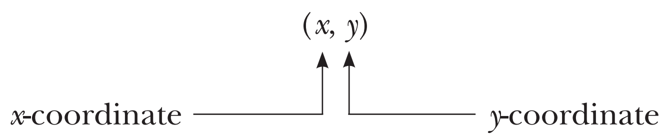
SKILL 1

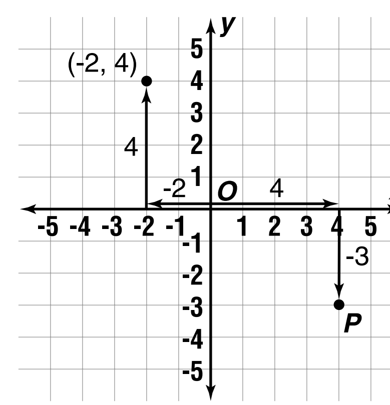
Name ______________________________ Date ____________

Ordered Pairs

A horizontal number line and a vertical number line meet at their zero points to form a **coordinate system**. The horizontal line is the ***x*-axis**. The vertical line is the ***y*-axis**. The location of a point in the coordinate system can be named using an ordered pair of numbers.



EXAMPLE ***Name the ordered pair for point P.***

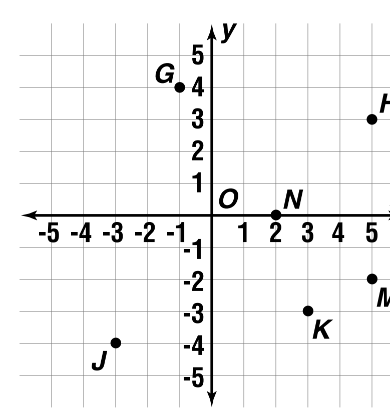


Start at *O*. Move along the *x*-axis until you are above point *P*. Then move down until you reach point *P*. Since you moved 4 units to the right and 3 units down, the ordered pair for point *P* is (4, −3).

Graph point (−2, 4).

Start at O. Move 2 units left on the *x*-axis. Then move 4 units up parallel to the *y*-axis to locate the point.

EXERCISES ***Name the ordered pair for each point.***



1. *G* **2.** *H*

3. *J* **4.** *K*

5. *M* **6.** *N*

Graph and label each point.

7. $A(-5, 5)$

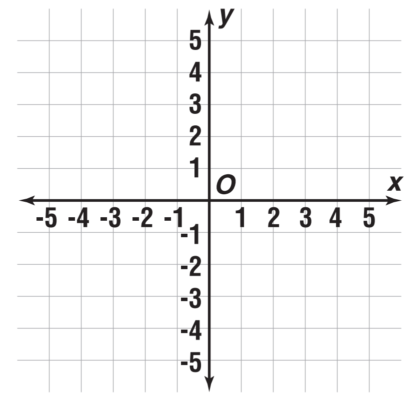
8. $B(2, 4)$

9. $C(0, 5)$

10. $D(-4, 0)$

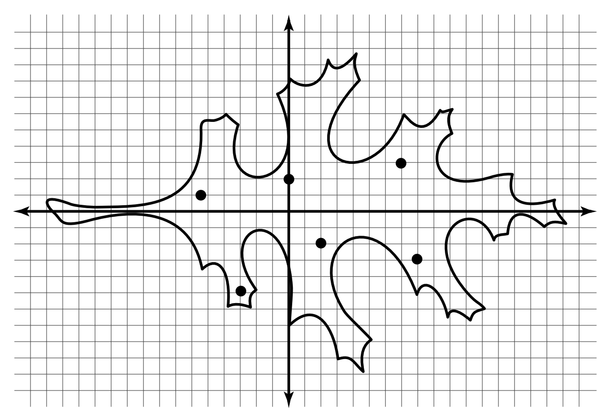
11. $E(2, 2)$

12. $F(4, -3)$



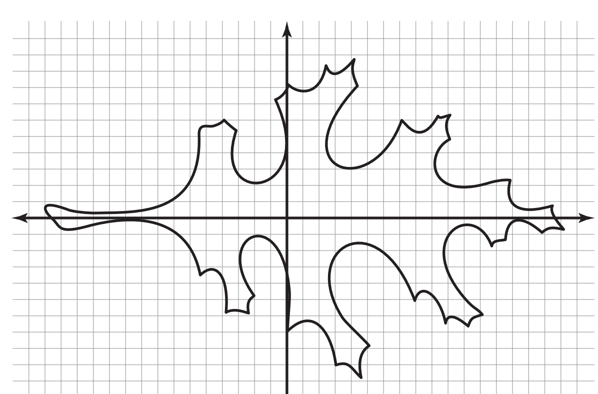
APPLICATIONS ***A botanist is interested in what part of a certain leaf is being infested by an insect that leaves black spots. She places a clear coordinate plane over several leaves that are about the same size and shape. Complete each of the following.***

13. Find the coordinates of the black spots on the leaf at the right.



14. Draw and label the spots having the following coordinates on the leaf at the right.

A(2, –3) *B*(3, –2) *C*(0, –4) *D*(–4, 0)
E(–5, 3) *F*(10, 2) *G*(2, 7) *H*(0, 5)



3. Norma's Repair Shop charges $35 for a service call and $25 an hour for each hour of labor. How much does she charge for an 8-hour service call?

APPLICATIONS

Jake and June Washington started a college fund for their daughter. They started the fund by depositing $800 at the beginning of the first month. They plan to add $75 to the fund at the end of every month. Use this information to answer Exercises 4–6.

4. How much will be in the account after

a. 1 month?

b. 6 months?

c. 1 year?

d. 2 years?

5. How can you extend your table from Exercise 4 to find out how much will be in the account after every year?

6. Suppose the Washingtons deposited $800 at the end of the first month and then $75 at the end of every month after that. How would this change your table?

7. Find out how much your long distance phone company charges for calls. How much would it cost you to make a 15-minute long distance phone call?

SKILL 3

Name ______________________ Date __________

Problem-Solving Strategies

There are many strategies that can be used to solve a problem. A few of these strategies are listed below.

- Draw a diagram
- Use a matrix or chart
- Make a list
- Use logical reasoning
- Draw a picture
- Guess and check

For each problem you solve, you must decide which strategy would work best for you.

EXAMPLE ***A 1-inch spool holds 100 inches of line, a 2-inch spool holds 400 inches of line, and a 3-inch spool holds 900 inches of line. How many inches of line are on a 5-inch spool?***

First make a chart.

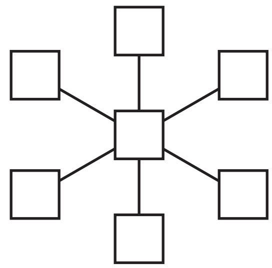
Spool Size	Inches of Line
1 in.	100
2 in.	400
3 in.	900

Study the chart. You know that $1^2 = 1$, $2^2 = 4$, and $3^2 = 9$. Using logical reasoning, a 5-inch spool holds $5^2 \times 100$ or 2,500 inches.

EXERCISES ***Solve using any strategy.***

1. Juan has a mixture of pennies and dimes worth $2.28. He has between 39 and 56 pennies. How many dimes does Juan have?

2. Arrange the digits 1 through 7 in the squares so that the sum along any line is 10.



3. There are three cubes each measuring a different whole number of inches on an edge. When the cubes are stacked, the stack is six inches high. What is the length of the edge of each cube?

APPLICATIONS

4. Mr. Patel asked five of his students to line up by height. Juan is *not* the shortest and is *not* standing next to Pamela. Tad is the tallest and is *not* standing next to Juan. Marco is taller than Pamela and Caroline is next to Tad. Who is standing in the middle?

5. If it takes 20 seconds to inflate a balloon with helium from a tank, how many balloons can be inflated in 6 minutes?

6. A vending machine dispenses products that each cost 60¢. It accepts quarters, dimes, and nickels only. If it only accepts exact change, how many different combinations of coins must the machine be programmed to accept?

7. The bus leaves the downtown for the mall at 7:35 A.M., 8:10 A.M., 8:45 A.M., and 9:20 A.M.. If the bus continues to run on this schedule, what time does the bus leave between 10:00 A.M. and 11:00 A.M.?

8. Bob needs to go to the bank, the post office, and the bicycle shop. In how many different orders can he do his errands?

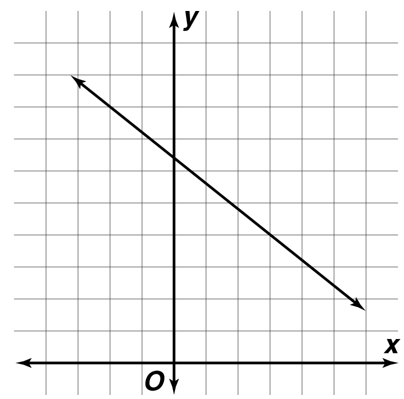
9. Ronda spent 22 minutes on the telephone talking long-distance to her cousin. If the rate is $0.20 for each of the first 3 minutes and $0.15 for each minute after that, how much did the call cost?

SKILL 4

Name ______________________ Date __________

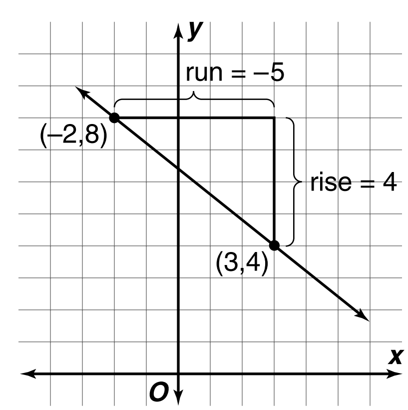
Slope of a Line

The graph of a line is shown below.



EXAMPLE

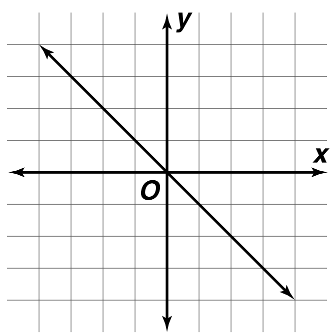
Find the slope of the line.
Follow these steps to find the slope.



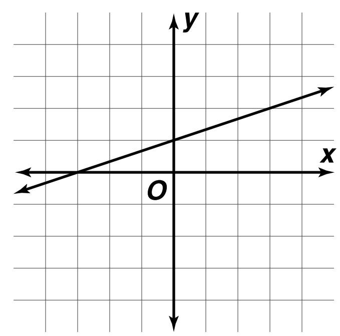
1. Choose any two points on the line. The points chosen at the right have coordinates (3, 4) and (–2, 8).
2. Draw a vertical line and then a horizontal line to connect the two points.
3. Find the length of the vertical line to find the rise. The rise is 4 units up or 4.
4. Find the length of the horizontal line to find the run. The run is 5 units to the left or –5.
5. slope $= \frac{\text{rise}}{\text{run}} = \frac{4}{-5}$

EXERCISES *Find the slope of each line shown.*

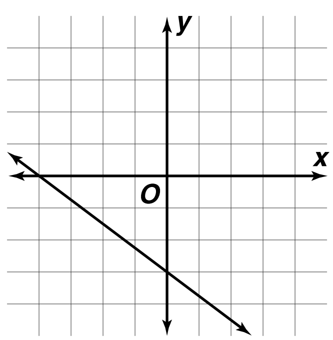
1.



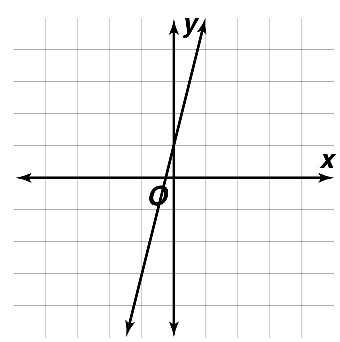
2.



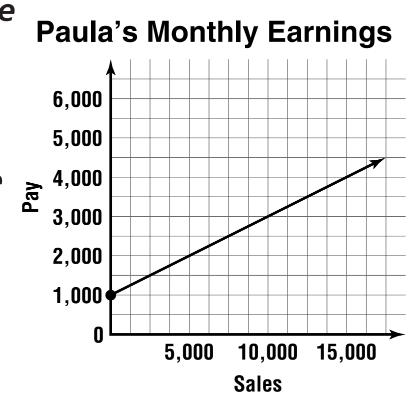
3.



4.



APPLICATIONS ***Paula works as a sales representative for a computer manufacturer. She earns a base pay of $1,000 each month. She also earns a commission based on her sales. The graph at the right shows her possible monthly earnings. Use the graph to answer Exercises 5–8.***



5. What is the slope of the line?

6. What information is given by the slope of the line?

7. If Paula's base pay changed to $1,100, would it change

a. the graph? Why or why not?

b. the slope? Why or why not?

8. If Paula's rate of commission changed to 25%, would it change the graph? Why or why not?

SKILL 5

Name ______________________ **Date** __________

Solve Two-Step Equations

To solve two-step equations, you need to add or subtract first. You also need to multiply or divide.

EXAMPLES ***Solve each equation.***

$$7v - 3 = 25$$

$$7v - 3 + 3 = 25 + 3 \quad \textit{Add 3 to each side.}$$

$$7v = 28$$

$$\frac{7v}{7} = \frac{28}{7} \quad \textit{Divide each side by 7.}$$

$$v = 4$$

The solution is 4.

$$\frac{1}{6}(r - 3) = -5$$

$$6 \times \frac{1}{6}(r - 3) = 6 \times -5 \quad \textit{Multiply each side by 6.}$$

$$r - 3 = -30$$

$$r - 3 + 3 = -30 + 3 \quad \textit{Add 3 to each side.}$$

$$r = -27$$

The solution is –27.

EXERCISES ***Name the first step in solving each equation. Then solve each equation.***

1. $6n - 2 = 22$

2. $\frac{1}{2}(y - 3) = 12$

Solve each equation.

3. $-5t - 5 = -5$

4. $4x - 5 = 15$

5. $24 = 17 - 2c$

6. $-5h - 6 = 24$

7. $6 - 3b = -9$

8. $12 - 4n = 4$

9. $7 + \frac{k}{4} = 9$

10. $\frac{5}{7}(d + 20) = -10$

11. $\frac{2}{3}(a - 18) = -6$

Translate each sentence into an equation. Then solve the equation.

12. Six less than a number divided by 3 is 12.

13. The sum of a number and four, times 3, is negative twelve.

14. Three times a number plus negative five is negative eleven.

APPLICATIONS

15. On a July day in Detroit, Michigan, the temperature rose to 80°F. Find this temperature in degrees Celsius. ($F = \frac{9}{5}C + 32$)

16. Aardvark Taxis charge $1.50 for the first half mile and then $0.25 for each additional quarter of a mile. What would the cost be for a 2-mile trip?

17. Three pens cost $1.55 including $0.08 sales tax. How much did each pen cost?

SKILL 6

Name ______________________ **Date** __________

Slope-Intercept Form

An equation in **slope-intercept form** looks like the following.

$$y = mx + b$$

The variables are x and y. The m and b are used to represent constants. In a particular equation, m and b would be specific numbers. An example of an equation in slope-intercept form is $y = 3x + 8$.

The **slope** is m. This can also be thought of as, the value y changes each time the value of x increases by 1. In a graph of the equation, the slope is how steep the line is.

The ***y*-intercept** is b, or the value of y when $x = 0$. In a graph of the equation, the y-intercept is the place where the line crosses the y axis

EXAMPLE ***Rewrite the equation 3y −15x = 4 in slope-intercept form.***

Rearrange the equation by doing the same thing to each side until the equation is in the form you want.

$3y - 15x = 4$

$3y = 15x + 4$	Add $15x$ to each side.
$\frac{3}{3}y = \left(\frac{15}{3}\right)x + \frac{4}{3}$	Divide each side by 3.
$y = 5x + \frac{4}{3}$	Simplify.

EXERCISES ***Rewrite each equation in slope-intercept form.***

1. $x + y = 16$

2. $8x - y = 5$

3. $x + 2y = 12$

4. $5y - x = 10$

5. $y + 5x = -20$

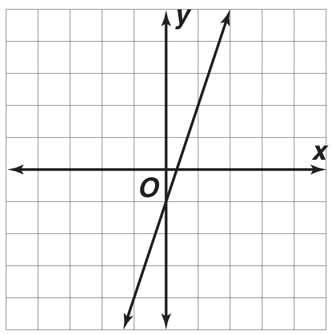
6. $y - 2 = 4x$

7. $9x - 4y = -12$

8. $15x = 18 - 9y$

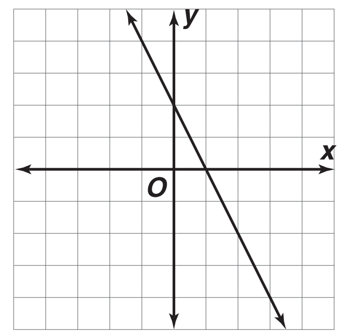
EXERCISES ***Identify the slope and y-intercept of each graph. Then write the equation that corresponds to each graph. Write the equations in slope-intercept form.***

9.



Slope (m)	
y-intercept (b)	
Equation	

10.



Slope (m)	
y-intercept (b)	
Equation	

APPLICATIONS ***Kate can walk 4 miles per hour. She can go 13 miles per hour on her bicycle. Use this information to answer Exercises 11–13.***

11. Does Kate travel faster on foot or on her bicycle?

12. Complete the input-output tables. In the following tables, d represents the distance she travels, and t represents the time it takes her to go that distance.

Distance Kate Can Travel (in miles) on Foot				
Time (t)	0 hours	1 hour	2 hours	3 hours
Distance (d)				

Distance Kate Can Travel (in miles) by Bicycle				
Time (t)	0 hours	1 hour	2 hours	3 hours
Distance (d)				

13. If you graphed the data from each input-output table, what would you label the horizontal axis, t or d?

14. What does the slope of the graph represent in the story? Why does one of the graphs have a steeper slope than the other?

SKILL 7

Name ______________________ Date ____________

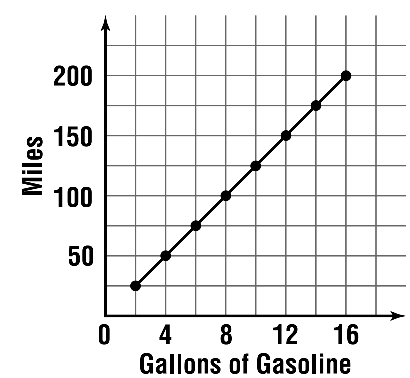
Graphing Functions

Carin's motor home averages about 25 miles on two gallons of gasoline. The function table at the right shows this relationship.

Gallons of Gasoline	Miles
2	25
4	50
6	75
8	100
10	125
12	150
14	175
16	200

EXAMPLE ***Graph the function.***

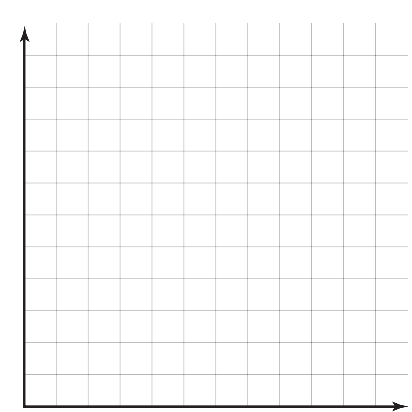
To graph the function, first label the axes and graph the points named by the data. Then connect the points as shown in the graph at the right.



EXERCISES ***Graph each function.***

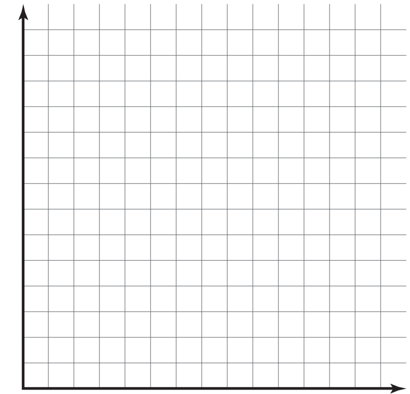
1.

Length of Side (cm)	Area (sq cm)
1	1
2	4
3	9
4	16
5	25
6	36
7	49



2.

Time (years)	Savings (dollars)
1	100
2	250
3	150
4	300
5	600
6	550
7	650

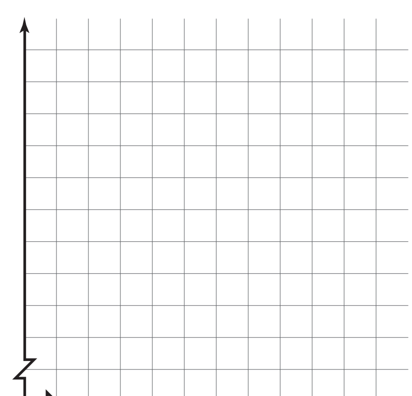


APPLICATIONS

The function table at the right shows the apparent temperature for the given room temperatures for a relative humidity of 80%. Use the data to answer Exercises 3–5.

Room Temperature (in °F)	Apparent Temperature (in °F)
69	70
70	71
71	73
72	74
73	75
74	76
75	77

3. Graph the function.



4. If this pattern continues, what would you expect the apparent temperature to be for a room temperature of 68°F?

5. Where does a change in the pattern of the function occur? Why do you think this change occurs?

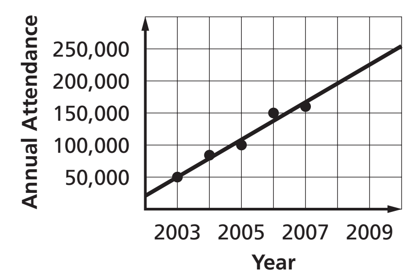
Name ______________________________ Date ____________

Using Statistics to Make Predictions

When real-life data are collected in a statistical experiment, the points graphed usually do not form a straight line. They may, however, approximate a linear relationship. A **best-fit line** can be used to show such a relationship. A best-fit line is a line that is very close to most of the data points.

EXAMPLE ***Use the best-fit line to predict the annual attendance at Fun Times Amusement Park in 2009.***

Draw a line so that the points are as close as possible to the line. Extend the line so that you can find the *y* value for an *x* value of 2009. The *y* value for 2009 is about 225,000.



So, the predicted annual attendance at Fun Times Amusement Park in 2009 is 225,000 people.

You can also write an equation of a best-fit line.

EXAMPLE ***Use the information from the example above. Write an equation in slope-intercept form for the best-fit line and then predict the annual attendance in 2008.***

Step 1 First, select two points on the line and find the slope. Choose (2004, 90,000) and (2006, 150,000).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Definition of slope*

$= \frac{150{,}000 - 90{,}000}{2006 - 2004}$ $x_1 = 2004,\ y_1 = 90{,}000$

$x_2 = 2006,\ y_2 = 150{,}000$

$= 30{,}000$ *Simplify.*

Step 2 Find the *y*-intercept.

$y = mx + b$ *Slope-intercept form*

$90{,}000 = 30{,}000(2004) + b$ $y = 90{,}000,\ m = 30{,}000,\ x = 2004$

$b = -60{,}030{,}000$ *Simplify.*

Step 3 Write the equation.

$y = mx + b$ *Slope-intercept form*

$y = 30{,}000x - 60{,}030{,}000$ $m = 30{,}000,\ b = -60{,}030{,}000$

Step 4 Solve the equation.

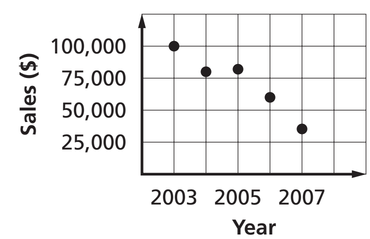
$y = 30{,}000(2008) - 60{,}030{,}000$ $x = 2008$

$= 210{,}000$ *Simplify.*

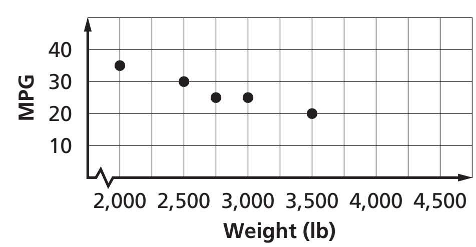
The predicted annual attendance at Fun Times Amusement Park in 2008 is 210,000.

EXERCISES

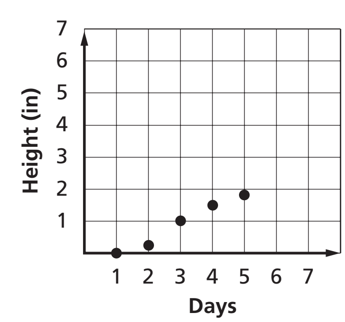
1. Predict the sales figure for 2009.



2. What is the gas mileage for a car weighing 4,500 pounds?



3. How tall would a tomato plant be ten days after planting the seed?



APPLICATIONS

4. Use the graph in Exercise 2. Determine the equation of the best-fit line. Use it to predict the gas mileage for a car weighing 4,000 pounds.

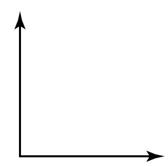
5. Use the graph in Exercise 3. Determine the equation of the best-fit line. Use it to predict the height of the tomato plant 8 days after planting.

SKILL 9

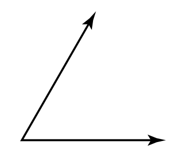
Name ______________________ Date __________

Angles

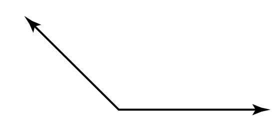
An angle is formed by two rays with a common endpoint called the **vertex**. Angles are often measured in **degrees** and can be classified according to their measure.



Right angles
measure 90°.



Acute angles
measure between
0° and 90°.



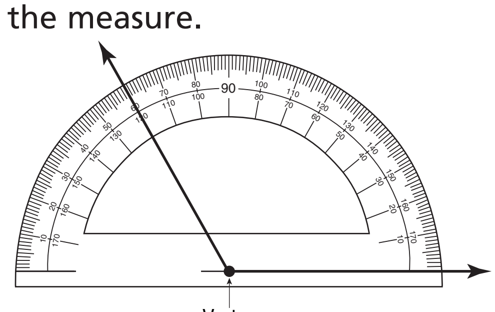
Obtuse angles
measure between
90° and 180°.

A protractor may be used to measure an angle.

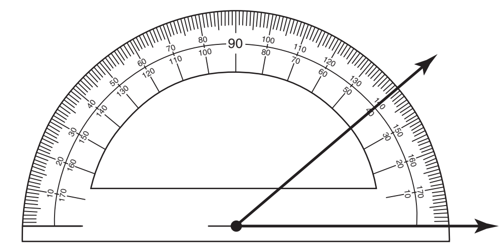
EXAMPLE

Measure each angle. Classify each angle as right, acute, or obtuse.

To measure an angle, place the center of the protractor on the vertex of the angle. Place the zero mark of the scale along one side of the angle. Find the place where the other side crosses the scale and read the measure.



The measure of the angle is 120°.
Therefore, it is obtuse.



The measure of the angle is 40°.
Therefore, it is acute.

EXERCISES ***Use the figure at the right to find the measure of each angle. Then classify each angle as right, acute, or obtuse.***

1. $\angle FPJ$

2. $\angle DPJ$

3. $\angle BPJ$

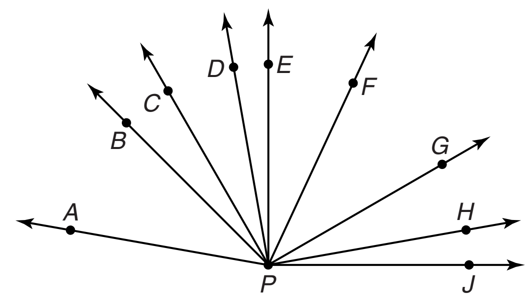
4. $\angle EPJ$

5. $\angle BPF$

6. $\angle APG$

7. $\angle DPH$

8. $\angle EPG$



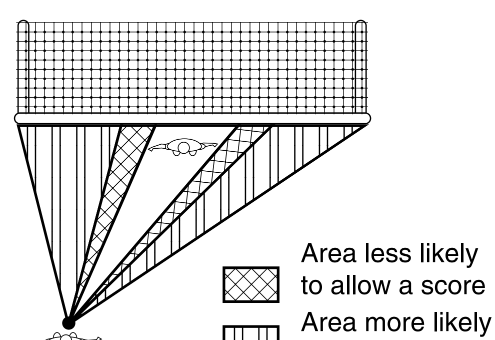
Use a protractor to draw angles with the following measures.

9. 80°

10. 125°

11. 90°

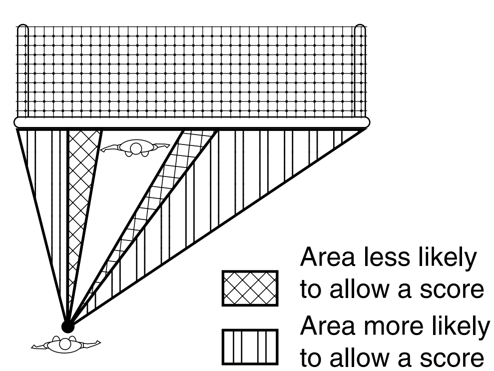
APPLICATIONS ***Soccer is a game of angles. Study the diagrams of a player shooting against a goalkeeper to answer Exercises 12–14.***



12. In the first diagram, would the player increase her chances of scoring a goal by moving closer to the goalkeeper?

13. The player forces the goalkeeper to the left as shown in the second diagram. Will the player have a better chance of making a goal?

14. If the goalkeeper moves closer to the player, will the player have a better chance of making a goal?

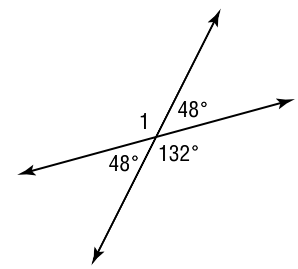


Name ______________________ Date __________

Angle Relationships

When two lines intersect, they form two pairs of opposite angles called **vertical angles.** Vertical angles have the same measure and are therefore congruent.

EXAMPLE ***Find $m\angle 1$.***

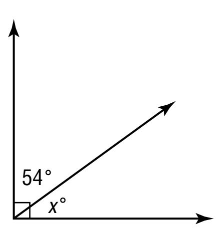


Angle 1 and the angle whose measure is 132° are vertical angles. Therefore, they are congruent.

Thus, $m\angle 1 = 132°$.

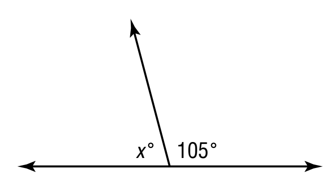
Two angles are **complementary** if the sum of their measures is 90°. Two angles are **supplementary** if the sum of their measures is 180°.

EXAMPLE ***Find x in each figure.***



The two angles form a right angle, which measures 90°. Therefore, the angles are complementary.

$$\begin{aligned} 54 + x &= 90 \\ 54 + x - 54 &= 90 - 54 \\ x &= 36 \end{aligned}$$

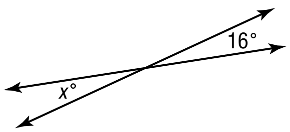


The two angles form a straight line, which measures 180°. Therefore, the angles are supplementary.

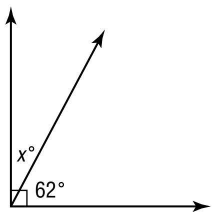
$$\begin{aligned} x + 105 &= 180 \\ x + 105 - 105 &= 180 - 105 \\ x &= 75 \end{aligned}$$

EXERCISES ***Find the value of x in each figure.***

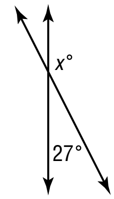
1.



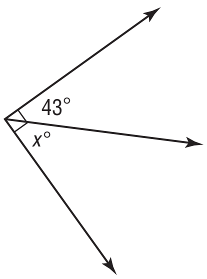
2.



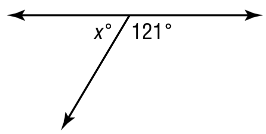
3.



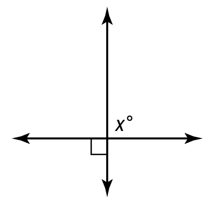
4.



5.



6.



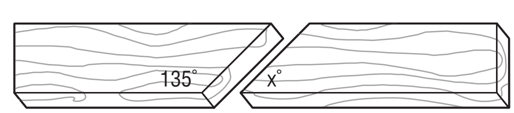
7. Angles A and B are vertical angles. If $m\angle A = 63°$ and $m\angle B = (x + 15)°$, find the value of x.

8. Angles P and Q are supplementary angles. If $m\angle P = (x - 25)°$ and $m\angle Q = 102°$, find the value of x.

9. Angles Y and Z are complementary. If $M\angle Y = (4x + 2)°$ and $m\angle Z = (5x - 2)°$, find the value of x.

APPLICATIONS

10. A carpenter uses a power saw to cut a piece of lumber at a 135° angle. What is the measure of the other angle formed by the cut?

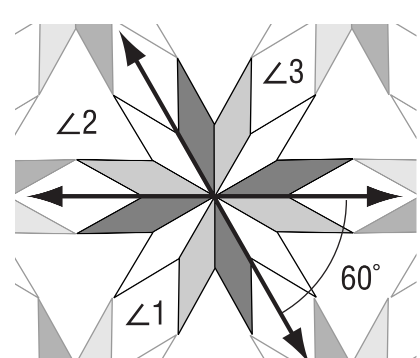


11. Megan is making a quilt using the pattern shown at the right.

a. What is $m\angle 1$?

b. What is $m\angle 2$?

c. What is $m\angle 3$?

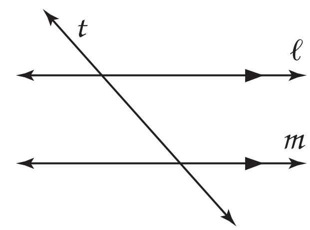


SKILL 11

Name ______________________ **Date** __________

Parallel Lines and Angle Relationships

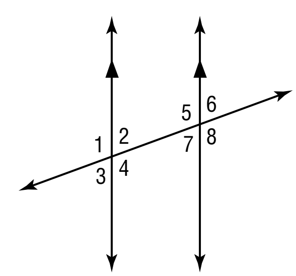
Two lines in a plane that never intersect or cross are called **parallel lines.** Arrowheads on the lines indicate that they are parallel.



Lines ℓ and m are parallel. This can be written as $\ell \parallel m$. Line t is a transversal.

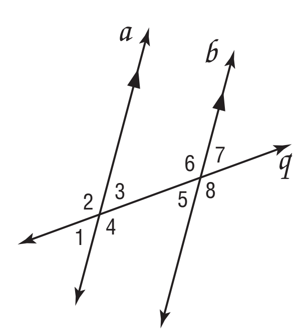
A line that intersects two or more parallel lines is called a **transversal.**

When two parallel lines are cut by a transversal, then the following are true.



- **Alternate interior angles,** those on opposite sides of the transversal and inside the other two lines are congruent. In the figure, $\angle 2 \cong \angle 7$ and $\angle 4 \cong \angle 5$.
- **Alternate exterior angles,** those on opposite sides of the transversal and outside the other two lines, are congruent. In the figure, $\angle 3 \cong \angle 6$ and $\angle 1 \cong \angle 8$.
- **Corresponding angles,** those in the same position on the two lines in relation to the transversal, are congruent. In the figure,$\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.
- **Consecutive interior angles,** those on the same side of the transversal and inside the other two lines, are supplementary. In the figure, $\angle 2$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 7$ are supplementary.

EXAMPLES ***Find the measure of each angle.***



If $m\angle 4 = 115°$, $m\angle 6 =$ ______.
Since $\angle 4$ and $\angle 6$ are alternate interior angles, they are congruent. So, $m\angle 6 = 115°$

If $m\angle 7 = 50°$, $m\angle 1 =$ ______.
Since $\angle 7$ and$\angle 1$ are alternate exterior angles, they are congruent. So, $m\angle 1 = 50°$

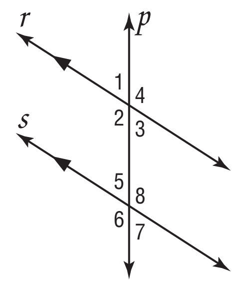
If $m\angle 1 = 62°$, $m\angle 5 =$ ______.
Since $\angle 1$ and $\angle 5$ are corresponding angles, they are congruent. So, $m\angle 5 = 62°$

If $m\angle 3 = 34°$, $m\angle 6 =$ ______.
Since $\angle 3$ and $\angle 6$ are consecutive interior angles, they are supplementary.

$34 + m\angle 6 = 180$ *Definition of supplementary angles*

$m\angle 6 = 146$ *Subtract 34 from each side.*

EXERCISES ***Find the measure of each angle.***



1. $m\angle 1 = 42°$, $m\angle 7 =$ ______

2. $m\angle 3 = 58°$, $m\angle 5 =$ ______

3. $m\angle 2 = 110°$, $m\angle 5 =$ ______

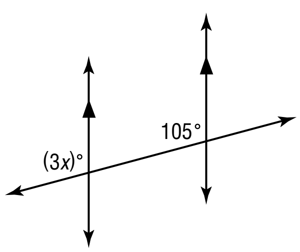
4. $m\angle 6 = 127°$, $m\angle 4 =$ ______

5. $m\angle 8 = 150°$, $m\angle 2 =$ ______

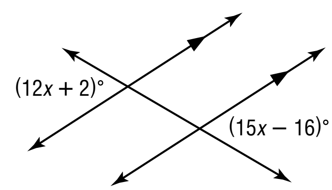
6. $m\angle 7 = 60°$, $m\angle 3 =$ ______

Find the value of x in each figure.

7.

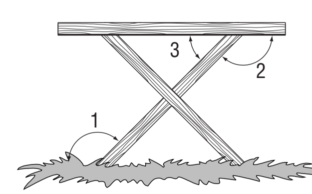


8.

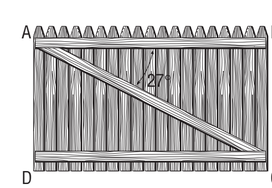


APPLICATIONS

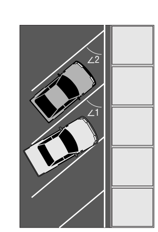
9. Brooke is building a bench to place in her yard. The top of the bench will be parallel to the ground. If $m\angle 1 = 135°$, find $m\angle 2$ and $m\angle 3$.



10. A section of fencing is reinforced with a diagonal brace as shown. What is $m\angle ACD$?



11. City planners angled the parking spaces at City Hall. All of the lines marking the parking spaces are parallel. If $m\angle 2 = 40°$, what is $m\angle 1$? Explain.



SKILL 12

Name ______________________ **Date** __________

Percents as Fractions and Decimals

A stereo is on sale for $33\frac{1}{3}\%$ off the original price.

EXAMPLE ***Write this percent as a fraction in simplest form and as a decimal.***

To express a percent as a fraction in simplest form, express the percent in the form $\frac{r}{100}$ and simplify.

$$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$$

$$= \frac{100}{3} \cdot \frac{1}{100}$$ *Write $33\frac{1}{3}$ as $\frac{100}{3}$ and multiply by the reciprocal of 100.*

$$= \frac{1}{3}$$

To express a percent as a decimal, express the percent in the form $\frac{r}{100}$ and then express the fraction as a decimal.

$$33\frac{1}{3}\% = \frac{1}{3}$$ *You found this in the example above.*

$$\approx 0.33$$

EXERCISES ***Write each percent as a fraction in simplest form and as a decimal.***

1. 40% **2.** 8% **3.** 29%

4. 55% **5.** 25% **6.** 81%

7. $66\frac{2}{3}\%$

8. 98%

9. 16.5%

10. 30%

11. 240%

12. 0.05%

APPLICATIONS ***Between 1980 and 1990, the population of New Hampshire increased by 20.5%. Use this information to answer Exercises 13–17.***

13. Write this percent as a fraction in simplest form.

14. Write this percent as a decimal.

15. When is it best to use the percent instead of the fraction or the decimal?

16. When is it best to use the fraction instead of the percent or the decimal?

17. When is it best to use the decimal instead of the percent or the fraction?

18. Between 1975 and 1985, the disposable personal income in the United States more than doubled. Does this mean the income has increased by more than 200%? Explain.

SKILL 13

Name ______________________ **Date** ____________

Percent of a Number

To find the percent of a number, you can either change the percent to a fraction and then multiply, or change the percent to a decimal and then multiply.

EXAMPLE

In the Washington County championship basketball game, Lee made 55% of his 20 attempted field goals. How many field goals did he make?

Find 55% of 20.

Method 1
Change the percent to a fraction.

$55\% = \frac{55}{100} = \frac{11}{20}$

$\frac{11}{20} \times 20 = 11$

Method 2
Change the percent to a decimal.

$55\% = \frac{55}{100}$ or 0.55

$0.55 \times 20 = 11$

Lee made 11 field goals.

EXERCISES ***Find the percent of each number.***

1. 25% of 200 **2.** 30% of 55 **3.** 3% of 610

4. 5.5% of 25 **5.** 13% of 85 **6.** 97% of 12

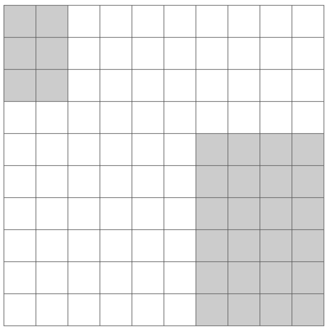
7. 1% of 25 **8.** 140% of 125 **9.** 100% of 50

10. Which of the following does not belong?
a. 25% of 80
b. 80% of 25
c. 2.5% of 800
d. 8% of 2,500

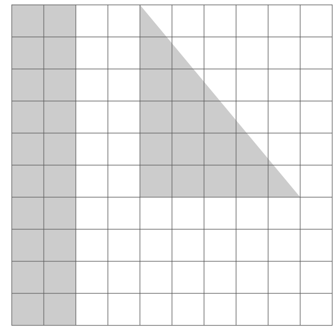
11. Hannah's basketball team won 75% of their games this season. They played 28 games this year. How many games did they win?

Write a percent to represent the shaded area.

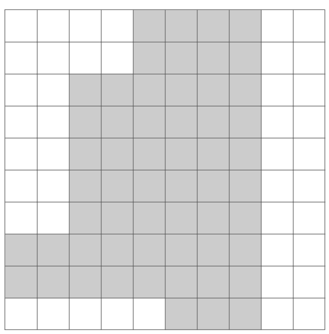
12.



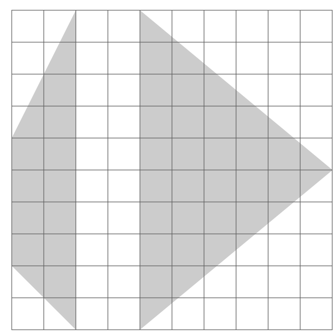
13.



14.



15.



APPLICATIONS

16. Kleema owns 40 music CD's. Fifteen of her CD's are recordings done by rap groups. What percent of her CD collection is rap music?

17. The Polletta's went out to dinner, and the food bill was $35.00. The standard rate for tipping is 15%.

a. What is the decimal value of this percent?
b. What should their tip be?
c. What is their total food and tip bill?

18. Angie wants to put a winter coat in layaway at a store. To do so, she must pay the store 20% of the cost of the coat so they will hold it. If the coat costs $48.99, about how much of a deposit does Angie need to pay the store?

19. Mrs. Saunders made $600 last week, and she put 15% of that amount into her savings account. How much did she save?

SKILL 14

Name ______________________ Date __________

Percent Proportion

Use the percent proportion to solve problems dealing with percent.

$\frac{P}{B} = \frac{r}{100}$ P = percentage B = base $\frac{r}{100}$ = rate

EXAMPLES

37.2 is what percent of 186?

$$\frac{P}{B} = \frac{r}{100}$$
$$\frac{37.2}{186} = \frac{r}{100}$$
$$(37.2)(100) = (186)(r)$$
$$3{,}720 = 186r$$
$$20 = r$$

37.2 is 20% of 186.

What number is 15% of 280?

$$\frac{P}{B} = \frac{r}{100}$$
$$\frac{P}{280} = \frac{15}{100}$$
$$(P)(100) = (280)(15)$$
$$100P = 4{,}200$$
$$P = 42$$

42 is 15% of 280.

EXERCISES ***Tell whether each number is the percentage, base, or rate.***

1. 12 is what percent of 30?

2. 6.25% of 190 is what number?

3. What percent of 49 is 7?

4. 40% of what number is 82?

Write a proportion for each problem. Then solve. Round answers to the nearest tenth.

5. What number is 10% of 230?

6. 25% of what number is 38?

7. Find 15% of 160.

8. 24 is 20% of what number?

9. 36 is 75% of what number?

10. 36% of what number is 18?

11. What percent of 224 is 28?

12. What number is 40% of 250?

13. 15% of 290 is what number?

14. 50% of what number is 74?

15. Use a proportion to find $55\frac{1}{2}\%$ of 66. Round to the nearest tenth.

16. Use a proportion to find $19\frac{1}{4}\%$ of 45. Round to the nearest tenth.

APPLICATIONS

17. In Juan's math class, there are 16 boys and 9 girls. What percent of Juan's class is girls?

18. To the nearest whole percent, 44% of the seventh-graders at King Middle School are girls. There are 425 seventh-graders. What is the number of girls in the seventh grade?

19. If 69% of the 247 students in the seventh grade ride the bus to school, about how many students do not ride the bus to school?

20. There are 20 students running for student council at Pine Bluff High School. If the school will elect a president, vice president, treasurer, and secretary, what percent of the students running will win in the election?

21. There were 102,269 tickets available for a rock concert. If The Ticket Company sold 72.5% of the tickets available, about how many tickets did they sell for the concert?

SKILL 15

Name ______________________ Date ____________

Percent of Change

The population of Iowa in 1980 was 2,913,808. The population in 1990 was 2,776,755.

EXAMPLE ***Find the percent of decrease in the population.***

To find the percent of decrease, you can follow these steps.

1. Subtract to find the amount of decrease.

$2{,}913{,}808 - 2{,}776{,}755 = 137{,}053$

2. Solve the percent proportion. Compare the amount of decrease to the original amount.

$$\frac{137{,}053}{2{,}913{,}808} = \frac{r}{100}$$

$$137{,}053 \cdot 100 = 2{,}913{,}808 \cdot r$$

$$13{,}705{,}300 = 2{,}913{,}808r$$

$$\frac{13{,}705{,}300}{2{,}913{,}808} = \frac{2{,}913{,}808r}{2{,}913{,}808}$$

$$5 \approx r$$

The population of Iowa decreased by about 5%.

EXERCISES ***Find the percent of change. Round to the nearest whole percent.***

1. old: $5
new: $7

2. old: 45 students
new: 50 students

3. old: 32 dogs
new: 30 dogs

4. old: $56
new: $52

5. old: 345 adults
new: 450 adults

6. old: $648
new: $635

7. old: 150 pounds
new: 138 pounds

8. old: 9.5 hours
new: 8 hours

APPLICATIONS ***Last year, the value of Paul's used car was $19,990. Use this information to answer Exercises 9–11.***

9. This year, the value of his car is $11,994. What was the percent change in the car's value?

10. The year before last the value of his car was $24,500. What was the percent change in the car's value? How does this change compare to the change from last year to this year?

11. What was the total percent change in the car's value over the two years? Can you find the answer to this question by simply adding the answers to Exercises 9 and 10? Why or why not?

12. A clothing store has a 65% markup on blazers. But, the blazers did not sell well at the listed price. So, the blazers were put on sale at 65% off the listed price. Did the store break even, make a profit, or lose money? Explain.

SKILL 16

Name ______________________ Date ____________

Powers and Exponents

An expression like $3 \times 3 \times 3 \times 3 \times 3$ can be written as a power. A power has two parts, a **base** and an **exponent**. The expression $3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5.

EXAMPLE ***Write the expression $m \cdot m \cdot m \cdot m \cdot m \cdot m$ using exponents.***

The base is *m*. It is a factor 6 times, so the exponent is 6.

$$m \cdot m \cdot m \cdot m \cdot m \cdot m = m^6$$

You can also use powers to name numbers that are less than one by using exponents that are negative integers. The definition of a negative exponent states that $a^{-n} = \frac{1}{a^n}$ for $a \neq 0$ and any integer n.

EXAMPLE ***Write the expression 4^{-3} using a positive exponent.***

$$4^{-3} = \frac{1}{4^3}$$

EXERCISES ***Write each expression using exponents.***

1. $2 \cdot 2 \cdot 2 \cdot 2$

2. $(-3)(-3)(-3)(-3)(-3)$

3. 9

4. $x \cdot x \cdot x$

5. $c \cdot c \cdot d \cdot d \cdot d \cdot d \cdot d$

6. $8 \cdot a \cdot a \cdot a \cdot b$

7. $(k - 2)(k - 2)$

8. $4 \cdot 4 \cdot 4 \cdot 4 \cdot h \cdot h$

9. $(-w)(-w)(-w)(-w)(-w)$

10. $6 \cdot 6 \cdot 6 \cdot y \cdot y \cdot y \cdot y$

Evaluate each expression if $m = 3$, $n = 2$, and $p = -4$.

11. m^4

12. n^6

13. $3p^2$

14. mn^2

15. $m^2 + p^3$

16. $(p + 3)^5$

17. $n^2 - 3n + 4$

18. $-2mp^2$

19. $5(n - 4)^3$

Write each expression using a positive exponent.

20. 6^{-1}

21. 4^{-3}

22. $(-2)^{-4}$

23. d^{-7}

24. m^{-5}

25. $3b^{-6}$

26. 10^{-2}

27. $\frac{1}{x^{-5}}$

28. $\frac{7}{p^{-4}}$

Write each fraction as an expression using a negative exponent other than −1.

29. $\frac{1}{4^{-5}}$

30. $\frac{1}{3^8}$

31. $\frac{1}{7^3}$

32. $\frac{1}{64}$

33. $\frac{1}{27}$

34. $\frac{1}{1{,}000}$

Evaluate each expression if a = −2 and b = 3.

35. 5^a

36. b^{-4}

37. a^{-3}

38. $(-3)^{-b}$

39. ab^{-2}

40. $(ab)^{-2}$

APPLICATIONS

41. The area of a square is found by multiplying the length of a side by itself. If a square swimming pool has a side of length 45 feet, write an expression for the area of the swimming pool using exponents.

42. A molecule of a particular chemical compound weighs one millionth of a gram. Express this weight using a negative exponent.

43. A needle has a width measuring 2^{-5} inch. Express this measurement in standard form.

SKILL 17

Name ____________________ Date __________

Scientific Notation

A number is expressed in scientific notation when it is written as the product of a factor and a power of ten. The factor must be greater than or equal to 1 and less than 10.

EXAMPLES ***Express each number in standard form.***

$8.26 \times 10^5 = 8.26 \times 100{,}000$		$10^5 = 100{,}000$
$= 826{,}000$		*Move the decimal point 5 places to the right.*
$3.71 \times 10^{-4} = 3.71 \times 0.0001$		$10^{-4} = 0.0001$
$= 0.000371$		*Move the decimal point 4 places to the left.*

Express each number in scientific notation.

$68{,}000{,}000 = 6.8 \times 10{,}000{,}000$	*The decimal point moves 7 places.*
$= 6.8 \times 10^7$	*The exponent is positive.*
$0.000029 = 2.9 \times 0.00001$	*The decimal point moves 5 places.*
$= 2.9 \times 10^{-5}$	*The exponent is negative.*

EXERCISES ***Express each number in standard form.***

1. 7.24×10^3

2. 1.09×10^{-5}

3. 9.87×10^{-7}

4. 5.8×10^6

5. 3.006×10^2

6. 4.999×10^{-4}

7. 2.875×10^{-5}

8. 6.3×10^4

9. 4.003×10^6

10. 1.28×10^{-2}

Express each number in scientific notation.

11. 7,500,000

12. 291,000

13. 0.00037

14. 12,600

15. 0.0000002

16. 0.004

17. 60,000,000

18. 40,700,000

19. 0.00081

20. 12,500

Choose the greater number in each pair.

21. 3.8×10^{3}, 1.7×10^{5}

22. 0.0015, 2.3×10^{-4}

23. 60,000,000, 6.0×10^{6}

24. 4.75×10^{-3}, 8.9×10^{-6}

25. 0.00145, 1.2×10^{-3}

26. 7.01×10^{3}, 7,000

APPLICATIONS

27. The distance from Earth to the Sun is 1.55×10^{8} kilometers. Express this distance in standard form.

28. In 2001, the population of Asia was approximately 3,641,000,000. Express this number in scientific notation.

29. A large swimming pool under construction at the Greenview Heights Recreation Center will hold 240,000 gallons of water. Express this volume in scientific notation.

30. A scientist is comparing two chemical compounds in her laboratory. Compound A has a mass of 6.1×10^{-7} gram, and compound B has a mass of 3.6×10^{-6} gram. Which of the two compounds is heavier?

SKILL 18

Name ______________________ Date __________

Exponential Growth and Decay

In **exponential change** a quantity is repeatedly multiplied by the same factor.

If the quantity is increasing, the situation is called **exponential growth**. It happens when quantity is repeatedly multiplied by a number greater than 1.
If the quantity is decreasing, the situation is called **exponential decay**. It happens when quantity is repeatedly multiplied by a number between 0 and 1.
An exponential relationship between two variables is represented by an equation like this one:

$$y = b \cdot c^x$$

In this equation, b and c are constants. The variable b represents the amount you started with (when $x = o$, $y = b$). The variable c is the factor by which b is repeatedly multiplied. The variable x tells you how many times to multiply by b.

EXAMPLE

Laurie deposited \$500 in a savings account at the bank. The money in her account earns 3% interest each month. Write an equation to represent the amount of money in Laurie's account after *n* months. Is this a situation of exponential growth, exponential decay, or neither?

The amount of money in the account after month one is the initial \$500 plus 3% of \$500, or 103% of \$500. To calculate the interest in the account after 1 month, find 103% of \$500.

$$1.03 \cdot 500 = 515$$

After two months, the amount of money in the account is \$515 plus 3% of \$515.

Months Since Opening the Account (*n*)	Money in the Account (*d*)
0	\$500
1	\$515
2	\$530.45
3	\$546.36
4	\$562.75

You can use a table to help you see how the amount of money in the account changes over time. The equation that represents this situation is $d = 500 \times 1.03^n$.

In this equation, d is the amount of money in the account, n is the number of months that have passed since the account was opened. 500 is the amount of money in the account when it was opened. The value 1.03 is the **growth factor** or how much the amount of money increases or decreases each month.

EXERCISES ***For each equation, state whether it represents an exponential relationship. If it does, tell whether that relationship involves growth or decay.***

1. 5×5^x

2. 0.3×24^x

3. $67 \times x^4$

4. $8 \times (\frac{1}{3})^x$

For each table, state whether the relationship described could be exponential. If it could be, tell whether it would involve growth or decay. Explain your reasoning.

5.

x	y
1	10
2	20
3	40
4	80
5	160

6.

x	y
25	50
26	45
27	40
26	35
27	30

APPLICATIONS ***Jasmine's school set up a phone tree to notify students and parents about snow days. Each parent was given the name of four other parents to call. If school is cancelled, the principal calls the first four parents on the list. Each of those parents calls four other parents, and so on.***

7. The principal makes the first round of phone calls, calling four parents. Those four parents make the second round of phone calls. How many parents receive a phone call in the second round?

8. How many parents receive a phone call in the third round?

9. At the end of the third round, how many parents have been notified, total?

10. Write an equation to show how many parents are notified in a given round of phone calls. Use r to represent the number of rounds that have been completed, and p to represent the number of parents notified in the rth round of calls.

SKILL 19

Name ____________________ Date __________

Square Roots

If $a^2 = b$, then a is the **square root** of b.

EXAMPLE

Joanna wants to buy a house. The realtor told her that the family room in a certain house has a floor area of 144 square feet. What is the length of a side of the room if all four sides of the room are the same length?

If all four sides of the room are the same length, then the room is shaped like a square. The area of a square is given by the formula $A = s^2$. Use this formula to find the length of the sides of the room.

$$A = s^2$$

$$144 = s^2$$

$$\sqrt{144} = \sqrt{s^2}$$ *To solve this equation, find the square root of each side.*

$$12 = s$$ *The square root of 144 is 12.*

The length of a side of the room is 12 feet.

EXERCISES ***Find each square root.***

1. $\sqrt{9}$	**2.** $\sqrt{25}$	**3.** $\sqrt{81}$	**4.** $\sqrt{169}$
5. $\sqrt{36}$	**6.** $\sqrt{16}$	**7.** $\sqrt{64}$	**8.** $\sqrt{121}$
9. $\sqrt{100}$	**10.** $\sqrt{400}$	**11.** $\sqrt{900}$	**12.** $\sqrt{10{,}000}$
13. $\sqrt{196}$	**14.** $\sqrt{0.09}$	**15.** $\sqrt{0.81}$	**16.** $\sqrt{1.44}$

17. $\sqrt{0.49}$ **18.** $\sqrt{0.04}$ **19.** $\sqrt{2.25}$ **20.** $\sqrt{0.16}$

21. $\sqrt{\frac{4}{9}}$ **22.** $\sqrt{\frac{16}{25}}$ **23.** $\sqrt{\frac{49}{100}}$ **24.** $\sqrt{\frac{25}{36}}$

APPLICATIONS ***The area of a square picture is 64 square inches. Use this information to answer Exercises 25–27.***

25. What is the length of each side of the picture?

26. What is the length of each side of a picture frame for the picture if the area of the picture and the frame is 121 square inches?

27. Will a square mat with an area of 81 square inches be large enough on which to mount the picture? Why or why not?

28. A square dog run with an area of 289 square feet is fenced in on all sides. What is the length of the fencing along one side?

29. What is the diameter of a pizza that has an area of 254 square inches?

30. The area of the bottom of a pizza box is 100 square inches. If a circular pizza fits in the box with the pizza touching the sides of the box at their midpoints, what is the diameter of the pizza?

SKILL 20

Name ______________________ Date __________

nth Roots

The nth root of a number is the number that, when raised to the nth power, equals the original number. For example, the fourth root of 100 is the number that equals 100 when it is multiplied by itself 4 times. The nth root is written as $\sqrt[n]{}$

When n is odd, there is only one possible $\sqrt[n]{}$ for each value of x. If x is positive, $\sqrt[n]{}$ is also positive. If x is negative, $\sqrt[n]{}$ is also negative.

When n is even, there are two possible nth roots of x for each value of x. The two nth roots have the same numerical value; one is negative and the other is positive.

The nth root of 0 is always 0.

EXAMPLE ***Find the fourth roots of 256 without using a calculator.***

If you do not know where to begin, factoring 256 will help you figure out what the fourth roots could be.

$256 \div 2 = 128$

$128 \div 2 = 64$

At this point, you may recognize 64 as 8×8.

So, $256 = 2 \times 2 \times 8 \times 8$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 4 \times 4 \times 4 \times 4$

$= 4^4$

So, $\sqrt[4]{256} = 4$. The second fourth root of 256 is $-\sqrt[4]{256}$ or -4.

EXERCISES ***Evaluate without using a calculator.***

1. $\sqrt[3]{27}$

2. Find the fourth roots of 625.

3. Find the square roots of 144.

4. $\sqrt[5]{243}$

5. $-\sqrt[6]{64}$

6. Find the cube root of $-1{,}000$.

7. $\sqrt[4]{1296}$

8. $\sqrt[3]{343}$

9. Find the fourth roots of 0.0016.

10. Find the cube root of $\frac{27}{125}$.

11. $\sqrt[3]{-1000000}$

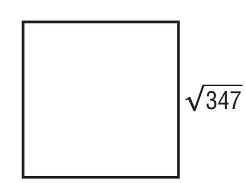
12. Find the square roots of $\frac{121}{169}$.

13. $\sqrt[5]{4^5}$

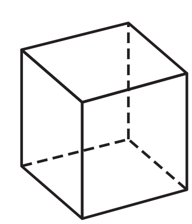
14. $\sqrt[7]{12^7}$

APPLICATIONS ***Use the diagram and given information to find the missing information.***

15. The sides of this square are $\sqrt{347}$ cm long. What is the area of the square?



16. The volume of this cube is 8,000 cm^3. What is the length of one side of the cube?



SKILL 21

Name ______________________ Date __________

Order of Operations

When you evaluate an expression in mathematics, you must do the operations in a certain order. This order is called the **order of operations**.

EXAMPLE ***Evaluate 56 ÷ (17 − 9) + 7 × 3.***

$56 \div (17 - 9) + 7 \times 3 =$

$56 \div 8 + 7 \times 3 =$ *Do all the operations within the grouping symbols.*

$7 + 21 =$ *Do multiplication and division from left to right.*

28 *Do addition and subtraction from left to right.*

Therefore, $56 \div (17 - 9) + 7 \times 3 = 28$.

EXERCISES ***Evaluate each expression.***

1. $2 \times 9 + 5 \times 3$

2. $(9 - 4) \div 5$

3. $10 - 4 + 1$

4. $15 - 18 \div 9 + 3$

5. $30 \div (12 - 6) + 4$

6. $(72 - 12) \div 2$

7. $2(16 - 9) - (5 + 1)$

8. $(43 - 23) - 2 \quad 5$

9. $90 - 45 - 24 \div 2$

10. $81 \div (13 - 4)$

11. $7 \times 8 - 2 \times 8$

12. $71 + (34 - 34)$

13. $9 - 4 \div 2 + 16$

14. $(24 - 10) - 3 \times 3$

15. $4(22 - 18) - 3 \times 5$

16. $12(5 - 5) + 3 \times 5$

17. $18(4 - 3) \div 3 + 3$

18. $(34 + 46) \div 20 + 20$

19. $92 - 66 - 12 \div 4$

20. $(16 - 8) \div 4 + 10$

21. $60 \div 12 \times (4 - 1)$

22. $(100 - 25) \times 2 + 25$

23. $3 \times 7 - 5 + 4$

24. $9 \times 4 \div 2 - 10$

25. $150 \div 10 - 3 \times 5$

26. $5(35 - 18) + 1$

APPLICATIONS ***Use the price list at the right to answer Exercises 27–29.***

Sam's Sporting Supplies Price List	
Ping Pong Balls	5 for \$2
Ping Pong Paddles	\$8
Softballs	\$5
Soccer Balls	\$20

27. Alfred wants to buy 15 ping pong balls and 4 ping pong paddles. What is the cost of this purchase?

28. Ali plans to buy 6 softballs and 3 soccer balls for the teen club. If he has a coupon for \$8 off his purchase, how much will he pay for the balls?

29. What is the cost of 20 ping pong balls, 2 ping pong paddles, 3 softballs, and 1 soccer ball?

30. Tickets for the play cost \$12 for adults and \$8 for children. How much would 3 adult tickets and 5 children tickets cost?

31. Use operation symbols, parentheses, and the numbers 1, 2, 3, and 4 to express the numbers from 1 to 15. For example, $2 + 3 - (4 \times 1) = 1$.

SKILL 22

Name ______________________ Date __________

Multiplication Properties

The table shows the properties for multiplication.

Property	Examples
Commutative The product of two numbers is the same regardless of the order in which they are multiplied.	$21 \cdot 2 = 2 \cdot 21$ $42 = 42$
Associative The product of three or more numbers is the same regardless of the way in which they are grouped.	$5 \cdot (3 \cdot 6) = (5 \cdot 3) \cdot 6$ $5 \cdot 18 = 15 \cdot 6$ $90 = 90$
Identity The product of a number and 1 is the number.	$81 \times 1 = 81$
Inverse (Reciprocal) The product of a number and its reciprocal is 1.	$\frac{7}{8} \times \frac{8}{7} = 1$
Distributive The sum of two addends multiplied by a number is equal to the sum of the products of each addend and the number.	$2 \cdot (9 + 3) = (2 \cdot 9) + (2 \cdot 3)$ $2 \cdot 12 = 18 + 6$ $24 = 24$

EXERCISES ***Name the multiplicative inverse, or reciprocal, of each number.***

1. $\frac{6}{11}$ **2.** $\frac{19}{3}$ **3.** $\frac{1}{8}$ **4.** 9

Name the property shown by each statement.

5. $67 \cdot 89 = 89 \cdot 67$

6. $1 \cdot 45 = 45$

7. $\frac{11}{12} \cdot 1 = \frac{11}{12}$

8. $\left(\frac{1}{5} \cdot \frac{2}{3}\right) \cdot \frac{5}{9} = \frac{1}{5} \cdot \left(\frac{2}{3} \cdot \frac{5}{9}\right)$

9. $\frac{3}{4} \cdot \frac{5}{6} = \frac{5}{6} \cdot \frac{3}{4}$

10. $\frac{3}{5}\left(\frac{1}{3} + \frac{5}{7}\right) = \left(\frac{3}{5} \cdot \frac{1}{3}\right) + \left(\frac{3}{5} \cdot \frac{5}{7}\right)$

11. $\frac{1}{4} \cdot 4 = 1$

12. $45(23 + 3) = (45 \cdot 23) + (45 \cdot 3)$

13. $\frac{9}{4} \cdot \frac{4}{9} = 1$

14. $\frac{4}{5} \cdot \frac{3}{4} = \frac{3}{4} \cdot \frac{4}{5}$

APPLICATIONS

15. Jill runs for $1\frac{3}{4}$ as long as Eva. Find Jill's running time if Eva runs for 48 minutes.

16. A chihuahua is 6 inches tall. The height of a German shepherd is $3\frac{2}{3}$ the height of the chihuahua. Find the height of the German shepherd.

SKILL 23

Name ______________________ Date __________

Solve a Simpler Problem

EXAMPLE ***Find the sum of the whole numbers from 1 to 300.***

This would be a tedious problem to solve using a calculator or adding the numbers yourself. The problem is easier to solve if you solve simpler problems. First consider the partial sums indicated below.

1, 2, 3, 4, 5, . . . , 150, 151, . . . , 296, 297, 298, 299, 300

$150 + 151 = 301$

.
.
.

$5 + 296 = 301$

$4 + 297 = 301$

$3 + 298 = 301$

$2 + 299 = 301$

$1 + 300 = 301$

Notice that each sum is 301. There are 150 of these partial sums.

$$301 \times 150 = 45{,}150$$

The sum of the whole numbers from 1 to 300 is 45,150.

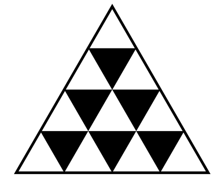
EXERCISES ***Solve by solving a simpler problem.***

1. Find the sum of the whole numbers from 1 to 150.

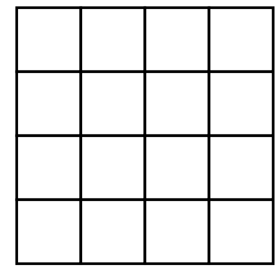
2. Find the sum of the whole numbers from 101 to 300.

3. Find the sum of the even numbers from 2 to 200.

4. What is the total number of triangles of any size in the figure at the right?

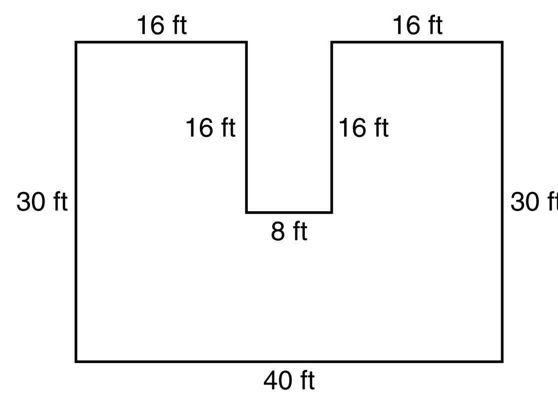


5. What is the total number of squares of any size in the figure at the right?



APPLICATIONS

6. Shea is planning to carpet a large area in her basement as shown at the right. How much carpet will she need to carpet this area?



7. Cliff heard a funny joke on the radio on Sunday. On Monday (day 1), he told the joke to Sarah, Rich, and Claire. These people each told the joke to 3 more people on Tuesday (day 2), who told the joke to 3 more people on Wednesday (day 3). This pattern continued. How many people heard the joke on the sixth day?

8. How many days passed before at least 100 people had heard the joke in Exercise 7?

9. By the end of the day 6, how many people altogether had heard the joke in Exercise 7? (Remember to count Cliff!)

10. A summer camp has 7 buildings arranged in a circle. Paths must be constructed joining every building to every other building. How many paths are needed?

SKILL 24

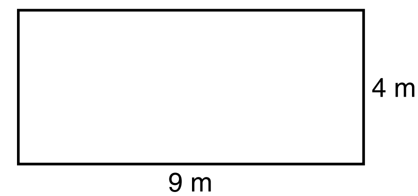
Name ____________________ Date __________

Area of Rectangles

Area is the number of square units needed to cover a surface. The area of a rectangle is the product of its length (ℓ) and its width (w).

$$A = \ell w$$

EXAMPLE ***Find the area of the rectangle at the right.***

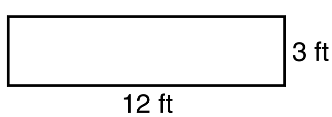


$A = \ell w$
$A = 9 \times 4$
$A = 36$

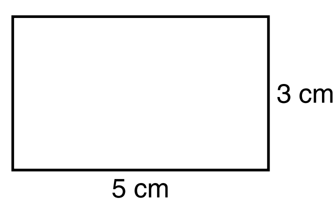
The area of the rectangle is 36 square meters.

EXERCISES ***Find the area of each rectangle.***

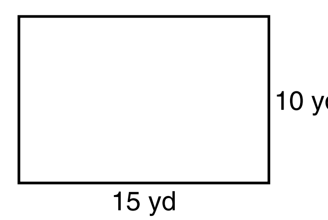
1.



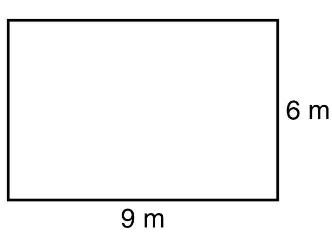
2.



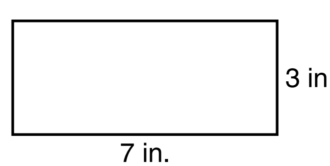
3.



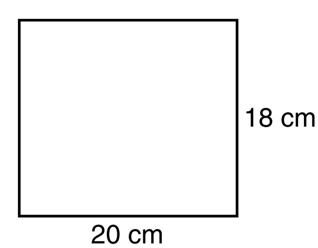
4.



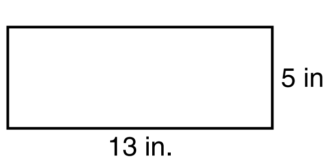
5.



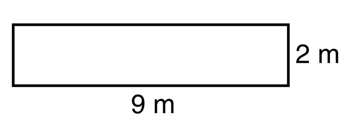
6.



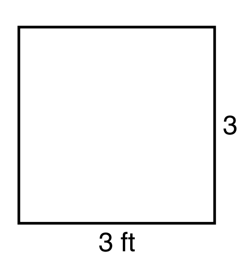
7.



8.

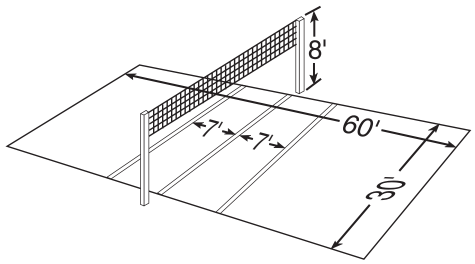


9.

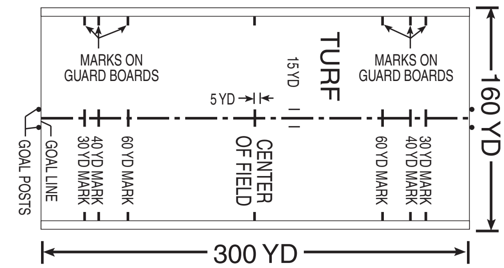


APPLICATIONS ***Find the area of each playing field.***

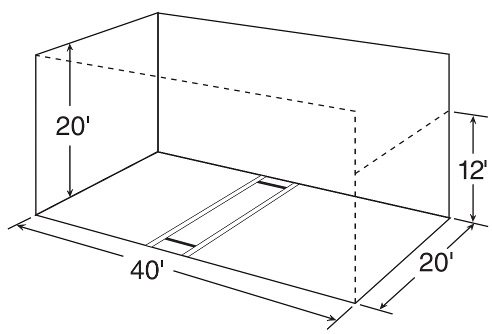
10. volleyball court



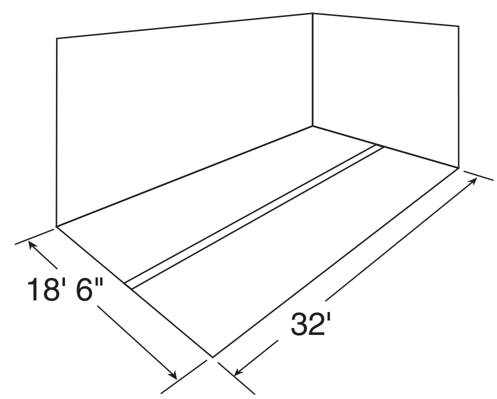
11. polo field



12. four-wall handball court



13. squash court



The maximum and minimum sizes of a soccer field are given at the right. Use this information to answer Exercises 14–16.

Soccer Field Size	
Maximum	225 ft by 360 ft
Minimum	195 ft by 330 ft

14. What is the maximum area of a soccer field?

15. What is the minimum area of a soccer field?

16. What is the difference between the maximum area of a soccer field and the minimum area of a soccer field?

17. Henry wants to carpet a rectangular room that is 6 yards by 5 yards. If the carpet costs $29.50 a square yard, how much will it cost to carpet the room?

SKILL 25

Name ______________________ Date __________

Adding and Subtracting Decimals

To add decimals, line up the decimal points. Then add the same way you add whole numbers.

EXAMPLE ***4.76 + 3.62***

$$\begin{array}{r} 4.76 \\ +\ 3.62 \\ \hline 8.38 \end{array}$$

The sum is 8.38.

12.8 + 3.467 + 8.56

$$\begin{array}{r} 12.800 \\ 3.467 \\ +\ 8.560 \\ \hline 24.827 \end{array}$$

Annex zeros.

The sum is 24.827.

To subtract decimals, line up the decimal points. Then subtract the same way you subtract whole numbers.

EXAMPLE ***15.05 − 4.86***

$$\begin{array}{r} 15.05 \\ -\ 4.86 \\ \hline 10.19 \end{array}$$

The difference is 10.19.

35 − 13.631

$$\begin{array}{r} 35.000 \\ -\ 13.631 \\ \hline 21.369 \end{array}$$

Annex zeros.

The difference is 21.369.

EXERCISES ***Add or subtract.***

1. $\begin{array}{r} 45.9 \\ +\ 12.7 \\ \hline \end{array}$

2. $\begin{array}{r} 6.83 \\ -\ 3.77 \\ \hline \end{array}$

3. $\begin{array}{r} 43.89 \\ +\ 56.32 \\ \hline \end{array}$

4. $\begin{array}{r} 205.7 \\ -\ 98.8 \\ \hline \end{array}$

5. $\begin{array}{r} 6.7 \\ +\ 3.56 \\ \hline \end{array}$

6. $\begin{array}{r} 18.75 \\ -\ 7.2 \\ \hline \end{array}$

7. $\begin{array}{r} 17.93 \\ +\ 33.5 \\ \hline \end{array}$

8. $\begin{array}{r} 77 \\ -\ 12.66 \\ \hline \end{array}$

9. $\begin{array}{r} 6.5 \\ +\ 7.547 \\ \hline \end{array}$

10. 4.7 − 0.89

11. 15.6 + 7.89

12. 25 − 4.76

13. 6.43 + 7.8 + 13

14. 9.857 − 4.5

15. 65.8 + 15.75 + 7.854

16. 408.7 − 56.78

17. 7.9 + 1.22 + 6.1 + 11

18. 73.56 − 29

19. 11.444 + 5.9 + 13.93

APPLICATIONS

The results of the 1948 presidential election is given at the right. Use this information to answer Exercises 20–22.

Candidate	Percent of Popular Vote
Truman	49.5
Dewey	45.12
Thurmond	2.4
Wallace	2.38
Other	0.6

20. What percent of the vote was cast for Truman or Dewey?

21. How many more percentage points did Truman receive than Dewey?

22. What percent of the vote was not cast for Truman or Dewey?

23. Albert had $284.73 in his checking account. He wrote checks for $55.86 and $25.00. He deposited a check for $113.76. What is his new balance in his checking account?

24. For lunch, Connie buys a sandwich for $2.35 and a small lemonade for $0.79. If she gives the cashier a five-dollar bill, how much change should she receive?

25. Tony drove 12.7 kilometers to the computer store. Then he drove 5.2 kilometers to the library, and finally 6.7 kilometers to his house. What was the total distance Tony drove?

SKILL 26

Name ______________________ Date __________

Multiplying and Dividing Decimals

EXAMPLE ***Multiply 1.45 by 0.68.***

$$\begin{array}{r l}
1.45 & \leftarrow \text{ 2 decimal places} \\
\times\ 0.68 & \leftarrow \text{ 2 decimal places} \\
\hline
1160 & \\
+\ 870\ \ & \\
\hline
0.9860 & \leftarrow \text{ 4 decimal places}
\end{array}$$

The sum of the decimal places in the factors is 4, so the product has 4 decimal places.

The product is 0.9860.

EXAMPLE ***Divide 38.22 by 2.6.***

$$\begin{array}{r}
14.7 \\
2.6.\overline{)38.2.2} \\
-\ 26\ \ \ \ \\
\hline
12\ 2\ \ \\
-\ 10\ 4\ \ \\
\hline
1\ 8\ 2 \\
-\ 1\ 8\ 2 \\
\hline
0
\end{array}$$

Change 2.6 to 26 by moving the decimal point one place to the right.

Move the decimal point in the dividend one place to the right.

Divide as with whole numbers, placing the decimal point above the new point in the dividend.

The quotient is 14.7.

EXERCISES ***Multiply.***

1. $\begin{array}{r} 4.9 \\ \times\ 35 \\ \hline \end{array}$

2. $\begin{array}{r} 53 \\ \times\ 3.7 \\ \hline \end{array}$

3. $\begin{array}{r} 2.8 \\ \times\ 3.5 \\ \hline \end{array}$

4. $\begin{array}{r} 18.9 \\ \times\ 3.7 \\ \hline \end{array}$

5. $\begin{array}{r} 0.014 \\ \times\ 0.65 \\ \hline \end{array}$

6. $\begin{array}{r} 53.98 \\ \times\ 71.2 \\ \hline \end{array}$

7. $\begin{array}{r} 4.55 \\ \times\ 41.8 \\ \hline \end{array}$

8. $\begin{array}{r} 0.133 \\ \times\ 4.2 \\ \hline \end{array}$

9. $\begin{array}{r} 3.91 \\ \times\ 8.5 \\ \hline \end{array}$

Divide.

10. $6\overline{)8.52}$

11. $23\overline{)64.4}$

12. $53\overline{)19.61}$

13. $1.6\overline{)134.4}$

14. $0.52\overline{)37.96}$

15. $0.23\overline{)36.8}$

16. $1.7\overline{)0.0374}$

17. $0.112\overline{)72.8}$

18. $7.4\overline{)37.962}$

APPLICATIONS

The prices at Martha's Meat Market are given at the right. Use this information to answer Exercises 19–21.

Martha's Meat Market	
Specials of the Week	
Ground Beef	\$1.90/lb
Chicken	\$1.15/lb
Turkey Breast	\$1.75/lb

19. What is the cost of a chicken that weighs 3.4 pounds?

20. Willy buys a package of ground beef for \$6.84. How many pounds of ground beef did he buy?

21. A turkey breast costs \$8.05. How much does the turkey breast weigh?

22. One centimeter on a map represents 56 kilometers. If a distance between two towns on the map is 3.2 centimeters, what is the actual distance between the towns?

SKILL 27

Name ______________________________ Date ____________

Adding and Subtracting Fractions

To add fractions, you must have a common denominator.

EXAMPLE ***Find each sum.***

a. $\frac{2}{7} + \frac{3}{7}$

$$\begin{array}{r} \frac{2}{7} \\ + \frac{3}{7} \\ \hline \frac{5}{7} \end{array}$$

The sum $\frac{5}{7}$.

b. $\frac{1}{4} + \frac{5}{6}$

$$\begin{array}{rcr} \frac{1}{4} & = & \frac{3}{12} \\ + \frac{5}{6} & = & + \frac{10}{12} \\ \hline & & \frac{13}{12} = 1\frac{1}{12} \end{array}$$

The sum is $1\frac{1}{12}$.

To subtract fractions, you must have a common denominator.

EXAMPLE ***Find each difference.***

a. $\frac{11}{12} - \frac{2}{12}$

$$\begin{array}{r} \frac{11}{12} \\ - \frac{2}{12} \\ \hline \frac{9}{12} = \frac{3}{4} \end{array}$$

The difference is $\frac{3}{4}$.

b. $\frac{5}{8} - \frac{1}{2}$

$$\begin{array}{rcr} \frac{5}{8} & = & \frac{5}{8} \\ - \frac{1}{2} & = & - \frac{4}{8} \\ \hline & & \frac{1}{8} \end{array}$$

The difference is $\frac{1}{8}$.

EXERCISES ***Add or subtract. Write each answer in simplest form.***

1. $\begin{array}{r} \frac{7}{9} \\ - \frac{4}{9} \\ \hline \end{array}$

2. $\begin{array}{r} \frac{3}{8} \\ + \frac{1}{8} \\ \hline \end{array}$

3. $\begin{array}{r} \frac{5}{6} \\ - \frac{1}{6} \\ \hline \end{array}$

4. $\begin{array}{r} \frac{4}{5} \\ -\ \frac{1}{2} \\ \hline \end{array}$

5. $\begin{array}{r} \frac{6}{7} \\ +\ \frac{1}{3} \\ \hline \end{array}$

6. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{6} \\ \hline \end{array}$

7. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{5}{12} \\ \hline \end{array}$

8. $\begin{array}{r} \frac{1}{7} \\ +\ \frac{4}{5} \\ \hline \end{array}$

9. $\begin{array}{r} \frac{13}{15} \\ -\ \frac{2}{3} \\ \hline \end{array}$

10. $\begin{array}{r} \frac{1}{2} \\ \frac{3}{5} \\ +\ \frac{2}{5} \\ \hline \end{array}$

11. $\begin{array}{r} \frac{1}{8} \\ \frac{3}{4} \\ +\ \frac{1}{2} \\ \hline \end{array}$

12. $\begin{array}{r} \frac{3}{5} \\ \frac{7}{10} \\ +\ \frac{1}{4} \\ \hline \end{array}$

APPLICATIONS

13. Reginald planted $\frac{2}{5}$ of his garden with tomatoes and $\frac{1}{4}$ of his garden with green beans. How much of his garden is planted with either tomatoes or green beans? How much of his garden is planted with other crops?

14. Tina rode her bicycle $\frac{2}{3}$ mile to the park and then $\frac{1}{2}$ mile to the library. Finally she rode her bicycle $\frac{3}{5}$ mile to her home. How far did Tina ride her bike?

15. In a survey, $\frac{2}{7}$ of the people said they preferred Brand A, and $\frac{1}{5}$ of the people said they preferred Brand B. What is the difference between the fraction of people who prefer Brand A and the fraction of people who prefer Brand B?

SKILL 28

Name ______________________________ Date ____________

Multiplying and Dividing Fractions

To multiply fractions, multiply the numerators and multiply the denominators.

EXAMPLE ***What is the product $\frac{3}{8}$ of and $\frac{2}{3}$?***

$$\frac{3}{8} \times \frac{2}{3} = \frac{3 \times 2}{8 \times 3}$$ *Multiply the numerators. Multiply the denominators.*

$$= \frac{6}{24} \text{ or } \frac{1}{4}$$ *Simplify.*

The product is $\frac{1}{4}$.

To divide by a fraction, multiply by its reciprocal.

EXAMPLE ***What is the quotient of $\frac{3}{5}$ and $\frac{1}{2}$?***

$$\frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \frac{2}{1}$$ *Multiply by the reciprocal of $\frac{1}{2}$.*

$$= \frac{3 \times 2}{5 \times 1}$$ *Multiply the numerators. Multiply the denominators.*

$$= \frac{6}{5} \text{ or } 1\frac{1}{5}$$ *Simplify.*

The quotient is $\frac{6}{5}$ or $1\frac{1}{5}$.

EXERCISES ***Multiply or divide. Write each answer in simplest form.***

1. $\frac{1}{2} \times \frac{2}{3}$

2. $\frac{1}{2} \div \frac{2}{3}$

3. $\frac{4}{5} \times \frac{1}{6}$

4. $\frac{5}{7} \div \frac{5}{6}$

5. $\frac{4}{5} \times \frac{3}{4}$

6. $\frac{3}{5} \div \frac{1}{3}$

7. $\frac{4}{7} \times \frac{2}{3}$

8. $\frac{5}{6} \div \frac{2}{3}$

9. $\frac{3}{4} \times \frac{5}{6}$

10. $\frac{1}{7} \div \frac{2}{3}$

11. $\frac{5}{6} \times \frac{1}{3}$

12. $\frac{7}{8} \div \frac{1}{6}$

13. $\frac{2}{5} \times \frac{3}{4}$

14. $\frac{1}{6} \div \frac{1}{9}$

15. $\frac{4}{5} \times \frac{1}{2}$

16. $\frac{7}{9} \div \frac{2}{3}$

17. $\frac{3}{8} \times \frac{4}{5}$

18. $\frac{8}{9} \div \frac{2}{3}$

19. $\frac{6}{7} \times \frac{2}{3}$

20. $\frac{3}{7} \div \frac{2}{3}$

21. $\frac{1}{8} \times \frac{6}{7}$

APPLICATIONS

22. Of the 48 NBA World Championship Series from 1947 to 1994, the Boston Celtics won $\frac{5}{16}$ of the championships. Two thirds of the Celtics' championships occurred before 1970. What fraction represents the championships that were won by the Celtics before 1970?

23. About $\frac{1}{11}$ of the land in the continental United States is in Texas. About $\frac{5}{9}$ of the land in Texas is used as rural pastureland. What fraction of the land in the continental United States is Texas pastureland?

24. Helen planted vegetables and flowers in her garden. Three fourths of her garden is planted in flowers. If $\frac{1}{10}$ of the total garden is planted in roses, what fraction of the flower garden is planted in roses?

25. One third of the videos at Vinnie's Video Store are appropriate for young children. If $\frac{2}{5}$ of the children's videos are cartoons, what fraction of the videos in the store are children's cartoons?

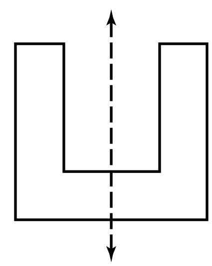
SKILL 29

Name ______________________ Date ____________

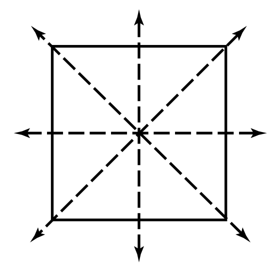
Line Symmetry

If a figure can be folded in half so that the two halves match exactly, the figure has a line of symmetry.

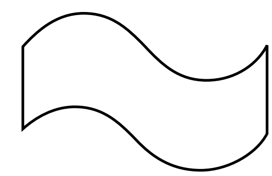
EXAMPLE ***Draw all lines of symmetry for each figure.***



one line of symmetry



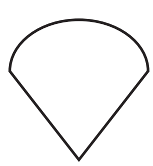
four lines of symmetry



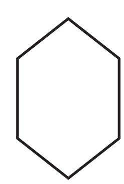
no lines of symmetry

EXERCISES ***Draw all lines of symmetry for each figure.***

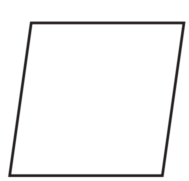
1.



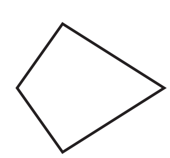
2.



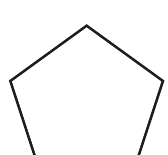
3.



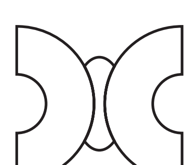
4.



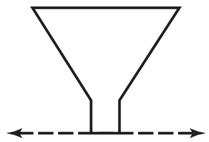
5.

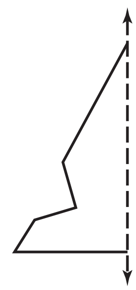


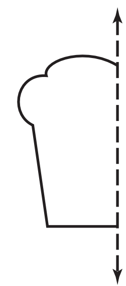
6.



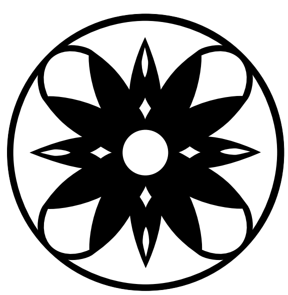
Complete each figure so that the dashed line is a line of symmetry.

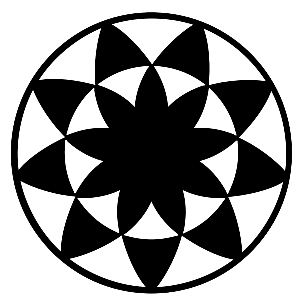
7. 

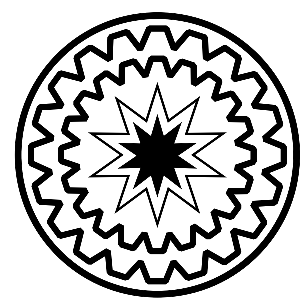
8. 

9. 

APPLICATIONS ***The following are designs from Navaho baskets. Determine the number of lines of symmetry for each of the designs.***

10. 

11. 

12. 

Printers use many fonts or styles of type. For Exercises 13–16, consider block capital letters.

13. List the letters that have a vertical line of symmetry.

14. List the letters that have a horizontal line of symmetry.

15. List the letters that have no line of symmetry.

16. List the letters that have more than one line of symmetry.

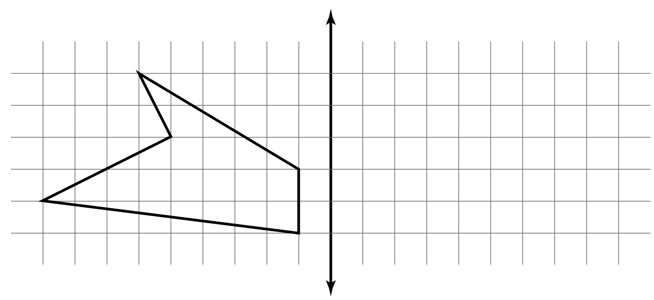
SKILL 30

Name ______________________ **Date** __________

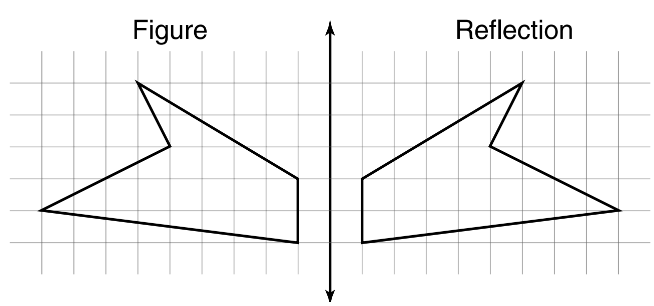
Reflections

In a **transformation**, every point in an image corresponds to exactly one point on the figure. **Reflections** are one type of transformation.

EXAMPLE ***Use the grid to reflect, or flip, the figure over the given line.***

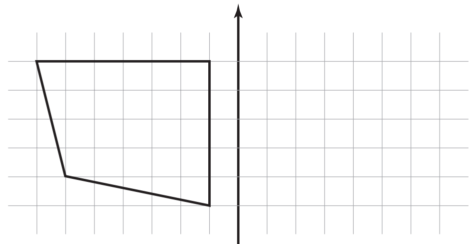


For each vertex on the figure, find the point that is exactly the same distance from the line of reflection, but on the other side of the line. Draw the completed image.

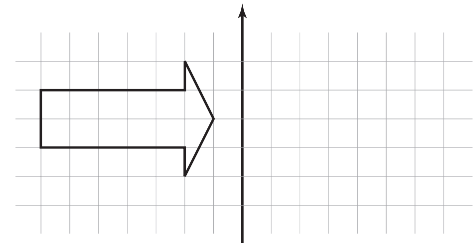


EXERCISES ***Reflect each figure over the given line.***

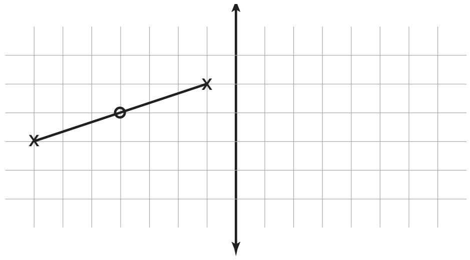
1.



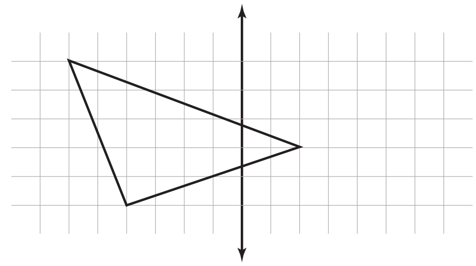
2.



3.



4.



Use your reflections to answer Exercises 5–8.

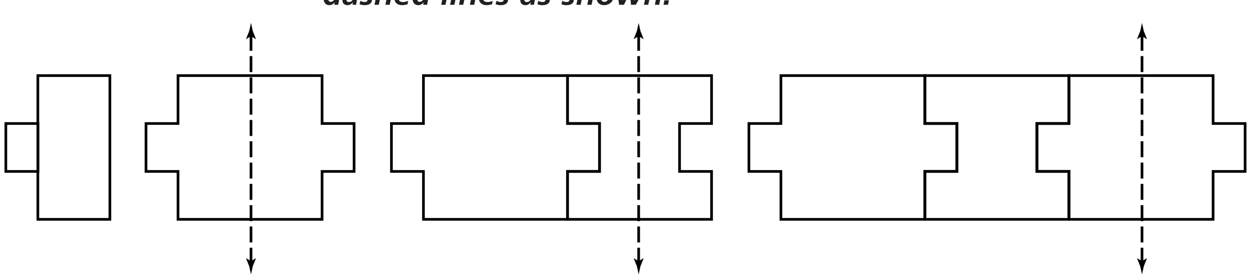
5. Are the reflections in Exercises 1–4 smaller, larger, or the same size as the original figures?

6. In Exercise 2, are the arrows pointing in the same direction? Do you think that direction is the same for a figure and its reflection?

7. In Exercise 3, the x's and the dot are in a straight line. In the reflection, are the x's and the dot in a straight line?

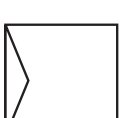
8. In Exercise 3, the dot is between the two x's. In the reflection, is the dot between the two x's?

APPLICATIONS ***M. C. Escher used transformations such as reflections to create interesting art. A simple example of his type of art starts with a square. A simple change is made and this change is reflected over the dashed line. Other reflections are made over other dashed lines as shown.***

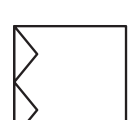


Make a drawing using reflections, squares, and the changes indicated.

9.



10.



11. Make your own design using reflections.

SKILL 31

Name ______________________ Date __________

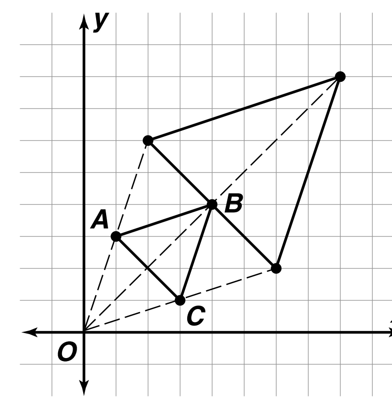
Dilations and Rotations

In mathematics, there are several ways that a figure may be moved or changed. Two of these ways are **dilations** and **rotations**.

EXAMPLES

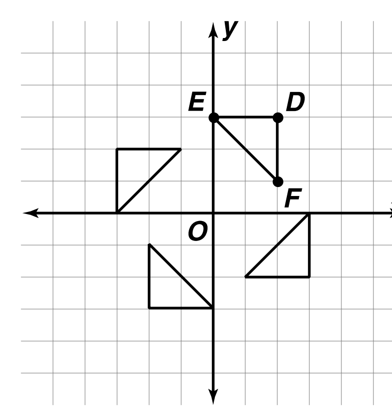
Draw the image of the triangle ABC for a dilation with a scale factor of 2.

Draw a dashed line from the origin of the coordinate plane to point *A*. Extend the dashed line so that its length is twice as long as the distance from the origin to point *A*. This is one vertex of the dilated triangle. Repeat the procedure for the other two vertices and draw the dilated triangle.



Draw three rotated images of triangle DEF. Rotate the image around the origin of the coordinate plane using 90° as the angle for each successive rotation.

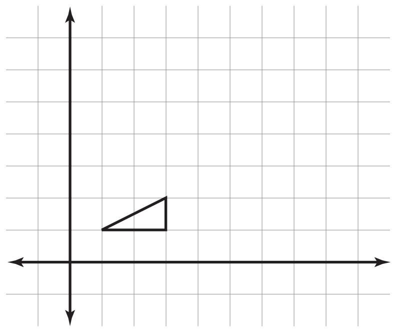
Visualize point *E* rotating around the origin clockwise 90°. Remember that the image point must be the same distance from the origin as the original point. In this case the image of (0, 3) is (3, 0). Find the image points for the other two vertices and draw the rotated triangle. Rotate the image two more times.



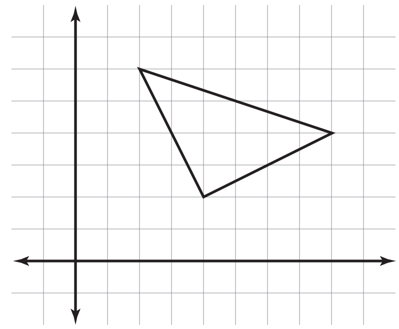
EXERCISES

Draw a dilation for the given scale drawing.

1. Scale factor: 3

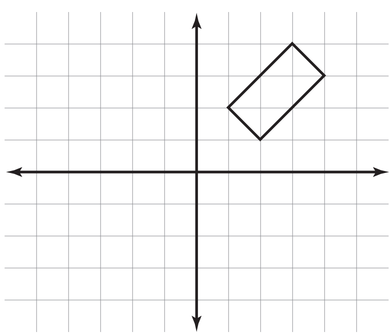


2. Scale factor: $\frac{1}{2}$

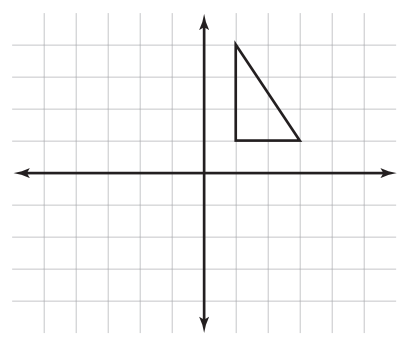


Draw three images using 90° rotations around the origin.

3.



4.



Answer each of the following.

5. Does a dilation form similar or congruent figures?

6. Does a rotation form similar or congruent figures?

APPLICATIONS

7. Does the movement of a Ferris wheel represent a dilation or a rotation?

8. Does an enlargement of a photograph represent a dilation or a rotation?

9. Make a design using rotations.

10. Make a design using dilations.

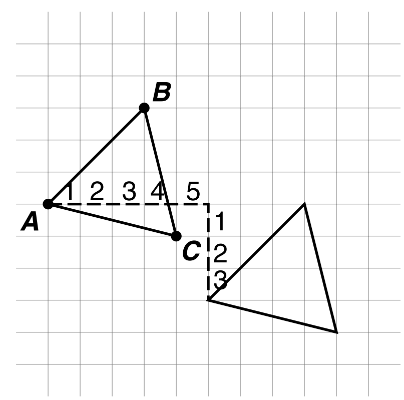
SKILL 32

Name ____________________ Date __________

Translations

A **translation** is a slide or movement of a figure from one place to another.

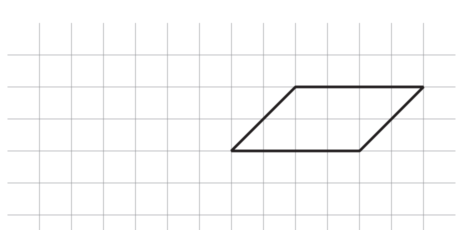
EXAMPLE ***Translate triangle ABC 5 units to the right and 3 units down.***



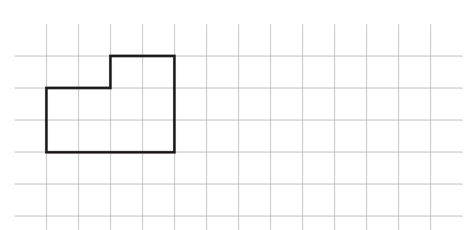
Move point *A* 5 units to the right and 3 units down. Move point *B* 5 units to the right and 3 units down. Finally, move point *C* 5 units to the right and 3 units down and draw the new triangle.

EXERCISES ***Translate each figure as indicated.***

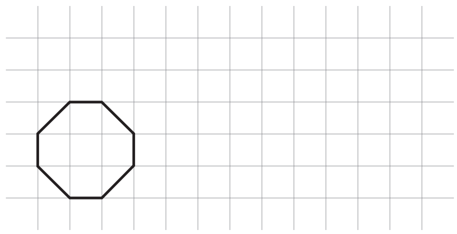
1. 7 units to the left



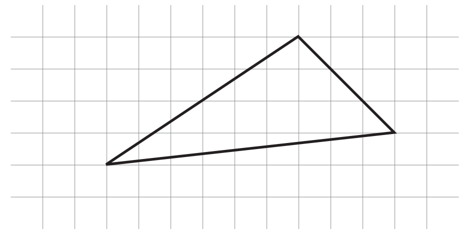
2. 8 units to the right and 2 units down



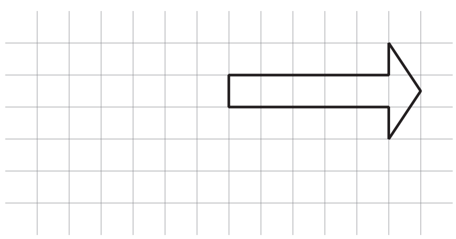
3. 5 units to the right and 2 units up



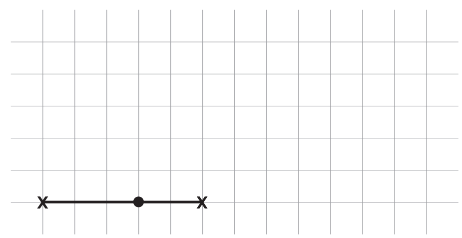
4. 2 units to the left and 1 unit down



5. 5 units to the left and 2 units down



6. 6 units to the right and 4 units up



Answer each question.

7. Are the translated figures congruent or similar to the original figures?

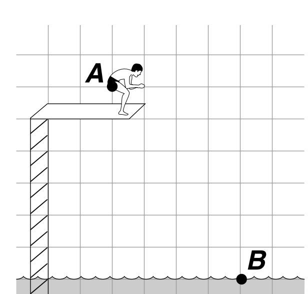
8. In Exercise 5, are the arrows pointing in the same direction? Is direction the same for a figure and its translation?

9. In Exercise 6, the x's and the dot are in a straight line. In the translation, are the x's and the dot in a straight line?

10. In Exercise 6, the dot is between the two x's. In the translation, is the dot between the two x's?

APPLICATIONS

11. Describe the dive from *A* to *B* in terms of a translation.



12. Describe a translation from your house to a friend's house.

SKILL 33

Name ______________________ Date __________

Scale Drawings

Chuck has a scale drawing of Detroit's Tiger Stadium. The scale of the drawing is $\frac{1}{4}$ inch equals 25 feet. On the drawing, the home-run distance from home plate to right field is $3\frac{1}{4}$ inches.

EXAMPLE

What is the actual home-run distance from home plate to right field?

Think of $\frac{1}{4}$ inch as 0.25 inch and $3\frac{1}{4}$ inches as 3.25 inches. Use the scale 0.25 inch equals 25 feet and write a proportion to find the actual distance.

$$\frac{\textit{drawing}}{\textit{actual distance}} \begin{matrix}\rightarrow\\ \rightarrow\end{matrix} \frac{0.25}{25} = \frac{3.25}{x} \begin{matrix}\leftarrow\\ \leftarrow\end{matrix} \frac{\textit{drawing}}{\textit{actual distance}}$$

$$0.25x = 25 \times 3.25 \quad \textit{Cross multiply.}$$

$$0.25x = 81.25$$

$$\frac{0.25x}{0.25} = \frac{81.25}{0.25} \quad \textit{Divide each side by 0.25.}$$

$$x = 325$$

The actual distance is 325 feet.

EXERCISES

On a map, the scale is 1 inch equals 150 miles. For each map distance, find the actual distance.

1. 3 inches

2. 8 inches

3. $\frac{1}{2}$ inch

4. 5 inches

5. $1\frac{1}{2}$ inches

6. $4\frac{1}{2}$ inches

On a scale drawing of a floor plan for a new building, the scale is $\frac{1}{4}$ inch equals 1 foot. Find the actual dimensions of the rooms if the measurements from the drawing are given.

7. 5 inches by 3 inches

8. 2 inches by 4 inches

9. 2 inches by $3\frac{1}{2}$ inches

10. $4\frac{1}{2}$ inches by $4\frac{1}{2}$ inches

11. $3\frac{1}{4}$ inches by $2\frac{1}{2}$ inches

12. $3\frac{3}{4}$ inches by $4\frac{1}{4}$inches

APPLICATIONS ***An igloo is a domed structure built of snow blocks traditionally used by the Inuit people of Canada. Sometimes several families built a cluster of igloos connected by passageways. Use the scale drawing of such a cluster to answer Exercises 13–17.***

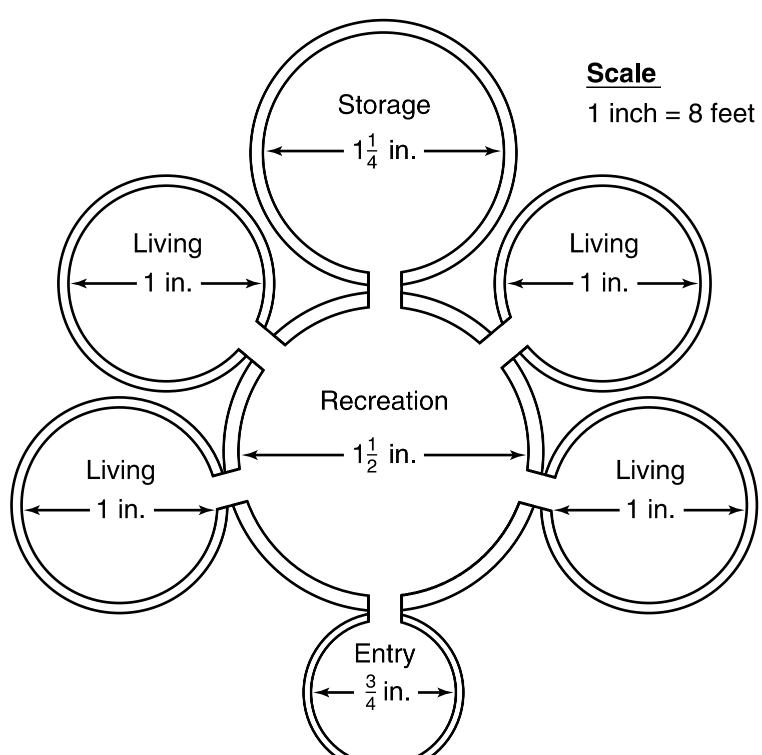
13. What is the actual diameter of the living chambers?

14. What is the actual diameter of the entry chamber?

15. What is the actual diameter of the recreation area?

16. What is the actual diameter of the storage area?

17. Estimate the actual distance from the entry chamber to the back of the storage chamber.



SKILL 34

Name ______________________ Date ____________

Use an Equation

Lucy bought some stickers that cost \$0.25 each and a sticker book for \$3.50. She spent \$6.00.

EXAMPLE ***How many stickers did Lucy buy?***

Let s equal the number of stickers. Write and solve an equation.

s stickers at \$0.25 each plus a \$3.50 book cost \$6.00.

$s \quad \times \quad \$0.25 \quad + \quad \$3.50 \quad = \quad \$6.00$

$$0.25s + 3.50 = 6.00$$

$$0.25s + 3.50 - 3.50 = 6.00 - 3.50 \qquad \textit{Subtract 3.50 from each side.}$$

$$0.25s = 2.50$$

$$0.25s \div 0.25 = 2.50 \div 0.25 \qquad \textit{Divide each side by 0.25.}$$

$$s = 10$$

Lucy bought 10 stickers.

EXERCISES ***Solve by using an equation.***

1. A number increased by 14 is 27. Find the number.

2. The product of a number and 5 is 80. Find the number.

3. A number is divided by 7. Then 6 is added to the result. The result is 26. What is the number?

4. Three times a number minus 17 is equal to 28. What is the number?

5. A number is multiplied by 12. Then 3 is added to the result. If the answer is 51, what is the original number?

6. Twelve less than 16 times a number is 2 less than the product of 10 and 15. What is the number?

APPLICATIONS

7. Ruiz earned $117. If his pay is $6.50 per hour, how many hours did he work?

8. There are 425 students at Dayville Elementary School. If 198 of the students are girls, how many students are boys?

9. Jason is driving to his grandmother's house 635 miles away. He drives 230 miles the first day and 294 miles the second day. How many miles must he drive the third day to reach his grandmother's house?

10. Pachee bought some baseballs for $4 each and a batting glove for $10. She spent $26. How many baseballs did she buy?

11. Fred has saved $490 toward the purchase of an $825 clarinet. His aunt gave him $75 to be used toward the purchase. How much more money must he save?

12. Cindy went to the hobby shop and bought 2 model sports cars at $8.95 each and some paints. If she spent $23.65, what was the cost of the paints?

13. Arlen drove for 3 hours at 52 miles per hour. How fast must he drive during the next 2 hours in order to have traveled a total of 254 miles?

14. Postage costs $0.29 for the first ounce and $0.23 for each additional ounce. Peter spent $1.44 to send a package. How much did it weigh?

SKILL 35

Name ______________________ **Date** __________

Work Backward

Rupesh earned some money mowing lawns one month. He put half of his money into savings. With the rest, he spent \$15 on a new CD, \$6 to see a movie, and \$3 on food. He still had \$24 left in his pocket.

EXAMPLE ***How much money did Rupesh earn mowing lawns?***

Work backward to answer this question. Undo each step.

Start with \$24.	\$24
Add the \$3 spent on food.	\$24 + \$3 = \$27
Add the \$6 spent to see the movie.	\$27 + \$6 = \$33
Add the \$15 spent on the CD.	\$33 + \$15 = \$48
Since Rupesh saved half of the money, multiply by 2.	\$48 × 2 = \$96

Rupesh made \$96 mowing lawns.

EXERCISES ***Solve by working backward.***

1. A number is added to 8, and the result is multiplied by 10. The final answer is 140. Find the number.

2. A number is divided by 8, and the result is added to 12. The final answer is 75. Find the number.

3. A number is decreased by 12. The result is multiplied by 5, and 30 is added to the new result. The final result is 200. What is the number?

4. Twenty five is added to a number. The sum is multiplied by 4, and 35 is subtracted from the product. The result is 121. What is the number?

5. Take a number, divide it by 3, add 14, multiply by 7, and double the answer. The result is 252. What is the number?

APPLICATIONS

6. Dwayne's weight is twice Beth's weight minus 24 pounds. Dwayne weighs 120 pounds. How much does Beth weigh?

7. Kara wants to buy a certain leather jacket, but she did not have enough money. The leather jacket went on sale and was reduced by $15.00, then by $13.50 more, and finally by an additional $12.15. Kara bought the jacket at the final sale price of $109.35. What was the original price?

8. James arrived for piano practice at 4:45 P.M. On the way from school, he stopped at the video store for 15 minutes and also made a call from the phone booth for 10 minutes. It usually takes 25 minutes to get from the school to the piano teacher's house. What time did James leave school?

9. Dave has 12 baseball cards left after trading cards. This is one third as many as he had yesterday, which is 8 less than the day before. How many cards did Dave have on the day before yesterday?

10. A fence is put around a dog run 10 feet wide and 20 feet long. Enough fencing is left over to also fence a square garden with an area of 25 square feet. If there is 3 feet left after the fencing is completed, how much fencing was available at the beginning?

SKILL 36

Name ______________________________ **Date** ____________

Solve Equations Involving Addition and Subtraction

Addition Property of Equality: If you add the same number to each side of an equation, the two sides remain equal.

EXAMPLE ***Solve $t - 57 = 46$.***

$$t - 57 = 46$$

$$t - 57 + 57 = 46 + 57 \qquad \textit{Add 57 to each side.}$$

$$t = 103$$

Check:

$$t - 57 = 46$$

$$103 - 57 \stackrel{?}{=} 46 \qquad \textit{Replace t with 103.}$$

$$46 = 46 \ \checkmark$$

The solution is 103.

Subtraction Property of Equality: If you subtract the same number from each side of an equation, the two sides remain equal.

EXAMPLE ***Solve $t + 24.4 = 25.1$.***

$$t + 24.4 = 25.1$$

$$t + 24.4 - 24.4 = 25.1 - 24.4 \qquad \textit{Subtract 24.4 from each side}$$

$$t = 0.7 \qquad \textit{of the equation.}$$

Check:

$$t + 24.4 = 25.1$$

$$0.7 + 24.4 \stackrel{?}{=} 25.1 \qquad \textit{Replace t with 0.7.}$$

$$25.1 = 25.1 \ \checkmark$$

The solution is 0.7.

EXERCISES ***Complete each statement.***

1. $y + 18 = 39$
$y + 18 - 18 = 39 -$ ____

2. $m - 23 = 17$
$m - 23 +$ _____ $=$ _____ $+$ _____

Solve each equation. Check your solution.

3. $w + 6 = 19$

4. $n - 4.7 = 8.4$

5. $m + 18 = 78$

6. $18.42 + t = 63$

7. $e - 0.9 = 17.4$

8. $b - 43 = 18$

9. $h - 32\frac{3}{5} = 44$

10. $947 = p - 43$

11. $7.36 + w = 8.94$

12. $g - 6.3 = 9.5$

13. $r - 18 = 36$

14. $2.17 + k = 4.19$

APPLICATIONS

Each of Exercises 15–18 can be modeled by one of these equations:

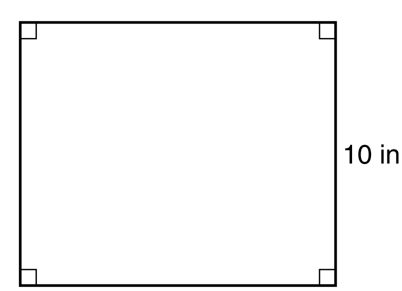
n + 2 = 10 n − 2 = 10

Choose the correct equation. Then solve the problem.

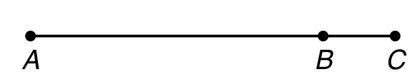
15. Jameel loaned two tapes to a friend. He has ten tapes left. How many tapes did Jameel originally have?

16. Ana needs \$2 more to buy a \$10 scarf. How much money does she already have?

17. The width of the rectangle shown at the right is 2 inches less than the length. What is the length?



18. In the figure at the right, the length of $\overline{AC}$ is 10 centimeters. The length of $\overline{BC}$ is 2 centimeters. What is the length of $\overline{AB}$?



SKILL 37

Name ______________________ Date __________

Solve Equations Involving Multiplication and Division

Division Property of Equality: If you divide each side of an equation by the same nonzero number, the two sides remain equal.

EXAMPLE ***Solve 156 = 4r.***

$$156 = 4r$$

$$\frac{156}{4} = \frac{4r}{4} \qquad \textit{Divide each side by 4.}$$

$$39 = r$$

Check: $156 = 4r$

$$156 \stackrel{?}{=} 4 \times 39 \qquad \textit{Replace r with 39.}$$

$$156 = 156 \ \checkmark$$ The solution is 39.

Multiplication Property of Equality: If you multiply each side of an equation by the same number, the two sides remain equal.

EXAMPLE ***Solve $\frac{w}{21} = 4.2$.***

$$\frac{w}{21} = 4.2$$

$$\frac{w}{21} \times 21 = 4.2 \times 21 \qquad \textit{Multiply each side by 21.}$$

$$w = 88.2$$

Check: $\frac{w}{21} = 4.2$

$$\frac{88.2}{21} \stackrel{?}{=} 4.2 \qquad \textit{Replace w with 88.2.}$$

$$4.2 = 4.2 \ \checkmark$$ The solution is 88.2.

EXERCISES ***Complete the solution of each equation.***

1. $12h = 48$

$\frac{12h}{12} = \frac{48}{\_\_\_}$

$h = \_\_\_$

2. $34 = \frac{r}{3}$

$34 \times \_\_\_ = \frac{r}{3} \times \_\_\_$

$\_\_\_ = r$

Solve each equation. Check your solution.

3. $3.6t = 11.52$

4. $\frac{n}{4} = 15$

5. $\frac{1}{2}w = \frac{3}{8}$

6. $1.4j = 0.7$

7. $4.1m = 13.12$

8. $\frac{c}{5} = 16$

9. $1.3z = 3.9$

10. $\frac{7}{8} = \frac{1}{2}f$

11. $\frac{d}{3.5} = 0.6$

12. $h \div 12 = 4.8$

13. $4.8g = 15.36$

14. $c \div \frac{1}{4} = \frac{1}{2}$

APPLICATIONS ***Each of Exercises 15–17 can be modeled by one of these equations:***

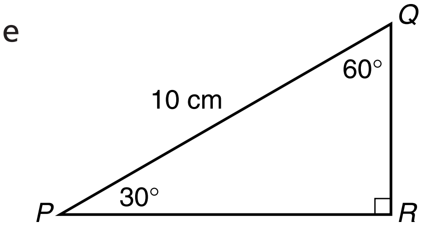
$$2n = 10 \qquad \frac{n}{2} = 10$$

Choose the correct equation. Then solve the problem.

15. Chad earned \$10 for working two hours. How much did he earn per hour?

16. Kathy and her brother won a contest and shared the prize equally. Each received \$10. What was the amount of the prize? $\boldsymbol{\frac{n}{2}}$

17. In the triangle at the right, the length of $\overline{PQ}$ is twice the length of $\overline{QR}$. What is the length of $\overline{QR}$?



SKILL 38

Name ______________________________ Date ____________

Solve Inequalities

Inequalities are sentences that compare two quantities that are not necessarily equal. The symbols below are used in inequalities.

Symbol	Words
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

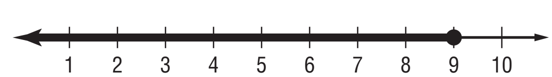
EXAMPLES ***Solve each inequality. Show the solution on a number line.***

$$\begin{aligned} 2n + 1 &> 5 & \\ 2n + 1 - 1 &> 5 - 1 & \textit{Subtract 1 from each side.} \\ 2n &> 4 & \\ \frac{2n}{2} &> \frac{4}{2} & \textit{Divide each side by 2.} \\ n &> 2 & \end{aligned}$$

To graph the solution on a number line, draw an open circle at 2. Then draw an arrow to show all numbers greater than 2.

$$\begin{aligned} 2p - 3 &\leq 15 & \\ 2p - 3 + 3 &\leq 15 + 3 & \textit{Add 3 to each side.} \\ 2p &\leq 18 & \\ \frac{2p}{2} &\leq \frac{18}{2} & \textit{Divide each side by 2.} \\ n &\leq 9 & \end{aligned}$$

To graph the solution on a number line, draw a closed circle at 9. Then draw an arrow to show all numbers less than 9.



EXERCISES

Solve each inequality. Graph the solution on a number line.

1. $a + 7 < 12$

2. $b - 3 > 8$

3. $2c - 7 \geq 9$

4. $5d + 7 \leq 32$

5. $e + 2 > 16$

6. $f + 12 < 18$

7. $\frac{g}{2} \geq 3$

8. $\frac{h}{2} + 6 < 8$

9. $\frac{j}{3} + 6 \leq 10$

10. $\frac{k}{4} + 2 > 3$

APPLICATIONS

19. Madison wants to earn at least $75 to spend at the mall this weekend. Her father said he would pay her $15 to mow the lawn and $5 an hour to work on the landscaping. If Madison mows the lawn, how many hours must she work on the landscaping to earn at least $75?

20. A rental car agency rents cars for $32 per day. They also charge $0.15 per mile driven. If you are taking a 5-day trip and have budgeted $250 for the rental car, what is the maximum number of miles you can drive and stay within your budget?

21. Mr. Stamos needs 1,037 valid signatures on a petition to become a candidate for the school board election. An official at the board of elections told him to expect that 15% of the signatures he collects will be invalid. What is the minimum number of signatures he should get to help ensure that he qualifies for the ballot?

SKILL 39

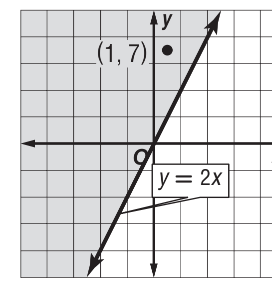
Name ______________________ **Date** ____________

Graphing Inequalities

EXAMPLE ***Graph the inequality y > 2x on a coordinate plane.***

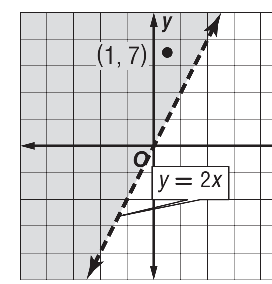
Find the equation that corresponds to the inequality. $y = 2x$

Graph the equation.



The inequality represents a range of possible points, all those whose y coordinate is more than 2 times their x coordinate.

Locate a point that satisfies the inequality. Try (1, 7).



Since $7 > 2$, this point satisfies the inequality. Therefore, it must be in the area of points that satisfy the inequality. All of the points on the same side of the $y = 2x$ line as this point are possible solutions.

Shade the area of the graph that includes the possible solutions.

What about points on the line $y = 2x$ itself? Are they solutions to $y > 2x$? No value for y can be equal to $2x$ and also larger than $2x$. Use a dotted line to indicate that the line $y = 2x$ is the boundary of the solution area, but that it is not included in the area.

If you were graphing $y \geq 2x$, the line would be included in the solution area, because the solutions to the equation are also solutions to the inequality. You would indicate by drawing a solid line instead of a dotted one.

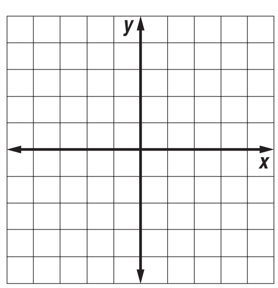
EXERCISES ***For Exercises 1 and 2 use the inequality $y \geq 5x + 3$.***

1. If you graphed this inequality, would you use a solid line or a dotted line for the edge of the solution area? Explain your answer.

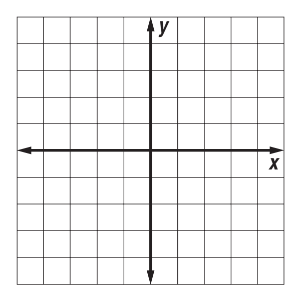
2. What part of the graph would you shade? Explain your answer.

Convert each inequality into linear form and then graph.

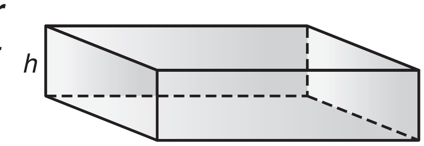
3. $y - 2x > 4$



4. $2y + 3x \geq 4x + y + 3$



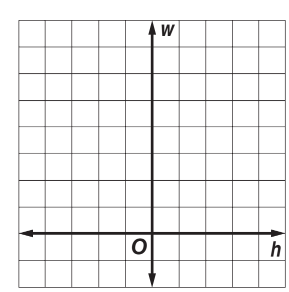
APPLICATIONS ***The volume of a box with a rectangular bottom varies depending on the height of the box.***



5. The area of the bottom of the box is 20 cm^2. Write an expression to represent the volume of the box, using h to represent the box's height.

6. Use your expression from Exercise 5 to write an inequality representing the amount of water that could be in the box. Use w to represent the amount of water.

7. Graph the inequality from Exercise 6.



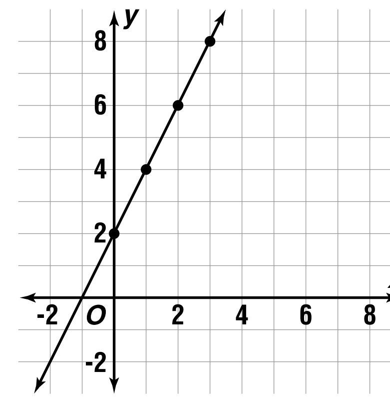
SKILL 40

Name ______________________ Date __________

Graphing Equations

EXAMPLE ***Graph the equation $y = 2x + 2$.***

Make a function table for $y = 2x + 2$. Then graph each ordered pair and complete the graph.



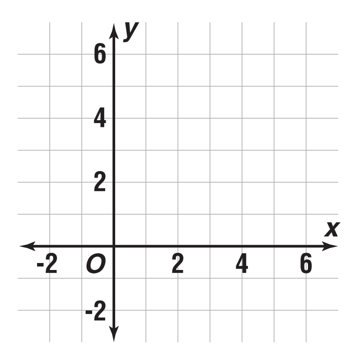
$y = 2x + 2$

x	$2x + 2$	y	(x, y)
0	$2(0) + 2$	2	(0, 2)
1	$2(1) + 2$	4	(1, 4)
2	$2(2) + 2$	6	(2, 6)
3	$2(3) + 2$	8	(3, 8)

EXERCISES ***Complete each function table. Then graph the equation.***

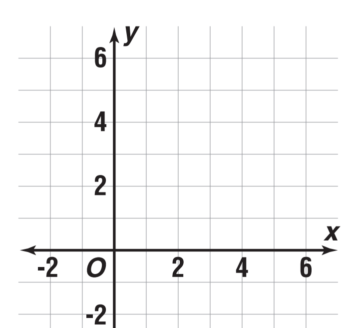
1. $y = x - 1$

x	$x - 1$	y	(x, y)
1			
2			
3			
4			
5			



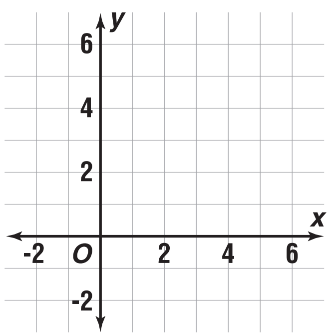
2. $y = 5 - x$

x	$5 - x$	y	(x, y)
0			
1			
2			
3			
4			

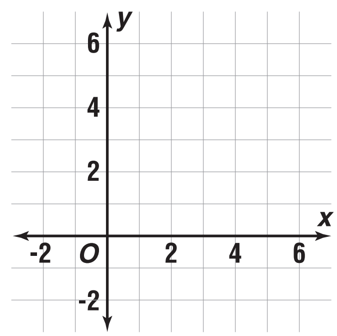


Graph each equation.

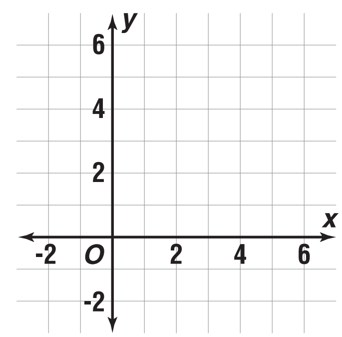
3. $y = x + 2$



4. $y = 3x$

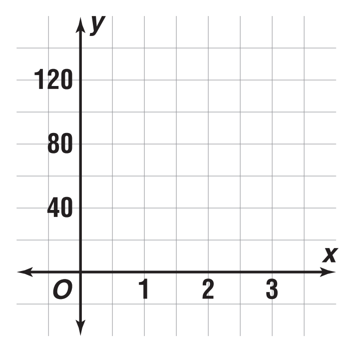


5. $y = \frac{1}{2}x + 1$

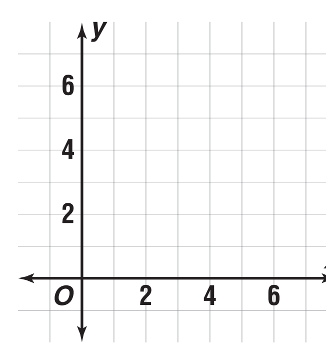


APPLICATIONS

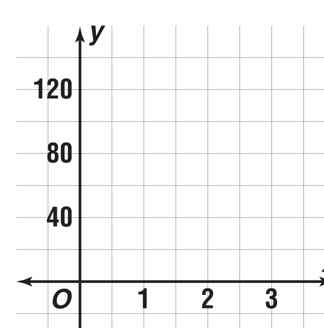
6. An electrician charges an initial fee of \$40, plus \$50 for every hour she works. Let x represent the number of hours she works and y represent the total fee. Write an equation to represent the total fee. Graph the equation.



7. A blizzard at the Slippery Ski Area deposited $\frac{1}{2}$ foot of snow per hour atop a 3-foot snow base. Let x represent the number of hours and y represent the total amount of snow. Write an equation to represent the total amount of snow. Graph the equation.



8. Yukari averages 40 miles per hour when she drives from Los Angles to San Francisco. Let x represent the number of hours and y represent the distance traveled. Write an equation to represent the distance traveled. Graph the equation.



SKILL 41

Name ______________________ Date __________

Solve Equations With Two Variables

An ordered pair that makes an equation true is a solution for the equation.

EXAMPLE ***Find four solutions for the equation $y = 5x - 1$.***

Choose values for x.	Calculate y values.	Write ordered pairs.
Let $x = -4$.	$y = 5(-4) - 1 = -21$	$(-4, -21)$
Let $x = -2$.	$y = 5(-2) - 1 = -11$	$(-2, -11)$
Let $x = 0$.	$y = 5(0) - 1 = -1$	$(0, -1)$
Let $x = 2$.	$y = 5(2) - 1 = 9$	$(2, 9)$

Four solutions are $(-4, -21)$, $(-2, -11)$, $(0, -1)$, and $(2, 9)$.

EXERCISES ***Complete the table for each equation. Then use the results to write four solutions for each equation. Write the solutions as ordered pairs.***

1. $y = 3x + 2$

x	$3x + 2$	y
1	$3(1) + 2$	
2	$3(2) + 2$	
3	$3(3) + 2$	
4	$3(4) + 2$	

2. $y = 4x$

x	$4x$	y
-1	$4(-1)$	
0	$4(0)$	
1	$4(1)$	
2	$4(2)$	

3. $y = -3x - 4$

x	$-3x - 4$	y
-1	$-3(-1) - 4$	
0	$-3(0) - 4$	
1	$-3(1) - 4$	
2	$-3(2) - 4$	

Find four solutions for each equation. Write your solutions as ordered pairs.

4. $y = x - 4$

5. $y = 3x + 1$

6. $y = -3$

7. $y = -2x - 2$

8. $y = 2.5x$

9. $y = -2x + 4$

10. $y = -\frac{1}{2}x - 4$

11. $y = \frac{1}{3}x + 1$

12. $y = \frac{1}{2}x + 3$

APPLICATIONS

13. One number is three more than half another number. Determine which ordered pairs in the set {(0, 3), (–2, 2), (4, –1), $(1, 3\frac{1}{2})$} are solutions for the two numbers.

14. An organization donates one third of all the money it raises for housing the homeless. How much will it donate if it raises $6,000?

15. You can show the distance in feet it takes a car to stop when traveling at a certain speed on a dry, concrete surface by using the formula $d = 0.042\ s^2 + 1.1s$. Complete the table to find the distance for each speed. Round the distances to the nearest foot.

speed in mph (*s*)	30	35	40	45	50	55	60	65	70	75
distance in feet (*d*)										

SKILL 42

Name ______________________ Date __________

Function Tables

The data at the right shows the shipping and handling charged by a catalog company.

Maximum Purchase (dollars)	Shipping and Handling (dollars)
50	6.95
100	9.95
150	12.95
200	15.95
250	
300	
350	

EXAMPLE ***Complete the table.***

First look for a pattern in the data that is already given. Each entry in the shipping and handling column is $3 greater than the previous entry. So, to complete the table, add $3 to each entry in the second column to get the next entry. The entries for the last 3 rows of the table are given below.

Maximum Purchase	Shipping and Handling
250	18.95
300	21.95
350	24.95

EXERCISES ***Complete each table.***

1.

Principal (dollars)	Interest (dollars)
1,000	10
1,500	15
2,000	20
2,500	25
3,000	
3,500	
4,000	
4,500	

2.

Distance (feet)	Time (seconds)
5	7.5
10	15
15	22.5
20	30
25	37.5
30	
35	
40	

3.

Purchase (dollars)	Tax (dollars)
10	0.60
20	1.20
30	1.80
40	2.40
50	
60	
70	
80	

4.

Length of call (minutes)	Cost (dollars)
1	1.00
2	1.35
3	1.70
4	2.05
5	
6	
7	
8	

APPLICATIONS

The table at the right shows the amount of Federal individual income tax for 1993 for different amounts of adjusted gross income between $22,100 and $53,500 for single taxpayers. Use the data to answer Exercises 5–7.

Adjusted Gross Income (dollars)	Income Tax (dollars)
25,000	7,000
30,000	8,400
35,000	9,800
40,000	
45,000	
50,000	

5. Complete the table.

6. Make a new table that includes 27,500, 32,500, 37,500, 42,500, 47,500, and 52,500 in the adjusted gross income column. Explain how you found the income tax for these amounts.

7. Do you think it would be useful to have a table that contains more data? Why or why not? How can you add more data to the table?

8. The rate for single taxpayers with an adjusted gross income between $53,500 and $115,000 is 31%. Make a table using adjusted gross incomes of $55,000, $60,000, $65,000, $70,000, $75,000, $80,000, $85,000, and $90,000.

9. Extend the table you made in Exercise 8 to include any additional data you think would be useful. Explain why you included the data you did.

SKILL 43

Name ______________________________ Date ____________

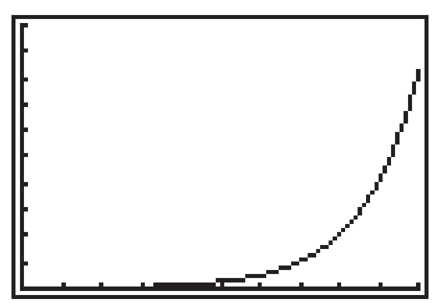
Graphing Exponential Equations

Jamie conducted an experiment that began with 400 bacteria. He found that the number of bacteria, y, after x hours was given by the equation $y = 400(2^x)$.

EXAMPLE ***Use a graphing calculator to graph this equation.***

Follow the steps below to graph the equation.

1. Press the [Y=] key. Then enter the equation by pressing 400 [×] 2 [^] [X,T,θ] .
2. Press [WINDOW] to view the current boundaries of the viewing window of the calculator. Set the boundaries at Xmin = 0, Xmax = 10, Xscl = 1, Ymin = 0, Ymax = 500000, and Yscl = 50000.
3. Press [GRAPH] to draw the graph shown below.



EXERCISES ***Use a graphing calculator to graph each equation. Make a sketch of each screen.***

1. $y = 5^x$

2. $y = 0.8^x$

3. $y = \left(\frac{1}{8}\right)^x$

4. $y = 2^{2x}$

5. $y = 30(0.5^x)$

6. $y = 500(0.25^x)$

APPLICATIONS

Carbon-14 has a half-life of 5,730 years. Manford has a sample that contains 200 g of carbon-14. The equation for the grams of carbon-14 in the sample, y, after x 5,730-year intervals is given by the equation $y = 200(0.5^x)$.

7. Use a graphing calculator to graph this equation.

8. How would you use the information shown on this graph?

9. Do you think this graph is the best way to display this information? Why or why not?

10. Jaunita conducted an experiment that began with 200 bacteria. She found that the number of bacteria, y, after x hours was given by the equation $y = 200(3^x)$. Use a graphing calculator to graph this equation. How would you use the information shown on this graph?

SKILL 44

Name ______________________ Date ____________

Quadratic Equations and Graphs

EXAMPLE

Graph the equation $y = \frac{x^2}{4}$.

Because there is a quadratic term in the equation, you should expect the shape of the graph to be a parabola (U-shape).

Find some points that satisfy the equation.

Choose a value for *x*.	$x = 0$
Substitute the *x*-value into the equation.	$y = \frac{0^2}{4}$
Determine the corresponding value of *y*.	$y = 0$

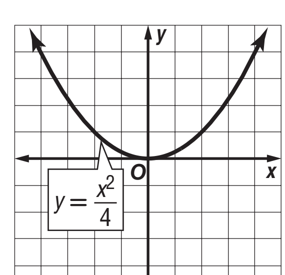
The point (0, 0) is on the graph.

An input-output table can help you keep track of the points you find and can also help you see patterns in the *x*- and *y*-values.

x	0	1	-1	2	-2	4	-4
y	0	$\frac{1}{4}$	$-\frac{1}{4}$	1	1	4	4

Try to pick points that will be easy to graph and give you useful information about the shape of the graph. For example, it is a good idea to try 0 for *x*, and to figure out (if possible) what values of *x* correspond to a *y*-value of 0. In this example, knowing that you will have to divide x^2 by 4, it makes sense to try even *x* values, so that you will be able to simplify the resulting fraction.

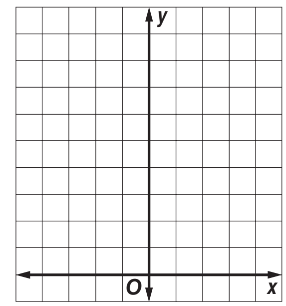
Graph each point. When you have enough points to get a sense of the shape of the graph, connect all your points with a smooth curve.



EXERCISES ***In each exercise, find some points that satisfy the equation. Record the points in the input-output table. Graph the points and connect them with a smooth curve.***

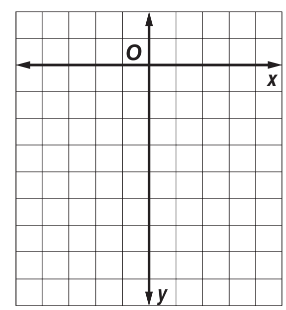
1. $y = 4x^2$

x							
y							



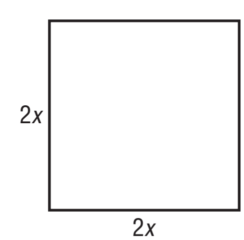
2. $y = -2x^2$

x							
y							

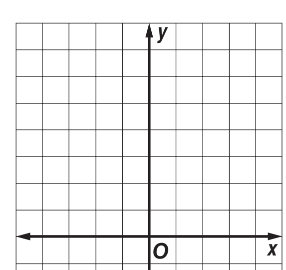


APPLICATIONS ***The sides of this square are the same length as the longer side of the rectangle.***

3. Write an equation to calculate the area of this square.



4. Make a graph showing how the area of the square changes as the value of x changes.



SKILL 45

Name ____________________ Date __________

Inverse Relationships

The variables x and y are **inversely proportional** when the product of x and y is always the same. This relationship is also called a **reciprocal relationship**.

EXAMPLE ***There are different families of equations that express inverse relationships. In the equations below, x and y are variables, a, b, and c are constants.***

Family A

$xy = a$ or $y = \frac{a}{x}$

In equations like this, x and y are inversely proportional.

Family B

$(x + b)y = 1$ or $y = \frac{1}{(x + b)}$

In this family of equations, y is not inversely proportional to x. Instead, y is inversely proportional to $(x + b)$.

Family C

$x(y - c) = 1$ or $y = \frac{1}{x + c}$

In this family of equations, x is inversely proportional to $(y - c)$

EXERCISES ***For each exercise, decide if the equation represents an inverse relationship. If it does not, tell what kind of relationship the equation does represent.***

1. $y = \frac{2}{x}$

2. $x + 3 = y$

3. $5xy = 7$

4. $y + 1 = \frac{1}{x}$

5. $y = x^2 + 2$

6. $y = \frac{1}{(x + 5)}$

7. $y = \frac{1}{x - 15}$

8. $\frac{1}{y} = 2x$

9. $xy = x + 3$

APPLICATIONS ***For each exercise, do parts a – c.***

a. Write an equation that belongs to the given family.

b. Complete the input-output table with points that fit the equation.

c. Graph the points on a pair of axes.

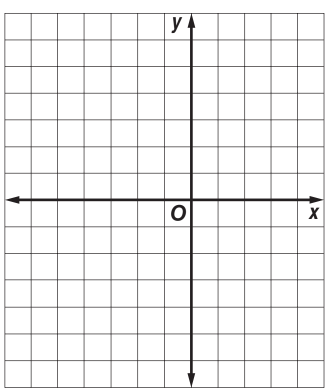
10. Family A

a. $z = \frac{3}{m}$

b.

m							
z							

c.



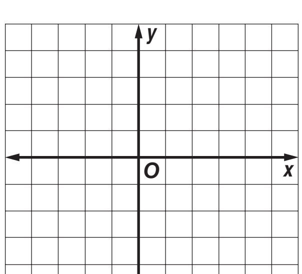
11. Family B

a. $t = \frac{1}{(s - 1)}$

b.

s							
t							

c.



SKILL 46

Name ______________________ Date __________

Prime Factorization

Evelyn has 105 books. She is trying to decide how to put them on the shelves of 3 separate bookcases.

EXAMPLE ***How can she arrange the books if she wants to have the same number of books on each shelf?***

To solve this problem, find the **prime factorization** of 105.

$$\begin{aligned}105 &= 3 \cdot 35 \\ &= 3 \cdot 5 \cdot 7\end{aligned}$$

She can put 5 books on each of 7 shelves or 7 books on each of 5 shelves.

EXERCISES ***Find the prime factorization of each number.***

1. 75 **2.** 36 **3.** 49 **4.** 72

5. 90 **6.** 42 **7.** 100 **8.** 121

9. 275 **10.** 385 **11.** 210 **12.** 147

13. 525 **14.** 66 **15.** 196 **16.** 500

17. 136 **18.** 495 **19.** 231 **20.** 1,001

21. 234 **22.** 84 **23.** 255 **24.** 252

APPLICATIONS

Monty's yard has dimensions of 35 feet by 35 feet. He wants to construct a rectangular garden in his yard. Use this information to answer Exercises 25–27.

25. Monty decides that the garden should have an area of 95 square feet. What are the whole number dimensions that are possible for this garden?

26. Monty changes his mind and decides that the garden should have an area of 100 square feet. What are the whole number dimensions that are possible for this garden?

27. Monty's neighbor asks Monty if he wants to construct a garden that they could share. One-half of the garden would be in Monty's yard and one-half would be in his neighbor's yard. His neighbor's yard has dimensions 40 feet by 35 feet. They decide to construct a rectangular garden with an area of 250 feet. What are the whole number dimensions that are possible for this garden?

SKILL 47

Name ______________________ Date __________

Greatest Common Factor (GCF)

Carlos is 20 years old and his brother Thomas is 24 years old. The **greatest common factor (GCF)** of their ages is the same as their niece Cristina's age.

EXAMPLE ***How old is Cristina?***

To find Cristina's age, find the GCF of 20 and 24. One way to find the GCF is to find the prime factorization of each number.

$$20 = 2 \cdot 10 \qquad 24 = 2 \cdot 12$$

$$= 2 \cdot 2 \cdot 5 \qquad = 2 \cdot 4 \cdot 3$$

$$= 2 \cdot 2 \cdot 2 \cdot 3$$

Then find the common prime factors.

$$20 = 2 \cdot 2 \cdot 5$$

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

The common prime factors are 2 and 2. So, the greatest common factor of 20 and 24 is $2 \cdot 2$, or 4. So Cristina is 4 years old.

EXERCISES ***Find the GCF for each set of numbers.***

1. 16, 24 **2.** 15, 18 **3.** 18, 36 **4.** 32, 48

5. 28, 70 **6.** 72, 96 **7.** 81, 48 **8.** 48, 36

9. 40, 56 **10.** 14, 28 **11.** 30, 18 **12.** 84, 154

13. 24, 64	**14.** 35, 25	**15.** 100, 80	**16.** 75, 120
17. 2, 4, 8	**18.** 8, 12, 16	**19.** 18, 30, 36	**20.** 15, 25, 30
21. 18, 12, 24	**22.** 24, 36, 48	**23.** 8, 16, 40	**24.** 12, 18, 72

APPLICATIONS ***Nita is making a baby quilt. She is using strips of material that are cut from pieces of material that are 36 inches wide and 48 inches wide. Use this information to answer Exercises 25 and 26.***

25. All of the strips are to be the same width and as wide as possible. How wide should the strips be? How many strips will Nita be able to cut from each piece of material?

26. Nita found another piece of material that she decided to use for the quilt. The piece of material is 54 inches wide. If all of the strips from the three pieces of material are to be the same width and as wide as possible, how wide should the strips be? How many strips will Nita be able to cut from each of the three pieces of material?

SKILL 48

Name ______________________________ Date ____________

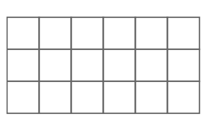
Perimeter and Area

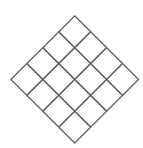
EXAMPLE ***Tova Albert wants to make a garden with a perimeter of 54 feet because that is the amount of fence that she has. She wants the least area possible because she doesn't have that much space in her yard. What should be the dimensions of her garden?***

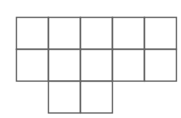
Dimensions	Perimeter	Area
1×26	54	26
2×25	54	50
3×24	54	72
4×23	54	92

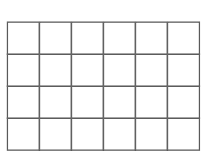
Notice that the perimeter stays 54 feet but the area continues to increase. Therefore, the least area with a perimeter of 54 feet is a garden with dimensions 1 foot by 26 feet.

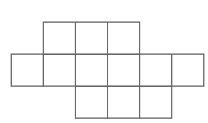
EXERCISES ***Find the perimeter and area of each figure.***

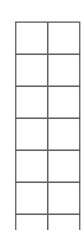
1. 

2. 

3. 

4. 

5. 

6. 

APPLICATIONS

7. A cardboard tube has a circumference of 7 inches and a length of 15 inches. When it is cut straight down its length, it becomes a rectangle. How much cardboard is used to make this tube?

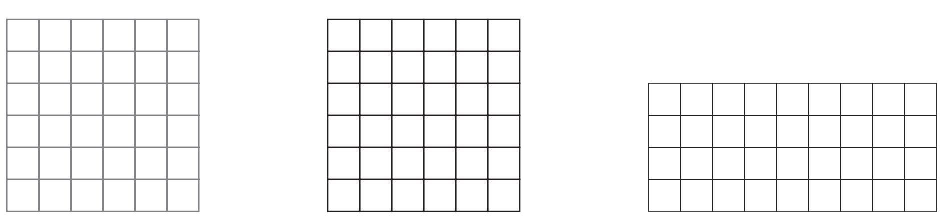
8. Ryan Allaire wants to build a deck onto the back of his house. He wants the area to be at least 240 square feet. There is space for the length to be up to 20 feet, but the width cannot be more than 15 feet.

a. Will he have room to build the size deck that he wants?

b. What is the largest deck that he can build?

c. If he wants the deck to be exactly 240 square feet, what are the whole number dimensions that are possible for him?

9. Using the large square below, show how to cut it into two pieces (cuts must be made along the grid lines) that can be rearranged to form a rectangle with a perimeter of 26 centimeters.



10. Bovinet Candy Company needs to have a box designed so that the bottom has an area of 96 square inches but has the least perimeter possible. What would be the whole number dimensions of the bottom of the box?

SKILL 49

Name ______________________ Date __________

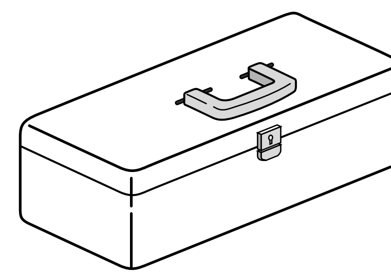
Volume of Rectangular Prisms

The **volume** (V) of a rectangular prism is found by multiplying the length (ℓ), the width (w), and the height (h).

$$V = \ell wh$$

EXAMPLE

Nicholas has been working with his dad in the evenings and on weekends in his dad's repair shop. For Nicholas' birthday, his dad bought him a new toolbox and some of the starting tools he would need. What is the volume of Nicholas' toolbox if it is 18 inches long, 8 inches tall, and 7.5 inches deep?



$V = \ell wh$

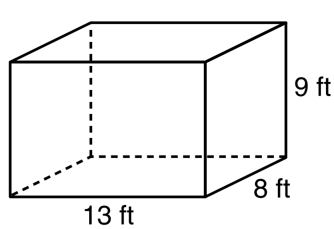
$V = 18 \times 7.5 \times 8$

$V = 1{,}080$ The volume of the toolbox is 1,080 cubic inches.

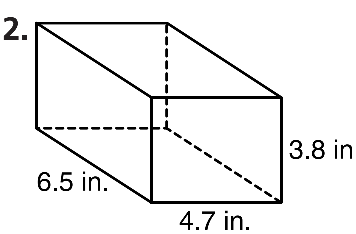
EXERCISES

Find the volume of each rectangular prism shown or described below. Round decimal answers to the nearest tenth.

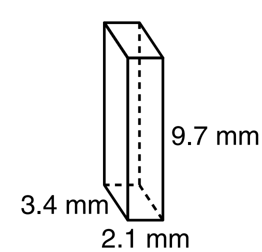
1.



2.



3.



4. length, 14 meters
width, 23 meters
height, 18 meters

5. length, $4\frac{1}{3}$ feet
width, $3\frac{3}{4}$ feet
height, 5 feet

6. cube:
side, 9.2 cm

7. Draw and label a rectangular prism whose length is 6 centimeters, width is 4 centimeters, and height is 10 centimeters. Find its volume.

8. How many different rectangular prisms can be formed with 18 cubes?

9. The surface area of a cube is 486 square inches. What is the volume of the cube?

10. A cube has a volume of 1,000 cubic inches. What is the surface area of the cube?

11. What is the height of a rectangular prism if the volume is 2,112 cubic yards, the length is 48 feet, and the width is 36 feet?

12. A rectangular prism has a volume of 36 cubic centimeters. Make a list showing all the possible whole-number dimensions of the prism.

APPLICATIONS

13. A bar of soap has the dimensions $2 \times 4 \times 1.5$ inches. A bathtub has the inside dimensions of $21 \times 50 \times 15$ inches. How many bars of soap would it take to fill the bathtub?

14. An aquarium is 3 feet long and $1\frac{1}{2}$ feet wide. It is filled with water to a height of 1 foot. How many gallons of water are in the aquarium? (Hint: 1 cubic foot $\approx$ 7.5 gallons.)

SKILL 50

Name ____________________ Date __________

Make a List

Pat's Pizza offers 7 different toppings: pepperoni, sausage, bacon, green peppers, onions, mushrooms, and anchovies. The Davis family wants to order a 3-topping pizza. Tommy Davis does not like anchovies.

EXAMPLE ***How many different pizzas can the Davis family order if they want to satisfy all members of the family?***

Let P = pepperoni, S = sausage, B = bacon, G = green peppers, O = onions, M = mushrooms, and A = anchovies. List the possible combinations that do not include anchovies.

PSB	PSG	PSO	PSM
PBO	PBM	PGO	PGM
SBG	SBO	SBM	SGO
SOM	BGO	BGM	BOM
PBG	POM	SGM	GOM

There are 20 different pizzas the Davis family can order.

EXERCISES ***Solve by making a list.***

1. How many different ways can a triangle, a square, and a circle be arranged in a row?

2. How many different four-digit numbers can be formed from the numbers 4, 5, 6, and 7 if all the digits must be different?

3. How many different three-digit numbers can be formed from the numbers 4, 5, 6, and 7 if all the digits must be different?

4. How many different two-digit numbers can be formed from the numbers 4, 5, 6, and 7 if both the digits must be different?

5. How many numbers between 77 and 103 are divisible by 3?

APPLICATIONS

6. A vendor at a rock concert sells T-shirts in three colors: red, blue, and yellow. The T-shirts come in 4 sizes: small, medium, large, and extra large. How many different T-shirts are available to the customers?

7. Four chairs are placed in a row on the stage. The three candidates for class president, Adrian, Toni, and Miwa, are seated on the stage. How many different ways can the candidates seat themselves?

8. Leslie wants to take a picture of her four dogs. She has a beagle, a terrier, a collie, and a poodle. How many ways can she arrange her dogs in a row if the beagle and terrier must be next to each other?

9. Using only dimes and nickels, how many different ways can a clerk make change for a dollar?

10. Earl attends a convention every three years. The year 1992 was a leap year, and Earl attended a convention. What is the next leap year that Earl will be attending a convention?

SKILL 51

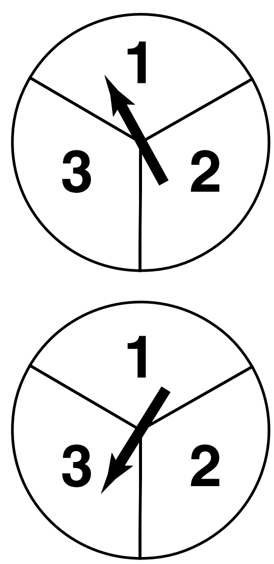
Name ______________________ **Date** __________

Probability of Independent Events

The **probability** of an event is the ratio of the number of ways an event can occur to the number of possible outcomes.

$$\text{Probability of an event} = \frac{\text{number of ways the event can occur}}{\text{number of possible outcomes}}$$

EXAMPLE ***Suppose you spin the two spinners. What is the probability that the sum of the numbers showing on the two spinners will be 4?***



Make a tree diagram to show all possible outcomes of these events.

First Spinner	Second Spinner	Sum
1	1	2
	2	3
	3	4
2	1	3
	2	4
	3	5
3	1	4
	2	5
	3	6

There are 3 outcomes that have a sum of 4 and there are 9 possible outcomes.

Probability of sum of 4 $= \frac{3}{9}$ or $\frac{1}{3}$

The probability that the sum will be 4 is $\frac{1}{3}$.

EXERCISES ***Use the spinners in the Example above to answer Exercises 1–4.***

1. What is the probability that the sum of the numbers showing on the two spinners is 3?

2. What is the probability that the sum of the numbers showing on the two spinners is greater than 3?

3. What is the probability that the sum of the numbers showing on the two spinners is an even number?

4. What is the probability that the sum of the numbers showing on the two spinners is *not* a 5?

5. Make a tree diagram showing the possible outcomes of tossing a penny and a dime.

6. What is the probability that a tossed penny and a tossed dime will both show heads?

7. What is the probability that a tossed penny and a tossed dime will both show one head and one tail?

8. What is the probability that a tossed penny and a tossed dime will show at least one tail?

APPLICATIONS ***Beau, Jiang, and Marci are playing a game that requires each player to toss two number cubes. Use this information to answer Exercises 9–12.***

9. Beau needs a sum of 4 on the number cubes to win. What is the probability that Beau will toss a 4?

10. Jiang needs a sum of 9 on the number cubes to win. What is the probability that Jiang will toss a 9?

11. Marci needs a sum of 7 on the number cubes to win. What is the probability that Marci will toss a 7?

12. Who is most likely to win the game?

SKILL 52

Name ____________________ Date __________

Expected Value of an Outcome

EXAMPLE ***Mr. Eugene has four different colored markers in a cup on his desk. Each day he pulls a marker out of the cup at random. How often could he expect to use a given marker in 8 days? in 16 days? in 40 days?***

The probability of choosing any one of the four different colored markers is $\frac{1}{4}$.

In 8 days, he could expect to use the given marker twice.

In 16 days, he could expect to use the given marker 4 times.

In 40 days, he could expect to use the given marker 10 times.

EXERCISES ***A number cube is rolled 12 times. How often would you expect to get each of the following outcomes?***

1. a 6

2. a 7

3. a prime number

4. an even number

5. a number greater than 2

6. a number less than 1

7. a multiple of 1

8. a multiple of 4

A coin is tossed 20 times. How often would you expect to get each of the following outcomes?

9. a head

10. a tail

11. a head or a tail

12. neither a head nor a tail

APPLICATIONS ***LeRoy has 15 different ties. He chooses a tie at random every day.***

13. How many times could he expect to wear a given tie in 45 days?

14. How many times could he expect to wear a given tie in 180 days?

15. How many times could he expect to wear a given tie in a year that is not a leap year?

16. Suppose LeRoy buys 5 more ties to add to his collection. How many times could he now expect to wear a given tie in 45 days? in 180 days? in a year that is not a leap year?

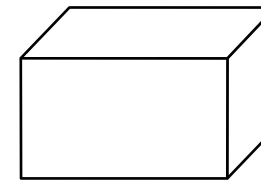
17. How many ties would LeRoy need to own in order to expect to wear each tie just 5 times in a year that is not a leap year?

SKILL 53

Name ______________________ **Date** ____________

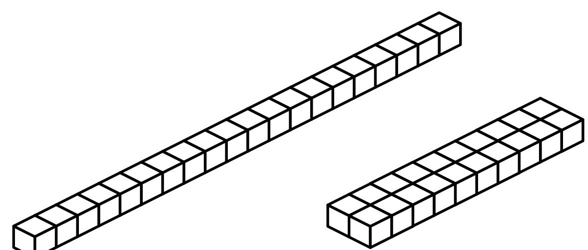
Make a Model

A box like the one at the right is a **rectangular prism**. It has six sides and each one is the shape of a rectangle.

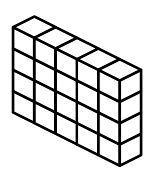


EXAMPLE ***How many different shapes of rectangular prisms can be formed using exactly 20 cubes?***

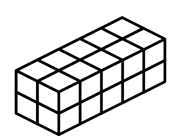
Use 20 cubes to model this problem. Make as many different shapes of rectangular prisms as you can.



$1 \times 1 \times 20$ $1 \times 2 \times 10$



$1 \times 4 \times 5$



$2 \times 2 \times 5$

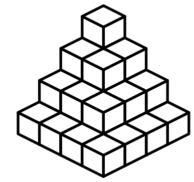
There are four different shapes of rectangular prisms that can be made.

EXERCISES ***Solve by making a model.***

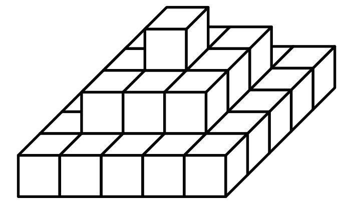
1. How many different shapes of rectangular prisms can be formed using exactly 12 cubes?

2. How many different shapes of rectangular prisms can be formed using exactly 24 cubes?

3. How many cubes are needed to make the display shown at the right?



4. How many cubes are needed to make the display shown at the right?



APPLICATIONS

5. Ronnie used blocks to build a “fort”. The blocks were cubes and were stacked five high. The top, front, and side views were all squares. How many blocks did Ronnie need to build his fort?

6. Twelve one-inch-tall square snack cakes are packed in a box. No two cakes are stacked on top of one another. What are the possible dimensions of the box if the top view of each cake is a two-inch by two-inch square?

7. The town playground is to have a hedge around it. The playground is in the shape of a pentagon with two sides of 40 feet, two sides of 60 feet, and one side of 70 feet. The bushes will be planted every 5 feet. How many bushes will be needed?

8. Rita collects miniature lamps. She is building a shelf around the rectangular family room to display them. If the family room is 15 feet wide and 18 feet long, how many feet of shelving will she need?

9. A carton is 8 inches by 4 inches by 12 inches. How many four-inch cubes can Brian pack in the carton?

SKILL 54

Name ______________________ Date __________

Classify Information

In 1980, the United States film industry took in $2,748,500,000 in box office receipts. The average admission charge was $2.69. In 1990, the box office receipts were $5,021,800,000, and the average admission charge was $4.75.

EXAMPLE ***How much more were the box office receipts in 1990 than in 1980?***

What is the question?
How much more were the receipts in 1990 than 1980?
What information is needed?
The total receipts in 1980 and 1990 are needed.
What information is *not* needed?
The average admission charges in 1980 and 1990 are *not* needed.
Solve the problem.

$$\begin{array}{r} 5{,}021{,}800{,}000 \\ -\ 2{,}748{,}500{,}000 \\ \hline 2{,}273{,}300{,}000 \end{array}$$

In 1990, the receipts were $2,273,300,000 more than in 1980.

EXERCISES ***Classify information in each problem by writing* "not enough information" *or* "too much information." *Then solve, if possible.***

1. The sum of three numbers is 78. If one of the numbers is 14, what are the other two numbers?

2. If the product of 56 and 77 is 4,312, what is the sum of the numbers?

3. If the sum of 18 and a number is 54 and their product is 648, what is their difference?

4. If the product of two numbers is 100, what is the difference of the numbers?

APPLICATIONS ***Classify information in each problem by writing* "not enough information" *or* "too much information." *Then solve, if possible.***

5. Phien bought 3 address books that cost $4.98 each. She gave the cashier a $20 bill. What was the total cost of the books?

6. Jimmy grew 3 inches last year and 2 inches so far this year. How tall is Jimmy now?

7. Carla, a carpenter, has two tape measures. The steel tape is 8 feet long. The cloth tape is marked in metric measure at one-centimeter intervals. How much longer is the steel tape than the cloth tape?

8. Jonathan bought 10 computer disks for $1.39 each. The disks usually sell for $1.99 each, or ten for $18. How much did he pay for the disks?

9. The Sheng family drove 1,287 miles on their vacation. About how many miles did they drive per day?

10. Gerda pays a delivery service $18 for priority delivery, $15 for standard delivery, and $21 for Saturday delivery. How much will she save by sending a package by standard delivery instead of Saturday delivery?

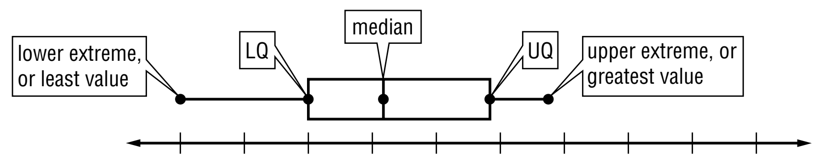
11. Alan ran the same number of miles for 6 days. How far did he run?

SKILL 55

Name ______________________ **Date** __________

Box-and-Whisker Plots

A **box-and-whisker plot** separates a set of data into four parts using the median and quartiles. A **box** is drawn around the quartile values, and **whiskers** extend from each quartile to the extreme data points.



EXAMPLE ***Draw a box-and-whisker plot for the set of test scores given below.***

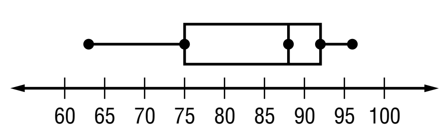
83 92 75 96 89 70 62 85 94 88 92

Step 1 Find the least and greatest numbers. Then draw a number line that covers the range of the data.
The lowest data value is 62 and the highest is 96. Draw a number line ranging from 60 to 100 with increments of 5.

Step 2 Find the median, the extremes, and the upper and lower quartiles. Mark these points above the number line.

The median for the data is 88. The extremes are 62 and 96. The upper quartile is 92 and the lower quartile is 75.

Step 3 Draw a box and the whiskers.



EXERCISES ***Draw a box-and-whisker plot for each set of data.***

1. 18, 24, 21, 19, 32, 22,
45, 21, 18, 28, 30

2. 132, 118, 145, 107, 125, 130,
105, 114, 122, 138, 117

3. 75, 88, 100, 92, 68, 73, 95, 84,
70, 85, 90, 66, 78, 89, 95

4. 6.8, 7.7, 8.3, 5.4, 6.9, 7.0,
9.1, 8.2, 7.1, 6.3, 5.5

5. 38, 42, 27, 19, 35, 40,
31, 24, 45, 37, 41

6. 5, 9, 7, 6, 5, 8, 4, 9,
7, 7, 6, 5, 8, 9, 6

7. $89, $74, $62, $83, $94, $66,
$80, $73, $88, $91, $70

8. 18, 21, 19, 18, 20,
22, 19, 24, 23

APPLICATIONS

9. The following data gives the total amount of snowfall (in inches) for a community in Ohio, over the past nine winters. Draw a box-and-whisker plot for the data.

   24.3, 18.6, 21.9, 34.1, 17.4, 25.5, 31.3, 22.7, 24.6

10. David is in charge of counting the money collected at his school each day for the annual fund-raiser. The data below shows the amounts collected each day during the past two weeks. Draw a box-and-whisker plot for the data.

    $12, $23, $18, $15, $9, $25, $14, $11, $21, $16

SKILL 56

Name ______________________________ Date ____________

Constructing and Interpreting Graphs

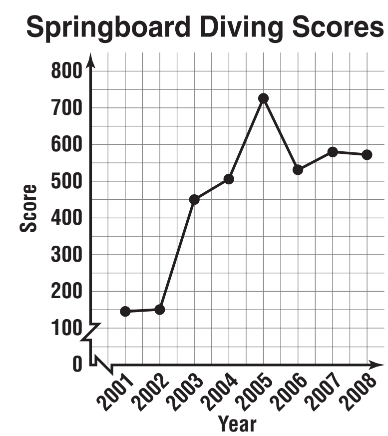
The chart at the right shows scores in the annual Springboard Diving event.

Springboard Diving Event	
Year	**Score**
2001	145.00
2002	150.77
2003	450.03
2004	506.19
2005	725.91
2006	530.70
2007	580.23
2008	572.40

EXAMPLE ***Construct and interpret a graph of the data.***

To graph the data, first label the axes and graph the points named by the data. Then connect the points as shown in the graph at the right.

The graph shows that the scores generally tended to increase with each successive year.



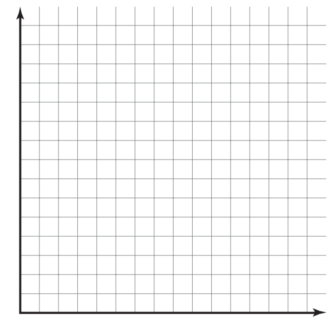
EXERCISES ***Construct and interpret a graph of each set of data.***

1.

Time (seconds)	Speed (mph)
5	5
10	8
15	20
20	18
25	30
30	40
35	55

2.

Distance (feet)	Speed (mph)
40	15
80	28
120	42
160	60
200	46
240	37
280	55



APPLICATIONS ***The chart at the right lists the winning times for the men's 110-meter hurdles at the state championships. Use the data to answer Exercises 3–6.***

Year	Time (seconds)
1994	13.6
1996	13.3
1998	13.24
2000	13.30
2002	13.20
2004	13.20
2006	12.98
2008	13.12

3. Construct a graph of the data.

4. Interpret the graph of the data.

5. Why do you think the times do *not* always show a consistent pattern?

6. What would you predict the time for this event to be in the next state championship? Explain why you chose this time.

7. Suppose you are driving down a street that has many traffic lights. What do you think a graph of your time versus your speed would look like? Why? Sketch your graph.

SKILL 57

Name ______________________ **Date** ____________

Adding and Subtracting Fractions

Lina is making trail mix for a hiking trip. She has $2\frac{1}{2}$ cups of peanuts, $3\frac{1}{4}$ cups of raisins, and $2\frac{2}{3}$ cups of carob chips.

EXAMPLE ***How many cups of trail mix will Lina have?***

$$2\frac{1}{2} = 2\frac{6}{12}$$

$$3\frac{1}{4} = 3\frac{3}{12}$$

$$+\ 2\frac{2}{3} = +\ 2\frac{8}{12}$$

$$7\frac{17}{12} = 7 + \frac{12}{12} + \frac{5}{12}$$

$$= 7 + 1 + \frac{5}{12}$$

$$= 8 + \frac{5}{12}$$

$$= 8\frac{5}{12}$$

Lina will have $8\frac{5}{12}$ cups of trail mix.

If Lina wants 15 cups of trail mix, how many more cups of trail mix does she have to make?

$$15 = 14 + 1 = 14 + \frac{12}{12} = 14\frac{12}{12}$$

$$15 = 14\frac{12}{12}$$

$$-\ 8\frac{5}{12} = -\ 8\frac{5}{12}$$

$$6\frac{7}{12}$$

She needs to make another $6\frac{7}{12}$ cups of trail mix.

EXERCISES ***Add or subtract. Write each answer in simplest form.***

1. $\frac{7}{12} + \frac{2}{12}$ **2.** $\frac{9}{10} - \frac{3}{10}$ **3.** $\frac{7}{9} + \frac{5}{9}$

4. $\frac{7}{16} - \frac{3}{16}$

5. $\frac{1}{6} + \frac{1}{2}$

6. $\frac{2}{3} - \frac{1}{2}$

7. $\frac{1}{4} + \frac{7}{8}$

8. $\frac{9}{10} - \frac{3}{5}$

9. $\frac{4}{5} + \frac{1}{12}$

10. $\frac{11}{15} - \frac{1}{3}$

11. $\frac{1}{9} + \frac{1}{6}$

12. $\frac{1}{2} - \frac{7}{16}$

13. $\frac{3}{10} + \frac{4}{5}$

14. $\frac{4}{5} - \frac{1}{6}$

15. $7\frac{1}{10} + 2\frac{1}{5}$

16. $9\frac{1}{2} - 5\frac{1}{6}$

17. $5\frac{3}{4} + 2\frac{5}{8}$

18. $9\frac{3}{4} - 2\frac{1}{6}$

APPLICATIONS

19. The route from Ramon's house to city hall and then to the school is $\frac{9}{10}$ mile. It is $\frac{3}{10}$ mile from city hall to the school. What is the distance from Ramon's house to city hall?

20. To make a salad, Henry used $\frac{3}{4}$ pound of Boston lettuce and $\frac{2}{3}$ pound of red lettuce. How much lettuce did he use?

21. Donna has $10\frac{3}{4}$ yards of ribbon. She needs $3\frac{1}{2}$ yards of ribbon to make a bow. How much ribbon will she have after she makes the bow?

22. Part of the daily diet of polar bears at the Bronx Zoo is $1\frac{1}{4}$ pounds of apples and a $1\frac{1}{2}$-pound mixture of oats and barley. What is the combined weight of these items?

23. Ani has two chores to do on Saturday. She has to wash the car which will take her $\frac{3}{4}$ hour and rake the leaves which will take her $1\frac{1}{2}$ hours. How much time should she plan to spend on these chores?

24. Mr. Vazquez wants to put a fence around his rectangular vegetable garden. If the garden is 18 $\frac{3}{4}$ feet long and $10\frac{1}{2}$ feet wide, how much fence will he need?

SKILL 58

Name ______________________________ Date ____________

Multiplying and Dividing Fractions

A new industrial park is being developed. The ABC Manufacturing Company owns a rectangular piece of property that is $\frac{2}{5}$ mile long and $\frac{1}{4}$ mile wide.

EXAMPLE

What is the area of the property owned by the ABC Manufacturing Company?

To find the area of a rectangle, you multiply the length by the width.

$\frac{2}{5} \times \frac{1}{4} = \frac{2 \times 1}{5 \times 4}$ *Multiply the numerators. Multiply the denominators.*

$= \frac{2}{20}$ or $\frac{1}{10}$ *Simplify.*

The ABC Manufacturing Company owns $\frac{1}{10}$ square mile of land.

The A to Z Distribution Company owns $\frac{1}{8}$ square mile of land in the industrial park. If the land is in the shape of a rectangle and the length of the land is $\frac{1}{3}$ mile, what is the width of their land?

To find the width, divide the area of the rectangle by the length.

$\frac{1}{8} \div \frac{1}{3} = \frac{1}{8} \times \frac{3}{1}$ *Multiply by the reciprocal of $\frac{1}{3}$.*

$= \frac{1 \times 3}{8 \times 1}$ *Multiply the numerators. Multiply the denominators.*

$= \frac{3}{8}$

The width of the land owned by A to Z Distributing Company is $\frac{3}{8}$ mile.

EXERCISES ***Multiply or divide. Write each answer in simplest form.***

1. $\frac{2}{3} \times \frac{1}{4}$

2. $\frac{1}{4} \div \frac{2}{5}$

3. $\frac{3}{7} \times \frac{1}{2}$

4. $\frac{5}{8} \div \frac{4}{5}$

5. $\frac{1}{3} \times \frac{3}{5}$

6. $\frac{2}{9} \div \frac{3}{5}$

7. $\frac{1}{2} \times \frac{6}{7}$

8. $\frac{2}{5} \div \frac{2}{3}$

9. $\frac{3}{8} \div \frac{1}{6}$

10. $\frac{1}{3} \div \frac{2}{5}$

11. $\frac{7}{10} \times \frac{5}{7}$

12. $\frac{2}{3} \div \frac{1}{2}$

13. $\frac{2}{3} \times \frac{5}{6}$

14. $\frac{3}{5} \div \frac{3}{10}$

15. $\frac{3}{4} \times \frac{1}{3}$

16. $\frac{1}{9} \div \frac{5}{6}$

17. $\frac{2}{3} \times \frac{5}{7}$

18. $\frac{1}{4} \div \frac{1}{2}$

19. $\frac{4}{7} \times \frac{5}{9}$

20. $\frac{1}{2} \div \frac{7}{8}$

21. $\frac{2}{3} \times \frac{2}{3}$

APPLICATIONS

22. About $\frac{1}{8}$ of the world's population lives in Africa. About $\frac{1}{13}$ of the population of Africa lives in Ethiopia. About what fraction of the world's population lives in Ethiopia?

23. About $\frac{1}{20}$ of the world's water supply is fresh water. If about $\frac{5}{7}$ of Earth's surface is covered with water, about what fraction of Earth is covered with fresh water?

24. Two thirds of Esma's garden is planted in flowers. If $\frac{1}{4}$ of the flowers are gladiolas, what fraction of the garden is planted in gladiolas?

25. One eighth of Jonas' garden is planted in green beans. If $\frac{3}{4}$ of his garden is planted in vegetables, what fraction of the vegetable garden is planted in green beans?

26. Three fourths of the books sold at Bernie's Book Store are paperbacks. If $\frac{1}{3}$ of the paperbacks sold are adventure stories, what fraction of the books sold are paperback adventure books?

27. A honeybee can produce $\frac{1}{10}$ pound of honey in its lifetime. How many honeybees does it take to make $\frac{1}{2}$ pound of honey?

SKILL 59

Name ______________________ **Date** __________

Algebraic Fractions

EXAMPLE

Find the sum of $\frac{3}{(x+1)} + \frac{4}{(2x+2)}$.

To add two algebraic fractions, convert them into fractions with a common denominator. Here are two strategies you could use to find a common denominator.

Strategy 1 Multiply the two denominators together and use that expression for the new denominator. This strategy will always work, but you may have do extra work when you simplify the fraction.

The new denominator is $(x + 1)(2x + 2)$.

$$\frac{3}{(x+1)} \times \frac{(2x+2)}{(2x+2)} = \frac{3(2x+2)}{(x+1)(2x+2)}$$

$$\frac{4}{(2x+2)} \times \frac{(x+1)}{(x+1)} = \frac{4(x+1)}{(2x+2)(x+1)}$$

So, $\frac{3}{(x+1)} + \frac{4}{(2x+2)}$ can be rewritten as $\frac{3(2x+2)}{(x+1)(2x+2)} + \frac{4(x+1)}{(2x+2)(x+1)}$.

Now that the two fractions have common denominators, you can add them and simplify the result.

$$\begin{aligned}\frac{3(2x+2)}{(x+1)(2x+2)} + \frac{4(x+1)}{(2x+2)(x+1)} &= \frac{3(2x+2)+4(x+1)}{(x+1)(2x+2)} \\ &= \frac{6x+6+4x+4}{(x+1)(2x+2)} \\ &= \frac{10x+10}{(x+1)(2x+2)} \\ &= \frac{10(x+1)}{(x+1)(2x+2)} \\ &= \frac{10}{(2x+2)} \\ &= \frac{5}{(x+1)}\end{aligned}$$

EXAMPLE

Strategy 2 Simplify one or both fractions until they have the same denominator. This strategy only works for fractions whose denominators share a common factor, and which can be simplified to get rid of the other factors in the denominator.

$$\frac{4}{(2x + 2)} = \frac{2 \cdot 2}{2(x + 1)} = \frac{2}{(x + 1)}$$

Now both fractions have the same denominator, $(x + 1)$. Add the fractions and simplify.

$$\frac{3}{(x + 1)} + \frac{2}{(x + 1)} = \frac{5}{(x + 1)}$$

These strategies also work for subtracting one algebraic fraction from another.

EXERCISES

Find each sum or difference. Simplify your answer as much as possible.

1. $\frac{1}{g} + \frac{3}{2g}$

2. $\frac{1}{(d + 2)} + \frac{9}{(3d + 6)}$

3. $\frac{x}{(x - 3)} - \frac{5}{(4x - 12)}$

4. $\frac{m}{(m + 1)} + \frac{p}{(p + 1)}$

5. $\frac{x}{(y + 1)} - \frac{y}{(x - 1)}$

6. $\frac{2}{(x^2 - 1)} - \frac{3}{(x + 1)}$

APPLICATIONS

Nikhil and Teresa are addressing newsletters to mail to the parents of all the students in the school. Nikhil can address 100 envelopes in x minutes. Teresa is a little faster so it takes her 1 minute less to address 100 envelopes than it takes Nikhil.

7. Write an expression for the number of envelopes Nikhil can address in 1 minute.

8. Write an expression for the number of envelopes Teresa can address in 1 minute.

9. Write an algebraic fraction to represent the number of envelopes can Teresa and Nikhil address in 1 minute, working together.