. $\frac{1}{3}, 2$

9. $0, -\frac{7}{2}$

Factor each polynomial.

10. $r^3 + 3r^2 - 54r$

11. $8a^2 + 2a - 6$

12. $c^2 - 49$

13. $x^3 + 8$

14. $16r^2 - 169$

15. $b^4 - 81$

Solve each equation by factoring.

16. $x^2 - 4x - 12 = 0$

17. $x^2 - 16x + 64 = 0$

18. $x^2 - 6x + 8 = 0$

19. $x^2 + 3x + 2 = 0$

20. $x^2 - 4x = 0$

21. $7x^2 = 4x$

22. $10x^2 = 9x$

23. $x^2 = 2x + 99$

24. $x^2 + 12x = -36$

25. $5x^2 - 35x + 60 = 0$

26. $36x^2 = 25$

27. $2x^2 - 8x - 90 = 0$

28. NUMBER THEORY Find two consecutive even positive integers whose product is 624.

29. NUMBER THEORY Find two consecutive odd positive integers whose product is 323.

30. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet.

31. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced?

Lesson 9 Practice

Designing a Study

Determine whether each situation calls for a *survey,* an *experiment*, or an *observational study.* Explain your reasoning.

1. You want to compare the health of students who walk to school to the health of students who ride the bus.

2. You want to find out if people who eat a candy bar immediately before a math test get higher scores than people who do not.

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

3. What is your current age?

4. Do you think teachers should be required to attend all home and away football games?

5. Do you agree or disagree with the following statement?
Teachers should not be required to not supervise students during lunch.

6. Most teenagers text message during class. Are you one of them?

7. A research group wants to conduct an experiment to test the claim that student who use laptops in class have higher standardized test scores. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Lesson 10 Practice

Distributions of Data

1. **KENNAL** The manager of a kennel records the weights for a sample of dogs currently being housed.

Weight (pounds)
31, 67, 8, 37, 12, 87, 14, 34, 105, 57, 42, 8, 16, 54, 17, 20, 72, 23, 27, 63, 24, 52, 14, 44, 27, 5, 28, 22, 33, 15, 6, 36, 41, 21, 46

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

2. **CAMP** The enrollment for a biannual computer camp over the past 15 years is shown.

Number of Participants
45, 68, 55, 25, 48, 36, 61, 52, 31, 8, 41, 58, 40, 55, 68, 47, 60, 28, 44, 63, 18, 68, 50, 57, 37, 16, 56, 40, 50, 68

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

3. **TEMPERATURES** The monthly average low temperatures for two cities are shown.

Astoria, OR	Boston, MA
36, 51, 37, 42, 54, 39, 53, 42, 46, 38, 50, 47	22, 57, 46, 24, 31, 41, 64, 50, 28, 59, 65, 38

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

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Lesson 11 Practice

Probability Distributions

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

1. the number of bytes in the memory of a computer

2. the world population

3. the mass of a banana

4. the speed of a car

5. COINS A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

a. P(2 pennies) **b.** P(2 dimes) **c.** P(1 nickel and 1 dime)

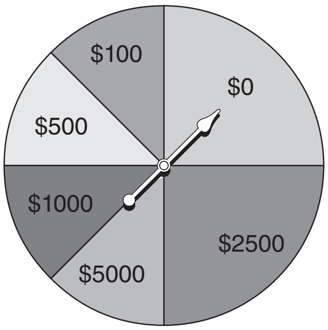
d. P(1 quarter and 1 penny) **e.** P(1 quarter and 1 nicke l) **f.** P(2 dimes and 1 quarter)

6. CARDS Chuck is drawing a card from a special deck that includes the following cards.

Card Value	1	2	3	4	5	6	7
Frequency	6	10	9	4	8	7	6

What is the expected value of the drawn card?

7. GAMES A contestant won two spins of the wheel.



a. Construct a relative-frequency table.

Sum ($)	0	100	200	500	600	1000	1100	1500	2000	2500
Relative Frequency										
Sum ($)	2600	3000	3500	5000	5100	5500	6000	7500	10,000	
Relative Frequency										

b. What is the expected value of two spins?

Lesson 12 Practice

The Binomial Distribution

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state *n*, *p*, and *q*.

1. You randomly remove one card from a deck to see if it is a heart. You place the card back in the deck and repeat the process five times.

2. A bag has 8 blue chips, 5 red chips, and 8 white chips. Four chips are removed without replacement to see how many red chips are removed.

3. BOARD GAME When Tarin and Sam play a certain board game, the probability that Tarin will win a game is $\frac{3}{4}$. If they play 5 games, find each probability.

a. P(Sam wins only once)

b. P(Tarin wins exactly twice)

c. P(Sam wins exactly 3 games)

d. P(Sam wins at least 1 game)

e. P(Tarin wins at least 3 games)

f. P(Tarin wins at most 2 games)

4. SAFETY In August 2001, the American Automobile Association reported that 73% of Americans use seat belts. In a random selection of 10 Americans in 2001, what is the probability that exactly half of them use seat belts?

5. HEALTH In 2001, the American Heart Association reported that 50 percent of the Americans who receive heart transplants are ages 50–64 and 20 percent are ages 35–49.

a. In a randomly selected group of 10 heart transplant recipients, what is the probability that at least 8 of them are ages 50–64?

b. In a randomly selected group of 5 heart transplant recipients, what is the probability that 2 of them are ages 35–49?

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Lesson 13 Practice

The Normal Distribution

A normal distribution has a mean of 186.4 and a standard deviation of 48.9.

1. What range of values represents the middle 99.7% of the data?

2. What percent of data will be more than 235.3?

3. What range of values represents the upper 2.5% of the data?

Find the missing variable. Indicate the position of *X* in the distribution.

4. σ if $\mu = 19, X = 21$, and $z = 1.3$

5. μ if σ $= 9.8, X = 55.4$, and $z = -1.32$

6. X if $z = -2.19, \mu = 68.2$, and $\sigma = 11.6$

7. z if $\mu = 112.4, X = 119.2$, and $\sigma = 11.9$

8. TESTING The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.

a. About what percent of the scores are between 70 and 80?

b. About what percent of the scores are between 85 and 115?

c. About what percent of the scores are over 115?

d. About what percent of the scores are lower than 90 or higher than 100?

e. If 80 people take the test, how many would you expect to score higher than 130?

f. If 75 people take the test, how many would you expect to score lower than 75?

9. TEMPERATURE The daily July surface temperature of a lake at a resort has a mean of 82˚ and a standard deviation of 4.2˚. If you prefer to swim when the temperature is at least 80˚, about what percent of the days does the temperature meet your preference?

Lesson 15 Practice

Confidence Intervals and Hypothesis Testing

Find a 99% confidence interval for each of the following. Round to the nearest tenth if necessary.

1. $\bar{x} = 56$, $s = 2$, and $n = 50$

2. $\bar{x} = 99$, $s = 22$, and $n = 121$

3. $\bar{x} = 34$, $s = 4$, and $n = 200$

4. $\bar{x} = 12$, $s = 4.5$, and $n = 100$

5. $\bar{x} = 37$, $s = 2.5$, and $n = 50$

6. $\bar{x} = 78$, $s = 2$, and $n = 225$

7. $\bar{x} = 36$, $s = 6$, and $n = 36$

8. $\bar{x} = 121$, $s = 2.5$, and $n = 100$

Test each null hypothesis at 1% significance. Write *reject* or *fail to reject*.

9. $H_0 \geq 200.1$, $H_a < 200.1$, $n = 50$, $\bar{x} = 200$, and $s = 2$

10. $H_0 \geq 75.6$, $H_a < 75.6$, $n = 100$, $\bar{x} = 77$, and $s = 7$

11. $H_0 \geq 89.3$, $H_a < 89.3$, $n = 100$, $\bar{x} = 89$ and $s = 1.5$

12 $H_0 \geq 75$, $H_a < 75$, $n = 150$, $\bar{x} = 74.2$, and $s = 2.5$

13. $H_0 \geq 121$, $H_a < 121$, $n = 64$, $\bar{x} = 120$, and $s = 2$

14. $H_0 \leq 198.5$, $H_a > 198.5$, $n = 100$, $\bar{x} = 200$, and $s = 7.5$

15. $H_0 \leq 38.5$, $H_a > 38.5$, $n = 50$, $\bar{x} = 40$, and $s = 4.5$

16. $H_0 \geq 112.5$, $H_a < 112.5$, $n = 100$, $\bar{x} = 110.5$, and $s = 10$

17. RUNNING Josh and his sister Megan run together each morning and do not use a stopwatch to keep track of their time. Josh thinks they usually run the mile under 7 minutes, while Megan thinks it takes them longer. They borrow a stopwatch and time themselves each day for 20 days. Their mean time to run one mile is 7.4 minutes with a standard deviation of 0.2 minutes. Test Megan's hypothesis at 10% significance.

18. QUALITY CONTROL Kim is a quality tester for a tropical fruit company. The company claims that their canned pineapple stays fresh for at least 16 hours after opening. Kim tests 15 different cans to see if they actually stay fresh for at least 16 hours. Use the data below to conduct a hypothesis test at 5% significance.

Number of Hours Each Can Stays Fresh				
12	14	7	12	10
12	12	13	16	9
5	11	19	18	6