SKILL 1

Name ____________________ Date __________

Variables and Expressions

Algebra is a language of symbols. In algebra, letters, called **variables**, are used to represent unknown quantities. A combination of one or more variables, numbers, and at least one operation is called an **algebraic expression**.

$x - 9$ means x minus 9.

$7m$ means 7 times m.

ab means a times b.

$\frac{h}{4}$ means h divided by 4.

To **evaluate** an algebraic expression, replace the variable or variables with known values and then use the order of operations.

EXAMPLE ***Evaluate $2c - 7 + d$ if $c = 8$ and $d = 5$.***

$2c - 7 + d = 2(8) - 7 + 5$	*Replace c with 8 and d with 5.*
$= 16 - 7 + 5$	*Multiply.*
$= 9 - 5$	*Subtract.*
$= 14$	*Add.*

EXERCISES ***Evaluate each expression if $x = 9$, $y = 5$, and $z = 2$.***

1. $x + 6$

2. $y - 3$

3. $z + 11$

4. $23 - x$

5. $6z$

6. $14 + y$

7. $4z + 5$

8. $24 - 2x$

9. $3y - 7$

10. $\frac{x}{3}$

11. $\frac{14}{z}$

12. $\frac{xy}{15}$

13. $4x - 2y$

14. $6z - x$

15. $18 - 2x$

16. $6y - (x + z)$

17. $3x - z$

18. $5(y + 7)$

19. $2x + y - z$

20. $5z - y$

21. $4x - (z + 2y)$

22. $\frac{2x + 3z}{12}$

23. $\frac{7z - Y}{x}$

24. $\frac{5y - 7}{x}$

25. $(11 - 3z) + x + y$

26. $7(x - z)$

27. $6y - 9z$

28. $\frac{xy}{3} - z$

29. $\frac{40}{y} + x$

30. $\frac{4(x - y)}{z}$

31. $3x - 2(y - z)$

32. $(14 - 6z) + x$

33. $10z - (x + y)$

APPLICATIONS

34. The weekly production costs at Jessica's T-Shirt Shack are given by the algebraic expression $75 + 7s + 12t$ where s represents the number of short-sleeve shirts produced during the week and t represents the number of long-sleeve shirts produced during the week. Find the production cost for a week in which 30 short-sleeve and 24 long-sleeve shirts were produced.

35. The perimeter of a rectangle can be found by using the formula $2l + 2w$, where l represents the length of the rectangle and w represents the width of the rectangle. Find the perimeter of a rectangular swimming pool whose length is 32 feet and whose width is 20 feet.

SKILL 2

Name ______________________ Date __________

Writing Expressions and Equations

Translating verbal phrases and sentences into algebraic expressions and equations is an important skill in algebra. Key words and phrases play an essential role in this skill.

The first step in translating a verbal phrase into an algebraic expression or a verbal sentence into an algebraic equation is to choose a variable and a quantity for the variable to represent. This is called **defining a variable**.

The following table lists some words and phrases that suggest addition, subtraction, multiplication, and division. Once a variable is defined, these words and phrases will be helpful in writing the complete expression or equation.

Addition	Subtraction	Multiplication	Division
plus	minus	times	divided
sum	difference	product	quotient
more than	less than	multiplied	per
increased by	subtract	each	rate
in all	decreased by	of	ratio
together	less	factors	separate

EXAMPLES

Translate the phrase "three times the number of students per class" into an algebraic expression.

Words three times the number of students per class

Variable Let s represent the number of students per class.

Expression $3s$

Translate the sentence "The weight of the apple increased by five is equal to twelve ounces." into an algebraic equation.

Words The weight of the apple increased by five is equal to twelve ounces.

Variable Let w represent the weight of the apple.

Equation $w + 5 = 12$

EXERCISES *Translate each phrase into an algebraic expression.*

1. seven points less than yesterday's score
2. the number of jelly beans divided into nine piles
3. the morning temperature increased by sixteen degrees
4. six times the cost of the old book
5. two times the difference of a number and eight

Translate each sentence into an algebraic equation.

6. The sum of four and a number is twenty.
7. Fourteen is the product of two and a number.
8. Nine less than a number is three.
9. The quotient of a number and five is eleven.
10. Fifteen less than the product of a number and three is six.

APPLICATIONS

11. Sierra purchased an ice cream cone for herself and three friends. The cost was $8. Define a variable and then write an equation that can be used to find how much Sierra paid for each ice cream cone.

12. Nicholas weighed 83 pounds at his most recent checkup. He had gained 9 pounds since his last checkup. Define a variable and then write an equation to find Nicholas' weight at the previous checkup.

13. There are three times as many people at the amusement park today than there were yesterday. Today's attendance is 12,000. Define a variable and then write an equation to find yesterday's attendance.

SKILL 3

Name ______________________ Date __________

Simplifying Expressions and Equations

When an algebraic expression is separated into parts by addition and subtraction signs, each part is called a **term**. The numerical part of a term that contains a variable is called the **coefficient** of the variable. **Like terms** are terms that contain the same variables, such as $3a$ and $7a$ or $9mn$ and $2mn$. A term without a variable is called a **constant**. Constant terms are also like terms. An algebraic expression is in **simplest form** if it has no like terms and no parentheses.

EXAMPLE ***Simplify the expression x + 5(y + 2x).***

$x + 5(y + 2x)$	$= x + 5(y) + 5(2x)$	*Distributive Property*
	$= x + 5y + 10x$	*Multiply.*
	$= 1x + 5y + 10x$	*Identity Property*
	$= 1x + 10x + 5y$	*Commutative Property*
	$= (1 + 10)x + 5y$	*Distributive Property*
	$= 11x + 5y$	*Simplify.*

When solving equations, sometimes it is necessary to simplify the equation by combining like terms before the equation can be solved.

EXAMPLES ***Solve each equation.***

$6a - 2a + 5 = 17$	
$4a + 5 = 17$	*Combine like terms.*
$4a + 5 - 5 = 17 - 5$	*Subtract 5 from each side.*
$4a = 12$	*Simplify.*
$\frac{4a}{4} = \frac{12}{4}$	*Divide each side by 4.*
$a = 3$	*Simplify.*

$4(2x - 1) = -6(x + 3)$	
$8x - 4 = -6x - 18$	*Distributive Property*
$8x - 4 + 6x = -6x - 18 + 6x$	*Add 6x to each side.*
$14x - 4 = -18$	*Simplify.*
$14x - 4 + 4 = -18 + 4$	*Add 4 to each side.*
$14x = -14$	*Simplify.*
$\frac{14x}{14} = -\frac{14}{14}$	*Divide each side by 14.*
$x = -1$	*Simplify.*

EXERCISES *Simplify each expression.*

1. $6y + 9y$
2. $-4m + 2m$
3. $13v - 9v$
4. $7z + 5 - 3z + 2$
5. $2p - 11p$
6. $3g - 6 + 6$

Solve each equation.

7. $18p - 2p + 6 = 9 + 5$
8. $10b - 4 - 6b = 24 - 4$
9. $8n + 6 = 19 + 7n$
10. $-3m + 8m = 11 - 4 - 2m$
11. $6(3w + 5) = 2(10w + 10)$
12. $5(3x + 1) = 2(13x - 3)$
13. $3a + 4 - 2a - 7 = 4a + 3$
14. $4(8 - 3w) = 32 - 8(w + 2)$

APPLICATIONS

15. Suppose you buy 5 videos that each cost *c* dollars, a DVD for \$30, and a CD for \$20. Write an expression in simplest form that represents the total amount spent.

16. Malik earned *d* dollars raking leaves. His friend, Isaiah, earned three times as much. A third friend, Daniel, earned five dollars less than Malik. Write an expression in simplest form that represents the total amount earned by the three friends.

17. A rectangle has length $2x - 3$ and width $x + 1$. Write an expression in simplest form that represents the perimeter of the rectangle.

SKILL 4

Name ______________________ Date __________

Adding and Subtracting Decimals

To add decimals, line up the decimal points. Then add the same way you add whole numbers.

EXAMPLES ***16.45 + 18.62*** ***77.3 + 88.45 + 90***

$$\begin{array}{r} 16.45 \\ +\ 18.62 \\ \hline 35.07 \end{array}$$

The sum is 35.07.

$$\begin{array}{r} 77.30 \\ 88.45 \\ +\ 90.00 \\ \hline 255.75 \end{array}$$

Annex zeros.

The sum is 255.75.

To subtract decimals, line up the decimal points. Then subtract the same way you would subtract whole numbers.

EXAMPLES ***45.63 − 15.47*** ***134 − 105.67***

$$\begin{array}{r} 45.63 \\ -\ 15.47 \\ \hline 30.16 \end{array}$$

The difference is 30.16.

$$\begin{array}{r} 134.00 \\ -\ 105.67 \\ \hline 28.33 \end{array}$$

Annex zeros.

The difference is 28.33.

EXERCISES ***Find each sum or difference.***

1. $\begin{array}{r} 8.22 \\ +\ 6.83 \\ \hline \end{array}$

2. $\begin{array}{r} 17.532 \\ -\ \ 8.173 \\ \hline \end{array}$

3. $\begin{array}{r} 47.9 \\ +\ 134.2 \\ \hline \end{array}$

4. $\begin{array}{r} 1.36 \\ -\ 0.48 \\ \hline \end{array}$

5. $\begin{array}{r} 0.817 \\ -\ 0.6824 \\ \hline \end{array}$

6. $\begin{array}{r} 68.7 \\ +\ \ 1.47 \\ \hline \end{array}$

7. $\begin{array}{r} 46 \\ -\ 4.49 \\ \hline \end{array}$

8. $\begin{array}{r} 1.0349 \\ +\ 10.08 \\ \hline \end{array}$

9. $\begin{array}{r} 23 \\ -\ 4.093 \\ \hline \end{array}$

10. 47.9 + 32.422

11. 52.5 + 8.62

12. 3 + 24.15 + 56.052

13. 36 + 215.5 + 4.63

14. 16.2 − 5.59

15. 58 − 0.232

16. 23 − 1.59

17. 15.6 − 0.423

18. 38 + 3.65

19. 3.56 + 0.49

20. 170 − 67.34

21. 43.896 − 22.75

APPLICATIONS

The results of the 2000 Presidential election are given at the right. Use this information to answer Exercises 22–24.

2000 Presidential Elections	
Candidate	**Percent (%) of Popular Vote**
Browne	0.36
Buchanan	0.42
Bush	47.87
Gore	48.38
Hagelin	0.08
Harris	0.01
Nader	2.74
Phillips	0.09
Write-In	0.02
Other	0.03

Source: *The World Almanac*

22. What percent of the vote was cast for Bush or Gore?

23. How many more percentage points did Gore receive than Bush?

24. What percent of the vote was cast for listed candidates other than Gore or Bush?

25. Three pieces of cardboard are 0.125 inch, 0.38 inch, and 0.0634 inch thick. What is the combined thickness of all three pieces?

26. A weightlifter lifted 46.8 kilograms on his first lift. His next lift was 50 kilograms. How much more did he lift on his second lift than his first?

27. In a race, the first place finisher had a time of 29.14 seconds. The last-place finisher had a time of 35 seconds. What was the difference between the times?

SKILL 5

Name ______________________ Date __________

Multiplying and Dividing Decimals

EXAMPLE ***Multiply 2.56 by 1.03.***

$$\begin{array}{r} 2.56 \\ \times\ 1.03 \\ \hline 768 \\ 000 \\ 256 \\ \hline 2.6368 \end{array}$$

2.56 ← 2 decimal places

1.03 ← 2 decimal places

2.6368 ← 4 decimal places

The sum of the decimal places in the factors is 4, so the product has 4 decimal places.

The product is 2.6368.

EXAMPLE ***Divide 0.201 by 0.3.***

$$\begin{array}{r} 0.67 \\ 0.3.\overline{)\,0.2.01} \\ 0 \\ \hline 2\ 0 \\ 1\ 8 \\ \hline 21 \\ 21 \\ \hline 0 \end{array}$$

Change 0.3 to 3 by moving the decimal point one place to the right.

Move the decimal point in the dividend one place to the right.

Divide as with whole numbers, placing the decimal point above the new point in the dividend.

The quotient is 0.67.

EXERCISES ***Multiply.***

1. $\begin{array}{r} 2.5 \\ \times\ 1.3 \\ \hline \end{array}$

2. $\begin{array}{r} 6.92 \\ \times\ 53 \\ \hline \end{array}$

3. $\begin{array}{r} 46.89 \\ \times\ 0.06 \\ \hline \end{array}$

4. $\begin{array}{r} 925.1 \\ \times\ 30.2 \\ \hline \end{array}$

5. $\begin{array}{r} 45.21 \\ \times\ 3.2 \\ \hline \end{array}$

6. $\begin{array}{r} 164.24 \\ \times\ 6.15 \\ \hline \end{array}$

7. $\begin{array}{r} 20.03 \\ \times\ 1.86 \\ \hline \end{array}$

8. $\begin{array}{r} 10.26 \\ \times\ 30.5 \\ \hline \end{array}$

9. $\begin{array}{r} 49.76 \\ \times\ 5.17 \\ \hline \end{array}$

Divide.

10. $0.04\overline{)\,.092}$

11. $0.7\overline{)\,0.245}$

12. $0.06\overline{)\,0.204}$

13. $0.63\overline{)\,7.56}$

14. $4.6\overline{)\,115}$

15. $8.1\overline{)\,132.03}$

16. $4.7\overline{)\,43.381}$

17. $0.68\overline{)\,4.42}$

18. $0.84\overline{)\,25.62}$

APPLICATIONS

19. Members of the student body ran 87.75 miles on a 0.25 mile track to raise money for charity. How many laps did they run?

20. A factory manager needs 3.25 yards of material to make a skirt. How many yards of fabric must be used to make 200 skirts?

21. Samantha worked 40.5 hours this week. She makes $9.50 per hour. How much money did she earn this week?

22. Batting averages are calculated to the nearest thousandth. Hikiro has 85 hits in 200 at bats. What is his batting average?

23. Joshua took a 37.5-mile boat trip. It took him 2.5 hours. What was the average speed of the boat?

24. Julia bought 3.5 pounds of mixed nuts that cost $7.49 per pound. How much did 3.5 pounds of nuts cost?

SKILL 6

Name ______________________ Date __________

Adding and Subtracting Fractions

To add or subtract fractions with unlike denominators, rename the fractions so that they have a common denominator.

EXAMPLES ***Find each sum or difference.***

a.
$$\begin{array}{rcr} \frac{1}{4} & = & \frac{2}{8} \\ +\frac{5}{8} & = & +\frac{5}{8} \\ \hline & & \frac{7}{8} \end{array}$$
The sum is $\frac{7}{8}$.

b.
$$\begin{array}{rcr} \frac{1}{6} & = & \frac{5}{30} \\ +\ \frac{7}{10} & = & +\ \frac{21}{30} \\ \hline & & \frac{26}{30} = \frac{13}{15} \end{array}$$
The sum is $\frac{13}{15}$.

c.
$$\begin{array}{rcr} 16\frac{1}{2} & = & 16\frac{7}{14} \\ +\ 14\frac{5}{7} & = & +14\frac{10}{14} \\ \hline & & 30\frac{17}{14} = 31\frac{3}{14} \end{array}$$
The sum is $31\frac{3}{14}$.

d. $\frac{8}{9} - \frac{1}{3}$
$$\begin{array}{rcr} \frac{8}{9} & = & \frac{8}{9} \\ -\frac{1}{3} & = & -\ \frac{3}{9} \\ \hline & & \frac{5}{9} \end{array}$$
The difference is $\frac{5}{9}$.

e. $\frac{5}{6} - \frac{3}{8}$
$$\begin{array}{rcr} \frac{5}{6} & = & \frac{20}{24} \\ -\frac{3}{8} & = & -\ \frac{9}{24} \\ \hline & & \frac{11}{24} \end{array}$$
The difference is $\frac{11}{24}$.

f. $6 - 3\frac{2}{5}$
$$\begin{array}{rcr} 6 & = & 5\frac{5}{5} \\ -3\frac{2}{5} & = & -3\frac{2}{5} \\ \hline & & 2\frac{3}{5} \end{array}$$
The difference is $2\frac{3}{5}$.

EXERCISES ***Find each sum or difference.***

1. $\begin{array}{r} \frac{1}{5} \\ +\ \frac{1}{4} \\ \hline \end{array}$

2. $\begin{array}{r} \frac{5}{12} \\ +\ \frac{1}{3} \\ \hline \end{array}$

3. $\begin{array}{r} \frac{1}{6} \\ +\ \frac{3}{5} \\ \hline \end{array}$

4. $\begin{array}{r} \frac{7}{8} \\ -\ \frac{1}{4} \\ \hline \end{array}$

5. $\begin{array}{r} \frac{7}{10} \\ -\ \frac{3}{8} \\ \hline \end{array}$

6. $\begin{array}{r} \frac{11}{12} \\ -\ \frac{1}{6} \\ \hline \end{array}$

7. $\begin{array}{r} 5\frac{1}{4} \\ +\ 7\frac{1}{3} \\ \hline \end{array}$

8. $\begin{array}{r} 11\frac{3}{4} \\ +\ 8\frac{2}{3} \\ \hline \end{array}$

9. $\begin{array}{r} 13 \\ +9\frac{7}{8} \\ \hline \end{array}$

10. $\begin{array}{r} 15\frac{1}{2} \\ +\ 9\frac{4}{5} \\ \hline \end{array}$

11. $\begin{array}{r} 12\frac{1}{2} \\ -\ 8\frac{2}{3} \\ \hline \end{array}$

12. $\begin{array}{r} 14\frac{5}{8} \\ -6\frac{5}{6} \\ \hline \end{array}$

13. $18\frac{7}{8} - 13$

14. $11 - 3\frac{5}{9}$

15. $16\frac{2}{5} - 13\frac{3}{4}$

16. $\frac{3}{10} + \frac{4}{15}$

17. $\frac{3}{8} + \frac{5}{12}$

18. $18\frac{5}{18} - 8\frac{1}{9}$

19. $2\frac{1}{4} + 3\frac{1}{2} + 5\frac{5}{6}$

20. $15\frac{3}{4} + 12\frac{5}{16} + 10\frac{3}{8}$

21. $21 + 8\frac{7}{10} + 14\frac{3}{4}$

APPLICATIONS

22. Ashley spends $\frac{1}{4}$ of her study time studying math and $\frac{1}{6}$ of her time studying history. How much of her study time does she spend on math and history?

23. Hinto repaired her bike for $\frac{5}{6}$ hour and then rode it for $\frac{3}{5}$ hour. How much more time did she spend repairing her bike?

24. A tailor buys some cloth to make pants. He buys $3\frac{5}{6}$ yards of one type of fabric and $4\frac{7}{36}$ yards of another. How much fabric did he buy in all?

25. A park ranger led a group of campers on a $5\frac{1}{2}$-mile hike. They have already hiked $2\frac{1}{3}$ miles. How far do they have yet to hike?

SKILL 7

Name ____________________ Date __________

Multiplying and Dividing Fractions

To multiply fractions, multiply the numerators and multiply the denominators.

EXAMPLE ***What is the product of $\frac{5}{6}$ and $\frac{9}{10}$?***

$\frac{5}{6} \times \frac{9}{10} = \frac{5 \times 9}{6 \times 10}$ *Multiply the numerators. Multiply the denominators.*

$= \frac{45}{60}$ or $\frac{3}{4}$ *Simplify.*

The product is $\frac{3}{4}$.

To divide by a fraction, multiply by its reciprocal.

EXAMPLE ***What is the quotient of $\frac{4}{15}$ and $\frac{2}{5}$?***

$\frac{4}{15} \div \frac{2}{5} = \frac{4}{15} \times \frac{5}{2}$ *Multiply by the reciprocal of $\frac{2}{5}$, which is $\frac{5}{2}$.*

$= \frac{4}{15} \times \frac{5}{2}$ *Multiply the numerators. Multiply the denominators.*

$= \frac{20}{30}$ or $\frac{2}{3}$ *Simplify.*

The quotient is $\frac{2}{3}$.

EXERCISES ***Multiply. Express each answer in simplest form.***

1. $\frac{2}{3} \times \frac{1}{4}$

2. $\frac{3}{7} \times \frac{1}{2}$

3. $\frac{7}{10} \times \frac{5}{7}$

4. $\frac{5}{8} \times \frac{1}{4}$

5. $\frac{1}{6} \times \frac{3}{5}$

6. $\frac{4}{5} \times \frac{9}{10}$

7. $6 \times \frac{2}{3}$

8. $\frac{3}{5} \times 10$

9. $12 \times \frac{5}{16}$

Divide. Express each answer in simplest form.

10. $\frac{3}{4} \div \frac{1}{2}$

11. $\frac{1}{5} \div \frac{1}{4}$

12. $\frac{3}{8} \div \frac{3}{4}$

13. $\frac{4}{5} \div \frac{2}{5}$

14. $\frac{7}{8} \div \frac{1}{4}$

15. $\frac{4}{7} \div \frac{8}{9}$

16. $\frac{4}{9} \div \frac{2}{3}$

17. $\frac{5}{9} \div 5$

18. $20 \div \frac{3}{10}$

Find each product or quotient. Express each answer in simplest form.

19. $\frac{2}{3} \times \frac{5}{9}$

20. $\frac{1}{6} \div \frac{2}{9}$

21. $\frac{9}{10} \div \frac{1}{4}$

22. $\frac{1}{15} \times 15$

23. $\frac{15}{16} \div \frac{15}{16}$

24. $\frac{4}{5} \times \frac{15}{24}$

APPLICATIONS

25. A piece of lumber 12 feet long is cut into pieces that are each $\frac{2}{3}$ foot long. How many short pieces are there?

26. About $\frac{1}{20}$ of the population of the world lives in South America. If $\frac{1}{35}$ of the population of the world lives in Brazil, what fraction of the population of South America lives in Brazil?

27. There is $\frac{1}{3}$ pound of peanuts in 2 pounds of mixed nuts. What part of the mixed nuts are peanuts?

28. Three fourths of an apple pie is left over from dessert. If the pie was originally cut in $\frac{1}{16}$ pieces, how many pieces are left?

29. A recipe calls for $\frac{1}{8}$ cup of sugar. Christopher is making half the recipe. How much sugar will he need?

30. Ms. Valdez has 2 dozen golf balls. She lost $\frac{1}{3}$ of them. How many golf balls does she have left?

SKILL 8

Name ______________________ **Date** __________

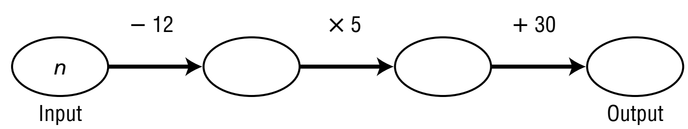
Work Backward

Some problems start with the end result and ask for something that happened earlier. The strategy of **working backward**, or **backtracking**, can be used to solve problems like this. To use this strategy, start with the end result and undo each step.

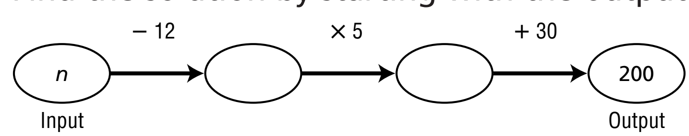
EXAMPLE

A number is decreased by 12. The result is multiplied by 5, and 30 is added to the new result. The final result is 200. What is the number?

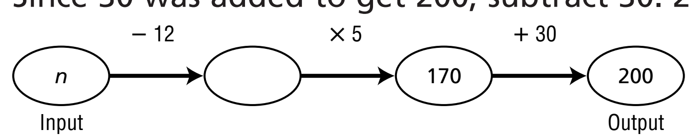
Use a flowchart to show the steps in the computation.



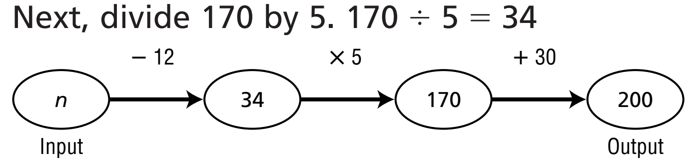
Find the solution by starting with the output.



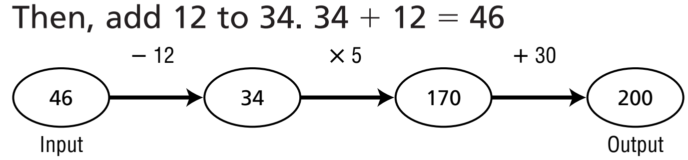
Since 30 was added to get 200, subtract 30. $200 - 30 = 170$



Next, divide 170 by 5. $170 \div 5 = 34$



Then, add 12 to 34. $34 + 12 = 46$



Thus, the number is 46.

EXERCISES

Solve by working backward.

1. A number is added to 12, and the result is multiplied by 6. The final answer is 114. Find the number.

2. A number is divided by 3, and the result is added to 20. The result is 44. What is the number?

3. A number is divided by 8, and the result is added to 12. The final answer is 78. Find the number.

4. Twenty five is added to a number. The sum is multiplied by 4, and 35 is subtracted from the product. The result is 121. What is the number?

5. A number is divided by three, and 14 is added to the quotient. The sum is multiplied by 7. The product is doubled. The result is 252. What is the number?

APPLICATIONS

6. A bacteria population doubles every 8 hours. If there are 1,600 bacteria after 2 days, how many bacteria were there at the beginning?

7. Each school day, Alexander takes 35 minutes to get ready for school. He takes 5 minutes to walk to Jaaron's house. The two boys take 15 minutes to walk from Jaaron's house to school. School starts at 8:10 A.M. If the boys want to get to school at least 10 minutes before school starts, what is the latest Alexander must get out of bed?

8. A fence is put around a dog pen 10 feet wide and 20 feet long. Enough fencing is left over to also fence a square garden with an area of 25 square feet. If there are 3 feet left after the fencing is completed, how much fencing was available at the beginning?

SKILL 9

Name ______________________ Date __________

Properties

The table shows the properties for addition and multiplication.

<table>
<tr><th>Property</th><th colspan="2">Examples</th></tr>
<tr><td>Commutative
The sum or product of two numbers is the same regardless of the order in which they are added or multiplied.</td><td>Addition
$2 + 3 = 3 + 2$
$5 = 5$</td><td>Multiplication
$4 \times 6 = 6 \times 4$
$24 = 24$</td></tr>
<tr><td>Associative
The sum or product of three or more numbers is the same regardless of the way in which they are grouped.</td><td>Addition
$(5 + 2) + 6 = 5 + (2 + 6)$
$7 + 6 = 5 + 8$
$13 = 13$</td><td>Multiplication
$(3 \cdot 4) \cdot 7 = 3 \cdot (4 \cdot 7)$
$12 \cdot 7 = 3 \cdot 28$
$84 = 84$</td></tr>
<tr><td>Distributive
The sum of two addends multiplied by a number is equal to the products of each addend and the number.</td><td colspan="2">$5 \cdot (6 + 2) = (5 \cdot 6) + (5 \cdot 2)$
$5 \cdot (8) = 30 + 10$
$40 = 40$</td></tr>
<tr><td>Identity Property of Addition
The sum of a number and 0 is the number.</td><td colspan="2">$9 + 0 = 9$</td></tr>
<tr><td>Identity Property of Multiplication
The product of a number and 1 is the number.</td><td colspan="2">$15 \times 1 = 15$</td></tr>
<tr><td>Inverse Property of Addition
The sum of a number and its additive inverse (opposite) is 0.</td><td colspan="2">$4 + (-4) = 0$</td></tr>
<tr><td>Inverse Property of Multiplication
The product of a number and its multiplicative inverse (reciprocal) is 1.</td><td colspan="2">$2 \times \frac{1}{2} = \frac{2}{1} \times \frac{1}{2}$
$= 1$</td></tr>
</table>

EXERCISES ***Name the additive inverse, or opposite of each number.***

1. 8 **2.** 5 **3.** $\frac{3}{4}$ **4.** $1\frac{1}{2}$

Name the multiplicative inverse, or reciprocal of each number.

5. 4 **6.** 7 **7.** $\frac{2}{5}$ **8.** $\frac{7}{16}$

Name the property shown by each statement.

9. $34 + 42 = 42 + 34$

10. $8 \times (53 + 12) = (8 \times 53) + (8 \times 12)$

11. $\frac{1}{16} \times 16 = 1$

12. $16 \cdot (5 \cdot 15) = (16 \cdot 5) \cdot 15$

13. $\frac{2}{5} \cdot \frac{5}{3} = \frac{5}{3} \cdot \frac{2}{5}$

14. $(32 + 48) + 52 = 32 + (48 + 52)$

15. $256 + 0 = 256$

16. $\frac{3}{10} \cdot \frac{10}{3} = 1$

17. $1 \times 143 = 143$

18. $81 + (-81) = 0$

APPLICATIONS

19. Michael rides his bike $2\frac{3}{5}$ as long as Jacob. Find Michael's riding time if Jacob rides for 45 minutes.

20. A daisy is 24 inches tall. The height of a sunflower is $3\frac{1}{2}$ times the height of the daisy. Find the height of the sunflower.

21. Jasmine buys an apple for $0.45, an orange for $0.55, and a pear for $0.99. Write an expression you could use to mentally calculate her total. What is her total?

22. The distance from the library to the park is 1.2 miles, and the distance from the park to the pool is 0.5 mile. The park is between the library and the pool. Show that the distance from the library to the pool is the same as the distance from the pool to the library.

23. Greeting cards cost $2 each and wrapping paper costs $3 per roll. Write an expression you could use to find the total cost of buying 6 greeting cards and 6 rolls of wrapping paper. What is the total cost?

SKILL 10

Name ______________________ Date __________

Function Tables

A student ticket to the Franklin School of Music's annual concert costs \$3.00. The equation that can be used to find the costs of x tickets is $y = 3x$.

EXAMPLE ***Make a function table showing the total cost of 2, 4, 6, 8, or 10 tickets.***

$y = 3x$

x	$3x$	y
2	3(2)	6
4	3(4)	12
6	3(6)	18
8	3(8)	24
10	3(10)	30

EXERCISES ***Complete each function table.***

1. $y = x - 7$

x	$x - 7$	y
10	10 − 7	3
14	14 − 7	
20		
25		
50		

2. $y = x \div 2$

x	$x \div 2$	y
4		
8		
10		
30		
100		

3. $y = 4x - 8$

x	$4x - 8$	y
5	4(5) − 8	12
10	4(10) − 8	
20		
50		
100		

4. $y = 6x + 1$

x	$6x + 1$	y
2		
4		
8		
20		
100		

5. $y = 2x - 2$

x	$2x - 2$	y
1		
2		
3		
4		
8		

6. $y = 2.5x + 1$

x	$2.5x + 1$	y
0		
2		
4		
10		
25		

APPLICATIONS

The cost per hour of operating appliances is listed at the right. Use this information to make a function table for the cost of operating each appliance for 1, 2, 3, 5, or 10 hours.

Appliance	Cost per Hour
Television	12¢
Microwave Oven	14¢
Vacuum Cleaner	7¢
Computer	24¢

7. Television
$y = 12x$

x	$12x$	y
1		
2		
3		
5		
10		

8. Microwave Oven
$y = 14x$

x	$14x$	y
1		
2		
3		
5		
10		

9. Vacuum Cleaner
$y = 7x$

x	$7x$	y
1		
2		
3		
5		
10		

10. Computer
$y = 24x$

x	$24x$	y
1		
2		
3		
5		
10		

SKILL 11

Name ______________________ Date __________

Problem-Solving Strategies

Three possible strategies for solving problems are listed below.

• Guess and Check • Look for a Pattern • Eliminate the Possibilities

EXAMPLE ***The Connor family bought some tickets to the zoo. Admission is $12 for adults and $7 for children under 12. They spent $71 for admission. How many adult tickets and how many children's tickets did the Connor family buy?***

Use the guess-and-check strategy to solve the problem. Suppose your first guess is 2 adults and 5 children.

$$2 \times 12 + 5 \times 7 = 59$$

This guess is too low. Try 3 adults and 6 children.

$$3 \times 12 + 6 \times 7 = 78$$

This guess is too high. Try 3 adults and 5 children.

$$3 \times 12 + 5 \times 7 = 71$$

The Connor family bought 3 adult tickets and 5 children tickets.

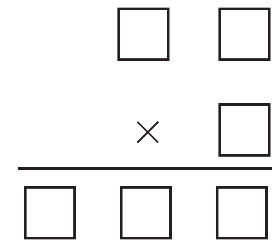
EXAMPLE ***Kwan gave the clerk $60 to pay for a purse that costs $32.95 and a hat that costs $17.50. Should she expect about $10, $20, or $30 in change?***

In this problem, an exact answer is not needed. Use the eliminate-the-possibilities strategy.

First estimate the answer by rounding $32.95 to $33 and $17.50 to $18. Kwan spent about $33 + $18 or $51. Since $60 − $51 = $9, you can eliminate $20 and $30 as possible answers.

The correct answer is about $10.

EXERCISES *Solve.*



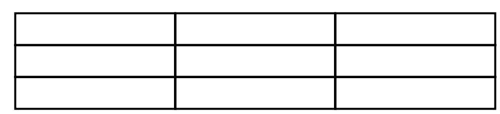
1. Fill in the boxes at the right with the digits 0, 1, 3, 4, 5, and 7 to make a correct multiplication problem. Use each digit exactly once.

2. Write the next two numbers in the pattern.

   $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

3. Does 81.4×0.68 equal 553.52, 55.352, or 5.5352? Do *not* actually compute.

4. What is the total number of rectangles in the figure at the right?



APPLICATIONS

5. Abby's test scores were 95, 82, 78, 84, and 88. Is the best estimate of her average test score 90, 85, 70, or 75?

6. Erica has some quarters and dimes in her pocket. The value of the coins is \$1.65. If she has a total of 9 coins, how many quarters and how many dimes does Erica have?

7. James wants to work up to doing 40 sit-ups a day. He plans to do 5 sit-ups the first day, 9 sit-ups the second day, 13 sit-ups the third day, and so on. On what day will he do 45 sit-ups?

8. The school bell rings at 8:05 A.M., 8:47 A.M., 8:50 A.M., 9:32 A.M., 9:35 A.M., 10:17 A.M., 10:20 A.M., and 11:02 A.M. If the pattern continues, what are the next three times the bell will ring?

9. Armando bought a car. He paid \$3,000 down and will pay \$350 per month for 48 months. Does the car cost closer to \$30,000, \$25,000, \$20,000, or \$15,000?

10. Jennifer bought some cookies for 55¢ each and some bottles of fruit drink for 80¢ each. She spent \$5.70. How many cookies and bottles of fruit drink did she buy?

11. The length of a rectangle is 6 more inches than the width. The area of the rectangle is 216 square inches. What are the dimensions of the rectangle?

SKILL 12

Name ____________________ Date __________

Divisibility Rules

Sometimes we need to know if a number is divisible by another number. In other words, does a number divide evenly into another number. You can use divisibility rules.

A number is divisible by:
- 2 if the ones digit is divisible by 2.
- 3 if the sum of the digits is divisible by 3.
- 5 if the ones digit is 0 or 5.
- 6 if the number is divisible by 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the ones digit is zero.

EXAMPLE ***Determine whether 2,346 is divisible by 2, 3, 5, 6, 9, or 10.***

2: The ones digit is 6 which is divisible by 2.
So 2,346 is divisible by 2.
3: The sum of the digits (2 + 3 + 4 + 6 = 15) is divisible by 3.
So 2,346 is divisible by 3.
5: The ones digit is *not* 0 or 5.
So 2,346 is *not* divisible by 5.
6: The number is divisible by 2 and 3.
So 2,346 is divisible by 6.
9: The sum of the digits (2 + 3+ 4 + 6 = 15) is *not* divisible by 9.
So 2,346 is *not* divisible by 9.
10: The ones digit is not 0.
So 2,346 is *not* divisible by 10.

2,346 is divisible by 2, 3, and 6.

EXERCISES ***Use the divisibility rules to determine whether the first number is divisible by the second number.***

1. 3,465,870; 5

2. 5,653,121; 3

3. 34,456,433; 9

4. 6,432; 10

5. 42,981; 2

6. 73,125; 3

7. 3,469; 6

8. 3,522; 6

Determine whether each number is divisible by 2, 3, 5, 6, 9, or 10.

9. 660

10. 5,025

11. 5,091

12. 356

13. 240

14. 657

15. 8,760

16. 3,408

17. 4,605

18. 7,800

19. 8,640

20. 432

21. 1,700,380

22. 4,937,728

APPLICATIONS

23. Ms. Vescelius wants to divide her class into cooperative learning groups. If there are 28 students in the class and she wants all the groups to have the same number of students, how many students should she put in each group?

24. The Kennedy High School band has 117 members. The band director is planning rectangular formations for the band. What formations could he make with all the band members?

25. Fisher Mountain Bike Company wants to produce between 1,009 and 1,030 mountain bicycles per month. Since the demand for the bicycles is great everywhere, they want to ship equal numbers to each of their 6 stores. Find the possible number of bicycles Fisher should ship.

26. Name the greatest 4-digit number that is divisible by 2, 3, and 5.

SKILL 13

Name ____________________ Date __________

Multiples

Bryan noticed that every time he spent $1 at the department store, he paid 8¢ in sales tax. He decided to make a table of the amount of sales tax charged on whole-dollar purchases.

EXAMPLE ***Can you help him make the table?***

The amount of sales tax charged on whole-dollar purchases can be found using **multiples** of 8. A multiple of a number is the product of that number and any whole number.

Amount of Purchase	Amount of Sales Tax
$1	8¢
$2	16¢
$3	24¢
$4	32¢
$5	40¢
$6	48¢
$7	56¢
$8	64¢
$9	72¢
$10	80¢

EXERCISES ***List the first four multiples of each number.***

1. 10 **2.** 9 **3.** 15

4. 7 **5.** 18 **6.** 12

7. 20 **8.** 25 **9.** 16

Determine whether the first number is a multiple of the second number.

10. 56; 7

11. 42; 14

12. 81; 18

13. 45; 11

14. 100; 20

15. 72; 36

16. 95; 19

17. 225; 25

18. 110; 21

APPLICATIONS

Kyle is planning a trip. He plans to drive 55 miles per hour. Use this information to answer Exercises 19 and 20.

19. How far will Kyle travel in

- **a.** 1 hour?
- **b.** 2 hours?
- **c.** 3 hours?
- **d.** 4 hours?
- **e.** 5 hours?
- **f.** 6 hours?

20. Suppose after Kyle's trip he determines that he actually averaged 60 miles per hour. How could you use your answers to Exercise 19 to determine the distance at this rate?

21. Tia is laying a pattern of tiles in rows. One row has tiles that are 4 inches long, and the next row has tiles that are 5 inches long. In how many inches will the ends of the two rows be even and the pattern start to repeat?

SKILL 14

Name ______________________ Date __________

Greatest Common Factor

The **greatest common factor (GCF)** of two or more numbers is the greatest number that is a factor of each number. One way to find the greatest common factor is to list the factors of each number and then choose the greatest common factors.

EXAMPLE ***Find the GCF of 36 and 48.***

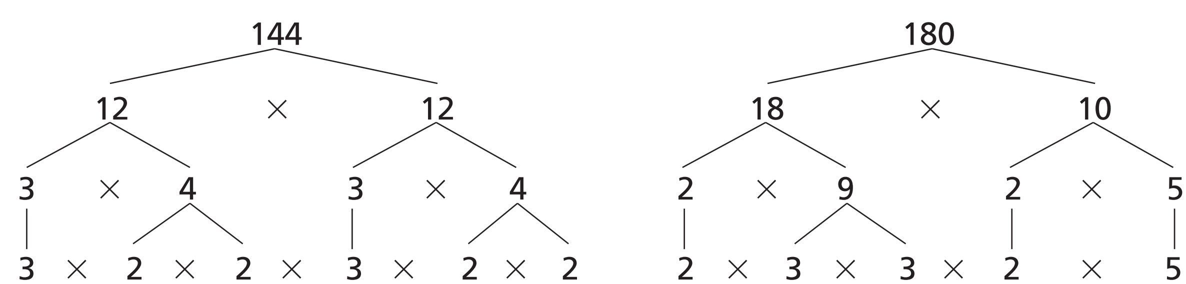
factors of 36: **1**, **2**, **3**, **4**, **6**, 9, **12**, 18, 36
factors of 48: **1**, **2**, **3**, **4**, **6**, 8, **12**, 16, 24, 48

common factors: 1, 2, 3, 4, 6, 12

The GCF of 36 and 48 is 12.

Another way to find the GCF is to use the prime factorization of each number. Then identify all common prime factors and find their product.

EXAMPLE ***Find the GCF of 144 and 180.***



common prime factors: 2, 2, 3, 3

The GCF of 144 and 180 is $2 \times 2 \times 3 \times 3$, or 36.

EXAMPLES ***Find the GCF for each set of numbers.***

1. 18, 24 **2.** 64, 40 **3.** 60, 75

4. 28, 52 **5.** 54, 72 **6.** 48, 72

7. 63, 81 **8.** 84, 144 **9.** 72, 170

10. 96, 216 **11.** 225, 500 **12.** 121, 231

13. 240, 320 **14.** 350, 140 **15.** 162, 243

16. 256, 640 **17.** 9, 18, 12 **18.** 30, 45, 15

19. 81, 27, 108 **20.** 16, 20, 36 **21.** 98, 168, 196

APPLICATIONS

22. Sharanda is tiling the wall behind her bathtub. The area to be tiled measures 48 inches by 60 inches. What is the largest square tile that Sharanda can use and not have to cut any tiles?

23. Mr. Mitchell is a florist. He received a shipment of 120 carnations, 168 daisies, and 96 lilies. How many mixed bouquets can he make if there are the same number of each type of flower in each bouquet, and there are no flowers left over?

24. Students at Washington Middle School collected 126 cans of fruit, 336 cans of soup, and 210 cans of vegetables for a food drive. The students are making care packages with at least one of each type of canned good. If the students divide each type of canned good evenly among the care packages, what is the greatest number of care packages if there are no canned goods remaining?

SKILL 15

Name ____________________ Date ____________

Least Common Multiple

A **multiple** of a number is the product of that number and any whole number. The least nonzero multiple of two or more numbers is the **least common multiple (LCM)** of the numbers.

EXAMPLE ***Find the least common multiple of 6 and 8.***

positive multiples of 6: 6, 12, 18, **24**, 30, 36, 42, . . .
positive multiples of 8: 8, 16, **24**, 32, 40, 48, 56, . . .

The LCM of 6 and 8 is 24.

Prime factorization can also be used to find the LCM.

EXAMPLE ***Find the least common multiple of 9, 15, and 21.***

$$9 = 3 \times 3$$
$$15 = 3 \times 5$$
$$21 = 3 \times 7$$
$$3 \times 3 \times 5 \times 7 = 315$$

The LCM of 9, 15, and 21 is 315.

Find prime factors of each number.
Circle all sets of common factors.
Multiply the common factors and any other factors.

EXERCISES ***Find the LCM of each set of numbers by listing the multiples of each number.***

1. 3, 4 **2.** 10, 25 **3.** 18, 24, 48

Find the LCM of each set of numbers by writing the prime factorization.

4. 35, 49 **5.** 27, 36 **6.** 10, 12, 15

Find the LCM of each set of numbers.

7. 16, 24

8. 56, 16

9. 28, 20

10. 64, 72

11. 63, 77

12. 110, 120

13. 66, 78, 90

14. 40, 60, 108

15. 132, 144, 156

16. 125, 275, 400

17. 196, 225, 256

18. 120, 450, 1500

19. Find the GCF and LCM of 36 and 54.

20. Find the two smallest numbers whose GCF is 7 and whose LCM is 98.

21. List the first five multiples of $6p$.

APPLICATIONS

22. Suppose that your taxes, car insurance, and health club membership fees are all due in August. The taxes are due every three months, car insurance is due every six months, and health club membership is due every two months. Name the next month that all three bills will be due in the same month.

23. Antoine is buying hamburgers and buns for a class picnic. Hamburgers come in packages of 15 patties and buns come in packages of 8. Antoine wants to have the same number of hamburger patties and buns. What is the least number of hamburger patties and buns he can buy?

24. Members of the U.S. House of Representatives are elected every 2 years. United States Senators are elected every 6 years. The President of the United States is elected every 4 years. If a citizen voted for a representative, a senator, and the president in 2004, what is the next year in which the voter can vote for all three in the same year?

SKILL 16

Name ______________________ Date ____________

Powers and Exponents

An expression like $3 \times 3 \times 3 \times 3 \times 3$ can be written as a power. A power has two parts, a **base** and an **exponent**. The expression $3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5.

EXAMPLE ***Write the expression m · m · m · m · m · m using exponents.***

The base is m. It is a factor 6 times, so the exponent is 6.

$$m \cdot m \cdot m \cdot m \cdot m \cdot m = m^6$$

You can also use powers to name numbers that are less than one by using exponents that are negative integers. The definition of a negative exponent states that $a^{-n} = \frac{1}{a^n}$ for $a \neq 0$ and any integer n.

EXAMPLE ***Write the expression 4^{-3} using a positive exponent.***

$$4^{-3} = \frac{1}{4^3}$$

EXERCISES ***Write each expression using exponents.***

1. $2 \cdot 2 \cdot 2 \cdot 2$

2. $(-3)(-3)(-3)(-3)(-3)$

3. 9

4. $x \cdot x \cdot x$

5. $c \cdot c \cdot d \cdot d \cdot d \cdot d \cdot d$

6. $8 \cdot a \cdot a \cdot a \cdot b$

7. $(k - 2)(k - 2)$

8. $4 \cdot 4 \cdot 4 \cdot 4 \cdot h \cdot h$

9. $(-w)(-w)(-w)(-w)(-w)$

10. $6 \cdot 6 \cdot 6 \cdot y \cdot y \cdot y \cdot y$

Evaluate each expression if $m = 3$, $n = 2$, and $p = -4$.

11. m^4

12. n^6

13. $3p^2$

14. mn^2

15. $m^2 + p^3$

16. $(p + 3)^5$

17. $n^2 - 3n + 4$

18. $-2mp^2$

19. $5(n - 4)^3$

Write each expression using a positive exponent.

20. 6^{-1}

21. 4^{-3}

22. $(-2)^{-4}$

23. d^{-7}

24. m^{-5}

25. $3b^{-6}$

26. 10^{-2}

27. $\frac{1}{x^{-5}}$

28. $\frac{7}{p^{-4}}$

Write each fraction as an expression using a negative exponent other than −1.

29. $\frac{1}{4^{-5}}$

30. $\frac{1}{3^8}$

31. $\frac{1}{7^3}$

32. $\frac{1}{64}$

33. $\frac{1}{27}$

34. $\frac{1}{1,000}$

Evaluate each expression if a = −2 and b = 3.

35. 5^a

36. b^{-4}

37. a^{-3}

38. $(-3)^{-b}$

39. ab^{-2}

40. $(ab)^{-2}$

APPLICATIONS

41. The area of a square is found by multiplying the length of a side by itself. If a square swimming pool has a side of length 45 feet, write an expression for the area of the swimming pool using exponents.

42. A molecule of a particular chemical compound weighs one millionth of a gram. Express this weight using a negative exponent.

43. A needle has a width measuring 2^{-5} inch. Express this measurement in standard form.

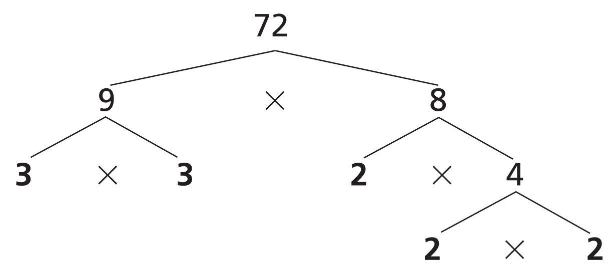
SKILL 17

Name ______________________ Date __________

Prime Factorization

Every composite number can be written as the product of prime numbers. This product is called the **prime factorization** of the number. One way to find the prime factorization of a number is to use a **factor tree**.

EXAMPLE ***Find the prime factorization of 72.***



Write 72 as the product of two factors. Keep factoring until all the factors are prime numbers

The prime factorization of 72 is $2 \times 2 \times 2 \times 3 \times 3$, or $2^3 \times 3^2$.

EXERCISES ***Find the prime factorization of each number.***

1. 18 **2.** 24 **3.** 27

4. 32 **5.** 38 **6.** 45

7. 68 **8.** 75 **9.** 84

10. 115 **11.** 132 **12.** 144

13. 165 **14.** 196 **15.** 225

16. 360 **17.** 400 **18.** 576

19. 888 **20.** 1,470 **21.** 2,340

APPLICATIONS ***A new rectangular picnic area is being built at Springfield City Park.***

22. If the picnic area is to cover an area of 260 square yards, what are the whole number dimensions that are possible for the picnic area?

23. Suppose the park manager decides to build the picnic area to cover an area of 300 square yards. What are the whole number dimensions that are possible for this picnic area?

23. If the original picnic area covered 180 square yards and the new picnic area is to cover twice as much area, what are the whole number dimensions that are possible for the new picnic area?

SKILL 18

Name ______________________ Date __________

Multiplying by Powers of Ten

The exponent in a power of ten is the same as the number of zeros in the number.

Powers of Ten

10^0	1
10^1	10
10^2	100
10^3	1,000
10^4	10,000
10^5	100,000

To multiply by a power of ten, move the decimal point to the right the number of places shown by the exponent or the number of zeros. Annex zeros if necessary.

EXAMPLES ***Find each product.***

0.08×10^4

$0.0800 = 800$ *Move the decimal point 4 places to the right.*

The product is 800.

$6.25 \times 1{,}000$

$6.250 = 6{,}250$ *Move the decimal point 3 places to the right.*

The product is 6,250.

EXERCISES ***Choose the correct product.***

1. 2.48×100; 0.0248 or 248

2. 0.9×10^0; 9 or 0.9

3. 0.039×10^2; 3.9 or 39

4. 1.5×10^4; 150,000 or 15,000

Multiply.

5. 15.24×10

6. 0.702×100

7. $5.149 \times 1{,}000$

8. 0.52×100

9. 2.587×10^0

10. 0.2674×100

11. 6.8×10^2

12. 9.57×10^4

13. 6.2×10^5

Solve each equation.

14. $d = 0.92 \times 100$

15. $12.43 \times 10^3 = h$

16. $h = 3.68 \times 10^6$

17. $a = 0.004 \times 10^2$

18. $0.23 \times 1{,}000 = j$

19. $1.89 \times 10^0 = v$

20. $d = 10{,}000 \times 7.07$

21. $0.014 \times 10^2 = k$

22. $v = 589 \times 10^1$

APPLICATIONS

23. What is the length of the Amazon River if it can be represented by 3.9×10^3 miles long? How much longer is it than the Wood River which is 5.7×10^2?

24. The United States spends 37.3×10^9 dollars on research and development in the military. Germany spends 1.4×10^9 dollars on research and development in the military. How much money do these two countries spend altogether?

25. The diameter of Neptune is about 4.95×10^4 kilometers. The diameter of Venus is about 1.21×10^4 kilometers. About how much greater is Neptune's diameter?

SKILL 19

Name ______________________ Date __________

Dividing by Powers of Ten

The exponent in a power of ten is the same as the number of zeros in the number.

Powers of Ten	
10^0	1
10^1	10
10^2	100
10^3	1,000
10^4	10,000
10^5	100,000

To divide by a power of ten, move the decimal point to the left the number of places shown by the exponent or the number of zeros.

EXAMPLES ***Find each quotient.***

$8 \div 10^4 = 0.0008$ *Move the decimal point 4 places to the left.*

The quotient is 0.0008.

$62.5 \div 1{,}000 = 0.0625$ *Move the decimal point 3 places to the left.*

The quotient is 0.0625.

EXERCISES ***Choose the correct quotient.***

1. $2.48 \div 100$; 0.0248 or 248

2. $0.9 \div 10^0$; 9 or 0.9

3. $0.39 \div 10^2$; 0.039 or 0.0039

4. $1.5 \div 10^4$; 0.00015 or 15,000

Divide.

5. $15.24 \div 10$

6. $0.702 \div 100$

7. $514.9 \div 1{,}000$

8. $5.2 \div 100$ **9.** $2.587 \div 10^0$ **10.** $267.4 \div 100$

11. $68 \div 10^2$ **12.** $9.57 \div 10^4$ **13.** $6,245 \div 10^5$

Solve each equation.

14. $d = 92 \div 100$ **15.** $12.43 \div 10^3 = h$ **16.** $h = 36.8 \div 10^6$

17. $a = 0.004 \div 10^2$ **18.** $2,358 \div 1,000 = j$ **19.** $1.89 \div 10^0 = v$

20. $d = 76.9 \div 10,000$ **21.** $8,714 \div 10^2 = k$ **22.** $v = 589 \div 10^1$

APPLICATIONS

23. Mr. Fraley bought 1,000 postage stamps for $290 for use in his office. How much did each stamp cost?

24. Mary donated 100 cans of soup to the local food pantry. It cost her $23 to buy the soup. How much did each can of soup cost?

25. George has $245.60 that he wants to split evenly with his 10 nieces and nephews. How much money will each one receive?

26. The planet Saturn is an average distance of about 887,000,000 miles from the sun. If a space ship could travel that distance in 10,000 hours, how fast would it be going?

SKILL 20

Name ____________________ Date __________

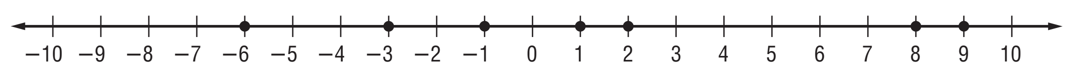
Integers

Numbers greater than zero are called **positive numbers**. Numbers less than zero are called **negative numbers**. The set of numbers that includes positive and negative numbers, and zero are called **integers**.

EXAMPLE

Emily recorded the temperature at noon for a week. The temperatures she recorded were 9°F, 8°F, −6°F, −3°F, −1°F, 2°F, and 1°F. What was the lowest and highest temperature recorded?

To answer the question, locate the temperatures on a number line.



On a number line, values increase as you move to the right.

Since −6 is furthest to the left, −6°F is the coldest temperature. 9 is the farthest number to the right, so 9°F is the highest temperature.

The **absolute value** of a number is the positive number of units a number is from zero on a number line.

EXAMPLE

Refer to the table. Which city's population changed the most?

Population Change, 1990–2000	
Atlanta, GA	+22,457
Baltimore, MD	−84,860
Columbus, OH	+78,560
Detroit, MI	−76,704
Indianapolis, IN	+49,974
Philadelphia, PA	−68,027

Find the absolute value of each number.

$|+22{,}457| = 22{,}457$
$|-84{,}860| = 84{,}860$
$|+78{,}560| = 78{,}560$
$|-76{,}704| = 76{,}704$
$|+49{,}974| = 49{,}974$
$|-68{,}027| = 68{,}027$

Since the absolute value of −84,860 is the greatest, Baltimore, Maryland, had the greatest population change.

EXERCISES

Fill in each blank with <, >, or = to make a true sentence.

1. 5 _____ −5 **2.** −4 _____ 3 **3.** 0 _____ −2

4. −6 _____ −12 **5.** −35 _____ −16 **6.** 19 _____ −22

7. 34 _____ 21 **8.** 23 _____ 23 **9.** −45 _____ −52

Write each set of integers in order from least to greatest.

10. {45, −23, 55, 0, −12, −37}

11. {56, −22, 34, −34, 12, −12}

12. {−450, −100, 254, 564, −356}

13. {1,276, −3,456, −943, −237, −467}

Find the absolute value.

14. $|-3|$ **15.** $|-5|$ **16.** $|16|$ **17.** $|27|$

18. $|156|$ **19.** $|-359|$ **20.** $|-821|$ **21.** $|1{,}436|$

APPLICATIONS

Write an integer to describe each situation.

22. Julio finished the race 3 seconds ahead of the second place finisher.

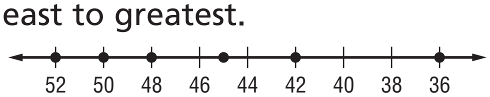
23. Matthew ended his round of golf 4 under par.

24. Denver is called the Mile High City because its elevation is 5,280 feet above sea level.

For Exercises 25–27, refer to the table.

Record Low Temperatures	
California	−45°F
Illinois	−36°F
Maine	−48°F
Nevada	−50°F
New York	−52°F
Pennsylvania	−42°F
Washington	−48°F

25. Use a number line to order the temperatures from least to greatest.



26. The record low temperature for Michigan is −51°F. Which states have higher record low temperatures?

27. Indiana's record low temperature is −36°F. Which states in the table have lower record low temperatures?

SKILL 21

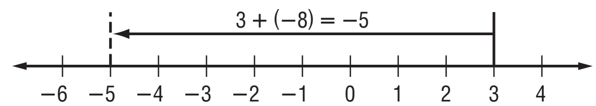
Name ______________________ Date __________

Adding and Subtracting Integers

You can use a number line to add integers. Locate the first addend on the number line. Move right if the second addend is positive. Move left if the second addend is negative.

EXAMPLE ***Find 3 + (−8).***

Start at 3. Since −8 is negative, move left 8 units.



Therefore, 3 + (−8) = − 5.

When you add integers, remember:

- The sum of two positive integers is positive.
- The sum of two negative integers is negative.
- The sum of a positive and negative integer is:
  positive if the positive integer has the greater absolute value.
  negative if the negative integer has the greater absolute value.

To subtract an integer, add its opposite.

EXAMPLE ***Find 4 −7.***

4 − 7 = 4 + (−7) To subtract 7, add −7.

= − 3

Find 5 − (−6).

5 − (−6) = 5 + (+ 6) To subtract −6, add + 6.

= 11

EXERCISES ***Find each sum or difference.***

1. $15 + (-10)$

2. $-20 + (-9)$

3. $16 - (-3)$

4. $-11 - (-6)$

5. $65 - (-45)$

6. $-11 + (-19)$

7. $12 + 15$

8. $-2 - 16$

9. $8 - 17$

10. $16 + (-8)$

11. $-8 + 34$

12. $-12 + (-37)$

13. $23 - 17$

14. $-9 - 25$

15. $14 + 98$

16. $-63 + 53$

17. $(-27) - (-18)$

18. $31 - 74$

19. $81 + 62$

20. $41 - (-35)$

21. $-55 - 23$

22. $20 + (-50)$

23. $-16 - (-16)$

24. $-125 + 79$

APPLICATIONS ***Great Adventures Outdoor Shop reported profits and losses for a five-month period as shown in the table.***

Profit and Loss	
May	profit of $800
June	loss of $1,400
July	loss of $900
August	profit of $500
September	profit of $1,200

25. How much more money was lost in June than in July?

26. How much more were the total profits for the last two months than for the first three months?

27. From May through September, did the store have an overall loss or gain and how much?

28. How much did the store lose in October if the overall loss from May through October was $500?

SKILL 22

Name ____________________ Date __________

Multiplying and Dividing Integers

When multiplying or dividing integers:
If two integers have the same sign, their product or quotient is positive.
If two integers have different signs, their product or quotient is negative.

EXAMPLE ***Solve each equation.***

$a = 8 \times (-4)$	*One factor is positive and the other is negative.*
$a = -32$	*The product is negative.*
The solution is -32.	
$b = -3 \times (-12)$	*Both factors are negative.*
$b = 36$	*The product is positive.*
The solution is 36.	
$c = -63 \div (-7)$	*Both factors are negative.*
$c = 9$	*The quotient is positive.*
The solution is 9.	
$d = -52 \div 4$	*The factors have different signs.*
$d = -13$	*The quotient is negative*
The solution is -13.	

EXERCISES ***Tell whether the product or quotient is positive or negative. Then find the product or quotient.***

1. 8×9

2. $4 \times (-5)$

3. $-81 \div (-9)$

4. $-16 \div 4$

5. -5×7

6. $27 \div 3$

7. $56 \div (-8)$

8. $-3 \times (-6)$

9. $-42 \div 7$

10. 6×8

Solve each equation.

11. $a = -16 \times 4$

12. $b = 120 \div 20$

13. $c = -240 \div (-4)$

14. $d = -64 \div 8$

15. $e = 14 \times (-8)$

16. $f = 144 \div 6$

17. $g = -80 \div (-16)$

18. $h = 14 \times 36$

19. $j = -11 \times 11$

20. $k = -16 \times (-9)$

21. $m = 240 \div (-8)$

22. $n = -315 \div 9$

23. $p = 14 \times 12$

24. $q = 18 \times 0$

25. $r = 285 \div (-15)$

26. $s = -33 \times (-9)$

APPLICATIONS ***A full 60-gallon water storage tank drains at a rate of 3 gallons per minute.***

27. How much water is in the tank after 4 minutes?

28. How much water is in the tank after 8 minutes?

29. How long does it take to drain 15 gallons of water?

30. How long does it take to drain the entire tank?

31. Suppose water is added to the tank at a rate of 2 gallons a minute. How long will it take to drain the tank?

SKILL 23

Name ______________________________ Date ____________

Metric Units of Measure

Metric units of length are millimeters, centimeters, meters, and kilometers.

Length
1 centimeter (cm) = 10 millimeters (mm)
1 meter (m) = 100 centimeters
1 meter = 1,000 millimeters
1 kilometer (km) = 1,000 meters
Capacity
1 liter (L) = 1,000 milliliters (mL)
1 kiloliter (kL) = 1,000 liters
Mass
1 gram (g) = 1,000 milligrams (mg)
1 kilogram (kg) = 1,000 grams

Metric units of capacity are milliliters, liters, and kiloliters.

Metric units of mass are milligrams (mg), grams (g), and kilograms (kg).

When changing from a smaller unit to a larger unit, divide.
When changing from a larger unit to a smaller unit, multiply.

EXAMPLE ***Change 65 meters to centimeters.***

65 = 5 _____ cm *You are changing from a larger unit (m) to a smaller unit (cm), so multiply.*

65 × 100 = 6,500 *Since 1 meter = 100 centimeters, multiply by 100.*

65 m = 6,500 cm

Change 500 milliliters to liters.

500 mL = _____ L *You are changing from a smaller unit (mL) to a larger unit (L), so divide.*

500 ÷ 1,000 = 0.5 *Since 1 liter = 1,000 milliliters, divide by 1,000.*

Change 4,500 grams to kilograms.

4,500 g = _____ kg *You are changing from a smaller unit (g) to a larger unit (kg), so divide.*

4,500 ÷ 1,000 = 4.5 *Since 1 kilogram = 1,000 grams, divide by 1,000.*

4,500 g = 4.5 kg

EXERCISES *Complete.*

1. 55 mm = ______ cm	**2.** 71 cm = ______ mm	**3.** 7 km = __________ m
4. 750 m = ______ km	**5.** 210 cm = ______ m	**6.** 1.4 m = ______ cm
7. 5 m = ________ mm	**8.** 900 mm = ______ m	**9.** 3 km = ____________ mm
10. 70 L = __________ mL	**11.** 9000 mL = ______ L	**12.** 0.6 kL = ______ L
13. 52 L = _________ kL	**14.** 70,000 mL = ______ kL	**15.** 90 mL = ____________ kL
16. 16 kg = __________ g	**17.** 66 g = ________ kg	**18.** 1.2 g = __________ kg

APPLICATIONS *Choose the best estimate.*

19.	length of a race	5 cm	5 m	5 km
20.	wingspan of an eagle	2.4 cm	2.4 m	2.4 km
21.	length of a computer disk	90 mm	90 cm	90 m
22.	capacity of can of soft drink	355 mL	355 L	355 kL
23.	capacity of a bathtub	80 mL	80 L	80 kL
24.	amount of vanilla in a cookie recipe	3 mL	3 L	3 kL
25.	mass of a nickel	5 mg	5 g	5 kg
26.	mass of an adult human	60 mg	60 g	60 kg
27.	mass of an apple	0.2 mg	0.2 g	0.2 kg

28. The average shower uses 19 liters of water per minute. If you take a five-minute shower each day, how many kiloliters of water do you use in a 30-day month?

29. Soft drinks are sold in 2 liter containers. How many milliliters of soft drink are in one of these containers?

30. The mass of a collie is 33,000 grams, and the mass of a basset hound is 26 kilograms. Which dog is bigger?

SKILL 24

Name ______________________ **Date** ____________

Scientific Notation

A number is expressed in scientific notation when it is written as the product of a factor and a power of ten. The factor must be greater than or equal to 1 and less than 10.

EXAMPLE ***Express each number in standard form.***

$8.26 \times 10^5 = 8.26 \times 100{,}000$ — $10^5 = 100{,}000$

$= 826{,}000$ — *Move the decimal point 5 places to the right.*

$3.71 \times 10^{-4} = 3.71 \times 0.0001$ — $10^{-4} = 0.0001$

$= 0.000371$ — *Move the decimal point 4 places to the left.*

Express each number in scientific notation.

$68{,}000{,}000 = 6.8 \times 10{,}000{,}000$ — *The decimal point moves 7 places.*

$= 6.8 \times 10^7$ — *The exponent is positive.*

$0.000029 = 2.9 \times 0.00001$ — *The decimal point moves 5 places.*

$= 2.9 \times 10^{-5}$ — *The exponent is negative.*

EXERCISES ***Express each number in standard form.***

1. 7.24×10^3

2. 1.09×10^{-5}

3. 9.87×10^{-7}

4. 5.8×10^6

5. 3.006×10^2

6. 4.999×10^{-4}

7. 2.875×10^{-5}

8. 6.3×10^4

9. 4.003×10^6

10. 1.28×10^{-2}

Express each number in scientific notation.

11. 7,500,000

12. 291,000

13. 0.00037

14. 12,600

15. 0.0000002

16. 0.004

17. 60,000,000

18. 40,700,000

19. 0.00081

20. 12,500

Choose the greater number in each pair.

21. 3.8×10^3, 1.7×10^5

22. 0.0015, 2.3×10^{-4}

23. 60,000,000, 6.0×10^6

24. 4.75×10^{-3}, 8.9×10^{-6}

25. 0.00145, 1.2×10^{-3}

26. 7.01×10^3, 7,000

APPLICATIONS

27. The distance from Earth to the Sun is 1.55×10^8 kilometers. Express this distance in standard form.

28. In 2001, the population of Asia was approximately 3,641,000,000. Express this number in scientific notation.

29. A large swimming pool under construction at the Greenview Heights Recreation Center will hold 240,000 gallons of water. Express this volume in scientific notation.

30. A scientist is comparing two chemical compounds in her laboratory. Compound A has a mass of 6.1×10^{-7} gram, and compound B has a mass of 3.6×10^{-6} gram. Which of the two compounds is heavier?

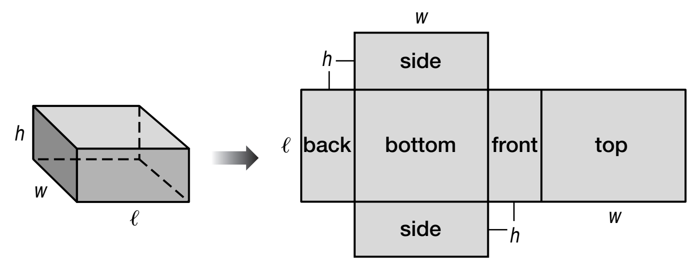
SKILL 25

Name ____________________ Date __________

Surface Area of Rectangular Prisms and Cylinders

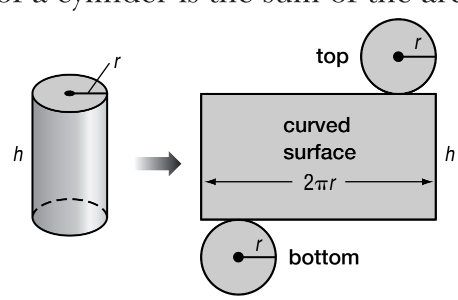
The sum of the areas of all the surfaces of a solid is called the **surface area.**

The surface area of a rectangular prism is the sum of the areas of each of its six faces.



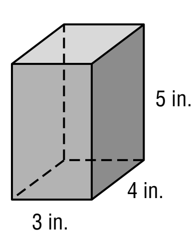
top and bottom: $(\ell \times w) + (\ell \times w) = 2\ell w$
front and back: $(\ell \times h) + (\ell \times h) = 2\ell h$
two sides: $(w \times h) + (w \times h) = 2wh$
surface area $= 2\ell w + 2\ell h + 2wh$

The surface area of a cylinder is the sum of the areas of the two bases and the curved surface.

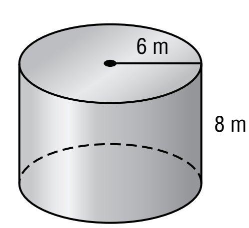


top and bottom: $\pi r^2 + \pi r^2 = 2\pi r^2$
curved surface: $(2\pi r) \times h = 2\pi rh$
surface area $= 2\pi r^2 + 2\pi rh$

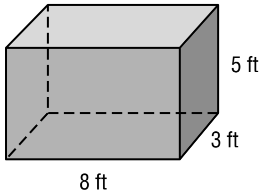
EXAMPLE ***Find the surface area of each solid.***



surface area $= 2\ell w + 2\ell h + 2wh$
surface area $= 2 \times 3 \times 4 + 2 \times 3 \times 5 + 2 \times 4 \times 5$
surface area $= 94$
The surface area is 94 square inches.



surface area $= 2\pi r^2 + 2\pi rh$
surface area $= 2\pi r^2 \times 6^2 + 2\pi \times 6 \times 8$
surface area ≈ 527.8
The surface area is about 527.8 square meters.

1.


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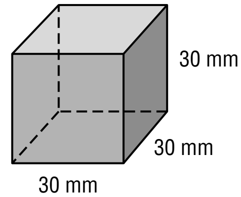

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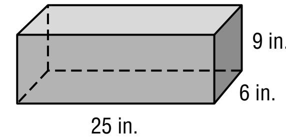

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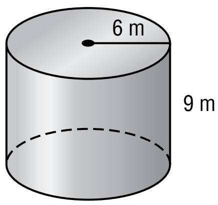

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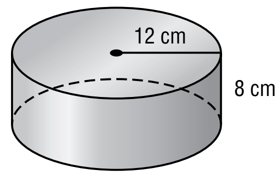

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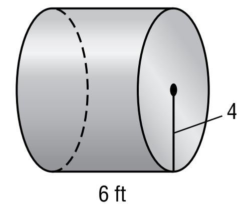

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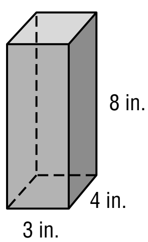

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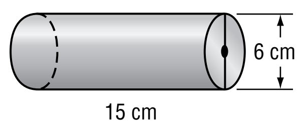

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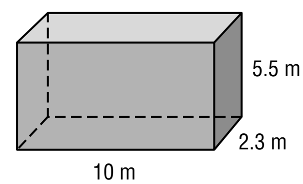
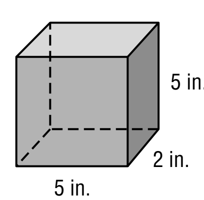
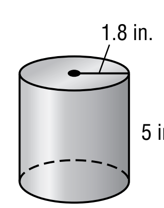


APPLICATIONS

10. A box company is making rectangular boxes that are 10 centimeters by 8 centimeters by 5 centimeters. How many of these boxes can the company make using 200,000 square centimeters of cardboard?

11. The two boxes have about the same volume. Which box takes less material to manufacture?





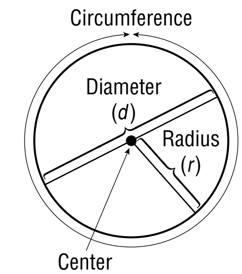
12. A wheel of cheese is sealed in a wax covering. The wheel of cheese is in the shape of a cylinder that has a diameter of 25 centimeters and a height of 20 centimeters. What is the surface area of the cheese that needs to be covered in wax.

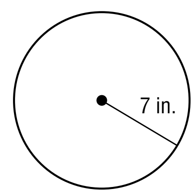
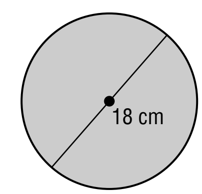
SKILL **26**

Name ______________________ **Date** ____________

Circumference and Area of Circles

The parts of a circle are illustrated at the right. Notice that the radius is one-half of the diameter.



	Circumference	**Area**
Formula	$C = \pi d$ or $C = 2\pi r$ (d = diameter and r = radius)	$A = \pi r^2$ (r = radius)
Example	 $C = 2\pi r$ $C = 2\pi \times 7$ $C \approx 44.0$ Use a calculator The circumference is about 44.0 inches.	 The radius is half of 18 or 9 centimeters $A = \pi r^2$ $A = \pi \times 9^2$ $A \approx 254.5$ Use a calculator. The area is about 254.5 square centimeters.

EXERCISES ***Find the circumference and area of each circle. Round to the nearest tenth.***

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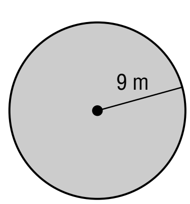

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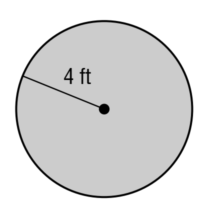

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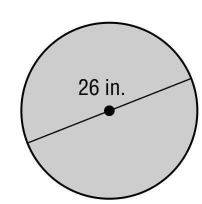

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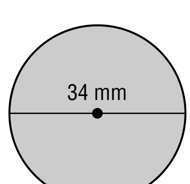

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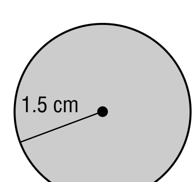

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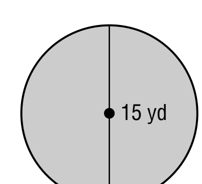
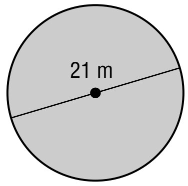
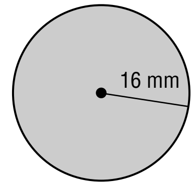
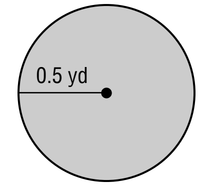


7. 

8. 

9. 

APPLICATIONS

10. The approximate diameter of Earth is 3,960 miles. What is the distance around the equator?

11. A circular garden has a diameter of 28 feet. The garden is to be covered with peat moss. If each bag of peat moss covers 160 square feet, how many bags of peat moss will be needed?

12. What is the area of a pizza with a diameter of 14 inches?

13. The stage of a theater is a semicircle. If the radius of the stage is 32 feet, what is the area of the stage?

14. A Ferris wheel has a diameter of 212 feet. How far will a passenger travel in one revolution of the wheel?

15. A water sprinkler produces a spray that goes out 25 feet. If it sprays the water in a circular pattern, what is the area of the lawn that it waters?

16. The diameter of a bicycle wheel is 26 inches. How many feet will the bicycle travel if the wheel turns 20 times?

17. The Roman Pantheon is 142 feet in diameter. What is the distance around the Pantheon?

18. The diameter of a dime is 17.9 millimeters. What is the area of one side of the coin?

SKILL 27

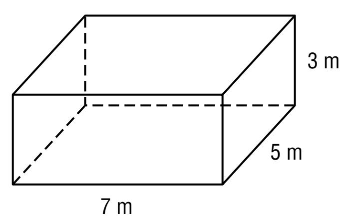
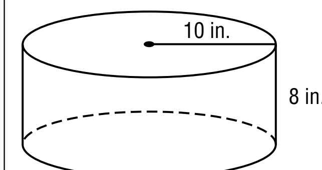
Name ______________________________ **Date** ____________

Volume of Rectangular Prisms and Cylinders

The volume of an object is the amount of space a solid contains. Volume is measured in cubic units. The volume of a prism or a cylinder can be found by multiplying the area of the base times the height.

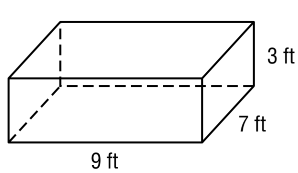
$$V = Bh$$

For a rectangular prism, the area of the base equals the length times the width. For cylinders, the area of the base equals pi times the radius squared.

	Rectangular Prism	Cylinder
Volume Formula	$V = \ell wh$ (ℓ = length, w = width, and h = height)	$V = \pi r^2 h$ (r = *radius and h* = height)
Example	 $V = \ell wh$ $V = 7 \times 5 \times 3$ $V = 105$ The volume is 105 cubic meters.	 $V = \pi r^2 h$ $V = \pi \times 10^2 \times 8$ $V \approx 2{,}513.3$ Use a calculator. The volume is about 2,513.3 cubic inches.

EXERCISES ***Find the volume of each rectangular prism.***

1.



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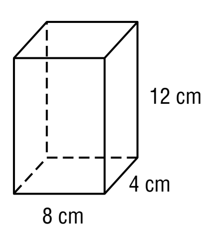

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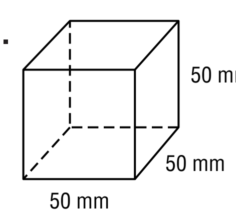

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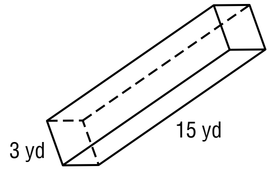

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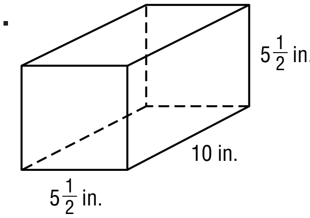

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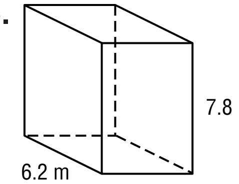
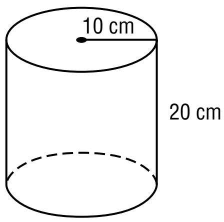


Find the volume of each cylinder. Round to the nearest tenth.

7.



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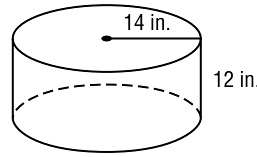

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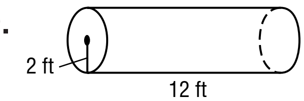

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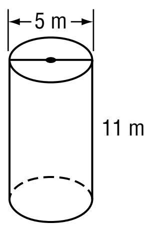

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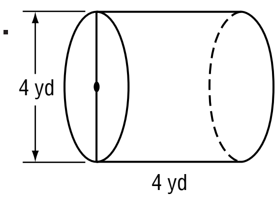

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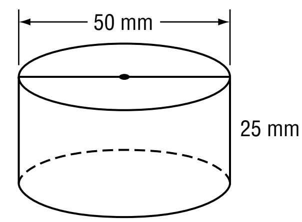
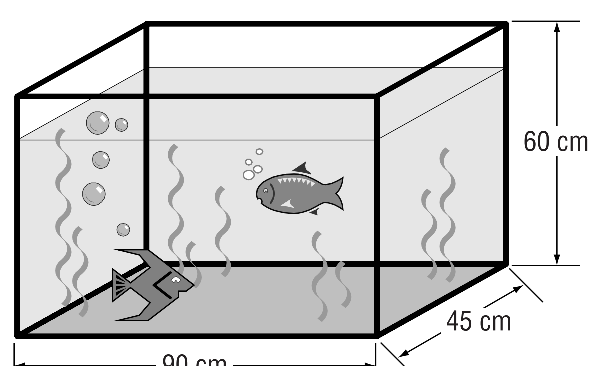


APPLICATIONS

13. Mariah is making cylindrical candles. The candles she plans to make have a diameter of 3 inches and a height of 8 inches. If she has 200 cubic inches of wax, how many candles can Mariah make.

14. A fish tank is shown at the right.



a. What is the volume of the tank?

b. If the tank is filled to the height of 50 centimeters, what is the volume of the water in the tank?

15. A square cake pan is 8 inches long on each side. It is 2 inches deep. A round cake pan has a diameter of 8 inches. It is 2 inches deep.

a. Which pan can hold more cake batter?

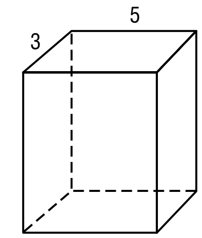
b. How much greater is the volume of the pan that holds more batter?

16. How many cubic inches are in a cube that is 1 foot on each side?

SKILL 28

Name ______________________________ Date __________

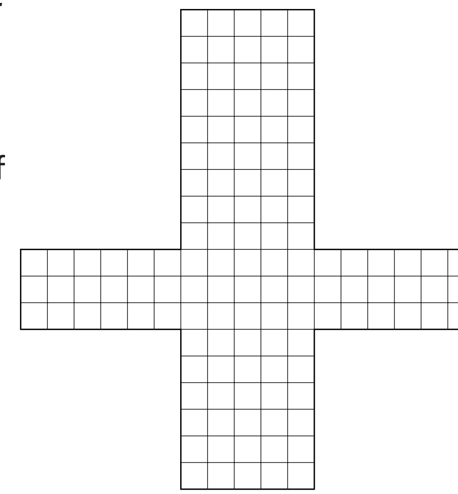
Nets and Solids



A **net** is the shape that is formed by "unfolding" a three-dimensional figure. A net shows all the faces that make up the surface area of the figure.

EXAMPLE

Draw a net for a rectangular prism that has length 5 units, width 3 units, and height 6 units.



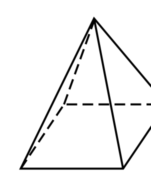
The net will be made up of three sets of congruent shapes:

1) the top and bottom of the prism—rectangles 3 units by 5 units

2) the two sides of the prism—rectangles 3 units by 6 units

3) the front and back of the prism—rectangles 5 units by 6 units.

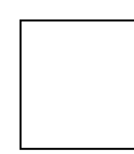
Three-dimensional figures can also be portrayed by drawing their top, side, and front views separately.



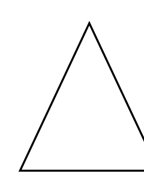
EXAMPLE

Draw a top, a side, and a front view for the pyramid

If you look at the figure from directly above, you would see the square base.



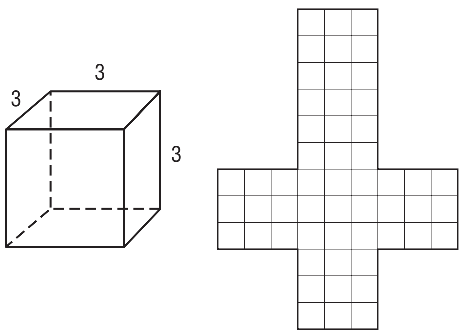
Looking at the figure from the sides, front, and the rear you would see a triangle.



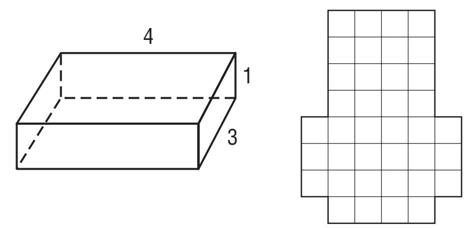
EXERCISES

Draw a net for each figure.

1.

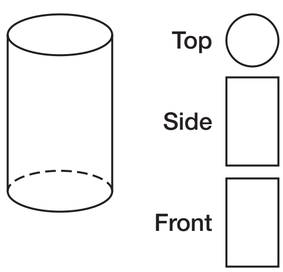


2.

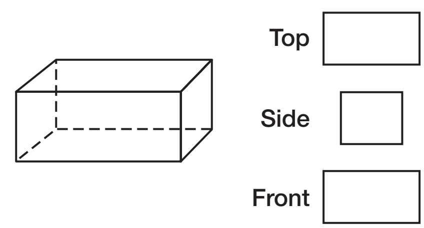


Draw a top, a side, and a front view of each figure.

3.

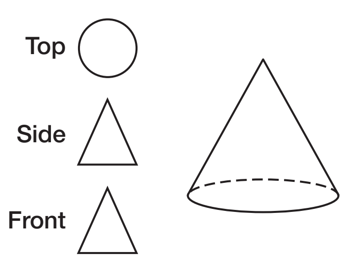


4.

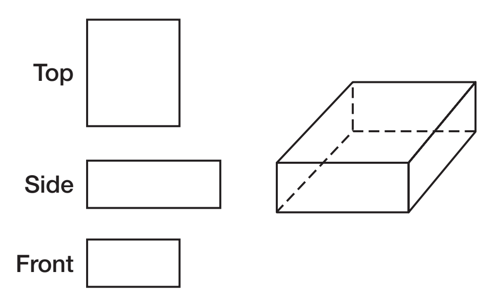


Make a perspective drawing of each figure by using the top, side, and front views as shown.

5.

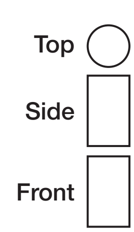


6.

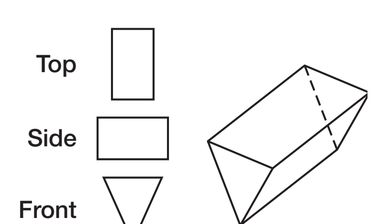


APPLICATIONS

7. Javier finds a container of oatmeal that is in the shape of a cylinder. Draw a top, a side, and a front view of the container.



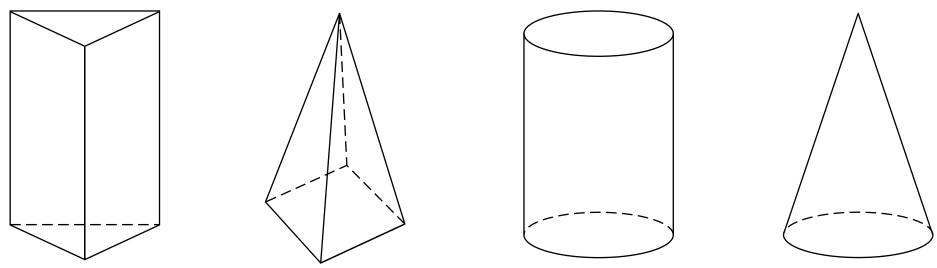
8. Victoria has borrowed a block from her little sister's building block collection. The top, side, and front views of the block are given. Draw the block.



Name ______________________________ Date ____________

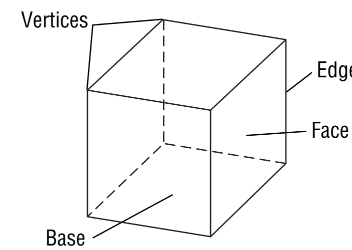
Three-Dimensional Figures

Common three-dimensional figures, or solids, are shown below.



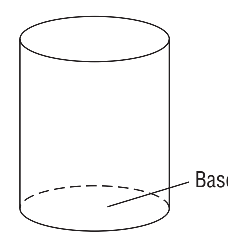
Prism **Pyramid** **Cylinder** **Cone**

The flat surfaces of a prism or pyramid are called **faces**. The faces intersect to form **edges**. The edges intersect to form **vertices**.

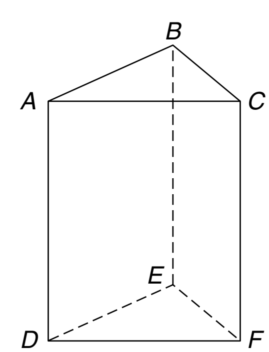


Prisms and pyramids are named by their bases. The prism at the right is a square prism.

At least two faces of a prism must be parallel and congruent polygons. These are called the **bases**. The base of a pyramid can be any polygon. All other faces are triangles. The bases of cylinders and cones are circles.



EXAMPLES ***Refer to the triangular prism.***



List the faces of the prism. Identify the bases.

Faces: $\triangle ABC$, $\triangle DEF$, $ACFD$, $ABED$, and $BCFE$

Bases: $\triangle ABC$ and $\triangle DEF$

List the edges of the prism.

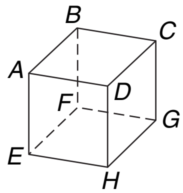
$\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{AD}$, $\overline{BE}$, $\overline{CF}$, $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$

List the vertices of the prism.

A, B, C, D, E, F

EXERCISES *Name each solid. Identify the bases.*

1.



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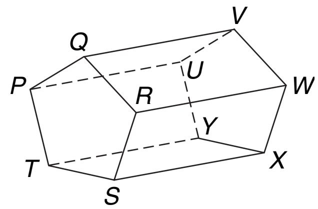
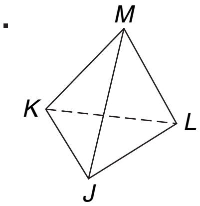
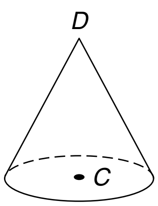


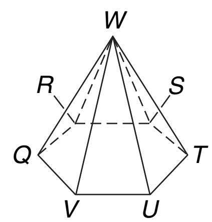
3.



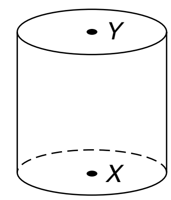
4.



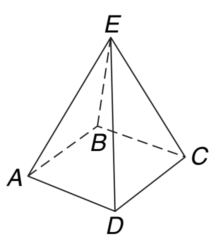
5.



6.



7.



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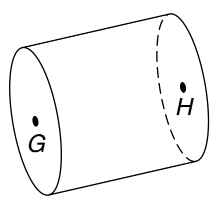
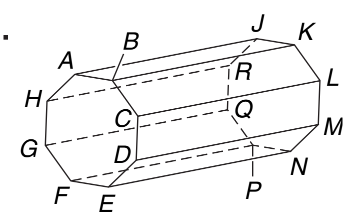
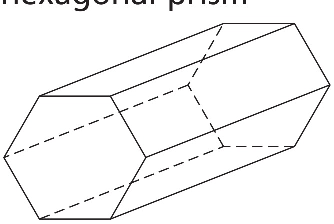


9.

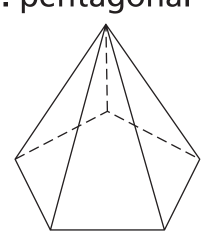


Draw each solid.

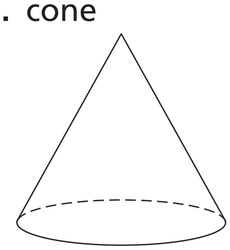
10. hexagonal prism



11. pentagonal pyramid



12. cone



APPLICATIONS *Name two real-world examples of each solid.*

13. rectangular prism

14. square pyramid

15. cylinder

16. triangular pyramid

17. pentagonal prism

18. cone

SKILL 30

Name ______________________________ Date ____________

Weight and Mass

This table shows the relationship among different units of mass in the metric system.

	milligrams	centigrams	decigrams	grams	decagrams	hectograms	kilograms
1 milligram	1 milligram	0.1 centigram	0.01 decigram	0.001 gram	0.0001 decagram	0.00001 hectogram	0.000001 kilogram
1 centigram	10 milligrams	1 centigram	0.1 decigram	0.01 gram	0.001 decagram	0.0001 hectogram	0.00001 kilogram
1 decigram	100 milligrams	10 centigrams	1 decigram	0.1 gram	0.01 decagram	0.001 hectogram	0.0001 kilogram
1 gram	1,000 milligrams	100 centigrams	10 decigrams	1 gram	0.1 decagram	0.01 hectogram	0.001 kilogram
1 decagram	10,000 milligrams	1,000 centigrams	100 decigrams	10 grams	1 decagram	0.1 hectogram	0.01 kilogram
1 hectogram	100,000 milligrams	10,000 centigrams	1,000 decigrams	100 grams	10 decagrams	1 hectogram	0.1 kilogram
1 kilogram	1,000,000 milligrams	100,000 centigrams	10,000 decigrams	1,000 grams	100 decagrams	10 hectograms	1 kilogram

This table shows the relationship among different units of weight in the U.S. Customary system.

	ounces	pounds	tons
1 ounce	1 ounce	1/16 pound	1/32,000 ton
1 pound	16 ounces	1 pound	1/2,000 ton
1 ton	32,000 ounces	2,000 pounds	1 ton

EXAMPLES ***Convert each mass into the units given.***

1. 2,000 milligrams = ______ grams = ________ kilograms

2. 35 grams = ______ decigrams = ________ centigrams

EXERCISES ***Convert each weight into the units given.***

3. 24 ounces = _________ pounds

4. 4,000 pounds = _________ ounces = _________ tons

5. 80 ounces = _________ pounds = _________ tons

6. $1\frac{1}{2}$ tons = _________ pounds = _________ ounces

7. 10 pounds = _________ ounces = _________ tons

8. $\frac{1}{4}$ ton = _________ pounds = _________ ounces

APPLICATIONS

9. Which is heavier, 1.5 kilograms or 23,000 milligrams?

10. The mass of a medium-sized mouse is about 20 grams. The mass of a medium-sized cat is about 6 kilograms. How many mice would balance one cat on a scale?

11. The weight of a penny is 2.5 grams. The weight of a liter of water is 1 kilogram (not including the container). If a 1-liter bottle of water costs $2.00, and you paid for it in pennies, would the pennies weigh more than the water?

12. Which is heavier, $2\frac{1}{4}$ pounds or 40 ounces?

13. Tony wants to buy 5 pounds of rice, but the store only sells rice in 10-ounce packages. How many packages does he need?

14. A box of crackers weighs 12 ounces. A crate holds 37.5 pounds of crackers. How many boxes are in a crate?

SKILL 31

Name ______________________ Date ____________

Simplify and Use Logical Reasoning

Two possible strategies for solving problems are listed below.

- Solve a Simpler Problem
- Use Logical Reasoning

EXAMPLE ***Find the sum of the whole numbers from 1 to 200.***

This would be a tedious problem to solve using a calculator or adding the numbers yourself. The problem is easier to solve if you solve a simpler problem. First, consider the partial sums indicated below.

1, 2, 3, …, 100, 101, …, 198, 199, 200

$$100 + 101 = 201$$
$$3 + 198 = 201$$
$$2 + 199 = 201$$
$$1 + 200 = 201$$

Notice that each sum is 201. There are 100 of these partial sums.

$$201 \times 100 = 20{,}100$$

The sum of the whole numbers from 1 to 200 is 20,100.

EXAMPLE ***Neva, Justin, Toshiro, and Sydney are lining up by height. Toshiro is not standing next to Sydney. Neva is the shortest and is not standing next to Toshiro. List the students from shortest to tallest.***

Use logical reasoning to solve this problem.

- Since Neva is the shortest, write Neva in the first position.
- Since Toshiro and Sydney are not standing next to each other, write Justin in the third position.
- Since Neva and Toshiro are not standing next to each other, write Toshiro in the fourth position.
- Since Sydney is the only student left, write Sydney in the second position.

1. Neva **2.** Sydney **3.** Justin **4.** Toshiro

The students from shortest to tallest are Neva, Sydney, Justin, and Toshiro.

EXERCISES *Solve.*

1. Find the sum of the whole numbers from 1 to 400.

2. Find the sum of the even numbers from 2 to 100.

3. There are three boards each a different odd number of feet long. If the boards are placed end to end, the total length is 9 feet. What are the lengths of the boards?

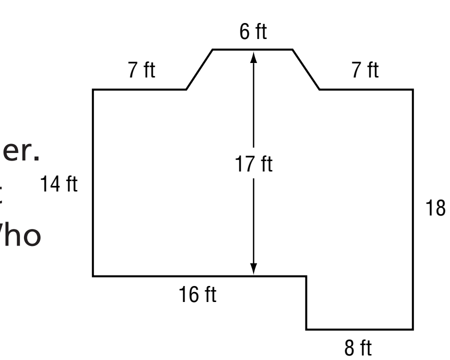
4. A total of 492 digits are used to print all the page numbers of a book beginning with page 1. How many pages are in the book?

5. Anna, Iris, and Oki each have a pet. The pets are a fish, cat, and a bird. Anna is allergic to cats. Oki's pet has 2 legs. Whose pet is the fish?

APPLICATIONS

6. Connie, Kristina, and Roberta are the pitcher, catcher, and shortstop for a softball team, but not necessarily in that order. Kristina is not the catcher. Roberta and Kristina share a locker with the shortstop. Who plays each position?

7. Mr. Lee wants to carpet the room shown at the right. How much carpet will he need?



8. Doug, Louann, and Sandy have lockers next to each other. Louann rides the bus with the person whose locker is at the right. Doug's locker is not next to Luann's locker. Who has the locker at the left?

9. A rectangular field is fenced on two adjacent sides by a brick wall. The field is 63 yards long with an area of 1,323 square yards. How much fencing is needed on the two sides of the field?

10. A vending machine sells items that cost 80¢. It only accepts quarters, dimes, and nickels. If it only accepts exact change, how many different combinations of coins must the machine be programmed to accept?

SKILL 32

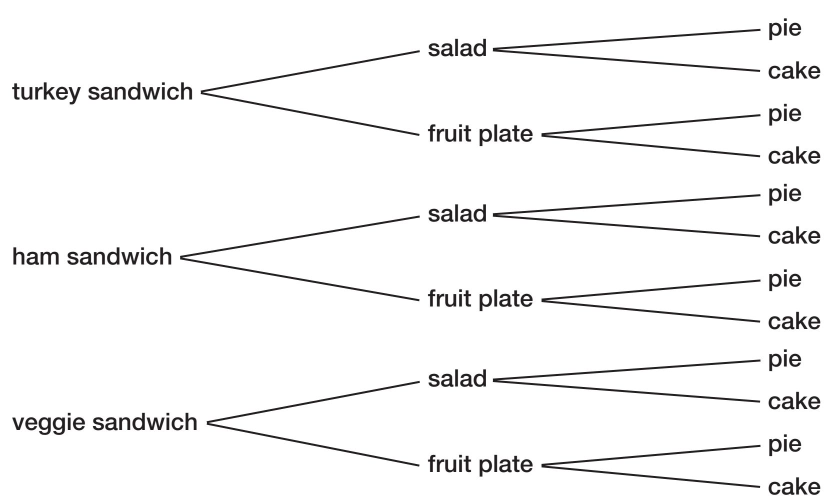
Name ______________________ Date __________

Counting Outcomes and Tree Diagrams

An organized list can help you determine the number of possible outcomes for a situation. One type of organized list is a **tree diagram.**

EXAMPLE ***The lunch special at Morgan's Diner is a choice of a turkey, ham, or veggie sandwich, salad, or fruit plate, and either pie or cake. If you wish to order the lunch special, how many different choices do you have?***

To answer this question, make a tree diagram.



There are 12 choices for the lunch special.

The **Fundamental Counting Principle** states that if an event M can occur m ways and it is followed by an event N that can occur n ways, then the event M followed by event N can occur $m \times n$ ways.

EXAMPLE ***If two quarters are tossed, find the total number of outcomes.***

Use the Fundamental Counting Principle.

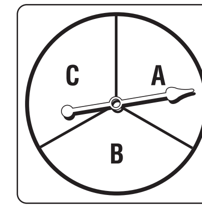
There are 2 possible outcomes when tossing a coin, heads or tails.
outcomes for quarter 1 × outcomes for quarter 2 = possible outcomes
$2 \times 2 = 4$
There are 4 possible outcomes if 2 quarters are tossed.

EXERCISES

Draw a tree diagram to find the number of outcomes for each situation.

1. A coin and a number cube are tossed.

2. The spinner is spun twice.



Use the Fundamental Counting Principle to find the number of outcomes for each situation.

3. 5 types of juice come in 3 different sized containers.

4. T-shirts come in 3 sizes and 6 colors.

5. Baseball hats come in 2 styles and 3 sizes for each of 12 teams.

6. Pizzas come in 5 sizes, with 2 different crusts and 14 available toppings.

APPLICATIONS

7. Mrs. Jenkins' history test has 10 questions. Seven of the questions are multiple-choice with four answer choices. Two of the questions are true-false. How many possible sets of answers are there for the test?

8. Ryan is buying a new bicycle. He can choose from a mountain bike, a stunt bike, or a BMX bike. Each of the bikes comes in 6 colors. The bikes offer a choice of 2 types of tires and 3 types of seats. How many different bicycles can Ryan select?

9. Lonan is choosing a new password for his email account. The password must contain eight characters. The first two characters of his password must be letters and the last 6 digits must be any digit 0–9. How many possible passwords can Lonan create?

SKILL 33

Name ______________________ Date ____________

Permutations

An arrangement or listing in which order is important is called a **permutation**. $P(n, r)$ stands for the number of permutations of n things taken r at a time.

EXAMPLE ***There are 6 runners in a race. How many permutations of first, second, and third place are possible?***

There are 6 choices for first place, then 5 choices for second place, and finally 4 choices for third place. Find $P(6, 3)$

$$\begin{aligned} P(6, 3) &= 6 \times 5 \times 4 \\ &= 120 \end{aligned}$$

The number of permutations is 120.

The expression $6 \times 5 \times 4 \times 3 \times 2 \times 1$ can be written as 6!. It is read *six factorial.* In general, $n!$ is the product of whole numbers starting at n and counting backward to 1. To find the number of permutations involving all members of a group, $P(n, n)$, find $n!$.

EXAMPLE ***There are 6 runners in a race. In how many ways can they finish the race?***

There are 6 choices for first, 5 choices for second, and so on.

$P(6, 6) = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

There are 720 ways in which the runners can finish the race.

EXERCISES ***Find the value of each expression.***

1. $P(5, 2)$ **2.** $P(8, 3)$ **3.** $P(4, 3)$ **4.** $P(7, 4)$

5. $P(10, 2)$ **6.** $P(4, 4)$ **7.** $P(6, 1)$ **8.** $P(9, 5)$

9. 4! **10.** 5! **11.** 8! **12.** 10!

13. $\frac{6!}{3!}$ **14.** $\frac{5!}{4!}$ **15.** $\frac{6!}{2!}$ **16.** $\frac{8!}{5!}$

APPLICATIONS

17. How many ways can a winner and a runner-up be chosen from 8 show dogs at a dog show?

18. In how many ways can 5 horses in a race cross the finish line?

19. In how many ways can a president, vice-president, secretary, and treasurer be chosen from a club with 12 members?

20. A shelf has a history book, a novel, a biography, a dictionary, a cookbook, and a home-repair book. In how many ways can 4 of these books be rearranged on another shelf?

21. In how many ways can 8 people be seated at a counter that has 8 stools in a row?

22. Eight trained parrots fly onto the stage but find there are only 5 perches. How many different ways can the parrots land on the perches if only one parrot is on each perch?

23. Seven students are running for class president. In how many different orders can the candidates make their campaign speeches?

24. In how many different ways can a coach name the first three batters in a nine-batter softball lineup?

25. How many different flags consisting of 4 different-colored vertical stripes can be made up from blue, green, red, black, and white?

26. In how many ways can the gold, silver, and bronze medals be awarded to 10 swimmers?

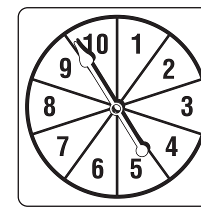
SKILL 34

Name ______________________ Date ____________

Probability

The **probability** of an event is the ratio of the number of ways an event can occur to the number of possible outcomes. The probability of one event occurring is called a **simple probability**.

EXAMPLE ***The spinner has ten equally likely outcomes. Find the probability of spinning a number less than 7.***



Numbers less than 7 are 1, 2, 3, 4, 5, and 6.
There are 10 possible outcomes.

probability of a number less than 7 $= \frac{6}{10}$ or $\frac{3}{5}$

The probability of spinning a number less than 7 is $\frac{3}{5}$.

A **compound event** consists of two or more simple events. **Independent events** occur when the outcome of one event does *not* affect the outcome of another event. If the outcome of one event affects the outcome of another event, the events are called **dependent events**.

EXAMPLES ***Tiles numbered 1 through 25 are placed in a box. Two tiles are selected at random. Find each probability.***

drawing an even number, replacing the tile, and then randomly drawing a multiple of 3
The events are independent since the outcome of one drawing does not affect the other.

$P(\text{even number}) = \frac{12}{25}$ $\qquad P(\text{multiple of } 3) = \frac{8}{25}$

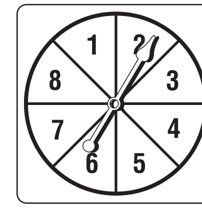
$P(\text{even number, multiple of } 3) = \frac{12}{25} \cdot \frac{8}{25}$ or $\frac{96}{625}$

drawing a number greater than 10, and then drawing a number less than 10 without replacing the first tile
The events are dependent since there is one less tile from which to choose on the second draw.

$P(n > 10) = \frac{15}{25}$ or $\frac{3}{5}$ $\qquad P(n < 10) = \frac{9}{24}$ or $\frac{3}{8}$

$P(n > 10, n < 10) = \frac{3}{5} \cdot \frac{3}{8}$ or $\frac{9}{40}$

EXERCISES ***The spinner shown is equally likely to stop on each of the sections. Find each probability.***

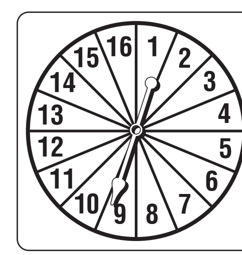


1. $P(n < 5)$
2. P(even number)
3. P(multiple of 4)
4. P(factor of 6)

A bag of marbles contains 3 yellow, 6 blue, 1 green, 12 red, and 8 orange marbles. Find each probability.

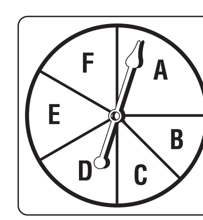
5. P(red)
6. P(blue or yellow)
7. P(not orange)
8. P(yellow then blue, with replacement)
9. P(green then red without replacement)
10. P(yellow, yellow, orange, without replacement)

The spinner shown is equally likely to stop on each of the sections. The spinner is spun twice. Find each probability.



11. P(multiple of 2, multiple of 3)
12. $P(n > 10, n > 12)$
13. P(product is even)
14. $P(\text{sum} = 20)$

A number cube is rolled and the spinner is spun. Find each probability.



15. P(6 and B)
16. P(odd number and E)
17. $P(n > 3$ and A, B, or C$)$
18. $P(n < 3$ and vowel$)$

APPLICATIONS ***Brianna, Mai-Lin, and Camila are playing a board game in which two number cubes are tossed to determine how far a player's game piece is to move.***

19. Brianna needs to move her piece 9 spaces to return it to base. What is the probability that she will roll 9?
20. If Mai-Lin rolls doubles, then she gets to roll again. What is the probability that Mai-Lin will get to roll twice on her next turn?

SKILL 35

Name ____________________ Date ____________

Theoretical and Experimental Probability

The **theoretical probability** of an event is the ratio of the number of ways the event can occur to the number of possible outcomes.

The **experimental probability** of an event is the ratio of the number of successful trials to the number of trials.

EXAMPLE ***Sean wants to determine the probability of getting a sum of 7 when rolling two number cubes. The sample space, or all possible outcomes, for rolling two number cubes is shown below.***

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

What is the theoretical probability of rolling a sum of 7? What is the experimental probability of rolling a sum of 7 if Sean rolls the number cubes 20 times and records 4 sums of 7?

There are 6 sums of 7 shown in the sample space above. So, the theoretical probability of rolling a sum of 7 is $\frac{6}{36}$ or $\frac{1}{6}$.
Since Sean rolled 4 sums of 7 on 20 rolls, the experimental probability is $\frac{4}{20}$ or $\frac{1}{5}$.

EXERCISES ***Find the theoretical probability of each of the following.***

1. getting tails if you toss a coin

2. getting a 6 if you roll a number cube

3. getting a sum of 2 if you roll two number cubes

4. getting a sum less than 6 if you roll two number cubes

5. Amanda rolled one number cube 30 times and got 8 sixes.

a. What is her experimental probability of getting a six?

b. What is her experimental probability of *not* getting a six?

6. Ramón rolled two number cubes 36 times and got 3 sums of 11.

a. What is his experimental probability of getting a sum of 11?

b. What is his experimental probability of *not* getting a sum of 11?

APPLICATIONS ***While playing a board game, Akira rolled a pair of number cubes 48 times and got doubles 10 times.***

7. What was his experimental probability of rolling doubles?

8. How does his experimental probability compare to the theoretical probability of rolling doubles?

9. How do you think the experimental probability compares to the theoretical probability in most experiments?

10. Do you think the experimental probability is ever equal to the theoretical probability? Explain?

SKILL 36

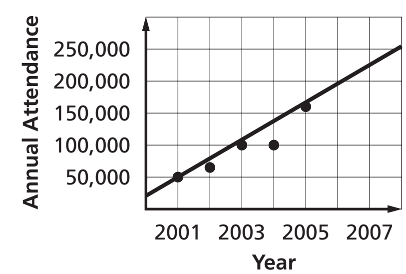
Name ______________________ Date __________

Using Statistics to Make Predictions

When real-life data are collected in a statistical experiment, the points graphed usually do not form a straight line. They may, however, approximate a linear relationship. A **best-fit line** can be used to show such a relationship. A best-fit line is a line that is very close to most of the data points.

EXAMPLE ***Use the best-fit line to predict the annual attendance at Fun Times Amusement Park in 2007.***

Draw a line so that the points are as close as possible to the line. Extend the line so that you can find the y value for an x value of 2007. The y value for 2007 is about 225,000.



So, the annual attendance at Fun Times Amusement Park in 2007 is 225,000 people.

You can also write an equation of a best-fit line.

EXAMPLE ***Use the information from the example above. Write an equation in slope-intercept form for the best-fit line and then predict the annual attendance in 2008.***

Step 1 First, select two points on the line and find the slope. Choose (2001, 50,000) and (2003, 100,000).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Definition of slope*

$= \frac{100{,}000 - 50{,}000}{2003 - 2001}$ $x_1 = 2001$, $y_1 = 50{,}000$

$x_2 = 2003$, $y_2 = 100{,}000$

$= 25{,}000$ *Simplify.*

Step 2 Find the y-intercept.

$y = mx + b$ *Slope-intercept form*

$50{,}000 = 25{,}000(2001) + b$ $y = 50{,}000$, $m = 25{,}000$, $x = 2001$

$-49{,}975{,}000 = b$ *Simplify.*

(Continued on the next page.)

Step 3 Write the equation.

$y = mx + b$ Slope-intercept form

$y = 25{,}000x - 49{,}975{,}000$ $m = 25{,}000$, $b = -49{,}975{,}000$

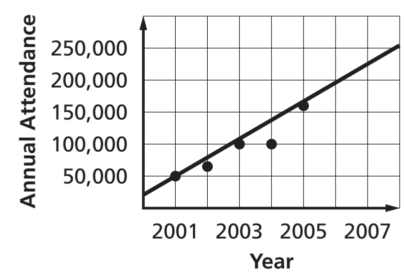
Step 4 Solve the equation.

$y = 25{,}000(2008) - 49{,}975{,}000$ $x = 2008$

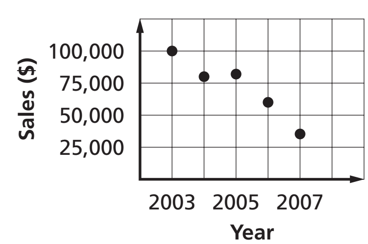
$-225{,}000$ *Simplify.*

The predicted annual attendance at Fun Times Amusement Park in 2008 is 225,000.

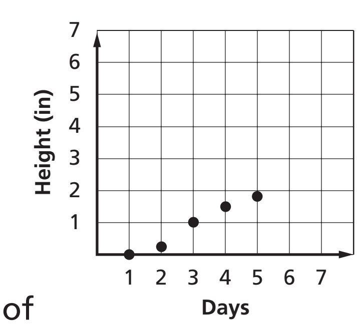
EXERCISES



1. Predict the sales figure for 2008.



2. What is the gas mileage for a car weighing 4,500 pounds?



3. How tall would a tomato plant be ten days after planting the seed?

APPLICATIONS

4. Use the graph in Exercise 2. Determine the equation of the best-fit line. Use it to predict the gas mileage for a car weighing 4,000 pounds.

5. Use the graph in Exercise 3. Determine the equation of the best-fit line. Use it to predict the height of the tomato plant 8 days after planting.

SKILL 37

Name ______________________ Date __________

Mean, Median, and Mode

You can analyze a set of data by using three measures of central tendency: **mean, median,** and **mode.**

EXAMPLE ***Tim Duncan, 2003's Most Valuable Player in the National Basketball Association, helped the San Antonio Spurs win the NBA Championship. In winning the six games of the series, Duncan scored 32, 19, 21, 23, 29, and 21 points. Find the mean, median, and mode of his scores.***

Mean: $\frac{32 + 19 + 21 + 23 + 29 + 21}{6} \approx 24.167$

The mean is about 24 points.

Median: 19, 21, 21, 23, 29, 32

$\frac{21 + 23}{2} = 22$

The median is 22 points.

Mode: The mode is 21 since it is the number that appears the most times.

EXERCISES ***Find the mean, median, and mode for each set of data.***

1. 2, 3, 7, 8, 10, 3, 1, 7, 5

2. 17, 18, 20, 13, 23, 37, 20, 16

3. 4.8, 6.4, 7.2, 4.5, 2.3, 6.0, 3.5

4. 82, 77, 82, 76, 79, 78, 81, 86

5. 40, 42, 41, 43, 41, 40, 40, 42, 43

6. \$7.50, \$7.00, \$8.50, \$7.50, \$4.50, \$6.50, \$8.00, \$6.00, \$4.50

7. 1.78, 1.45, 1.33, 1.72, 1.94, 1.73, 1.14

8. 3, −3, 1, 4, 5, 0, −4, −1, 2, −1

9. 90%, 98%, 96%, 85%, 91%, 90%, 88%, 87%, 88%, 90%

10. 5.8 cm, 8.9 cm, 8.8 cm, 8.6 cm, 8.8 cm, 8.8 cm, 8.9 cm

11. $50,000, $37,500, $43,900, $76,900, $46,000, $48,580

12. 29.1°F, 33.9°F, 38.2°F, 46.5°F, 55.4°F, 62.0°F, 63.6°F, 62.3°F, 56.1°F, 47.2°F, 37.3°F, 32.0°F

APPLICATIONS

The data at the right shows the ages of U.S. Presidents from 1900–2004 at the time of their inaugurations. Use this data to answer Exercises 13–16.

58	42	51	56	55
51	54	51	60	62
43	55	56	61	52
69	64	46	54	

13. What is the mode of the data?

14. What is the mean of the data?

15. What is the median of the data?

16. If the age of each President was 1 year older, would it change
a. the mean? Why or why not?

b. the median? Why or why not?

c. the mode? Why or why not?

SKILL 38

Name ______________________________ Date ____________

Frequency Tables

There are four frequency values that can be considered.

absolute frequency: the frequency for an individual interval
relative frequency: the ratio of an interval's absolute frequency to the total number of elements
cumulative frequency: the sum of the absolute frequency of an interval and all previous absolute frequencies
relative cumulative frequency: the ratio of an interval's cumulative frequency to the total number of elements

EXAMPLE ***The base prices of new cars from one manufacturer are listed below.***

$24,625	***$16,200***	***$22,225***	***$40,450***	***$35,050***	***$33,565***	***$44,535***
$22,075	***$24,370***	***$20,465***	***$9,995***	***$21,560***	***$25,330***	***$24,695***
$28,105	***$21,630***	***$22,145***	***$41,995***	***$28,905***	***$30,655***	***$10,700***
$18,995	***$20,060***	***$37,900***	***$22,080***			

Organize this information in a frequency table. Determine the absolute frequencies, relative frequencies, cumulative frequencies and relative cumulative frequencies for the data. Then find the range of the data.

Use intervals of 10,000 to organize the data.

New Car Prices

Interval	Tally	Absolute Frequency	Relative Frequency	Cumulative Frequency	Relative Cumulative Frequency
$0–$9,999	|	1	$\frac{1}{25} = 0.04$	1	$\frac{1}{25} = 0.04$
$10,000–$19,999	|||	3	$\frac{3}{25} = 0.12$	4	$\frac{4}{25} = 0.16$
$20,000–$29,999	~~||||~~ ~~||||~~ ||||	14	$\frac{14}{25} = 0.56$	18	$\frac{18}{25} = 0.72$
$30,000–$39,999	||||	4	$\frac{4}{25} = 0.16$	22	$\frac{22}{25} = 0.88$
$40,000–$49,999	|||	3	$\frac{3}{25} = 0.12$	25	$\frac{25}{25} = 1.00$

To determine the range, find the difference between the highest price and lowest price.
$44,535 − $9,995 = $34,540

EXAMPLE ***Organize the information in a frequency table. Determine the absolute frequencies, relative frequencies, cumulative frequencies, and relative cumulative frequencies for the data.***

1. Test scores of students in a classroom.
94, 81, 85, 59, 83, 73, 75, 96, 72, 87, 77, 88, 90, 65, 71, 82, 86, 89, 68, 96

Score	Tally	Absolute Frequency	Relative Frequency	Cumulative Frequency	Relative Cumulative Frequency
50–59	|	**1**	$\frac{1}{20} = 0.05$	**1**	$\frac{1}{20} = 0.05$
50–59	||	**2**	$\frac{2}{20} = 0.10$	**3**	$\frac{2}{20} = 0.15$
50–59	𝍸	**5**	$\frac{5}{20} = 0.25$	**8**	$\frac{8}{20} = 0.40$
50–59	𝍸 |||	**8**	$\frac{8}{20} = 0.45$	**16**	$\frac{16}{20} = 0.80$
50–59	||||	**4**	$\frac{4}{20} = 0.20$	**20**	$\frac{20}{20} = 1.00$

APPLICATIONS ***Use a frequency table to determine the absolute frequencies, relative frequencies, cumulative frequencies, and relative cumulative frequencies for the data.***

2. World Series champions from 1990–2003.

1990 Cincinnati Reds
1991 Minnesota Twins
1992 Toronto Blue Jays
1993 Toronto Blue Jays
1994 *Not Held*
1995 Atlanta Braves
1996 New York Yankees
1997 Florida Marlins
1998 New York Yankees
1999 New York Yankees
2000 New York Yankees
2001 Arizona Diamondbacks
2002 Anaheim Angels
2003 Florida Marlins

SKILL 39

Name ____________________ Date __________

Circle Graphs

The air surrounding Earth is referred to as the atmosphere. Without air there would be no life on Earth. Air is a mixture of gases. By volume, dry air is composed of 78% nitrogen, 21% oxygen, and 1% other gases.

EXAMPLE ***Make a circle graph to show the composition of the Earth's atmosphere.***

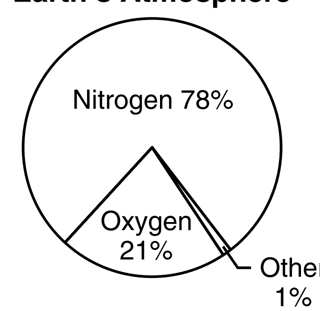
To make a circle graph, first find the number of degrees that correspond to each percent. Use a calculator and round to the nearest degree.

Nitrogen: 78% of 360° ≈ 281°
Oxygen: 21% of 360° ≈ 76°
Other: 1% of 360° ≈ 4°

Use a compass and a protractor to draw the circle graph.

Note that the sum of the degrees is *not* 360° because of rounding.

Earth's Atmosphere



EXERCISES ***Make a circle graph to show the data in each chart.***

1.

Favorite TV Shows	
Movies	12%
Sports	20%
News	4%
Drama	16%
Comedy	20%
Music	28%

2.

Daily Activities	
Sleeping	8 hours
Eating	1 hours
School	6 hours
Homework	3 hours
Team practice	2 hours
Miscellaneous	4 hours

APPLICATIONS ***Make a circle graph to show the data in each chart.***

3.

Area of Continents	
Continent	**Area in Millions of Square Miles**
Europe	3.8
Asia	17.4
North America	9.4
South America	6.9
Africa	11.7
Oceania and Australia	3.3
Antarctica	5.4

4.

World Cup Winners	
Country	**Number of Wins**
Argentina	2
Brazil	4
England	1
Italy	3
Uruguay	2
West Germany	3

5.

Area of New England States	
State	**Area in Square Miles**
Maine	33,215
New Hampshire	9,304
Vermont	9,609
Massachusetts	8,257
Connecticut	5,009
Rhode Island	1,214

6. Make a circle graph showing how you spent your time last Saturday.

SKILL 40

Name ______________________ Date __________

Stem-and-Leaf Plots

A **stem-and-leaf** plot is one way to organize a list of numbers. The **stems** represent the greatest place value in the numbers. The **leaves** represent the next place value.

EXAMPLE ***The fourteen states with the most representatives in the House of Representatives are listed below. Make a stem-and-leaf plot for this data.***

State	Representatives	State	Representatives
California	52	New Jersey	13
Florida	23	New York	31
Georgia	11	North Carolina	12
Illinois	20	Ohio	19
Indiana	10	Pennsylvania	21
Massachusetts	10	Texas	30
Michigan	16	Virginia	11

The stem will be the tens place and the leaves will be the ones place.

1	00112369
2	013
3	01
4	
5	2

1|0 means 10 representatives.

EXERCISES ***Make a stem-and-leaf plot for each set of data.***

1. 56, 65, 57, 69, 58, 55, 52, 55, 66, 60, 53, 63

2. 230, 350, 260, 370, 240, 380, 290, 270, 220, 350, 300, 280

3. 4.5, 6.8, 5.2, 5.9, 5.1, 6.7, 4.0, 4.4, 6.0, 6.9

7. 1,900, 2,000, 2,600, 3,000, 2,500, 1,800, 2,200, 2,700, 1,600, 1,700, 2,000, 2,300

APPLICATIONS ***Each number below represents the age of workers at Fred's Fast Food.***
20 52 21 39 40 58 27 48 36 20 51 26
45 30 49 22 59 50 33 35 28 43 55 20
Use this data to answer Exercises 5–10.

5. Make a stem-and-leaf plot of the data.

6. How many people work at Fred's Fast Food?

7. What is the difference in the ages between the oldest and youngest workers at Fred's?

8. What is the most common age for a worker?

9. Which age group is most widely represented?

10. How many workers are older than 35 years?

11. Measure the length of your classmates' shoes in centimeters. Record the numbers and make a stem-and-leaf plot.

12. What is the most common length of your classmates' shoes?

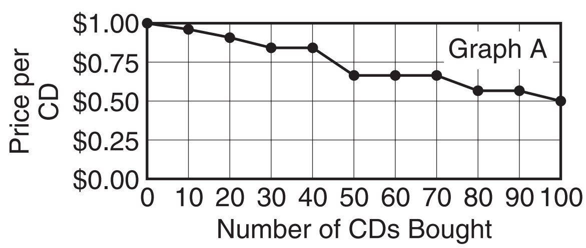
SKILL 41

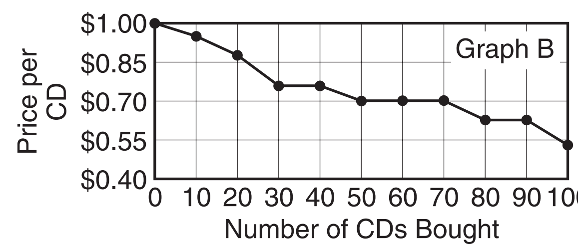
Name ______________________ Date __________

Misleading Graphs

Graphs can be used to present data in ways that are misleading.

EXAMPLE ***A company that sells computer CDs wants to encourage customers to buy more by showing how much the price drops as they buy more CDs. Which of the following graphs is misleading? Which graph should the company use to encourage customers to buy more CDs?***

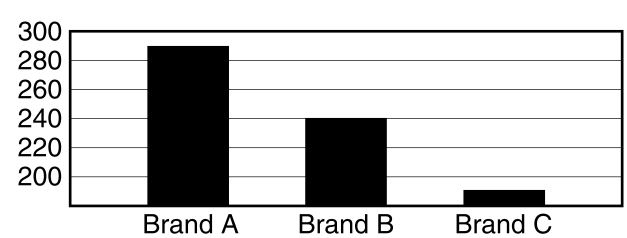




The vertical scale for Graph B does *not* begin with zero. Therefore, the drop in the cost of the CDs seems to be greater than the actual drop as shown in Graph A. Graph B is misleading.

Since the drop in the cost of the CDs seems greater in Graph B, this graph is more likely to encourage customers to buy more CDs.

EXERCISES ***Use the graph at the right to answer Exercises 1–3.***

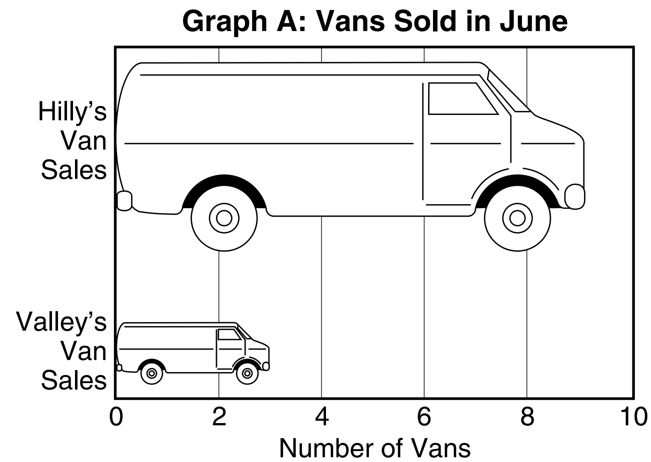


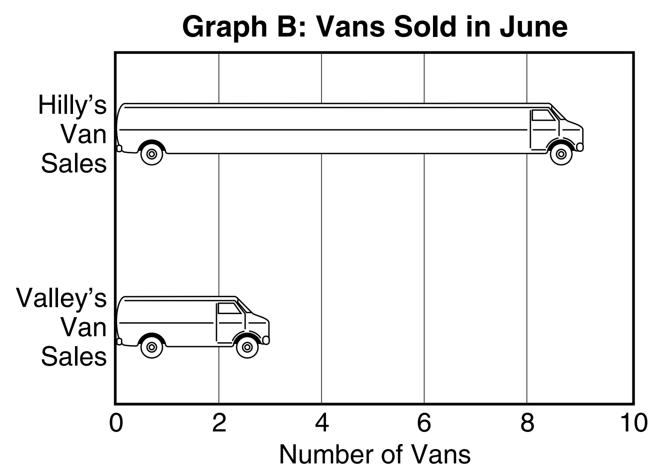
1. Which brand is the favorite of the greatest number of people?

2. Which brand is the favorite of the least number of people?

3. Why is this graph misleading?

APPLICATIONS ***Use the graphs at the right to answer Exercises 4–11.***





4. Do Graphs A and B give the same information on sales?

5. Find the ratio of Hilly's sales to Valley's sales.

6. In Graph A, the Hilly van is about 2.5 centimeters high by 6 centimeters long. What is its approximate area?

7. In Graph A, the Valley van is about 0.75 centimeters high and 2 centimeters long. What is its approximate area?

8. In Graph B, both vans are about 0.75 centimeter high. The Hilly van is about 6 centimeters long. What is its approximate area?

9. In Graph B, the Valley van is about 2 centimeters long. What is its approximate area?

10. Compute the following ratios.

Graph A: $\dfrac{\text{Area of Hilly}}{\text{Area of Valley}}$

Graph B: $\dfrac{\text{Area of Hilly}}{\text{Area of Valley}}$

11. Compare the results of Exercises 5 and 10. Which graph is misleading? Explain your answer.

SKILL 42

Name ______________________ Date ____________

Visualizing Information

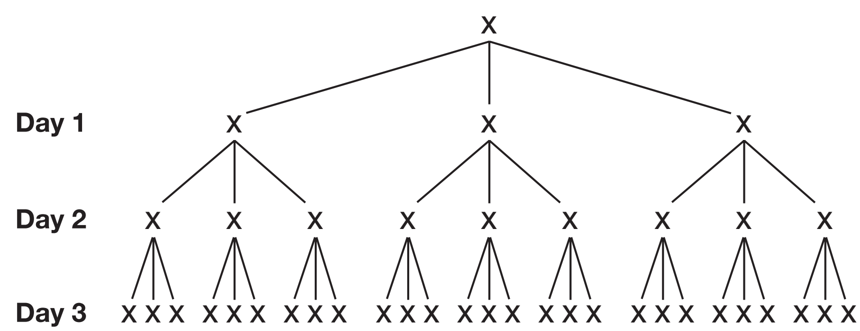
Three possible ways to visualize information to solve problems are listed below.

- Draw a Diagram
- Use a Graph
- Make a Model

EXERCISES

On the first day, Derrick e-mailed a joke to 3 of his friends. On the second day, each of these friends e-mailed 3 other people. On the third day, each of the people who read the joke on the second day e-mailed 3 more people. By the end of the third day, how many people read the joke?

To solve the problem, draw a diagram.

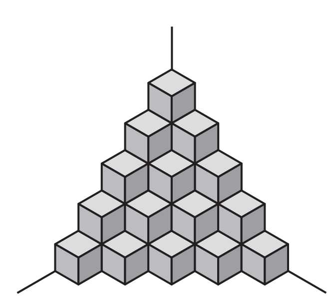


Count the number of Xs in Day 1, Day 2, and Day 3. By the end of the third day 39 people read the joke. Note that the first person e-mailed the joke, but did not read it, during the three days.

EXAMPLE

Identical boxes are stacked in the corner of a room as shown below. How many boxes are there altogether?

Make a model using cubes and count the number of cubes.



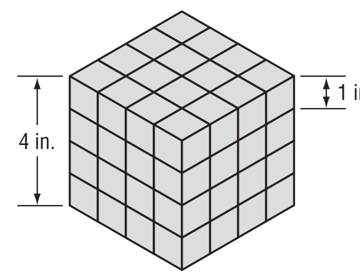
If you modeled the problem correctly, there should be 35 boxes.

EXERCISES *Solve.*

1. How many different shapes of rectangular prisms can be formed using exactly 18 cubes?

2. Six points are marked around a circle. How many straight lines must you draw to connect every point with every other point?

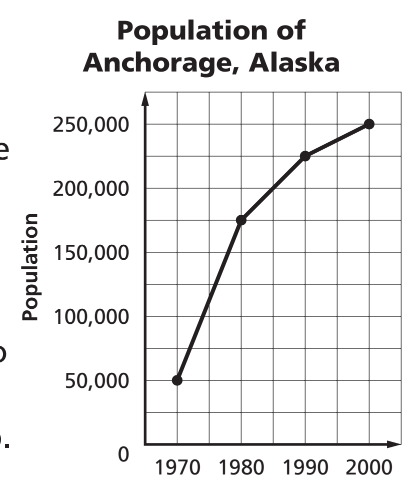
3. A cube with edges 4 inches long is painted on all six sides. Then, the cube is cut into smaller cubes with edges 1 inch long as shown at the right.
   a. How many of the smaller cubes are painted on only one side?
   b. How many of the smaller cubes are painted on exactly two sides?
   c. How many of the smaller cubes have no sides painted?



APPLICATIONS

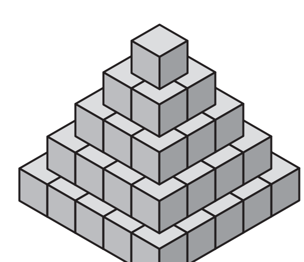
4. Coach Robinson is the tennis coach. He wants to schedule a round-robin tournament where every player plays every other player in singles tennis. If there are 8 members on the team, how many matches should the coach schedule?

5. The graph shows the population growth of Anchorage, Alaska.
   a. During what 10-year period did Anchorage show the greatest growth in population?
   b. What would you estimate the population will be in 2010?



6. Halfway through her plane flight from New York City to Orlando, Emma fell asleep. When she awoke, she still had to travel half the distance she traveled when asleep. For what fraction of the flight was Emma asleep?

7. Emilio wants to make a pyramid-shaped display of soccer balls for his sporting goods store. How many boxes of soccer balls will he need to make a display like the one at the right?



SKILL 43

Name ______________________ Date __________

Compare and Order Rational Numbers

EXAMPLE ***Here are strategies to compare two rational numbers and determine which is greater.***

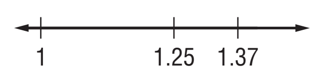
Strategy 1: Convert both numbers to decimals and see which is greater. This strategy is useful when the numbers you want to compare are decimals, fractions whose denominators are powers of 10, or fractions whose decimal equivalents you already know.

Example: Which is greater, $\frac{5}{4}$ or $1\frac{37}{100}$?

$\frac{5}{4}$ is the same as $1\frac{1}{4}$ or 1.25.

$1\frac{37}{100}$ in decimal form is 1.37.

1.25 is less than 1.37 because the two numbers both have 1 one, but 1.37 has 3 tenths, while 1.25 only has 2 tenths. So 1.37 is greater.



Strategy 2: Compare both numbers to a benchmark number. This strategy is useful when you can find a benchmark number that you know is greater than one of the numbers you are comparing and less than the other.

Example: Which is greater, -0.2 or $-\frac{3}{4}$?

Find a benchmark number that you can compare to both numbers.

$-\frac{1}{2}$ is less than -0.2. $-\frac{1}{2}$ is also greater than $-\frac{3}{4}$.

Since $-0.2 > -\frac{1}{2} > -\frac{3}{4}$, -0.2 must be greater than $-\frac{3}{4}$.

Strategy 3: Rewrite both numbers as fractions with a common denominator and compare them. This strategy is useful for fractions that are not easy to rewrite as decimals.

Example: Which is greater, $\frac{3}{12}$ or $\frac{5}{21}$?

Factor 12 and 21 to find the least common denominator.

Rewrite $\frac{3}{12}$ and $\frac{5}{21}$ with a common denominator of 84.

$\frac{3}{12} \times \frac{7}{7} = \frac{21}{84}$ $\qquad$ $\frac{5}{21} \times \frac{4}{4} = \frac{20}{84}$

$\frac{21}{84} > \frac{20}{84}$, so $\frac{3}{12} > \frac{5}{21}$

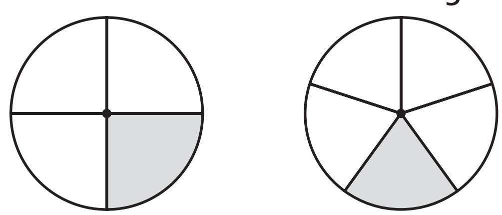
EXAMPLE

Strategy 4: Reason about the relative sizes of the numbers being compared.

Example: Which is greater, $\frac{3}{4}$ *or* $\frac{4}{5}$?

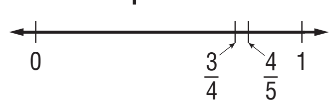
You know that fourths are larger than fifths, because if you cut an object into 5 equal pieces, the pieces must be smaller than if you cut the same object into only
4 pieces.

$\frac{3}{4} = 1 - \frac{1}{4}$ $\frac{4}{5} = 1 - \frac{1}{5}$



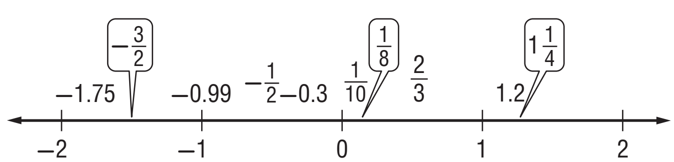
$\frac{4}{5}$ has less missing; it is closer to 1 than $\frac{3}{4}$.

So, $\frac{4}{5}$ is greater than $\frac{3}{4}$.



EXERCISES

Show the approximate place of each number on the number line.



1. 1.2 **2.** $-\frac{1}{2}$ **3.** 0.1

4. -0.99 **5.** $-\frac{3}{2}$ **6.** $\frac{1}{8}$

7. -1.75 **8.** $\frac{2}{3}$ **9.** -0.3

SKILL 44

Name ______________________ Date __________

Approximate Irrational Numbers

EXAMPLE

Estimate the decimal equivalent to $\sqrt{13}$. Write your answer with two digits to the right of the decimal point.

Find the two perfect squares that are closest to 13.

$3^2 = 9$
$4^2 = 16$

Since $9 < 13 < 16$, $\sqrt{13}$ must be greater than $\sqrt{9}$ and less than $\sqrt{16}$. So, $3 < \sqrt{13} < 4$.

Approximate. Since 13 is a little closer to 16 than to 9, so you could start with 3.7 as your first estimate.

Test your estimate.
$(3.7)^2 = 13.69$

Revise your estimate. 3.7 was close, but greater than 13. What about 3.6?
$(3.6)^2 = 12.96$

3.6 is a very good estimate, but it only has one digit to the right of the decimal point. So you need to refine your answer again.

3.6 was only a tiny bit less than 13, so you could try 3.61 as your next estimate.

$(3.61)^2 = 13.03$

3.61 is just a little closer than 3.60, so your final estimate is 3.61.

EXERCISES *Estimate each irrational number. Write your answers with two digits to the right of the decimal point.*

1. $\sqrt{7}$

2. $\sqrt{115}$

3. $\sqrt{30}$

4. $\sqrt{3}$

5. $\sqrt{90}$

6. $\sqrt{65}$

7. $\sqrt{21}$

8. $\sqrt{83}$

9. $\sqrt{42}$

10. $\sqrt{175}$

APPLICATIONS *In each exercise below, place all of the numbers in their approximate locations on the number line*

11. $\sqrt{45}$, 6.01, $\sqrt{8}$, $\frac{47}{10}$, $\sqrt{81}$, $\sqrt{27}$, $\sqrt{50}$, π, $\frac{10}{3}$

12. $\sqrt{34}$, $5\frac{5}{8}$, $\sqrt{30}$, $\sqrt{28}$, 5.23, $\sqrt{25}$, $5\frac{3}{4}$, $\sqrt{26}$

SKILL 45

Name ______________________________ Date ____________

Square Roots

If $a^2 = b$, then a is the **square root** of b. Square roots use the symbol $\sqrt{}$.
Thus, the square root of b would be written $\sqrt{b}$.

EXAMPLE ***Find the square root of each number.***

36
Since $6^2 = 36$, $\sqrt{36} = 6$.

100
Since $10^2 = 100$, $\sqrt{100} = 10$.

Numbers like 4, 9, 25, and 49 are called **perfect squares** because their square roots are whole numbers.

You can find an estimate for numbers that are not perfect squares.

EXAMPLE ***Estimate $\sqrt{95}$ to the nearest whole number.***

The closest perfect square less than 95 is 81.
The closest perfect square greater than 95 is 100.

$$81 < 95 < 100$$
$$\sqrt{81} < \sqrt{95} < \sqrt{100}$$
$$\sqrt{9^2} < \sqrt{95} < \sqrt{10^2}$$
$$9 < \sqrt{95} < 10$$

So, $\sqrt{95}$ is between 9 and 10. Since 95 is closer to 100 than to 81, the best whole number estimate for $\sqrt{95}$ is 10.

EXERCISES ***Find each square root.***

1. $\sqrt{25}$ **2.** $\sqrt{49}$ **3.** $\sqrt{16}$ **4.** $\sqrt{196}$

5. $\sqrt{256}$ **6.** $\sqrt{121}$ **7.** $\sqrt{225}$ **8.** $\sqrt{484}$

9. $\sqrt{529}$ **10.** $\sqrt{144}$ **11.** $\sqrt{576}$ **12.** $\sqrt{900}$

13. $\sqrt{39}$ **14.** $\sqrt{106}$ **15.** $\sqrt{71}$ **16.** $\sqrt{30}$

17. $\sqrt{15}$ **18.** $\sqrt{155}$ **19.** $\sqrt{200}$ **20.** $\sqrt{250}$

21. $\sqrt{500}$ **22.** $\sqrt{297}$ **23.** $\sqrt{340}$ **24.** $\sqrt{422}$

25. $\sqrt{803}$ **26.** $\sqrt{644}$ **27.** $\sqrt{975}$ **28.** $\sqrt{2{,}018}$

APPLICATIONS

The area of the floor of a square room is 324 square feet. Use this information to answer Exercises 29–31.

29. What is the length of each side of the floor?

30. If a square carpet with an area of 144 square feet is placed in the center of the room, what is the width of the floor that is uncovered on each side of the carpet?

31. How many 9-inch square tiles would be required to cover the entire floor?

32. The area of a square is 1,225 square centimeters. What is the perimeter of the square?

33. A bag of Super Green Lawn Fertilizer covers 9,500 square feet. What is the largest square lawn that can be fertilized using one bag of fertilizer? Round to the nearest foot.

34. Trees in orchards are often planted evenly spaced apart in square plots. How many rows of trees are in a plot that contains 1,024 trees?

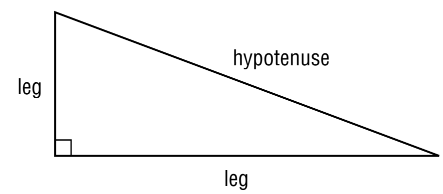
35. A square playground has an area of 750 square meters. Approximately how much fencing would be required to enclose the playground?

SKILL 46

Name ______________________ Date ____________

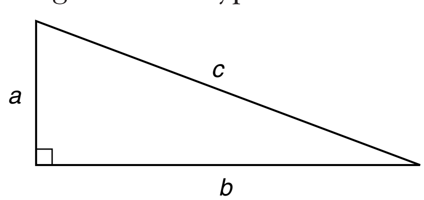
The Pythagorean Theorem

In a right triangle, the sides that form a right angle are called **legs**. The side opposite the right angle is the **hypotenuse**.



The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse. This theorem is true for any right triangle.

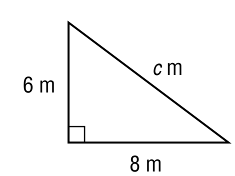
The Pythagorean Theorem states that if a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$, where a and b represent the lengths of the legs and c represents the length of the hypotenuse.

If you know the lengths of two sides of a right triangle, you can use the Pythagorean Theorem to find the length of the third side. This is called **solving a right triangle**.

EXAMPLE ***Find the length of the hypotenuse of the right triangle.***

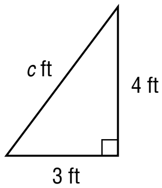


$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$c^2 = 6^2 + 8^2$	*Replace a with 6 and b with 8.*
$c^2 = 36 + 64$	*Evaluate 6^2 and 8^2.*
$c^2 = 100$	*Add 36 and 64.*
$\sqrt{c^2} = \sqrt{100}$	*Take the square root of each side.*
$c = 10$	*Simplify.*

The length of the hypotenuse is 10 meters.

EXERCISES ***In Exercises 1–6, find the length of the missing side.***

1.



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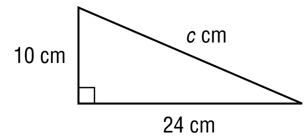

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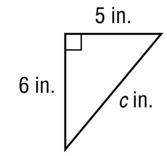

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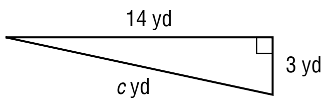

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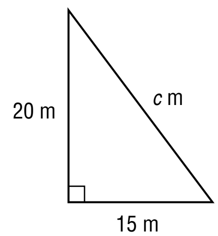

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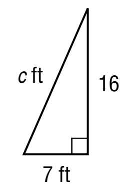


If c is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

7. $a = 12, b = ?, c = 18$

8. $a = ?, b = 7, c = 12$

9. $a = ?, b = 24, c = 32$

10. $a = 8, b = ?, c = 15$

11. $a = 42, b = ?, c = 60$

12. $a = ?, b = 16, c = 20$

APPLICATIONS

13. Zach is working on a hat which involves a pattern made by fitting together pieces of fabric in the shape of right triangles. Each of the pieces of fabric has legs measuring 8 inches and 10 inches. Find the hypotenuse of each piece of fabric. Round to the nearest tenth.

14. Breanna is building a new house on a plot of land that is shaped like a right triangle. One of the legs of the plot measures 48 feet, and the hypotenuse measures 82 feet. Find the length of the other leg. Round to the nearest tenth.

SKILL 47

Name ______________________ Date ____________

Triangles and Quadrilaterals

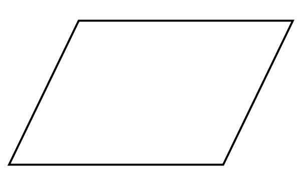
A triangle is a polygon with three angles and three sides. Triangles may be classified by the measures of their angles or by the lengths of their sides.

Triangles			
Classification by Angles		**Classification by Sides**	
Acute	all angles acute	Scalene	all sides different lengths
Obtuse	one obtuse angle	Isosceles	at least two sides the same length
Right	one right angle	Equilateral	three sides the same length

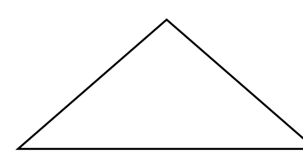
A quadrilateral is a polygon with four angles and four sides. Sides and angles are also used to classify quadrilaterals.

Quadrilaterals	
Trapezoid	only one pair of parallel sides
Parallelogram	both pairs of opposite sides parallel
Rectangle	parallelogram with four right angles
Rhombus	parallelogram with four sides the same length
Square	parallelogram with four right angles and four sides the same length

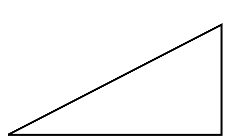
EXAMPLE ***Identify each polygon.***



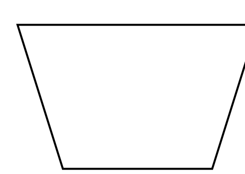
There are two pairs of opposite parallel sides. This quadrilateral is a parallelogram.



One of the angles is obtuse and two of the sides are the same length. This triangle is obtuse and isosceles.



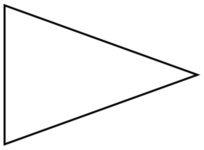
One of the angles is a right angle and none of the sides are the same length. This triangle is right and scalene.

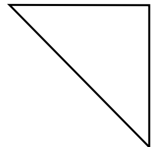


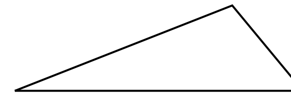
One pair of sides is parallel. This quadrilateral is a trapezoid.

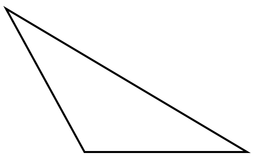
EXERCISES

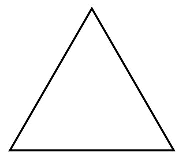
Classify each triangle by its angles and by its sides.

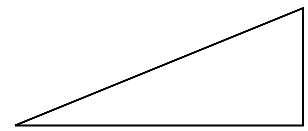
1. 

2. 

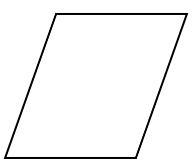
3. 

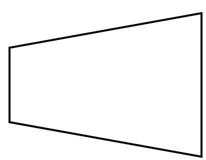
4. 

5. 

6. 

Name every quadrilateral that describes each figure. Then state which name best describes the figure.

7. 

8. 

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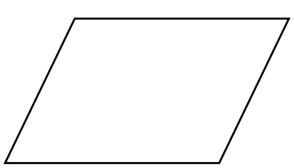
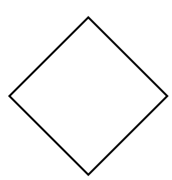
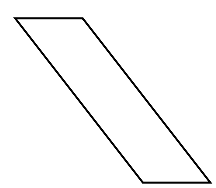
10. 

11. 

12. 

APPLICATIONS

Name two real-world examples of each figure.

13. equilateral triangle

14. trapezoid

15. rectangle

16. right scalene triangle

17. obtuse triangle

18. rhombus

19. square

20. acute isosceles triangle

SKILL 48

Name ______________________________ Date ______________

Ratio and Proportion

A **ratio** is a comparison of two numbers by division.

EXAMPLE ***In a class of 25 students there are 12 girls and 13 are boys. Write the relationship of the number of girls to the number of boys as a ratio.***

The ratio of girls to boys can be written as 12 to 13, 12:13, or $\frac{12}{13}$.

A **proportion** is a statement that two ratios are equal. In symbols, this can be shown by $\frac{a}{b} = \frac{c}{d}$. The cross products of a proportion, ad and bc, are equal.

EXAMPLE ***Determine if the ratios $\frac{3}{5}$ and $\frac{12}{20}$ form a proportion.***

Find the cross products of $\frac{3}{5} = \frac{12}{20}$.

$\frac{3}{5} \stackrel{?}{=} \frac{12}{20}$	*Write the proportion.*
$3(20) \stackrel{?}{=} 5(12)$	*Cross multiply.*
$60 = 60$	*Simplify.*

So, $\frac{3}{5}$ and $\frac{12}{20}$ form a proportion.

If one term of a proportion is not known, you can use the cross products to set up an equation to solve for the unknown term. This is called **solving the proportion**.

EXAMPLE ***Solve the proportion $\frac{8}{12} = \frac{x}{15}$.***

$\frac{8}{12} = \frac{x}{15}$	*Write the proportion.*
$8(15) = 12(x)$	*Cross multiply.*
$120 = 12(x)$	
$\frac{120}{12} = \frac{12(x)}{12}$	*Divide each side by 12.*
$10 = x$	

EXERCISES

Express each ratio as a fraction in simplest form.

1. 12 pennies to 18 coins

2. 15 bananas out of 25 fruits

3. 32 footballs to 40 basketballs

4. 6 cups to 14 pints

5. 8 clarinets out of 15 instruments

6. 16 tulips out of 24 flowers

7. 12 novels out of 27 books

8. 9 poodles to 12 beagles

Solve each proportion.

9. $\frac{a}{12} = \frac{3}{9}$

10. $\frac{8}{b} = \frac{12}{21}$

11. $\frac{24}{36} = \frac{c}{15}$

12. $\frac{27}{6} = \frac{18}{d}$

13. $\frac{7}{8} = \frac{e}{56}$

14. $\frac{27}{36} = \frac{6}{f}$

APPLICATIONS

15. If 8 gallons of gasoline cost $11.20, how much would 10 gallons cost?

16. A recipe for punch calls for 4 cups of lemonade for every 6 quarts of fruit juice. How many quarts of fruit juice should Elizabeth use if she has already added 10 cups of lemonade?

17. On a map, the scale is 1 inch equals 160 miles. What is the actual distance if the map distance is $3\frac{1}{2}$ inches?

18. One bag of jelly beans contains 14 red jelly beans. How many red jelly beans would be found in 4 bags of jelly beans?

SKILL 49

Name ______________________________ Date ____________

Proportional Reasoning

Super Value Grocery has a special on oranges this week. The price is 99¢ for 6 oranges.

EXAMPLE ***How many oranges can Daniel buy for $3.30?***

$$\frac{\text{oranges}}{\text{cost}(¢)} \begin{matrix}\longrightarrow \\ \longrightarrow\end{matrix} \frac{6}{99} = \frac{x}{330} \begin{matrix}\longleftarrow \\ \longleftarrow\end{matrix} \frac{\text{oranges}}{\text{cost}(¢)}$$ *Write a proportion.*

$$(6)(330) = 99(x)$$ *Cross multiply.*

$$1{,}980 = 99x$$ *Simplify.*

$$\frac{1{,}980}{99} = \frac{99x}{99}$$ *Divide each side by 99.*

$$20 = x$$ *Simplify.*

Daniel can buy 20 oranges.

EXERCISES ***Write a proportion to solve each problem. Then solve.***

1. 32 ounces of juice are required to make 2 gallons of punch.
6 gallons of punch require *n* ounces of juice.

2. 29 students for every teacher.
348 students for *t* teachers.

3. 374 miles driven using 22 gallons of gasoline.
1,122 miles driven using *g* gallons of gasoline.

4. 21 bolts connect 3 panels.
b bolts connect 8 panels.

5. 32 pages for 2 sections of newspaper.
p pages for 5 sections of newspaper.

6. $2.49 for 3 bottles of water.
$8.30 for *w* bottles of water.

7. 3 girls for every 2 boys.
261 girls and *b* boys.

8. 8 packages in 2 cases.
p packages in 7 cases.

9. \$11.50 earned in one hour.
d earned in 6.5 hours.

10. **1.5 inches represents 10 feet.**
5 inches represents *x* feet.

11. 18 candy bars in 3 boxes.
900 candy bars in *x* boxes.

12. $\frac{1}{2}$ gallon of paint covers 112 square feet.
n gallons of paint covers 560 square feet.

APPLICATIONS ***Farmers often express their crop yield in bushels per acre. The table at the right shows Mr. Decker's average yields. Use this data to answer Exercises 13–16.***

Mr. Decker's Yield (Bushels per acre)	
Corn	98
Soybeans	48
Wheat	45

13. How many bushels of corn should Mr. Decker harvest from 80 acres?

14. How many bushels of wheat should Mr. Decker expect from 105 acres?

15. If Mr. Decker plants soybeans on 90 acres, how many bushels can he expect to harvest?

16. Ms. Holleran harvested 3,815 bushels of corn from 35 acres. Is this yield more or less than Mr. Decker's yield?

17. Ms. Galvez paid \$150 for 600 square feet of roofing. If she needs 240 square feet more, what is the extra cost?

18. A picture measuring 25 centimeters long is enlarged on a copying machine to 30 centimeters long. If the width of the original picture is 15 centimeters, what is the width of the enlarged copy?

SKILL 50

Name ______________________________ Date ____________

Ratios and Rates

A **ratio** is a comparison of two numbers by division. A ratio can be written in several different ways. If there are 5 roses in a bouquet of 12 flowers, then the ratio of roses to total number of flowers in the bouquet can be written as 5 to 12, 5:12, or $\frac{5}{12}$.

EXAMPLE ***Express the ratio 8 dimes out of 28 coins as a fraction in simplest form.***

$\frac{8}{28} = \frac{2}{7}$

The ratio of dimes to coins is 2 to 7. This means that for every 7 coins, 2 of them are dimes.

A **rate** is a ratio of two measurements having different kinds of units, such as $25 for 2 dozen. When a rate is simplified so that it has a denominator of 1, it is called a **unit rate**.

EXAMPLE ***Express the ratio 252 miles in 4 hours as a unit rate.***

$\frac{252 \text{ miles}}{4 \text{ hours}} = \frac{63 \text{ miles}}{1 \text{ hour}}$

The unit rate is 63 miles per hour.

EXERCISES ***Express each ratio as a fraction in simplest form.***

1. 6 strawberries out of 14 pieces of fruit

2. 15 girls to 18 boys

3. 12 blue marbles to 18 green marbles

4. 21 red blocks out of 96 blocks

5. 14 ounces to 35 pounds

6. 15 puppies to 60 kittens

Express each ratio as a unit rate. Round to the nearest tenth, if necessary.

7. \$10 for 5 loaves of bread

8. 64 feet in 16 seconds

9. 132 miles on 6 gallons

10. \$32 for 5 books

11. 140 meters in 48 seconds

12. 1,400 miles in 4 days

13. \$66 for 4 shirts

14. 350 words in 8 minutes

APPLICATIONS

15. The table below shows the size, in ounces, and the cost of several brands of apple juice. Find the unit cost to determine which brand is the best buy.

Brand	Size (ounces)	Cost
Sweeties Apple Juice	16	\$1.89
Sunshine Apple Juice	32	\$3.49
Peter's Apple Juice	64	\$5.09

16. A runner training for a marathon ran 18 miles in 150 minutes. Find the length of time it takes the runner to cover 1 mile. Round to the nearest tenth.

17. Alyssa spent \$780 on 40 square yards of carpeting for her family room. Find the cost per square yard for the carpet Alyssa selected.

18. During a winter snow storm, a total of 14 inches of snow fell over a period of 8 hours. Find the rate of snowfall per hour. Round to the nearest tenth.

SKILL 51

Name ______________________ **Date** __________

Organizing Information

Two possible ways to organize information to solve problems are listed below.

- Make a List
- Use a Matrix or Table

EXAMPLE

On Saturday, Omar plans to go to the library, the discount store, and his grandmother's house. He cannot decide in which order to go to these locations. How many choices does he have?

Make a list to show the different orders.

library, discount store, grandmother's house
library, grandmother's house, discount store
discount store, library, grandmother's house
discount store, grandmother's house, library
grandmother's house, library, discount store
grandmother's house, discount store, library

Omar has 6 choices for the order he can go to the locations.

EXAMPLE

Kimi is offered two jobs. Job A has a starting salary of $24,000 per year with a guaranteed raise of $1,200 per year. Job B has a starting salary of $26,000 with a guaranteed raise of $800 per year. In how many years will both jobs pay the same amount of money?

Make a table to show the effect of each option over a 6-year period.

Year	Job A	Job B
1	$24,000	$26,000
2	$25,200	$26,800
3	$26,400	$27,600
4	$27,600	$28,400
5	$28,800	$29,200
6	$30,000	$30,000

In 6 years, the two jobs will pay the same amount of money.

EXERCISES *Solve.*

1. How many different four-digit numbers can be formed using each of the digits 1, 2, 3, and 4 once?
2. How many different two-digit numbers can be formed using 1, 2, 3, 4, and 5 if the digits can be used more than once?
3. How many ways can you give a clerk 65¢ using quarters, dimes, and/or nickels?

APPLICATIONS

4. Rebecca, Lisa, and Courtney each have one pet. The pets are a dog, a cat, and a parrot. Courtney is allergic to cats. Rebecca's pet has two legs. Whose pet is the dog?
5. Mr. Ramos is starting a college fund for his daughter. He starts out with $700. Each month he adds $80 to the fund. How much money will he have in a year?
6. The Centerville Civic Association is selling pizzas. They can add pepperoni, green peppers, and/or mushrooms to their basic cheese pizzas. How many different kinds of pizzas can they sell?
7. Kyle, Gabrielle, Spencer, and Stephanie each play a sport. The sports are basketball, gymnastics, soccer, and tennis. No one's sport starts with the same letter as his or her name. Gabrielle and the soccer player live next door to each other. Stephanie practices on the balance beam each day. Gabrielle does not own a racket. Which student plays each sport?
8. A deli sells 5 different soft drinks in 3 different sizes. How many options does a customer have to buy a soft drink?
9. In the World Series, two teams play each other until one team wins 4 games.
   - **a.** What is the greatest number of games needed to determine a winner?
   - **b.** What is the least number of games needed to determine a winner?

Name ______________________ Date __________

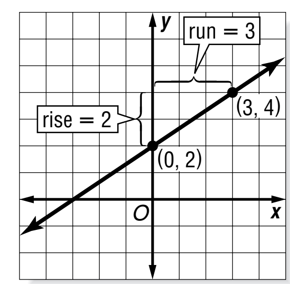
Slope of a Line

Slope describes the steepness of a line. The slope of a line can be expressed as a ratio of the **rise**, vertical change, to the **run**, horizontal change.

$$\text{slope} = \frac{\text{rise}}{\text{run}} \begin{matrix}\leftarrow \\ \leftarrow\end{matrix} \frac{\text{vertical change}}{\text{horizantalchange}}$$

EXAMPLE ***Find the slope of the line.***

Choose two points on the line.
The points chosen at the right have coordinates (0, 2) and (3, 4).



Draw a vertical line and a horizontal line to connect the points.

Find the length of the vertical segment to find the rise. The rise is 2 units up.

Find the length of the horizontal segment to find the run. The run is 3 units to the right.

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{3}$

The slope m of a line passing through points at (x_1, y_1) and (x_2, y_2) can be found using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

EXAMPLE ***Find the slope of the line that passes through A(−5, −3) and B(10, −6).***

Let $A(-5, -3) = (x_1, y_1)$ and let $B(10, -6) = (x_2, y_2)$.

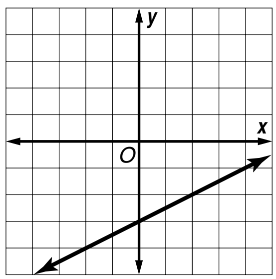
$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Definition of slope.*

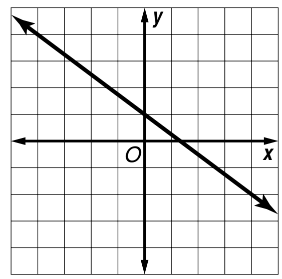
$= \frac{-6 - (-3)}{10 - (-5)}$ $x_1 = -5, y_1 = -3, x_2 = 10, y_2 = -6$

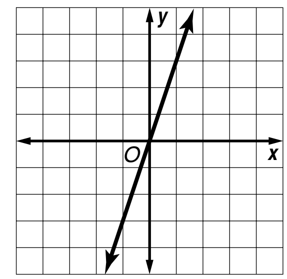
$= \frac{-3}{15}$ or $\frac{1}{-5}$ *Simplify.*

The slope is $\frac{1}{-5}$

EXERCISES *Find the slope of each line.*

1. 

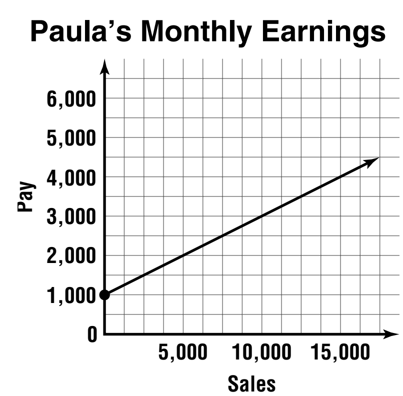
2. 

3. 

Find the slope of the line that passes through each pair of points.

4. $A(-2, -1)$, $B(3, 9)$
5. $C(0, -2)$, $D(3, -3)$
6. $E(-5, 20)$, $F(-8, 32)$
7. $G(-10, 2)$, $H(10, 8)$
8. $J(2, -1)$, $K(6, -11)$
9. $M(-3, -14)$ $N(-9, -30)$

APPLICATIONS ***Paula works as a sales representative for a computer store. She earns a base pay of $1,000 each month. She also earns a commission based on her sales. The graph at the right shows her possible monthly earnings. Use the graph to answer Exercises 10–13.***



10. What is the slope of the line?

11. What information is given by the slope of the line?

12. **a.** If Paula's base pay changed to $1,100, would it change the graph? Why or why not?

    b. would it change the slope? Why or why not?

13. If Paula's rate of commission changed to 25%, would it change the graph? Why or why not?

Name ______________________________ Date ____________

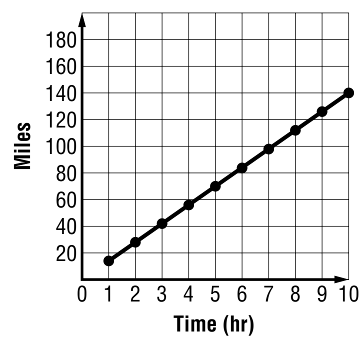
Graphing Functions

A function table can be used to graph a function.

EXAMPLE ***Chun is riding his bike at an average rate of 14 miles per hour. The function table at the right shows this relationship. Graph the function.***

Time (Hours)	Miles
1	14
2	28
3	42
4	56
5	70
6	84
7	98
8	112
9	126
10	140

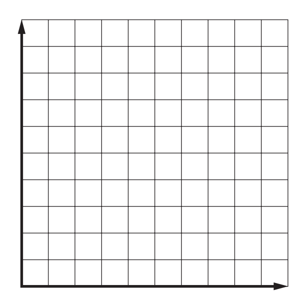
To graph the function, first label the axes and graph the points named by the data. Then connect the points.



EXERCISES ***Graph each function.***

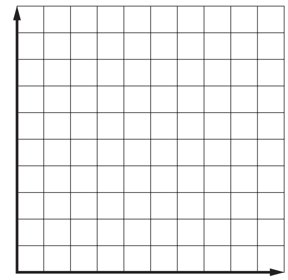
1.

Time (min)	Temperature (°C)
0	2
1	5
2	8
3	11
4	14
5	17
6	20
7	23
8	26
9	29
10	32



2.

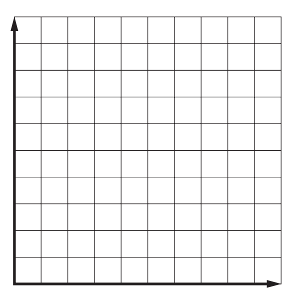
Radius (in.)	Area (sq in.)
1	3.14
2	12.57
3	28.27
4	50.27
5	78.54
6	113.10
7	153.94
8	201.06



APPLICATIONS ***The function table at the right shows the height of a golf ball above the ground after it is hit from ground level. Use the data to answer Exercises 3–6.***

Time (s)	Height (m)
0	0
0.25	4.0
0.5	7.5
0.75	10.25
1.0	12.5
1.25	14.0
1.5	15.0
1.75	15.25
2.0	15.0
2.25	14.0
2.5	12.5

3. Graph the function.



4. If the pattern continues, how high above the ground would you expect the golf ball to be after 3.25 seconds?

5. Where does the change in the function occur? Why do you think this change occurs?

6. How long will it take for the ball to hit the ground?

SKILL 54

Name ______________________________ Date ______________

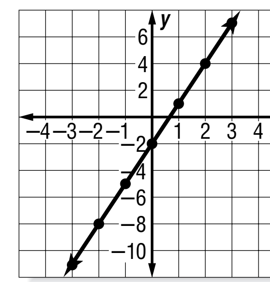
Graphing Linear Equations

Linear equations can be graphed in the same way that you graph functions.

EXAMPLE ***Graph the equation $y = 3x - 2$.***

Make a function table for $y = 3x - 2$. Then graph each ordered pair and complete the graph

$y = 3x - 2$.

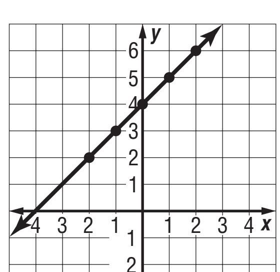


x	$3x - 2$	y	(x, y)
−3	3(−3) − 2	−11	(−3, −11)
−2	3(−2) − 2	−8	(−2, −8)
−1	3(−1) − 2	−5	(−1, −5)
0	3(0) − 2	−2	(0, −2)
1	3(1) − 2	1	(1,1)
2	3(2) − 2	4	(2,4)
3	3(3) − 2	7	(3,7)

EXERCISES ***Complete each function table. Then graph the equation.***

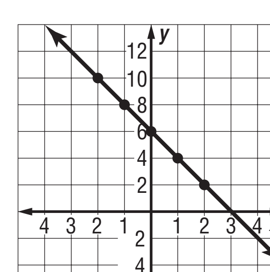
1. $y = x + 4$

x	$x + 4$	y	(x, y)
−2			
−1			
0			
1			
2			



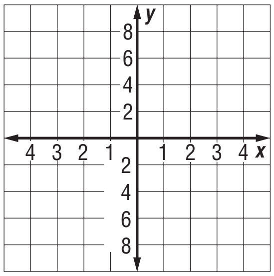
2. $y = 6 - 2x$

x	$6 - 2x$	y	(x, y)
−2			
−1			
0			
1			
2			

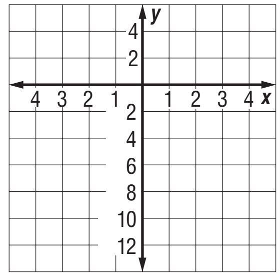


Graph each equation.

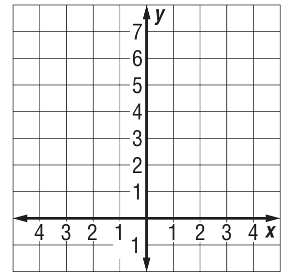
3. $y = -2x$



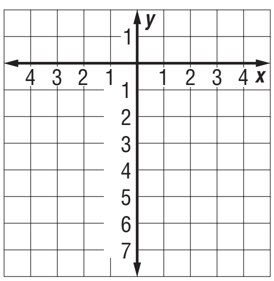
4. $y = 3x - 7$



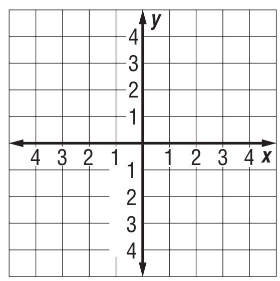
5. $y = -x + 6$



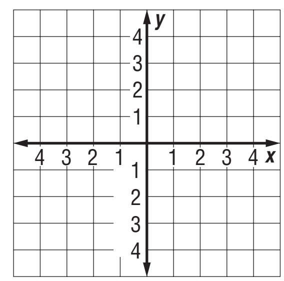
6. $y = \frac{1}{2}x - 5$



7. $y = -\frac{2}{3}x + 2$

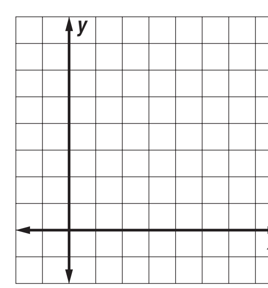


8. $y = \frac{4}{3}x + 1$

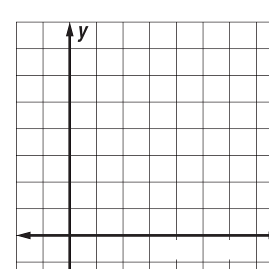


APPLICATIONS

9. A snow storm at Pine Tree Ski Resort deposited $\frac{1}{2}$ foot of snow per hour on top of a 3-foot snow base. Let x represent the number of hours and y represent the total amount of snow. Write an equation to represent the total amount of snow. Graph the equation.



10. Alaqua averages 40 miles per hour when she drives from Los Angeles to San Francisco. Let x represent the number of hours and y represent the distance traveled. Write an equation to represent the distance traveled. Graph the equation.



SKILL 55

Name ______________________ Date __________

Solve Equations in Two Variables

A **linear equation in two variables** is an equation in which the variables appear in separate terms and neither variable contains an exponent other than 1. **Solutions** of a linear equation in two variables are ordered pairs, (x, y) that make the equation true.

EXAMPLE ***Find four solutions of $y = -3x + 2$. Write the solutions as ordered pairs.***

Choose four values of x. Then substitute each value into the equation and solve for y.

x	$y = -3x + 2$	y	(x, y)
-1	$y = -3(-1) + 2$	5	$(-1, 5)$
0	$y = -3(0) + 2$	2	$(0, 2)$
1	$y = -3(1) + 2$	-1	$(1, -1)$
2	$y = -3(2) + 2$	-4	$(2, -4)$

Four solutions are $(-1, 5)$, $(0, 2)$, $(1, -1)$, and $(2, -4)$.

EXERCISES ***Find four solutions of each equation. Write the solutions as ordered pairs.***

1. $y = x - 3$

2. $y = 2x$

3. $y = 5 - x$

4. $y = 4x - 3$

5. $y = -2x + 4$

6. $y = -x$

7. $x + y = 5$

8. $2x + y = 9$

9. $y = -4$

10. $x = 3$

APPLICATIONS

11. The equation $y = 3x$ describes the number of eggs (y) required to make x batches of brownies. Find the number of eggs required to make 1, 2, 3, and 4 batches of brownies. Express your answers as ordered pairs.

12. The equation $y = 3x - 1$ describes the number of employees needed at a restaurant for every 10 customers (x). Find the number of employees required for 10, 20, 30, and 40 customers. Express your answers as ordered pairs.

13. The equation $y = 4x + 9$ describes the expenses incurred by a pizza shop (y) when x pizzas are made. Find the expense for making 4, 5, 6, and 7 pizzas. Express your answers as ordered pairs.

SKILL 56

Name ______________________ Date __________

Solve Equations Involving Addition and Subtraction

To solve an equation means to find a value for the variable that makes the equation true. To solve an equation, you need to get the variable by itself.

Addition Property of Equality: If you add the same number to each side of an equation, the two sides remain equal.

EXAMPLE ***Solve $s - 46 = 12$.***

$$s - 46 = 12$$
$$s - 46 + 46 = 12 + 46 \quad \textit{Add 46 to each side.}$$
$$s = 58$$

Check:
$$s - 46 = 12$$
$$58 - 46 \stackrel{?}{=} 2 \quad \textit{Replace s with 58.}$$
$$12 = 12 \checkmark$$

The solution is 58.

Subtraction Property of Equality: If you subtract the same number from each side of an equation, the two sides remain equal.

EXAMPLE ***Solve $d + 22 = 60$.***

$$d + 22 = 60$$
$$d + 22 - 22 = 60 - 22 \quad \textit{Subtract 22 from each side.}$$
$$d = 38$$

Check:
$$d + 22 = 60$$
$$38 + 22 \stackrel{?}{=} 60 \quad \textit{Replace d with 38.}$$
$$60 = 60 \checkmark$$

The solution is 38.

EXERCISES

Solve each equation. Check your solution.

1. $a - 91 = 20$	**2.** $1.5 + b = 3$	**3.** $c - 3.5 = 1.25$
4. $d + 140 = 300$	**5.** $5.6 + e = 7$	**6.** $f - 65 = 21$
7. $g + 35 = 62$	**8.** $h - 12 = 52$	**9.** $j + 16 = 47$
10. $k - 12 = 13$	**11.** $16 = m + 9$	**12.** $n + 16 = 34$
13. $20 + p = 40$	**14.** $22 = q - 12$	**15.** $r - 75 = 156$
16. $15.6 + s = 52.1$	**17.** $312 = t - 64$	**18.** $u - 71 = 23$

APPLICATIONS

19. Alexis sold 170 tickets for her school play. She has 290 tickets remaining. How many tickets were available?

20. Hector owns 87 CDs and DVDs. If he has 41 CDs, how many DVDs does Hector own?

21. Brandon is saving to buy a new computer game that costs $49.98. He still needs to save $21.50. How much has Brandon saved so far?

22. There are 34 students in Ms. Kim's class. Twelve of the students wear braces. How many students do not wear braces?

23. Taylor is downloading files from the Internet. She has transferred 8 of the 18 files she has selected. How many files have yet to be transferred?

24. A recipe calls for $2\frac{1}{2}$ cups of flour. Terrence has $1\frac{1}{3}$ cups available. How much more flour does Terrence need?

SKILL 57

Name ______________________________ **Date** ____________

Solve Equations Involving Multiplication and Division

Division Property of Equality: If you divide each side of an equation by the same nonzero number, the two sides remain equal.

EXAMPLE ***Solve 14x = 84.***

$$14x = 84$$

$$\frac{14x}{14} = \frac{84}{14} \quad \textit{Divide each side by 14.}$$

$$x = 6$$

Check:

$$14x = 84$$

$$14 \times 6 \stackrel{?}{=} 84 \quad \textit{Replace x with 6.}$$

$$84 = 84 \checkmark$$

The solution is 6.

Multiplication Property of Equality: If you multiply each side of an equation by the same number, the two sides remain equal.

EXAMPLE ***Solve*** $15 = \frac{y}{7}$.

$$15 = \frac{y}{7}$$

$$7(15) = 7\left(\frac{y}{7}\right) \quad \textit{Multiply each side by 7.}$$

$$105 = y$$

Check:

$$15 = \frac{y}{7}$$

$$15 \stackrel{?}{=} \frac{105}{7} \quad \textit{Replace y with 105.}$$

$$15 = 15 \checkmark$$

The solution is 105.

EXERCISES ***Solve each equation. Check your solution.***

1. $99 = 3a$

2. $0.5b = 3$

3. $\frac{c}{6} = 12$

4. $4 = \frac{d}{22}$

5. $\frac{e}{0.3} = 150$

6. $5 = 4f$

7. $\frac{g}{12} = 16$

8. $1.2h = 3.6$

9. $19 = \frac{j}{0.4}$

10. $\frac{k}{14} = 39$

11. $\frac{m}{5} = 16.4$

12. $8n = 9.6$

13. $1.2p = 2.76$

14. $72 = \frac{q}{1.8}$

15. $9r = 729$

16. $21s = 147$

17. $18t = 3.6$

18. $\frac{u}{17} = 3.4$

APPLICATIONS

19. City Center Parking Garage charges $0.75 an hour for parking. How long can Andrew park in the garage if he only has $6 for parking?

20. Elena is 5 times older than her youngest brother. Elena is 15 years old. How old is her brother?

21. Four friends split the cost of lunch equally. If each person pays $7.50, what is the total cost of lunch?

22. A bag of 20 oranges costs $6.99. What is the cost of each orange? Round to the nearest cent.

23. The area of a rectangle is 168 square centimeters. If the length of the rectangle is 12 centimeters, what is the measure of the width?

SKILL 58

Name ______________________ Date ____________

Solve Two-Step Equations

To solve two-step equations, you need to add or subtract first. Then you need to multiply or divide.

EXAMPLES ***Solve each equation.***

$$6x - 3 = 21$$

$$6x - 3 + 3 = 21 + 3 \qquad \textit{Add 3 to each side.}$$

$$6x = 24$$

$$\frac{6x}{6} = \frac{24}{6} \qquad \textit{Divide each side by 6.}$$

$$x = 4$$

The solution is 4.

$$\frac{y}{10} + 2.5 = 7.5$$

$$\frac{y}{10} + 2.5 - 2.5 = 7.5 - 2.5 \qquad \textit{Subtract 2.5 from each side.}$$

$$\frac{y}{10} = 5$$

$$10\left(\frac{y}{10}\right) = 10(5) \qquad \textit{Multiply each side by 10.}$$

$$y = 50$$

The solution is 50.

EXERCISES ***Solve each equation. Check your solution.***

1. $2a + 7 = 15$

2. $\frac{b}{7} + 10 = 40$

3. $8 - 1.2c = 2$

4. $\frac{d}{7} - 13 = 12$

5. $6e - 12 = 72$

6. $7f + 8.4 = 16.8$

7. $\frac{g}{2} + 11 = 16$

8. $\frac{h}{0.2} + 0.5 = 10$

9. $8 + 5j = 53$

10. $50 - 3k = 35$

11. $\frac{m}{3} - 5 = 2$

12. $6n + 4 = 58$

13. $\frac{p}{4} - 2 = 0.8$

14. $7q - 9.4 = 11.6$

15. $4 = \frac{r}{5} - 16$

16. $15 + \frac{s}{8} = 27$

17. $8t - 4.6 = 68.2$

18. $0.93 = 0.15 + 0.4u$

APPLICATIONS

19. Austin's doctor recommended that he take 4 doses of antibiotics the first day and two doses per day until all the medicine was gone. If the prescription was for 24 doses, how many days did Austin take the medicine?

20. A carpet store has carpet for $13.99 per square yard and charges $50 for installation. If a customer paid $364.78, approximately how many square yards of carpet were purchased?

21. To convert a temperature in degrees Celsius to degrees Fahrenheit you can use the formula $F = \frac{9}{5}C + 32$. If the outside temperature is 63°F, what is the temperature in degrees Celsius? Round to the nearest whole degree.

22. A wireless phone company charges $34.99 a month for phone service. They also charge $0.48 per minute for long distance calls. If Vanessa's bill at the end of the billing period is $64.75, how many minutes of long distance calls did she make?

SKILL 59

Name ______________________ Date __________

Solve Inequalities

Inequalities are sentences that compare two quantities that are not necessarily equal. The symbols below are used in inequalities.

Symbol	Words
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

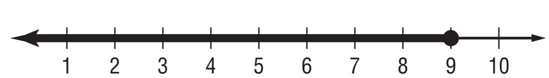
EXAMPLES ***Solve each inequality. Show the solution on a number line.***

$$\begin{aligned} 2n + 1 &> 5 \\ 2n + 1 - 1 &> 5 - 1 && \textit{Subtract 1 from each side.} \\ 2n &> 4 \\ \frac{2n}{2} &> \frac{4}{2} && \textit{Divide each side by 2.} \\ n &> 2 \end{aligned}$$

To graph the solution on a number line, draw an open circle at 2. Then draw an arrow to show all numbers greater than 2.

$$\begin{aligned} 2p - 3 &\leq 15 \\ 2p - 3 + 3 &\leq 15 + 3 && \textit{Add 3 to each side.} \\ 2p &\leq 18 \\ \frac{2p}{2} &\leq \frac{18}{2} && \textit{Divide each side by 2.} \\ n &\leq 9 \end{aligned}$$

To graph the solution on a number line, draw a closed circle at 9. Then draw an arrow to show all numbers less than 9.



EXERCISES ***Solve each inequality. Graph the solution on a number line.***

1. $a + 7 < 12$

2. $b -$

3. $2c - 7 \geq 9$

4. $5d + 7 \leq 32$

5. $e + 2 > 16$

6. $f + 12 < 18$

7. $\frac{g}{2} \geq 3$

8. $\frac{h}{2} + 6 < 8$

9. $\frac{j}{3} + 6 \leq 10$

10. $\frac{k}{4} + 2 > 3$

APPLICATIONS

19. Madison wants to earn at least $75 to spend at the mall this weekend. Her father said he would pay her $15 to mow the lawn and $5 an hour to work on the landscaping. If Madison mows the lawn, how many hours must she work on the landscaping to earn at least $75?

20. A rental car agency rents cars for $32 per day. They also charge $0.15 per mile driven. If you are taking a 5-day trip and have budgeted $250 for the rental car, what is the maximum number of miles you can drive and stay within your budget?

21. Mr. Stamos needs 1,037 valid signatures on a petition to become a candidate for the school board election. An official at the board of elections told him to expect that 15% of the signatures he collects will be invalid. What is the minimum number of signatures he should get to help ensure that he qualifies for the ballot?

SKILL 60

Name ______________________ Date __________

Scale Drawings

A **scale drawing** is used to represent an object that is too large to be drawn or built at actual size.

EXAMPLE

Carlos is drawing plans for a new shopping center. The scale of the drawing is $\frac{1}{2}$ inch equals 5 feet. On the drawing, the front of the shopping center is $18\frac{1}{2}$ inches. What is the actual length of the front of the shopping center?

Express $\frac{1}{2}$ inch as 0.5 inch and $18\frac{1}{2}$ inches as 18.5 inches. Use the scale 0.5 inch = 5 feet to write a proportion.

$$\frac{\text{drawing}}{\text{actual length}} \begin{matrix}\rightarrow\\\rightarrow\end{matrix} \frac{0.5}{5} = \frac{18.5}{x} \begin{matrix}\leftarrow\\\leftarrow\end{matrix} \frac{\text{drawing}}{\text{actual length}}$$

$0.5x = (5)(18.5)$ *Cross multiply.*

$0.5x = 92.5$ *Simplify.*

$\frac{0.5x}{0.5} = \frac{92.5}{0.5}$ *Divide each side by 0.5.*

$x = 185$ *Simplify.*

The actual length of the front of the shopping center is 185 feet.

EXERCISES ***On a map, the scale is 1 inch equals 40 miles. For each map distance, find the actual distance.***

1. $2\frac{1}{2}$ inches **2.** 12 inches **3.** $\frac{3}{4}$ inch

4. $7\frac{1}{4}$ inches **5.** $8\frac{1}{2}$ inches **6.** $4\frac{3}{8}$ inches

On a blueprint of a new house, the scale is $\frac{1}{4}$ inch equals 2 feet. Find the dimensions of the rooms on the blueprint if the actual measurements of the rooms are given.

7. 20 feet by $16\frac{3}{4}$ feet

8. 17 feet by $12\frac{3}{4}$ feet

9. $11\frac{1}{2}$ feet by $10\frac{1}{4}$ feet

10. 11 feet by $9\frac{1}{2}$ feet

11. 19 feet by 14 feet

12. $10\frac{3}{4}$ feet by $11\frac{1}{4}$feet

APPLICATIONS

An igloo is a domed structure built of snow blocks by Eskimos. Sometimes several families built a cluster of igloos connected by passageways. Use the scale drawing of such a cluster to answer Exercises 13–17.

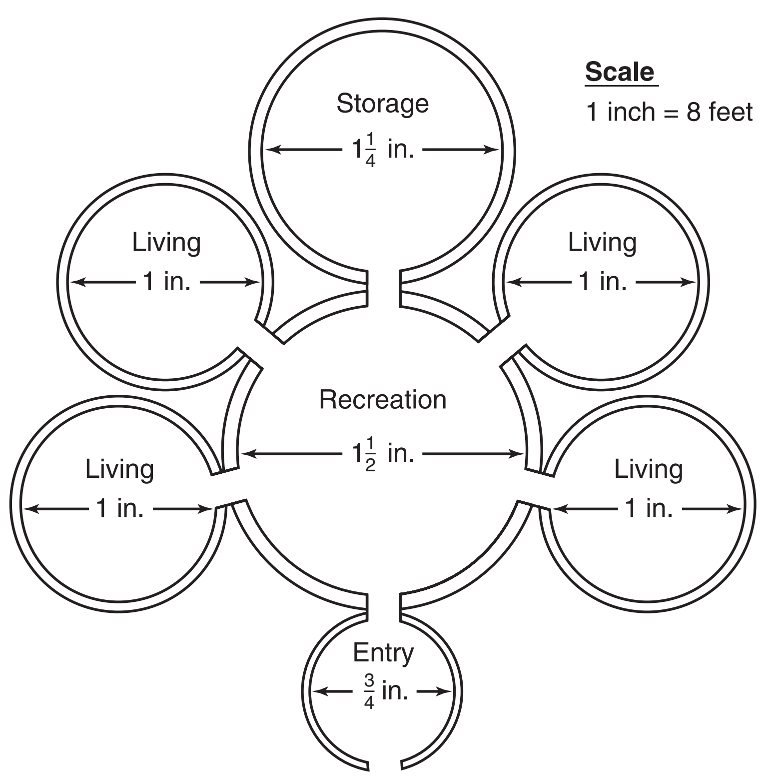
13. What is the actual diameter of each living chamber?

14. What is the actual diameter of the entry chamber?

15. What is the actual diameter of the recreation area?

16. What is the actual diameter of the storage area?

17. Estimate the actual distance from the entry chamber to the back ofthe storage chamber.



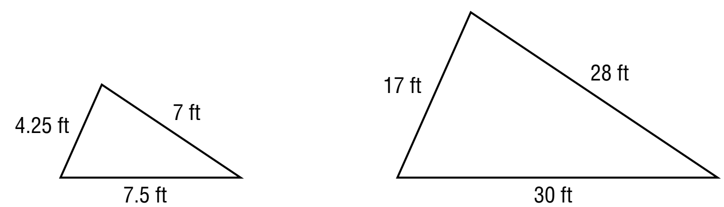
SKILL 61

Name ______________________ Date ____________

Similar Figures

Figures that have the same shape but not necessarily the same size are similar. You can use ratios to determine whether two figures are similar.

EXAMPLE ***Determine if the triangles are similar.***



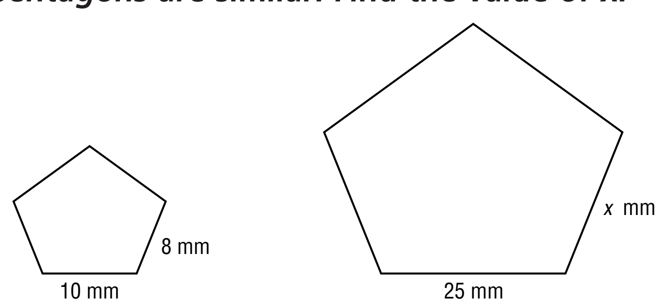
Write ratios comparing the sides of one triangle to the corresponding sides of the other triangle.

$\dfrac{\text{side measure of first triangle}}{\text{side measure of second triangle}}$ $\quad \dfrac{4.25}{17} = \dfrac{1}{4} \quad \dfrac{7}{28} = \dfrac{1}{4} \quad \dfrac{7.5}{30} = \dfrac{1}{4}$

The ratios of the corresponding sides all equal $\frac{1}{4}$. Therefore, the triangles are similar.

Proportions can be used to determine the measures of the sides of similar figures.

EXAMPLE ***The pentagons are similar. Find the value of x.***

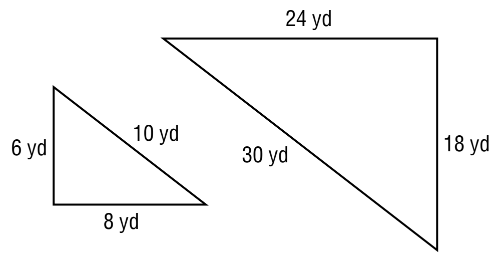


$\dfrac{10}{25} = \dfrac{8}{x}$		*Write a proportion.*
$(10)(x) = (25)(8)$		*Cross multiply.*
$10x = 200$		*Simplify.*
$\dfrac{10x}{10} = \dfrac{200}{10}$		*Divide each side by 10.*
$x = 20$		*Simplify.*

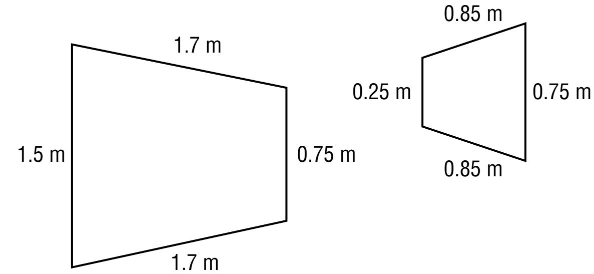
EXERCISES

Determine if each pair of figures is similar.

1.

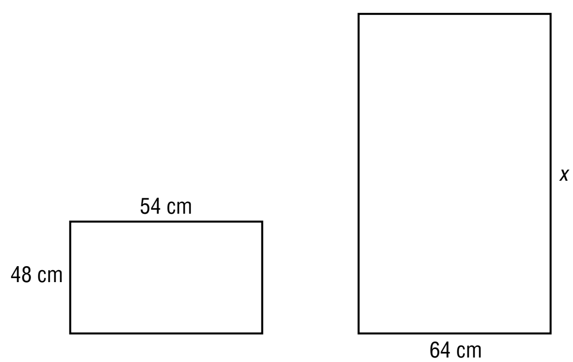


2.

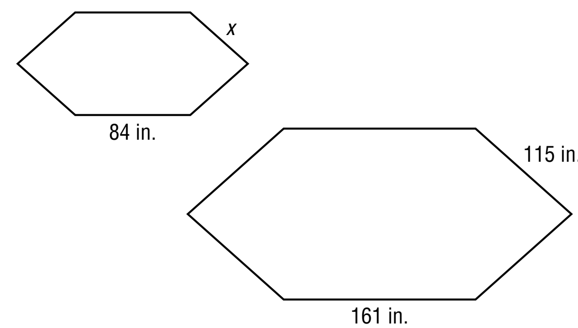


Find the value of x in each pair of similar figures.

3.

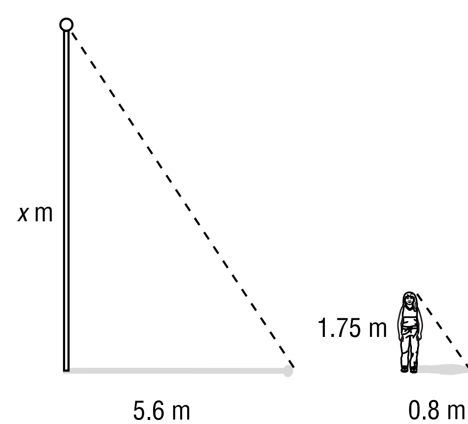


4.

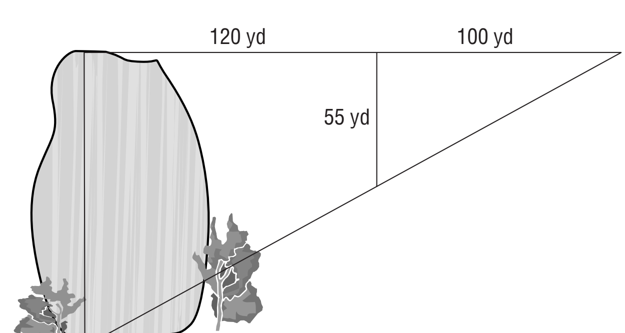


APPLICATIONS

5. A flagpole casts a shadow 5.6 meters long. Isabel is 1.75 meters tall and casts a shadow 0.8 meter long. How tall is the flagpole?



6. Will and Kayla want to know how far it is across a pond. They made the sketch at the right. How far is it across the pond?



SKILL 62

Name ______________________________ Date ____________

Percents as Fractions and Decimals

To write a percent as a fraction, write a fraction with the percent in the numerator and with a denominator of 100, $\frac{r}{100}$. Then write the fraction in simplest form.

EXAMPLES ***Express each percent as a fraction.***

a. 40%

$$40\% = \frac{40}{100}$$

$$= \frac{2}{5}$$

Therefore, $40\% = \frac{2}{5}$.

b. $87\frac{1}{2}\%$

$$87\frac{1}{2}\% = \frac{87\frac{1}{2}}{100}$$

$$= \frac{\frac{175}{2}}{100}$$

$$= \frac{175}{2} \times \frac{1}{100}$$

$$= \frac{175}{200}$$

$$= \frac{7}{8}$$

Therefore, $87\frac{1}{2}\% = \frac{7}{8}$.

To express a percent as a decimal, first express the percent as a fraction with a denominator of 100. Then express the fraction as a decimal.

EXAMPLES ***Express each percent as a decimal.***

a. 51%

$$51\% = \frac{51}{100}$$

$$= 0.51$$

Therefore, $51\% = 0.51$.

b. 90.2%

$$90.2\% = \frac{90.2}{100}$$

$$= \frac{90.2 \times 10}{100 \times 10}$$

$$= \frac{902}{1{,}000}$$

$$= 0.902$$

Therefore, $90.2\% = 0.902$.

EXERCISES ***Express each percent as a fraction.***

1. 75% **2.** 84% **3.** 90% **4.** $18\frac{1}{2}$

5. 38% **6.** $33\frac{1}{3}$% **7.** 56% **8.** 60%

Express each percent as a decimal.

9. 82% **10.** 61.5% **11.** 8.9% **12.** $48\frac{1}{2}$%

13. **70%** **14.** $27\frac{1}{4}$% **15.** 3% **16.** 0.25%

Write each percent as a fraction in simplest form and write as a decimal.

17. 18% **18.** 22%

19. $82\frac{1}{2}$% **20.** $\frac{5}{8}$%

21. $91\frac{2}{3}$ % **22.** 19.6%

23. 0.5625% **24.** **4.9%**

APPLICATIONS

25. The average household in the United States spends 15% of its money on food. Express 15% as a decimal.

26. Bananas grow on plants that can be 30 feet tall. A single banana may be 75% water. Express 75% as a fraction and as a decimal.

27. In the United States, showers usually account for 32% of home water use. Express this percent as a fraction and as a decimal.

28. Only 2% of earthquakes in the world occur in the United States. Express this percent as a fraction and as a decimal.

SKILL 63

Name ______________________________ Date ____________

Percent of a Number

To find the percent of a number, you can either change the percent to a fraction and then multiply, or change the percent to a decimal and then multiply.

EXAMPLE ***Yankee Stadium in New York has a capacity of about 57,500. If attendance for one baseball game was about 90%, approximately how many people attended the game?***

Change the percent to a decimal.

$90\% = \frac{90}{100}$ or 0.9

Multiply the number by the decimal.

$57{,}500 \times 0.9 = 51{,}750$

About 51,750 people attended the game.

EXERCISES ***Find the percent of each number.***

1. 50% of 48

2. 25% of 164

3. 70% of 90

4. 60% of 125

5. 55% of 960

6. 35% of 600

7. 15% of 120

8. 6% of 50

9. 200% of 13

10. 55% of 84

11. 16% of 48

12. 150% of 60

13. 45% of 80

14. 60% of 40

15. 18% of 300

16. 5% of 16

17. 15% of 50

18. 100% of 47

21. 0.5% of 180

22. 0.1% of 770

23. 1.4% of 40

24. 1.05% of 62

25. $12\frac{1}{2}$% of 70

26. $5\frac{3}{8}$% of 200

27. $2\frac{1}{4}$% of 150

28. $33\frac{1}{3}$% of 45

APPLICATIONS ***Sarah has a part-time job. Each week she budgets her money as shown in the table. Use this data to answer Exercises 29–31.***

Sarah's Budget	
Savings	40%
Lunches	25%
Entertainment	15%
Clothes	20%

29. If Sarah made \$90 last week, how much can she plan to spend on entertainment?

30. If Sarah made \$105 last week, how much should she plan to save?

31. If Sarah made \$85 last week, how much can she plan to spend on lunches?

32. The population of the U.S. was about 290 million people in 2004. The population of the New York Metropolitan area was about 7.3% of the total. About how many people lived in the New York area in 2004?

33. Ninety percent of the seats of a flight are filled. There are 240 seats. How many seats are filled?

34. Of the people Joaquin surveyed, 60% had eaten lunch in a restaurant in the past week. If Joaquin surveyed 150 people, how many had eaten lunch in a restaurant in the past week?

35. A car that normally sells for \$25,900 is on sale for 84.5% of the usual price. What is the sale price of the car?

SKILL 64

Name ______________________ **Date** __________

Percent Proportion

You can use the percent proportion to solve problems involving percents.

$\frac{a}{b} = \frac{p}{100}$ a = part b = base p = percent

EXAMPLES

23.4 is what percent of 65?

The part is 23.4 and the base is 65.

$\frac{a}{b} = \frac{p}{100}$

$\frac{23.4}{65} = \frac{p}{100}$

$23.4 \cdot 100 = 65 \cdot p$

$2{,}340 = 65p$

$36 = p$

23.4 is 36% of 65.

55% of what number is 33?

The part is 33 and the percent is 55% or $\frac{55}{100}$.

$\frac{a}{b} = \frac{p}{100}$

$\frac{33}{b} = \frac{55}{100}$

$33 \cdot 100 = 55 \cdot b$

$3{,}300 = 55b$

$60 = b$

55% of 60 is 33.

EXERCISES ***Tell whether each number is the part, base, or percent.***

1. What number is 25% of 20?

2. What percent of 10 is 5?

3. 14% of what number is 63?

4. 7 is what percent of 28?

5. 78% of what number is 50?

6. 72 is 24% of what number?

Write a proportion for each problem. Then solve. Round answers to the nearest tenth.

7. What percent of 25 is 5?

8. 9.3% of what number is 63?

9. 30% of what number is 27?

10. 126 is 39% of what number?

11. 61.6 is what percent of 550?

12. 108 is 18% of what number?

13. What percent of 84 is 20?

14. What percent of 400 is 164?

15. 29.7 is 55% of what number?

16. 18% of 350 is what number?

17. 61.5 is what percent of 600?

18. 72.4 is 23% of what number?

19. What number is 31% of 13?

20. $33\frac{1}{3}$% of what number is 15?

21. Use a proportion to find $12\frac{2}{3}$% of 462. Round to the nearest hundredth.

22. Use a proportion to determine what percent of 512 is 56. Round to the nearest hundredth.

23. Use a proportion to determine 23% of what number is 81.3. Round to the nearest hundredth.

APPLICATIONS

24. There are 18 girls and 15 boys in Tyler's homeroom. What percent of Tyler's homeroom are boys? Round to the nearest tenth.

25. If 32% of the 384 students in the eighth grade walk to school, about how many eighth graders walk to school?

26. At North Middle School, 53% of the students are girls. There are 927 students at the school. How many of the students are girls?

SKILL 65

Name ______________________ **Date** __________

Percent of Change

A **percent of change** tells the percent an amount has increased or decreased. When an amount increases, the percent of change is a **percent of increase.**

EXAMPLE

According to the U.S. Department of Labor, there were approximately 126,708,000 people employed in 1996. In 2002, there were about 136,485,000 people employed. Find the percent of increase in the number of people employed.

To find the percent of increase, you can follow these steps.

1. Subtract to find the amount of change.
new − original
136,485,000 − 126,708,000 = 9,777,000   *new* − original

2. Write a ratio that compares the amount of change to the original amount. Express the ratio as a percent.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

$$= \frac{9{,}777{,}000}{126{,}708{,}000} \quad \textit{Substitution}$$

$$\approx 0.0772$$

The number of people employed increased about 7.72%.

When the amount decreases, the percent of change is a **percent of decrease.** Percent of decrease can be found using the same steps.

EXAMPLE

A handheld computer that originally sells for $249 is on sale for $219. What is the percent of decrease of the price of the computer?
original price − new price

249 − 219 = 30

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

$$= \frac{30}{249} \quad \textit{Substitution}$$

$$\approx 0.12$$

The percent of decrease in the price of the handheld computer is about 12%.

EXERCISES ***Find the percent of change. Round to the nearest tenth.***

1. old: \$14.50
new: \$13.05

2. old: 237 students
new: 312 students

3. old: 27.4 inches of snow
new: 22.8 inches of snow

4. old: 12,000 cars per hours
new: 14,300 cars per hour

5. old: 2.3 million bushels
new: 3.1 million bushels

6. old: \$119.50
new: \$79.67

7. old: \$7,082
new: \$10,189

8. old: 37.5 hours
new: 42.0 hours

9. old: 74.8 million acres
new: 67.5 million acres

10. old: 5.7 liters
new: 4.8 liters

APPLICATIONS

11. At the beginning of the day, the stock market was at 10,120.8 points. At the end of the day, it was at 10,058.3 points. What was the percent of change in the stock market value?

12. An auto manufacturer suggests a selling price of \$32,450 for its sport coupe. The next year it suggests a selling price of \$33,700. What is the percent of change in the price of the car?

13. The U.S. Consumer Price Index in 1990 was 391.4. By 2000 the Consumer Price Index was 515.8. Find the percent of change.

14. During the past school year, there were 2,856 students at Main High School. The next year there were 3,042 students. What was the percent of change?

15. During a clearance sale, the price of a television is reduced from \$1,099 to \$899 the first week. The next week, the price of the television is lowered to \$739. What is the percent of change each week? What is the percent of change from the original price to the final price?

SKILL 66

Name ______________________ Date ____________

Unit Rate

EXAMPLE ***Mr. Lee's car burned 6 gallons of gas when he drove 120 miles. Ms. Mendoza drove her car 100 miles and used 4 gallons of gas. Which car gets more miles per gallon of gas?***

Miles per gallon is a *unit rate*. This unit rate means how many miles a car can drive using 1 gallon of gas.

To find the unit rate for each, set up a ratio.

miles driven/gallons of gas

Mr. Lee's Car	Ms. Mendoza's Car
120 miles/6 gallons	100 miles/4 gallons

Divide the numerator by the denominator to find how many miles the car can drive on 1 gallon of gas.

Mr. Lee's Car	Ms. Mendoza's Car
120 miles/6 gallons	100 miles/4 gallons
120 ÷ 6 = 20 miles/ gallon	100 ÷ 4 = 25 miles/gallon

Now you can compare the unit rates. Ms. Mendoza's car gets 25 miles per gallon, while Mr. Lee's car gets only 20 mile per gallon. So Ms. Mendoza's car gets more miles per gallons than Mr. Lee's.

EXERCISES ***Calculate a unit rate for each situation.***

1. 5 pounds of apples cost $7.25. How much do apples cost per pound?

2. 245 busses carried 8575 students to school. How many students were there per bus?

3. An airplane flew 1692 miles in 3 hours. What was the plane's speed in miles per hour?

4. T-shirts are on sale at 5 for $33. What is the unit rate per shirt?

EXERCISES ***Use unit rates to solve each problem.***

5. The SuperLaser printer prints 13 pages in 3 minutes. The PhotoFlash printer prints 26 pages in 5 minutes. Find the unit rate per page. Which printer prints faster?

6. At QuickShop, 6 cans of cat food cost $10. At Hopper's Grocery, cat food costs $7.50 for 4 cans. Find the price per can at each store. Which store gives you a better deal?

7. Jane walked 3 miles in 45 minutes. Alexis walked 5 miles in 1 hour and 40 minutes. Find the rate for each walker. Who walked faster?

8. SonicBoom is having a sale on CDs. Buy any 8 CDs for $46. What is the unit rate of each CD?

APPLICATIONS ***At Sheffield Farms, you can pick your own fruit. Strawberries cost $3/quart, raspberries cost $4.50/quart, and blueberries cost $2.50/quart. Mark picked 4 quarts of each kind of berry.***

9. Which cost more: 4 quarts of strawberries, or 4 quarts of raspberries?

10. How much did all 12 quarts cost together?

11. What was the average (mean) price per quart that Mark paid for his berries?

Mark mixed all the berries together and put them in the blender with milk and ice to make smoothies. Each quart of berries made 1.5 quarts of smoothie. He sold the smoothies at his town's Summer Fair. He wanted to make a profit, so he sold the smoothies for more than it cost to make them.

12. How much did it cost Mark to make 1 quart of smoothie?

13. What price should Mark charge for the smoothies in order to make a profit?

14. If Mark sells 3 quarts of smoothie for $7.35, will he make or lose money? Explain your reasoning.

SKILL 67

Name ______________________ Date __________

Using Rates to Convert Currencies

EXAMPLE

Karen lives in the U.S. and is planning a trip to Denmark. She used the Internet to look up the prices of Danish hotels. One hotel in Copenhagen listed its price as 1,095.00 krone per night. When Karen checked the exchange rate of U.S. dollars to Danish krone, it was:

$1 = 4.798 DKK.

If Karen books a room right away, what amount in U.S. dollars will be charged to her credit card?

To convert the price in krone to a price in dollars set up a proportion. The exchange rate between krone and dollars is the proportional relationship between the price in krone and the price in dollars. That is, it tells you how many dollars each krone in the hotel's price is worth.

$$\frac{\text{(number of krone in 1 dollar)}}{\text{(1 dollar)}} = \frac{\text{(price in krone)}}{\text{(price in dollars)}}$$

$$\frac{4.798 \text{ DKK}}{1 \text{ dollar}} = \frac{1{,}095 \text{ DKK}}{x \text{ dollars}}$$

Now, solve for x to determine the price of the hotel in dollars.

$$\begin{aligned} x \text{ dollars} \bullet 4.798 \text{ DKK} &= 1{,}095 \text{ DKK per dollar} \\ x \text{ dollars} &= \frac{1{,}095 \text{ DKK per dollar}}{4.798 \text{ DKK}} \\ x &= \frac{1{,}095}{4.798} \\ &= 228.22. \end{aligned}$$

So the hotel room costs $228.22 per night.

EXERCISES ***Use this table of exchange rates to solve the following problems. For each problem, convert the given price into the new currency.***

	1 U.S. Dollar	1 Brazilian Real	1 Chinese Yuan	1 Euro	1 Hong Kong Dollar
U.S. Dollar	1	0.60	0.14	1.55	0.13
Brazilian Real	1.66	1	0.24	2.57	0.21
Chinese Yuan	6.98	4.17	1	10.74	0.9
Euro	0.64	0.39	0.09	1	0.08
Hong Kong Dollar	7.79	4.65	1.12	11.98	1
Indian Rupee	40.8				
Thai Baht	31.67				

1. A shirt costs 450 rupees. What is the price in U.S. dollars?
2. A meal in a restaurant costs 20 euros. What is the price in yuan?
3. A train ticket costs 155 Hong Kong dollars. What is the price in real?
4. A pair of sneakers costs 280 yuan. What is the price in real?
5. A book costs 50 yuan. What is the price in U.S. dollars?
6. A haircut costs 30 U.S. dollars. What is the price in Hong Kong dollars?
7. A CD costs 25 real. What is the price in euros?
8. Washing and drying a load of laundry costs 3 U.S. dollars. What is the price in real?
9. A cab ride costs 42 real. What is the price in yuan?

APPLICATIONS ***The exchange table in the Exercises section is incomplete. It does not include columns to show how to convert from Rupees or Baht to other currencies.***

10. Explain how you could use the information you have to figure out how many U.S. dollars 1 Rupee is worth.