

Glencoe McGraw-Hill



New York

# IMPACT *Mathematics*

## Implementation Guide



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# Program Philosophy

In developing *IMPACT Mathematics* we have relied on our collective experiences as teachers, parents, and former students. Our main goal is to offer a curriculum that respects the background and knowledge of middle school teachers, recognizes the competence and energy of middle school students, and addresses the need for intellectually challenging and inclusive mathematics materials. With *IMPACT Mathematics*, we have combined the best of what is known as “reform” curricula with the best of “traditional” curricula, incorporating more active involvement on the part of students in making sense of important mathematical ideas.

With middle grades teachers and students in mind, we have created a comprehensive curriculum for Grades 6 through 8 that completes a full year of algebra by the end of Grade 8. While the number and operations, geometry, and data and probability strands were created especially for this program, the algebra strand is based on the highly successful Australian program, *Access to Algebra*, developed by Curriculum Corporation.

The rewarding and interesting introduction to algebra offered by this program can help develop and maintain students’ ongoing interest in all areas of mathematics. The materials created for *IMPACT Mathematics* follow the *Access to Algebra* material in style: use of narrative and realistic contexts, personalization in the form of cartoons in which middle grades students explain how they approach problems, and opportunities for students to choose or create their own problems.

To find  $3\frac{2}{3} - 1\frac{3}{4}$ , I changed both mixed numbers to fractions.

$$3\frac{2}{3} - 1\frac{3}{4} = \frac{11}{3} - \frac{7}{4}$$

I tried to subtract the whole parts first and then the fractions; but since  $\frac{3}{4}$  is greater than  $\frac{2}{3}$ , I couldn't subtract the fractions.

$$\begin{array}{r} 3\frac{2}{3} \\ - 1\frac{3}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 3 \\ - 1 \\ \hline 2 \end{array} \quad \begin{array}{r} \frac{2}{3} \\ - \frac{3}{4} \\ \hline ? \end{array}$$

Then I saw that if I changed  $3\frac{2}{3}$  to  $2\frac{5}{3}$ , I could subtract the fraction parts.

$$\begin{array}{r} 3\frac{2}{3} \\ - 1\frac{3}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 2\frac{5}{3} \\ - 1\frac{3}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 2 \\ - 1 \\ \hline 1 \end{array} \quad \begin{array}{r} \frac{5}{3} \\ - \frac{3}{4} \\ \hline \end{array}$$

## Conceptual Understanding and Basic Skills

Discussions regarding mathematics learning in both professional circles and the popular media might lead you to believe that teaching for conceptual understanding and teaching basic skills are mutually exclusive. But, in fact, the opposite is true. Conceptual understanding and basic skills are not opposing interests; they go hand in hand and support each other.

*IMPACT Mathematics* makes the big ideas as well as the important skills of mathematics accessible to middle school students. It presents mathematical ideas intact, not broken down into bite-sized bits that lack the big idea. *IMPACT Mathematics* helps students both build new mathematical ideas and see how these new ideas relate to ideas they have already developed. In this way, *IMPACT Mathematics* takes a conceptual approach.

At the same time, *IMPACT Mathematics* recognizes that for students to be able to use the new ideas and procedures effectively, they need practice. Practice need not be the enemy of learning; the enemy of learning is mindless drill. Instead, practice can encourage students to stay interested in the mathematical concepts. *IMPACT Mathematics* provides plenty of opportunity for practice, but with variety and contrast to keep students' attention focused.

## Algebraic Focus in a Comprehensive Program

*IMPACT Mathematics* is a comprehensive program including number and operations, proportional reasoning, geometry, probability, and data, with a focus on the development of algebraic thinking. The program takes a developmental approach to algebra. Student understanding of the algebra strand—interwoven with and related to the other mathematical strands—evolves over a three-year period, allowing the ideas and skills to develop and become familiar over time.

Most students develop strong algebraic ideas in the early years of elementary school, but they do not acquire ways of expressing and manipulating them in algebraic terms until later, when algebra is formally taught. For example, young children know how to share \$36 among three people by first distributing the ten dollar bills and then distributing the ones. Later, if children learn a standard method for dividing  $3\overline{)36}$  they may see again that the process is like dividing  $3\overline{)30}$ , then dividing  $3\overline{)6}$ , and finally adding the results. If this process is written out as  $\frac{36}{3} = \frac{30}{3} + \frac{6}{3}$ , that concise statement contains an important idea about adding fractions and an even more general algebraic idea. Students who understand *why*  $\frac{36}{3} = \frac{30}{3} + \frac{6}{3}$  know that the sum of  $\frac{30}{3}$  and  $\frac{6}{3}$  must be  $\frac{36}{3}$ , and not  $\frac{36}{6}$ . The idea, expressed more generally, is  $\frac{a}{3} + \frac{b}{3} = \frac{a+b}{3}$ , and even more generally, is  $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ , and so leads to the distributive law of division over addition.

Our approach in *IMPACT Mathematics* is to start with algebra as a notation for “generic” arithmetic, a description of processes that students understand. Later, algebra also becomes a handy language for “unlocking secrets” (equation solving) and building mathematical models. By the end of Course 3, students will have learned both to express functions using variables and to graph these functions. They will have also learned how to use variables to set up and solve equations, as well as how to factor some familiar polynomials, and to understand the origin and use of the quadratic formula.

### Use of Manipulatives and Calculators

Manipulatives and calculators can be powerful tools for teaching and learning mathematics. There is, however, much discussion and controversy about the appropriateness of their use. As the authors of *IMPACT Mathematics*, we believe that when manipulatives and calculators are used, they must be used to support the content learning. More specifically, we consider the important mathematical ideas first and then determine whether manipulatives or calculators can be used in learning those ideas more completely.

We believe it is critical that students develop good number sense and calculation skills before they work extensively with calculators. For example, we incorporate graphing calculators in Course 3 to explore families of functions, but only *after* students have a firm idea of how to graph “parent” functions by hand. Graphing technology can then be used to allow students to graph more complex functions, analyze their behavior, and compare representations. Similar to our philosophy of integrating skills with understanding, we believe that students need experiences with pencil and paper along with graphing technology.

### Organization by Content

*IMPACT Mathematics* often uses applications to help develop a particular mathematical concept or place it in context. However, *IMPACT Mathematics* remains organized by mathematical content, not by contexts. This organization helps both teacher and student keep the mathematical ideas at the fore, easily recognizable and never buried or lost in the settings. While the mathematical focus shifts with each chapter, the *IMPACT Mathematics* approach offers opportunities to connect topics to one another so that earlier learning is not abandoned as new ideas are introduced.





## Developing Concepts in Varied Contexts

The contexts used for developing concepts and practicing skills include real-world applications, as well as mathematical settings such as number puzzles, and the world of the imagination such as a factory that uniformly resizes objects using stretching machines. Sometimes, *IMPACT Mathematics* provides exercises that are *not* set in contexts or integrated into word problems precisely so that students can focus on the mathematical ideas, undistracted by surrounding material.

## A Final Note

The unique power of mathematics stems from the world of the imagination in which one envisions triangles with perfectly straight sides, or two-dimensional objects embedded in perfectly smooth planes. In the real world, all objects are three dimensional (even a line drawn on paper has thickness, or it wouldn't be visible!), all lines are irregular, and all surfaces are pitted. Likewise, all measurements are only approximations, and no physical object can have an irrational length. Our minds reason well precisely because we can ignore irregularities and focus instead on the essential features. We can reason about quantities that no physical ruler can measure but that we can “measure” with our mental rulers. In sum, we reason well because we can abstract reality.

We, the authors of *IMPACT Mathematics*, recognize that all people, from early childhood on, do reason abstractly, and that what grows over time is both their ability to recognize the abstractions, and the formality with which they are able to express abstractions. We also recognize that mathematics, while not simply common sense, is rooted in common sense. Mathematics is a human product that has developed as an extension and a codification of ways of thinking that are natural to us all. Students must not think of mathematics as a departure from natural, logical thinking. To that aim *IMPACT Mathematics* is written to help students use and sharpen their own logical thinking, learn to be comfortable with the abstractions that give mathematics its power, develop their ideas and mathematical imagination, and acquire the skills that support all that good thinking and the ability to express it clearly to others.

We hope you will enjoy teaching and learning with these materials.



# Research Support

## **The Research Used to Develop *IMPACT Mathematics***

To attain excellence in mathematics learning, there is a need for high-quality curricula that allow students to think deeply about mathematics, require them to explain their ideas, and connect their understanding to other contexts both within and outside of mathematics. *IMPACT Mathematics* relied heavily on key research about mathematics education during its development. Some of these are discussed below.

***Principles and Standards for School Mathematics*** (National Council of Teachers of Mathematics, 1989)

## ***Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics***

(NCTM, 2006)

The National Council of Teachers of Mathematics produced a set of national math standards in 1989 and reshaped those standards in 2006 with the intent of improving mathematics education on a national level. These publications emphasize the belief that all students can and should learn and understand important mathematical ideas. *IMPACT Mathematics* is both a comprehensive program, including the strands of number and operations, proportional reasoning, geometry, probability, and data, as well as a program focused on the development of algebraic thinking.

## ***Algebra for Everyone*** (NCTM, 1990)

*Algebra for Everyone* put forth the view of algebra as the gateway course, a course that must be part of the basic knowledge of all. Therefore, algebra must be taught on a broadened scale, where students come into it with the appropriate mathematical background and disposition.

## ***Algebra in the K–12 curriculum: Dilemmas and possibilities*** (NCTM Algebra Working Group, 1995)

This report examined the research evidence pointing to the inaccessibility of the traditional algebra curriculum, generally taught as a stand-alone course in the 9th grade. In *IMPACT Mathematics*, student understanding of the algebra strand—interwoven with and related to the other mathematical strands—evolves over a three-year period, allowing such important ideas as patterns, functions, proportional reasoning, and algebraic structure and skills to develop and become familiar over time. The algebra strand is based on the highly successful Australian program, *Access to Algebra*, developed by Curriculum Corporation. This program provided an algebra curriculum relevant to students' lives, more inclusive of the interests and experiences of middle school students.

## ***The National Forum to Accelerate Middle-Grades Reform***

The Forum identified three components of academically excellent curricula: academic rigor, equity, and developmental appropriateness. The main goal of *IMPACT Mathematics* is to offer a curriculum that respects the background and knowledge of middle school teachers, recognizes the competence and energy of middle school

students, and addresses the need for intellectually challenging and inclusive materials.

*IMPACT Mathematics* has combined the best of research on “reform” curricula with the best of “traditional” curricula, incorporating more active involvement on the part of students in making sense of important mathematical ideas.

## Evidence of Effectiveness:

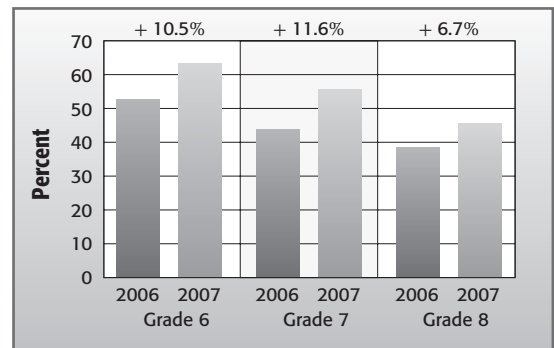
### New York City Mathematics Results

Since the adoption of *IMPACT Mathematics* in grades 6–8, New York City students have seen their test scores improve dramatically. When compared against other students in the state, the positive trend is immediately apparent.

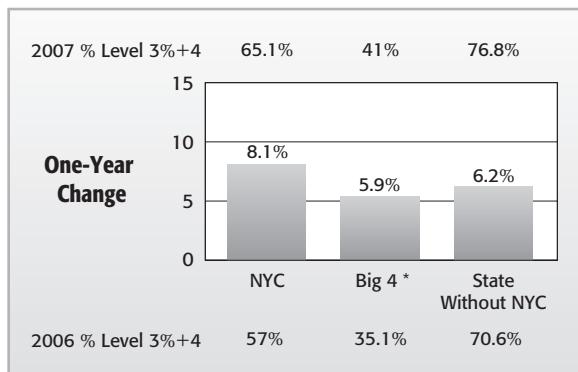
The graph in Figure 1 shows the performance on New York mathematics tests of students who used *IMPACT Mathematics*.

- In **Grade 6** in 2007, **10.5%** more students performed at Levels 3 and 4 on the state test than in the previous year.
- In **Grade 7**, **11.6% more** students performed at the highest levels.
- In **Grade 8**, **6.7% more** students performed at higher levels than their 2006 peers.

**Figure 1**  
Students’ Performance in Levels 3 and 4  
Percent Increase in Grades 6–8



**Figure 2**  
New York State Mathematics  
Achievement Comparisons  
(Grades 3–8 at Levels 3 and 4)



\*Buffalo, Rochester, Syracuse, and Yonkers

The graph Figure 2 shows the comparison of students in New York City (where *IMPACT Mathematics* is the core mathematics curriculum) to their peers in other large cities within the state of New York and to New York State on the whole.

In 2007, the number of students using *IMPACT Mathematics* in New York City who performed at the top levels on state tests increased by 8.1% from 2006. The rest of the state (where other math programs are used) did not see such gains.

# Mathematics Strands

Although *IMPACT Mathematics* provides the equivalent of a first-year algebra program, it is a truly integrated curriculum, incorporating content not only from algebra but also from number and operation, geometry, and probability and statistics. Although most chapters emphasize particular strands, the strands are connected and integrated throughout the program.

## Algebra

*IMPACT Mathematics* covers the equivalent of a first year algebra course. All three courses cover symbolic manipulation of expressions, multiple representations of algebraic relationships, and equation solving. The program makes algebra accessible to middle school students by using a unique exposure-to-mastery approach. Ideas are introduced informally at early stages and then revisited with increasing formality and depth. The following example shows how ideas about graphical representations of functions are carefully developed over the three courses.

## Geometry

In Course 1, the focus is on two-dimensional geometry. Students learn terminology and basic properties associated with polygons and develop formulas for area and perimeter of two-dimensional figures. In Course 2, the focus shifts to three-dimensional geometry. Students gain experience visualizing and representing three-dimensional figures and they develop formulas for surface area and volume. Course 3 introduces students to symmetry and geometric transformations both on and off the coordinate plane.

## Number and Operation

The number and operation strand is most prominent in Courses 1 and 2 of *IMPACT Mathematics*. Course 1 builds a thorough understanding of fractions, decimals, and percents and the relationships among these representations. Course 1 also helps students develop and apply algorithms for fraction operations, and reviews methods for finding products and quotients of decimals. In Course 2, students investigate algorithms for operating with signed numbers. They also learn about operating with integer exponents and develop a sense of very large numbers. In addition, Course 2 emphasizes ratio, proportion, and percent and includes problems that require students to apply proportional reasoning. Course 3 extends student understanding of exponents, and makes connections between exponents and roots.

## Probability and Statistics

In Course 1, students determine and compare experimental and theoretical probabilities, devise game winning strategies, learn to design simple simulations, and explore measures of center and statistical displays. In Course 2, students are introduced to probabilities of dependent events, focus on sampling, and circle graphs. They analyze games to determine whether they are fair. In Course 3, students develop strategies for counting outcomes to determine the size of a sample space and look at probability distributions for various situations.

# NCTM Standards

## Process Standards

### Problem Solving

The *IMPACT Mathematics* curriculum is centered around problem sets that students work on individually or in groups. Many of the problems are open-ended, allowing students to choose or develop solution strategies. The Think & Discuss and Share & Summarize features allow students to reflect on their strategies and to learn about a variety of strategies presented by other students. Cartoons and Examples throughout the book model problem-solving strategies. Optional Inquiry Investigations provide more extended explorations, in which students make predictions based on observations or collected data, and then use mathematics to either support or disprove their predictions.

### Reasoning and Proof

Throughout *IMPACT Mathematics*, students are asked to make conjectures based on patterns they observe and to develop convincing mathematical arguments. This begins early in Course 1, in which students try to explain why certain number “tricks” work. By Course 2, students are able to use their knowledge of algebra to write more formal proofs. In *IMPACT Mathematics*, students learn that proving a conjecture means showing it is true in all cases, not just in specific instances. They also learn that a single counterexample shows that a conjecture is not true.

### Communication

*IMPACT Mathematics* provides many opportunities for students to reflect upon, critique, and communicate their ideas. Many of the exercise sets are done in groups. The Think & Discuss and Share & Summarize features provide opportunities for students to share their ideas with both the teacher and the class. Written communication is also an important part of

*IMPACT Mathematics*. Many problems and homework exercises ask students to explain their answers and strategies. Every homework section includes an “In Your Own Words” feature that asks students to summarize a key idea from the chapter in writing.

### Connections

Making connections is an important part of *IMPACT Mathematics*. The concepts in each chapter connect to, and build on, concepts developed in earlier chapters and courses. A Big Picture chart at the beginning of each Teacher Guide chapter shows how the main ideas in a particular chapter connect to past and future work. The program also connects the various mathematical strands. For example, students look at geometric representations of algebraic concepts such as the distributive property and use algebra to generalize geometric patterns. Connections are also made between mathematics and other academic disciplines and between mathematics and the real world. For example, students explore how similar types of repeating patterns occur in the weather, ocean tides, and the movement of a bicycle tire.

### Representation

*IMPACT Mathematics* emphasizes a variety of mathematical representations including written descriptions, equations, graphs, and tables. For example, in addition to algebraic methods for solving equations, students learn how to estimate solutions using graphs and tables. They learn how to move smoothly among representations and to choose the representation that best suits a particular purpose. Modeling is also a focus of *IMPACT Mathematics*. Students model real-life situations with equations, graphs, and physical models. And, they create and run simulations of situations involving chance.

# Content Standards

## COURSE 1

	Number & Operations	Algebra	Geometry	Measurement	Data Analysis and Probability
Polygons, Angles, and Circles					
Fractions and Decimals					
Patterns, Numbers, and Rule					
Fraction and Decimal Operations					
Rate, Ratio, and Proportion					
Percents					
Area, Volume, and Capacity					
Coordinate Plane					
Equations					
Data and Probability					

## COURSE 2

	Number & Operations	Algebra	Geometry	Measurement	Data Analysis and Probability
Expressions					
Exponents					
Signed Numbers					
Magnitude of Numbers					
Geometry in Three Dimensions					
Data and Probability					
Real Numbers					
Linear Relationships					
Equations					
Proportional Reasoning and Percents					

## COURSE 3

	Number & Operations	Algebra	Geometry	Measurement	Data Analysis and Probability
Linear Relationships					
Lines and Angles					
Percents and Proportions					
Exponents and Exponential Variation					
Algebraic Expressions					
Transformational Geometry					
Inequalities and Linear Systems					
Quadratic and Inverse Relationships					
Solve Quadratic Equations					
Functions and Their Graphs					
Data and Probability					
Algebraic Fractions					

Primary Focus
  Secondary Focus

# NCTM Focal Points Correlation Charts

The Curriculum Focal Points identify key mathematical ideas for Grade K-8. They are not discrete topics or a checklist to be mastered; rather, they provide a framework for the majority of instruction at a particular grade level and the foundation for future mathematical study. The complete document can be found at [www.nctm.org/focalpoints](http://www.nctm.org/focalpoints) Also, see pages 14–19 for a complete listing of the NCTM Focal Points and an explanation of the abbreviations used below.

## Course 1

Chapter	Focal Points and Connections
Chapter 1	G6-FP1, G6-FP6C
Chapter 2	G6-FP1, G6-FP4C
Chapter 3	G6-FP3, G6-FP5C
Chapter 4	G6-FP1, G6-FP4C
Chapter 5	G6-FP2
Chapter 6	G6-FP1
Chapter 7	G6-FP1, G6-FP6C
Chapter 8	G6-FP3
Chapter 9	G6-FP3
Chapter 10	G6-FP2, G6-FP4C

## Course 2

Chapter	Focal Points and Connections
Chapter 1	G7-FP3
Chapter 2	G7-FP3
Chapter 3	G7-FP3
Chapter 4	G7-FP3
Chapter 5	G7-FP2, G7-FP4C
Chapter 6	G7-FP6C, G7-FP7C
Chapter 7	G7-FP3, G7-FP5C
Chapter 8	G7-FP1, G7-FP3, G7-FP5C
Chapter 9	G7-FP3, G7-FP5C
Chapter 10	G7-FP1, G7-FP4C

## Course 3

Chapter	Focal Points and Connections
Chapter 1	G8-FP1, G8-FP5C
Chapter 2	G8-FP2
Chapter 3	G8-FP4C
Chapter 4	G8-FP4C, G8-FP7C
Chapter 5	G8-FP1, G8-FP2
Chapter 6	G8-FP2
Chapter 7	G8-FP1
Chapter 8	G8-FP4C
Chapter 9	G8-FP4C
Chapter 10	G8-FP1
Chapter 11	G8-FP3, G8-FP6C
Chapter 12	G8-FP1, G8-FP4C

## NCTM Focal Points Grade 6

The Curriculum Focal Points identify key mathematical ideas for this grade. They are not discrete topics or a checklist to be mastered; rather, they provide a framework for the majority of instruction at a particular grade level and the foundation for future mathematics study. The complete document may be viewed at [www.nctm.org/focalpoints](http://www.nctm.org/focalpoints).

### **G6-FP1 Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals**

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.

### **G6-FP2 Number and Operations: Connecting ratio and rate to multiplication and division**

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.

### **G6-FP3 Algebra: Writing, interpreting, and using mathematical expressions and equations**

Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. They construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as  $3x = y$ ) shown in a table.



## Connections to the Focal Points

**G6-FP4C Number and Operations:** Students' work in dividing fractions shows them that they can express the result of dividing two whole numbers as a fraction (viewed as parts of a whole). Students then extend their work in grade 5 with division of whole numbers to give mixed number and decimal solutions to division problems with whole numbers. They recognize that ratio tables not only derive from rows in the multiplication table but also connect with equivalent fractions. Students distinguish multiplicative comparisons from additive comparisons.

**G6-FP5C Algebra:** Students use the commutative, associative, and distributive properties to show that two expressions are equivalent. They also illustrate properties of operations by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are  $x + 3$  by 5). Sequences, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to develop formulas.

**G6-FP6C Measurement and Geometry:** Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend the students' work in grade 5 on area and volume and provide a context for applying new work with equations.

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## NCTM Focal Points Grade 7

The Curriculum Focal Points identify key mathematical ideas for this grade. They are not discrete topics or a checklist to be mastered; rather, they provide a framework for the majority of instruction at a particular grade level and the foundation for future mathematics study. The complete document may be viewed at [www.nctm.org/focalpoints](http://www.nctm.org/focalpoints).

### **G7-FP1 Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity**

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ( $\frac{y}{x} = k$ , or  $y = kx$ ) from other relationships, including inverse proportionality ( $xy = k$ , or  $y = \frac{k}{x}$ ).

### **G7-FP2 Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes**

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes (*Volume = Area of base  $\times$  Height*). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.

### **G7-FP3 Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations**

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

## Connections to the Focal Points

**G7-FP4C Measurement and Geometry:** Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.

**G7-FP5C Number and Operations:** In grade 4, students used equivalent fractions to determine the decimal representations of fractions that they could represent with terminating decimals. Students now use division to express any fraction as a decimal, including fractions that they must represent with infinite decimals. They find this method useful when working with proportions, especially those involving percents. Students connect their work with dividing fractions to solving equations of the form  $ax = b$ , where  $a$  and  $b$  are fractions. Students continue to develop their understanding of multiplication and division and the structure of numbers by determining if a counting number greater than 1 is a prime, and if it is not, by factoring it into a product of primes.

**G7-FP6C Data Analysis:** Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs.

**G7-FP7C Probability:** Students understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.

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## NCTM Focal Points Grade 8

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### **G8-FP1 Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations**

Students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize a proportion ( $y/x = k$ , or  $y = kx$ ) as a special case of a linear equation of the form  $y = mx + b$ , understanding that the constant of proportionality ( $k$ ) is the slope and the resulting graph is a line through the origin. Students understand that the slope ( $m$ ) of a line is a constant rate of change, so if the input, or  $x$ -coordinate, changes by a specific amount,  $a$ , the output, or  $y$ -coordinate, changes by the amount  $ma$ . Students translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and  $y$ -intercept appear in different representations. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.

### **G8-FP2 Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle**

Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.

### **G8-FP3 Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets**

Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose.

## NCTM Focal Points

- G8-FP4C Algebra:** Students encounter some nonlinear functions (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions. They view arithmetic sequences, including those arising from patterns or problems, as linear functions whose inputs are counting numbers. They apply ideas about linear functions to solve problems involving rates such as motion at a constant speed.
- G8-FP5C Geometry:** Given a line in a coordinate plane, students understand that all “slope triangles”—triangles created by a vertical “rise” line segment (showing the change in  $y$ ), a horizontal “run” line segment (showing the change in  $x$ ), and a segment of the line itself—are similar. They also understand the relationship of these similar triangles to the constant slope of a line.
- G8-FP6C Data Analysis:** Building on their work in previous grades to organize and display data to pose and answer questions, students now see numerical data as an aggregate, which they can often summarize with one or several numbers. In addition to the median, students determine the 25th and 75th percentiles (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information. Students make scatterplots to display bivariate data, and they informally estimate lines of best fit to make and test conjectures.
- G8-FP7C Number and Operations:** Students use exponents and scientific notation to describe very large and very small numbers. They use square roots when they apply the Pythagorean theorem.

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# Correlation of *IMPACT Mathematics, Course 1* to the New York State Grade 6 Standards

Pre-March section		
<b>Chapter 1 Polygons, Angles, and Circles</b>		
1-3	6.G.5, 6.G.6, 6.G.7, 6.G.9	Measure Around
<b>Chapter 2 Fractions and Decimals</b>		
2-1	6.N.14, 6.N.21, 6.N.27	Patterns in Fractions
2-2	6.N.14	Patterns in Decimals
2-3	6.N.20, 6.N.21, 6.N.22	Fraction and Decimal Equivalents
<b>Chapter 3 Patterns, Numbers, and Rules</b>		
3-1	6.N.1, 6.N.23, 6.N.24	Number Sense
3-2	6.N.4, 6.N.22, 6.N.23, 6.N.25, 6.G.2	Patterns
3-3	6.N.5, 6.N.25, 6.A.1, 6.A.2	Variables and Rules
3-4	6.N.2, 6.N.3, 6.N.4	Apply Properties
<b>Chapter 4 Fraction and Decimal Operations</b>		
4-1	6.N.16, 6.N.18	Add and Subtract Fractions
4-2	6.N.17, 6.N.18, 6.N.19, 6.N.27	Multiply and Divide Fractions
4-3	6.N.17	Multiply and Divide Decimals
4-4	6.S.5, 6.S.6	What is Typical?
<b>Chapter 5 Rate, Ratio, and Proportion</b>		
5-1	6.N.6, 6.N.8	Ratios and Rates
5-2	6.N.7, 6.N.9, 6.N.10	Proportions
5-3	6.G.1	Similarity and Congruence
<b>Chapter 6 Percents</b>		
6-1	6.N.11, 6.N.14, 6.N.15, 6.N.21, 6.N.26	Use Percents
6-2	6.N.11	Percent of a Quantity
6-3	6.N.12	Percents and Wholes

Chapter 7 Area, Volume, and Capacity		
7-1	6.N.22, 6.N.23, 6.G.2	Squares
7-2	6.A.6, 6.G.2, 6.G.3, 6.G.5, 6.G.7, 6.G.8	Calculate Areas
7-3	6.G.4, 6.M.1, 6.M.7, 6.M.8	Surface Area and Volume
7-4	6.M.2, 6.M.3, 6.M.4, 6.M.5, 6.M.6, 6.M.9	Capacity
<b>Chapter 8 Coordinate Plane</b>		
8-1	6.S.7, 6.S.8	Interpret Graphs
8-3	6.N.13	Graph in Four Quadrants
<b>Chapter 10 Data and Probability</b>		
10-1	6.G.4, 6.G.7	Data Displays
10-2	6.S.8	Collect and Analyze Data

Optional Lessons section		
Chapter 1 Polygons, Angles, and Circles		
1-1	5.G.4, 5.G.6 (prerequisite for Course 1)	Patterns in Geometry
1-2	5.M.8 (prerequisite for Course 1)	Angles

Post-March section		
<b>Chapter 3 Patterns, Numbers, and Rules</b>		
3-3	6.A.2, 6.A.3	Variables and Rules
<b>Chapter 5 Rate, Ratio, and Proportion</b>		
5-2	6.A.5	Proportions
<b>Chapter 8 Coordinate Plane</b>		
8-1	6.S.4	Interpret Graphs
8-2	6.G.10, 6.S.4	Draw and Label Graphs
8-3	6.G.10, 6.G.11	Graph in Four Quadrants
<b>Chapter 9 Equations</b>		
9-1	6.A.3	Understand Equations
9-2	6.A.3, 6.A.4	Backtracking
9-3	6.A.3, 6.A.4	Guess-Check-and-Improve
<b>Chapter 10 Data and Probability</b>		
10-1	6.S.3	Data Displays
10-2	6.S.1, 6.S.2, 6.S.4	Collect and Analyze Data
10-3	6.S.10	The Language of Chance
10-4	6.S.9, 6.S.10, 6.S.11	Make Matches

# Correlation of the New York State Grade 6 Standards to *IMPACT Mathematics, Course 1*

Pre-March section		
Standard	Performance Indicator	Course 1
6.N.1	Read and write whole numbers to trillions	Lesson 3.1
6.N.2	Define and identify the commutative and associative properties of + and $\times$	Lesson 3.4
6.N.3	Define and identify the distributive property of multiplication over addition	Lesson 3.4
6.N.4	Define and identify the identity and inverse properties of addition and multiplication	Lesson 3.2, Lesson 3.4
6.N.5	Define and identify the zero property of multiplication	Lesson 3.3
6.N.6	Understand the concept of rate	Lesson 5.1
6.N.7	Express equivalent ratios as a proportion	Lesson 5.2
6.N.8	Distinguish the difference between rate and ratio	Lesson 5.1
6.N.9	Solve proportions using equivalent fractions	Lesson 5.2
6.N.10	Verify the proportionality using the product of the means equals the product of the extremes	Lesson 5.2
6.N.11	Read, write, and identify percents of a whole (0% to 100%)	Lesson 6.1, Lesson 6.2
6.N.12	Solve percent problems involving percent, rate, and base	Lesson 6.3
6.N.13	Define absolute value and determine the absolute value of rational numbers (including positive and negative)	Lesson 8.3

6.N.14	Locate rational numbers on a number line (including positive and negative)	Lesson 2.1, Lesson 2.2, Lesson 6.1
6.N.15	Order rational numbers (including positive and negative)	Lesson 6.1
6.N.16	Add and subtract fractions with unlike denominators	Lesson 4.1
6.N.17	Multiply and divide fractions with unlike denominators	Lesson 4.2, Lesson 4.3
6.N.18	Add, subtract, multiply and divide mixed numbers with unlike denominators	Lesson 4.1, Lesson 4.2
6.N.19	Identify the multiplicative inverse (reciprocal) of a number	Lesson 4.2
6.N.20	Represent fractions as terminating or repeating decimals	Lesson 2.3
6.N.21	Find multiple representations of rational numbers (fractions, decimals, and percents 0 to 100)	Lesson 2.1, Lesson 2.3, Lesson 6.1
6.N.22	Evaluate numerical expressions using order of operations (may include exponents of two and three)	Lesson 2.3, Lesson 3.2, Lesson 7.1
6.N.23	Represent repeated multiplication in exponential form	Lesson 3.1, Lesson 3.2, Lesson 7.1
6.N.24	Represent exponential form as repeated multiplication	Lesson 3.1
6.N.25	Evaluate expressions having exponents where the power is an exponent of one, two, or three	Lesson 3.2, Lesson 3.3
6.N.26	Estimate a percent of quantity (0% to 100%)	Lesson 6.1

6.N.27	Justify the reasonableness of answers using estimation (including rounding)	Lesson 2.1, Lesson 4.2
6.A.1	Translate two-step verbal expressions into algebraic expressions	Lesson 3.3
6.A.2	Use substitution to evaluate algebraic expressions (may include exponents of one, two and three)	Lesson 3.3
6.A.6	Evaluate formulas for given input values (circumference, area, volume, distance, temperature, interest, etc.)	Lesson 7.2
6.G.1	Calculate the length of corresponding sides of similar triangles, using proportional reasoning	Lesson 5.3
6.G.2	Determine the area of triangles and quadrilaterals (squares, rectangles, rhombi, and trapezoids) and develop formulas	Lesson 3.2, Lesson 7.1, Lesson 7.2
6.G.3	Use a variety of strategies to find the area of regular and irregular polygons	Lesson 7.2
6.G.4	Determine the volume of rectangular prisms by counting cubes and develop the formula	Lesson 7.3, Lesson 10.1
6.G.5	Identify radius, diameter, chords and central angles of a circle	Lesson 1.3, Lesson 7.2
6.G.6	Understand the relationship between the diameter and radius of a circle	Lesson 1.3
6.G.7	Determine the area and circumference of a circle, using the appropriate formula	Lesson 1.3, Lesson 7.2, Lesson 10.1

6.G.8	Calculate the area of a sector of a circle, given the measure of a central angle and the radius of the circle	Lesson 7.2
6.G.9	Understand the relationship between the circumference and the diameter of a circle	Lesson 1.3
6.M.1	Measure capacity and calculate volume of a rectangular prism	Lesson 7.3
6.M.2	Identify customary units of capacity (cups, pints, quarts, and gallons)	Lesson 7.4
6.M.3	Identify equivalent customary units of capacity (cups to pints, pints to quarts, quarts to gallons)	Lesson 7.4
6.M.4	Identify metric units of capacity (liter and milliliter)	Lesson 7.4
6.M.5	Identify equivalent metric units of capacity (milliliter to liter and liter to milliliter)	Lesson 7.4
6.M.6	Determine the tool and technique to measure with an appropriate level of precision: capacity	Lesson 7.4
6.M.7	Estimate volume, area, and circumference (see figures identified in geometry strand)	Lesson 7.3
6.M.8	Justify the reasonableness of estimates	Lesson 7.3
6.M.9	Determine personal references for capacity	Lesson 7.4
6.S.5	Determine the mean, mode, and median for a given set of data	Lesson 4.4
6.S.6	Determine the range for a given set of data	Lesson 4.4
6.S.7	Read and interpret graphs	Lesson 8.1
6.S.8	Justify predictions made from data	Lesson 8.1, Lesson 10.2

Post-March section		
6.A.2	Use substitution to evaluate algebraic expressions (may include exponents of one, two and three)	Lesson 3.3
6.A.3	Translate two-step verbal sentences into algebraic equations	Lesson 3.3, Lesson 9.2, Lesson 9.3
6.A.4	Solve and explain two-step equations involving whole numbers using inverse operations	Lesson 9.1, Lesson 9.2, Lesson 9.3
6.A.5	Solve simple proportions within context	Lesson 5.2
6.G.10	Identify and plot points in all four quadrants	Lesson 8.2, Lesson 8.3
6.G.11	Calculate the area of basic polygons drawn on a coordinate plane (rectangles and shapes composed of rectangles having sides with integer lengths)	Lesson 8.3
6.S.1	Develop the concept of sampling when collecting data from a population and decide the best method to collect data for a particular question	Lesson 10.2
6.S.2	Record data in a frequency table	Lesson 10.2
6.S.3	Construct Venn diagram to sort data	Lesson 10.1

6.S.4	Determine and justify the most appropriate graph to display a given set of data (pictograph, bar graph, line graph, histogram, or circle graph)	Lesson 8.1, Lesson 8.2, Lesson 10.2
6.S.9	List possible outcomes for compound events	Lesson 10.4
6.S.10	Determine the probability of dependent events	Lesson 10.3, Lesson 10.4
6.S.11	Determine the number of possible outcomes for a compound event by using the fundamental counting principle and use this to determine the probabilities of events when the outcomes have equal probability	Lesson 10.4



# Correlation of *IMPACT Mathematics*, Course 2 to the New York State Grade 7 Standards

Pre-March section		
Chapter 1	Expressions	
1-1	7.N.11, 7.A.1	Variables and Expressions
1-2	7.G.1, 7.G.7, 7.A.1, 7.A.6	Expressions and Formulas
Chapter 2	Exponents	
2-1	7.N.8, 7.N.9, 7.N.10	Factors and Multiples
2-2	7.N.4, 7.N.11	Exponent Machines
2-3	7.N.4	More Exponent Machines
Chapter 3	Signed Numbers	
3-1	7.N.12, 7.A.5, 7.N.13, 7.N.19, 7.G.10	Add and Subtract with Negative Numbers
3-2	7.N.12, 7.S.4	Multiply and Divide with Negative Numbers
Chapter 4	Magnitude of Numbers	
4-1	7.N.5, 7.N.6, 7.N.7, 7.M.10	Scientific Notation
4-2	7.N.5, 7.N.6, 7.N.14	Negative Exponents
Chapter 5	Geometry in Three Dimensions	
5-1	7.G.2, 7.G.4, 7.M.11	Surface Area and Volume
5-2	7.G.2, 7.G.3, 7.G.4	Nets and Solids
5-3	7.M.2, 7.M.3, 7.M.4, 7.M.9, 7.M.12, 7.M.13	Mass and Weight
Chapter 6	Data and Probability	
6-1	7.S.1, 7.S.8, 7.S.10	Dependence
6-2	7.S.1, 7.S.8, 7.S.9, 7.S.11, 7.S.12	Make Predictions
6-3	7.S.1, 7.S.2, 7.S.3, 7.S.5, 7.S.6, 7.S.7, 7.M.8	Data Graphs

Chapter 7	Real Numbers	
7-1	7.N.1, 7.N.3	Rational Numbers
7-2	7.N.2, 7.N.15, 7.N.16, 7.N.17, 7.N.18, 7.N.19	Irrational Numbers
Chapter 9	Equations	
9-2	7.A.1	A Model for Solving Equations
9-3	7.A.5, 7.G.10	Solve Equations

Optional Lessons section		
Chapter 10	Proportional Reasoning and Percents	
10-1	6.N.6, 6.N.7, 6.N.8	Ratios
10-3	8.N.3, 8.N.4, 8.N.5	Percents and Proportions

Post-March section		
Chapter 1	Expressions	
1-1	7.A.8	Variables and Expressions
1-3	7.A.2, 7.A.3, 7.A.4	The Distributive Property
Chapter 7	Real Numbers	
7-3	7.G.5, 7.G.6, 7.G.8, 7.G.9	The Pythagorean Theorem
Chapter 8	Linear Relationships	
8-1	7.A.7, 7.A.10	Rates
8-2	7.A.7, 7.A.8, 7.A.10	Speed and Slope
8-3	7.A.9, 7.A.10	Recognize Linear Relationships
Chapter 9	Equations	
9-1	7.A.4	Find a Solution Method
9-3	7.A.4	Solve Equations
9-4	7.A.4	Solve Equations with Parentheses
Chapter 10	Proportional Reasoning and Percents	
10-2	7.M.1	Proportions and Similarity
10-4	7.M.5, 7.M.6, 7.M.7	Rates

# Correlation of the New York State Grade 7 Standards to *IMPACT Mathematics, Course 2*

Pre-March section		
Standard	Performance Indicator	Course 2
7.N.1	Distinguish between the various subsets of real numbers (counting/natural numbers, whole numbers, rational numbers, and irrational numbers)	Lesson 7.1
7.N.2	Recognize the difference between rational and irrational numbers (explore different approximations of $\pi$ )	Lesson 7.2
7.N.3	Place rational and irrational numbers (approximations) on a number line and justify the placement of the numbers	Lesson 7.1
7.N.4	Develop the laws of exponents for multiplication and division	Lesson 2.2, Lesson 2.3
7.N.5	Write numbers in scientific notation	Lesson 4.1, Lesson 4.2
7.N.6	Translate numbers from scientific notation into standard form	Lesson 4.1, Lesson 4.2
7.N.7	Compare numbers written in scientific notation	Lesson 4.1
7.N.8	Find the common factors and greatest common factor of two or more numbers	Lesson 2.1
7.N.9	Determine multiples and least common multiple of two or more numbers	Lesson 2.1
7.N.10	Determine the prime factorization of a given number and write in exponential form	Lesson 2.1
7.N.11	Simplify expressions using order of operations (expressions may include absolute value and/or integral exponents greater than 0)	Lesson 1.1, Lesson 2.2

7.N.12	Add, subtract, multiply, divide integers	Lesson 3.1, Lesson 3.2
7.N.13	Add and subtract two integers (with and without the use of a number line)	Lesson 3.1
7.N.14	Develop a conceptual understanding of negative and zero exponents with a base of ten and relate to fractions and decimals ( $10^2 = .01 = 1/100$ )	Lesson 4.2
7.N.15	Recognize and state the value of the square root of a perfect square (up to 225)	Lesson 7.2
7.N.16	Determine the square root of non-perfect squares using a calculator	Lesson 7.2
7.N.17	Classify irrational numbers as non-repeating/non-terminating decimals	Lesson 7.2
7.N.18	Identify the two consecutive whole numbers between which the square root of a non-perfect square whole number less than 225 lies (with and without the use of a number line)	Lesson 7.2
7.N.19	Justify the reasonableness of answers using estimation	Lesson 3.1, Lesson 7.2
7.A.1	Translate two-step verbal expressions into algebraic expressions	Lesson 1.1, Lesson 1.2
7.A.5	Solve one-step inequalities (positive coefficients only) (See 7.G.10)	Lesson 3.1, Lesson 9.3
7.A.6	Evaluate formulas for given input values (surface area, rate, and density problems)	Lesson 1.2
7.G.1	Calculate the radius or diameter, given the circumference or area of a circle	Lesson 1.2

7.G.2	Calculate the volume of prisms and cylinders, using a given formula and a calculator	Lesson 5.1, Lesson 5.2
7.G.3	Identify the two-dimensional shapes that make up the faces and bases of three-dimensional shapes (prisms, cylinders, cones and pyramids)	Lesson 5.2
7.G.4	Determine the surface area of prisms and cylinders using a calculator and a variety of methods	Lesson 5.1, Lesson 5.2
7.G.7	Find a missing angle when given angles of a quadrilateral	Lesson 1.2
7.G.10	Graph the solution set of an inequality (positive coefficients only) on a number line	Lesson 3.1, Lesson 9.3
7.M.2	Convert capacities and volumes within a given system	Lesson 5.3
7.M.3	Identify customary and metric units of mass	Lesson 5.3
7.M.4	Convert mass within a given system	Lesson 5.3
7.M.8	Draw central angles in a given circle using a protractor (circle graphs)	Lesson 6.3
7.M.9	Determine the tool and technique to measure with an appropriate level of precision: mass	Lesson 5.3
7.M.10	Identify the relationship between relative error and magnitude when dealing with large numbers (money, population)	Lesson 4.1
7.M.11	Estimate surface area	Lesson 5.1
7.M.12	Determine personal references for customary/metric units of mass	Lesson 5.3

7.M.13	Justify the reasonableness of the mass of an object	Lesson 5.3
7.S.1	Identify and collect data using a variety of methods	Lesson 6.1, Lesson 6.2, Lesson 6.3
7.S.2	Display data in a circle graph	Lesson 6.3
7.S.3	Convert raw data into double bar graphs and double line graphs	Lesson 6.3
7.S.4	Calculate the range for a given set of data	Lesson 3.2
7.S.5	Select the appropriate measure of central tendency	Lesson 6.3
7.S.6	Read and interpret data represented graphically (pictograph, bar graph, histogram, line graph, double line/bar graphs, or circle graphs)	Lesson 6.3
7.S.7	Identify and explain misleading statistics and graphs	Lesson 6.3
7.S.8	Interpret data to provide the basis for predictions and to establish experimental probabilities	Lesson 6.1, Lesson 6.2
7.S.9	Determine the validity of sampling methods to predict outcomes	Lesson 6.2
7.S.10	Predict the outcome of experiment	Lesson 6.1
7.S.11	Design and conduct and experiment to test predictions	Lesson 6.2
7.S.12	Compare actual results to predicted results	Lesson 6.2

Post-March section		
7.A.2	Add and subtract monomials with exponents of one	Lesson 1.3
7.A.3	Identify a polynomial as an algebraic expression containing one or more terms	Lesson 1.3
7.A.4	Solve multi-step equations by combining like terms, using the distributive property, or moving variables to one side of the equation	Lesson 1.3, Lesson 9.1, Lesson 9.3, Lesson 9.4
7.A.7	Draw the graphic representation of a pattern from an equation or from a table of data	Lesson 8.1, Lesson 8.2
7.A.8	Create algebraic patterns using charts/tables, graphs, equations, and expressions	Lesson 1.1, Lesson 8.2
7.A.9	Build a pattern to develop a rule for determining the sum of the interior angles of polygons	Lesson 8.3
7.A.10	Write an equation to represent a function from a table of values	Lesson 8.1, Lesson 8.2, Lesson 8.3
7.G.5	Identify the right angle, hypotenuse, and legs of a right triangle	Lesson 7.3

7.G.6	Explore the relationship between the lengths of the three sides of a right triangle to develop the Pythagorean Theorem	Lesson 7.3
7.G.8	Use the Pythagorean Theorem to determine the unknown length of a side of a right triangle	Lesson 7.3
7.G.9	Determine whether a given triangle is a right triangle by applying the Pythagorean Theorem and using a calculator	Lesson 7.3
7.M.1	Calculate distance using a map scale	Lesson 10.2
7.M.5	Calculate unit price using proportions	Lesson 10.4
7.M.6	Compare unit prices	Lesson 10.4
7.M.7	Convert money between different currencies with the use of an exchange rate table and a calculator	Lesson 10.4

# Correlation of *IMPACT Mathematics, Course 3* to the New York State Grade 8 Standards

Pre-March section		
<b>Chapter 1</b>	<b>Linear Relationships</b>	
1-1	8.A.3, 8.A.15, 8.A.16	Direct Variation
1-2	8.A.16	Slope
<b>Chapter 2</b>	<b>Lines and Angles</b>	
2-1	8.A.4	Lines
2-2	8.G.1, 8.G.2, 8.G.3, 8.G.4, 8.G.5, 8.G.6	Angle Relationships
<b>Chapter 3</b>	<b>Percents and Proportions</b>	
3-1	8.N.3, 8.N.4, 8.N.6	Understand Percents
3-2	8.N.4	Work with Percents
<b>Chapter 4</b>	<b>Exponents and Exponential Variation</b>	
4-1	8.N.1, 8.N.2	Exponents
4-2	8.N.2	Exponential Relationships
<b>Chapter 5</b>	<b>Algebraic Expressions</b>	
5-1	8.A.5, 8.A.7, 8.A.12	Rearrange Algebraic Expressions
5-2	8.A.5, 8.A.6, 8.A.8, 8.A.9	Monomials, Binomials, and Trinomials
5-3	8.A.8	Special Products
<b>Chapter 6</b>	<b>Transformational Geometry</b>	
6-1	8.G.7, 8.G.9	Symmetry and Reflection

6-2	8.G.7, 8.G.8, 8.G.12	Rotation
6-3	8.G.7, 8.G.10, 8.G.11, 8.G.12	Translations, Dilations, and Combined Transformations
<b>Chapter 7</b>	<b>Inequalities and Linear Systems</b>	
7-1	8.N.4, 8.N.5, 8.M.1	Equations
7-2	8.A.1, 8.A.2	Inequalities
<b>Chapter 8</b>	<b>Quadratic and Inverse Relationships</b>	
8-1	8.A.15	Use Graphs and Tables to Solve Equations
8-2	8.A.3, 8.A.15	Quadratic Relationships
8-3	8.A.3, 8.A.15	Families of Quadratics
8-4	8.A.15	Inverse Variation
<b>Chapter 9</b>	<b>Solve Quadratic Equations</b>	
9-2	8.A.5, 8.A.10, 8.A.11	Factoring
<b>Chapter 10</b>	<b>Functions and Their Graphs</b>	
10-1	8.A.15	Functions

Post-March section		
<b>Chapter 1</b>	<b>Linear Relationships</b>	
1-2	8.G.13, 8.G.17	Slope
1-3	8.A.19, 8.G.13, 8.G.14, 8.G.15, 8.G.16, 8.G.17	Write Equations
<b>Chapter 2</b>	<b>Lines and Angles</b>	
2-1	8.G.17	Lines
2-3	8.G.0	Constructions
<b>Chapter 7</b>	<b>Inequalities and Linear Systems</b>	
7-2	8.A.13, 8.A.14, 8.G.19	Inequalities
7-3	8.G.18	Solve Systems of Equations
<b>Chapter 8</b>	<b>Quadratic and Inverse Relationships</b>	
8-2	8.G.20	Quadratic Relationships
8-3	8.G.21	Families of Quadratics
8-5	8.PS.5	Conjectures
<b>Chapter 10</b>	<b>Functions and Their Graphs</b>	
10-1	8.A.17, 8.A.18	Functions
10-2	8.G.20, 8.G.21	Graphs of Functions

Optional Lessons section		
<b>Chapter 4</b>	<b>Exponents and Exponential Variation</b>	
4-3	A.N.2, A.N.3	Radicals
<b>Chapter 9</b>	<b>Solve Quadratic Equations</b>	
9-1	A.A.8, A.A.25	Backtracking
9-3	A.A.19, A.A.28	Completing the Square
9-4	A.A.8, A.A.28	The Quadratic Formula
<b>Chapter 11</b>	<b>Data and Probability</b>	
11-1	A.N.7, A.N.8	Counting Strategies
11-2	A.N.8	Modeling with Data
<b>Chapter 12</b>	<b>Algebraic Fractions</b>	
12-1	A.N.3, A.A.1, A.A.16	Work with Algebraic Fractions
12-2	A.A.1, A.A.16	Add and Subtract Algebraic Fractions

# Correlation of the New York State Grade 8 Standard to *IMPACT Mathematics, Course 3*

Pre-March section		
Standard	Performance Indicator	Course 3
8.N.1	Develop and apply the laws of exponents for multiplication and division	Lesson 4.1
8.N.2	Evaluate expressions with integral exponents	Lesson 4.1, Lesson 4.2
8.N.3	Read, write, and identify percents less than 1% and greater than 100%.	Lesson 3.1
8.N.4	Apply percents to: Tax percent increase/decrease, simple interest, sale price, commission, interest rates, and gratuities	Lesson 3.1, Lesson 3.2, Lesson 7.1
8.N.5	Estimate a percent of a quantity, given an application	Lesson 7.1
8.N.6	Justify the reasonableness of answers using estimation	Lesson 3.1
8.A.1	Translate verbal sentences into algebraic inequalities	Lesson 7.2
8.A.2	Write verbal expressions that match given mathematical expressions	Lesson 7.2
8.A.3	Describe a situation involving relationships that matches a given graph	Lesson 1.1, Lesson 8.2, Lesson 8.3
8.A.4	Create a graph given a description or an expression for a situation involving a linear or nonlinear relationship	Lesson 2.1

8.A.5	Use physical models to perform operations with polynomials	Lesson 5.1, Lesson 5.2, Lesson 9.2
8.A.6	Multiply and divide monomials	Lesson 5.2
8.A.7	Add and subtract polynomials (integer coefficients)	Lesson 5.1
8.A.8	Multiply a binomial by a monomial or binomial (integer coefficients)	Lesson 5.2, Lesson 5.3
8.A.9	Divide a polynomial by a monomial (integer coefficients) Note: The degree of the denominator is less than or equal to the degree of the numerator for all variables.	Lesson 5.2
8.A.10	Factor algebraic expressions using the GCF	Lesson 9.2
8.A.11	Factor a trinomial in the form $ax^2 + bx + c$ ; $a = 1$ and $c$ having no more than 3 sets of factors	Lesson 9.2
8.A.12	Apply algebra to determine the measure of angles formed by or contained in parallel lines cut by a transversal and by intersecting lines	Lesson 5.1
8.A.15	Understand that numerical information can be represented in multiple ways: arithmetically, algebraically, and graphically	Lesson 1.1, Lesson 8.1, Lesson 8.2, Lesson 8.3, Lesson 8.4, Lesson 10.1

8.A.16	Find a set of ordered pairs to satisfy a given linear numerical pattern (expressed algebraically); then plot the ordered pairs and draw the line	Lesson 1.1, Lesson 1.2
8.G.1	Identify pairs of angles as congruent	Lesson 2.2
8.G.2	Identify pairs of supplementary and complementary angles	Lesson 2.2
8.G.3	Calculate the missing angle in a supplementary or complementary pair	Lesson 2.2
8.G.4	Determine angle pair relationship when given two parallel lines cut by a transversal	Lesson 2.2
8.G.5	Calculate the missing angle measurements when given two parallel lines cut by a transversal	Lesson 2.2
8.G.6	Calculate the missing angle measurements when given two intersecting lines and an angle	Lesson 2.2
8.G.7	Describe and identify transformations in the plane, using proper function notation (rotations, reflections, translations, and dilations)	Lesson 6.1, Lesson 6.2, Lesson 6.3

8.G.8	Draw the image of a figure under rotations of 90 and 180 degrees	Lesson 6.2
8.G.9	Draw the image of a figure under a reflection over a given line	Lesson 6.1
8.G.10	Draw the image of a figure under a translation	Lesson 6.3
8.G.11	Draw the image of a figure under a dilation	Lesson 6.3
8.G.12	Identify the properties preserved and not preserved under a reflection, rotation, translation, and dilation	Lesson 6.2, Lesson 6.3
8.M.1	Solve equations/proportions to convert to equivalent measurements within metric and customary measurement systems. Note: Also allow Fahrenheit to Celsius and vice versa	Lesson 7.1

Post-March section		
8.A.13	Solve multi-step inequalities and graph the solution set on a number line	Lesson 7.2
8.A.14	Solve linear inequalities by combining like terms, using the distributive property, or moving variables to one side of the inequality (include multiplication or division of inequalities by a negative number)	Lesson 7.2
8.A.17	Define and use correct terminology when referring to function (domain and range)	Lesson 10.1

8.A.18	Determine if a relation is a function	Lesson 10.1
8.A.19	Interpret multiple representations using equation, table of values, and graph	Lesson 1.3
8.G.0	Construct the following using a straight edge and compass: Segment congruent to a segment; angle congruent to an angle; perpendicular bisector; and angle bisector	Lesson 2.3
8.G.13	Determine the slope of a line from a graph and explain the meaning of the slope as a constant rate of change	Lesson 1.2, Lesson 1.3
8.G.14	Determine the y-intercept of a line from a graph and be able to explain the y-intercept	Lesson 1.3
8.G.15	Graph a line using a table of values	Lesson 1.3
8.G.16	Determine the equation of a line given the slope and y-intercept	Lesson 1.3
8.G.17	Graph a line from an equation in slope-intercept form ( $y = mx + b$ )	Lesson 1.2, Lesson 1.3, Lesson 2.1
8.G.18	Solve a system of equations graphically (only linear, integral solutions, $y = mx + b$ format, no vertical/horizontal lines)	Lesson 7.3

8.G.19	Graph the solution set of an inequality on a number line	Lesson 7.2
8.G.20	Distinguish between linear and nonlinear equations $y = ax^2 + bx + c$ ; $a = 1$ (only graphically)	Lesson 8.2, Lesson 10.2
8.G.21	Recognize the characteristics of quadratics in tables, graphs, equations, and situations	Lesson 8.3, Lesson 10.2

# Scope & Sequence

## Algebra

TOPICS		Course 1										Course 2										Course 3												
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12	
Coordinate Graphs	<b>Algebraic Representations</b>	Develop										Develop										Develop												
	Plotting and reading points							F	F									C	C			C	C					C	C	C				
	Make predictions							F	F			C							F			F	F					C	C	C				
	Modeling situations				F			F	F			C							F			C	F	F					F	F	F			
	Relate to tables and written descriptions				F			C	F	C									F			F	F						F	F	F			
	Relate to equations and expressions							C	C	C										F		F	F						F	F	F			
	Solve equations, approximate solutions								C	F										F		F	F						F	F	F			
	Graphing equations								C	C										F		F	F						F	F	F			
	Direct, indirect, and inverse variation																			F		C	F	C					C					
	Slope and rates of change								C											F		C	F	F					C		C			
	Distance formula																			F														
	Coordinate models of transformations																											C		F		F		
	Tables and Graphs	Analyze change vs. time data							F	F									C	F		F	C						C	F				
Making predictions and generalizations		F	C			C		F	F	C	C	C	C	C	C	F		F	F		F	F						C	F	C				
Modeling data patterns		C	C		F			F	C	F	F	C	C	C	F		C	C	F		F	C						C	C					
Relate to written descriptions				F	F			F	F	F									F	F	C	F						F	C					
Relate to equations and expressions								C	C	C	F	C	C		C				F	F	F					C		F	F	C	F			
Solve equations, approximate solutions				C					F	F										F	F	F	F						F	C	C	F		
"What's My Rule?"		C	F						C	F										F	F													
Patterns and Numeric Forms	<b>Algebraic Representations</b>	Develop										Develop										Develop												
	Find and describe patterns	F	F			C	F	C	C	C	F	C	C	F	F	F	F	F	C	F	F	C	F	F	C			F	F	C				
	Extend and generalize patterns	F	F			C	F	C			C	F	C	C	F	F	F	F	C	F	F	C	F	F	C			F	F	C				
	Create and verify patterns	F	F	F			C	F			C	F	C	C	F	F	F	F	F		F		F	C				F						
	Express patterns as algebraic rules	C	C						C	F	F				F				F	F	F							F	C					
	"What's My Rule?"		F						C	F										F	F													
	Equivalent expressions		F	C					C	F	C	C	C	C					C	F	F	F					F	F	C	C	C	C		
	Factoring, expanding, simplifying									F										C	F	F	F					C	F	C	C	F		
	Signed numbers and operations																			F								F						
	Absolute value and opposites								F											F									C					
	Exponents, roots, and radicals			F					C						C	F	C											F			C			
	Scientific notation			C																F									F					
	Evaluating expressions			C					C	F	F	C	C						F	F	C						C	F	C	C	C	F		
	Flowcharts and backtracking								F	F										F		C							C	F				
	Verify solutions by substitution								F	F										F		F	F						F	F				

### KEY

**F** = This topic is a Focus of Instruction in this chapter.

**C** = This topic is Connected to the content of the chapter and is either reviewed in this chapter or informally introduced.

**Expose:** Ideas are introduced at an informal concrete level and will be fully developed later in the program.

**Develop:** Ideas are formalized and fully developed.

**Apply:** Ideas are reviewed and used to extend understanding of related ideas.





# Geometry

TOPICS		Course 1										Course 2										Course 3											
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12
<b>Two-Dimensional Shapes</b>		Develop										Apply										Apply											
<b>Polygons</b>	Definitions and properties	F			C	C										C								C				C					
	Classification and naming	F			C	C										C																	
	Reflectional symmetry	C																										F					
	Rotational symmetry																											F					
	Angle sums of polygons	F																															
	Area and perimeter	F	C	C				C								C	C			C								C					
	On the coordinate plane								C																			C					
<b>Quadrilaterals</b>	Definitions and properties	F			C										C												C						
	Determining uniqueness	C			F																												
	Interior angle sum	F								C													C										
<b>Triangles</b>	Acute, right, obtuse	F																										C					
	Equilateral, isosceles, scalene	F													C													C					
	Properties of special triangles	F													C																		
	Triangle inequality	F																															
	Triangle Sum Theorem	C																									C						
	Pythagorean Theorem																	F															
<b>Angles</b>	Estimate and measure	F				C	C																										
	Classify acute, right, obtuse	F																									C						
<b>Geometric Relationships</b>		Expose										Develop										Apply											
<b>Congruence</b>	Definition and properties				F	C																	C				C						
	Identifies, determines congruence				F														C				F				C						
	Identify corresponding parts				F														C				F				C						
	As special case of similarity																		C								C						
	Represented by transformation							C															F				F						
<b>Similarity</b>	Definition and properties				F														C				C				C						
	Identifies, determines similarity				F														C								C						
	Properties of similar figures				F														C								C						
	Scale factors				F														F	C							F		C				
	Congruence as scale factor of 1				F																						C						
	Relation to area and perimeter				C	C													C								C						
	Relation to surface area and volume							C							C				C														
	Relation to ratio and rates				F														F	C							C						
	Proportional reasoning	C			C	C				C									F	C							F						
	AA Similarity Property				F																						C						
	Indirect measurement				C																												
	Scale drawings and maps				C													C	C									F					

TOPICS		Course 1										Course 2										Course 3													
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12		
<b>Three-Dimensional Figures</b>		Expose										Develop										Apply													
<b>Spatial Visualization</b>	Visualizing structures						<b>F</b>										<b>F</b>																		
	Creating and drawing structures						<b>C</b>										<b>F</b>																		
	Types of 3-D drawings																<b>C</b>																		
	Determining unique structures																<b>F</b>																		
<b>3-D Solids</b>	Definitions and properties						<b>C</b>										<b>F</b>																		
	Prisms and pyramids						<b>F</b>										<b>F</b>																		
	Cones and cylinders																<b>F</b>																		
	Nets						<b>C</b>										<b>F</b>																		
	Volume and surface area						<b>F</b>										<b>F</b>																		
<b>Measurement</b>		Develop										Develop										Apply													
<b>Perimeter and Area</b>	Approximation and estimation	<b>F</b>	<b>C</b>	<b>C</b>			<b>F</b>											<b>C</b>		<b>C</b>															
	Formulas for squares and rectangles	<b>F</b>					<b>F</b>										<b>C</b>																		
	Areas of parallelograms						<b>F</b>										<b>C</b>																		
	Areas of triangles						<b>F</b>										<b>C</b>																		
	Areas of circles and circumference	<b>F</b>					<b>F</b>										<b>C</b>																		
	Relating perimeter and area	<b>F</b>					<b>C</b>																												
	Changes due to scale factor					<b>C</b>															<b>F</b>								<b>C</b>						
	Maximizing and minimizing	<b>F</b>																																	
	Pythagorean Theorem																	<b>F</b>																	
<b>Surface Area and Volume</b>	Approximation and estimation						<b>F</b>																												
	Formulas for prisms and cylinders						<b>F</b>										<b>F</b>																		
	Formulas for pyramids and cones																<b>F</b>																		
	Relating surface area and volume						<b>F</b>										<b>F</b>																		
	Changes due to scale factor																<b>F</b>																		
	Maximizing and minimizing																<b>F</b>																		
	Capacity						<b>F</b>										<b>F</b>																		
<b>Coordinate Geometry</b>		Develop										Develop										Develop													
<b>Coordinate Representations</b>	Plotting and reading points						<b>F</b>										<b>C</b>	<b>C</b>		<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>	<b>C</b>		
	Graphing equations						<b>C</b>		<b>C</b>									<b>F</b>		<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>		
	Relating graphs, tables, equations						<b>C</b>	<b>C</b>	<b>C</b>									<b>F</b>		<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>		
	Written descriptions of graphs						<b>F</b>											<b>C</b>		<b>F</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>		
	Approximating and finding solutions						<b>C</b>		<b>C</b>											<b>C</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>		
	Direct and indirect variation																		<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>		
	Slope of a line and rates of change						<b>C</b>												<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>		
	Equations of lines						<b>C</b>												<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>		
	Slopes of parallel lines																		<b>C</b>		<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>		
	Slopes of perpendicular lines																		<b>C</b>		<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>	<b>C</b>	<b>F</b>		
	Distance formula																	<b>F</b>																	
<b>Transformations</b>	Reflectional symmetry																											<b>F</b>							
	Rotational symmetry																											<b>F</b>							
	Translations																					<b>C</b>	<b>C</b>					<b>F</b>		<b>C</b>					
	Changes of magnitude (dilations)																											<b>F</b>		<b>C</b>					
	Rotations																											<b>F</b>							
	Compound transformations																											<b>F</b>		<b>C</b>					
	Congruence and similarity																											<b>F</b>							
	Relating to equations																					<b>F</b>						<b>C</b>			<b>F</b>				

# Number and Operations

TOPICS		Course 1										Course 2										Course 3												
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12	
Whole Numbers	<b>Numbers and Number Sense</b>	Develop										Develop										Apply												
	Magnitude of large, small numbers			F									C		F												C							
	Operations on odd and even numbers											C	C																					
	Factors and multiples	C		F	C								F									C			C			C						C
	Common factors and multiples				C								F									C			C									C
	Factor pairs and area models				C							C	F															C						C
	Greatest common factor				C								F															C						C
	Least common multiple				C								F									C						C						C
	Prime and composite												F																					
	Relatively prime												F															C						
	Prime factorization												F																					
	Fundamental Theorem of Arithmetic												F																					
	Factor trees												F																					
Signed Numbers	Meaning and basic concepts								C				F	C																				
	Concrete and number line models								F				F					F	C										C					
	Four quadrant coordinate model								F				C																					
	Comparison								F				F	C																				
	Order								F				F																					
	Absolute value and opposites								F				F																					
	Addition and subtraction												F																					
	Multiplication and division												F								C													
	Meaning in exponential notation													F													F							
	Negative slope																		F					F	C									
Exponents and Roots	Meaning and basic concepts												F														C							
	Positive and negative exponents												F	F													F							
	Scientific notation												F														F							
	Laws of exponents												F	F													F							
	Used in expressions and equations												F	F													F	C		C	C			
	Geometric interpretation of square												F																					
	Geometric interpretation of cube												F																					
Fraction and Decimal Concepts	<b>Rationals and Irrationals</b>	Develop										Apply										Apply												
	Rational and irrational numbers	F																									F						C	
	Geometric and number line models	F		C	C									F													F							
	Fractional relationships to a whole	F		F	C	F															C						C						C	
	Comparison	F		F	F	F		C							C												C							
	Benchmarks for comparison	F		F	F	F																					F							
	Order	F		F	F	F		C						C	C												F							
	Naming equivalent fractions	F		C	C	C			C																		C					C	F	
	Naming equivalent decimals	F		F	F	F																					F							
	Fraction and decimal equivalencies	F		C	C	C																					C							
	Fraction and decimal approximations	F		C	C	C								C													C							
	Fractions as indicated division	F		C	C	C																					F		C				C	
	Repeating decimals	F		C	C	C																						F						
	Relating fractions and decimals	F		C	C	C																					F		C	C				
	Converting fractions to decimals	F		C	C	C																					C		C	C				
	Converting decimals to fractions	F		C	C	C																					C		C	C				

TOPICS		Course 1										Course 2										Course 3														
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12			
Meaning and Representations	<b>Percents</b>	Develop										Develop										Apply														
	Definition and basic concepts					F																														
	Greater than 100 or less than 1					F																														
	Common scale comparisons					F																														
	Relating to fractions and decimals					F																														
	Relating to ratios and proportions					C																														
	Converting to a decimal					F																														
	Converting to a fraction					F																														
	Estimate percent using benchmarks					F																														
	Calculate percent					F																														
	Solve percent problems (all cases)					C																														
	Relating to proportional reasoning					F																														
	Percent increase and decrease					F																														
	<b>Ratios and Rates</b>	Expose										Develop										Apply														
Meaning and Representations	Basic concepts	C	C	F	C					C											F	F														
	Comparison statements				F	C																F	F													
	Part-to-part comparisons				F																		F	F				C								
	Part-to-whole comparisons	C			F	F											C						F	F				C			C					
	Relating unlike quantities or units				F	C																F	F													
	Types and uses of ratio notation				F																		F	F												
	Equivalent ratios and ratio tables				F																		F	F												
	Comparing ratios	C	C	F	C												C						F	F												
	Equivalent rates				F																		F	F	F											
	Rate tables and rate graphs				F																		F	F	F	F										
	Slope and rates of change																						F	F	C	F										
Proportions	Definition and basic concepts					C																C	F	F				C								
	Proportions as equations								C														F	F	F				F					C		
	Checking if a proportion exists																						F	F	C				F							
	Methods of solving proportions																							F	F				F						C	
	Proportional relationships																						F	F	F				F			C				
	Proportional reasoning	C				C				C													C	F				F	C		C	C		C		
	<b>Algorithms and Operations</b>	Develop										Apply										Apply														
Fractions	Generating equivalent fractions	F	C	C	C																		C				C						C			
	Fraction, mixed number conversions	F		C																													C			
	Percent, decimal conversion	F		C	F																						F									
	Addition and subtraction			F			C									C																	C			
	Multiplication and division			F			C									C																	C			
Decimals	Fraction, percent conversions	F		F	F																					F										
	Addition and subtraction			C			C									C	C																			
	Multiplication and division			C			C									C	C																			
Signed Numbers	Addition and subtraction																																			
	Multiplication and division																															F				

# Probability and Statistics

TOPICS		Course 1										Course 2										Course 3												
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12	
<b>Data Analysis</b>		Develop										Develop										Develop												
<b>Graphs and Displays</b>	Line plots				<b>F</b>	<b>C</b>					<b>F</b>																							
	Stem-and-leaf plots										<b>F</b>																							
	Box plots																																	<b>F</b>
	Venn Diagrams	<b>C</b>									<b>F</b>																							
	Circle graphs						<b>C</b>				<b>C</b>																							
	Frequency tables						<b>C</b>				<b>F</b>																							
	Histograms										<b>F</b>																							
	Scatter plots									<b>C</b>	<b>F</b>																							
	Choose appropriate displays										<b>F</b>																							
	Skewness and symmetry										<b>F</b>																							
	<b>Modeling and Analysis</b>	Identify patterns and trends		<b>C</b>	<b>F</b>					<b>C</b>	<b>F</b>																							
Identify clusters, gaps, and outliers										<b>F</b>																								
Describe shape and scatter										<b>F</b>																								
Compare data sets										<b>F</b>																								
Fit a line by "eyeballing"										<b>C</b>																								
Make predictions and generalizations										<b>C</b>																								
Misleading graphs										<b>C</b>																								
Law of large numbers										<b>C</b>																								
Outlier affect on measures of center											<b>F</b>																							
Compare measures of center											<b>F</b>																							
Choose appropriate measure of center											<b>C</b>																							
<b>Statistical Measures</b>	Frequency									<b>C</b>																								
	Range									<b>F</b>																								
	Mode									<b>F</b>																								
	Median									<b>F</b>																								
	Mean									<b>F</b>																								
	Quartiles																																	
<b>Surveys and Sampling</b>	Sample survey vs. census																																	
	Randomization									<b>C</b>																								
	Simple random sample																																	
	Sample size									<b>C</b>																								
	Law of large numbers									<b>F</b>																								
	Bias in survey methods									<b>C</b>																								
	Population and sample identification									<b>C</b>																								

TOPICS		Course 1										Course 2										Course 3											
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11	12
<b>Probability</b>		Develop										Develop										Develop											
<b>Basic Concepts and Rules</b>	Measure between 0 and 1										<b>F</b>																					<b>F</b>	
	Empirical and theoretical probabilities										<b>F</b>																					<b>F</b>	
	Equally likely										<b>F</b>																					<b>F</b>	
	Compound events										<b>C</b>																					<b>F</b>	
	Mutually exclusive events										<b>C</b>																					<b>F</b>	
	Independent events										<b>F</b>																					<b>C</b>	
	Dependent events										<b>F</b>																					<b>C</b>	
	Confidence										<b>C</b>																						
	Expected value										<b>C</b>																						
	<b>Counting Methods</b>	Counting trees										<b>F</b>																					<b>F</b>
Combinations																																<b>F</b>	
Permutations																																<b>F</b>	
<b>Experiments and Simulations</b>	Estimate likelihood										<b>C</b>																					<b>C</b>	
	Relate to theoretical probabilities										<b>F</b>																					<b>C</b>	
	Analyze fairness										<b>C</b>																					<b>C</b>	

# Chapter Summaries

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## **Chapter 1: Polygons, Angles, and Circles**

Students will explore various characteristics of polygons, angles, and circles.

Topics include classifying figures as polygons, measuring angles, classifying angles, exploring line symmetry, identifying angle relationships of vertical angles, and calculating perimeter and circumference.

## **Chapter 2: Fractions and Decimals**

Students review important topics related to fractions and decimals.

Topics include the meanings of fractions and decimals, moving smoothly between fraction and decimal representations of rational numbers, and patterns in fractions and decimals.

## **Chapter 3: Patterns, Numbers, and Rules**

Students look for, describe, extend, and generalize geometric and numeric patterns.

Topics include writing both recursive and general rules to describe patterns, following rules to create patterns, finding rules that relate input and output in “What’s My Rule?” games, determining informally if expressions are equivalent, writing a rule in two different ways, and applying order of operations.

## **Chapter 4: Fraction and Decimal Operations**

Students develop and apply algorithms for adding, subtracting, multiplying, and dividing fractions and review methods for finding products and quotients of decimals.

Topics include estimating and computing sums and differences of fractions (including mixed numbers) and decimals, developing algorithms for sums and differences of fractions, understanding the meaning of fraction multiplication and division, developing algorithms for products and quotients of fractions (including mixed numbers), and estimating and computing products and quotients of fractions and decimals.

## **Chapter 5: Ratio, Rate, and Proportion**

Students solve problems by using proportional thinking and write and interpret ratio and percent comparisons.

Topics include investigating types of ratios such as part-to-part and part-to-whole; writing, interpreting, and scaling ratios; interpreting percent as a ratio of a number to 100; solving percent problems; determining whether two ratios form a proportion; setting up and solving proportions; applying ideas of similarity and proportion to solve real problems.



## **Chapter 6: Percents**

Students develop an understanding of percent and solve problems involving percents.

Topics include using percents to make comparisons; finding the percent one number is of another; finding a given percent of a number; finding the whole quantity when told what percent of the whole a given part represents; and moving smoothly among fraction, decimal, and percent representations of rational numbers.

## **Chapter 7: Area, Volume, and Capacity**

Students explore measurement of two-and three-dimensional figures.

Topics include finding and estimating perimeters and area of polygons and shapes with curved sides; developing formulas for areas of rectangles, parallelograms, and triangles; and finding capacity in both metric and customary units.

## **Chapter 8: Coordinate Plane**

Students develop skill at interpreting and creating coordinate graphs.

Topics include matching graphs with written descriptions, writing stories to match graphs, giving coordinates of points, plotting points, making predictions based on graphs, determining when it is appropriate to connect points, and interpreting graphs of real data.

## **Chapter 9: Equations**

Students develop skill at solving equations by using the backtracking and guess-check-and-improve methods.

Topics include determining whether a given value is a solution of an equation, solving simple equations mentally, solving equations by backtracking, solving equations by using guess-check-and-improve, and choosing an appropriate solution method for a given equation.

## **Chapter 10: Data and Probability**

Students are introduced to the basics of probability and various ways to display data.

Topics include finding experimental probabilities, understanding that probability describes behavior over the long run, calculating theoretical probabilities, finding geometric probabilities, using probabilities to devise game-winning strategies, and understanding and designing simple simulations.



# Chapter Summaries

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## Chapter 1: Expressions

Students represent situations with algebraic expressions and use the distributive property to rewrite expressions.

Topics include writing algebraic expressions to represent situations, making up situations to match algebraic expressions, exploring concrete models of the distributive property, using the distributive property to rewrite expressions, and backtracking to solve equations.

## Chapter 2: Exponents

Students develop an understanding of exponents and scientific notation and develop a sense of large numbers.

Topics include prime and composite numbers, common factors and multiples, the laws of exponents, and exponential notation for positive exponents.

## Chapter 3: Signed Numbers

Students develop and apply algorithms for operations with signed numbers and extend their knowledge of coordinate graphs to include all four quadrants.

Topics include adding, subtracting, multiplying, and dividing signed numbers; and graphing in four quadrants.

## Chapter 4: Magnitude of Numbers

Students expand their knowledge of exponents to very large and small numbers.

Topics include exploring situations involving very large numbers, developing benchmarks for understanding very large numbers, the meaning of negative exponents, scientific notations, and powers of ten.

## Chapter 5: Geometry in Three Dimensions

Students gain experience visualizing three-dimensional structures and explore surface area and volume.

Topics include creating and interpreting views of block structures; finding surface area and volume of block structures, prisms, and cylinders; exploring different surface areas for a given volume and different volumes for a given surface area; creating nets for three-dimensional figures; and determining whether a net can fold to form a given figure, and exploring mass and weight.

## **Chapter 6: Data and Probability**

Students explore sampling techniques and investigate the probability concepts of fairness and independence.

Topics include understanding and applying sampling techniques, extrapolating from sample data, understanding the importance of random samples, exploring how sample size is related to the reliability of conclusions, and interpreting data collected from probability experiments, creating and interpreting data graphs including double-bar graphs, circle graphs, and stem-and-leaf graphs.

## **Chapter 7: Real Numbers**

Students learn more about the real numbers including rational and irrational numbers

Topics include ordering and comparing rational numbers, approximating irrational numbers, finding squares and square roots, and using the Pythagorean Theorem.

## **Chapter 8: Linear Relationships**

Students investigate graphs, tables, and equations for linear relationships.

Topics include understanding and describing rates; creating graphs, tables, and equations for linear situations; exploring direct variation; recognizing a linear relationship from a table, graph, or equation; and using graphs to make predictions.

## **Chapter 9: Equations**

Students use balance and ribbon models to develop the “doing-the-same-thing-to-both-sides” method for solving equations.

Topics include reviewing backtracking and guess-check-and-improve methods for solving equations, using balances to model equations, solving balance problems by doing the same thing to both sides, solving equations by doing the same thing to both sides, and simplifying equations before solving them.

## **Chapter 10: Proportional Reasoning and Percents**

Students explore ratios, unit rates, and proportions.

Topics include comparing and scaling ratios, finding unit rates, converting currencies, drawing map scales, exploring similar objects, and using proportions to determine heights.

# Chapter Summaries

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## Chapter 1: Linear Relationships

Students review ideas of linear variation. Topics include reviewing linear variation; understanding slope; relating linear equations and graphs; finding the equation of a line given a point and the slope, or two points.

## Chapter 2: Lines and Angles

Students explore properties of lines and the angles formed by intersecting lines. Topics include fitting lines to data, finding angle measures of supplementary, complementary, and vertical angles, and constructing line segments and angles.

## Chapter 3: Percents and Proportions

Students understand and work with percents. Topics include percents as a common scale, finding percent increase and decrease, calculating percents of percents, and interpreting and comparing discounts.

## Chapter 4: Exponents and Exponential Variation

Students review exponents and investigate exponential growth. Topics include reviewing positive and negative exponents and scientific notation, understanding the meaning of roots, and exploring situations that show exponential growth.

## Chapter 5: Algebraic Expressions

Students explore techniques for rewriting algebraic expressions. Topics include combining like terms, using area models to explore the distributive property, applying the distributive property to expand expressions, recognizing and rewriting a difference of two squares, and recognizing and rewriting a perfect square trinomial.

## Chapter 6: Transformational Geometry

Students recognize and describe symmetries and apply and describe geometric transformations both on and off the coordinate plane. Topics include recognizing reflectional, rotational, and translational symmetry; identifying lines of symmetry, angles of rotation, and direction and magnitude of translations; performing reflections, rotations, translations, and dilations; and writing and interpreting algebraic rules that describe transformations such as  $(x, y) \mapsto (x, -y)$  is a reflection over the  $x$ -axis.

## Chapter 7: Inequalities and Linear Systems

Students solve equations and systems of equations symbolically and by using tables and graphs. Topics include reviewing how to solve equations by backtracking and by doing the same thing to both sides; solving inequalities; solving equations with tables; solving equations with graphs; and solving systems of equations graphically, by substitution, and by elimination.

## **Chapter 8: Quadratic and Inverse Relationships**

Students explore graphs, tables, and equations for quadratic and reciprocal relationships. Topics include exploring patterns and situations that show quadratic variation; exploring characteristics of parabolas; exploring patterns and situations that show reciprocal variation; and exploring characteristics of hyperbolas.

## **Chapter 9: Solving Quadratic Equations**

Students develop and apply techniques for solving quadratic equations. Topics include solving quadratic equations with no  $x$  terms by backtracking, learning some factoring shortcuts, solving quadratics given in factored form, solving quadratics by factoring, completing the square, and solving quadratics by applying the quadratic formula.

## **Chapter 10: Functions and Their Graphs**

Students solve quadratic equations by graphing and investigate relationships between quadratic equations and graphs. Topics include solving quadratic equations by graphing; using graphs to understand why a quadratic equation has zero, one, or two solutions; connecting the  $x$ -intercepts of a graph with the factored form of the equation; exploring characteristics of parabolas; using technology to examine the effects of constants and coefficients on the graphs of quadratic equations; and solving linear and quadratic systems of equations.

## **Chapter 11: Data and Probability**

Students find probabilities that involve permutations and combinations and use graphs to organize and analyze data. Topics include developing counting strategies, determining which counting strategy is the most appropriate for a given situation, solving probability problems involving permutations and combinations, exploring probability distributions, understanding and creating simulations and determining which types of tables, graphical displays, and statistics are the most appropriate for a given set of data and a given purpose.

## **Chapter 12: Algebraic Fractions**

Students expand the use of algebra and fractions to include simplifying algebraic fractions. Topics include rewriting algebraic fractions, adding and subtracting algebraic fractions, and exploring real situations that can be represented by algebraic fractions.

# Expectations

## Entrance Expectations for Course 1

*What students should know as they begin Course 1*

### **Algebra**

- Are familiar with some relationships in tabular form such as input/output boxes
- Have some limited experience with variables

### **Geometry**

- Know the names of common geometric figures
- Identify figures with line symmetry
- Measure lengths and are familiar with both customary and metric measures of length
- Find the perimeter of figures with straight-line sides
- Find the areas of rectangles

### **Number and Operation**

- Are proficient with whole number arithmetic
- Are proficient with decimal addition and subtraction
- Have multiplied decimals but are not proficient
- Know the algorithm for finding equivalent fractions but may not understand why it works
- Are proficient with writing decimals as fractions
- Know decimal equivalents for  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$  and for fractions with denominators that are powers of 10
- Add and subtract fractions with the same denominator
- Have been exposed to addition and subtraction of fractions with unlike denominators but may not be proficient
- Have been exposed to multiplication of fractions but are not proficient
- Have seen percents but know only simple things about them

### **Data and Probability**

- Interpret and create bar graphs and pictographs

# Exit Expectations for Course 1

## Entrance Expectations for Course 2

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*What students should know as they finish Course 1;  
What students should know as they begin Course 2*

### **Algebra**

- Understand the concept of a variable
- Solve simple one- and two-step equations with the variable on one side only

### **Geometry**

- Understand area and perimeter and have committed important formulas to memory
- Calculate volume of a rectangular prism and understand capacity
- Give reasonable estimates for angle measures and measure angles with a protractor
- Plot points in four quadrants

### **Number and Operation**

- Are proficient with fraction and decimal operations
- Move efficiently among fraction, decimal, and percent representations

### **Data and Probability**

- Conduct simple experiments to determine experimental probabilities
- Calculate theoretical probabilities in simple situations with a small number of equally likely outcomes
- Calculate measures of central tendency
- Interpret bar graphs, line graphs, Venn Diagrams, line plots, and stem-and-leaf plots

## Exit Expectations for Course 2

## Entrance Expectations for Course 3

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*What students should know as they finish Course 2;  
What students should know as they begin Course 3*

### **Algebra**

- Write algebraic expressions to represent situations and patterns
- Apply the distributive property to expand expressions and to factor out a common monomial factor (includes combining like terms)
- Solve single-variable linear equations in which the variable appears on both sides (by doing the same thing to both sides)
- Recognize a linear relationship from a written description, a table, a graph, or an equation
- Have a thorough understanding of slope (rise/run, rate of change, constant change, and so on)

### **Geometry**

- Understand volume and surface and have important formulas committed to memory
- Understand and apply ideas about similarity and scale factor including map scales
- Understand and apply the relationship between scale factor, area, and volume
- Plot points in all four quadrants
- Apply the distance formulas

### **Number and Operation**

- Are proficient with operations with signed numbers
- Are proficient in working with positive and negative integer exponents
- Are proficient with percent operations, including calculating percent increase and percent decrease
- Understand ratios, rates, and proportions and solve problems that require comparing ratio or solving proportions and find unit rates
- Understand the distinction between rational and irrational numbers and use the Pythagorean Theorem

### **Data and Probability**

- Calculate probabilities in situations involving multipart outcomes (tossing four coins, spinning two spinners, and so on)
- Conduct simple simulations to find probabilities
- Interpret circle graphs, stem-and-leaf plots, and identify misleading graphs
- Understand the purpose of sampling and the importance of selecting a random sample



# Exit Expectations for Course **3**

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*What students should know as they finish Course 3*

## **Algebra**

- Solve linear inequalities
- Solve linear systems
- Write a linear equation given two points or a point and a slope
- Understand how the graph of  $y = x^2$  is changed if a constant is added to  $x^2$  or if  $x^2$  is multiplied by a constant
- Recognize a quadratic relationship from a table, a graph, or an equation
- Multiply binomials
- Solve quadratic equations graphically, by using the quadratic formula and (in fairly simple cases) by factoring
- Understand the exponential growth pattern and recognize this pattern from a table, graph, or equation
- Understand inverse variation and recognize inverse variation from a table, graph, or equation
- Solve simple equations involving rational expressions and radical expressions
- Understand the meaning of function
- Use technology to graph functions and identify solutions to equations, maximum and minimum points, intercepts, and lines of symmetry

## **Geometry**

- Recognize and describe reflectional and rotational symmetry (including identifying lines of symmetry and specifying angles of rotation)
- Write and recognize algebraic rules for similarity transformations, translations, simple reflections (over the  $x$ -axis, over the  $y$ -axis, over the line  $y = x$ ), and simple rotations ( $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ )

## **Number and Operation**

- Understand square roots and manipulate expressions involving square roots

## **Data and Probability**

- Perform computations involving combinations or permutations
- Solve probability problems that require using combinatorics to count outcomes
- Fit a line to a set of linear data (by eyeballing) and then use the graph or equation of the line to make predictions
- Interpret box-and-whisker plots

# The Instructional Cycle

*IMPACT Mathematics* is designed to actively engage students in their own learning. To facilitate the learning and teaching process, *IMPACT Mathematics* is designed around a three-step instructional cycle.

## Introduce

Each multiday lesson begins with a class discussion, activity, or problem designed to introduce the mathematics and help set a context for learning. To help guide the introduction, **Explore** activities and **Think & Discuss** questions are provided in the student materials.

## Develop

Each lesson in *IMPACT Mathematics* is composed of in-class **Investigations** that provide a mix of worked-out examples, direct modeling through cartoons, and interactive problem sets. During Investigations, the mathematics, not an artificial format, determines the approach and the day's activity. Most Investigations are designed to last about 45 minutes or one class period.

The **Share & Summarize** questions signal the end of each Investigation. These questions offer students an opportunity to share what they did and what was learned. For teachers they offer an important assessment opportunity.

## Assign & Assess

Independent assignments and opportunities to assess what students have learned are a regular part of the curriculum. The **On Your Own Exercises** at the end of each lesson are an integral part of program instruction and are intended for individual work done primarily outside of class. You will find three types of problems in each set of On Your Own Exercises.

- *Practice & Apply* problems provide opportunities for students to reinforce and directly apply the skills and concepts they have learned in each of the Investigations.
- *Connect & Extend* problems relate student learning in the lesson to other mathematical topics and strands, and sometimes require students to stretch their thinking.
- *Mixed Review* problems are an important part of the instructional and assignment structure. Frequent review of previously learned skills helps students maintain mastery and replaces the need to reteach topics.

### Steps in the Instructional Cycle

#### INTRODUCE

The multiday lesson begins with a full-class discussion, activity, or problem. The teacher poses questions, orchestrates an activity, or monitors strategies students use to solve problems. **Explore activities** and **Think & Discuss** questions can be used to guide discussion.

#### DEVELOP

In-class **Investigations** provide a mix of worked out examples, direct modeling through cartoons, and interactive problem sets. Students may work independently or in small, collaborative groups. Investigations can be completed in one class period. The mathematical content of

# Assessment

The assessment tools in *IMPACT Mathematics* are broader than those in traditional mathematics programs. They encompass the processes of problem solving, reasoning, communication, connections, concepts, applications, representational strategies, and procedures.

## In the Student Edition

- **Share & Summarize** questions provide a forum for students to summarize and share their learning with the class.
- **On Your Own Exercises**, an integral part of daily instruction, are independent assignments intended for individual work outside of class.
- **Review & Self-Assessment** provides students with an opportunity to reflect on the important topics within the chapter and to prepare for formal assessment.

## In the Teacher's Guide

- **Troubleshooting** notes provide remedial work students might need in order to move on to the next Investigation successfully.
- **Additional Examples** can be used as on-the-run assessment tools.
- **Quick Checks** provide checklists of what students should be able to do at the end of each lesson.
- **Quick Quizzes** provide brief end-of-lesson assessment opportunities.

## In the Chapter Resource Masters

- A **Pretest** determines whether students have the prerequisite skills for the course.
- **Chapter Tests** provide a comprehensive evaluation of chapter content.
- **Performance Assessments** provide open-ended opportunities to measure student achievement. They can be used to supplement or replace items on chapter and semester tests, as take-home assignments, as group assessments, or as challenge or extra-credit problems.
- **Semester Tests** provide cumulative midyear and end-of-year evaluations.

the Investigations determines the approach and the day's activity. Homework or assignment guides are available for each Investigation, and each wraps up with **Share & Summarize** questions.

## ASSIGN & ASSESS

Independent assignments and assessment opportunities provide rich opportunities for students to demonstrate their learning. Each lesson concludes with **On Your Own Exercises**. The types of problems included in each set of On Your Own Exercises are:

- **Practice & Apply**, which are similar to the Investigations.
- **Connect & Extend**, which relate the lesson topics to other mathematical topics and strands.
- **Mixed Review**, which provides review of previously learned skills to maintain mastery.

# Student Materials

Each course of *IMPACT Mathematics* includes the components shown on pages 46–47. Together, the components provide a rich and rigorous mathematics curriculum for students, along with complete instructional support for teachers.

Every course provides middle grades students with an integrated curriculum, and mathematical content that is important for all middle grades students to experience. Each course includes chapters that focus on algebra, geometry, number and operation, and data and probability. By revisiting big ideas across the strands in every grade level, students of all ability levels reinforce and solidify their understanding without the need for endless reteaching.

Multiday lessons are designed around a three-step instructional cycle to keep mathematical ideas together and to engage middle grades students in their own learning, both in class and at home.

**LESSON 7.1** Squares

You know that the perimeter of a two-dimensional shape is the distance around the shape. The **area** of a two-dimensional shape is the amount of space inside the shape.

**Explore**  
Consider these shapes.

**Vocabulary**  
area

**Materials**  
• copies of the two shapes  
• scissors

**Real-World Link**  
You're in a room with square tiles. A square is a shape with four equal sides. A square is a special kind of rectangle. A square can be put together to form other shapes.

Which shape do you think is larger? That is, which shape do you think has the greater area?  
Cut out shape 1 along the lines. Rearrange the pieces to make a square. Do the same for shape 2.  
Of the two squares you made, which has the greater area? How can you tell?  
Do the original shapes have the same area as the squares? Why or why not?  
When determining which shape has the greater area, it is easier to compare the original shapes or the squares? Why?

Squares are the basic unit used for measuring areas. In this lesson, you will look closely at areas of squares and at a special operation associated with the areas of squares.

Course 1, Student Edition, p.398

**Share & Summarize**

- A shirt is on sale for \$16.80 as a mixed number.
- A big-screen television costs 100 times as much as the shirt. How much does the TV cost?
- A fresh-cooked pretzel costs  $\frac{1}{10}$  as much as the shirt. How much is the pretzel?

**Investigation 2 Measure with Decimals**

**Materials**  
• tape  
• centimeter ruler

**Math Link**  
The abbreviation for centimeter is cm.  
The abbreviation for millimeter is mm.  
The abbreviation for meter is m.

In the metric system, units of measure are based on the number 10. This makes converting from one unit to another as easy as moving a decimal point.  
The basic unit of length in the metric system is the meter. Each meter can be divided into 100 centimeters.

Each centimeter can be divided into 10 millimeters. (Note: Ruler is not to scale.)

**Think & Discuss**  
Fill in the blanks. Give your answers as both decimals and fractions.

1 cm = \_\_\_\_\_ m    1 mm = \_\_\_\_\_ cm    1 mm = \_\_\_\_\_ m  
 2 cm = \_\_\_\_\_ m    15 mm = \_\_\_\_\_ cm    15 mm = \_\_\_\_\_ m

Course 1, Student Edition, p.79

**On Your Own Exercises**  
Lesson 10.1

**Practice Apply**

- In Parts a–d, tell which graph could represent the situation.
  - 
  - 
  - 
  -
- a. a child's activity level from before a nap until after a nap  
 b. the populations of six cities  
 c. the change in water level from high tide to low tide  
 d. the change in the weight of a cat from birth until age two
- This table shows the number of people who visited Milo's Restaurant in the years from 1996 to 2007.
- The change in the number of customers is greatest from 2006 to 2007. Between which two years is the change in the number of customers least?
- Make a bar graph showing the number of customers during the years shown in the table.
- Describe what your graph indicates about the change in the number of customers during the time period. Choose high and low points, periods of greatest and least change, and any other patterns you see.

Year	Customers (thousands)
1996	39.4
1997	42.7
1998	43.3
1999	45.8
2000	44.8
2001	45.5
2002	46.5
2003	47.8
2004	46.4
2005	46.0
2006	50.9
2007	45.5

Course 1, Student Edition, p.592

## INTRODUCE

Explore or Think & Discuss questions help launch each lesson.

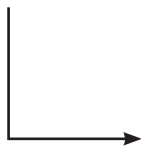
## DEVELOP

Investigations with Share & Summarize questions comprise each lesson.

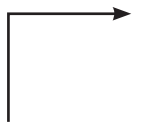
## ASSIGN & ASSESS

On Your Own Exercises provide comprehensive assessment.

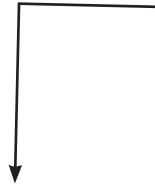
**Vocabulary** is highlighted and defined in context.



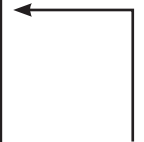
**Math Links** remind students about information they learned in previous lesson.



**Share & Summarize** allow students to conclude learning in investigations and make connections between concepts.



**Think & Discuss** provide examples and questions to prompt classroom discussion.



**Share & Summarize**

1. Explain the difference between an equation and an inequality. Give an example of each.
2. Give an example of an equation or an inequality that is false. Then explain how you could change the sentence to make it true.

**Investigation 2 Equations with Variables**

**Vocabulary**

- open sentence
- solution

You can determine whether the equations in Investigation 1 are true by finding the value of each side. But what if an equation contains a variable? For example, consider this equation.

$$3 \cdot n = 18$$

You cannot tell whether this equation is true or false unless you know the value of  $n$ . An equation or inequality that can be true or false depending on the value of the variable is called an **open sentence**.

**Think & Discuss**

For each open sentence, find a value of  $n$  that makes it true. Then find a value of  $n$  that makes it false.

$$3 \cdot n = 18 \qquad n \div 2 = 2.5 \qquad n + 5 = 25$$

**Math Link**

These are three ways to write "6 times."

$$6 \times n \quad 6 \cdot n \quad 6n$$

Finding the values of the variable or variables that make an equation true is called *solving* the equation. A value that makes an equation true is a **solution** of the equation. Consider this equation.


$$6 \cdot n - 1 = 29$$

Finding a solution of  $6 \cdot n - 1 = 29$  is the same as answering the following question.

For what value of  $n$  is  $6 \cdot n - 1$  equal to 29?

Course 1, Chapter 2, p.536

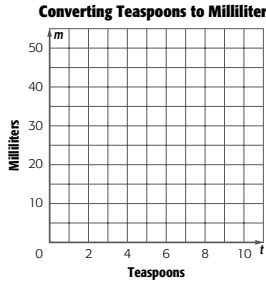
Other features, including *Explore*, *Examples*, *In Your Own Words*, and *On Your Own Exercises* complete the curriculum.



**Real-World Link**  
A woodpecker has a tough head. It can pound its beak against wood at a rate of 20 times per second.

4. On axes like those below, plot the data from your table.

**Converting Teaspoons to Milliliters**



5. Does it make sense to connect the points on your graph with a line? Explain. If it does make sense, do it.

6. Does it make sense to extend your graph beyond the points? Explain. If it does make sense, do it.

**Think & Discuss**

Darnell and Maya wrote rules relating pints and quarts.

1 quart = 2 pints. So, my rule is  $q = 2p$ .

If I have 2 quarts, I have 4 pints. Whatever number of quarts I have, the number of pints is 2 times that amount. My rule is  $p = 2q$ .

Whose rule is correct? Explain your answer.

Course 2, Chapter 8, p.370

**Real-World Links** engage student with interesting facts related to problem context.

**Cartoons** provide direct modeling of mathematical reasoning and problem solving strategies in an accessible and inviting format.

The **Think & Discuss** feature encourage classroom discussion and communication about mathematics.

# Teacher Materials

Each course includes a *Teacher's Guide*, with corresponding volumes of Chapter Resource Masters. These materials provide in-depth teacher support, including detailed lesson notes, blackline masters that facilitate classroom Investigation, and complete assessment notes and masters.

The *Chapter Planner* at the front of each chapter provides all chapter planning information in a convenient and easy-to-use format.

**CHAPTER 3**  
**Patterns, Numbers, and Rules**

**Chapter Overview**

In this chapter, students will study numbers in the millions and billions and learn to use different strategies to extend and generalize patterns in a variety of contexts, including Pascal's triangle. In studying a broad range of patterns, students will develop stronger understandings of the mathematical concepts underlying the patterns. Students also learn about some properties of real numbers and review the order of operations.

**The Big Picture**

Links to the Past	Chapter 3	Links to the Future
<b>Grade 5</b> Developing number sense for large numbers; multiplying whole numbers such as $5 \cdot 5$ .	<b>Lesson 3.1</b> (p. 110) Number Sense	<b>Course 2, Chapter 4</b> Magnitude of Numbers (pp. 172–209) <b>Course 3, Chapter 4</b> Exponents and Exponential Variation (pp. 144–203) <b>Course 3, Chapter 4</b> Exponents and Exponential Variation (pp. 144–203)
<b>Grade 4</b> Recognizing, describing, and extending repetitive patterns of numbers and shapes.	<b>Lesson 3.2</b> (p. 120) Patterns	<b>Course 2, Chapter 8</b> Linear Relationships (pp. 366–433) <b>Course 3, Chapter 1</b> Linear Relationships (pp. 2–63)
<b>Grade 5</b> Writing multiplication number models for rectangular arrays; using algebraic expressions such as $n + 2$ .	<b>Lesson 3.3</b> (p. 143) Variables and Rules	<b>Chapter 9</b> Equations (pp. 532–575) <b>Course 2, Chapter 1</b> Expressions (pp. 2–71) <b>Course 3, Chapter 10</b> Functions and Their Graphs (pp. 522–575)
<b>Grade 5</b> Reading a clock; using the distributive property to multiply mentally.	<b>Lesson 3.4</b> (p. 174) Apply Properties	<b>Course 2, Chapter 1</b> Expressions (pp. 2–71) <b>Course 3, Chapter 7</b> Inequalities and Linear Systems (pp. 310–371)

108A CHAPTER 3 Patterns, Numbers, and Rules

**Chapter Overview** states the high-level objectives and expectations.

**Links to the Past** shows where students previously covered concepts that will be introduced and developed in the current chapter.

**Links to the Future** demonstrate where students will revisit content learned in the current chapter.

Course 1, Chapter 3, p.108A

CHAPTER 3				
Planning Guide				
Lesson Resources				
	Pacing: 2 days	Pacing: 5 days	Pacing: 7 days	Pacing: 4 days
	Lesson 3.1	Lesson 3.2	Lesson 3.3	Lesson 3.4
Lesson Title	<b>Number Sense</b> (p. 110)	<b>Patterns</b> (p. 120)	<b>Variables and Rules</b> (p. 143)	<b>Apply Properties</b> (p. 174)
Lesson Objectives	<ul style="list-style-type: none"> <li>To understand the relationship between a million, a billion, and a trillion</li> <li>To write a number in exponential form</li> </ul>	<ul style="list-style-type: none"> <li>To create Pascal's triangle and describe patterns that aid in the creation</li> <li>To evaluate and compare expressions using the order of operations</li> <li>To write rules for a constructed pattern</li> </ul>	<ul style="list-style-type: none"> <li>To write symbolic rules for a number or dot sequence and show how each term relates to the term number</li> <li>To recognize when rules for a pattern are equivalent</li> <li>To use letters for variables to write rules for real-life situations</li> </ul>	<ul style="list-style-type: none"> <li>To apply the commutative and associative properties</li> <li>To perform numerical calculations using the distributive property</li> <li>To use the additive identity and the multiplicative identity</li> </ul>
Materials	calculators, one quarter, ruler	Lesson 3.2 Masters 1–3, Chapter 3 Master 3, *counters, scientific calculators, newspaper sections, computer with spreadsheet software, toothpicks	Chapter 3 Master 1–3 paper bags, *blocks	Lesson 3.4 Master 1
Quick Review Math Handbook	<b>Lesson 3.1</b> Powers and Exponents	<b>Lesson 1.3</b> Order of Operations	<b>Lesson 1.3</b> Order of Operations; (A)	<b>Lesson 5.2</b> Simplifying Expressions
Print Resources	<ul style="list-style-type: none"> <li>Study Guide and Intervention (p. 6)</li> <li>Skills Practice (p. 7)</li> <li>Problem-Solving Practice (p. 8)</li> <li>Enrichment (p. 9)</li> <li>Investigation Notebook and Reflection Journal</li> <li>Differentiation Handbook</li> </ul>	<ul style="list-style-type: none"> <li>Study Guide and Intervention (p. 11)</li> <li>Skills Practice (p. 12)</li> <li>Problem-Solving Practice (p. 13)</li> <li>Enrichment (p. 14)</li> <li>Investigation Notebook and Reflection Journal</li> <li>Differentiation Handbook</li> </ul>	<ul style="list-style-type: none"> <li>Study Guide and Intervention (p. 19)</li> <li>Skills Practice (p. 20)</li> <li>Problem-Solving Practice (p. 21)</li> <li>Enrichment (p. 22)</li> <li>Investigation Notebook and Reflection Journal</li> <li>Differentiation Handbook</li> </ul>	<ul style="list-style-type: none"> <li>Study Guide and Intervention (p. 24)</li> <li>Skills Practice (p. 25)</li> <li>Problem-Solving Practice (p. 26)</li> <li>Enrichment (p. 27)</li> <li>Investigation Notebook and Reflection Journal</li> <li>Differentiation Handbook</li> </ul>
Technology Resources	TeacherWorks Plus Classroom Presentation Toolkit ExamView Assessment Suite StudentWorks Plus Math Online > Brain Pops • Concepts in Motion	TeacherWorks Plus Classroom Presentation Toolkit ExamView Assessment Suite StudentWorks Plus Math Online > Brain Pops • Concepts in Motion	TeacherWorks Plus Classroom Presentation Toolkit ExamView Assessment Suite StudentWorks Plus Math Online > Brain Pops • Concepts in Motion	TeacherWorks Plus Classroom Presentation Toolkit ExamView Assessment Suite StudentWorks Plus Math Online > Brain Pops • Concepts in Motion

Lesson Objectives show the purpose of each lesson.

The Materials list helps prepare for class.

Pacing provides a guide for the length of each lesson.

Print Resources give ways to supplement instruction with clear indication of where each can be found.

Course 1, Chapter 3, p.108C

Technology Resources are additional options for differentiating instruction and assessing students.

\*Included in the Impact Mathematics Manipulative Kit (A) Lesson 5.1 Writing Expressions and Equations

108C CHAPTER 3 Patterns, Numbers, and Rules



### Teacher Tips

give practical suggestion for managing collaborative group work, teaching lessons, organizing materials, and more.

**Develop & Understand: C**

21. The expressions in the following statement involve division rather than multiplication.

$$a \div b = \frac{a}{b} + \frac{b}{a}$$

Choose some values for  $a$ ,  $b$ , and  $c$ . Test the statement to see whether it is true. For example, you might try  $a = 2$ ,  $b = 5$ , and  $c = 7$ . Try several values for each variable. Do you think the statement is true for all values of  $a$ ,  $b$ , and  $c$ ? The statement is true for all values of the variables, provided  $c \neq 0$ .

22. The expressions in the statement below are like those in Exercise 21, but they involve multiplication rather than addition.

$$\frac{a}{c} = \frac{a}{c} + \frac{b}{c}$$

Choose some values for  $a$ ,  $b$ , and  $c$ . Test the statement to see whether it is true. Try several values for each variable. Do you think the statement is true for all values of  $a$ ,  $b$ , and  $c$ ? The statement is not true.

23. Choose some values for  $a$  and  $b$ , and test this statement.

$$(a + b)^2 = a^2 + b^2$$

Do you think the statement is true for all values of  $a$  and  $b$ ? No (it works only when  $a = 0$  or  $b = 0$ .)

**Share & Summarize**

1. Make a dot diagram to show that  $6 \cdot 3 + 6 \cdot 2 = 6(3 + 2)$ .

2. Give examples of calculations that look difficult but are easy to do mentally by using the distributive property. For such examples, explain how the distributive property can be used to simplify the calculation. Answers will vary.

**Teacher Tips** This problem set addresses the misconception that any operation distributes over any other operation. This is an important consideration for students since it may affect their basic math computations when they solve more complex problems.

**Develop & Understand: C**

*Suggested Grouping: Pairs*

► **Exercise 21** Students determine whether division can be distributed over addition.

► **Exercise 22** Students find that division does not distribute over multiplication.

► **Exercise 23** Students look at expressions involving exponents, which do not distribute over addition.

**Wrap-Up** Have students share their answers to Exercises 21–23. You may want to point out which operation is distributing over another for each exercise.

**Share & Summarize**

These exercises focus on the basic concepts in this investigation. **Exercise 1** asks students to make a dot diagram. They can use Chapter 3 Master 2, Centimeter Dot Paper.

**Exercise 2** asks students to show how to use the distributive property to find answers mentally. You may want to provide students with additional practice by having students share the parts of their calculations that look difficult. Then have the class use the distributive property to simplify the calculation.

**Troubleshooting** If students are having difficulty showing the groupings of the distributive property, have them use concrete materials to model some of the problems in this investigation. Students may need to use bags and blocks in order for them to internalize the concepts. Once they have a basic grasp of the concepts, they can move on to symbolic representations.

**Reaching All Learners**

**Beyond Level** Students who finish early may want to explore distributing with other pairs of operations, such as addition and subtraction or division and subtraction. Encourage students to write an expression using variables, try values for the variables, and decide whether the statement is true for all values of the variables.

LEARN 1.4 Apply Properties 183

### Share & Summarize

helps wrap up each Investigation with guiding questions to assess student comprehension of lesson content.

### Reaching All Learners

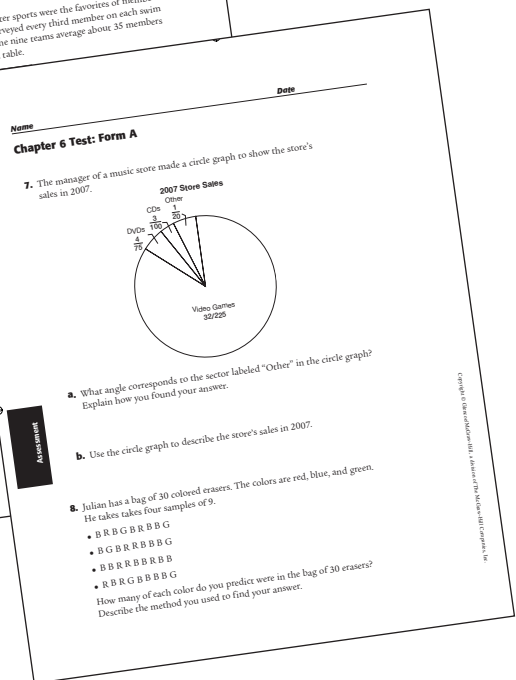
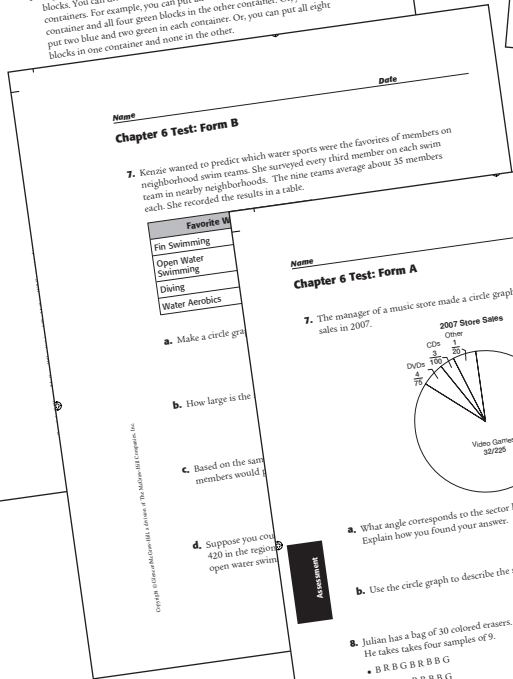
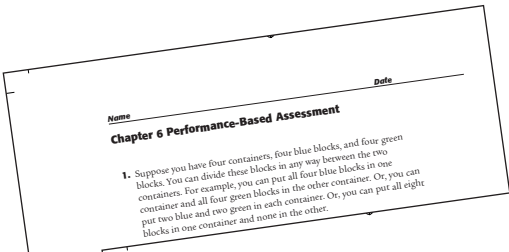
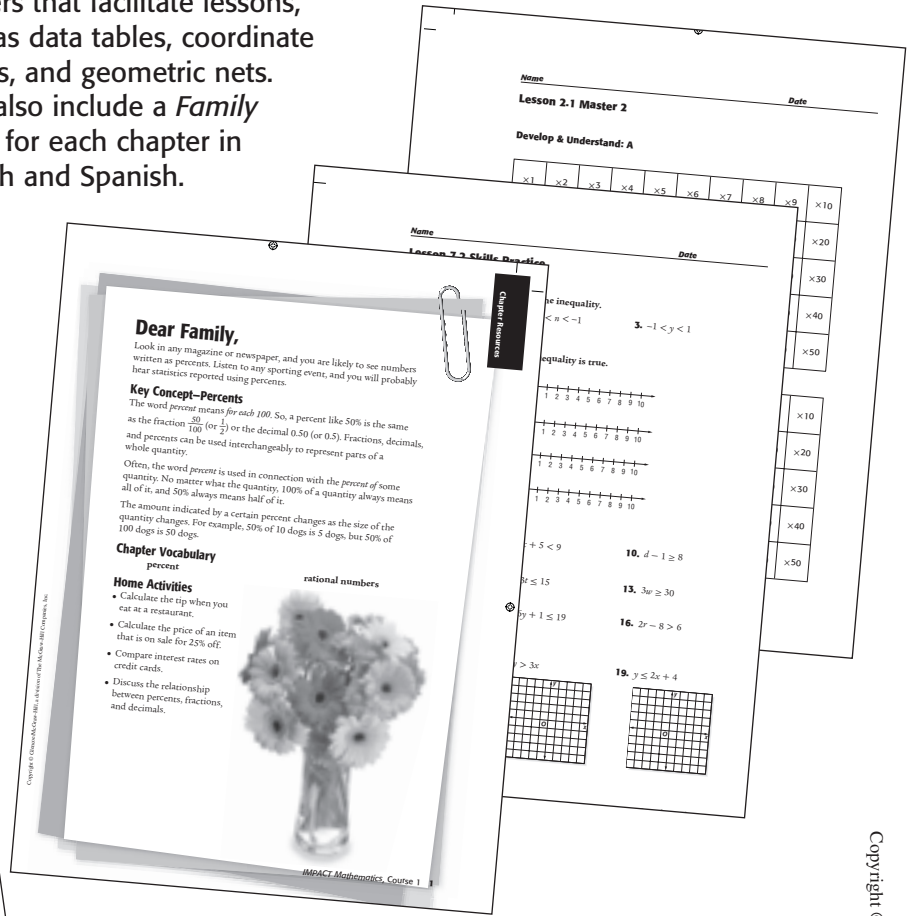
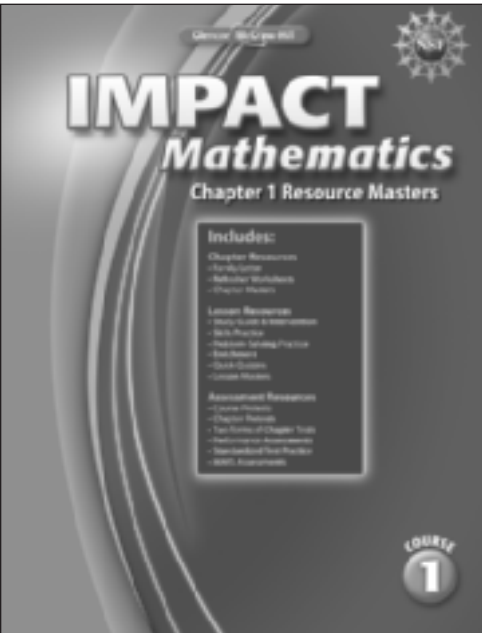
provides on the spot ideas for differentiation and ensuring that the curriculum is accessible to all students.

Course 1 Teacher Guide, p.183

**Troubleshooting** features are a first intervention step to quickly redirect students with misconceptions to successfully understand content.

# Chapter Resource Masters

Chapter Resource Masters for each chapter include skill practice worksheets, blackline masters that facilitate lessons, such as data tables, coordinate graphs, and geometric nets. They also include a *Family Letter* for each chapter in English and Spanish.



Assessment Resources in the Chapter Resource Masters include two equivalent chapter test forms as well as a performance assessment test for each chapter and each semester. They also include assessment forms to help track student progress in a variety of ways.

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# Other Components

The *IMPACT Mathematics* program has several optional components available to help teachers be successful implementing in the classroom. Manipulatives, print resources, technology resources, and online materials round out the textbook used in the classroom.

**IMPACT Mathematics Manipulative Kit** Contains conveniently packaged sets of manipulatives for use with all three courses of *IMPACT Mathematics*. Each package is appropriate for classrooms of up to 28 students. Includes 500 2-cm colored wooden cubes, 400 color tiles, 36 pair of numeral cubes, 14 GeoMirrors, 14 1-cm overhead grids, 600 two-color counters, and 6 sets of Linkage Strips.

**TeacherWorks Plus CD-ROM** Includes a lesson planner and an Interactive Teacher Edition enabling you to customize an entire course of study to meet your specific needs. Find all the resources electronically for lesson planning or printing. Assessments, enrichment materials, black-line masters, and more.

**StudentWorks Plus CD-ROM** Includes the entire text, formatted like the hardbound book, so students can study from just about anywhere. With StudentWorks, students can also access interactive assessments, links to the Internet, and many more exciting tools.

**Glencoe.com** Access e-games, tutorials, and interactive assessments online.

## ExamView Assessment Suite

- Create multiple versions of tests.
- Create modified tests for inclusion students with one mouse click.
- Edit existing questions and add your own questions.
- Build tests aligned with state standards using built-in state curriculum correlations.
- Change English test to Spanish with one mouse click and vice versa.



# About MARS

## Mathematics Assessment Resource Service

**MARS, the Mathematics Assessment Resource Service**, is a U.S.-based international team that created the performance-based assessments in the *IMPACT Mathematics* program. MARS is under the direction of a Mathematics Board that includes teachers and recognized United States and international experts in the mathematics education and assessment fields.

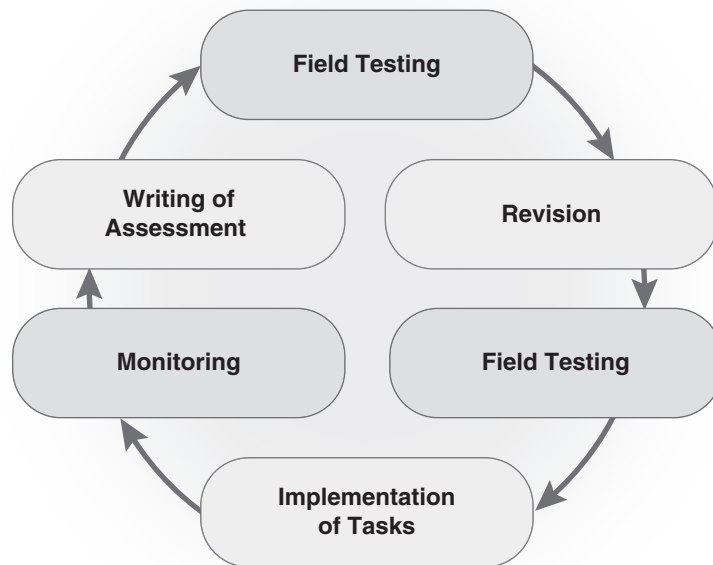
## Background

An NSF grant (National Science Foundation grant #ESI-9726403) has supported the many years of research, development, and evaluation that form the basis of the high-quality, performance-based assessments that comprise the assessment section of *IMPACT Mathematics*.



## Development Process

Each assessment task is carefully constructed to assess the broad domain of mathematical performance that national and local standards specify. Tasks go through a development and review process to ensure validity and usability for student evaluation and continued improvement in instruction.



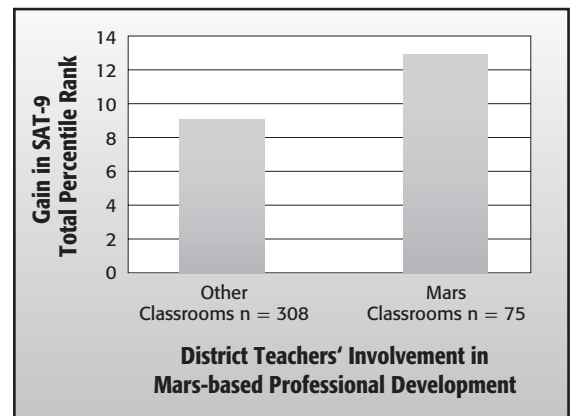
During the writing and revision process, the scoring rubric is refined and student work is collected. Rubrics, along with scored student work, accompany the MARS assessments, allowing teachers to evaluate student knowledge and progress and better inform instruction.

## Evaluative Evidence

Over the years, MARS performance-based assessments have been used both throughout the United States and internationally. Evidence shows that the MARS assessments test a broader range of skills and knowledge than many state tests and are comparably challenging overall.

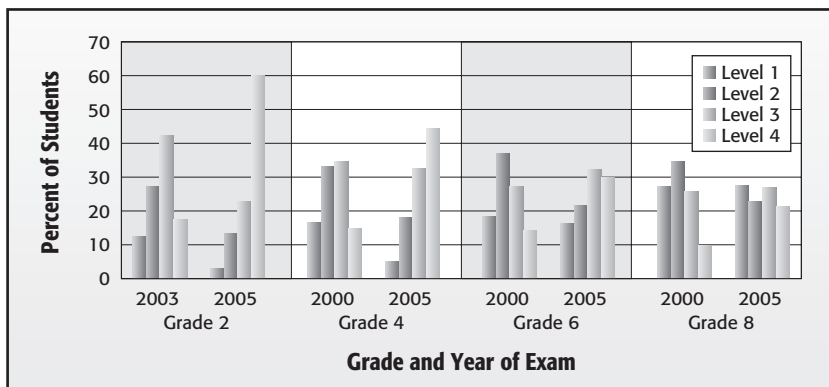
**Figure 1.** In one particular study, teachers used MARS assessments as part of a formative-assessment piece prior to taking the SAT-9 standardized test. The data show that students in classrooms receiving the MARS treatment outperformed control classrooms without MARS.

**Figure 1**  
Average Gain in Math Percentile Rank  
Between 1999 SAT-9 and 2000 SAT-9



Source: Noyce Foundation Annual Report 2000

**Figure 2**  
Student Achievement By Performance Levels on Mars Exam  
Comparison Between First Year and 2005



Source: Noyce Foundation Annual Report 2005

**Figure 2.** Over time, the evidence for MARS follows similar trends. The number of students performing at the highest levels (Level 4) of the MARS assessments climbs each year and at every grade level, while the number performing at the lower levels (Level 1) declines.

# MARS Performance-Based Assessment

Included with each *IMPACT Mathematics* unit is a high-quality, research-based performance assessment by the MARS group. Using such an assessment helps determine student progress toward mastery of critical mathematical concepts.

## What Is Performance-Based Assessment?

Performance assessments are an authentic form of assessment in which students are asked to perform tasks or solve problems. By placing these tasks in authentic (real-world) contexts, students are better able to see how mathematical concepts, skills, and problem solving abilities are useful outside of the classroom. Solving problems and performing tasks allows students to communicate their understanding of a concept more fully.

## The Assessment

The broader range and greater depth of the tasks presented enables teachers to recognize, and thus encourage, students' achievement in meeting these higher standards for mathematical performance, including concepts, skills, and problem solving. The tasks demand substantial chains of reasoning and non-routine problem solving, covering the content and the process areas specified in national mathematics standards.

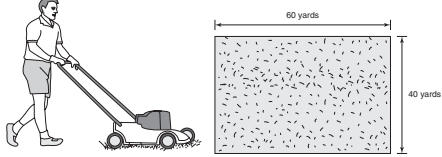
Name \_\_\_\_\_ Date \_\_\_\_\_

**Chapter 5 MARS Performance-Based Assessment**

**Lawn Mowing**

This problem gives you the chance to:

- solve a practical problem involving ratios
- use proportional reasoning



Dan and Andy take turns in cutting the grass. Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard? \_\_\_\_\_ square yards
2. Dan takes an hour to cut the lawn using an old mower. How many square yards does Dan cut in a minute? Show your work. \_\_\_\_\_ square yards
3. Alan only takes 40 minutes using a new mower. \_\_\_\_\_ square yards  
How many square yards does Alan cut in a minute?
4. If they each mow an area that is twice the length and twice the width as their lawn, how long do they take? Show how you figured it out.  
Dan \_\_\_\_\_ Alan \_\_\_\_\_

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## Rubrics and Scoring

Each assessment is supported by a scoring rubric and actual graded student work. The scoring rubric lists point values to assign for alternative approaches and student responses. The sample student work shows how real student responses have been scored as a guide for teachers.

Name \_\_\_\_\_ Date \_\_\_\_\_

**Chapter 5 MARS Performance-Based Assessment**  
**Rubric: Lawn Mowing**

The core elements of performance required by this task are:

- solve a practical problem involving ratios
- use proportional reasoning

Based on these, credit for specific aspects of performance should be assigned as follows:

Exercises	Points	Section Points
1. Gives correct answer: <b>2,400</b> square yards	1	1
2. Gives correct answer: <b>40 square yards per minute</b> Shows correct work such as: $(60 \times 40) \div 60$	1 1	2
3. Gives correct answer: <b>60 square yards per minute</b> Shows work such as: $(60 \times 40) \div 40$	1 1	2
4. Gives correct answer: Dan: <b>4 hours</b> Alan: <b>2 hours and 40 minutes</b> accept 160 minutes Gives correct explanation such as: The new lawn is 4 times as big: $4 \times 1 \text{ hour} =$ $4 \times 40 \text{ minutes} =$	1 1 1 1	4
<b>Total Points</b>		<b>9</b>

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## Sample Student Work


Examples of student work show how actual students have performed on the assessment. Each assessment includes samples from three different students: approaching level, on level, and beyond level. Teacher responses and scores (using the rubric as a guide) are also shown on the student sample pages.

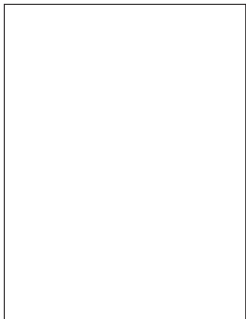
Name \_\_\_\_\_ Date \_\_\_\_\_

**Chapter 5 MARS Performance-Based Assessment**  
**Sample Student Papers**









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# Middle Grade Issues

## How Are Middle Grades Students Unique?

Middle grades students possess a special view of themselves and of the world, as any middle grades teacher can tell you. The middle school acts as a bridge between the elementary and the high school while nurturing the unique developmental needs of the adolescent student.

The middle grades are critical to the academic, emotional, and social success of the adolescent. This period is a pivotal point in students' lives, when they are being pulled in every direction, between excitement and confusion, between depending on others and making decisions on their own. A school environment that is tailored to this exciting and challenging time can help students become confident and well-grounded individuals. A successful curriculum is one that offers opportunities for students to take responsibility for their own learning, and to learn from the decisions they make.

***To make wise, informed decisions about their academic and social lives, students require strong decision-making skills.***

*IMPACT Mathematics* believes that successful mathematics education requires a curriculum that is balanced with respect to structured learning, direct instruction, and creative problem solving. It's critical that student discovery play as significant a role in the learning process as teacher-directed instruction. Students are focused on themselves at this age, and the *IMPACT Mathematics* program provides a methodology that encourages adolescents to make connections among themselves, mathematics, and the world.

Middle grades students make decisions that have extraordinary influence on their lives. To make wise, informed decisions about their academic and social lives, students require strong decision-making skills. They need to know how to gather and evaluate information, and must have confidence in their ability to make a decision or solve a problem based on that information. An effective school curriculum should provide students with the tools they need for informed decision making. *IMPACT Mathematics* provides students with opportunities to strengthen decision-making skills, and provides all students with opportunities for success. Additionally, the *IMPACT Mathematics* approach maintains students' interest, thereby encouraging them to continue their exploration of mathematics.

In our technological society, all students need mathematics, regardless of their academic and career pursuits. *IMPACT Mathematics* addresses the needs of all students no matter what their ability levels. Student differences are accommodated through the intensity in which investigations are pursued, the varying degrees of problem difficulty, and the intricacy of the applications. *IMPACT Mathematics* is a rigorous program that encourages students to reach beyond their textbooks to develop their understanding; *IMPACT Mathematics* encourages students to use a variety of sources and tools to solve problems.



## Middle School Models

Middle school classes are structured in a variety of ways, with a variety of learning models. The flexibility of *IMPACT Mathematics* allows for these variations, and the program can be implemented successfully in each setting.

### Departmentalized Middle Schools

In departmentalized middle schools, teachers are organized by the curricular area in which they teach. Usually there is a set bell schedule, though not all classes are necessarily the same length. In general, teachers in departmental middle schools might teach mathematics without a great deal of collaboration with teachers from other disciplines. Occasionally they may plan and teach a unit with other teachers to incorporate numerous disciplines. A teacher in a departmentalized, self-contained setting will find everything needed to teach mathematics completely contained in *IMPACT Mathematics*.

### Interdisciplinary Teams of Teachers

An interdisciplinary team of teachers shares common students, has common planning time, and has control over the daily schedule. Thematic units are created around various topics. Each team teacher contributes content from an existing curriculum, and pulls in materials from outside sources as appropriate. The school day's schedule is arranged to accommodate the day's activities. Sometimes students work in separate classes, sometimes they work together as one large group. Typically, students and teachers engage in a culminating activity to tie together the unit's instruction. Chapter Openers provide real-life examples of math applications that can easily lead to cross-disciplinary projects. There are many problems provided throughout *IMPACT Mathematics* that lend themselves to connections in other subject areas. These cross-discipline problems are clearly labeled.

### Block Scheduling

With block scheduling, classes are generally organized into large blocks of time. Responsibility for learning rests primarily on the students, who spend much of their time investigating problems and posing questions. Textbooks serve mostly as resources for students as they pursue investigations. Activities are used to demonstrate understanding of concepts and reinforce knowledge. Each lesson in *IMPACT Mathematics* contains several *Investigations* that address the lesson's objectives. In a block setting, two *Investigations* with accompanying discussion can be completed in a single day.

## Pedagogy

Effective teaching methods for middle grades students are varied and student-centered. *IMPACT Mathematics* encourages active learning through an assortment of teaching methods—collaborative problem solving, teacher-directed instruction, class discussion, and individual practice.

From the beginning of the program, *IMPACT Mathematics* students are expected to explain, justify, defend, hypothesize, and verify, both verbally and in writing. Additionally, *IMPACT Mathematics* maintains a balance between exploratory, problem-centered investigations and directed instruction. Important topics are revisited periodically with increasing depth and formality. The result is better student understanding without endless review and reteaching.

With *IMPACT Mathematics*, a comprehensive study of Algebra 1 becomes a realistic option for middle grades students. The following qualities ensure the successful coverage of Algebra 1 content:

- Exposure-to-mastery (informal-to-formal) approach
- Conceptual understanding combined with the practice of computational skills and symbolic manipulation skills
- Extensive problem solving, which includes critical-thinking proficiencies, applications, connections, and extensions

*IMPACT Mathematics* is an effective mathematics program that balances applications and theory. Each unit focuses on a key mathematical concept developed through real-world contexts and appropriate use of technology. Students develop conceptual understanding, then use technology to extend and broaden their understanding to solve more complex problems.

## Assessment

Valid assessment should provide a comprehensive picture of student development while giving teachers useful feedback about instructional needs. Assessment should be an integral part of a program, not tacked on at the end of a chapter. The assessment tools in *IMPACT Mathematics* are consistent with the recommendations of the National Council of Teachers of Mathematics and are broader than the assessment tools used in traditional mathematics programs. They encompass the processes of problem solving, reasoning, communication, connections, concepts, applications, representational strategies, and procedures.

The *IMPACT Mathematics* curriculum offers assessment options that can be used individually or in combination to develop a complete assessment package.

### In the Student Edition:

- Homework and assignment guides are available for every Investigation.
- *Share & Summarize* questions provide a forum for students to summarize and share their learning from Investigations with the class.
- *On Your Own Exercises*, an integral part of daily program instruction, are independent assignments intended for individual work primarily done outside of class.
- *Chapter Review* allows students an opportunity to reflect on the important topics within the chapter and to prepare for formal assessment.

### In the Teacher's Guide:

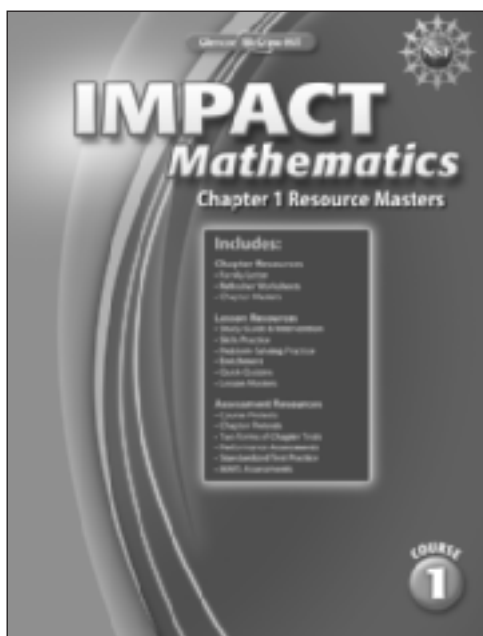
- The Problem Set Wrap-Up offers summary points, questions, and checkpoint notes to insure that students are making appropriate progress through an Investigation.
- Troubleshooting notes provide remedial work some students might need in order to move on to the next Investigation successfully.
- Additional Examples are worked out for use as an on-the-run assessment tool.
- Quick Check is an informal assessment that provides a checklist of what students should be able to do at the end of each lesson.
- Quick Quiz provides a brief end of lesson assessment opportunity.

### In the Chapter Resource Masters:

- Diagnostic Tests determine whether students have the prerequisite skills to begin a course.
- Refresher Worksheets help students review the skills they will need.
- Chapter Assessments come in two equivalent forms and provide a comprehensive test of chapter content.
- Performance Assessments at the end of each chapter provide more open-ended opportunities to measure student achievement. They can be used to supplement or replace items on a chapter test, as a take-home assignment, as a group assessment, or as challenge or extra-credit problems.
- Semester Assessments provide a cumulative semester test and a set of performance assessment items.

The flexibility and variety of assessment in *IMPACT Mathematics* addresses the various types of ability levels and learning styles of students, as well as the instructional needs of teachers.

Teachers may also want to use student journals, student notebooks, and portfolios to extend their assessment options. Journals provide an avenue for students to communicate what they do or do not understand about mathematics concepts. This method of self-assessment is nonthreatening to students and provides feedback to teachers regarding areas for reteaching. Student notebooks, showing problems and solutions worked on in class, can provide additional support for students when doing homework independently. Portfolios can assess student thinking, their ability to make mathematical connections, and their proficiency in applying the problem solving process.



## Home Involvement

Support and understanding from the home plays a crucial role in the successful implementation of *IMPACT Mathematics* and in the success of students.

Beginning even before students reach adolescence, many parents are concerned about college and careers. The *IMPACT Mathematics* curriculum is an alternative, current, standards-based road both to college and career opportunities in our modern, technological society.

Parents should receive information in a timely manner about *IMPACT Mathematics* and should be provided with the basis for the implementation of its concepts, methodology, and sequence. The more familiar parents are with the *IMPACT Mathematics* program, the more effectively they can support the teacher's efforts and carry over mathematical learning into the home.

Teachers may want to begin by sharing the following topics during an open house or curriculum night at the beginning of the school year:

- Review of student materials
- Discussion of mathematical content integration
- Visual overview of algebra coverage
- Review of scope and sequence and related concept mastery timeline
- Brief demonstrations of lessons which emphasize problem solving and informal-to-formal concept development
- Explanation of examples of comprehensive assessment tools and opportunities
- Explanation of the use of manipulatives, calculators, and other technology that will be integrated into the course

Parents are, in most instances, responsive to teachers' efforts to maintain open communication. In addition to sending informative letters home, conducting open houses, and holding parent-teacher conferences, teachers can help keep parents involved in other creative ways in their child's learning. With a letter home, the teacher may ask parents to be a guest speaker, and both parent and students can recognize how mathematics instruction in school connects to the real world.

Teachers might also periodically engage parents as student partners for homework or other assignments. Students and their parents can work together to find solutions to problems, instead of the more traditional approach of parents "helping with homework." This collaborative approach breaks down barriers to sharing schoolwork with parents, and can open lines of communication between the school, the student, and the home.

Some households communicate less than others. Schools and teachers should be sensitive to differences and adapt their approaches as necessary. But whatever the methods of connecting the school to the home, middle grades students will gain valuable insight when they begin to see how their education in the classroom is part of learning about the world at large.

# Frequently Asked Questions

**Q** *What should students know and be able to do before beginning **IMPACT Mathematics**?*

**A** Students should have a solid foundation of mathematical understanding from a K–5 program. Topics include number and arithmetic proficiency, exposure to data and probability; familiarity with geometry; exposure to algebra. For a detailed list of *IMPACT Mathematics* entrance expectations for Course 1, see Expectations, Course 1 on page 39 of this book.

**Q** *Is it necessary to begin **IMPACT Mathematics** in Grade 6?*

**A** No. Students can begin *IMPACT Mathematics*:

- in Grade 6 after completing any K–5 mathematics program.
- in Grade 7 after completing any K–6 program.
- after completing a K–5 program plus the first course of a typical middle school program.

**Q** *How do **IMPACT Mathematics, Course 1** and **Everyday Mathematics Grade 6** compare? How do we choose which program to use in the sixth grade?*

**A** Mathematically, both *IMPACT Mathematics* and *Everyday Mathematics Grade 6* are quite compatible. Both curricula are designed to revisit and develop important mathematical ideas over time, and to provide more mathematics to all students. The differences that do exist are minor. *Everyday Mathematics* devotes more lessons to decimal topics and provides a wider range of geometry topics. *IMPACT Mathematics* devotes a few more lessons to fractions and percents, and provides a more systematic development of early algebra concepts.

**Q** *Our school plans to implement a new middle school math program in Grades 6, 7, and 8 next year. Can our current seventh graders be successful in **IMPACT Mathematics Grade 8** next year?*

**A** Without having experienced *IMPACT Mathematics* in Grade 7, your first group of eighth-grade students may struggle with the algebra that was covered by *IMPACT Mathematics* at the end of Grade 7. During your first year of implementation, we recommend that you choose some topics from Grade 7 to integrate with your Grade 8 instruction.

**Q** *Our middle school uses a team-teaching model. Which resources are provided to support interdisciplinary teams?*

**A** Each *Chapter Opener* provides cross-discipline, real-life examples of the application of math content covered in the chapter. Teams can use these ideas to create interdisciplinary projects. In addition, there are many problems provided within each chapter that lend themselves to connections in other subject areas.

**Q** *In our district, only our high ability students take algebra in 8th grade. Is **IMPACT Mathematics** an honors course for middle school?*

**A** *IMPACT Mathematics* is not intended as an honors curriculum for middle school. Instead, the program makes algebra and other middle grades content accessible to all students. Many middle school programs spend an inordinate amount of time reviewing and reteaching whole-number computation, fraction and decimal computation, percents, ratio, and proportion without substantially advancing students' content knowledge in algebra and geometry. While these number and arithmetic topics are important, endless review and reteaching of computation is not necessary. A large majority of middle grades students will successfully complete algebra with *IMPACT Mathematics*.

**Q** *How can we document for our high schools that students completing **IMPACT Mathematics** have finished Algebra 1?*

**A** The scope and sequence for the program clearly outlines all of the important content of Algebra 1 covered in the program. Additionally, the assessment resources for Course 3 include an end-of-course algebra exam that can be used to demonstrate the algebra content covered in the program.

**Q** *Which courses in Grade 9 are recommended for students who have completed **IMPACT Mathematics**?*

**A** Students who successfully complete *IMPACT Mathematics* can take either the geometry course usually offered to sophomores who complete Algebra 1, Algebra 2, or the second course of an integrated program such as *Contemporary Mathematics in Context*.

**Q** *What technology is required to successfully teach **IMPACT Mathematics**?*

**A** Scientific calculators are necessary for all courses. Graphing calculators are optional in Courses 1 and 2 and necessary for in-class use in Course 3. Computer software such as spreadsheets or dynamic geometry programs is useful but not required.

# Spreading the Word

The following guidelines may help you present *IMPACT Mathematics* to the school board or other members of the community.

## Making Presentations to the School Board and Community

1. Make transparencies or handouts that describe the *IMPACT Mathematics* program.
2. Describe the specific ways that you have adapted or could adapt this program to fit the needs of your students.
3. Point out ways in which *IMPACT Mathematics* matches or fulfills the goals of your district.
4. Have students demonstrate an activity or ask the audience to participate in one.
5. Have samples of the student books available for people to examine.
6. Allow plenty of time for questions and answers.

## Involving Parents

1. A brief letter home at the beginning of each chapter will let parents know what to expect, and will encourage involvement in their children's learning.
2. Ask parents if they have a particular talent, level of expertise, contacts, or specific items that would add to the chapter.
3. Ask parents to help supervise activities that require help.
4. Adapt some student activities to include parents as participants. Offer alternative solutions to students whose parents are unable to participate.



# IMPACT Mathematics

## Goals:

- To provide a three-year mathematics curriculum that makes more mathematics accessible to more middle grades students
- To provide a unique alternative to existing middle grades programs

## How is the content unique?

- Standards based, integrated curriculum that incorporates topics from several strands
- Students moving out of 8th grade will not need to take algebra in 9th grade
- Exposure-to-mastery approach makes algebra accessible and appropriate for middle grades students
- Strong content progress from grade to grade with minimal reteaching of topics

## How is the pedagogy unique?

- Balance between exploratory, problem-centered investigations and direct instruction
- Important topics are revisited over time with greater depth and more formality
- Greater depth of topics means greater student understanding without endless review and reteaching

## Who can use this program?

- Appropriate for students who have used *Everyday Mathematics*, and for students from more traditional programs
- Ideal for heterogeneous classrooms
- Accessible for a wide range of ability levels
- Flexibility whether entering in 6th or in 7th grade

# IMPACT Mathematics

## Comparison to Traditional Programs

### IMPACT Mathematics

- Informal to formal development of concepts making mathematics accessible to middle grades students
- Important topics are revisited in greater depth
- Balance of open-ended problems and direct instruction
- Depth of development
- Meaningful integrated review
- Strong content progress from grade to grade
- Strong support for the teacher
- Direct, sequential lesson structure

#### Similarities

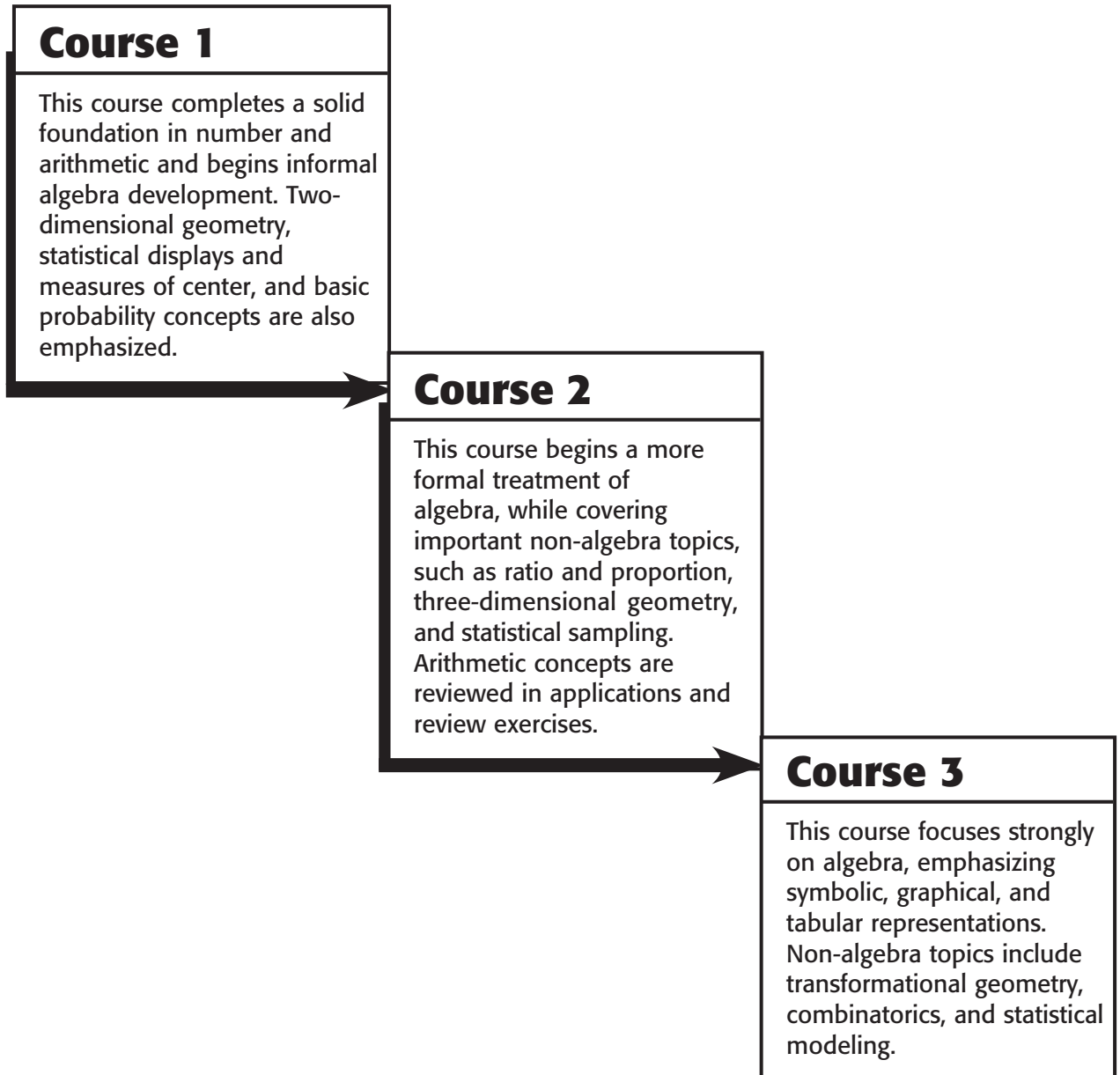
- Practice
- Applications
- Extension
- Mixed review
- Clear pacing
- Explicit homework

#### Differences

- Immediate formal development
- Show/tell, try, and practice
- Breadth of topics, limited depth
- Constant review and reteaching
- Limited content progress from grade to grade
- Teacher materials have lots of options with little actual support
- Busy, fragmented lessons

# IMPACT Mathematics

## Course-by-Course Progression



## Course 1

<b>Chapter 1:</b>	Polygons, Angles, and Circles
<b>Chapter 2:</b>	Fractions and Decimals
<b>Chapter 3:</b>	Patterns, Numbers, and Rules
<b>Chapter 4:</b>	Fractions and Decimals Operations
<b>Chapter 5:</b>	Rate, Ratio, and Proportion
<b>Chapter 6:</b>	Percents
<b>Chapter 7:</b>	Area, Volume, and Capacity
<b>Chapter 8:</b>	Coordinate Plane
<b>Chapter 9:</b>	Equations
<b>Chapter 10:</b>	Data and Probability

## Course 2

<b>Chapter 1:</b>	Expressions
<b>Chapter 2:</b>	Exponents
<b>Chapter 3:</b>	Signed Numbers
<b>Chapter 4:</b>	Magnitude of Numbers
<b>Chapter 5:</b>	Geometry in Three Dimensions
<b>Chapter 6:</b>	Data and Probability
<b>Chapter 7:</b>	Real Numbers
<b>Chapter 8:</b>	Linear Relationships
<b>Chapter 9:</b>	Equations
<b>Chapter 10:</b>	Proportional Reasoning and Percents

## Course 3

<b>Chapter 1:</b>	Linear Relationships
<b>Chapter 2:</b>	Lines and Angles
<b>Chapter 3:</b>	Percents and Proportions
<b>Chapter 4:</b>	Exponents and Exponential Variation
<b>Chapter 5:</b>	Algebraic Expressions
<b>Chapter 6:</b>	Transformational Geometry
<b>Chapter 7:</b>	Inequalities and Linear Systems
<b>Chapter 8:</b>	Quadratic and Inverse Relationships
<b>Chapter 9:</b>	Solve Quadratic Equations
<b>Chapter 10:</b>	Functions and Their Graphs
<b>Chapter 11:</b>	Data and Probability
<b>Chapter 12:</b>	Algebraic Fractions

# The Instructional Cycle

## **I N T R O D U C E**

The multiday lesson begins with a full-class discussion, activity, or problem. The teacher poses questions, orchestrates an activity, or monitors strategies students use to solve problems. **Explore** activities and **Think & Discuss** questions can be used to guide discussion.

## **D E V E L O P**

In-class **Investigations** provide a mix of worked out examples, direct modeling through cartoons, and interactive problem sets. Students may work independently or in small, collaborative groups. Investigations can be completed in one class period. The mathematical content of the Investigations determines the approach and the day's activity. Homework or assignment guides are available for each Investigation, and each Investigation wraps up with **Share & Summarize** questions.

## **A S S I G N &**

## **A S S E S S**

Independent assignments and assessments provide rich opportunities for students to demonstrate their learning. Each lesson concludes with **On Your Own Exercises**. The types of problems included in each set of **On Your Own Exercises** are:

- **Practice & Apply**, which are similar to the Investigations.
- **Connect & Extend**, which relate the lesson topics to other mathematical topics and strands.
- **Mixed Review**, which provides review of previously learned skills in order to maintain mastery.

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