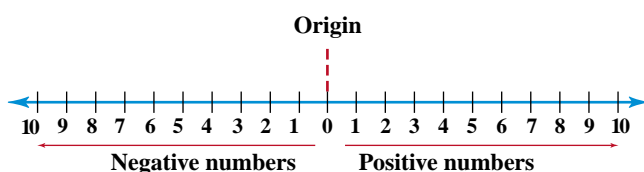


Mathematics is a language used in science to express and solve problems. Use this handbook to review basic math skills and to reinforce some math skills presented in the chapters in more depth.

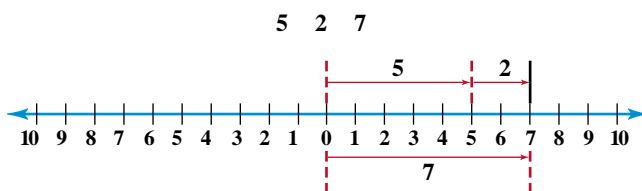
Arithmetic Operations

Calculations you perform during your study of chemistry require arithmetic operations, such as addition, subtraction, multiplication, and division, using numbers. Numbers can be positive or negative, as you can see in **Figure 1**. Examine the number line below. Numbers that are positive are greater than zero. A plus sign (+) or no sign at all indicates a positive number. Numbers that are less than zero are negative. A minus sign (−) indicates a negative number. Zero (the origin) is neither positive nor negative.



1. Addition and subtraction

Addition is an arithmetic operation. As you can see in **Table 1**, the result of addition is called a sum. When the signs of the numbers you are adding are alike, add the numbers and keep the same sign. Use the number line below to solve for the sum $5 + 2$ in which both numbers are positive. To represent the first number, draw an arrow that starts at the origin. The arrow that represents the second number starts at the arrowhead of the first arrow. The sum is at the head of the second arrow. In this case, the sum equals the positive number seven.



The process of adding two negative numbers is similar to adding two positive numbers. The negative sign indicates that you must move in the direction opposite to the direction that you moved to add two positive numbers. Use the number line below to verify that the sum below equals -7 . Notice that the sign of the resulting number when you add two negative numbers is always negative.

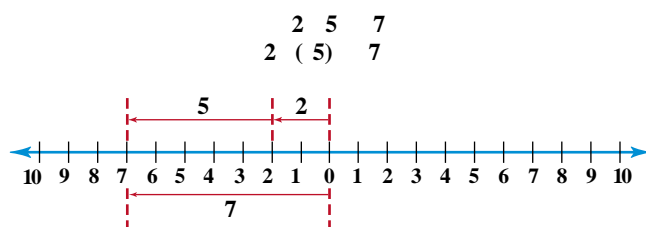


Figure 1

Water freezes at 32°F and 0°C . What temperature scale do you think was used on this sign? Explain.



Table 1

Arithmetic Operations		
Operation	Sign	Result
Addition	+	Sum
Subtraction	−	Difference
Multiplication	×	Product
Division	÷	Quotient

Most scientific calculators have a button labeled $\boxed{\log}$ and, in most cases, the number is simply entered and the log button is pushed to display the log of the number. Note that there is the same number of digits after the decimal in the log as there are significant figures in the original number entered.

$$\log 5.34 \times 10^5 = \log 5.34 + \log 10^5 = 0.728 + 5 = 5.728$$

Suppose the pH of the aqueous ammonia in **Figure 23** is 9.54 and you are asked to find the concentration of the hydrogen ions in that solution. By definition, $\text{pH} = -\log [\text{H}^+]$. Compare this to the general equation for the common log.

$$\begin{array}{ll} \text{Equation for pH:} & \text{pH} = -\log [\text{H}^+] \\ \text{General equation:} & y = \log 10^n \end{array}$$

To solve the equation for $[\text{H}^+]$, you must follow the reverse process and calculate the antilogarithm (antilog) of -9.54 to find $[\text{H}^+]$.

Antilogs are the reverse of logs. To find the antilog, use a scientific calculator to input the value of the log. Then, use the inverse function and press the log button.

$$\begin{aligned} \text{If } n &= \text{antilog } y, \text{ then } y = 10^n. \\ \text{Thus, } [\text{H}^+] &= \text{antilog}(-9.54) = 10^{-9.54} = 10^{0.46 + (-10)} \\ &= 10^{0.46} \times 10^{-10} \\ &= 2.9 \times 10^{-10} M \end{aligned}$$

Check the instruction manual for your calculator. The exact procedure to calculate logs and antilogs may vary.

Table 5

Properties of Exponents	
Exponential Notation	Logarithm
$10^A \times 10^B = 10^{A+B}$	$\log(A \times B) = \log A + \log B$
$10^A \div 10^B = 10^{A-B}$	$\log(A \div B) = \log A - \log B$
A^B	$(\log A) \times B$

PRACTICE PROBLEMS

21. Find the log of each of the following numbers:

a. 367

b. 4078

c. X^n

d. $\left(\frac{1}{2}\right)^{1/T}$

22. Find the antilog of each of the following logs:

a. 4.663

b. 2.367

c. 0.371

d. -1.588



Figure 23

Ammonia is a base. That means, its hydrogen ion concentration is less than 10^{-7} .



Figure 22

Carbon-dating of this skull is based on the radioactive decay of carbon-14 atoms which is measured in half-lives.

2. Multiplication and division

When multiplying fractions, the numerators and denominators are multiplied together as follows:

$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$$

Note the final answer is simplified by dividing the numerator and denominator by two.

When dividing fractions, the divisor is inverted and multiplied by the dividend as follows:

$$\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{2 \times 2}{3 \times 1} = \frac{4}{3}$$

PRACTICE PROBLEMS

20. Perform the indicated operation:

a. $\frac{2}{3} + \frac{3}{4}$

b. $\frac{4}{5} + \frac{3}{10}$

c. $\frac{1}{4} - \frac{1}{6}$

d. $\frac{7}{8} - \frac{5}{6}$

e. $\frac{1}{3} \times \frac{3}{4}$

f. $\frac{3}{5} \times \frac{2}{7}$

g. $\frac{5}{8} \div \frac{1}{4}$

h. $\frac{4}{9} \div \frac{3}{8}$

Logarithms and Antilogarithms

When you perform calculations, such as using half-life of carbon to determine the age of the skull in **Figure 22** or the pH of the products in **Figure 23**, you may need to use the log or antilog function on your calculator. A logarithm (log) is the power or exponent to which a number, called a base, must be raised in order to obtain a given positive number. This textbook uses common logarithms based on a base of 10. Therefore, the common log of any number is the power to which ten is raised to equal that number. Examine **Table 4**. Note the log of each number is the power of ten for the exponent of that number. For example, the common log of 100 is two and the common log of 0.01 is -2 .

$$\begin{aligned}\log 10^2 &= 2 \\ \log 10^{-2} &= -2\end{aligned}$$

A common log can be written in the following general form.

$$\text{If } 10^n = y, \text{ then } \log y = n.$$

In each example in **Table 4**, the log can be determined by inspection. How do you express the common log of 5.34×10^5 ? Because logarithms are exponents, they have the same properties as exponents. See **Table 5**.

$$\log 5.34 \times 10^5 = \log 5.34 + \log 10^5$$

Table 4

Comparison Between Exponents and Logs	
Exponent	Logarithm
$10^0 = 1$	$\log 1 = 0$
$10^1 = 10$	$\log 10 = 1$
$10^2 = 100$	$\log 100 = 2$
$10^{-1} = 0.1$	$\log 0.1 = -1$
$10^{-2} = 0.01$	$\log 0.01 = -2$

Percents

A percent is a ratio that compares a number to 100. The symbol for percent is %. You also are used to working with percents in your daily life. The number of correct answers on an exam may be expressed as a percent. If you answered 90 out of 100 questions correctly, you would receive a grade of 90%. Signs like the one in **Figure 20** indicate a reduction in price. If the item's regular price is \$100, how many dollars would you save? Sixty-five percent means 65 of every 100, so you would save \$65. How much would you save if the sign said 75% off?

The comparison between molar mass of hydrogen gas and the molar mass of oxygen gas described on the previous page also can be expressed as a percent by taking the fraction, converting it to decimal form, and multiplying by 100 as follows:

$$\frac{\text{molar mass H}_2}{\text{molar mass O}_2} \times 100 = \frac{2.00 \text{ g/mol}}{32.00 \text{ g/mol}} \times 100 = 0.0625 \times 100 = 6.25\%$$

Thus, diatomic hydrogen gas has 6.25% of the mass of diatomic oxygen gas.

Operations Involving Fractions

Fractions are subject to the same type of operations as other numbers. Remember that the number on the top of a fraction is the numerator and the number on the bottom is the denominator. See **Figure 21**.

1. Addition and subtraction

Before two fractions can be added or subtracted, they must have a common denominator. Common denominators are found by finding the least common multiple of the two denominators. Finding the least common multiple often is as easy as multiplying the two denominators together. For example, the least common multiple of the denominators of the fractions $1/2$ and $1/3$ is 2×3 or 6.

$$\frac{1}{2} + \frac{1}{3} = \left(\frac{3}{3} \times \frac{1}{2}\right) + \left(\frac{2}{2} \times \frac{1}{3}\right) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Sometimes, one of the denominators will divide into the other, which makes the larger of the two denominators the least common multiple. For example, the fractions $1/2$ and $1/6$ have 6 as the least common multiple denominator.

$$\frac{1}{2} + \frac{1}{6} = \left(\frac{3}{3} \times \frac{1}{2}\right) + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

In still other situations, both denominators will divide into a number that is not the product of the two. For example, the fractions $1/4$ and $1/6$ have the number 12 as their least common multiple denominator, rather than 24, the product of the two denominators. This can be deduced as follows:

$$\frac{1}{6} + \frac{1}{4} = \left(\frac{4}{4} \times \frac{1}{6}\right) + \left(\frac{6}{6} \times \frac{1}{4}\right) = \frac{4}{24} + \frac{6}{24} = \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$$

Because both fractions can be simplified by dividing numerator and denominator by 2, the least common multiple must be 12.

Figure 20

Would the savings be large at this sale? How would you determine the sale price?



Quotient $\frac{9}{3} \frac{10^8}{10^4}$

Dividend
(numerator)

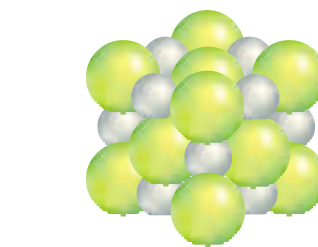
Divisor
(denominator)

Figure 21

When two numbers are divided, the one on top is the numerator and the one on the bottom is the denominator. The result is called the quotient. When you perform calculations with fractions, the quotient may be expressed as a fraction or a decimal.

Figure 19

a The mass of one lime would be one-twelfth the mass of one dozen limes. **b** In a crystal of table salt (sodium chloride), each sodium ion is surrounded by chloride ions, yet the ratio of sodium ions to chloride ions is one:one. The formula for sodium chloride is NaCl.



Ratios

You make comparisons using ratios in your daily life. For example, if the mass of a dozen limes is shown in **Figure 19a**, how does it compare to the mass of one lime? The mass of one dozen limes is twelve times larger than the mass of one lime. In chemistry, the chemical formula for a compound compares the elements that make up that compound. See **Figure 19b**. A ratio is a comparison of two numbers by division. One way it can be expressed is with a colon (:). The comparison between the molar masses of oxygen and hydrogen can be expressed as follows.

$$\begin{aligned} \text{molar mass (H}_2\text{)}:\text{molar mass (O}_2\text{)} \\ 2.00 \text{ g/mol}:32.00 \text{ g/mol} \\ 2.00:32.00 \\ 1:16 \end{aligned}$$

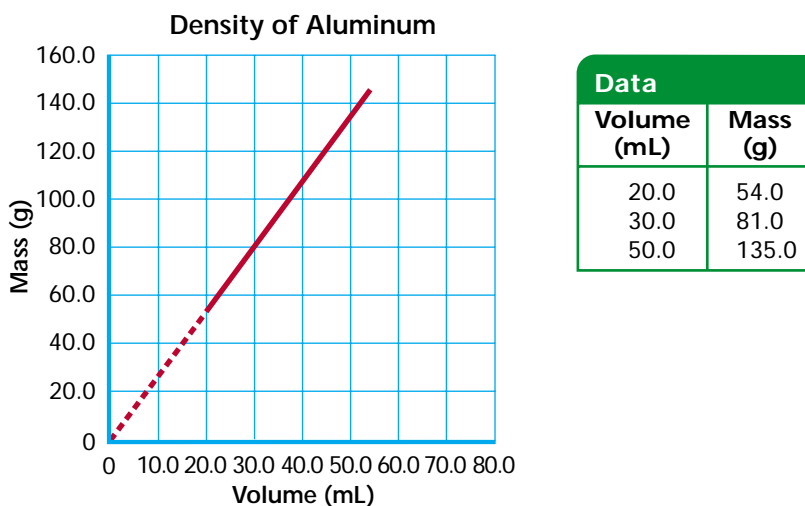
Notice that the ratio 1:16 is the smallest integer (whole number) ratio. It is obtained by dividing both numbers in the ratio by the smaller number, and then rounding the larger number to remove the digits after the decimal. The ratio of the molar masses is one to sixteen. In other words, the ratio indicates that the molar mass of diatomic hydrogen gas is sixteen times smaller than the molar mass of diatomic oxygen gas.

Fractions

Ratios are often expressed as fractions in simplest form. A fraction is a quotient of two numbers. To express the comparison of the molar masses as a fraction, place the molar mass of hydrogen over the molar mass of oxygen as follows:

$$\frac{\text{molar mass H}_2}{\text{molar mass O}_2} = \frac{2.00 \text{ g/mol}}{32.00 \text{ g/mol}} = \frac{2.00}{32.00} = \frac{1}{16}$$

In this case, the simplified fraction is calculated by dividing both the numerator (top of the fraction) and the denominator (bottom of the fraction) by 2.00. This fraction yields the same information as the ratio. That is, diatomic hydrogen gas has one-sixteenth the mass of diatomic oxygen gas.

**Figure 18**

Interpolation and extrapolation will help you determine the values of points you did not plot.

curve to the desired point. In **Figure 18**, you can determine that the mass at 10.0 cm³ equals 27 g. One caution regarding extrapolation—some straight-line curves do not remain straight indefinitely. So, extrapolation should only be done where there is a reasonable likelihood that the curve doesn't change.

PRACTICE PROBLEMS

19. Plot the data in each table. Explain whether the graphs represent direct or indirect relationships.

Table 4

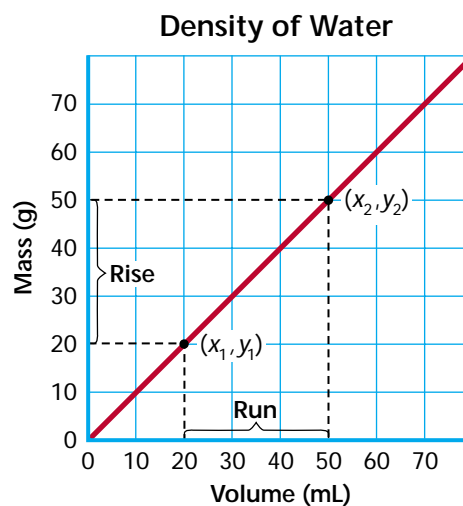
Effect of Pressure on Gas	
Pressure (mm Hg)	Volume (mL)
3040	5.0
1520	10.0
1013	15.0
760	20.0

Table 5

Effect of Pressure on Gas	
Pressure (mm Hg)	Temperature (K)
3040	819
1520	546
1013	410
760	273

Ratios, Fractions, and Percents

When you analyze data, you may be asked to compare measured quantities. Or, you may be asked to determine the relative amounts of elements in a compound. Suppose, for example, you are asked to compare the molar masses of the diatomic gases, hydrogen (H₂) and oxygen (O₂). The molar mass of hydrogen gas equals 2.00 g/mol; the molar mass of oxygen equals 32.00 g/mol. The relationship between molar masses can be expressed in three ways: a ratio, a fraction, or a percent.

**Figure 17**

A steep slope indicates that the dependent variable changes rapidly with a change in the independent variable. What would an almost flat line indicate?

The slope of a linear graph is the steepness of the line. Slope is defined as the ratio of the vertical change (the rise) to the horizontal change (the run) as you move from one point to the next along the line. See **Figure 17**. To calculate slope, choose any two points on the line, (x_1, y_1) and (x_2, y_2) . The two points need not be actual data points, but both must fall somewhere on the straight line. Once two points have been selected, calculate slope m using the equation

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2$$

where the symbol Δ stands for change, x_1 and y_1 are the coordinates or values of the first point, and x_2 and y_2 are the coordinates of the second point.

Choose any two points along the graph of mass vs. volume in **Figure 17** and calculate its slope.

$$m = \frac{135 \text{ g} - 54 \text{ g}}{50.0 \text{ cm}^3 - 20.0 \text{ cm}^3} = 2.7 \text{ g/cm}^3$$

Note that the units for the slope are the units for density. Plotting a graph of mass versus volume is one way of determining the density of a substance.

Apply the general equation for a straight line to the graph in **Figure 17**.

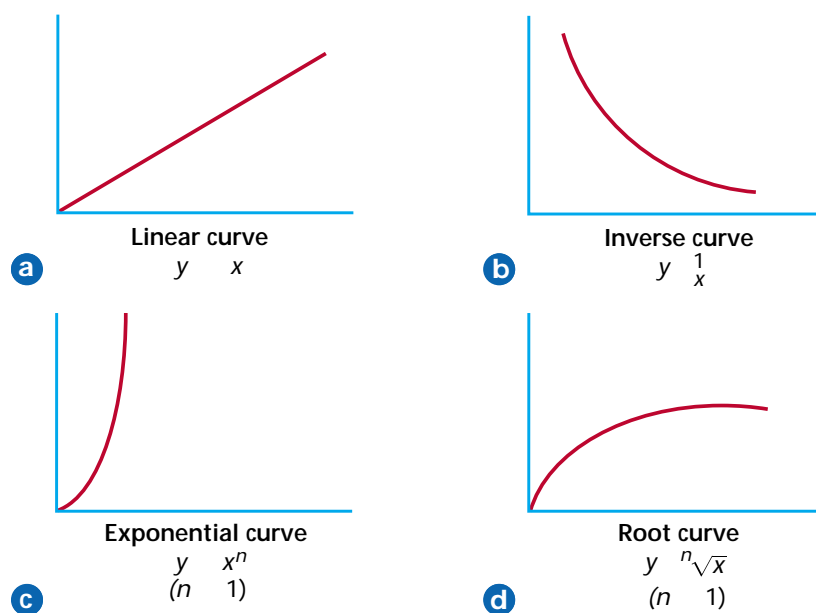
$$\begin{aligned} y &= mx + b \\ \text{mass} &= (2.7 \text{ g/cm}^3)(\text{volume}) + 0 \\ \text{mass} &= (2.7 \text{ g/cm}^3)(\text{volume}) \end{aligned}$$

This equation verifies the direct relationship between mass and volume. For any increase in volume, the mass also increases.

Interpolation and extrapolation

Graphs also serve functions other than determining the relationship between variables. They permit interpolation, the prediction of values of the independent and dependent variables. For example, you can see in the table in **Figure 18** that the mass of 40.0 cm^3 of aluminum was not measured. But, you can interpolate from the graph that the mass would be 108 g.

Graphs also permit extrapolation, which is the determination of points beyond the measured points. To extrapolate, draw a broken line to extend the

**Figure 15**

The shape of the curve formed by a plot of experimental data indicates how the variables are related.

Direct and inverse relationships

In your study of chemistry, the most common curves are the linear, representing the direct relationship ($y \propto x$), and the inverse, representing the inverse relationship ($y \propto 1/x$), where x represents the independent variable and y represents the dependent variable. In a direct relationship, y increases in value as x increases in value or y decreases when x decreases. In an inverse relationship, y decreases in value as x increases in value.

An example of a typical direct relationship is the increase in volume of a gas with increasing temperature. When the gases inside a hot air balloon are heated, the balloon gets larger. As the balloon cools, its size decreases. However, a plot of the decrease in pressure as the volume of a gas increases yields a typical inverse curve.

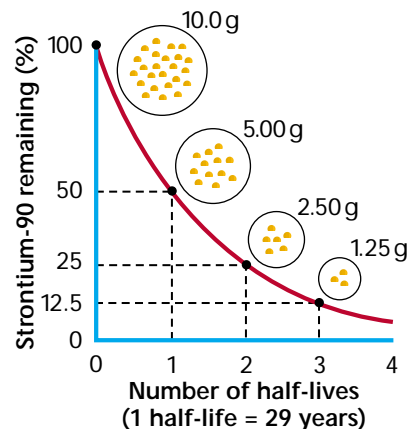
You also may encounter exponential and root curves in your study of chemistry. See **Figure 15c** and **d**. What types of relationships between the independent and dependent variables do the curves describe? How do the curves differ from **a** and **b**? An exponential curve describes a relationship in which one variable is expressed by an exponent. And, a root curve describes a relationship in which one variable is expressed by a root. What type of relationship is described by the curve in **Figure 16**?

The linear graph

The linear graph is useful in analyzing data because a linear relationship can be translated easily into equation form using the equation for a straight line

$$y = mx + b$$

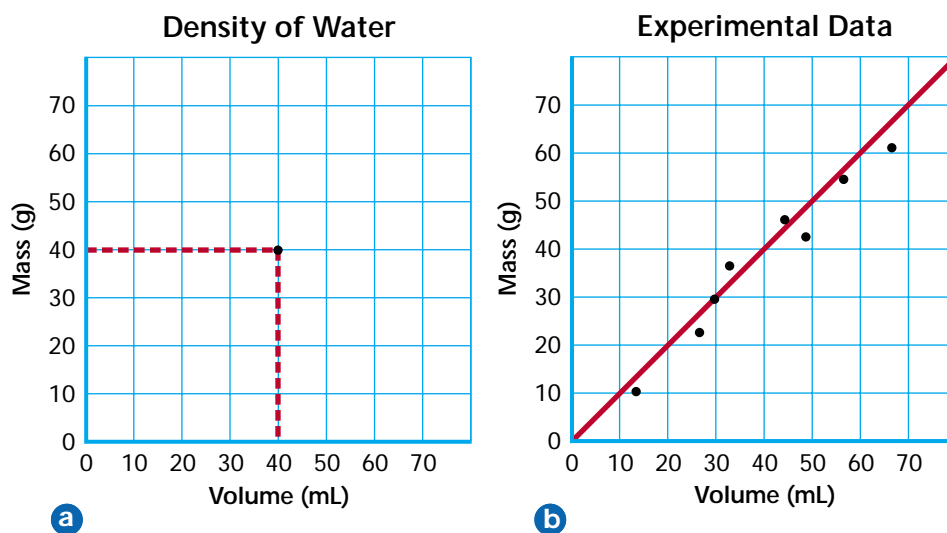
where y stands for the dependent variable, m is the slope of the line, x stands for the independent variable, and b is the y -intercept, the point where the curve crosses the y -axis.

**Figure 16**

Half-life is the amount of time it takes for half of a sample of a radioactive isotope to decay (Chapters 4 and 25). Notice that as the number of half-lives increases, the amount of sample decreases.

Figure 14

- a** To plot a point on a graph, place a dot at the location for each ordered pair (x,y) determined by your data. In this example, the dot marks the ordered pair (40 mL, 40 g).
- b** Generally, the line or curve that you draw will not include all of your experimental data points.



After the scales are chosen, plot the data. To graph or plot an ordered pair means to place a dot at the point that corresponds to the values in the ordered pair. The x -coordinate indicates how many units to move right (if the number is positive) or left (if the number is negative). The y -coordinate indicates how many units to move up or down. Which direction is positive on the y -axis? Negative? Locate each pair of x - and y -coordinates by placing a dot as shown in **Figure 14a**. Sometimes, a pair of rulers, one extending from the x -axis and the other from the y -axis, can ensure that data is plotted correctly.

Drawing a curve

Once the data is plotted, a straight line or a curve is drawn. It is not necessary to make it go through every point plotted, or even any of the points, as shown in **Figure 14b**. Graphing of experimental data is an averaging process. If the points do not fall along a line, the best-fit line or most probable smooth curve through the points should be drawn. When drawing the curve, do not assume it will go through the origin $(0,0)$.

Naming a graph

Last but not least, give each graph a title that describes what is being graphed. The title should be placed at the top of the page, or in a box on a clear area of the graph. It should not cross the data curve.

Once the data from an experiment has been collected and plotted, the graph must be interpreted. Much can be learned about the relationship between the independent and dependent variables by examining the shape and slope of the curve.

Using Line Graphs

The curve on a graph gives a great deal of information about the relationship between the variables. Four common types of curves are shown in **Figure 15**. Each type of curve corresponds to a mathematical relationship between the independent and dependent variables.



a

Drawing Line Graphs

Scientists, such as the one in **Figure 13a**, as well as you and your classmates, use graphing to analyze data gathered in experiments. Graphs provide a way to visualize data in order to determine the mathematical relationship between the variables in your experiment. Most often you use line graphs.

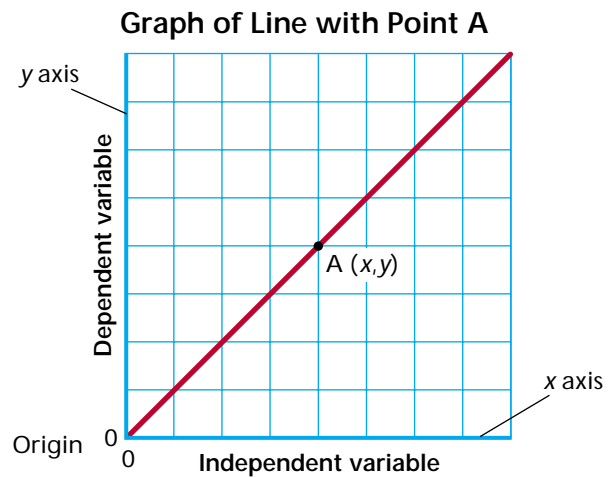
Line graphs are drawn by plotting variables along two axes. See **Figure 13b**. Plot the independent variable on the x -axis (horizontal axis), also called the abscissa. The independent variable is the quantity controlled by the person doing the experiment. Plot the dependent variable on the y -axis (vertical axis), also called the ordinate. The dependent variable is the variable that depends on the independent variable. Label the axes with the variables being plotted and the units attached to those variables.

Determining a scale

An important part of graphing is the selection of a scale. Scales should be easy to plot and easy to read. First, examine the data to determine the highest and lowest values. Assign each division on the axis (the square on the graph paper) with an equal value so that all data can be plotted along the axis. Scales divided into multiples of 1, 2, 5, or 10, or decimal values, often are the most convenient. It is not necessary to start at zero on a scale, nor is it necessary to plot both variables to the same scale. Scales must, however, be labeled clearly with the appropriate numbers and units.

Plotting data

The values of the independent and dependent variables form ordered pairs of numbers, called the x -coordinate and the y -coordinate (x,y) , that correspond to points on the graph. The first number in an ordered pair always corresponds to the x -axis; the second number always corresponds to the y -axis. The ordered pair $(0,0)$ always is the origin. Sometimes the points are named by using a letter. In **Figure 13b**, point A corresponds to the point (x,y) .



b

Figure 13

- a Once experimental data has been collected, it must be analyzed to determine the relationships between the measured variables.
- b Any graph of your data should include labeled x - and y -axes, a suitable scale, and a title.

You make unit conversions everyday when you determine how many quarters are needed to make a dollar or how many feet are in a yard. One unit that is often used in calculations in chemistry is the mole. Chapter 11 shows you equivalent relationships among mole, grams, and the number of representative particles (atoms, molecules, formula units, or ions). For example, one mole of a substance contains 6.02×10^{23} representative particles. Try the next example to see how this information can be used in a conversion factor to determine the number of atoms in a sample of manganese.

EXAMPLE PROBLEM 6

One mole of manganese (Mn) in the photo has a mass of 54.94 g. How many atoms are in two moles of manganese?

You are given the mass of one mole of manganese. In order to convert to the number of atoms, you must set up a conversion factor relating the number of moles and the number of atoms.

$$\frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ atoms}} \text{ and } \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mole}}$$

Choose the conversion factor that cancels units of moles and gives an answer in number of atoms.

$$\begin{aligned} 2.0 \text{ mole} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mole}} &= 12.04 \times 10^{23} \text{ atoms} \\ &= 1.2 \times 10^{24} \text{ atoms} \end{aligned}$$

The answer is expressed in the desired units (number of atoms). It is expressed in two significant figures because the number of moles (2.0) has the least number of significant figures.

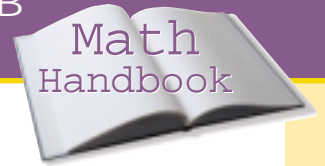


How many significant figures are in this measurement?

PRACTICE PROBLEMS

18. Convert the following measurements as indicated.

- a. 4 m = _____cm
- b. 50.0 cm = _____m
- c. 15 cm = _____mm
- d. 567 mg = _____g
- e. 4.6×10^3 m = _____mm
- f. 8.3×10^4 g = _____kg
- g. 7.3×10^5 mL = _____L
- h. 8.4×10^{10} m = _____km
- i. 3.8×10^4 m² = _____mm²
- j. 6.9×10^{12} cm² = _____m²
- k. 6.3×10^{21} mm³ = _____cm³
- l. 9.4×10^{12} cm³ = _____m³



Unit Conversion

Recall from Chapter 2 that the universal unit system used by scientists is called Le Système Internationale d'Unités or SI. It is a metric system based on seven base units—meter, second, kilogram, kelvin, mole, ampere, and candela—from which all other units are derived. The size of a unit in a metric system is indicated by a prefix related to the difference between that unit and the base unit. For example, the base unit for length in the metric system is the meter. One tenth of a meter is a decimeter where the prefix *deci-* means one tenth. And, one thousand meters is a kilometer. The prefix *kilo-* means one thousand.

You can use the information in **Table 3** to express a measured quantity in different units. For example, how is 65 meters expressed in centimeters? **Table 3** indicates one centimeter and one-hundredth meter are equivalent, that is, $1 \text{ cm} = 10^{-2} \text{ m}$. This information can be used to form a conversion factor. A conversion factor is a ratio equal to one that relates two units. You can make the following conversion factors from the relationship between meters and centimeters. Be sure when you set up a conversion factor that the measurement in the numerator (the top of the ratio) is equivalent to the denominator (the bottom of the ratio).

$$1 = \frac{1 \text{ cm}}{10^{-2} \text{ m}} \text{ and } 1 = \frac{10^{-2} \text{ m}}{1 \text{ cm}}$$

Recall that the value of a quantity does not change when it is multiplied by one. To convert 65 m to centimeters, multiply by a conversion factor.

$$65 \text{ m} \times \frac{1 \text{ cm}}{10^{-2} \text{ m}} = 65 \times 10^2 \text{ cm} = 6.5 \times 10^3 \text{ cm}$$

Note the conversion factor is set up so that the unit meters cancels and the answer is in centimeters as required. When setting up a unit conversion, use dimensional analysis to check that the units cancel to give an answer in the desired units. And, always check your answer to be certain the units make sense.

Table 3

Common SI Prefixes					
Prefix	Symbol	Exponential notation	Prefix	Symbol	Exponential notation
Peta	P	10^{15}	Deci	d	10^{-1}
Tera	T	10^{12}	Centi	c	10^{-2}
Giga	G	10^9	Milli	m	10^{-3}
Mega	M	10^6	Micro	μ	10^{-6}
Kilo	k	10^3	Nano	n	10^{-9}
Hecto	h	10^2	Pico	p	10^{-12}
Deka	da	10^1	Femto	f	10^{-15}

Dimensional Analysis

The dimensions of a measurement refer to the type of units attached to a quantity. For example, length is a dimensional quantity that can be measured in meters, centimeters, and kilometers. Dimensional analysis is the process of solving algebraic equations for units as well as numbers. It is a way of checking to ensure that you have used the correct equation, and that you have correctly applied the rules of algebra when solving the equation. It also can help you to choose and set up the correct equation, as you will see on the next page when you learn how to do unit conversions. It is good practice to make dimensional analysis a habit by always stating the units as well as the numerical values whenever substituting values into an equation.

EXAMPLE PROBLEM 5

The density (D) of aluminum is 2700 kg/m^3 . Determine the mass (m) of a piece of aluminum of volume (V) 0.20 m^3 .

The equation for density is

$$D = \frac{m}{V}$$

Multiply both sides of the equation by V and isolate m .

$$DV = \frac{mV}{V}$$

$$DV = \frac{V}{V} \times m$$

$$m = DV$$

Substitute the known values for D and V .

$$m = DV = (2700 \text{ kg/m}^3)(0.20 \text{ m}^3) = 540 \text{ kg}$$

Notice that the unit m^3 cancels out, leaving mass in kg, a unit of mass.



Aluminum is a metal that is useful from the kitchen to the sculpture garden.

PRACTICE PROBLEMS

Determine whether the following equations are dimensionally correct. Explain.

14. $v = s \times t$ where $v = 24 \text{ m/s}$, $s = 12 \text{ m}$ and $t = 2 \text{ s}$.
15. $R = \frac{nT}{PV}$ where R is in $\text{L}\cdot\text{atm}/\text{mol}\cdot\text{K}$, n is in mol , T is in K , P is in atm , and V is in L .
16. $t = \frac{v}{s}$ where t is in seconds, v is in m/s and s is in m .
17. $s = \frac{at^2}{2}$ where s is in m , a is in m/s^2 , and t is in s .

To see the difference order of operations makes, try replacing a with 2, b with 3, and c with 4.

$$\begin{aligned} a + (b \times c) &= 2 + (3 \times 4) = 14 \\ (a + b) \times c &= (2 + 3) \times 4 = 20 \end{aligned}$$

To solve algebraic equations, you also must remember the distributive property. To remove parentheses to solve a problem, any number outside the parentheses is “distributed” across the parentheses as follows:

$$6(x + 2y) = 6x + 12y$$

EXAMPLE PROBLEM 4

The temperature on a cold day was 25°F. What was the temperature on the Celsius scale?

Begin with the equation for the conversion from the Celsius to Fahrenheit temperature. Celsius temperature is the unknown variable.

$$^{\circ}\text{F} = \frac{9}{5}^{\circ}\text{C} + 32$$

Rearrange the equation to isolate $^{\circ}\text{C}$. Begin by subtracting 32 from both sides.

$$^{\circ}\text{F} - 32 = \frac{9}{5}^{\circ}\text{C} + 32 - 32$$

$$^{\circ}\text{F} - 32 = \frac{9}{5}^{\circ}\text{C}$$

Then, multiply both sides by 5.

$$5 \times (^{\circ}\text{F} - 32) = 5 \times \frac{9}{5}^{\circ}\text{C}$$

$$5 \times (^{\circ}\text{F} - 32) = 9^{\circ}\text{C}$$

Finally, divide both sides by 9.

$$\frac{5 \times (^{\circ}\text{F} - 32)}{9} = \frac{9^{\circ}\text{C}}{9}$$

$$^{\circ}\text{C} = \frac{5}{9} (^{\circ}\text{F} - 32)$$

Substitute in the known Fahrenheit temperature.

$$^{\circ}\text{C} = \frac{5}{9} (^{\circ}\text{F} - 32)$$

$$= \frac{5}{9} (25 - 32)$$

$$= -3.9^{\circ}\text{C}$$

The Celsius temperature is -3.9°C .

PRACTICE PROBLEMS

Isolate the indicated variable in each equation.

9. $PV = nRT$ for R

12. $\frac{2}{x} = 3 + y$ for x

10. $3 = 4(x + y)$ for y

13. $\frac{2x+1}{3} = 6$ for x

11. $z = x(4 + 2y)$ for y

Figure 12

When faced with an equation that contains more than one operation, use this flow chart to determine the order in which to perform your calculations.

Order of Operations

Examine all arithmetic operations.

Do all operations inside parentheses or brackets.

Do all multiplication and division from left to right.

Perform addition and subtraction from left to right.

When you solve algebraic equations, any operation that you perform on one side of the equal sign must be performed on the other side of the equation. Suppose you are asked to use the ideal gas law to find the pressure of the gas (P) in the bottle in **Figure 11**. To solve for, or isolate, P requires you to divide the left-hand side of the equation by V . This operation must be performed on the right-hand side of the equation as well.

$$PV = nRT$$

$$\frac{PV}{V} = \frac{nRT}{V}$$

The V 's on the left-hand side of the equation cancel each other out.

$$\frac{PV}{V} = \frac{nRT}{V}$$

$$P \times \frac{V}{V} = \frac{nRT}{V}$$

$$P \times 1 = \frac{nRT}{V}$$

$$P = \frac{nRT}{V}$$

The ideal gas law equation is now written in terms of pressure. That is, P has been isolated.

Order of operations

When isolating a variable in an equation, it is important to remember that arithmetic operations have an order of operations that must be followed. See **Figure 12**. Operations in parentheses (or brackets) take precedence over multiplication and division, which in turn take precedence over addition and subtraction. For example, in the equation

$$a + b \times c$$

variable b must be multiplied first by variable c . Then, the resulting product is added to variable a . If the equation is written

$$(a + b) \times c$$

the operation in parentheses or brackets must be done first. In the equation above, variable a is added to variable b before the sum is multiplied by variable c .

Figure 11

This beverage is bottled under pressure in order to keep carbon dioxide gas in the beverage solution.



PRACTICE PROBLEMS

7. Round off the following measurements to the number of significant figures indicated in parentheses.
- | | |
|-----------------|---------------------|
| a. 2.7518 g (3) | d. 186.499 m (5) |
| b. 8.6439 m (2) | e. 634 892.34 g (4) |
| c. 13.841 g (2) | f. 355 500 g (2) |
8. Perform the following operations.
- $(2.475 \text{ m}) + (3.5 \text{ m}) + (4.65 \text{ m})$
 - $(3.45 \text{ m}) + (3.658 \text{ m}) + (4.7 \text{ m})$
 - $(5.36 \times 10^{-4} \text{ g}) - (6.381 \times 10^{-5} \text{ g})$
 - $(6.46 \times 10^{12} \text{ m}) - (6.32 \times 10^{11} \text{ m})$
 - $(6.6 \times 10^{12} \text{ m}) \times (5.34 \times 10^{18} \text{ m})$
 - $\frac{5.634 \times 10^{11} \text{ m}}{3.0 \times 10^{12} \text{ m}}$
 - $\frac{(4.765 \times 10^{11} \text{ m})(5.3 \times 10^{-4} \text{ m})}{7.0 \times 10^{-5} \text{ m}}$

Solving Algebraic Equations

When you are given a problem to solve, it often can be written as an algebraic equation. You can use letters to represent measurements or unspecified numbers in the problem. The laws of chemistry are often written in the form of algebraic equations. For example, the ideal gas law relates pressure, volume, amount, and temperature of the gases the pilot in **Figure 10** breathes. The ideal gas law is written

$$PV = nRT$$

where the variables are pressure (P), volume (V), number of moles (n), and temperature (T). R is a constant. This is a typical algebraic equation that can be manipulated to solve for any of the individual variables.



Figure 10

Pilots must rely on additional oxygen supplies at high altitudes to prevent hypoxia—a condition in which the tissues of the body become oxygen deprived.

Table 2

Gas Pressures in Air	
	Pressure (kPa)
Nitrogen gas	79.10
Carbon dioxide gas	0.040
Trace gases	0.94
Total gases	101.3

remember that the result never can be more precise than the least precise measurement. That is, your answer cannot have more significant figures than the least precise measurement. Be sure to perform all calculations before dropping any insignificant digits.

The following rules determine how to use significant figures in calculations that involve measurements.

1. *To add or subtract measurements, first perform the mathematical operation, then round off the result to the least precise value.* There should be the same number of digits to the right of the decimal as the measurement with the least number of decimal digits.
2. *To multiply or divide measurements, first perform the calculation, then round the answer to the same number of significant figures as the measurement with the least number of significant figures.* The answer should contain no more significant figures than the fewest number of significant figures in any of the measurements in the calculation

EXAMPLE PROBLEM 2

Air contains oxygen (O_2), nitrogen (N_2), carbon dioxide (CO_2), and trace amounts of other gases. Use the known pressures in Table 2 to calculate the partial pressure of oxygen.

To add or subtract measurements, first perform the operation, then round off the result to correspond to the least precise value involved.

$$\begin{aligned} P_{O_2} &= P_{\text{total}} - (P_{N_2} + P_{CO_2} + P_{\text{trace}}) \\ P_{O_2} &= 101.3 \text{ kPa} - (79.11 \text{ kPa} + 0.040 \text{ kPa} + 0.94 \text{ kPa}) \\ P_{O_2} &= 101.3 \text{ kPa} - 80.090 \text{ kPa} \\ P_{O_2} &= 21.210 \text{ kPa} \end{aligned}$$

The total pressure (P_{total}) was measured to the tenths place. It is the least precise measurement. Therefore, the result should be rounded to the nearest tenth of a kilopascal. The leftmost dropped digit (1) is less than five, so the last two digits can be dropped.

The pressure of oxygen is $P_{O_2} = 21.2 \text{ kPa}$.

EXAMPLE PROBLEM 3

The reading on a tire-pressure gauge is 35 psi. What is the equivalent pressure in kilopascals?

$$\begin{aligned} P &= 35 \text{ psi} \times \frac{101.3 \text{ kPa}}{14.7 \text{ psi}} \\ P &= 241.1904762 \text{ kPa} \end{aligned}$$

There are two significant figures in the measurement, 35 psi. Thus, the answer can have only two significant figures. Do not round up the last digit to be kept because the leftmost dropped digit (1) is less than five.

The equivalent pressure is $P = 240 \text{ kPa} = 2.4 \times 10^2 \text{ kPa}$.

A small, hand-held pressure gauge can be used to monitor tire pressure.



PRACTICE PROBLEMS

6. Determine the number of significant figures in each measurement:

- | | |
|-------------------------|----------------------------|
| a. 35 g | m. 0.157 kg |
| b. 3.57 m | n. 28.0 mL |
| c. 3.507 km | o. 2500 m |
| d. 0.035 kg | p. 0.070 mol |
| e. 0.246 L | q. 30.07 nm |
| f. 0.004 m ³ | r. 0.106 cm |
| g. 24.068 kPa | s. 0.0076 g |
| h. 268 K | t. 0.0230 cm ³ |
| i. 20.040 80 g | u. 26.509 cm |
| j. 20 dozen | v. 54.52 cm ³ |
| k. 730 000 g | w. 2.40×10^6 kg |
| l. 6.751 g | x. 4.07×10^{16} m |

Rounding

Arithmetic operations that involve measurements are done the same way as operations involving any other numbers. But, the results must correctly indicate the uncertainty in the calculated quantities. Perform all of the calculations and then round the result to the least number of significant figures. To round a number, use the following rules.

1. *When the leftmost digit to be dropped is less than 5, that digit and any digits that follow are dropped. Then the last digit in the rounded number remains unchanged.* For example, when rounding the number 8.7645 to 3 significant figures, the leftmost digit to be dropped is 4. Therefore, the rounded number is 8.76.
2. *When the leftmost digit to be dropped is 5 or greater, that digit and any digits that follow are dropped, and the last digit in the rounded number is increased by one.* For example, when rounding the number 8.7676 to 3 significant figures, the leftmost digit to be dropped is 7. Therefore, the rounded number is 8.77.
3. *When rounding off a large number, that number should first be written in scientific notation.* For example, round the number 87 645 to 3 significant figures.

	87645
Express the number in scientific notation.	8.7645×10^4
Locate the leftmost digit to be dropped.	$8.76\underline{4}5 \times 10^4$
The digit is less than five.	$8.76\underline{4}5 \times 10^4$
Drop the last two digits.	8.76×10^4

Calculations with significant figures

Look at the glassware in **Figure 9**. Would you expect to measure a more precise volume with the beaker or the graduated cylinder? When you perform any calculation using measured quantities such as volume, it is important to



Figure 9

Compare the markings on the graduated cylinder **a** to the markings on the beaker **b**. Which piece of glassware will yield more precise measurements?

Examine each of the following measurements. Use the rules on the previous page to check that all of them have three significant figures.

245 K 18.0 L 308 km 0.006 23 g 186 000 m

Suppose you must do a calculation using the measurement 200 L. You cannot be certain which zero was estimated. To indicate the significance of digits, especially zeros, write measurements in scientific notation. In scientific notation, all digits in the decimal portion are significant. Which of the following measurements is most precise?

- 200 L has unknown significant figures.
- 2×10^2 L has one significant figure.
- 2.0×10^2 L has two significant figures.
- 2.00×10^2 L has three significant figures.

The greater the number of digits in a measurement expressed in scientific notation, the more precise the measurement is. In this example, 2.00×10^2 L is the most precise data.

EXAMPLE PROBLEM 1

How many significant digits are in the measurement 0.00302 g? 60 min? 5.620 m? 9.80×10^2 m/s²?

0.003 02 g

Not significant Significant
(Rule 4) (Rules 1 and 2)

The measurement 0.00302 g has three significant figures.

60 min
Unlimited significant figures
(Rule 5)

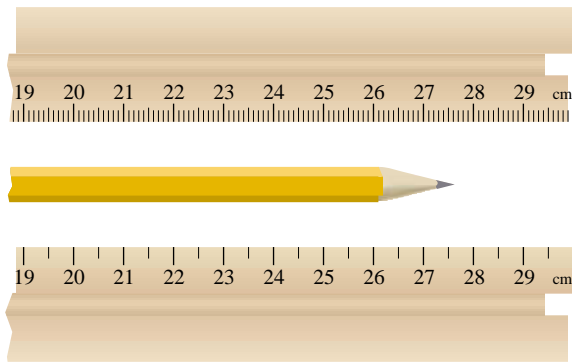
The measurement 60 min has unlimited significant figures.

5.620 m
Significant
(Rules 1 and 3)

The measurement 5.620 m has four significant figures.

9.80×10^2 m/s²
Significant
(Rules 1 and 3)

The measurement 9.8×10^2 m/s² has three significant figures.

**Figure 7**

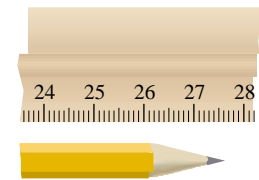
The estimated digit must be read between the millimeter markings on the top ruler. Why is the bottom ruler less precise?

Significant Figures

Much work in science involves taking measurements. Measurements that you take in the laboratory should show both accuracy and precision. Accuracy reflects how close your measurement comes to the real value. Precision describes the degree of exactness of your measurement. Which ruler in **Figure 7** would give you the most precise length? The top ruler with the millimeter markings would allow your measurements to come closer to the actual length of the pencil. The measurement would be more precise.

Measuring tools are never perfect, nor are the people doing the measuring. Therefore, whenever you measure a physical quantity, there will always be uncertainty in the measurement. The number of significant figures in the measurement indicates the uncertainty of the measuring tool.

The number of significant figures in a measured quantity is all of the certain digits plus the first uncertain digit. For example, the pencil in **Figure 8** has a length that falls between 27.6 and 27.7 cm. You can read the ruler to the nearest millimeter (27.6 cm), but after that you must estimate the next digit in the measurement. If you estimate that the next digit is 5, you would report the measured length of the pencil as 27.65 cm. Your measurement has four significant figures. The first three are certain and the last is uncertain. The ruler used to measure the pencil has precision to the nearest tenth of a millimeter.

**Figure 8**

If you determine that the length of this pencil is 27.65 cm, that measurement has four significant figures.

How many significant figures?

When a measurement is provided, the following series of rules will help you to determine how many significant figures there are in that measurement.

1. All nonzero figures are significant.
2. When a zero falls between nonzero digits, the zero is also significant.
3. When a zero falls after the decimal point and after a significant figure, that zero is significant.
4. When a zero is used merely to indicate the position of the decimal, it is not significant.
5. All counting numbers and exact numbers are treated as if they have an infinite number of significant figures.

3. Perform the following operations.

- $(5.0 \times 10^6) + (3.0 \times 10^7)$
- $(1.8 \times 10^9) + (2.5 \times 10^8)$
- $(3.8 \times 10^{12}) - (1.9 \times 10^{11})$
- $(6.0 \times 10^{-8}) - (4.0 \times 10^{-9})$

4. Perform the following operations.

- $(6.0 \times 10^{-4}) \times (4.0 \times 10^{-6})$
- $(4.5 \times 10^9) \times (7.0 \times 10^{-10})$
- $\frac{4.5 \times 10^{-8}}{1.5 \times 10^{-4}}$
- $\frac{9.6 \times 10^8}{1.6 \times 10^{-6}}$
- $\frac{(2.5 \times 10^6)(7.0 \times 10^4)}{1.8 \times 10^{-5}}$
- $\frac{(6.2 \times 10^{12})(5.8 \times 10^{-7})}{1.2 \times 10^6}$

5. See **Figure 5**. If you contain an average of 25 billion red blood cells, how many hemoglobin molecules are in your body?

Square and Cube Roots

A square root is one of two identical factors of a number. As you can see in **Figure 6a**, the number four is the product of two identical factors—two. Thus, the square root of four is two. The symbol $\sqrt{\quad}$, called a radical sign, is used to indicate a square root. Most scientific calculators have a square root key labeled $\sqrt{\square}$.

$$\sqrt{4} = \sqrt{2 \times 2} = 2$$

This equation is read “the square root of four equals two.” What is the square root of 9, shown in **Figure 6b**?

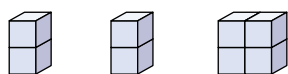
There may be more than two identical factors of a number, as shown in **Figure 6c**. You know that $4 \times 4 = 16$. Therefore, the square root of 16 equals 4. Are there any other factors of the number 16? It is the product of $2 \times 2 \times 2$. A cube root is one of three identical factors of a number. Thus, what is the cube root of 16? It is 2. A cube root also is indicated by a radical.

$$16 = \sqrt[3]{2 \times 2 \times 2} = 2$$

Check your calculator handbook for more information on finding roots.

Figure 6

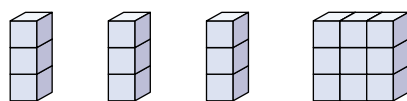
- The number four can be expressed as two groups of two. The identical factors are two.
- The number nine can be expressed as three groups of three. Thus, three is the square root of nine.
- Four is the square root of 16. What is the cube root of 16?



$$2 \times 2 = 4$$

$$2 = \sqrt{4}$$

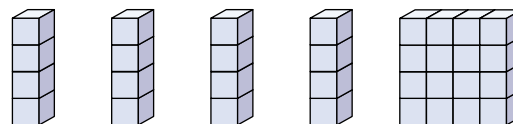
a



$$3 \times 3 = 9$$

$$3 = \sqrt{9}$$

b



$$4 \times 4 = 16$$

$$4 = \sqrt{16}$$

c

A negative exponent of ten is the number of places you move the decimal to the right until it is just past the first nonzero digit.

When converting a number that requires the decimal to be moved to the right, the exponent is decreased by the appropriate number. For example, the expression of 0.0098 in scientific notation is as follows:

$$\begin{array}{l} 0.0098 \times 10^0 \\ \underline{0.0098} \times 10^0 \quad 3 \\ 9.8 \times 10^{-3} \end{array} \quad \begin{array}{l} \text{The decimal point} \\ \text{moves} \\ \text{three places} \\ \text{to the right.} \end{array}$$

Thus, 0.0098 written in scientific notation is 9.8×10^{-3} .

Operations with scientific notation

The arithmetic operations performed with ordinary numbers can be done with numbers written in scientific notation. But, the exponential portion of the numbers also must be considered.

1. Addition and subtraction

Before numbers in scientific notation can be added or subtracted, the exponents must be equal. Remember that the decimal is moved to the left to increase the exponent and to the right to decrease the exponent.

$$\begin{aligned} (3.4 \times 10^2) + (4.57 \times 10^3) &= (0.34 \times 10^3) + (4.57 \times 10^3) \\ &= (0.34 + 4.57) \times 10^3 \\ &= 4.91 \times 10^3 \end{aligned}$$

2. Multiplication

When numbers in scientific notation are multiplied, only the decimal portion is multiplied. The exponents are added.

$$\begin{aligned} (2.00 \times 10^3)(4.00 \times 10^4) &= (2.00)(4.00) \times 10^{3+4} \\ &= 8.00 \times 10^7 \end{aligned}$$

3. Division

When numbers in scientific notation are divided, again, only the decimal portion is divided, while the exponents are subtracted as follows:

$$\frac{9.60 \times 10^7}{1.60 \times 10^4} = \frac{9.60}{1.60} \times 10^{7-4} = 6.00 \times 10^3$$

PRACTICE PROBLEMS

2. Express the following numbers in scientific notation.

- | | |
|------------|----------------|
| a. 5800 | d. 0.000 587 7 |
| b. 453 000 | e. 0.0036 |
| c. 67 929 | f. 0.000 087 5 |

Continued on next page



Figure 5

As blood passes through the body's tissues, red blood cells deliver oxygen and remove wastes. There are approximately 270 million hemoglobin molecules in one red blood cell.

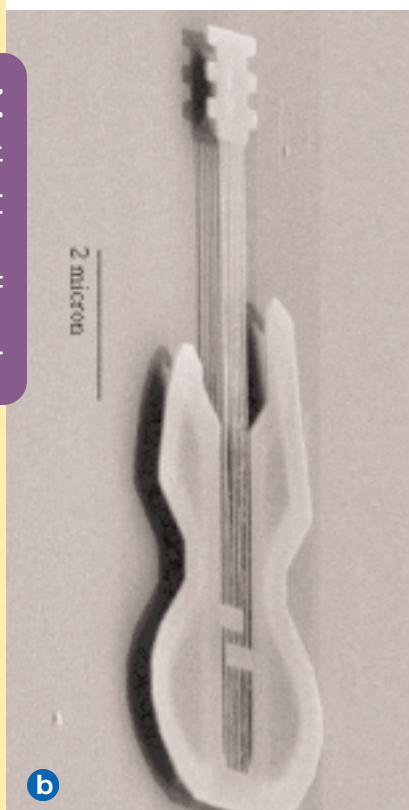


Figure 4

a Because of their short wavelengths (10^{-8} m to 10^{-13} m), X rays can pass through some objects. **b** A micron is 10^{-6} meter. Estimate the length of this nanoguitar in microns.

A number written in scientific notation has two parts.

$$N \times 10^n$$

The first part (N) is a number in which only one digit is placed to the left of the decimal point and all remaining digits to the right of the decimal point. The second part is an exponent of ten (10^n) by which the decimal portion is multiplied. For example, the number 2.53×10^6 is written in scientific notation.

$$2.53 \times 10^6$$

Number between
Exponent
one and ten
of ten

The decimal portion is 2.53 and the exponent is 10^6 .

Positive exponents

A positive exponent of ten (n) tells how many times a number must be multiplied by ten to give the long form of the number.

$$2.53 \times 10^6 = 2.53 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 2\,530\,000$$

You also can think of the positive exponent of ten as the number of places you move the decimal to the left until only one nonzero digit is to the left of the decimal point.

$$2\,530\,000.$$

The decimal point
moves
six places
to the left.

To convert the number 567.98 to scientific notation, first write the number as an exponential number by multiplying by 10^0 .

$$567.98 \times 10^0$$

(Remember that multiplying any number by 10^0 is the same as multiplying the number by 1.) Move the decimal point to the left until there is only one digit to the left of the decimal. At the same time, increase the exponent by the same number as the number of places the decimal is moved.

$$5\,67.98 \quad 10^0 \quad 2$$

The decimal point
moves
two places
to the left.

Thus, 567.98 written in scientific notation is 5.6798×10^2 .

Negative exponents

Measurements also can have negative exponents. See **Figure 4**. A negative exponent of ten tells how many times a number must be divided by ten to give the long form of the number.

$$6.43 \times 10^{-4} = \frac{6.43}{10 \times 10 \times 10 \times 10} = 0.000643$$

What happens when a negative number is multiplied by a negative number?
The product is always positive.

$$(-4)(-5) = 20$$

3. Division

Division is an arithmetic operation whose result is called a quotient. A quotient is expressed by \div or as a ratio. The quotient of numbers that have the same sign is always positive.

$$\frac{32}{8} = 4 \text{ and } \frac{-32}{-4} = 8$$

A negative number divided by a positive number, or a positive number divided by a negative number always yields a negative number.

$$\frac{35}{-7} = -5 \text{ and } \frac{-48}{8} = -6$$

PRACTICE PROBLEMS

1. Perform the following operations.

a. $8 + 5 - 6$

e. $0 - 12$

i. $(-16)(-4)$

b. $-4 - 9$

f. -6×5

j. $(-32) \div (-8)$

c. 6×8

g. $-14 + 9$

d. $56 \div 7$

h. $-44 \div 2$

Scientific Notation

Scientists must use extremely small and extremely large numbers to describe the objects in **Figure 3**. The mass of the proton at the center of a hydrogen atom is 0.000 000 000 000 000 000 000 001 673 kg. The length of the AIDS virus is about 0.000 000 11 m. The temperature at the center of the Sun reaches 15 000 000 K. Such small and large numbers are difficult to read and hard to work with in calculations. Scientists have adopted a method of writing exponential numbers called scientific notation. It is easier than writing numerous zeros when numbers are very large or small. It also is easier to compare the relative size of numbers when they are written in scientific notation.

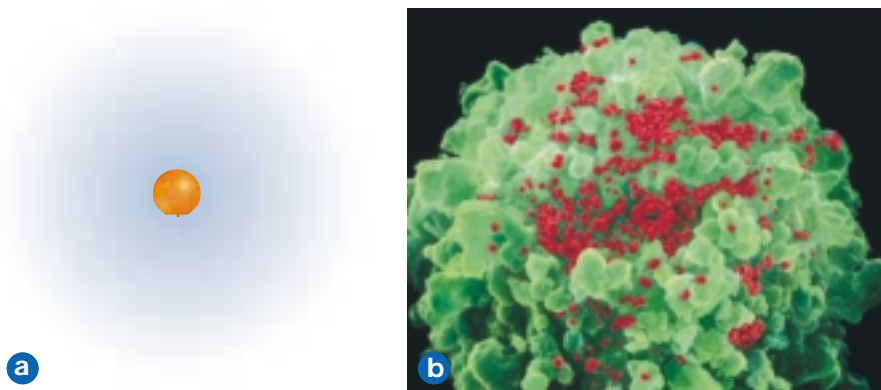


Figure 3

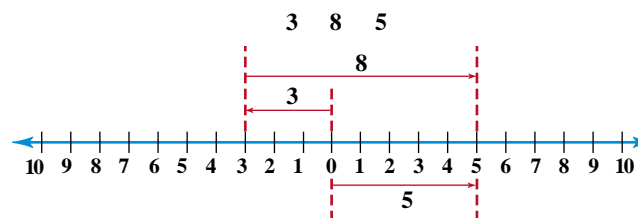
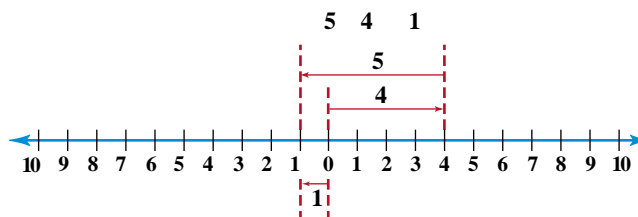
Scientific notation provides a convenient way to express data with extremely large or small numbers. Express the mass of a proton **a**, the length of the AIDS virus **b**, and the temperature of the Sun **c** in scientific notation.





Figure 2

The total mass of the eggs and the bowl is the sum of their individual masses. How would you determine the total mass of the eggs?

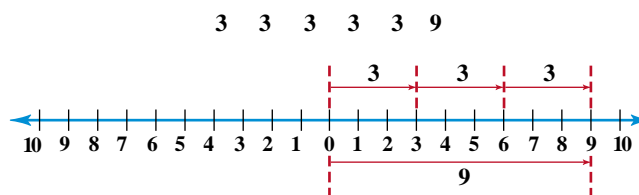


Suppose you must find the mass of the dozen eggs in **Figure 2**. You would measure the mass of the bowl alone. Then, subtract it from the mass of the eggs and the bowl. The result of this subtraction is called the difference. To find the difference between numbers, change the sign of the number being subtracted and follow the rules for addition. Use a number line to verify that the sign of the resulting number always will be the same as the larger number.

$$\begin{aligned} 4 - 5 &= 4 + (-5) = -1 \\ 4 - (-5) &= 4 + 5 = 9 \\ -4 - (-5) &= -4 + 5 = 1 \\ -4 - (+5) &= -4 - 5 = -9 \end{aligned}$$

2. Multiplication

The result of multiplication is called a product. The operation “three times three” can be expressed by 3×3 , $(3)(3)$, or $3 \cdot 3$. Multiplication is simply repeated addition. For example, $3 \times 3 = 3 + 3 + 3 = 9$.



Use a number line to show that a negative number multiplied by a positive number yields a negative number.

$$\begin{aligned} (3)(-3) &= -9 \\ (-3) + (-3) + (-3) &= -9 \end{aligned}$$