

## The public sector and equilibrium GDP

We can add a public sector to the model of GDP by assuming a fixed level of government expenditures,  $G$ , and lump-sum taxes,  $T$ . The addition of taxes to the model introduces another complication: disposable income no longer is identical to GDP. Rather,  $DI = Y - T$ . Here we again make use of the convention that “ $Y$ ” stands for GDP.

Equilibrium still requires that total production equals total purchases, or in this case,  $Y_e = C + I_g + X_n + G$ . As before, consumption is assumed to be a linear function of disposable income:  $C = a + bDI$ . Substituting  $Y - T$  for DI and inserting into the equilibrium relationship, we find

$$Y_e = a + b(Y_e - T) + I_g + X_n + G. \text{ Finally, we solve for } Y_e \text{ to find that } Y_e = \left( \frac{1}{1-b} \right) \cdot (a - bT + I_g + X_n + G).$$

Following the procedure we used in earlier *See the Math* notes, it is apparent that  $\frac{\Delta Y_e}{\Delta G} = \left( \frac{1}{1-b} \right)$

and that  $\frac{\Delta Y_e}{\Delta T} = \left( \frac{-b}{1-b} \right)$ . That is, the change in equilibrium GDP from a one dollar change in

government expenditures is equal to the standard multiplier, while a one dollar increase in lump-sum taxes *decreases* equilibrium GDP by  $b$  (the MPC) times the standard multiplier. Since the MPC is

between zero and one by assumption, it is clear that  $\left| \frac{\Delta Y_e}{\Delta T} \right| < \left| \frac{\Delta Y_e}{\Delta G} \right|$ . That is, the impact on equilibrium

GDP of a change in government spending exceeds the impact of an equal (but in the opposite direction) change in taxes.

Suppose for example that  $C = 97.5 + .75DI$ ,  $I_g = 20$ ,  $X_n = 0$ ,  $G = 20$ , and  $T = 20$ . Following the formula,  $Y_e = \left( \frac{1}{1-.75} \right) \cdot (97.5 - .75 \cdot (20) + 20 + 0 + 20) = 4 \cdot 122.5 = \$490$ . A \$10 increase in

government spending will increase equilibrium GDP by  $\left( \frac{1}{1-.75} \right) \cdot 10 = \$40$ , while a \$10 increase in

lump-sum taxes will *reduce* equilibrium GDP by  $\left( \frac{.75}{1-.75} \right) \cdot 10 = \$30$ .